
WILL EXPLAIN
HIGGS-BOSON
PARTICLE
FOR
FOOD
H/M

# BSM Higgs Lecture

*Sven Heinemeyer, IFT (CSIC, Madrid)*

Daejeon, 09-10/2025

1. The SM Higgs & Motivation for BSM Higgs

2. BSM Higgs sectors: RxSM, 2HDM, N2HDM, MSSM . . .

3. BSM Higgs sectors: collider phenomenology & more

4. Anomalies and possible explanations

# BSM Higgs Lecture

# BSM Higgs sectors: RxSM, 2HDM, N2HDM, MSSM, . . .

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1. BSM Higgs Introduction

2. Higgs Singlet and Doublet Extensions

3. The Higgs Sector of the (N)MSSM

4. How to distinguish BSM Higgs sectors experimentally

# 1. BSM Higgs Introduction

## The main questions:

- What are the couplings of this particle to other known elementary particles? Is its coupling to each particle proportional to that particle's mass, as required by the BEH mechanism?

- What are the mass, total width, spin and $\mathcal{CP}$ properties of this particle? Are there additional sources of $\mathcal{CP}$ violation in the Higgs sector?

- What is the value of the particle's self-coupling? Is this consistent with the expectation from the symmetry-breaking potential?

- Is this particle a single, fundamental scalar as in the SM, or is it part of a larger structure? Is it part of a model with additional scalar singlets/doublets/...?
  Or, could it be a composite state, bound by new interactions?

- Does this particle couple to new particles with no other couplings to the SM ("Higgs portal")? Is the particle mixed with new scalars of exotic origin, for example, the radion of extra-dimensional models?

# Models with extended Higgs sectors:

## Models with extended Higgs sectors:

**Q: Do you know any BSM Higgs models? :-)**

## Models with extended Higgs sectors:

1. SM with addional Higgs singlet (RxSM, . . . )
2. Two Higgs Doublet Model (2HDM): type I, II, III, IV
3. 2HDM with singlet extensions: N2HDM, S2HDM, 2HDMS, . . .
4. Minimal Supersymmetric Standard Model (MSSM)
5. MSSM with one extra singlet (NMSSM)
6. MSSM with more extra singlets ($\mu\nu$SSM)
7. SM/MSSM with Higgs triplets (GM, . . . )
8. . . .

## Extended Higgs sectors

Compatibility with the experimental results requires:

- A SM-like Higgs at $\sim 125$ GeV
- Properties of the other Higgs bosons (masses, couplings,...) have to be such that they are in agreement with the present bounds

The "sum rule": $\sum_i g^2_{h_i VV} = g^2_{H_\text{SM} VV}$ – and we know $g^2_{h_{125} VV} \sim g^2_{H_\text{SM} VV}$

$\Rightarrow$ not much room left for BSM Higgs couplings to gauge bosons

Sum rule "violated" only by triplets or higher representations . . .

# Model Motivation?

## Model Motivation? $\Rightarrow$ experimental data as guidance!

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Some “recent” measurements:

– top quark mass
– Higgs boson mass
– Higgs boson “couplings”
– Dark Matter (properties)

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Simple SUSY models predicted correctly:

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– Higgs boson "couplings"
– Dark Matter (properties)

**⇒ good motivation to look at SUSY! :-)**

But we cover everything (RxSM, 2HDM, N2HDM, . . . )

# 2. Higgs Singlet and Doublet Extensions

Models with extended Higgs sectors:

1. SM with addional Higgs singlet (RxSM, . . . ) ⇐ focus
2. Two Higgs Doublet Model (2HDM): type I, II, III, IV ⇐ focus
3. 2HDM with singlet extensions: N2HDM, S2HDM, 2HDMS, . . . ⇐ focus
4. Minimal Supersymmetric Standard Model (MSSM) ⇐ focus
5. MSSM with one extra singlet (NMSSM)
6. MSSM with more extra singlets ($\mu\nu$SSM)
7. SM/MSSM with Higgs triplets (GM, . . . )
8. . . .

## The simplest showcase: has all the interesting features

### SM plus one real singlet (RxSM):

Fields:

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \rho + i\eta) \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_S + \rho_S)$$

Lagrangian:

$$\mathcal{L}_{\mathsf{HxSM}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) + (\partial^\mu S)(\partial_\mu S) - V(\Phi, S)$$

Potential: (with an additional $Z_2$ symmetry)

$$\begin{aligned} V(\Phi, S) &= -m^2 \Phi^\dagger \Phi - \mu^2 S^2 + \left(\Phi^\dagger \Phi, S^2\right) \begin{pmatrix} \lambda_1 & \frac{1}{2}\lambda_3 \\ \frac{1}{2}\lambda_3 & \lambda_2 \end{pmatrix} \begin{pmatrix} \Phi^\dagger \Phi \\ S^2 \end{pmatrix} \\ &= -m^2 \Phi^\dagger \Phi - \mu^2 S^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 S^4 + \lambda_3 (\Phi^\dagger \Phi) S^2 \end{aligned}$$

