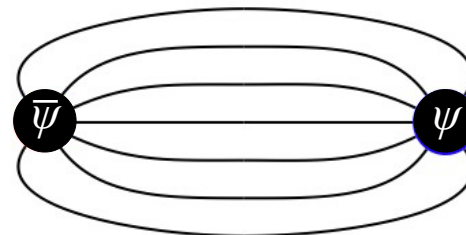


# Model Building in the Dynamical Dark Matter Framework: Thermal and Non-Thermal Realizations

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LAFAYETTE  
COLLEGE



**Based on work done in collaboration with:**

- Keith Dienes, Jacob Fennick, and Jason Kumar [arXiv:1601.05094]
- Keith Dienes, Fei Huang, and Shufang Su [arXiv:1610.04112]
- Keith Dienes, Jacob Fennick, and Jason Kumar [arXiv:1711.xxxxx]

Light Dark World 2017, Oct. 19th – 21st, 2017

# Dynamical Dark Matter

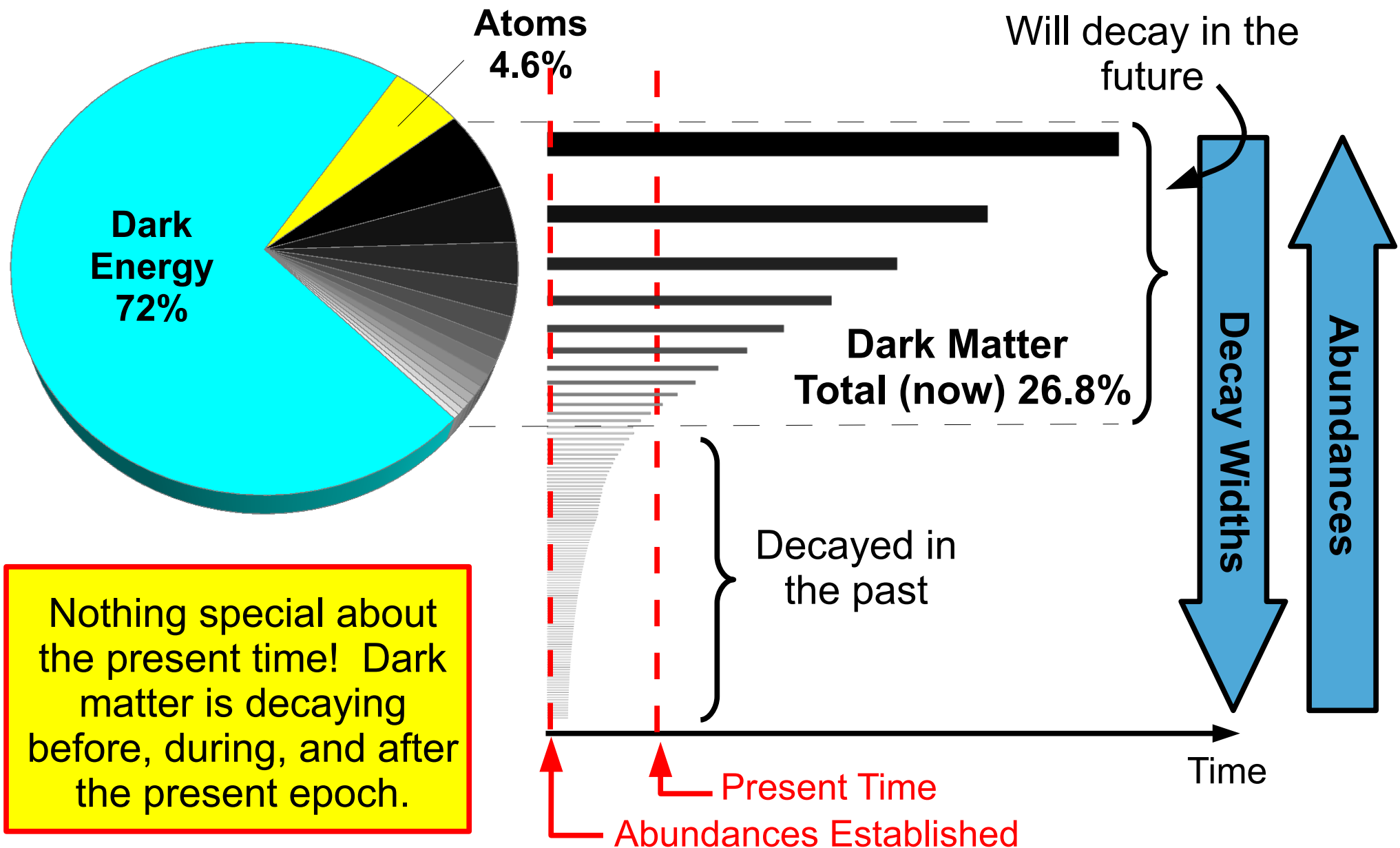
[Dienes, BT: 1106.4546]

Dynamical Dark Matter (DDM) is a theoretical framework in which constraints on dark matter can be satisfied without the hyperstability criterion ( $\tau_\chi \gtrsim 10^{26}$  s) typically required of traditional DM candidates.

In particular, in DDM scenarios...

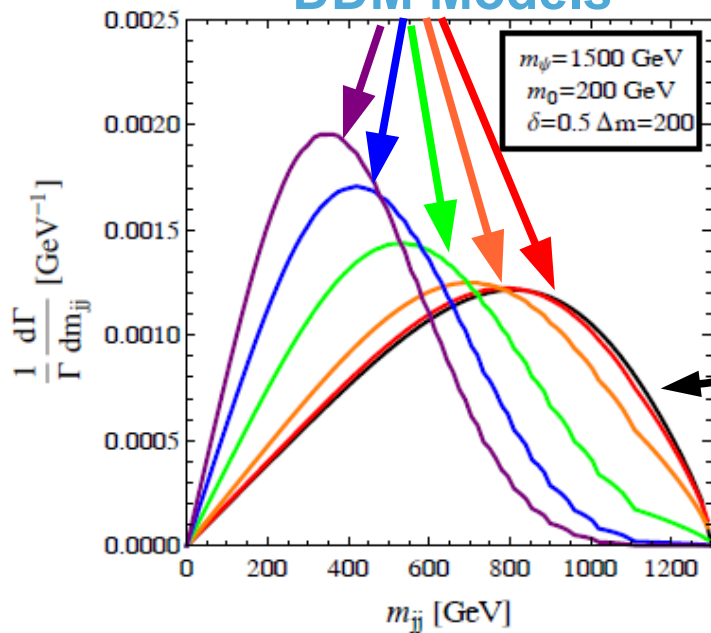
- The dark-matter candidate is an ensemble consisting of a potentially vast number of constituent particle species.
- The individual abundances of the ensemble constituents are balanced against decay rates across the ensemble such that constraints are satisfied.
- The DM abundance and equation of state also exhibit a non-trivial time-dependence beyond that associated with Hubble expansion.

# DDM Cosmology: The Big Picture



# DDM ensembles also exhibit a variety of distinctive and characteristic experimental signatures:

## DDM Models

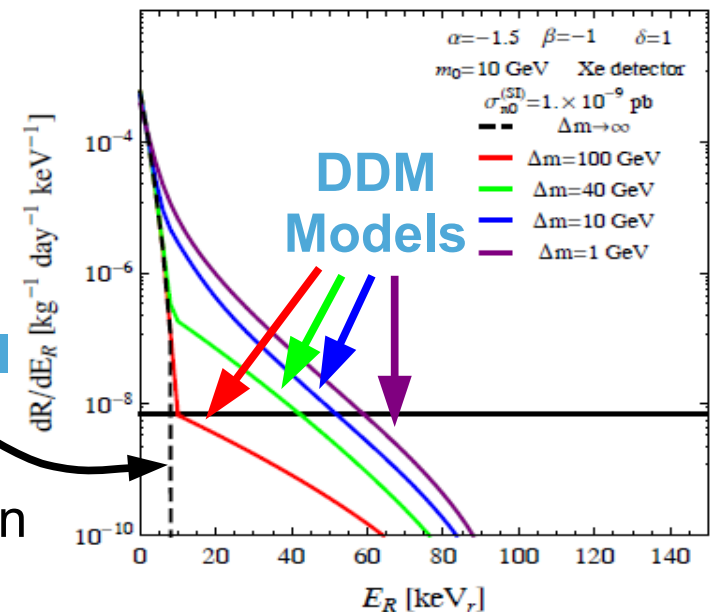


## At Colliders

- Characteristic features in kinematic distributions of SM particles produced alongside the ensemble constituents. Dienes, Su, BT [1204.4183, 1407.2606]

Traditional DM

Traditional DM



## At Direct-detection experiments

- DDM ensembles also give rise to distinctive features in recoil-energy spectra. Dienes, Kumar, BT [1208.0336]

## And at Indirect-Detection Experiments

- In the shape of the differential flux spectra of cosmic-ray particles produced from dark-matter annihilation or decay. Dienes, Kumar, BT [1306.2959]
- In characteristic features in the gamma-ray spectra of dwarf galaxies, the Galactic Center, etc. Boddy, Dienes, Kim, Kumar, Park, BT [1606.07440, 1609.09104]

# Important: Not *Just* Multi-component DM!

- A DDM ensemble is not simply an arbitrary assortment of dark-sector states!
- Its structure is determined by an organizing principle which specifies the properties of these states in terms of only a few underlying parameters.



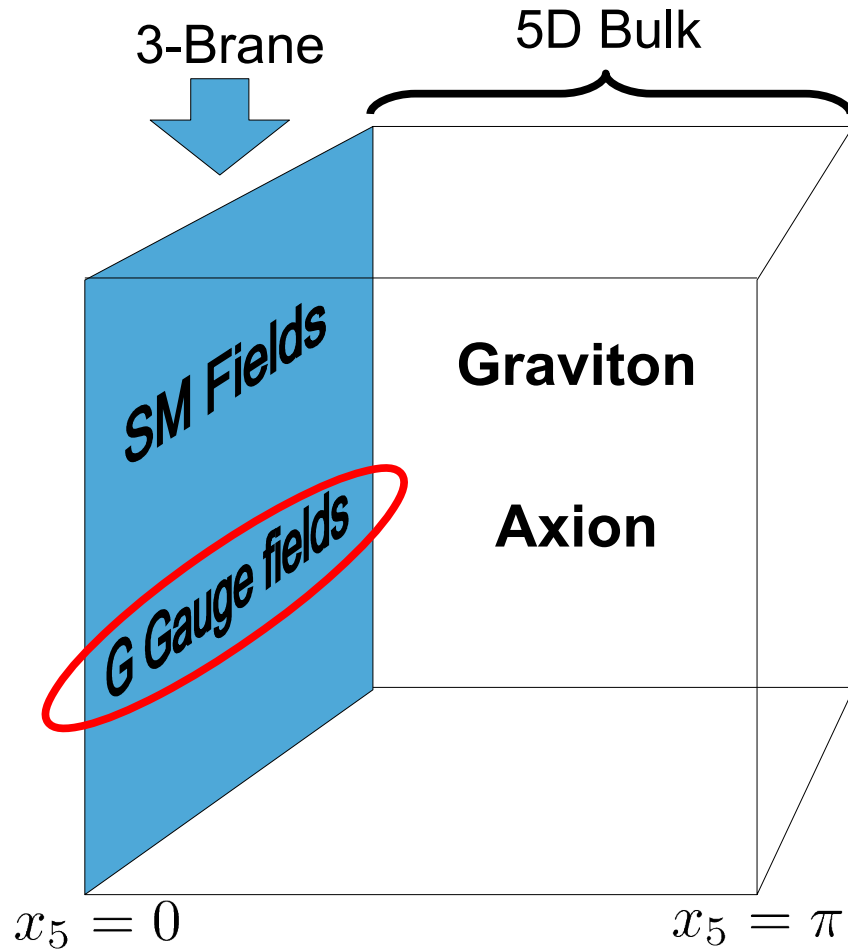
- This organizing principle includes three fundamental scaling relations which describe how masses, abundances, and decay widths scale in relation to each other across the ensemble:

- 1 Abundance  $\Omega(m)$  as a function of mass
  - 2 Decay width  $\Gamma(m)$  as a function of mass
  - 3 Density of states  $n(m)$  as a function of mass
- Depend on cosmology, couplings to external fields, etc.
- Reflects the internal structure of the ensemble itself

In a DDM context, then, model-building means identifying scenarios in which the appropriate scaling relations arise.

