The Axion Quality Problem

- Linda Carpenter Oct 2017

Axions are one of only a few solutions to strong CP, problem,why $\theta < 10^{-9}$, along with the disfavored mu=0, and the Nelson-Barr mechanism.

Axions may provide Dark Matter candidates and anomalous & spontaneously Broken U(1) may be built into renormalizable models

However the Axion Potential is extremely sensitive to Plank suppressed corrections

Unnaturalnesses

- Unexplianable small couplings

- Fine tunings, For example 32 orders of magnitude of Higgs mass squared.

- Sensitivity of potentials to small corrections

There seems to be a problem trading one source of unnaturalness for others

Limits on the axion decay Constant

Axion decay constant is strongly constrained from below from measurements of stars

$$f_a > 10^9 \text{ GeV}$$

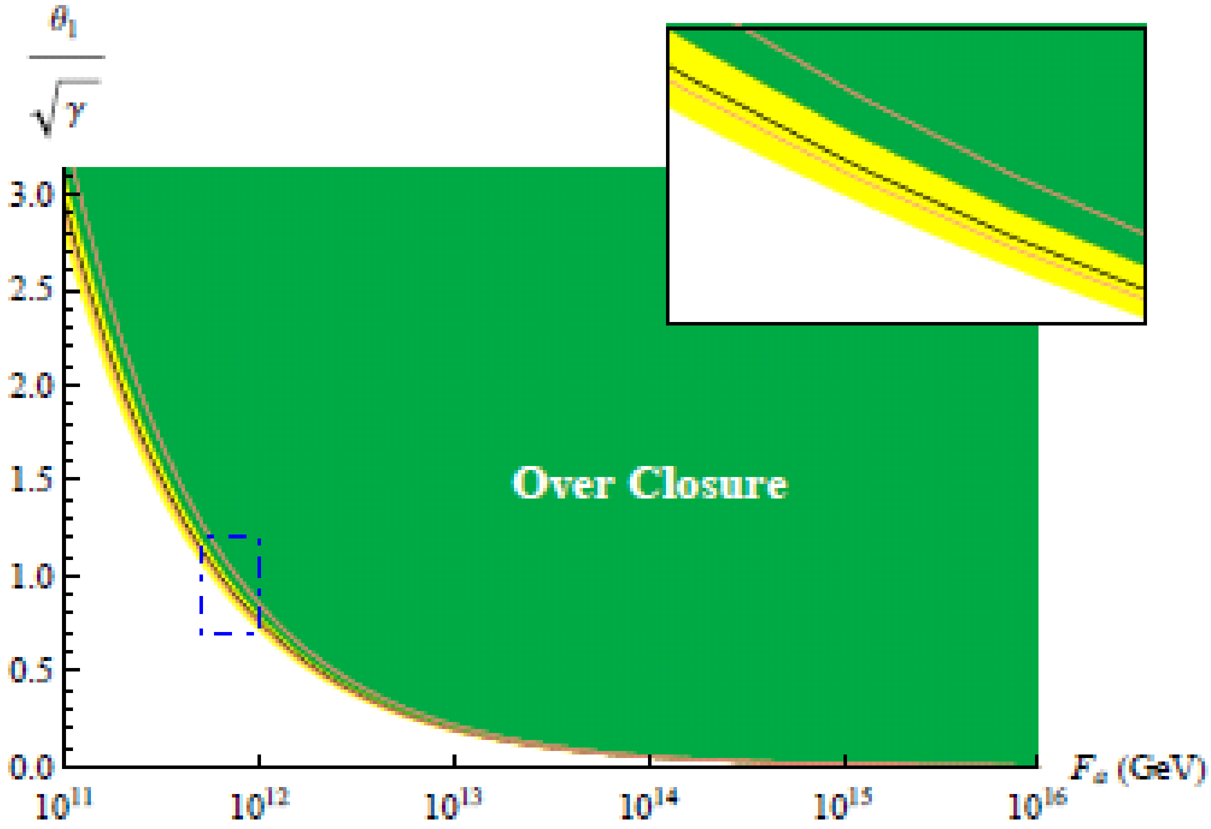
From above the constraint is from avoiding overclosure at the time of recombination. This depends on the initial misalignment angle

The initial axion energy density fraction is *

$$\Omega_a h \sim 0.7\ (\theta/\pi)^2\ (f_a/10^{12})^{7/6}$$

If the initial angle θ is order π the fraction f_a is bounded to be around 10^{12} to avoid overclosure.

fa vs Initial misalignment angle



from Covi et. Al.

If the misalignment angle is smaller, f_a can be raised. However unless the initial angle is tuned, we put and upper bound of 10^{15} GeV on it.

We see the tuning here is one part in 100 with a scale 2 orders of magnitude before the GUT scale.