Original basis: $\lambda_1$, $\lambda_2$, $\lambda_3$, $v$, $v_S$

Potential is bounded from below for

$$
\begin{aligned}
4\lambda_1\lambda_2 - \lambda_3^2 &> 0 \\
\lambda_1, \lambda_2 &> 0
\end{aligned}
$$

Unitary gauge:

$$
\begin{aligned}
\Phi &= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + \rho) \end{pmatrix} \\
S &= \frac{1}{\sqrt{2}}(v_S + \rho_S)
\end{aligned}
$$

Expansion around the minimum:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{mass}} &= (\rho, \rho_S) \quad \mathcal{M}^2 \quad \begin{pmatrix} \rho \\ \rho_S \end{pmatrix} \\
&= (\rho, \rho_S) \begin{pmatrix} 2\lambda_1 v^2 & \lambda_3 v v_S \\ \lambda_3 v v_S & 2\lambda_2 v_S^2 \end{pmatrix} \begin{pmatrix} \rho \\ \rho_S \end{pmatrix}
\end{aligned}
$$

The matrix is diagonalized with the angle $\alpha$:

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho \\ \rho_S \end{pmatrix}$$

Mass eigenvalues:

$$m^2_{h,H} = \lambda_1 v^2 + \lambda_2 v_S^2 \mp \sqrt{(\lambda_1 v^2 - \lambda_2 v_S^2)^2 + (\lambda_3 v v_S)^2}$$

One eigenvalue $\sim$ 125 GeV, the other one above or below

$\alpha$ is given by

$$\sin 2\alpha = \frac{\lambda_3 v v_S}{\sqrt{(\lambda_1 v^2 - \lambda_2 v_S^2)^2 + (\lambda_3 v v_S)^2}}$$

$$\cos 2\alpha = \frac{\lambda_2 v_S^2 - \lambda_1 v^2}{\sqrt{(\lambda_1 v^2 - \lambda_2 v_S^2)^2 + (\lambda_3 v v_S)^2}}$$

Physical basis: $m_h$, $m_H$, $\alpha$, $v$, $\tan\beta := v/v_S$ with $v \equiv v_{\mathrm{SM}} \sim 246$ GeV

Express original basis in terms of physical basis:

$$\lambda_1 = \frac{1}{2v^2}\left(m_h^2 \cos^2\alpha + m_H^2 \sin^2\alpha\right)$$
$$\lambda_2 = \frac{1}{2v_S^2}\left(m_h^2 \sin^2\alpha + m_H^2 \cos^2\alpha\right)$$
$$\lambda_3 = \frac{1}{2vv_S}\left(m_H^2 - m_h^2\right)\sin 2\alpha$$

Only the SM doublet couples to SM particles (with doublet component $\rho$)

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \rho \\ \rho_S \end{pmatrix}$$

$\Rightarrow$ All Higgs couplings are suppressed w.r.t. the SM

$$g_{h-\mathrm{SM}-\mathrm{SM}} = \cos\alpha\, g_{H_{\mathrm{SM}}-\mathrm{SM}-\mathrm{SM}}$$
$$g_{H-\mathrm{SM}-\mathrm{SM}} = \sin\alpha\, g_{H_{\mathrm{SM}}-\mathrm{SM}-\mathrm{SM}}$$

SM limit:

– $m_h \sim 125$ GeV: $\alpha \to 0$, $\cos\alpha \to 1$

– $m_H \sim 125$ GeV: $\alpha \to \pi/2$, $\sin\alpha \to 1$

Sum rule:

$$g^2_{hVV} + g^2_{HVV} = (\cos^2\alpha + \sin^2\alpha) g^2_{H_{\rm SM}VV} \equiv g^2_{H_{\rm SM}VV}$$

Decay widths:

$$\Gamma(h \to \text{SM}) = \cos^2\alpha\,\Gamma(H_{\rm SM} \to \text{SM}) \quad \text{with} \quad m_{H_{\rm SM}} \to m_h$$
$$\Gamma(H \to \text{SM}) = \sin^2\alpha\,\Gamma(H_{\rm SM} \to \text{SM}) \quad \text{with} \quad m_{H_{\rm SM}} \to m_H$$

[These mass setting are implicitely assumed in the following.]