# The Canonical Example: A Higher-Dimensional Axion

Dienes, BT [1106.4546,1107.0721]



- Single extra spatial dimension compactified on  $S_1/Z_2$  with radius  $R = 1/M_c$ .
- Global  $U(1)_X$  symmetry broken at scale  $f_X$  by a bulk scalar. The bulk axion is the PNGB.
- SM and an additional gauge group  $G$  are restricted to the brane.  $G$  confines at a scale  $\Lambda_G$ . Instanton effects lead to a brane-mass term  $m_X$  for the axion.
- Ensemble constituents are mass-eigenstate admixtures of the KK modes of this axion.

Three relevant scales:  $\{f_X, M_c, \Lambda_G\}$

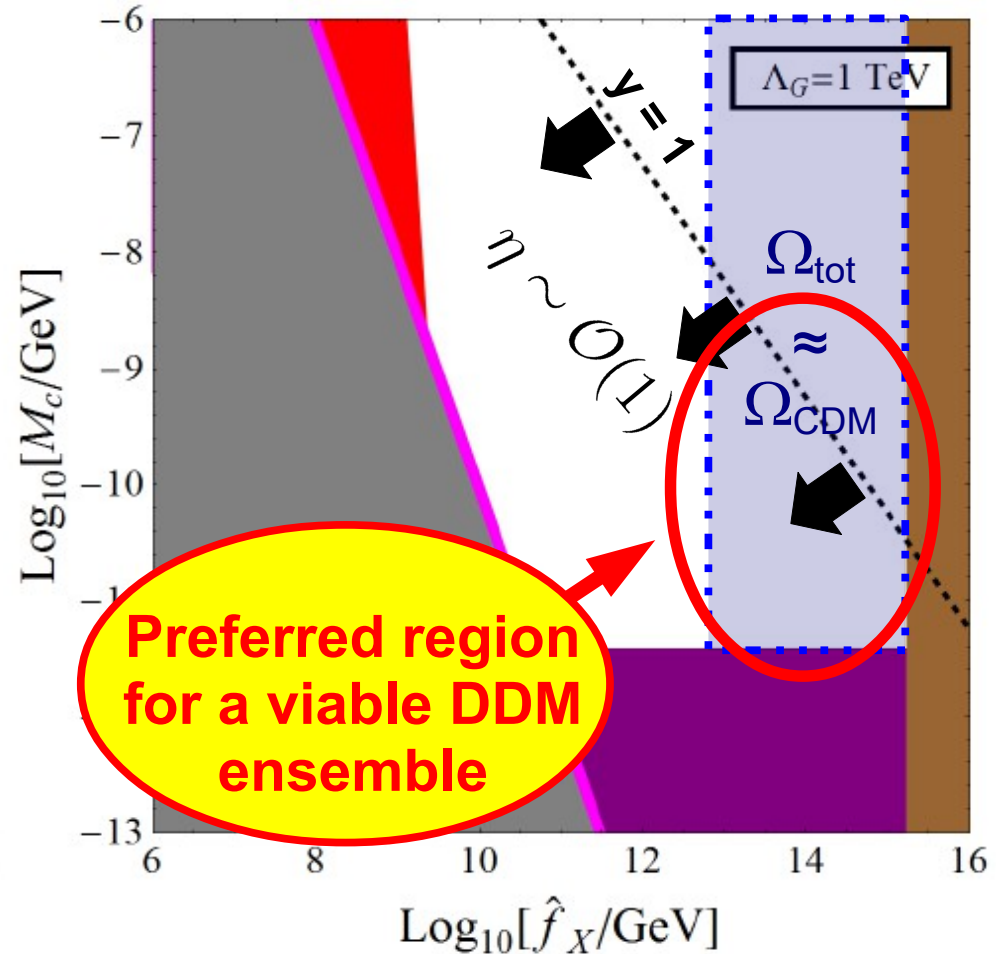
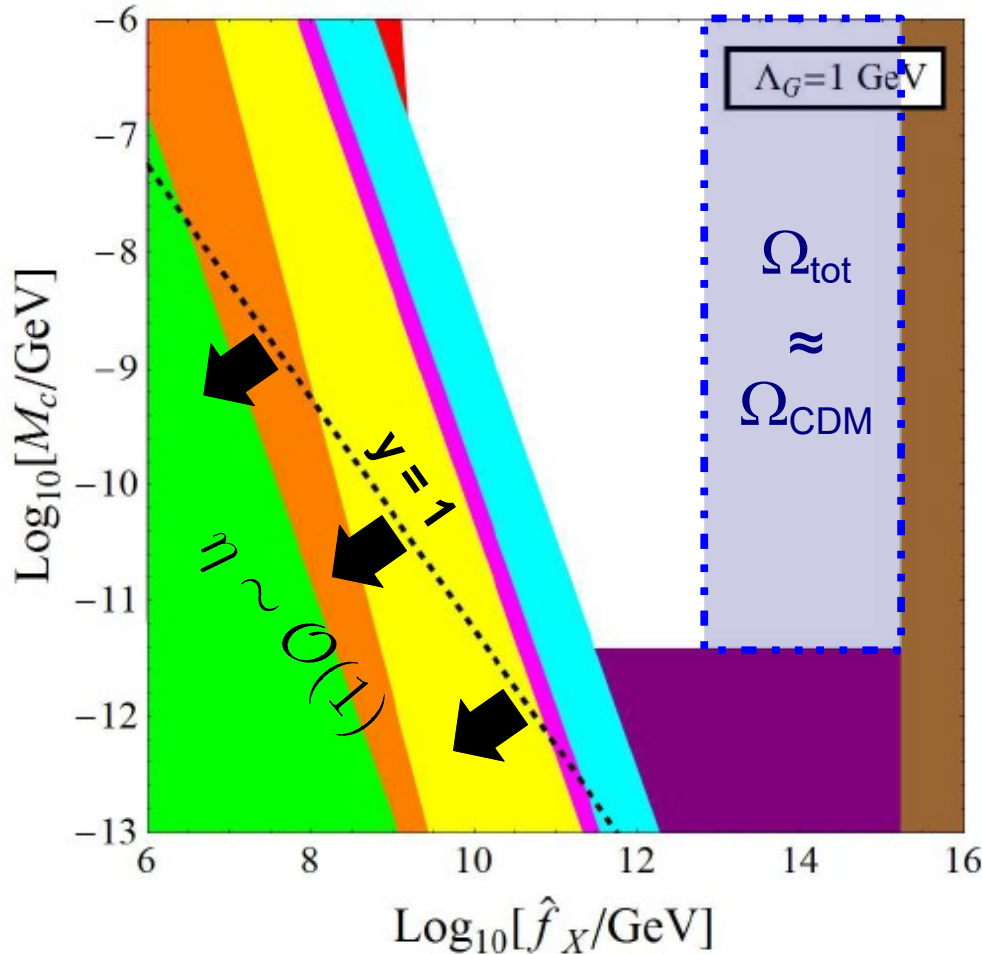
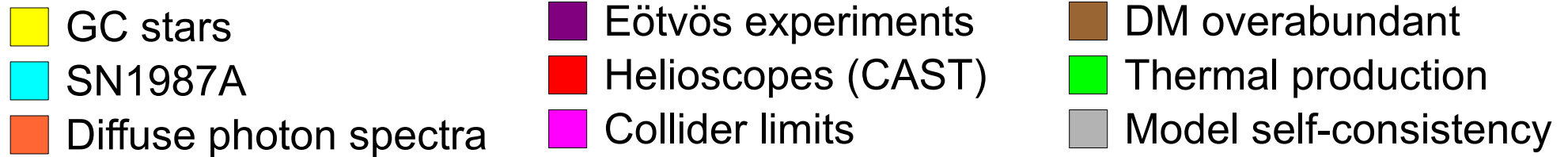
## Scaling Relations:

- Density of states  $\Rightarrow n(m) \sim [\text{const.}]$  (KK spectrum)
- Vacuum misalignment: natural mechanism for generating  $\Omega_n \Rightarrow \Omega(m) \sim m^{-2} \left[ 1 + \frac{m^2}{m_X^2} + \frac{\pi^2 m_X^2}{M_c^2} \right]^{-1}$



# The Result: A Viable DDM Ensemble

- While a great many considerations constrain scenarios involving light bulk axions, they can all be simultaneously satisfied while  $\Omega_{\text{tot}} \approx \Omega_{\text{CDM}}$  and  $\eta \sim \mathcal{O}(1)$ .



## In this talk...

- Axion DDM scenarios make use of a particular (non-thermal) mechanism for abundance-generation: misalignment production.



So are there other abundance-generation mechanisms naturally give rise to DDM ensembles in other contexts?



Yes there are. In fact, in this talk, I'll show that an appropriate spectrum of abundances for a DDM ensemble can even be generated thermally.

- I'll also provide two concrete examples of scenarios in which an appropriate set of scaling relations – and especially a density-of-states function  $n(m)$  – for a DDM ensemble arises in a thermal context:

1

Theories involving strongly-coupled dark sectors.

2

Theories with matter multiplets charged under additional, large symmetry groups which are spontaneously broken.

These serve as yet further examples of how simple, realistic DDM ensembles arise naturally in scenarios for new physics! They also expand the range of phenomenological possibilities for DDM.



# **Dynamical Dark Matter from Thermal Freeze-Out**

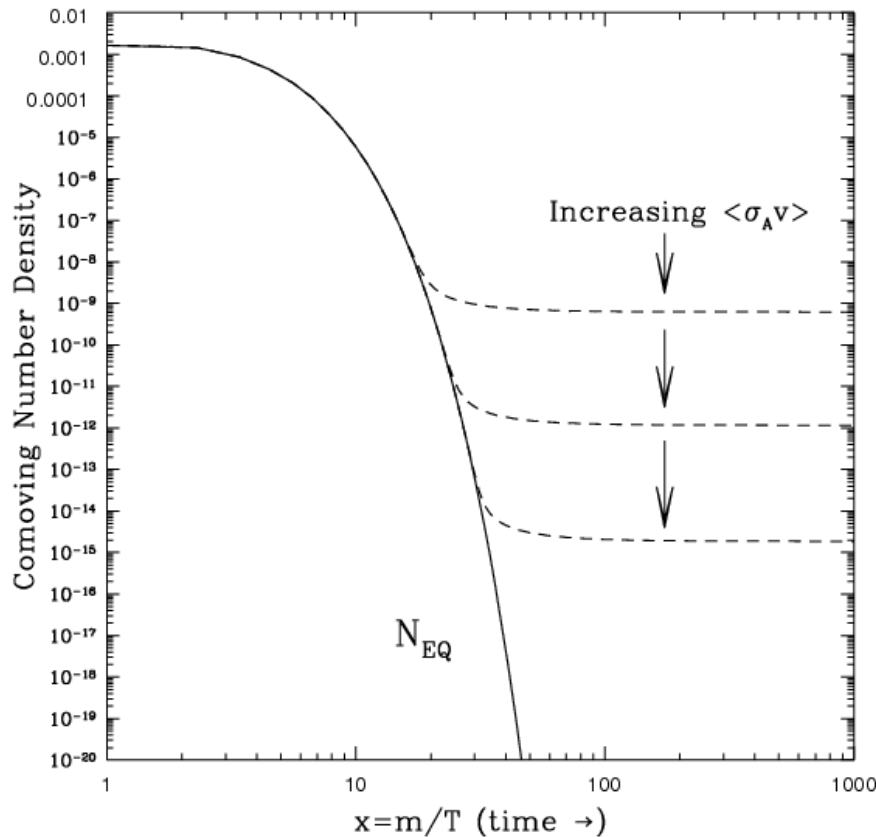


• **Keith Dienes, Jacob Fennick, and Jason Kumar [arXiv:1711.xxxxx]**

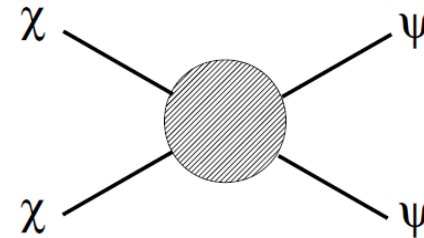
# Thermal Freezeout

- As an abundance-generation mechanism for dark matter, **thermal freeze-out** has a number of phenomenological advantages:

- Insensitivity to initial conditions
- Applicable to particles  $\chi$  with weak-scale masses and couplings sufficiently large (compared to, say, axions) as to be relevant for collider physics, direct detection, etc.



