Monte-Carlo methods for Multi-Higgs Production

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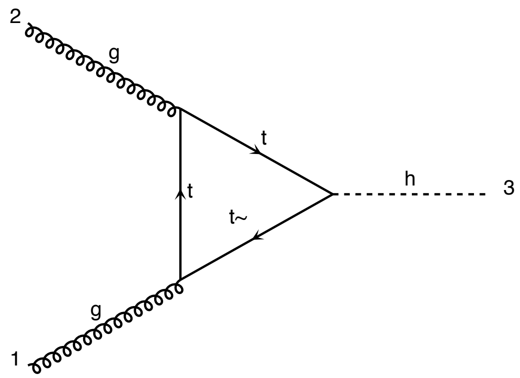
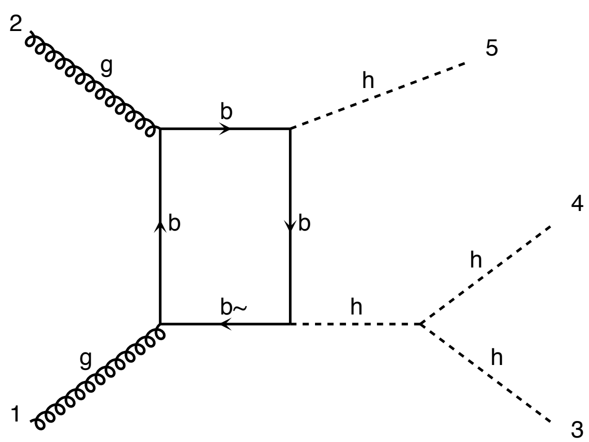
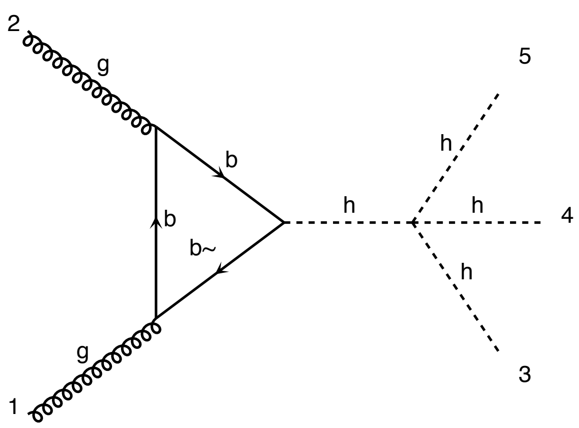
based on:
V. Hirschi, OM: arXiv:1507.00020
C.Degrande, V.Khoze, OM: 1605.06372
OM:1607.00763
M. Backović et al:1505.04190

Motivation







- Low multiplicity
  - ➡Measuring the Higgs potential is a deep test of the EWSB mechanism
  - ➡Multiple Higgs production is the only way to probe this experimentally
- Large multiplicity:
  - ➡Are we sensitive to the scalar unitary breaking of the perturbative expansion

Plan of the talk

- MG5aMC@NLO: short presentation
- Loop-induced computation in MG5aMC
  - ➡re-weighting and direct computation
  - ➡1507.00020/1607.00763
- High multiplicity Higgs production
  - ➡1605.06372

Red Line

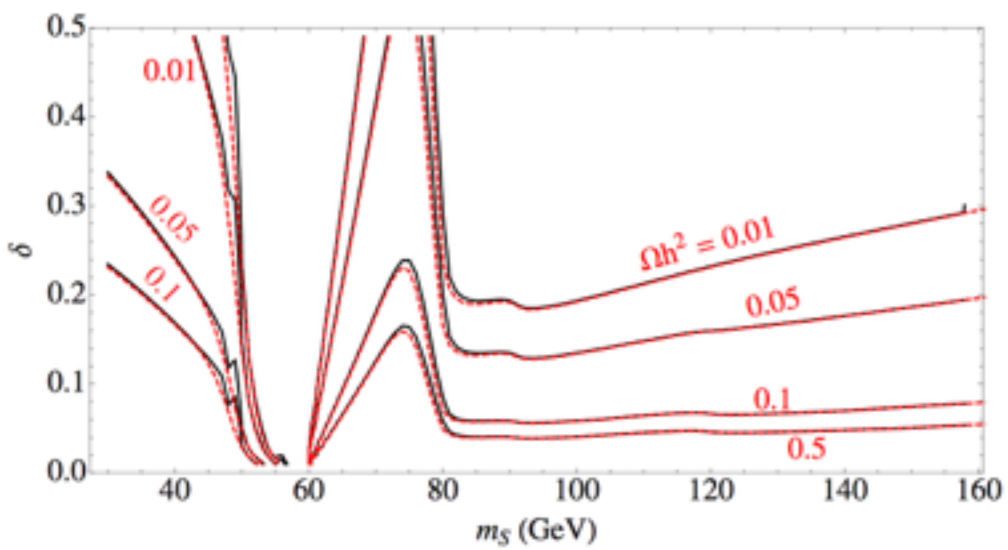
- Multi-Higgs production from one to hundred(s) of Higgs

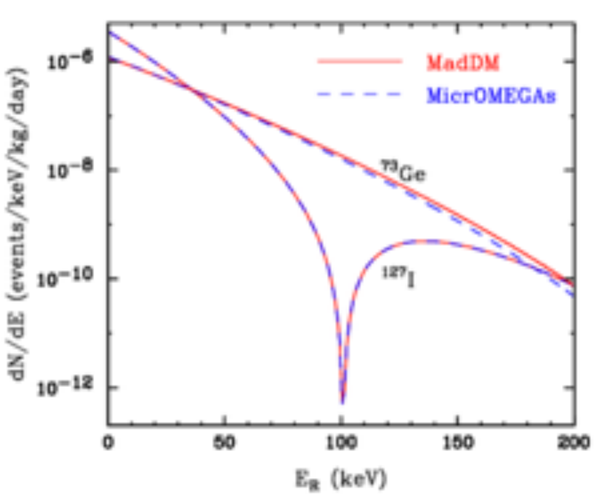
MG5_aMC@NLO

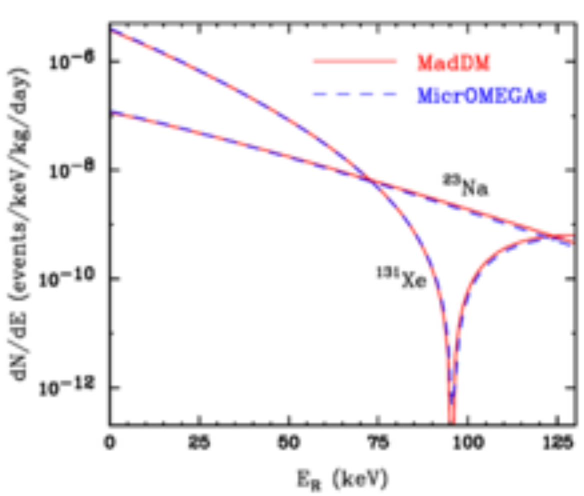
- MadGraph5_aMC@NLO
  - generate events/ compute cross-sections both at **LO/NLO in QCD** in SM/BSM
    - results available for EW corrections
      - NLO+PS in progress
  - is a **matrix-element provider**
    - new framework to customize output
  - is a **platform** allowing to run tools
    - parton-shower/re-weighting/plot/...