New possible decay mode (for $m_H > 2m_h$)

$$\Gamma(H \to hh) = \frac{|\mu'|^2}{8\,\pi\,m_H}\sqrt{1 - \frac{4m_h^2}{m_H^2}} \quad \text{with} \quad \mu' = \mu'(m_h, m_H, \alpha, v, \tan\beta)$$

Total decay widths:

$$\Gamma_{h,\text{tot}} = \cos^2\alpha\,\Gamma_{H_{\text{SM}},\text{tot}}$$

$$\Gamma_{H,\text{tot}} = \sin^2\alpha\,\Gamma_{H_{\text{SM}},\text{tot}} + \Gamma(H \to hh)$$

Branching ratios:

$$\text{BR}(h \to \text{SM}) = \frac{\Gamma(h \to \text{SM})}{\Gamma_{h,\text{tot}}} = \text{BR}(H_{\text{SM}} \to \text{SM})$$

$$\text{BR}(H \to \text{SM}) = \frac{\Gamma(H \to \text{SM})}{\Gamma_{H,\text{tot}}} = \sin^2\alpha\,\frac{\Gamma(H_{\text{SM}} \to \text{SM})}{\Gamma_{H,\text{tot}}} \, (= \text{BR}(H_{\text{SM}} \to \text{SM}) \text{ for}$$

$$\text{BR}(H \to hh) = \frac{\Gamma(H \to hh)}{\Gamma_{H,\text{tot}}} \qquad m_H < 2m_h)$$

# Two Higgs Doublet Model (2HDM):

Fields:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix}$$

Potential:

$$\begin{aligned} V \;=\; & m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + h.c.) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\ & + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + h.c.] \end{aligned}$$

Physical states: $h$, $H$, ($\mathcal{CP}$-even), $A$ ($\mathcal{CP}$-odd), $H^\pm$ (charged)

"Physical" input parameters:

$$c_{\beta-\alpha}\,, \quad \tan\beta\,, \quad v\,, \quad M_h, \quad M_H\,, \quad M_A\,, \quad M_{H^\pm}\,, \quad m_{12}^2$$

Assumption (for now): $h \sim h_{125}$

$Z_2$ symmetry to avoid FCNC:

$$\Phi_1 \to \Phi_1\,,\ \Phi_2 \to -\Phi_2\,,\ \Phi_S \to \Phi_S$$

Extension of the $Z_2$ symmetry to fermions determines four types:

| | $u$-type | $d$-type | leptons | |
|---|---|---|---|---|
| type I | $\Phi_2$ | $\Phi_2$ | $\Phi_2$ | |
| type II | $\Phi_2$ | $\Phi_1$ | $\Phi_1$ | $\to$ MSSM type |
| type III (lepton-specific) | $\Phi_2$ | $\Phi_2$ | $\Phi_1$ | |
| type IV (flipped) | $\Phi_2$ | $\Phi_1$ | $\Phi_2$ | |

Sum rule (with $h$ SM-like): $\sin(\beta - \alpha) \approx 1$, $\cos(\beta - \alpha) \approx 0$

Unitarity/perturbativity and EWPO: $\Rightarrow M_A \sim M_H \sim M_{H^\pm}$

# Higgs-boson couplings in the 2HDM

$$\mathcal{L} = - \sum_{f=u,d,l} \frac{m_f}{v} \left[ \xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H + i \xi_A^f \bar{f} \gamma_5 f A \right]$$

$$+ \sum_{h_i=h,H,A} \left[ g M_W \xi_{h_i}^W W_\mu W^\mu h_i + \frac{1}{2} g M_Z \xi_{h_i}^Z Z_\mu Z^\mu h_i \right],$$

| | I | II | III/Y/Flipped | IV/X/Lepton-Specific |
|---|---|---|---|---|
| $u$-quarks | $\Phi_2$ | $\Phi_2$ | $\Phi_2$ | $\Phi_2$ |
| $d$-quarks | $\Phi_2$ | $\Phi_1$ | $\Phi_1$ | $\Phi_2$ |
| leptons | $\Phi_2$ | $\Phi_1$ | $\Phi_2$ | $\Phi_1$ |

Table 1: Couplings to fermions for the four 2HDM types.

| | I | II | III/Y/Flipped | IV/X/Lepton-Specific |
|---|---|---|---|---|
| $\xi_h^u$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ |
| $\xi_h^d$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ | $s_{\beta-\alpha} - c_{\beta-\alpha} \tan\beta$ | $s_{\beta-\alpha} - c_{\beta-\alpha} \tan\beta$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ |
| $\xi_h^l$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ | $s_{\beta-\alpha} - c_{\beta-\alpha} \tan\beta$ | $s_{\beta-\alpha} + c_{\beta-\alpha} \cot\beta$ | $s_{\beta-\alpha} - c_{\beta-\alpha} \tan\beta$ |
| $\xi_H^u$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ |
| $\xi_H^d$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ | $c_{\beta-\alpha} + s_{\beta-\alpha} \tan\beta$ | $c_{\beta-\alpha} + s_{\beta-\alpha} \tan\beta$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ |
| $\xi_H^l$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ | $c_{\beta-\alpha} + s_{\beta-\alpha} \tan\beta$ | $c_{\beta-\alpha} - s_{\beta-\alpha} \cot\beta$ | $c_{\beta-\alpha} + s_{\beta-\alpha} \tan\beta$ |
| $\xi_A^u$ | $-\cot\beta$ | $-\cot\beta$ | $-\cot\beta$ | $-\cot\beta$ |
| $\xi_A^d$ | $\cot\beta$ | $-\tan\beta$ | $-\tan\beta$ | $\cot\beta$ |
| $\xi_A^l$ | $\cot\beta$ | $-\tan\beta$ | $\cot\beta$ | $-\tan\beta$ |

Table 2: Couplings to fermions for the four 2HDM types.