Also note the favored scale of $f_a \sim 10^{12}$-10^{15} GeV is fairly arbitrary

Axion Quality

The QCD potential is

$$V_{qcd} \approx -m_\pi^2 f_\pi^2 \cos(\frac{a}{f_a})$$

The axion potential is

$$V_a = Q f_a^4 \cos(\frac{a}{f_a} - \theta_0)$$

In order for QCD to be the biggest contribution to the potential we require

$$Q < 10^{-62} \left(\frac{10^{12}\mathrm{GeV}}{f_a} \right)^4$$

There is no good reason for these operators to go away as we don't expect Plank scale physics to respect global symmetries.

What is called a “folk theorem”, Quantum Gravity breaks all global symmetries

Plank suppressed operators

Plank suppressed operators add to the axion potential

$$\frac{\phi^{n+4}}{M_p^N}$$

which gives coefficient

$$Q = \left(\frac{f_a}{M_p}\right)^n$$

for $f_a \sim 10^{15}$ And require n>20 for proper suppression.

We may try to forbid this operator with a discreet symmetry, which would require Z_n of 24 or more!

Does Supersymmetry Help?

Superpotential operators

$$\delta W = \phi^3 \left(\frac{\phi}{M_p} \right)^{n-3}$$

Yields a scalar potential operator with constraint

$$f_a^2 |F| \left(\frac{f_a}{M_p} \right)^{n-3} < 10^{-14}\ \mathrm{GeV}^4$$

Choosing f_a and limiting the SUSY breaking term F, this requires n>13 symmetry to suppress, note this means Z_n>17. Holomorphy constraints on operators seem to help a little bit.

Kahler Potential Operators

$$K = \int d^4\theta \phi \phi^\dagger \left(\frac{\phi}{M_p} \right)^n$$

Yields a scalar Potential

$$|F||F^\dagger| \left(\frac{f_a}{M_p} \right)^n < 10^{-14}\ \mathrm{GeV}^4.$$

And requires n>8

There is no known good solution to axion quality, there isn't even an anthropic solution. Assume axions are dark matter and dark matter is anthropic . A simple constraint is that the 'axion' not to dominate the energy density at 1eV. Taking into account Plank suppressed operators ,the axion mass is given by

$$m_a^2 = h f_a^2 \left(\frac{f_a}{M_p}\right)^n$$

So the constraint becomes

$$10^{27} \left(\frac{f_a}{M_p}\right)^{\frac{n+10}{4}} < 1$$

.

Need n>8 is needed, not enough to explain the large Z_n

Past SUSY axion models built of renormalizible chiral operators , for example,

$$W = \chi(S_+S_- - \mu^2)$$

Where the axion lives in S+ and S-, and the decay constant is

$$f_a = \sqrt{|S_+|^2 + |S_-|^2}$$

This can be coupled to a sector that breaks SUSY

$$W_1 = \lambda_1 X(\phi_1\phi_{-1} - F) + m_1\phi_1^2 + m_2\phi_{-1}\phi_3.$$

Through intermediate fields (here called a and b)

$$W_2 = \chi(S_+S_- - \mu^2) + hS_+a_1\bar{a}_2 + yS_-\,b_1\bar{b}_2 + X(a_i\bar{a}_i + b_i\bar{b}_i)$$

One loop corrections fix the saxion vev,

$$S_+ = \mu\sqrt{\frac{y}{h}}e^{\frac{\phi}{\mu}}, \;\; S_- = \mu\sqrt{\frac{h}{y}}e^{-\frac{\phi}{\mu}} \qquad \phi = 0$$

The PQ anomaly is communicated through PQ messengers which couple to the axion, whose charges insure the PQ symmetry is anomalous, and SUSY breaking is communicated by its own set of messengers

$$W_3 = X(q_1\bar{q}_1 + \ell_1\bar{\ell}_1) + S_+q_2\bar{q}_2 + S_-\ell_2\bar{\ell}_2$$

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In principle, the scale of SUSY breaking and the scale of f_a are independent.

The saxion mass is given at 1 loop by

$$m_s^2 \sim \frac{1}{16\pi^2}\frac{F^2}{\mu^2}$$

While the two loop scalar mass contains a vev of x.

Like the axion the saxion as a pseudo-modulus begins with initial energy density fraction

$$f_a^2/M_p^2$$

And increases with 1/T.

At nucleosynthesis, we do not want the saxion energy density to exceed 1, so it must decay before. The width is given by

$$\Gamma_P = \frac{1}{16\pi}\frac{m_P^3}{f_a^2} \sim 10^{-25}\left(\frac{10^{12}}{f_a}\right)^2\left(\frac{m_P}{1\ \mathrm{GeV}}\right)^3 \mathrm{GeV}$$

This for f_a of 10^{12} this puts a limit on saxion mass to be 1 GeV.