Dark Matter

MadDM







- Computation of relic density

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -2\langle\sigma_{eff} v\rangle \left(n_\chi^2 - (n_\chi^{EQ})^2\right),$$

- Computation of direct detection
  - ➡ various type of experiment

- In progress: indirect detection

Backovic, Kong, McCaskey 1308.4955
Backovic, Martini, OM et al1505.04190

Multi-Higgs: Low Multiplicities

Computation method

ReWeighting

$$W_{sm} = \frac{|M_{sm}|^2}{|M_{heft}|^2} W_{heft}$$

- Start by an EFT theory
- change the weight associated to each event

Exact Integration

$$\int |M_{loop}|^2$$

- Easy extension to any loop-induced processes
- Allow BSM study
- No validity issue

BSM Re-Weighting

Re-weighting are everywhere

- scale and pdf uncertainties (available both for LO and NLO computation)
- matching/merging
- experimental re-weighting

BSM Re-weighting

- Change the events weights of a LHEF for various BSM theories.
- Re-use the same parton shower and detector simulation

LO

Re-Weighting

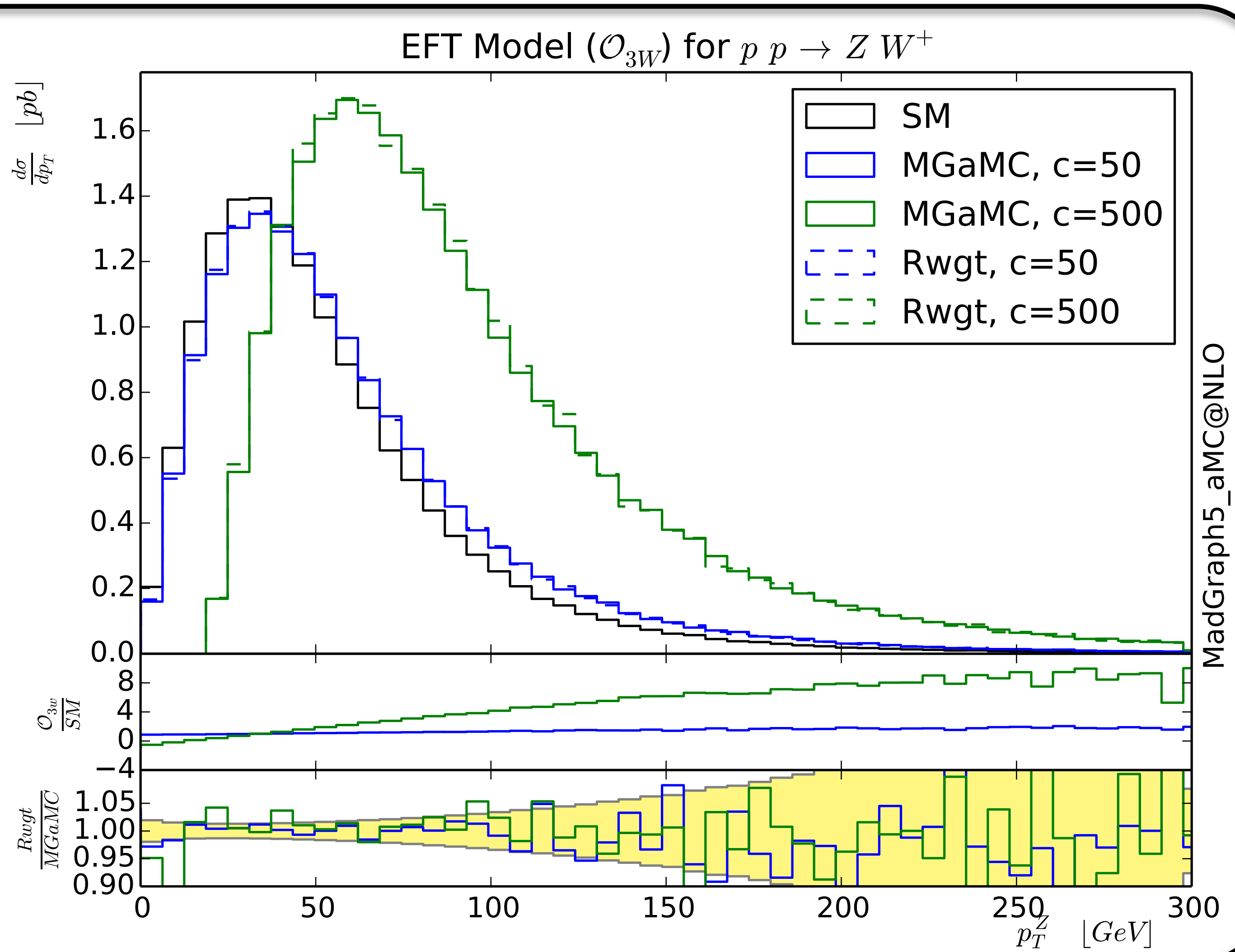
1404.7129
1607.00763

- Change the weight of the events

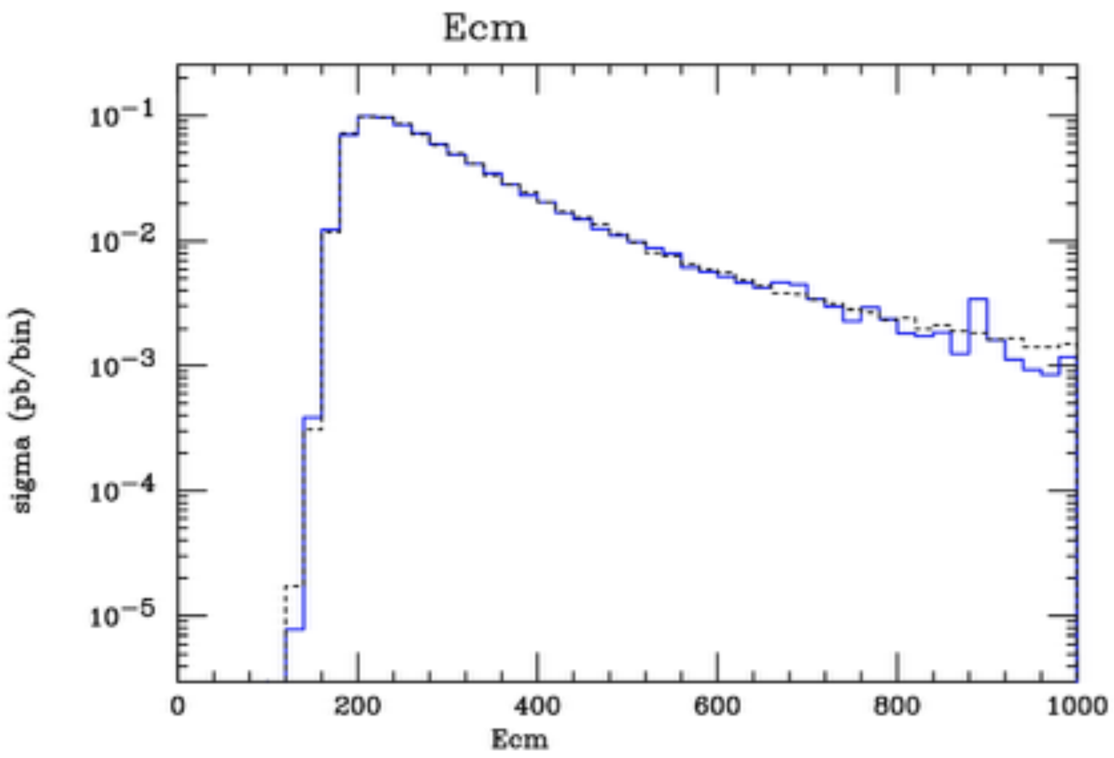
$$W_{new} = \frac{|M_{new}|^2}{|M_{old}|^2} * W_{old}$$

EFT Case

$$\mathcal{O}_{3W} = Tr\left[W_{\mu\nu}W^{\nu\rho}W_{\rho}{}^{\mu}\right]$$



Re-Weighting Limitation



$$\Delta\sigma_{new} = \frac{\sigma_{new}}{\sigma_{old}}\Delta\sigma_{new} + \mathrm{Var}_{wgt}\,\sigma_{old}$$

- statistical uncertainty can be enhanced by the re-weighting
- better to have wgt<1

- You need to have the same phase-space (more exactly a subset)
- Mass scan are possible only in special case
  - only for internal propagator
  - for small mass variation (order of the width)

NLO Re-Weighting

NLO method

- tracks the dependencies in the various matrix-elements (born, virtual, real)

$$d\sigma^\alpha = f_1(x_1,\mu_F) f_2(x_2,\mu_F) \Big[\mathcal{W}_0^\alpha + \mathcal{W}_F^\alpha \log\left(\mu_F/Q\right)^2 + \mathcal{W}_R^\alpha \log\left(\mu_R/Q\right)^2 \Big] d\chi^\alpha,$$

$$\mathcal{W}_\beta^\alpha = \mathcal{B} * \mathcal{C}_{\beta,B}^\alpha + \mathcal{B}_{CC} * \mathcal{C}_{\beta,B_{CC}}^\alpha + \mathcal{V} * \mathcal{C}_{\beta,V}^\alpha + \mathcal{R} * \mathcal{C}_{\beta,R}^\alpha$$