## Next-Two Higgs Doublet Model (N2HDM): $\rightarrow$ (nearly) NMSSM type

**Fields:**

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \; \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix}, \; \Phi_S = v_S + \rho_S$$

**Potential:**

$$\begin{aligned} V \;=\; & m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + h.c.) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\ & +\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + h.c.] \\ & +\frac{1}{2}m_S^2\Phi_S^2 + \frac{\lambda_6}{8}\Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger\Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger\Phi_2)\Phi_S^2 \end{aligned}$$

$Z_2$ symmetry: $\Phi_1 \rightarrow \Phi_1$ , $\Phi_2 \rightarrow -\Phi_2$ , $\Phi_S \rightarrow \Phi_S$

$Z_2'$ symmetry: $\Phi_1 \rightarrow \Phi_1$ , $\Phi_2 \rightarrow \Phi_2$ , $\Phi_S \rightarrow -\Phi_S$ (broken by $v_S \Rightarrow$ no DM)

Physical states: $h_1$, $h_2$, $h_3$ ($\mathcal{CP}$-even), $A$ ($\mathcal{CP}$-odd), $H^\pm$ (charged)

## Extension of the $Z_2$ symmetry to fermions determines four types:

| | $u$-type | $d$-type | leptons |
|---|---|---|---|
| type I | $\Phi_2$ | $\Phi_2$ | $\Phi_2$ |
| type II | $\Phi_2$ | $\Phi_1$ | $\Phi_1$ |
| type III (lepton-specific) | $\Phi_2$ | $\Phi_2$ | $\Phi_1$ |
| type IV (flipped) | $\Phi_2$ | $\Phi_1$ | $\Phi_2$ |

$\Rightarrow$ exactly as in 2HDM

## Three neutral $\mathcal{CP}$-even Higgses:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_S \end{pmatrix}, \quad R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -(c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & -(c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}$$

Coupling to massive gauge bosons: (identical for all four types)

| $c_{h_i VV} = c_\beta R_{i1} + s_\beta R_{i2}$ | |
|---|---|
| $h_1$ | $c_{\alpha_2} c_{\beta-\alpha_1}$ |
| $h_2$ | $-c_{\beta-\alpha_1} s_{\alpha_2} s_{\alpha_3} + c_{\alpha_3} s_{\beta-\alpha_1}$ |
| $h_3$ | $-c_{\alpha_3} c_{\beta-\alpha_1} s_{\alpha_2} - s_{\alpha_3} s_{\beta-\alpha_1}$ |

Coupling to fermions: (same pattern as in 2HDM)

| | $u$-type ($c_{h_i tt}$) | $d$-type ($c_{h_i bb}$) | leptons ($c_{h_i \tau\tau}$) |
|---|---|---|---|
| type I | $\frac{R_{i2}}{s_\beta}$ | $\frac{R_{i2}}{s_\beta}$ | $\frac{R_{i2}}{s_\beta}$ |
| type II | $\frac{R_{i2}}{s_\beta}$ | $\frac{R_{i1}}{c_\beta}$ | $\frac{R_{i1}}{c_\beta}$ |
| type III (lepton-specific) | $\frac{R_{i2}}{s_\beta}$ | $\frac{R_{i2}}{s_\beta}$ | $\frac{R_{i1}}{c_\beta}$ |
| type IV (flipped) | $\frac{R_{i2}}{s_\beta}$ | $\frac{R_{i1}}{c_\beta}$ | $\frac{R_{i2}}{s_\beta}$ |

“Physical” input parameters:

$$\alpha_{1,2,3}\,, \quad \tan\beta\,, \quad v\,, \quad v_S\,, \quad m_{h_{1,2,3}}\,, \quad m_A\,, \quad M_{H^\pm}\,, \quad m_{12}^2$$

# A model with triplets: Georgi-Machacek Model (GM)

[*Material taken from S. Chopra*]

## Field content:

In the GM model, the SM Higgs doublet $\phi$ with a complex triplet $\chi$ (hypercharge $Y = 1$) and a real triplet $\xi$ ($Y = 0$). These fields are represented as:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}. \tag{2.33}$$