- Characteristic dependence of the abundance when  $\chi$  annihilates (e.g., through light mediators or  $t$ -channel diagrams) into light fields  $\psi$ :



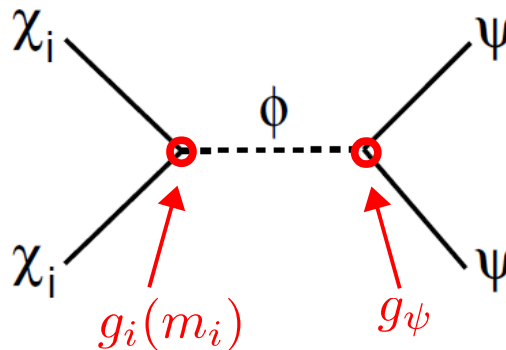
$$\langle \sigma v \rangle \sim \frac{g_\chi^2 g_\psi^2}{m_\chi^2} \quad \longrightarrow \quad \Omega_\chi \sim \frac{m_\chi^2}{g_\chi^2 g_\psi^2}$$

# Thermal DDM?

## The Question:

Can thermal freezeout naturally provide the correct balancing of decay widths against abundances for DDM?

- Typically,  $\Gamma_i$  scales with  $m_i$  to some positive power. For a viable ensemble,  $\Omega_i$  must scale with  $m_i$  to a sufficient **inverse power**.
- Consider an ensemble of dark-matter constituents  $\chi_i$  which all couple to a **common mediator**  $\phi$  which also couples to a light fields  $\psi$ .
- In the regime in which  $m_\phi > m_i$  for all  $\chi_i$ , all constituents annihilate primarily to  $\psi$  pairs via an  $s$ -channel  $\phi$ .



- Scaling of  $g(m_i)$  with  $m_i$  can depend on underlying theory structure, renormalization, etc. For simplicity we take  $g_i \equiv g_\chi$  to be **universal**.

# Annihilation Cross-Sections

- The way in which the annihilation cross-section scales with  $m_i$ ,  $m_\phi$ , and  $m_\psi$  is dictated by the structure of the pertinent Lagrangian operators:

$$\sigma_i \sim \frac{g_\chi^2 g_\psi^2}{m_i^2} v^{2r-1} \left( \frac{\mu}{m_i} \right)^{2(n_\chi + n_\psi)} \frac{(1 - m_\psi^2/m_i^2)^{s+1/2}}{(1 - m_\phi^2/4m_i^2)^2} \left( \frac{m_\psi}{m_i} \right)^t$$

## Operators (On the Dark-Matter Side)

$\chi_i$	$\phi$	coupling	$n_\chi$	$r$
spin-0	spin-0	S: $g_\chi \mu \chi^* \chi \phi$	1	0
spin-1/2	spin-0	S: $g_\chi \bar{\chi} \chi \phi$	0	1
spin-1/2	spin-0	P: $g_\chi \bar{\chi} \gamma_5 \chi \phi$	0	0
spin-0	spin-1 (time)	V: $g_\chi (\chi^* \partial_0 \chi) \phi^0$	—	—
spin-0	spin-1 (spatial)	V: $g_\chi (\chi^* \partial_i \chi) \phi^i$	0	1
spin-1/2	spin-1 (time)	V: $g_\chi \bar{\chi} \gamma_0 \chi \phi^0$	—	—
spin-1/2	spin-1 (spatial)	V: $g_\chi \bar{\chi} \gamma_i \chi \phi^i$	0	0
spin-1/2	spin-1 (time)	A: $g_\chi \bar{\chi} \gamma_0 \gamma_5 \chi \phi^0$	0	0
spin-1/2	spin-1 (spatial)	A: $g_\chi \bar{\chi} \gamma_i \gamma_5 \chi \phi^i$	0	1

$n_\chi$ : mass dimension of operator coefficient

$r$ : whether initial state can be  $L=0$  ( $r=0$ ) or only  $L=1$  ( $r=1$ )

# Annihilation Cross-Sections

- The way in which the annihilation cross-section scales with  $m_i$ ,  $m_\phi$ , and  $m_\psi$  is dictated by the structure of the pertinent Lagrangian operators:

$$\sigma_i \sim \frac{g_\chi^2 g_\psi^2}{m_i^2} v^{2r-1} \left( \frac{\mu}{m_i} \right)^{2(n_\chi + n_\psi)} \frac{(1 - m_\psi^2/m_i^2)^{s+1/2}}{(1 - m_\phi^2/4m_i^2)^2} \left( \frac{m_\psi}{m_i} \right)^t$$

## Operators (On the Light-Particle Side)

$\phi$	$\psi$	coupling	$n_\psi$	$s$	$t$
spin-0	spin-0	S: $g_\psi \mu \phi \psi^* \psi$	1	0	0
spin-0	spin-1/2	S: $g_\psi \phi \bar{\psi} \psi$	0	1	0
spin-0	spin-1/2	P: $g_\psi \phi \bar{\psi} \gamma_5 \psi$	0	0	0
spin-1 (time)	spin-0	V: $g_\psi \phi^0 (\psi^* \partial_0 \psi)$	—	—	—
spin-1 (spatial)	spin-0	V: $g_\psi \phi^i (\psi^* \partial_i \psi)$	0	1	0
spin-1 (time)	spin-1/2	V: $g_\psi \phi^0 \bar{\psi} \gamma_0 \psi$	—	—	—
spin-1 (spatial)	spin-1/2	V: $g_\psi \phi^i \bar{\psi} \gamma_i \psi$	0	0	0
spin-1 (time)	spin-1/2	A: $g_\psi \phi^0 \bar{\psi} \gamma_0 \gamma_5 \psi$	0	0	1
spin-1 (spatial)	spin-1/2	A: $g_\psi \phi^i \bar{\psi} \gamma_i \gamma_5 \psi$	0	1	0

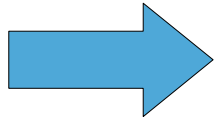
$n_\chi$ : mass dimension of operator coefficient

$s$ : whether final state can be  $L=0$  ( $s=0$ ) or only  $L=1$  ( $s=1$ )

$t$ : whether coupling is chirality-suppressed ( $t=1$ ) or not ( $t=0$ )

# Abundance Spectrum

- The corresponding spectrum of abundances  $\Omega_i$  for the ensemble is



$$\Omega_i \sim \frac{m_i^2}{g_\chi^2 g_\psi^2} m_i^{2(n_\chi + n_\psi) + t} \frac{(1 - m_\phi^2/4m_i^2)^2}{(1 - m_\psi^2/m_i^2)^{s+1/2}}$$

- Equivalently, we can parametrize this spectrum of abundances in terms of an ( $m_i$ -dependent) **scaling exponent**  $\gamma(m_i)$ :

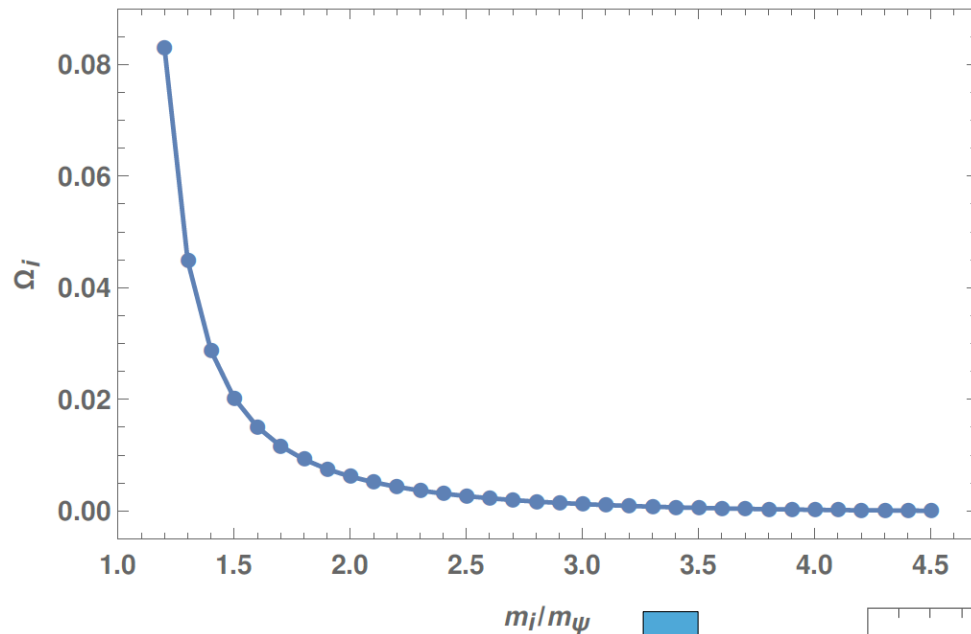
$$\Omega_i \sim m_i^{\gamma(m_i)}$$

$$\gamma(m_i) \equiv \frac{d \ln \Omega(m_i)}{d \ln m_i} = 2 + \Delta\gamma + \frac{1}{m_i^2/m_\phi^2 - 1/4} + \frac{2s+1}{1 - m_i^2/m_\psi^2}$$

Where we have defined  $\Delta\gamma \equiv 2(n_\chi + n_\psi) + t$



## Abundance Spectrum



- Spectrum of  $\Omega_i$  shown here for

$\phi$ : scalar  
 $\chi_i$ : fermion (S coupling)  
 $\psi$ : fermion (A coupling)