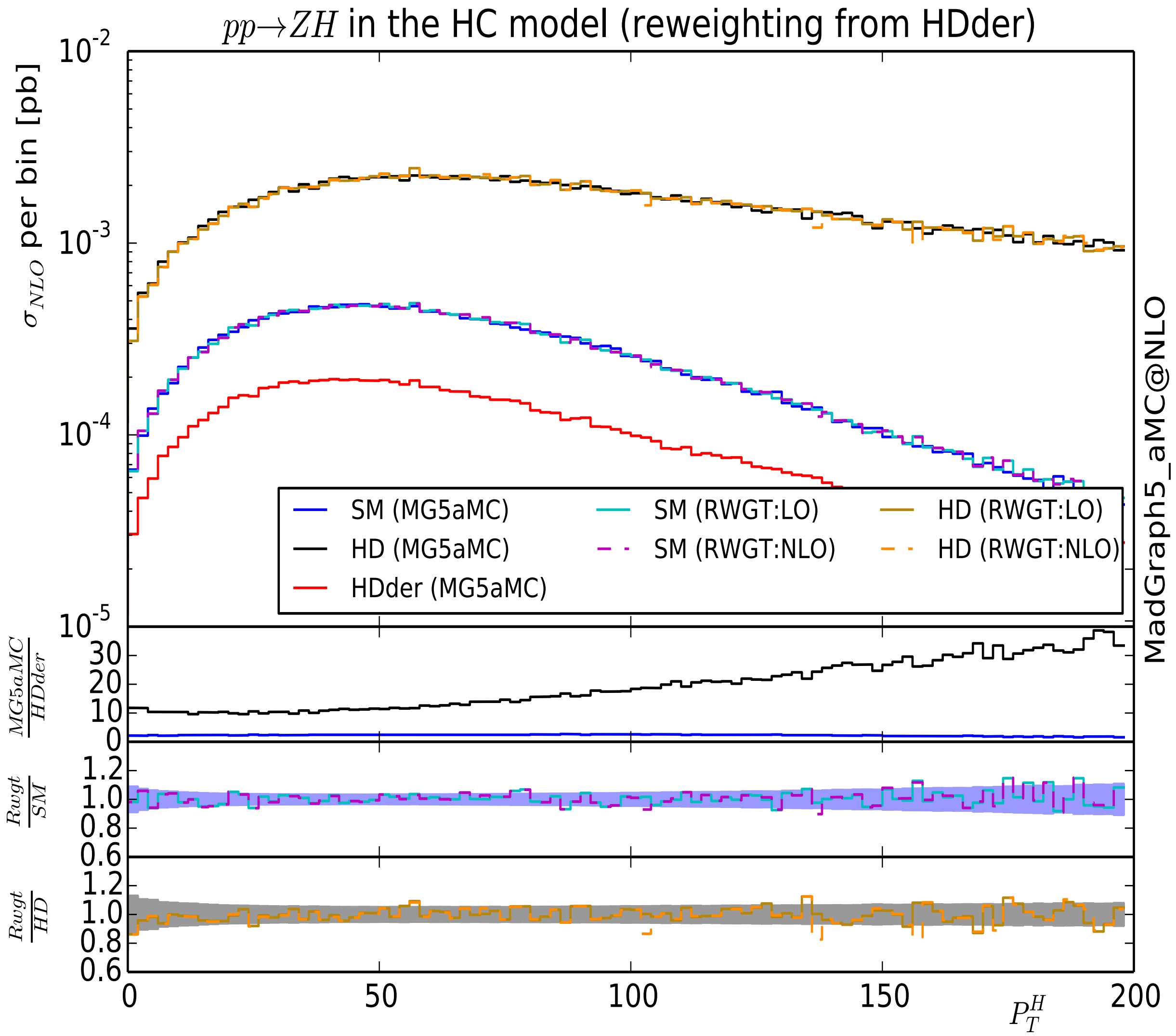
- re-weight each part according to the associated matrix-element
- compute the weight by summing each part

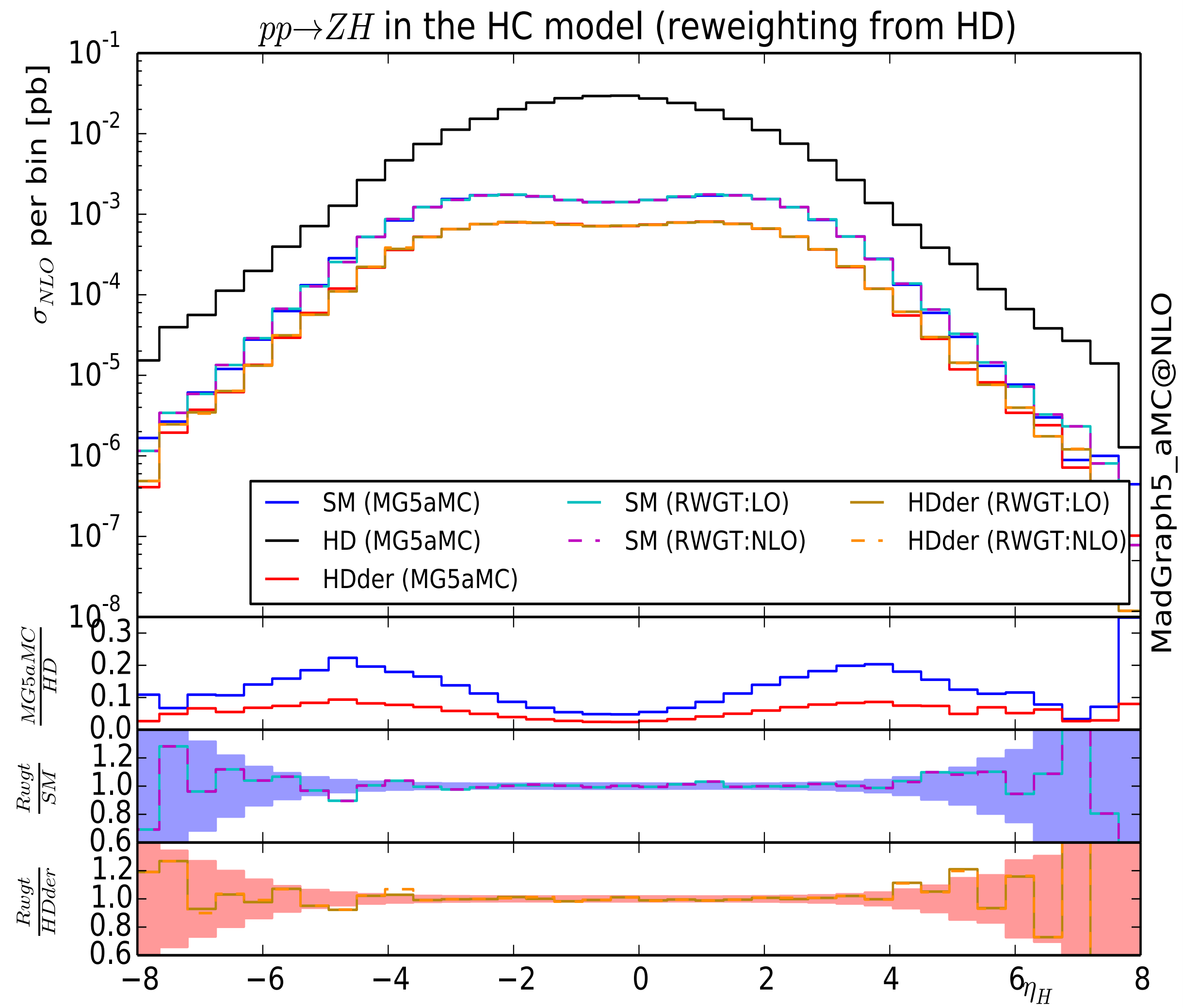
NLO Re-Weighting

EFT Example

$$\mathcal{L}_{HD} = -\frac{1}{4}\frac{1}{\Lambda}\kappa_{HWW} Z_{\mu\nu} Z^{\mu\nu} H$$

$$\mathcal{L}_{HDder} = -\frac{1}{\Lambda}\kappa_{H\partial Z} Z_{\nu}\partial_{\mu} Z^{\mu\nu} H + \left(-\frac{1}{\Lambda}\kappa_{H\partial W} W^{+}_{\nu}\partial_{\mu} W^{-\mu\nu} H + h.c.\right),$$





BSM Physics@NLO

Model Generation

- FeynRules
- NLOCT
  - ➡ private version for EFT

Duhr et al 1310.1921

Degrande 1406.3030

What new information?