## Neutral fields:

$$\phi^0 = \frac{1}{\sqrt{2}}(v_\phi + \phi_r^0 + i\phi_i^0), \quad \chi^0 = v_\chi + \frac{1}{\sqrt{2}}(\chi_r^0 + i\chi_i^0), \quad \xi^0 = v_\chi + \xi_r^0.$$

## Two vevs:

$$v^2 = v_\phi^2 + 8v_\chi^2 = \frac{4M_W^2}{g^2} \approx (246\,\mathrm{GeV})^2.$$

New mixing angle:

$$\cos\theta_H = \frac{v_\phi}{v}, \qquad \sin\theta_H = \frac{\sqrt{8}\,v_\chi}{v}.$$

Three neutral $\mathcal{CP}$-even Higgs bosons:

$$h^0 = \cos\alpha\,\phi^{0,r} - \sin\alpha\left(\frac{1}{\sqrt{3}}\chi^{0,r} + \sqrt{\frac{2}{3}}\,\xi^{0,r}\right),$$

$$H^0 = \sin\alpha\,\phi^{0,r} + \cos\alpha\left(\frac{1}{\sqrt{3}}\chi^{0,r} + \sqrt{\frac{2}{3}}\,\xi^{0,r}\right),$$

$$H_5^0 = \sqrt{\frac{2}{3}}\,\xi^{0,r} - \frac{1}{\sqrt{3}}\,\chi^{0,r},$$

Coupling modifiers of neutral Higgs bosons:

$$\kappa_V^{h^0} = \cos\theta_H \cos\alpha - \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \sin\alpha\,,$$

$$\kappa_V^{H^0} = \cos\theta_H \sin\alpha + \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \cos\alpha\,,$$

$$\kappa_{WW}^{H_5^0} = \frac{1}{\sqrt{3}} \sin\theta_H\,, \qquad \kappa_{ZZ}^{H_5^0} = -\frac{2}{\sqrt{3}} \sin\theta_H\,.$$

Difference w.r.t. 2HDM:

$$\left(\kappa_V^{h^0}\right)^2 + \left(\kappa_V^{H^0}\right)^2 = \cos^2\theta_H + \frac{8}{3}\sin^2\theta_H \geq 1$$

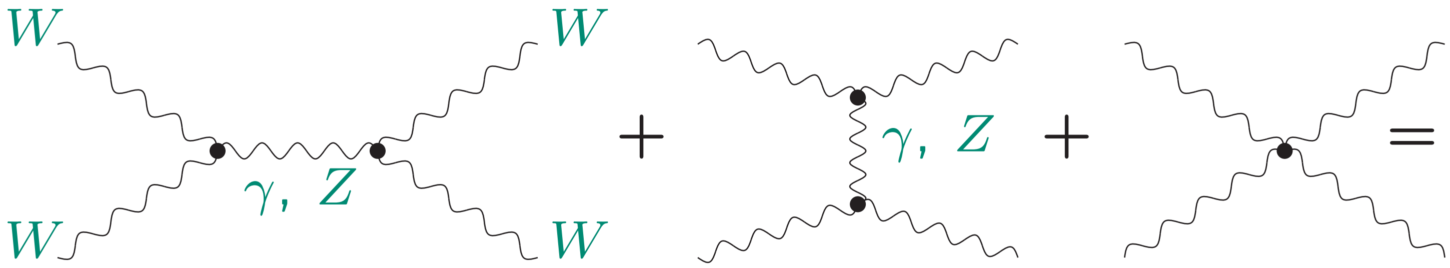
$\Rightarrow$ couplings larger than in SM possible!

# Sum rules - preservation of unitarity

[*Material taken from S. Chopra*]