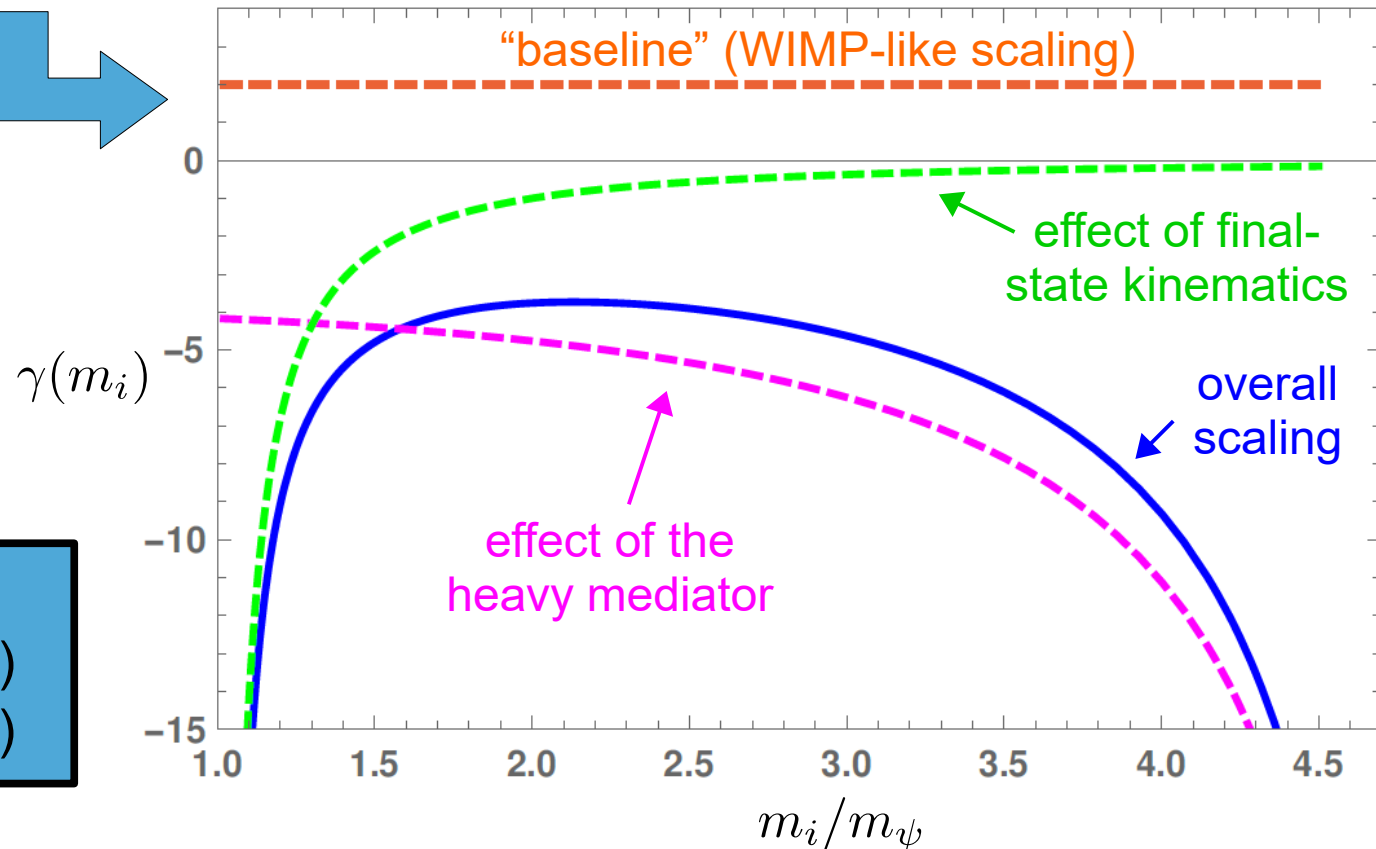
- Corresponds to the parameters:

$$n_\chi = n_\psi = t = r = 0 \quad s = 1$$

## Scaling Exponent

- Not a strong dependence on  $s$ , so curves basically the same for a simple  $Z'$  scenario where:

$\phi$ : vector  
 $\chi_i$ : fermion (V coupling)  
 $\psi$ : fermion (V coupling)



# Abundance Spectra

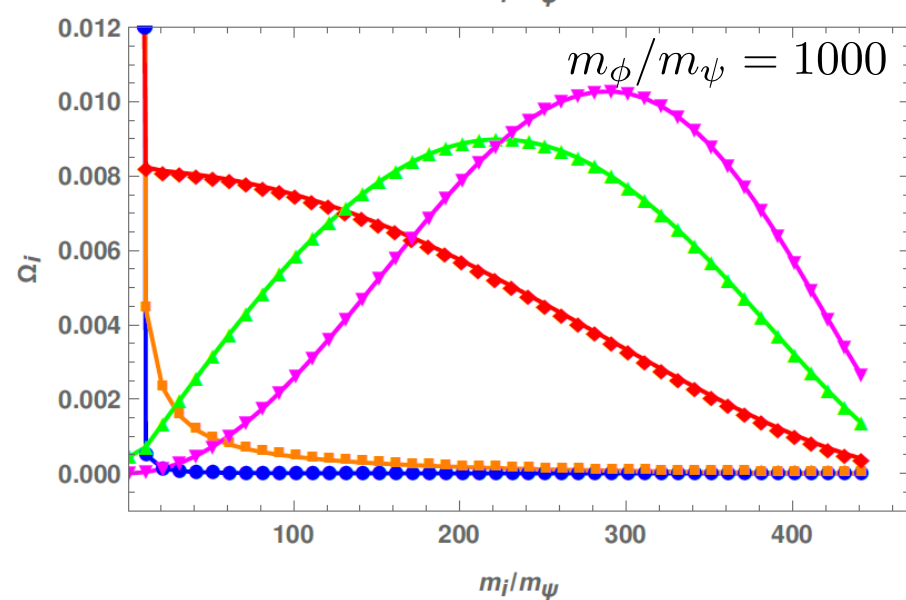
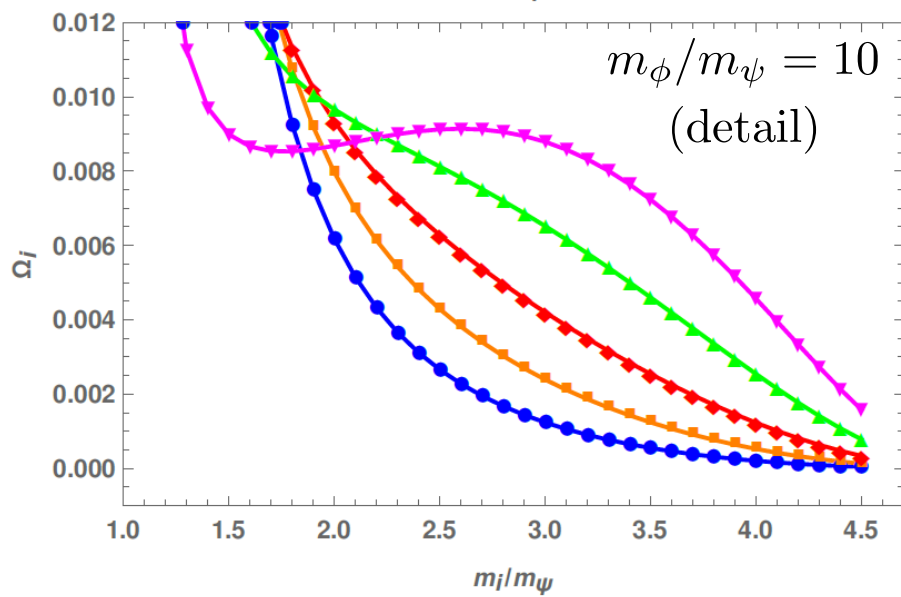
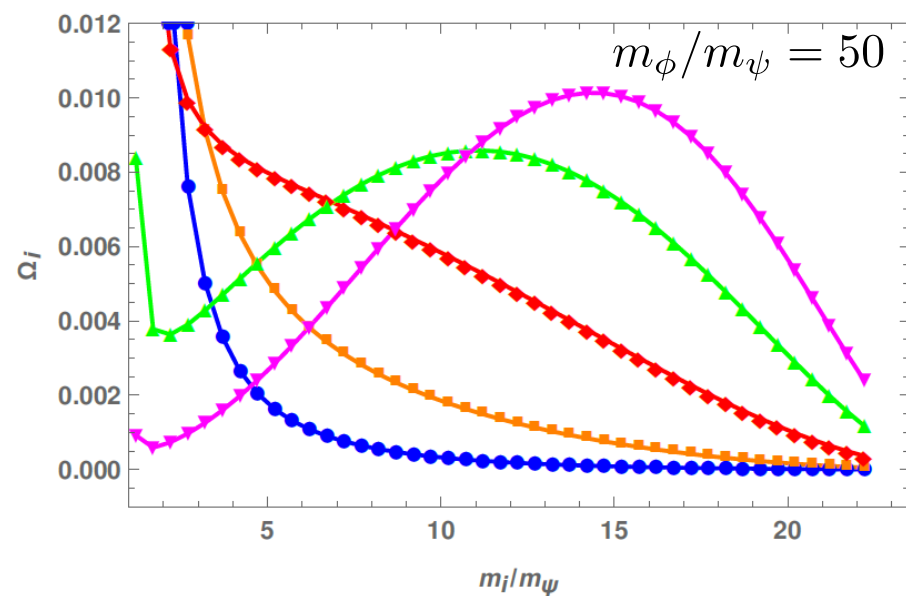
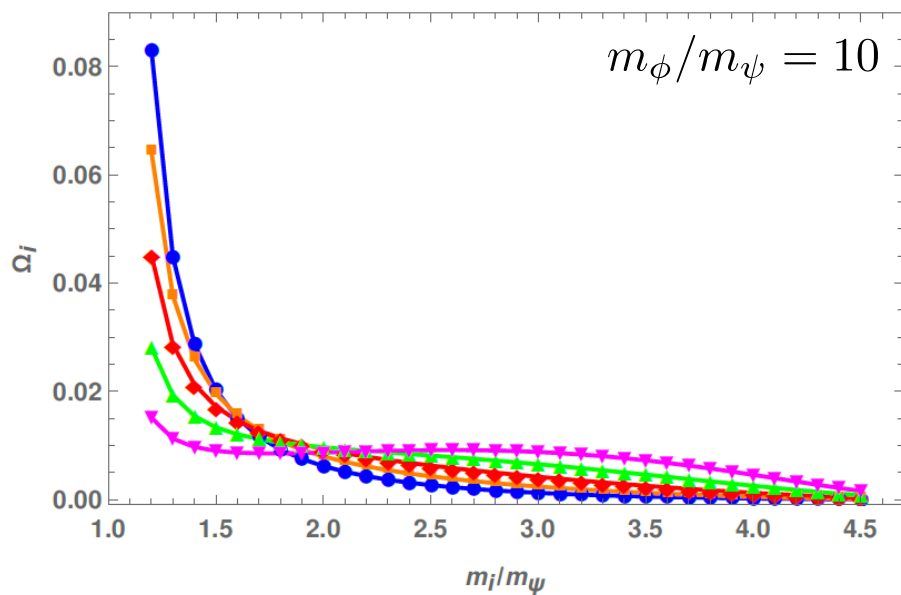
$\Delta\gamma = 0$

$\Delta\gamma = 1$

$\Delta\gamma = 2$

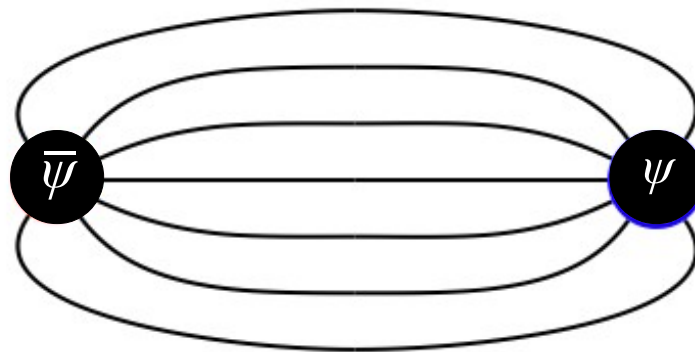
$\Delta\gamma = 3$

$\Delta\gamma = 4$



# Scenario I:

## Dynamical Dark Matter from Strongly-Coupled Dark Sectors



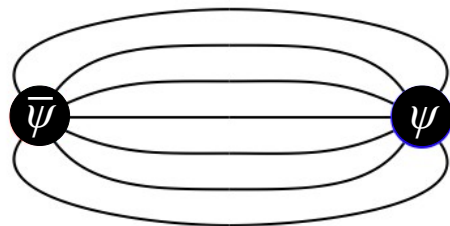
• Keith Dienes, Fei Huang, and Shufang Su [arXiv:1610.04112]

## The Basic Idea

- One natural context in which a large number of dark particles can naturally arise is in theories with strongly-coupled dark sectors.
- As an example, consider a dark sector consisting of a set of Dirac fermions (dark “quarks”) which are charged non-trivially under a non-Abelian gauge group  $G$ .

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + \sum_i \left( i\bar{\psi}_i \gamma^\mu D_\mu \psi_i - m\bar{\psi}_i \psi_i \right)$$

- Assume that  $G$  becomes confining below some critical temperature  $T_c$ .
- The degrees of freedom in the confined phase – the dark “hadrons” – will play the role of the DDM ensemble constituents.
- A great deal of information about the hadron spectrum can be gleaned by modeling the flux tubes associated with  $G$  as strings and bringing the full machinery of string theory to bear.