- UV counter-term (Renormalization)
- R2: Finite part of the one-loop computation
  - ➡ All computation are done in 4 dimension and not in d-dim
  - ➡ R2 is a process independent correction term to the loop
  - ➡ Epsilon part of the numerator

$$\int d^d l \, \frac{N(l,\tilde{l})}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \cdots \bar{D}_{m-1}}$$

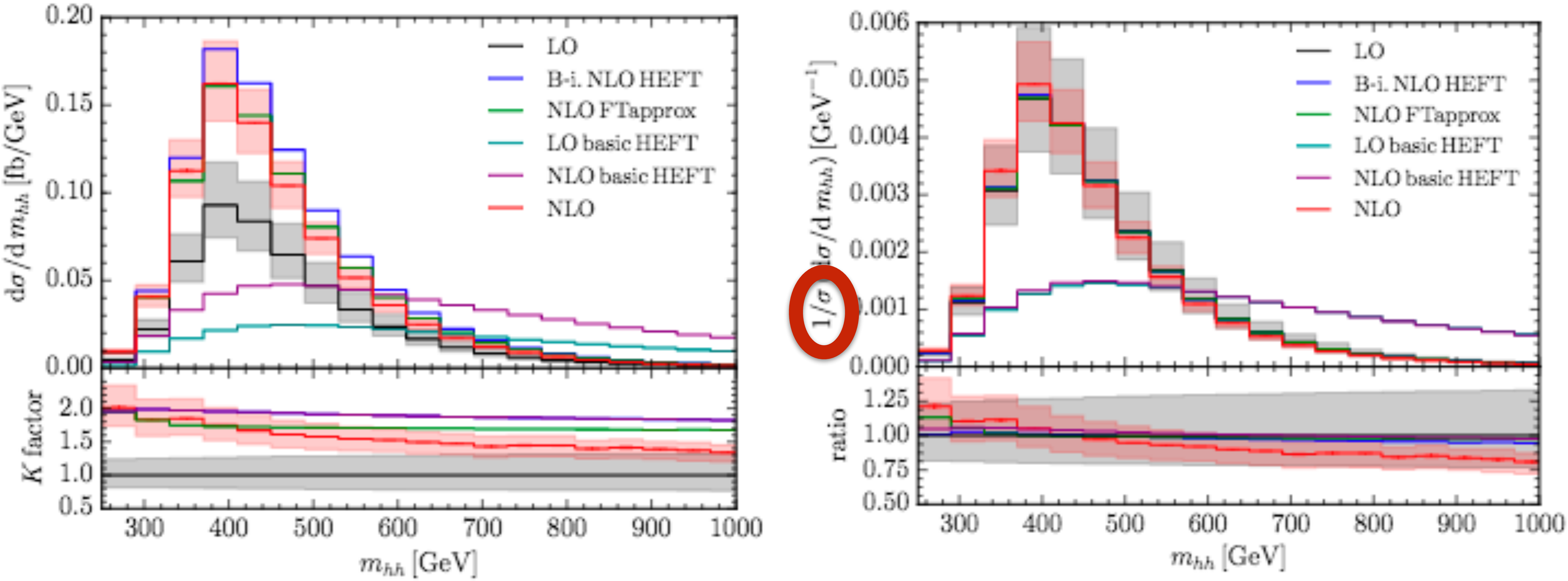
Loop-Improved

Loop-Improved re-weighting

1401.7340

- For loop-induced in absence of 2-loop
- Start by an EFT at NLO
- Use the NLO de-composition
- Re-weight the virtual part by the Born matrix-element
    - Allow to re-introduce mass-effect

Di-Higgs



B.i. NLO HEFT: All matrix-element re-weighted by Born ratio
NLO FTapprox: other name for loop-improved (re-weight real/ MC counter-term by the real

Borowka et al 1608.04798

Exact Integration

Difficulties?

- The phase-space integration is based on the tree diagrams
- Need Leading Color information for writing events
- Loop evaluation are extremely slow

New Solution

- Contract the loop to have tree-level diagrams which drive the integration multi-channel
- Compute the loop with the color flow algebra
- Use Monte-Carlo over helicity
- Increase parallelization

Available since 2.3.0

SM Tables

Process		Syntax	Cross section (pb)	$\Delta_{\hat{\mu}}$	Δ_{PDF}	Ref.
Single boson + jets			$\sqrt{s} = 13$ TeV			
a.1	$pp \to H$	`p p > h [QCD]`	17.79 ± 0.060	$^{+31.3\%}_{-23.1\%}$	$^{+0.5\%}_{-0.9\%}$	[49]
a.2	$pp \to Hj$	`p p > h j [QCD]`	12.86 ± 0.030	$^{+42.3\%}_{-27.7\%}$	$^{+0.6\%}_{-0.9\%}$	[49]
a.3	$pp \to Hjj$	`p p > h j j QED=1 [QCD]`	6.175 ± 0.020	$^{+61.8\%}_{-35.6\%}$	$^{+0.7\%}_{-0.9\%}$	[49]
*a.4	$gg \to Zg$	`g g > z g [QCD]`	43.05 ± 0.060	$^{+43.7\%}_{-28.4\%}$	$^{+0.7\%}_{-1.0\%}$	[34]
*a.5	$gg \to Zgg$	`g g > z g g [QCD]`	20.85 ± 0.030	$^{+64.5\%}_{-36.5\%}$	$^{+1.0\%}_{-1.1\%}$	[50]
†a.6	$gg \to \gamma g$	`g g > a g [QCD]`	75.61 ± 0.200	$^{+73.8\%}_{-41.6\%}$	$^{+0.7\%}_{-1.1\%}$	[–]
†a.7	$gg \to \gamma gg$	`g g > a g g [QCD]`	14.50 ± 0.030	$^{+76.2\%}_{-40.7\%}$	$^{+0.6\%}_{-1.0\%}$	[–]

Process		Syntax	Cross section (pb)	$\Delta_{\hat{\mu}}$	Δ_{PDF}	Ref.
Double bosons + jet			$\sqrt{s} = 13$ TeV			
b.1	$pp \to HH$	`p p > h h [QCD]`	$1.641 \pm 0.002 \cdot 10^{-2}$	$^{+30.2\%}_{-21.7\%}$	$^{+1.1\%}_{-1.2\%}$	[48]
b.2	$pp \to HHj$	`p p > h h j [QCD]`	$1.758 \pm 0.003 \cdot 10^{-2}$	$^{+45.7\%}_{-29.2\%}$	$^{+1.2\%}_{-1.2\%}$	[51]
*b.3	$pp \to H\gamma j$	`p p > h a j [QCD]`	$4.225 \pm 0.006 \cdot 10^{-3}$	$^{+38.6\%}_{-25.9\%}$	$^{+0.4\%}_{-0.7\%}$	[52]
*b.4	$gg \to HZ$	`g g > h z [QCD]`	$6.537 \pm 0.030 \cdot 10^{-2}$	$^{+29.4\%}_{-21.3\%}$	$^{+1.0\%}_{-1.1\%}$	[53]
*b.5	$gg \to HZg$	`g g > h z g [QCD]`	$5.465 \pm 0.020 \cdot 10^{-2}$	$^{+46.0\%}_{-29.4\%}$	$^{+1.2\%}_{-1.3\%}$	[52]
b.6	$gg \to ZZ$	`g g > z z [QCD]`	1.313 ± 0.004	$^{+27.1\%}_{-20.1\%}$	$^{+0.7\%}_{-1.0\%}$	[42]
*b.7	$gg \to ZZg$	`g g > z z g [QCD]`	0.6361 ± 0.002	$^{+45.4\%}_{-29.1\%}$	$^{+1.0\%}_{-1.2\%}$	[54]
b.8	$gg \to Z\gamma$	`g g > z a [QCD]`	1.265 ± 0.0007	$^{+30.2\%}_{-22.2\%}$	$^{+0.6\%}_{-1.0\%}$	[42]
*b.9	$gg \to Z\gamma g$	`g g > z a g [QCD]`	0.4604 ± 0.001	$^{+43.7\%}_{-28.4\%}$	$^{+0.8\%}_{-1.1\%}$	[55]
b.10	$gg \to \gamma\gamma$	`g g > a a [QCD]`	$5.182 \pm 0.010 \cdot 10^{+2}$	$^{+72.3\%}_{-43.4\%}$	$^{+1.0\%}_{-1.3\%}$	[42]
*b.11	$gg \to \gamma\gamma g$	`g g > a a g [QCD]`	19.22 ± 0.030	$^{+59.7\%}_{-35.7\%}$	$^{+0.7\%}_{-1.0\%}$	[56]
b.12	$gg \to W^+W^-$	`g g > w+ w- [QCD]`	4.099 ± 0.010	$^{+26.5\%}_{-19.7\%}$	$^{+0.7\%}_{-1.0\%}$	[57]
*b.13	$gg \to W^+W^-g$	`g g > w+ w- g [QCD]`	1.837 ± 0.004	$^{+45.2\%}_{-29.0\%}$	$^{+0.9\%}_{-1.1\%}$	[58]