## Scattering of longitudinal $W$ bosons: $W_L W_L \to W_L W_L$

$\mathcal{M}_V =$

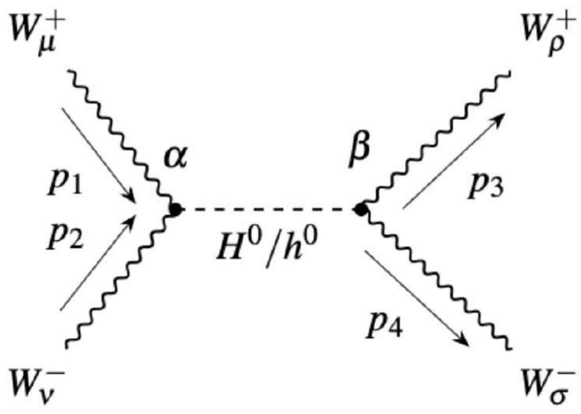



$= \frac{i}{v^2}(s+t) + \mathcal{O}(1)$

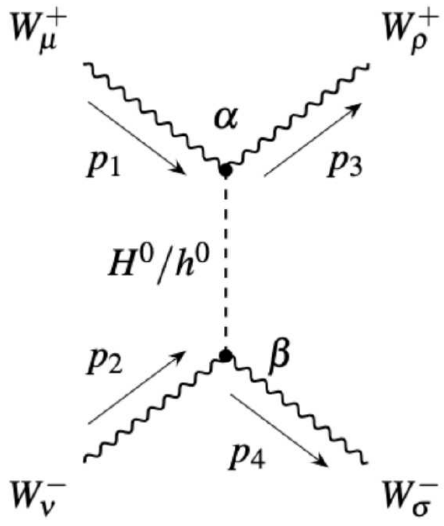
for $E \to \infty$

## Case I: 2HDM

## Additional Higgs contribution in the 2HDM:




s-channel




t-channel

## Additional Higgs contribution in the 2HDM:

$$i\mathcal{M} = -i\frac{g^2}{4m_W^2}\left[\left(\kappa_{WW}^{h^0}\right)^2\left(\frac{s^2}{s-m_{h^0}^2}+\frac{t^2}{t-m_{h^0}^2}\right)+\left(\kappa_{WW}^{H^0}\right)^2\left(\frac{s^2}{s-m_{H^0}^2}+\frac{t^2}{t-m_{H^0}^2}\right)\right],$$

$$\simeq -\frac{i}{v^2}\left[\left(\left(\kappa_{WW}^{h^0}\right)^2+\left(\kappa_{WW}^{H^0}\right)^2\right)(s+t)+2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2+2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2+\cdots\right].$$

Do the $(s+t)$ contribution cancel?

## Coupling modifiers in the 2HDM:

$$\kappa_{WW}^{h^0} = \sin(\beta-\alpha), \qquad \kappa_{WW}^{H^0} = \cos(\beta-\alpha),$$

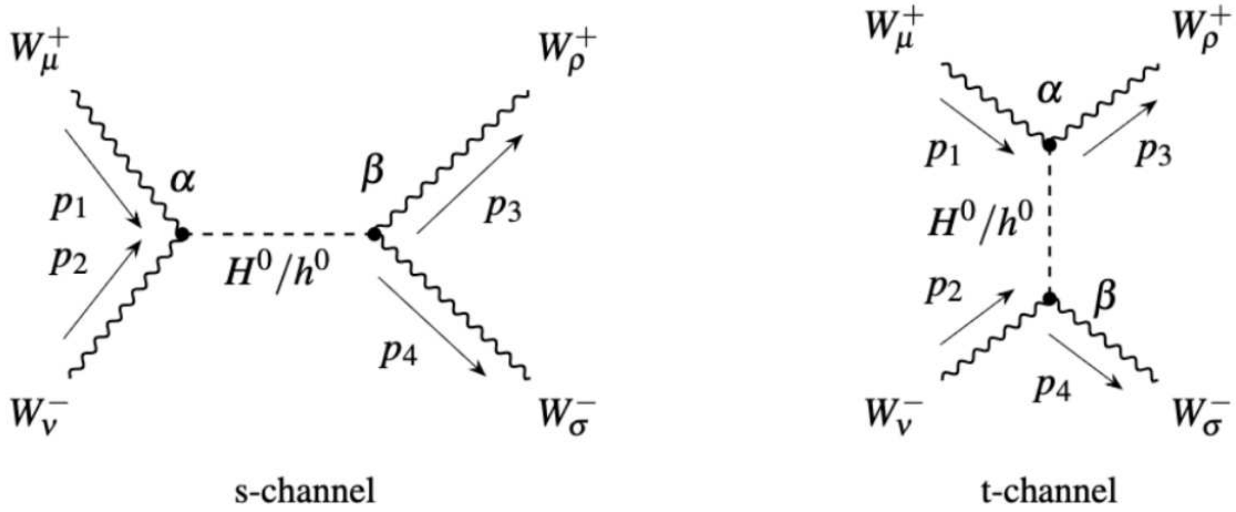
$\Rightarrow$ the $(s+t)$ contributions cancel:

$$\left(\kappa_{VV}^{h^0}\right)^2+\left(\kappa_{VV}^{H^0}\right)^2 = 1, \quad V = W, Z \text{ or}$$