So the F term must be at least

$$\sqrt{F} > 10^{7.5}$$

This is about what we need for SUSY scalar masses to come out right,

$$m_s = \frac{1}{16\pi^2}\frac{|F|}{f_a},$$

However, all of these scales are higher than normal for usual gauge mediation, and possible more arbitrary, and F cannot be much higher or flavor violating SUGRA effects will be a problem.

Try a composite axion: SUSY example

Plan to use large gauge groups SU(n) x SU(m) hidden sectors which also have global U(1) symmetries

The Gauge symmetry should protect the axion operators, superpotential is generally entirely protected

Kahler potential then requires only very small Z_n to work

Review 4-1 model

SU(4) xU(1) Superpotential is very restricted,

$$W = \lambda S_4 F_{-3} \bar{F}_{-1} + \frac{\Lambda_4^5}{\left(\bar{F}_i F^j A^{ik} A^{lm} \epsilon_{jklm}\right)^{1/2}}.$$

The scalar potential is

$$V_F = \lambda^{6/5} \Lambda^4 \left(|b|^4 + \left| 2bc - \frac{1}{ab^2} \right|^2 + \left| \frac{1}{a^2 b} \right|^2 \right)$$

With D terms

$$D_1 = g_1 \frac{\Lambda^2}{\lambda^{2/5}} (2|a|^2 - 4|b|^2 + 4|c|^2)$$

- F and D terms cannot be simultaneously set to zero SUSY is broken

- No runaway directions exist, there are no pseudomoduli

Model Building Plan

Use such a SUSY breaking sector with large gauge groups and additional global U(1) SUSY breaks and a U(1) charged condensate of fields get a vev. Identify it as the axion.

The Superpotenital is naturally protected by gauge symmetry, only Kahler potential operators effect axion quality

No field identifiable as the saxion exists, no light pseudomoduli

10^{12} GeV is not so unnatural of a scale to achieve with a large gauge group.

Models of these types have been extensively studied. Some like SU(2k) don't break SUSY. Some like the 4-1 model fail to spontaneously break the extra U(1), some do not posses the extra global U(1)

One must search the catalog of SUSY models to identify possible models

SU(2k+1)

- The theory posses symmetries

	$U(1)_Q$	$U(1)_{\bar{Q}}$	$U(1)_A$	$U(1)_R$
Q	1	0	0	0
$\bar{Q}$	0	1	0	0
A	0	0	1	0
Λ^{b_0}	N_f	$N-4+N_f$	$N-2$	$6-2N_f$.

- For anomaly free theory field content is One antisymmetric tensor, Nf fundamentals and 2k-3+nf anti-fundamentals

- The composite states of the theory are

$$
\begin{aligned}
M_i^a &= \bar{Q}_i^\alpha \, Q_\alpha^a \\
X_{ij} &= A_{\alpha\beta} \, \bar{Q}_i^\alpha \, \bar{Q}_j^\beta \\
Y^a &= Q^a_{\alpha_{2k+1}} \, \epsilon^{\alpha_1 \ldots \alpha_{2k+1}} \, A_{\alpha_1\alpha_2} \ldots A_{\alpha_{2k-1}\alpha_{2k}} \\
Z &= \epsilon^{\alpha_1 \ldots \alpha_{2k+1}} \, A_{\alpha_1\alpha_2} \ldots A_{\alpha_{2k-3}\alpha_{2k-2}} \, Q^a_{\alpha_{2k-1}} \, Q^b_{\alpha_{2k}} \, Q^c_{\alpha_{2k+1}} \, \epsilon_{abc} \, .
\end{aligned}
$$

- One flavor is integrated out and nonperturbative effects add a term to the S.P.

$$
W_{N_f=2} = \frac{\Lambda_{(2)}^{4k+3}}{\epsilon_{ac} \, Y^a \, M^c_{j_1} \, \epsilon^{j_1 \ldots j_{2k-1}} \, X_{j_2 j_3} \ldots X_{j_{2k-2} j_{2k-1}}}
$$

- We add a perturbation

$$
\delta W = m_c^i M_i^c
$$

Because the fields are subject to a constraint

$$Y \cdot M^2 \cdot X^{k-1} \; - \; c\, Z \, \mathrm{Pf} X \; = \; \Lambda^{4k+2}$$

The fields Y which are charged under the U(1) get an expectation value and may be identified with the axion.

The larger the number k, the more suppressed PQ violating operators in the Kahler potential will be.

For k=1, Y has three fields and the first allowed operator is $Y^2/Mp^4 = 10^{24}/Mp^4$ is $10^{-48.}$

We need 62 orders of magnitude, thus a Z3 will be enough to suppress this operator.

Future Work

It seems only SU(2k+1) and SU(2k)xU(1) models work, perhaps we can expand the number of functioning models

This has not been built into a more complete model which perhaps also includes GMSB

Summary

Models with large gauge groups in the hidden sector may break SUSY and protect the axion mass with gauge symmetry

Lack of flat direction mean there is no saxion

Gauge dynamics may explain the intermediate scale