1507.00020

SM Tables

Process		Syntax	Cross section (pb)	$\Delta_{\hat{\mu}}$	Δ_{PDF}	Ref.
Triple bosons			$\sqrt{s} = 13$ TeV			
*c.1	$pp \to HHH$	`p p > h h h [QCD]`	$3.968 \pm 0.010 \cdot 10^{-5}$	$^{+31.8\%}_{-22.6\%}$	$^{+1.4\%}_{-1.4\%}$	[59]
†c.2	$gg \to HHZ$	`g g > h h z [QCD]`	$5.260 \pm 0.009 \cdot 10^{-5}$	$^{+31.2\%}_{-22.2\%}$	$^{+1.3\%}_{-1.3\%}$	[–]
†c.3	$gg \to HZZ$	`g g > h z z [QCD]`	$1.144 \pm 0.004 \cdot 10^{-4}$	$^{+31.1\%}_{-22.2\%}$	$^{+1.2\%}_{-1.3\%}$	[–]
†c.4	$gg \to HZ\gamma$	`g g > h z a [QCD]`	$6.190 \pm 0.020 \cdot 10^{-6}$	$^{+29.3\%}_{-21.2\%}$	$^{+1.0\%}_{-1.2\%}$	[–]
†c.5	$pp \to H\gamma\gamma$	`p p > h a a [QCD]`	$6.058 \pm 0.004 \cdot 10^{-6}$	$^{+30.3\%}_{-21.8\%}$	$^{+1.1\%}_{-1.3\%}$	[–]
*c.6	$gg \to HW^+W^-$	`g g > h w+ w- [QCD]`	$2.670 \pm 0.007 \cdot 10^{-4}$	$^{+31.0\%}_{-22.2\%}$	$^{+1.2\%}_{-1.3\%}$	[60]
†c.7	$gg \to ZZZ$	`g g > z z z [QCD]`	$6.964 \pm 0.009 \cdot 10^{-5}$	$^{+30.9\%}_{-22.1\%}$	$^{+1.2\%}_{-1.3\%}$	[–]
†c.8	$gg \to ZZ\gamma$	`g g > z z a [QCD]`	$3.454 \pm 0.010 \cdot 10^{-6}$	$^{+28.7\%}_{-20.9\%}$	$^{+0.9\%}_{-1.1\%}$	[–]
*c.9	$gg \to Z\gamma\gamma$	`g g > z a a [QCD]`	$3.079 \pm 0.005 \cdot 10^{-4}$	$^{+28.0\%}_{-20.9\%}$	$^{+0.7\%}_{-1.0\%}$	[61]
†c.10	$gg \to ZW^+W^-$	`g g > z w+ w- [QCD]`	$8.595 \pm 0.020 \cdot 10^{-3}$	$^{+26.9\%}_{-19.5\%}$	$^{+0.6\%}_{-0.6\%}$	[–]
†c.12	$gg \to \gamma W^+W^-$	`g g > a w+ w- [QCD]`	$1.822 \pm 0.005 \cdot 10^{-2}$	$^{+28.7\%}_{-20.9\%}$	$^{+0.9\%}_{-1.1\%}$	[–]

Process		Syntax	Partial width (GeV)	Ref.
Bosonic decays				
g.1	$H \to jj$	`h > j j [QCD]`	$1.740 \pm 0.0006 \cdot 10^{-4}$	[49]
*g.2	$H \to jjj$	`h > j j j [QCD]`	$3.413 \pm 0.010 \cdot 10^{-4}$	[49]
†g.3	$H \to jjjj$	`h > j j j j QED=1 [QCD]`	$1.654 \pm 0.004 \cdot 10^{-4}$	[–]
g.4	$H \to \gamma\gamma$	`h > a a [QED]`	$9.882 \pm 0.002 \cdot 10^{-6}$	[67]
†g.5	$H \to \gamma\gamma jj$	`h > a a j j [QCD]`	$7.448 \pm 0.030 \cdot 10^{-13}$	[–]
†g.7	$H \to \gamma\gamma\gamma\gamma$	`h > a a a a [QED]`	$1.546 \pm 0.006 \cdot 10^{-14}$	[–]
*g.8	$Z \to ggg$	`z > g g g [QCD]`	$3.986 \pm 0.010 \cdot 10^{-6}$	[34]

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SM Tables

Process		Syntax	Cross section (pb)	$\Delta_{\hat{\mu}}$	Δ_{PDF}	Ref.
Selected $2 \to 4$			$\sqrt{s} = 13$ TeV			
†d.1	$pp \to Hjjj$	`p p > h j j j QED=1 [QCD]`	2.519 ± 0.005	$^{+75.1\%}_{-39.8\%}$	$^{+0.6\%}_{-0.6\%}$	[62]
⋆d.2	$pp \to HHjj$	`p p > h h j j QED=1 [QCD]`	$1.085 \pm 0.002 \cdot 10^{-2}$	$^{+62.1\%}_{-35.8\%}$	$^{+1.2\%}_{-1.3\%}$	[63]
†d.3	$pp \to HHHj$	`p p > h h h j [QCD]`	$4.981 \pm 0.008 \cdot 10^{-5}$	$^{+46.3\%}_{-29.6\%}$	$^{+1.4\%}_{-1.4\%}$	[–]
†d.3	$pp \to HHHH$	`p p > h h h h [QCD]`	$1.080 \pm 0.003 \cdot 10^{-7}$	$^{+33.3\%}_{-23.4\%}$	$^{+1.7\%}_{-1.7\%}$	[–]
d.4	$gg \to e^+e^-\mu^+\mu^-$	`g g > e+ e- mu+ mu- [QCD]`	$2.022 \pm 0.003 \cdot 10^{-3}$	$^{+26.4\%}_{-19.4\%}$	$^{+0.7\%}_{-1.1\%}$	[64]
†d.5	$pp \to HZ\gamma j$	`g g > h z a g [QCD]`	$4.950 \pm 0.008 \cdot 10^{-6}$	$^{+45.8\%}_{-29.3\%}$	$^{+1.2\%}_{-1.3\%}$	[–]
Non-hadronic processes			$\sqrt{s} = 500$ GeV, no PDF			
⋆e.1	$e^+e^- \to ggg$	`e+ e- > g g g [QED]`	$2.526 \pm 0.004 \cdot 10^{-6}$	$^{+31.2\%}_{-22.0\%}$		[65]
†e.2	$e^+e^- \to HH$	`e+ e- > h h [QED]`	$1.567 \pm 0.003 \cdot 10^{-5}$			[–]
†e.3	$e^+e^- \to HHgg$	`e+ e- > h h g g [QED]`	$6.629 \pm 0.010 \cdot 10^{-11}$	$^{+19.2\%}_{-14.8\%}$		[–]
⋆e.4	$\gamma\gamma \to HH$	`a a > h h [QED]`	$3.198 \pm 0.005 \cdot 10^{-4}$			[66]
Miscellaneous			$\sqrt{s} = 13$ TeV			
†f.1	$pp \to tt$	`p p > t t [QED]`	$4.045 \pm 0.007 \cdot 10^{-15}$	$^{+0.2\%}_{-0.8\%}$	$^{+0.9\%}_{-1.0\%}$	[–]