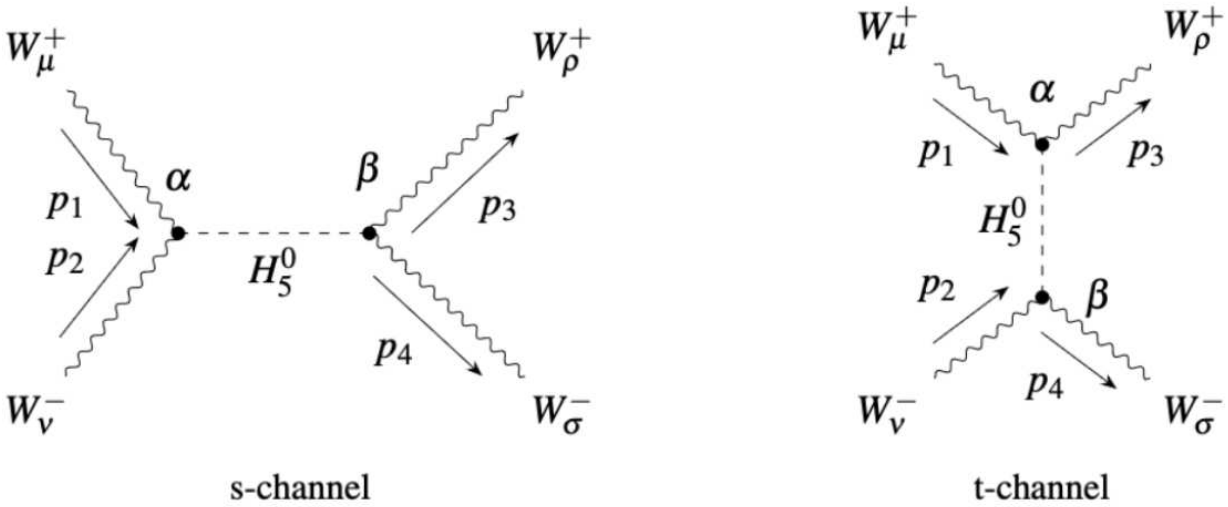
$$\sum_i g_{h_i VV}^2 = g_{H_{\rm SM}VV}^2, \quad h_i = h^0, H^0$$

# Case II: GM

## Additional neutral Higgs contribution in the GM:




s and t-channel $h^0/H^0$ diagrams **(CP even states)**



## Additional neutral Higgs contribution in the GM:

$$\mathcal{M} \sim -\frac{i}{v^2}\left[\left(\kappa^{h}_{WW}\right)^2 + \left(\kappa^{H}_{WW}\right)^2 + \left(\kappa^{H_5}_{WW}\right)^2\right](s+t)$$

Coupling modifiers of neutral Higgs bosons:

$$\kappa_V^{h^0} = \cos\theta_H \cos\alpha - \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \sin\alpha,$$

$$\kappa_V^{H^0} = \cos\theta_H \sin\alpha + \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \cos\alpha,$$

$$\kappa_{WW}^{H_5^0} = \frac{1}{\sqrt{3}} \sin\theta_H, \qquad \kappa_{ZZ}^{H_5^0} = -\frac{2}{\sqrt{3}} \sin\theta_H.$$

$$\left[\left(\kappa^{h}_{WW}\right)^2 + \left(\kappa^{H}_{WW}\right)^2 + \left(\kappa^{H_5}_{WW}\right)^2\right] = \cos^2\theta_H + 3\sin^2\theta_H$$

$\Rightarrow$ unitarity is NOT preserved??

# GM is a model with triplets

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GM contains doubly charged Higgs bosons

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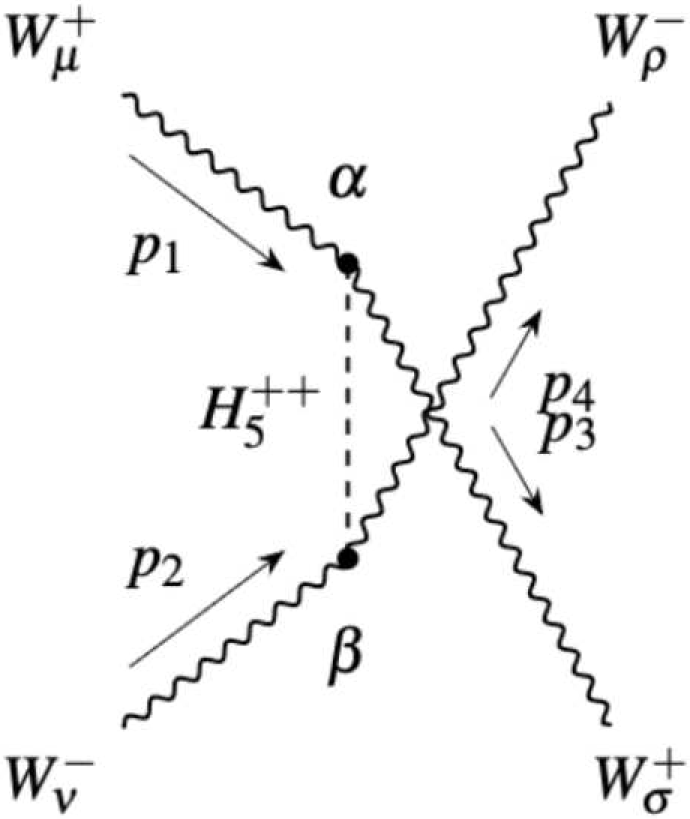
$\Rightarrow$ additional contribution to $W_L W_L \to W_L W_L$: $(u \sim -(s+t))$