$\approx$



# Modeling Dark Hadrons with Strings

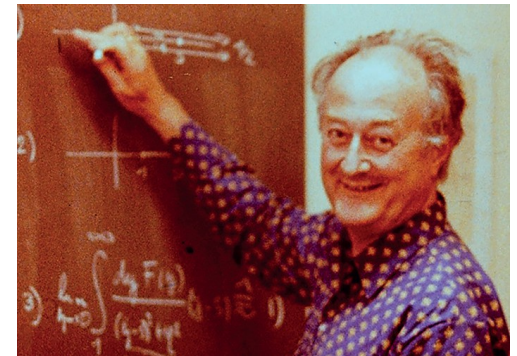
- The **mass spectrum** of the theory consists of a tower of dark-“hadron” states labeled by an integer  $n = 0, 1, 2, \dots$

$$M_n^2 = nM_s^2 + M_0^2 \quad \text{where} \quad M_s = \frac{1}{\sqrt{\alpha'}} \quad \text{“Regge Slope”}$$

- The **degeneracy of states**  $g_n$  at each energy level  $n$  has a generic functional form, parametrized by two constants  $B$  and  $C$ :

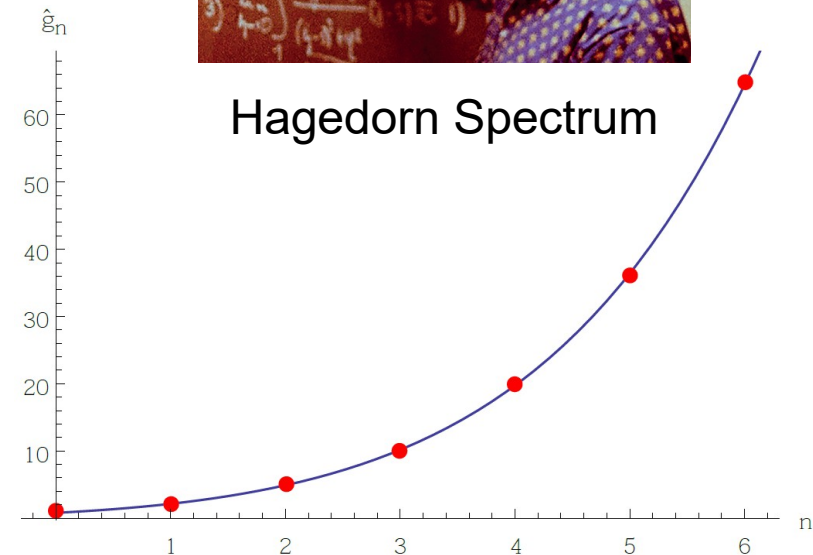
$$g_n \propto 2\pi \left( \frac{16\pi n}{C^2} - 1 \right)^{\frac{1}{4}-B} I_{|2B-\frac{1}{2}|} \left( C \sqrt{n - \frac{C^2}{16\pi n}} \right)$$

$$\approx \frac{1}{\sqrt{2}} \left( \frac{C}{4\pi} \right)^{2B-1} n^{-B} e^{C\sqrt{n}}$$



Hagedorn Spectrum

**The upshot:** the degeneracy of states rises exponentially with  $n$ . This is the well-known **Hagedorn spectrum**.



# String Consistency Conditions

- The constants  $B$  and  $C$  have physical meaning:

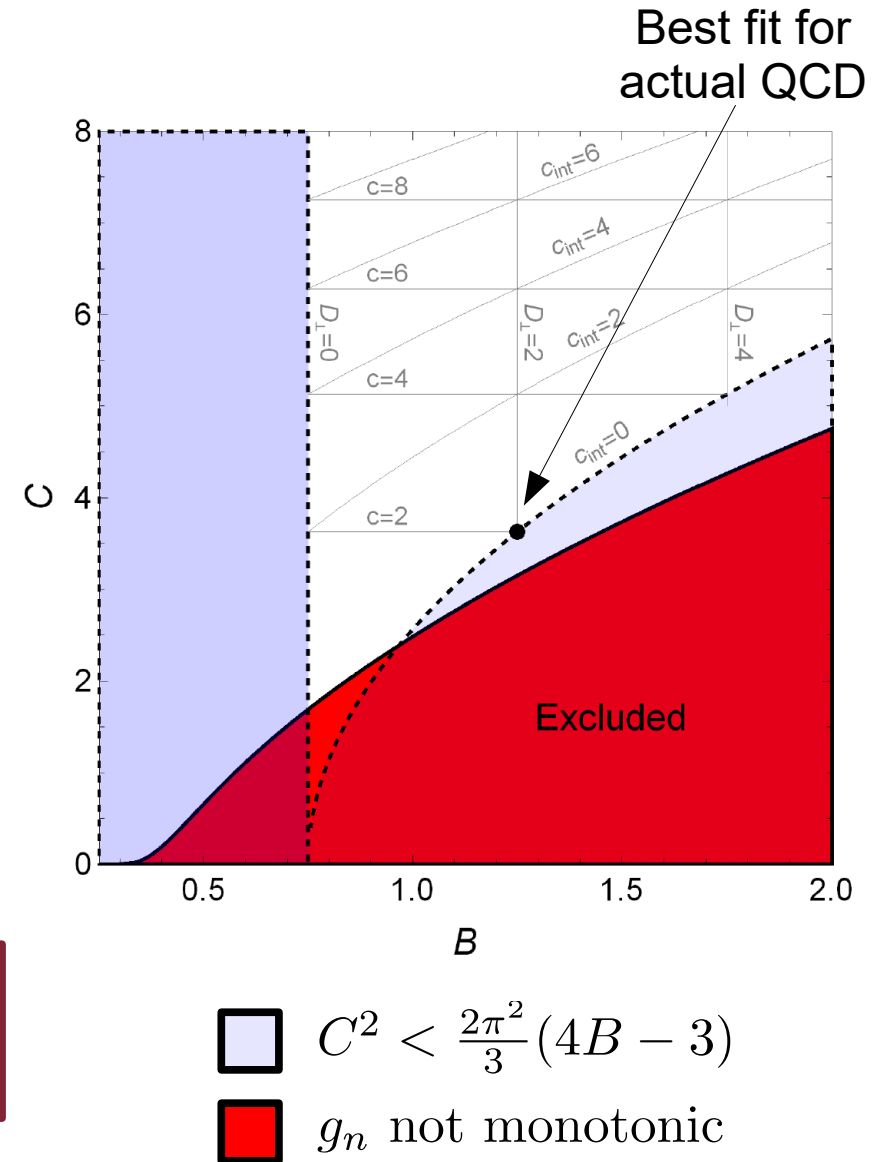
$$B = \frac{1}{4}(3 + D_{\perp}) \quad C = \pi\sqrt{2c/3}$$

- \*  $D_{\perp}$  : number of dimensions transverse to the string.
- \*  $c$  : central charge.

- In any self-consistent string model, these quantities are constrained by

$$\begin{cases} D_{\perp} > 0 \\ D_{\perp} \in \mathbb{Z} \\ c \geq D_{\perp} \end{cases} \quad \Rightarrow \quad C^2 \geq \frac{2\pi^2}{3}(4B - 3)$$

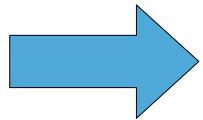
- Furthermore,  $g_n$  should rise monotonically with  $n$ .





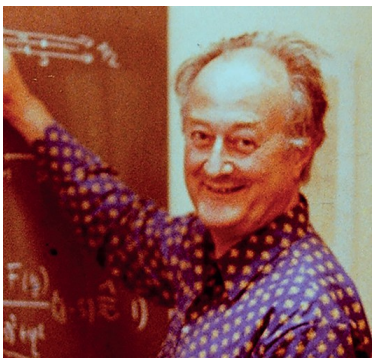
# Abundances: Boltzmann vs. Hagedorn

- At early times/high temperatures, the theory is the unconfined phase.
- However, when the temperature in the dark sector drops below some **critical temperature**  $T_C$ , the dark gauge group  $G$  becomes confining.
- Residual  $G$  interactions maintain thermal equilibrium among the hadronic states of the confining phase at temperatures just below  $T_C$ .



Primordial abundances are **Boltzmann suppressed**:

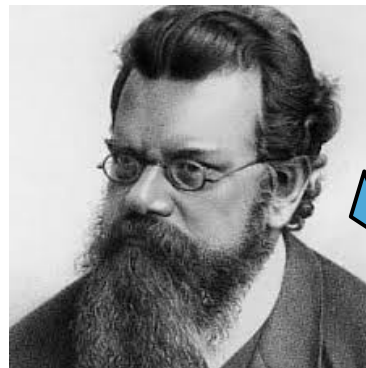
$$\Omega_n \approx \frac{1}{3M_P^2 H^2(T_C)} \int \frac{d^3\mathbf{p}}{(2\pi)^3} E_{\mathbf{p}} e^{-E_{\mathbf{p}}/T_C}$$



**Hagedorn**

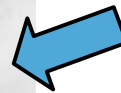
Exponentially *rising*  
density of states.

vs.



**Boltzmann**

Exponentially *falling*  
abundance spectrum



If Boltzmann wins, we have a sensibly defined ensemble of dark “hadrons.”

The criterion for this is  $T_C \leq T_H$ ,  
where  $T_H = M_s/C$  is the  
**Hagedorn temperature**.

# Decays and Constraints from Cosmology

- In general, these dark “hadrons” are unstable and **decay**. There are two possibilities, depending on the string coupling  $g_s$ : (an independent parameter):

$$\begin{cases} \text{Large } g_s \longrightarrow \text{decays to lighter dark “hadrons” dominate} \\ \text{Small } g_s \longrightarrow \text{decays to other states dominate} \end{cases}$$

focus on this case

- Parametrize the widths of the constituents as follows:

Width of lightest constituent

$$\Gamma_n = \Gamma_0 \left( \frac{M_n}{M_0} \right)^\xi$$

General scaling exponent

- These widths, and the other properties of the DDM ensemble are subject to a number of **cosmological and astrophysical constraints**:

- Total abundance:  $\Omega_{\text{tot}}(t_{\text{now}}) \approx 0.26$

[CMB data, Type Ia supernovae]

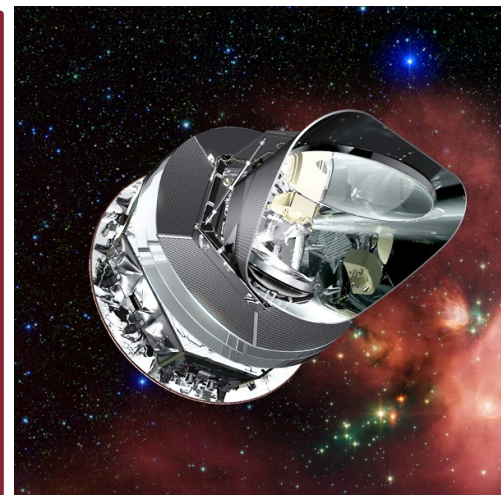
- Equation of state:  $w_{\text{eff}}(t_{\text{now}}) < 0.05$

[CMB data, Type Ia supernovae, reionization, etc.]