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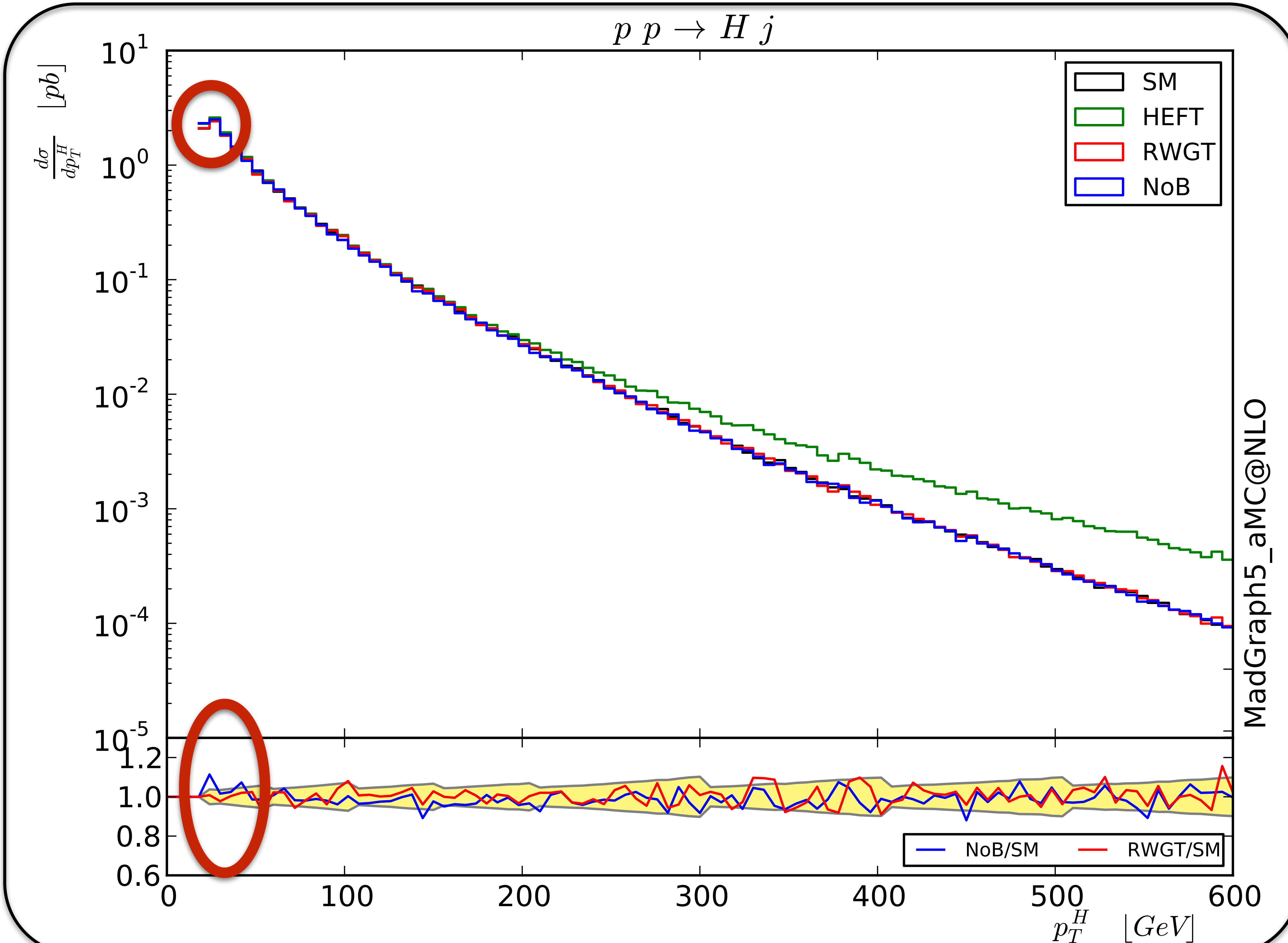
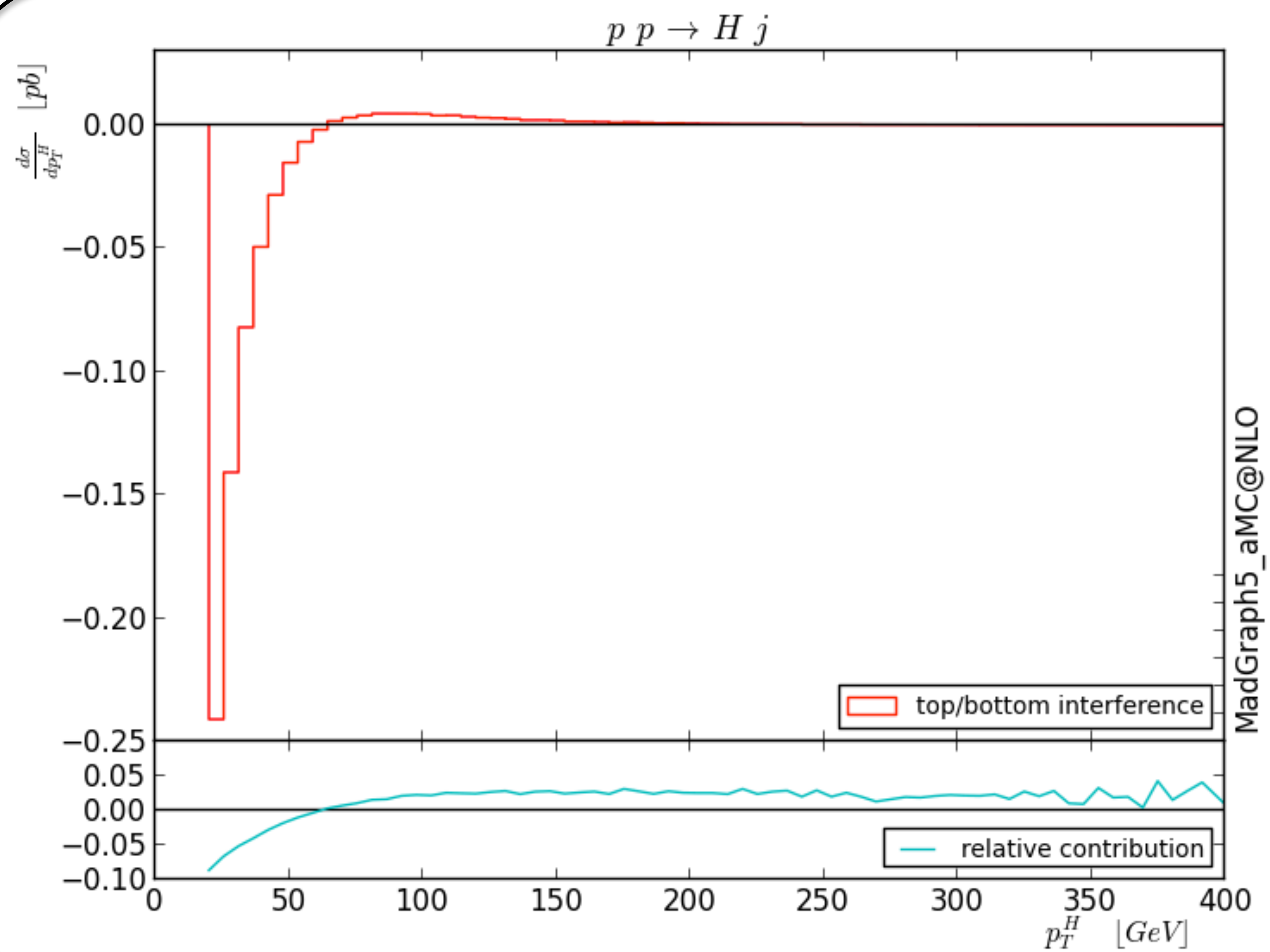
Validation p p > h j

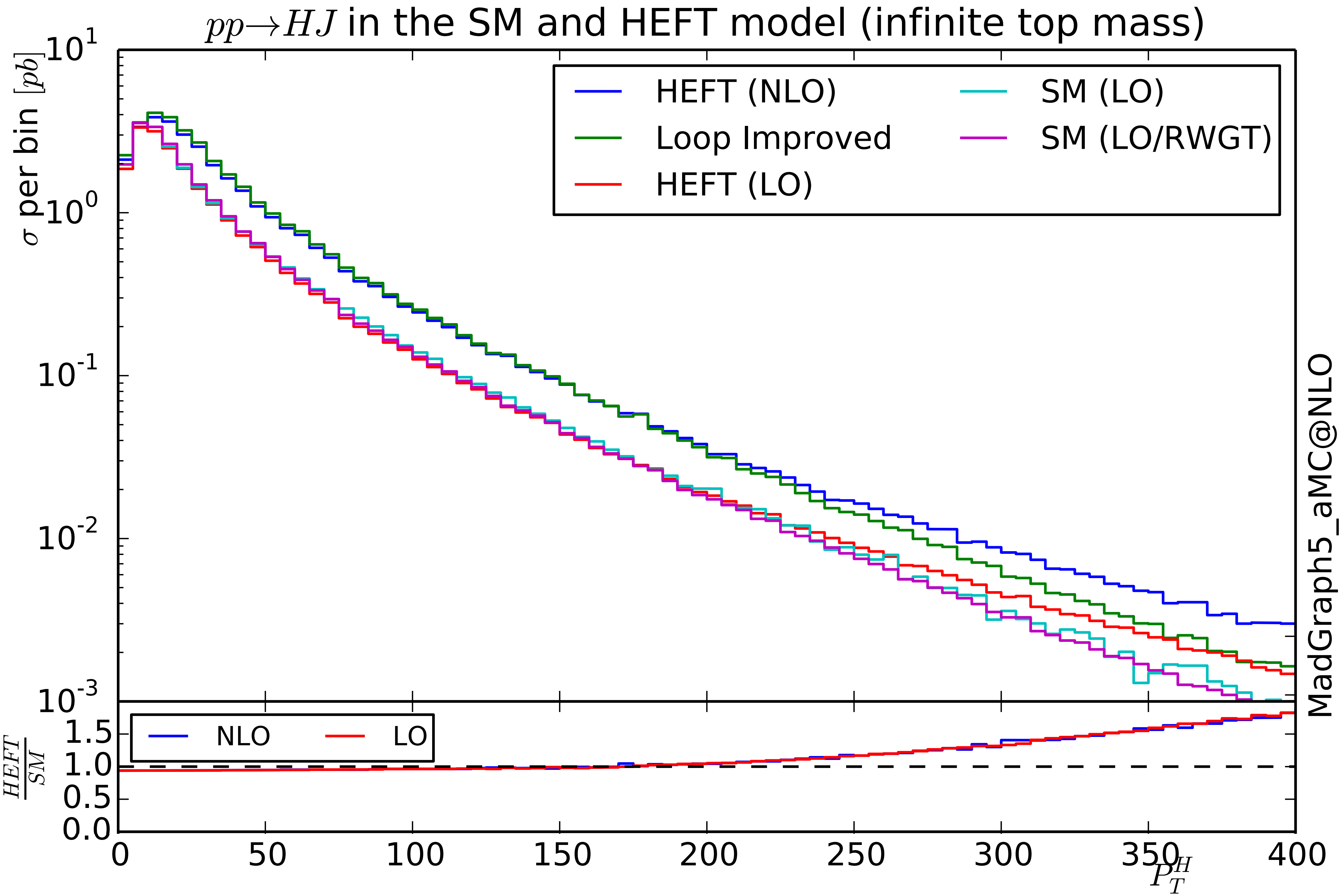


Validation p p > h j



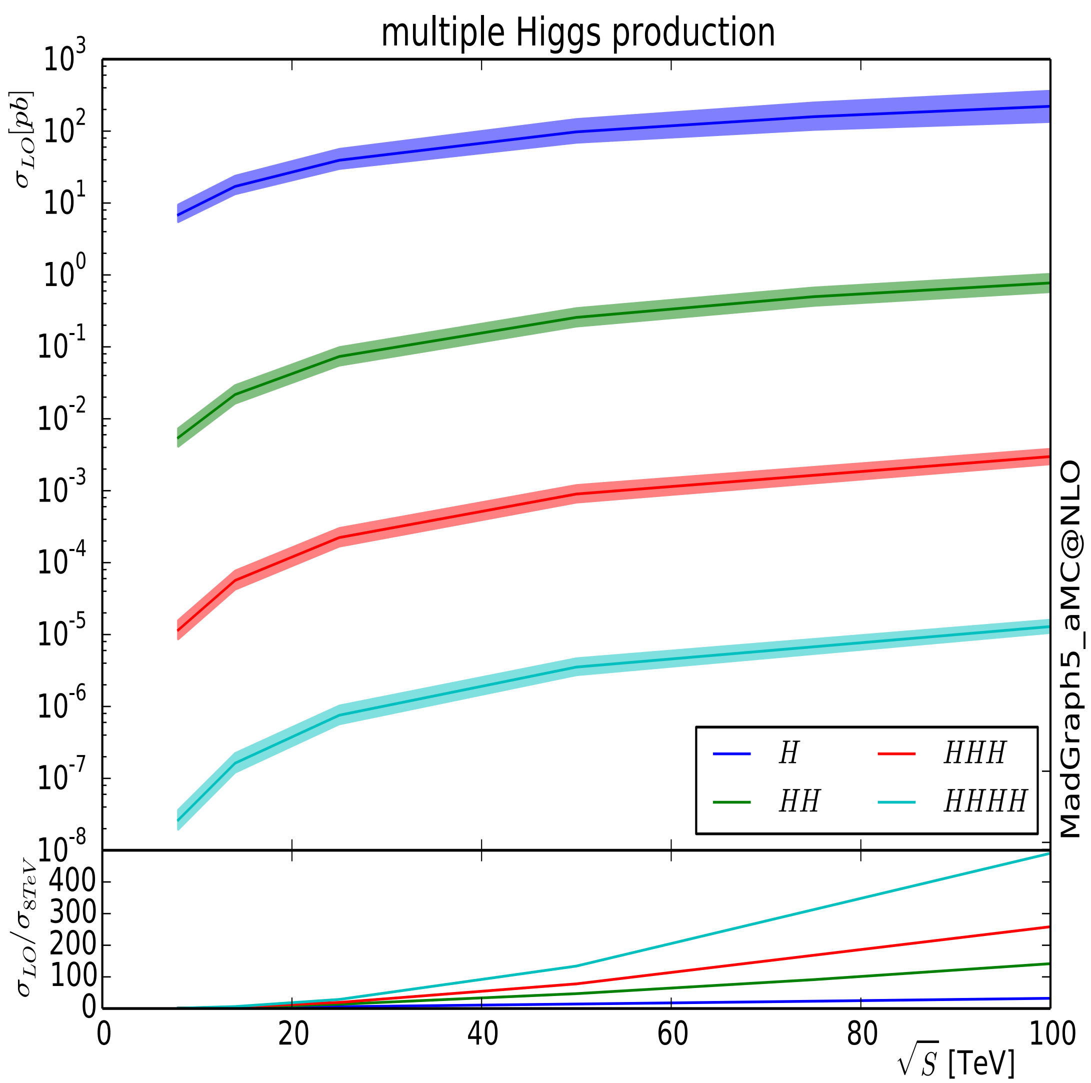
- b effect only important at low pt
- at large pt, this is just a re-scaling