- Mass of lightest constituent:  $M_0 \gtrsim \mathcal{O}(\text{keV})$

[BBN, small-scale structure]

rough, heuristic limit



# Viable DDM Ensembles of Dark Hadrons

- Parameter space includes six parameters:

$$\{B, C, r, s, \tau_0, \xi\}$$

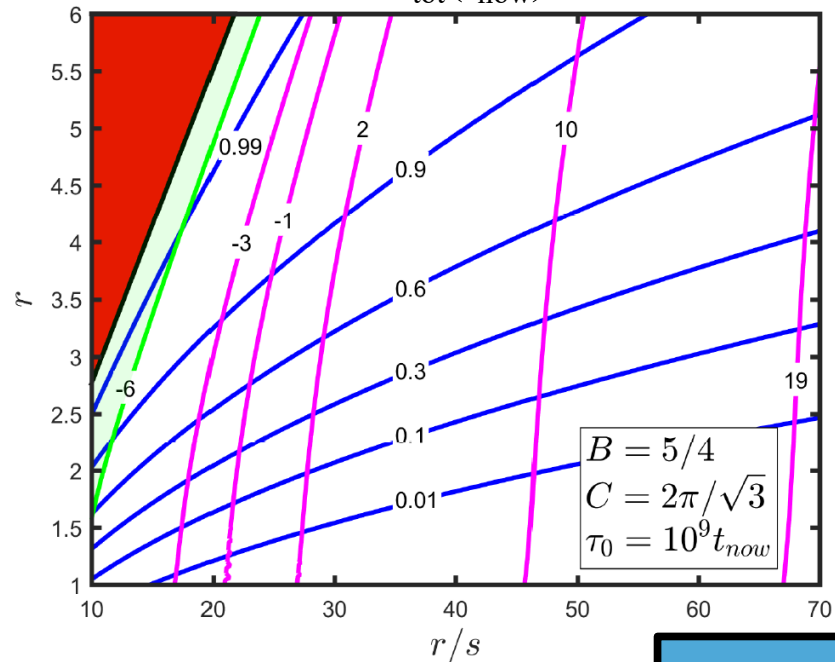
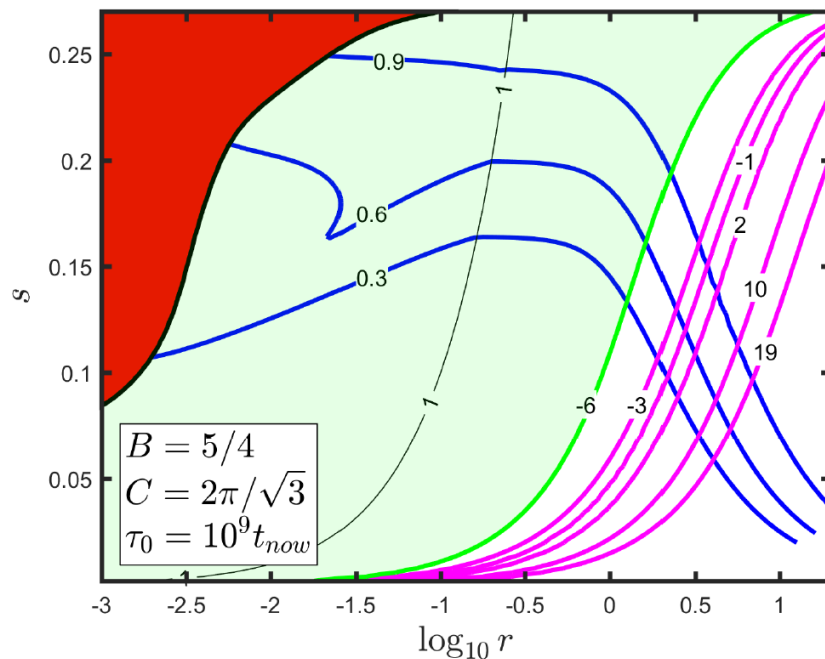
Where we have defined:

$$r \equiv \frac{M_0}{M_s} \quad s \equiv \frac{T_C}{M_s} \quad \tau_0 \equiv \frac{1}{\Gamma_0}$$



## Parameter Space for Dark-Hadron DDM

with primordial abundance chosen such that  $\Omega_{\text{tot}}(t_{\text{now}}) = 0.26$



Excluded:  $M_0 < \mathcal{O}(\text{keV})$

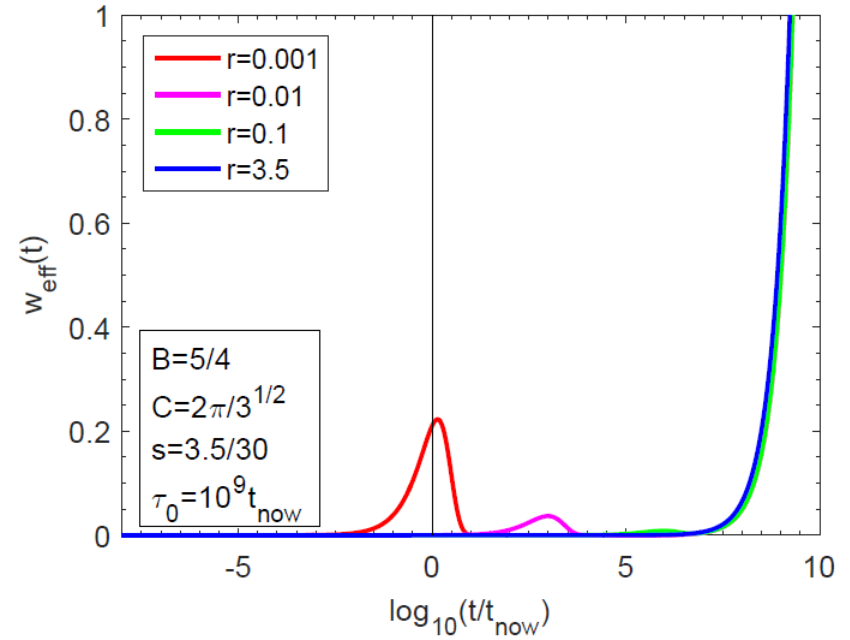
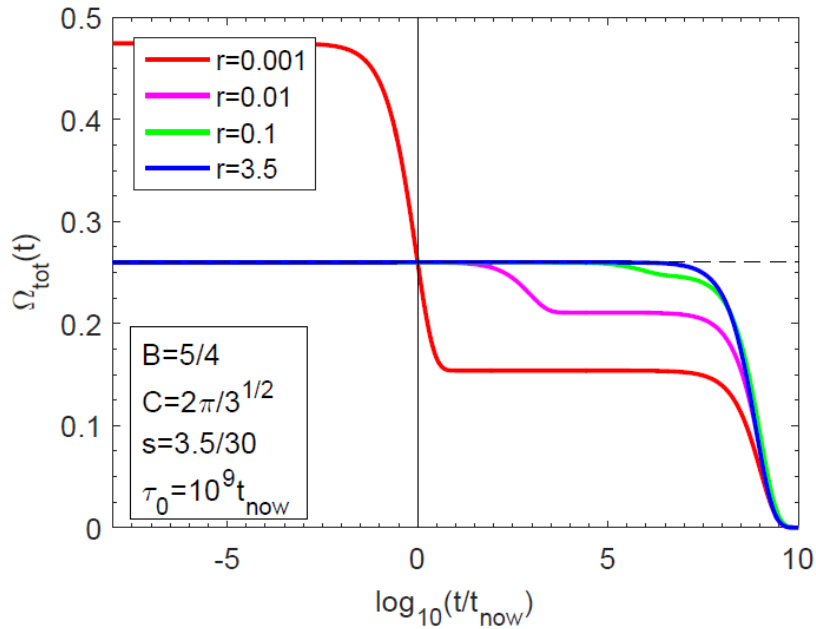
Excluded:  $w_{\text{eff}} < 0.05$

—  $\eta(t_{\text{now}})$

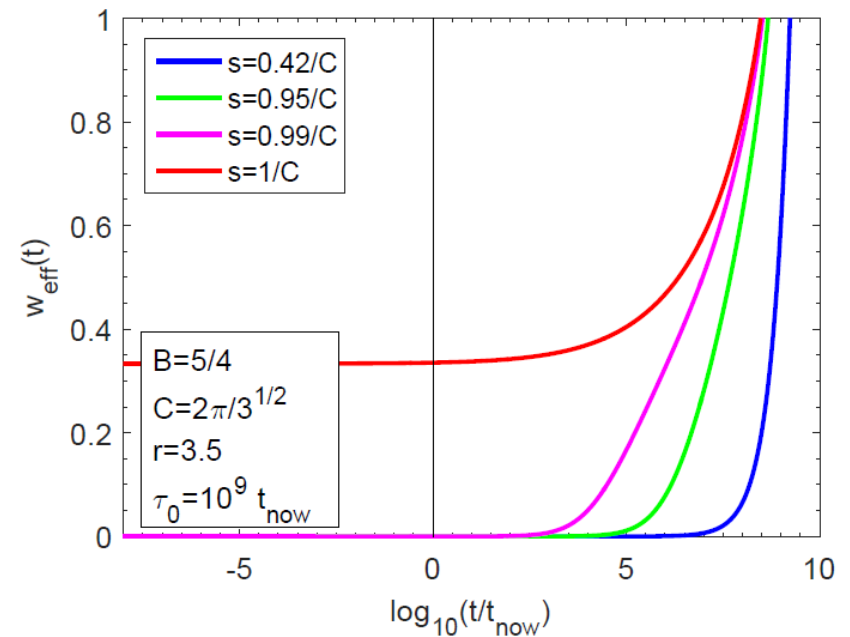
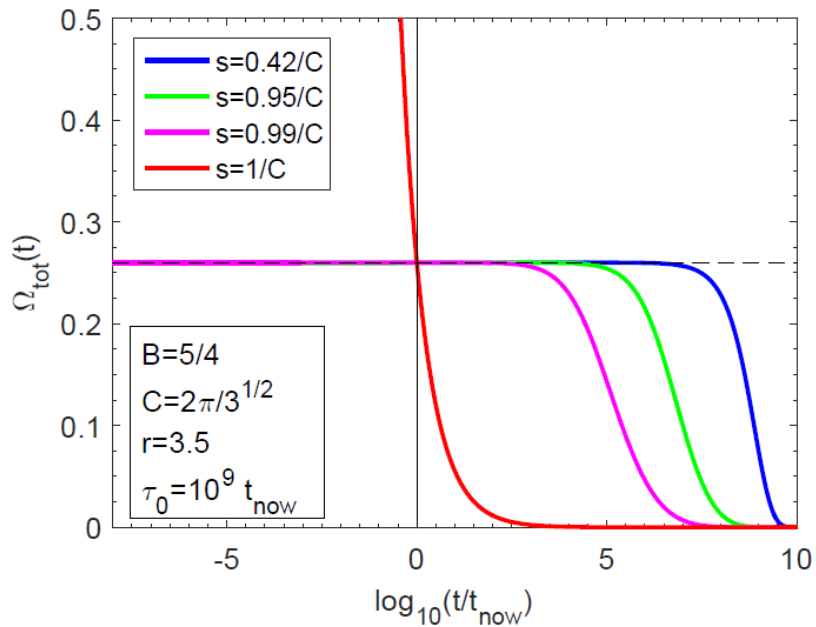
—  $\log_{10}(M_s/\text{GeV})$

Fraction of  $\Omega_{\text{tot}}$   
carried by all  
but the lightest  
constituent

## Evolution of $\Omega_{\text{tot}}$ and $w_{\text{eff}}$ : Dependence on $r$



## Evolution of $\Omega_{\text{tot}}$ and $w_{\text{eff}}$ : Dependence on $s$

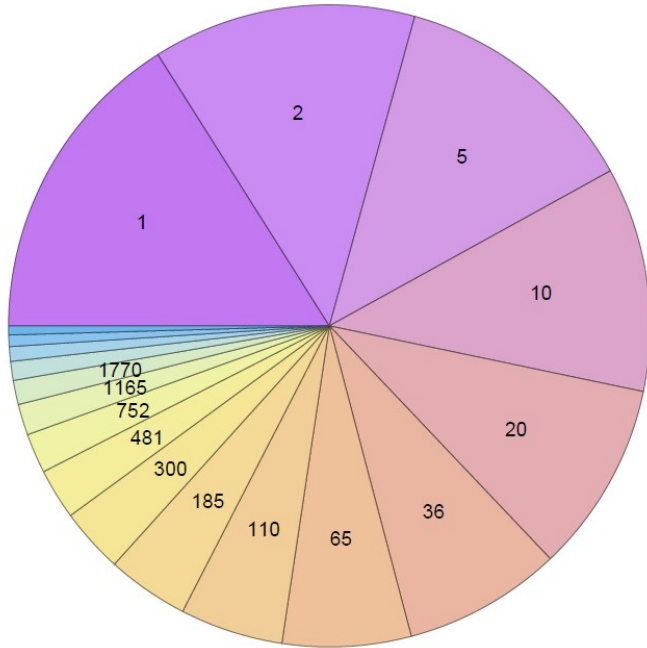


# Distribution of the Abundance

- It's also interesting to examine, in a more detailed way, how the total dark-matter abundance is distributed across the ensemble.