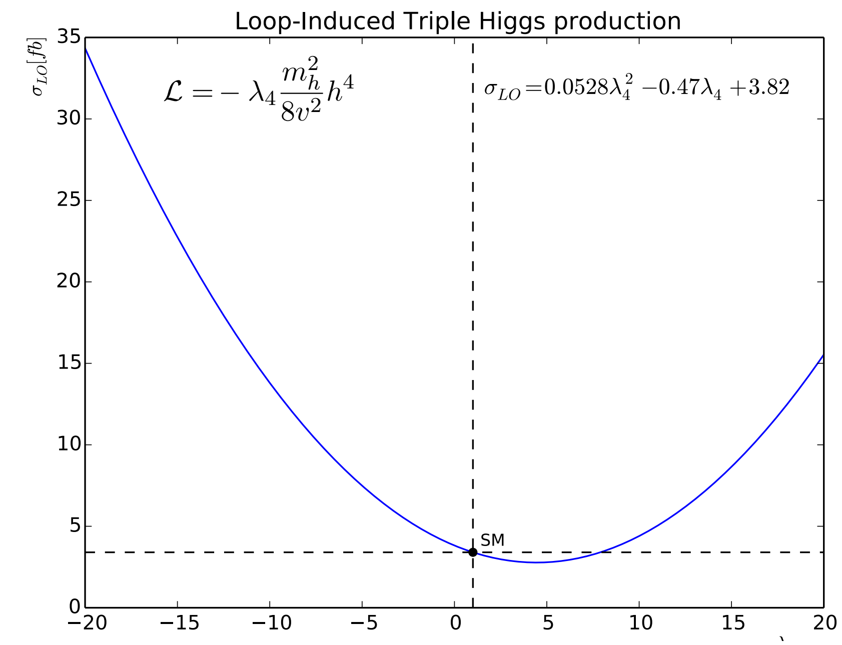
Higgs + Jet

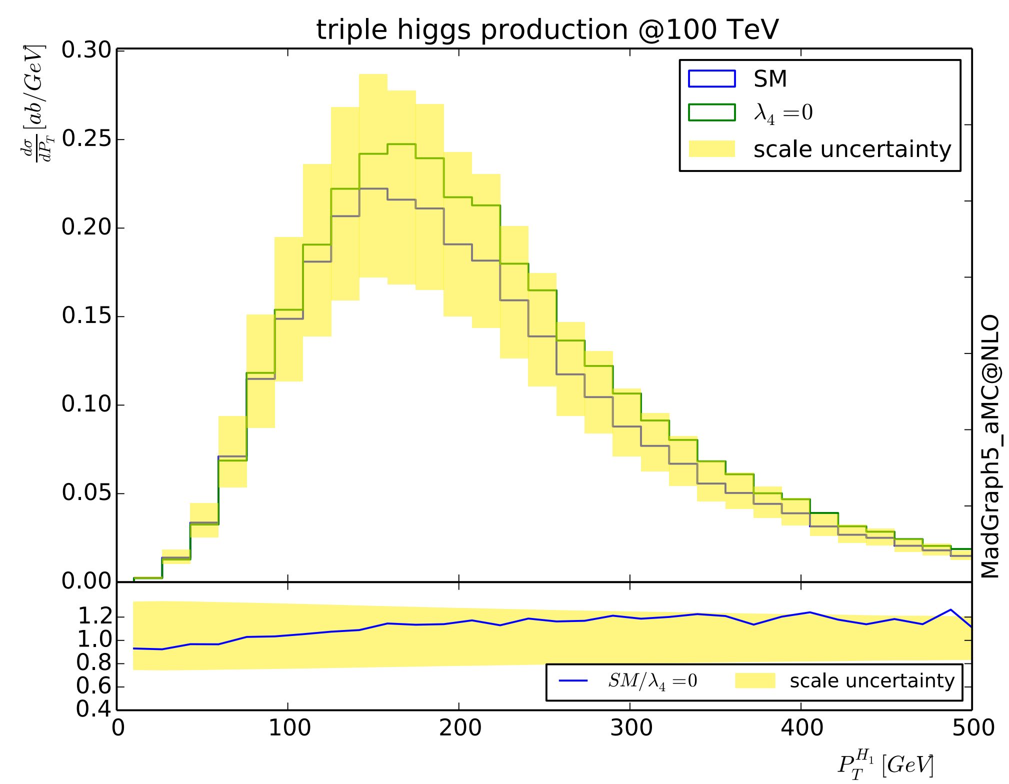


$pp \to HJ$ in the SM and HEFT model (infinite top mass)
σ per bin $[pb]$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
HEFT (NLO)
Loop Improved
HEFT (LO)
SM (LO)
SM (LO/RWGT)
MadGraph5_aMC@NLO
$\frac{HEFT}{SM}$
1.5
1.0
0.5
0.0
NLO
LO
0
50
100
150
200
250
300
350
400
P_T^H

Multi-Higgs







$$\lambda_4 \in [\sim -4, \sim +16]$$

1508.06524/1510.07697/1510.04013

Large Multiplicities

h > N h

Motivation

- Amplitude for off-shell Higgs to N on-shell Higgs is know to break perturbativity
- form of the amplitude know analytically
  - ➡ recently fitted thanks to MG5aMC
- What is the impact for LHC and future collider?
  - ➡ should we care about this?

Scaling

No PDF	$\sigma_{gg\to hh}$	$\sigma_{gg\to hhh}$	$\sigma_{gg\to hhhh}$
Triangles	$y_t^2 \frac{m_t^2 M_h^2}{s^3} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^4}{v^4}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^6}{v^6}$
Boxes	$y_t^4 \frac{1}{s}$	$y_t^4 \frac{1}{s} \frac{M_h^2}{v^2}$	$y_t^4 \frac{1}{s} \frac{M_h^4}{v^4}$
Pentagons	–	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right)$	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$
Hexagons	–	–	$y_t^8 \frac{1}{s}$

MadGraph5_aMC@NLO

- At High-Energy only even loop contributes

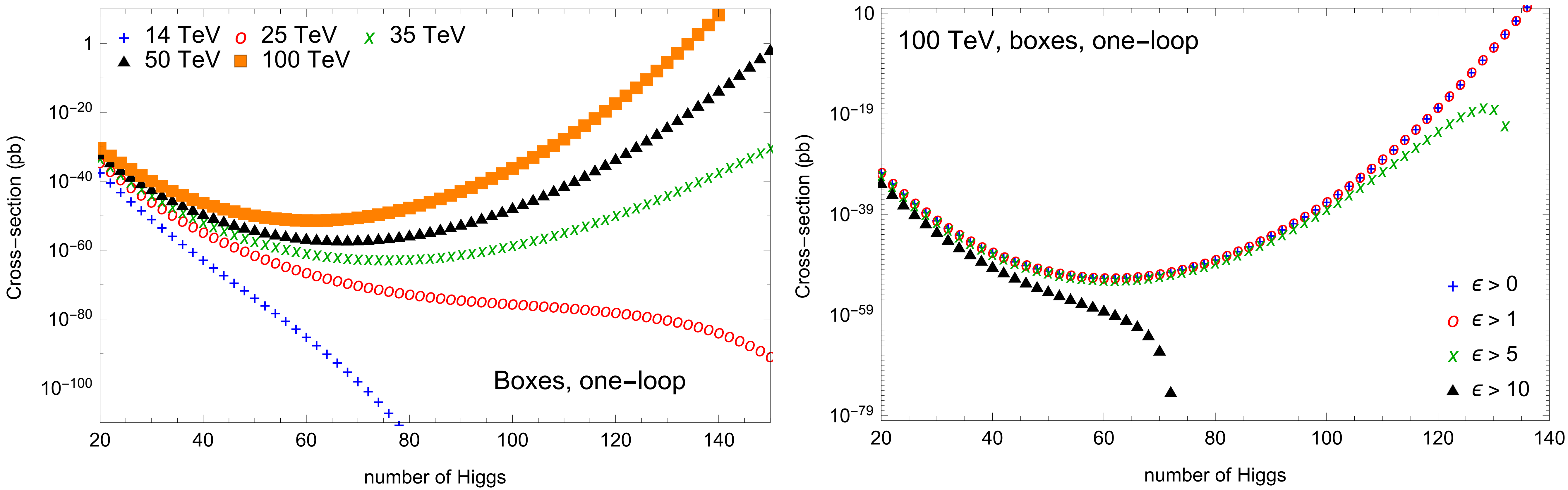
- For high Higgs multiplicity, the boxes is expected to dominate (all even loop are expected to be of the same order for threshold)

$$\varepsilon := \frac{\sqrt{s} - nM_h}{nM_h}.$$

$$\mathcal{A}^k_{gg>n\times h} \approx \left(\frac{1}{1+\epsilon}\right)^{k-2}$$

1605.06372

Multi-Higgs



- Approximation using semi-classical solution
- Perturbative theory breaks down
  - ➡ Not for 14 TeV
  - ➡ For 50/100 TeV regime

Conclusion

- Loop-induced computation
  - Fully available for any NLO model
  - All SM cases have been tested
- Re-weighting method
  - Available both at LO and NLO
- High multiplicity Higgs production
  - At 100 TeV collider, we should be sensitive to a breaking of unitarity in

$$h^* \to n \times h$$