$$r/s = 25$$

$$s = 3.5$$



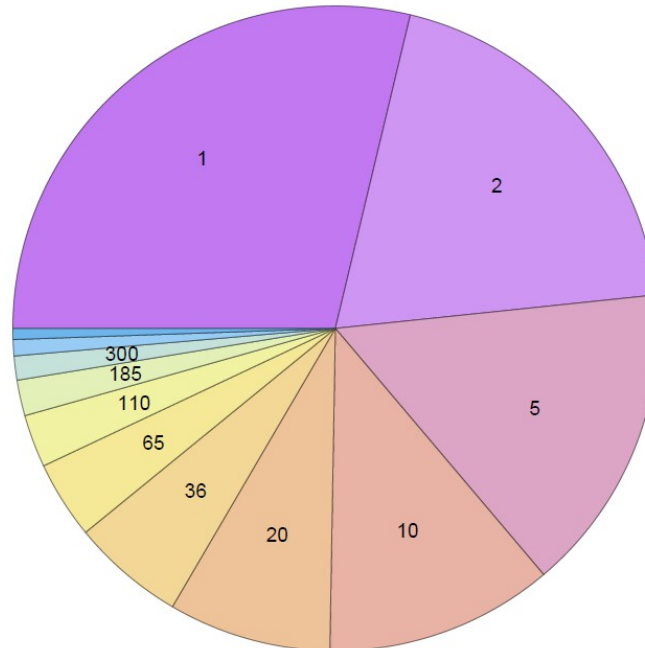
$$M_0 = 704.73 \text{ MeV}$$

$$T_C = 20.19 \text{ MeV}$$

$$M_s = 201.35 \text{ MeV}$$

$$r/s = 30$$

$$s = 3.5$$



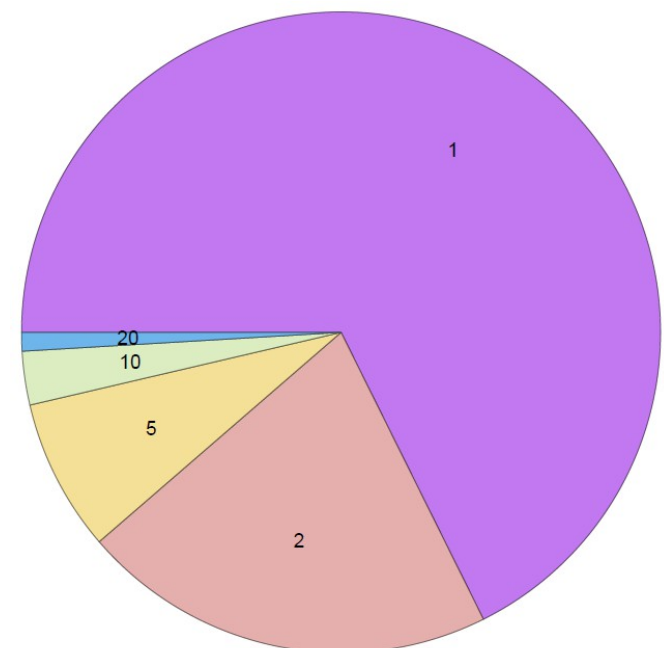
$$M_0 = 531.94 \text{ GeV}$$

$$T_C = 17.73 \text{ GeV}$$

$$M_s = 151.98 \text{ GeV}$$

$$r/s = 50$$

$$s = 3.5$$



$$M_0 = 428.9 \text{ EeV}$$

$$T_C = 0.6 \text{ EeV}$$

$$M_s = 122.2 \text{ EeV}$$

The distribution of the abundance across the DDM ensemble tends to be **more democratic** when the mass scales involved are **lower**.

# **Scenario III:**

## **Dynamical Dark Matter from Symmetry-Breaking Dynamics**



• Keith Dienes, Jacob Fennick, and Jason Kumar [arXiv:1601.05094]



# A Concrete Example

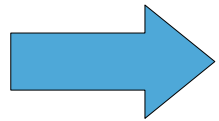
- Consider a pair of fields  $\phi$  and  $\eta$  which transform respectively as a fundamental and an adjoint under a **SU(N) symmetry group with large N** (which could in principle be either global or local).
- Symmetry structures of this sort are common features in **GUTs** and in **string theory**.
- If each of these fields transforms non-trivially under a different  $Z_2$  symmetry, the most general potential is

$$V_0 = \frac{1}{2}M^2\phi^\dagger\phi + \frac{1}{2}\mu^2\text{Tr}[\eta^2] + \frac{\xi_\phi}{4}(\phi^\dagger\phi) + \xi_\eta(\text{Tr}[\eta^2])^2 + \frac{\xi_1}{2}\phi^\dagger\eta^2\phi + \xi_2\text{Tr}[\eta^4]$$

- For this talk, I'll focus on the parameter regime in which:
  - $\mu^2 < 0$  and  $\xi_\eta > 0$
  - $\xi_2$  is small (negligible effect on vacuum structure)
  - Other parameters are such that  $\langle\phi\rangle = 0$

# Randomness and Vacuum Structure

- In this regime, the potential is minimized for **any** set of VEVs  $v_a$  for the components of  $\eta$  which satisfy

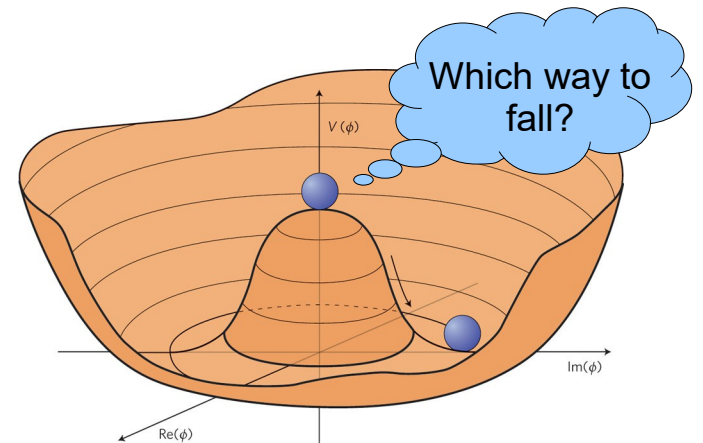


$$\sum_{a=1}^{N^2-1} v_a^2 \equiv v^2 = -\frac{\mu^2}{2\xi_\eta}$$

**But otherwise, the particular values for the  $v_a$  are essentially random!**

- The mass-squared matrix for the component fields  $\phi_i$  after the symmetry is broken takes the form

$$\mathcal{M}_{ij}^2 = M^2 \mathbb{I}_{ij} + \xi_1 \underbrace{\sum_{a=1}^{N^2-1} \sum_{b=1}^{N^2-1} v_a v_b (T_a T_b)_{ij}}_{\text{dynamically-generated contribution}}$$



This dynamically-generated contribution to the mass matrix is also **essentially random**, subject to the above constraint on the VEVs and conditions implied by the structure of the generator matrices  $T_a$ .



# DDM Ensembles and Random-Matrix Ensembles



Eugene Wigner

- The properties of ensembles of random matrices have been studied for a long time.
- The properties of particular ensembles of random matrices  $X$  and their eigenvalues  $\lambda_i$  are well known:

**Gaussian Unitary Ensemble (GUE):**

Complex Hermitian matrices,  
Gaussian textures

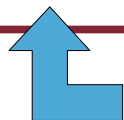
**“Fixed-Trace ” Ensemble (FTE):**

Complex Hermitian matrices,  
Gaussian textures,  $\text{Tr}[X^\dagger X] = [\text{const.}]$

- However, the dynamically-generated piece of the mass-squared matrix for our DDM ensemble is drawn from a different matrix ensemble:

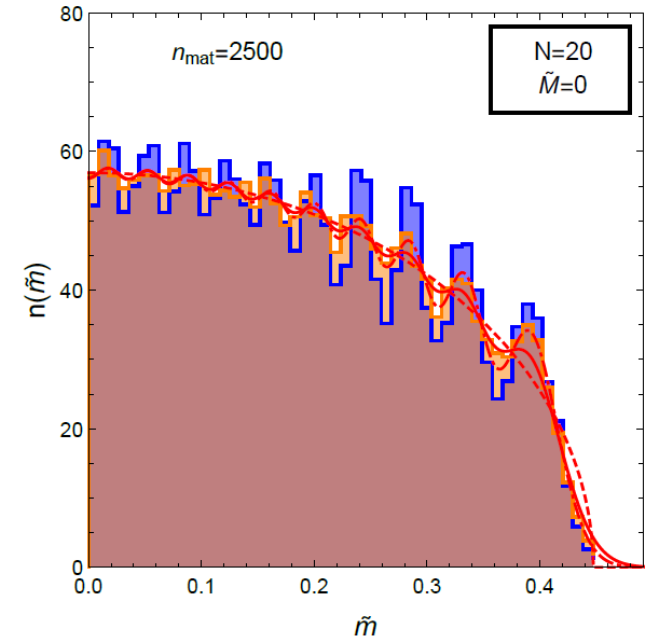
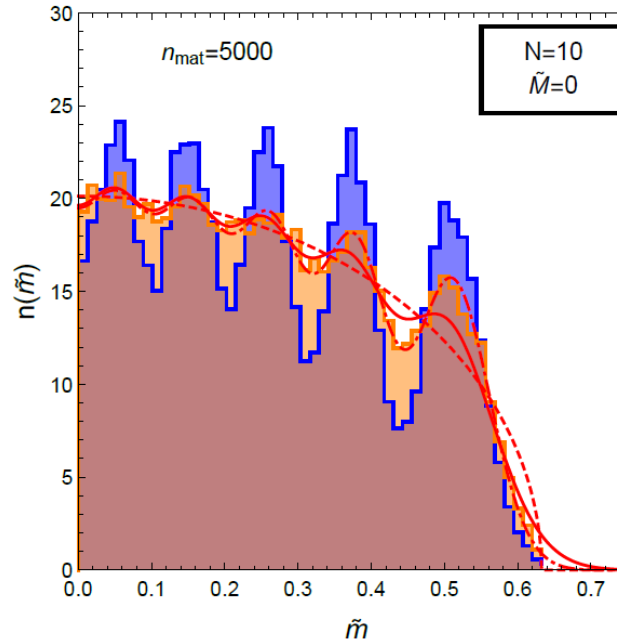
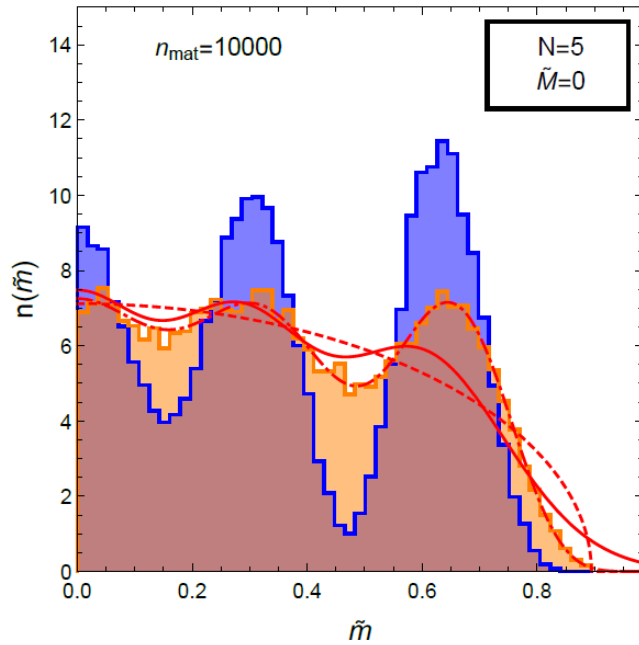
**SU(2) Matrix Ensemble:**

Complex Hermitian matrices, Gaussian textures,  $\text{Tr}[X^\dagger X] = [\text{const.}]$ ,  $\text{Tr}[X] = 0$



**We need to investigate the properties of this ensemble!**

# Density of states



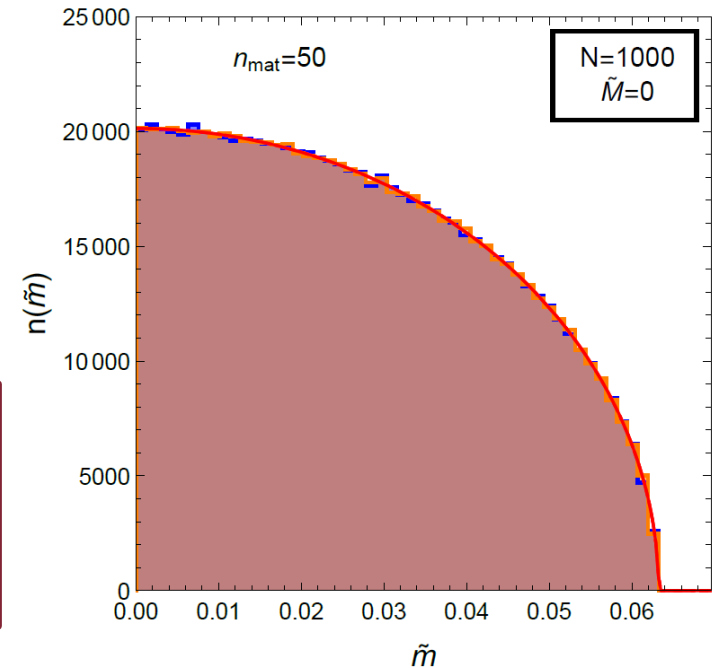
■ Fixed-Trace Ensemble  
■ SU(N) Matrix Ensemble

—  $n_{GUE}(\tilde{m})$   
- - -  $n_{FTE}(\tilde{m})$   
. . .  $n_{WS}(\tilde{m})$

where  $\tilde{m} \equiv \sqrt{2}m/(\sqrt{\xi_1}v)$

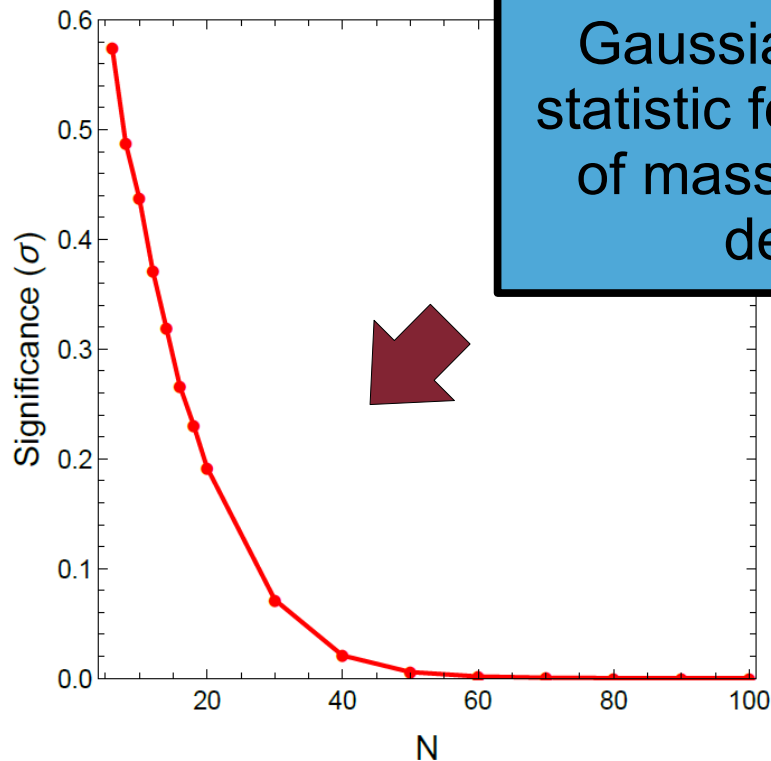
- For (moderately) large  $N$ , the density-of-states functions for **all** of these ensembles converge to the **Wigner semicircle distribution**:

$$n_{WS}(\tilde{m}) = \begin{cases} \frac{2N^{3/2}}{\pi} \sqrt{\frac{\tilde{m}^2}{\tilde{m}^2 - \tilde{M}^2} - \frac{N\tilde{m}^2}{4}} & \tilde{M} \leq \tilde{m} < \sqrt{\tilde{M}^2 + \frac{4}{N}} \\ 0 & \text{otherwise} \end{cases}$$



# A (Statistically) Predictive Mass Spectrum

- Even for moderate values of  $N$ , a robust statistical prediction for the distribution of mass eigenvalues emerges.



Gaussian-equivalent significance for the average  $\chi^2$  statistic for the goodness of fit between the distribution of masses for a single set of  $N$  eigenvalues and the density-of-states function  $n_{\text{SU}(N)}(m)$  itself.

## The Upshot:

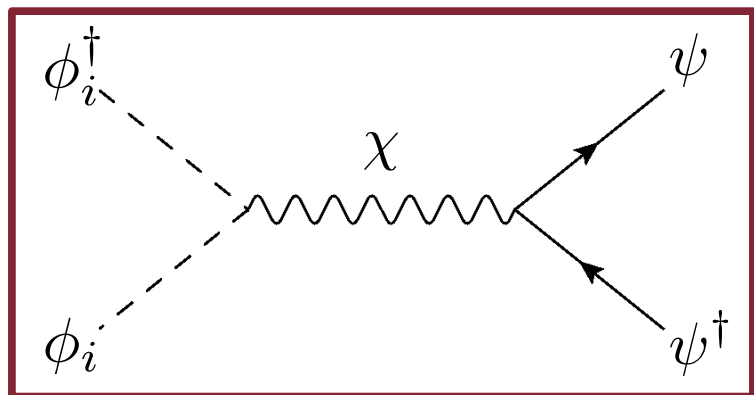
Despite the inherent randomness in the mass matrix, the eigenvalue distribution – and the density of states for the ensemble of  $\phi_i$  – are in a real sense, predictive!

Thus, theories with large symmetry groups naturally give rise to ensembles of particles with a characteristic density of states that decreases with  $m$ .

**Ideal for DDM!**

# Abundance Generation: An Example

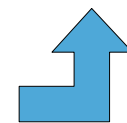
- A spectrum of abundances for the ensemble constituents  $\phi_i$  can arise naturally via **thermal freeze-out**. Dienes, Fennick, Kumar, BT [1711.xxxxx]
- For example, let's say that the  $\phi_i$  couple to a mediator – a massive U(1)' gauge field  $\chi^\mu$  – which also couples to some lighter fermion species  $\psi$  (which could in principle be an SM particle). In this case, the freeze-out contribution to the abundance for each ensemble constituent scales like:



$$\Omega_i \propto \langle \sigma_A v \rangle_i^{-1} \propto \frac{m_i^2}{g_i^2 g_\psi^2} \left( 1 - \frac{m_\chi^2}{4m_i^2} \right)^2$$

For  $m_\chi \gg m_i$ , we have  $\Omega_i \propto m_i^{-2}$ .

**Compatible with a wide variety of density-of-states functions, including the Wigner semicircle!**



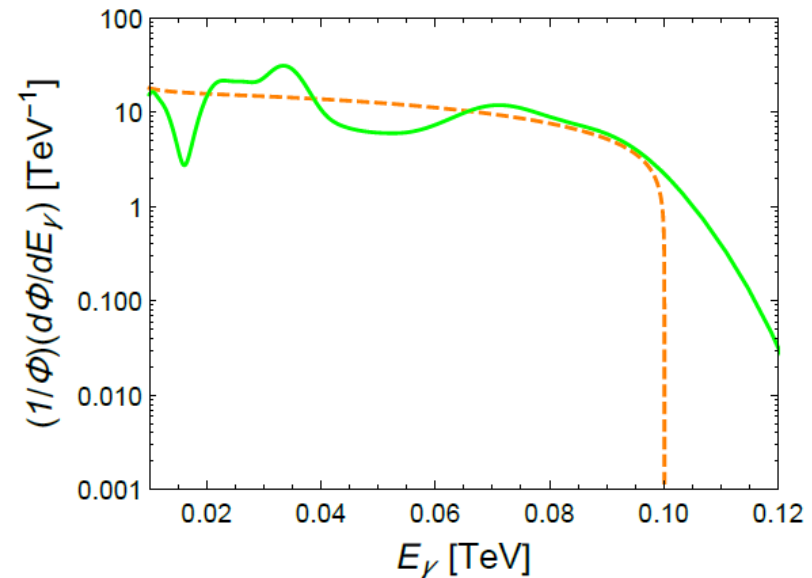
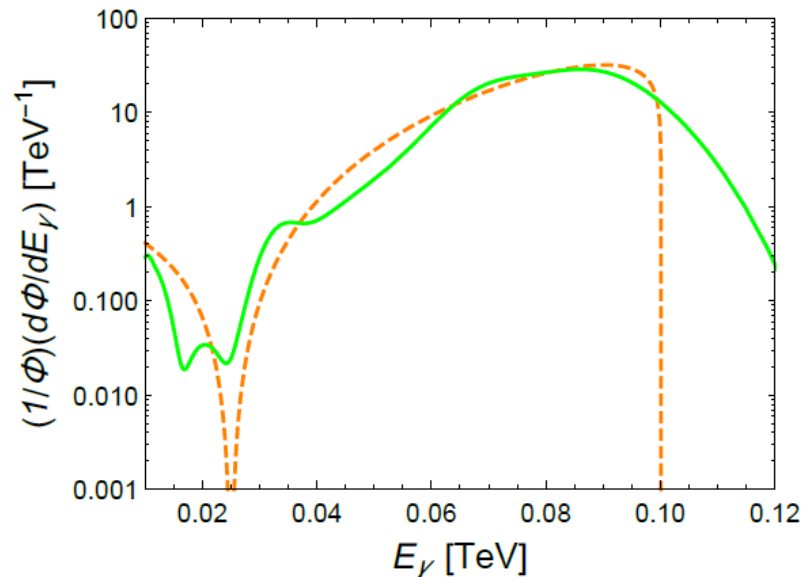
- Coannihilation, annihilation to on-shell  $\chi^\mu$  via  $t$ -channel processes, and processes involving the gauge fields associated with the SU(N) symmetry group (if it's local) in principle also contribute, but these contributions can be rendered negligible by an appropriate choice of parameters.



# Potential Signals

- If there exists a portal between the dark and visible sectors, distinctive signatures of such “statistical” DDM ensembles can arise.
- For example, if an interaction with the photon field of the form  $\phi_i F^{\mu\nu} F_{\mu\nu}$  is generated via controlled breaking of the  $Z_2$  symmetry under which the  $\phi_i$  transform, distinctive **indirect-detection** signatures can arise.

## Example Photon Spectra



$$N = 100$$

$$M = 10 \text{ GeV}$$

$$\sqrt{\xi_1} v^2 = \sqrt{2} \text{ TeV}$$

# Summary

- DDM ensembles emerge naturally in a variety of BSM contexts. In such scenarios, the properties of all constituent particles are determined, through a set of scaling relations, by only a handful of parameters.
- In this talk, I have shown the appropriate scaling relations for DDM can arise in scenarios in which the dark-matter abundance is generated via thermal freeze-out.
- We have examined one class of scenarios in which the ensemble constituents are composite states (dark “hadrons”) in the confining phase of a strongly-coupled dark sector.
- We have also examined another class of scenarios in which the masses of the ensemble constituents are generated by random symmetry-breaking dynamics in the early universe.
- Despite the randomness inherent in the breaking of such symmetries, a characteristic form for the density-of-states function emerges which leads to robust statistical predictions even for moderately large values of  $N$ .