: u-channel $H_5^{++}$ diagram

## Total Higgs contribution in GM:

$$\mathcal{M} \simeq -\frac{1}{v^2}\left[\left(\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\right)^2 - \left(\kappa_{WW}^{H_5^{++}}\right)^2\right)(s+t)\right.$$
$$\left. + 2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2 + 2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2 + 2\left(\kappa_{WW}^{H_5^0}\right)^2 m_5^2 + \left(\kappa_{WW}^{H_5^{++}}\right)^2 m_5^2\right]$$

## Sum rule in GM:

$$\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\right)^2 - \left(\kappa_{WW}^{H_5^{++}}\right)^2 = 1,$$

$$\sum_i g_{h_iWW}^2 - g_{h^{++}WW}^2 = g_{H_{\rm SM}WW}^2, \quad \text{where} \quad h_i = \{h^0, H^0, H_5^0\}, \quad h^{\pm\pm} = H_5^{\pm\pm}$$

$$c_H^2 + \frac{8}{3}s_H^2 + \frac{1}{3}s_H^2 - 2s_H^2 = c_H^2 + s_H^2 = 1$$

⇒ unitarity IS preserved in GM!

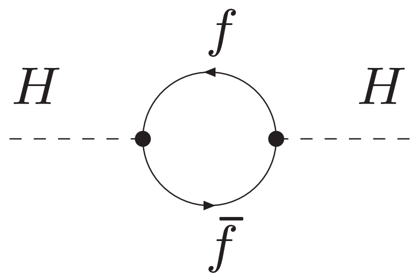
## 3. The Higgs sector of the (N)MSSM

### Motivation for BSM physics: the Hierarchy problem

Mass is what determines the properties of the free propagation of a particle

Free propagation: 

inverse propagator: $i(p^2 - M_H^2)$

Loop corrections: 

inverse propagator: $i(p^2 - M_H^2 + \Sigma_H^f)$

QM: integration over all possible loop momenta $k$

dimensional analysis:

$$\Sigma_H^f \;\sim\; N_f\,\lambda_f^2 \int d^4k \left( \frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

$$\text{for } \Lambda \to \infty: \qquad \Sigma_H^f \;\sim\; N_f\,\lambda_f^2 \left( \underbrace{\int \frac{d^4k}{k^2}}_{\sim\,\Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim\,\ln\Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_{\text{Pl}}$:

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{\text{Pl}}^2 \quad \Rightarrow \quad \delta M_H^2 \approx 10^{30}\, M_H^2$$

(for $M_H \lesssim 1$ TeV)

– no additional symmetry for $M_H = 0$

– no protection against large corrections

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_H^2 \approx M_{\text{GUT}}^2$

Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

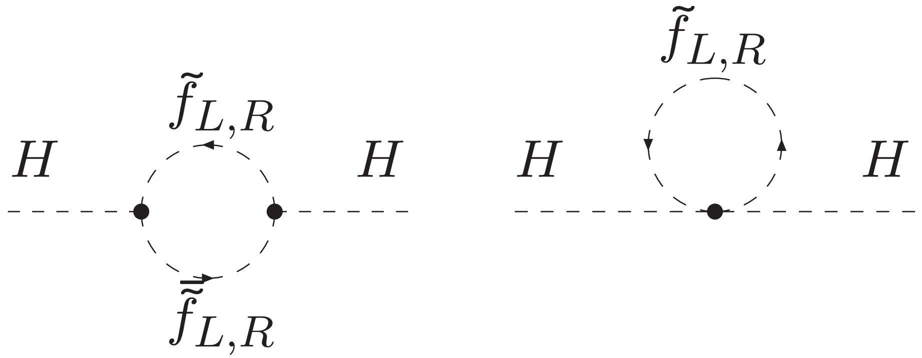
## Supersymmetry:

Symmetry between fermions and bosons

$$
\begin{aligned}
Q|\text{boson}\rangle &= |\text{fermion}\rangle \\
Q|\text{fermion}\rangle &= |\text{boson}\rangle
\end{aligned}
$$

Effectively: SM particles have SUSY partners (e.g. $f_{L,R} \to \tilde{f}_{L,R}$)

## SUSY: additional contributions from scalar fields:




$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2 \int d^4k \left( \frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{ terms without quadratic div.}$$

for $\Lambda \to \infty$: $\quad \Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2\, \Lambda^2$

$\Rightarrow$ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda_{\tilde{f}}^2 = \lambda_f^2$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m_{\tilde{f}}^2 = m_f^2 + \Delta^2, \quad \lambda_{\tilde{f}}^2 = \lambda_f^2$

$$\Rightarrow \Sigma_H^{f+\tilde{f}} \sim N_f \, \lambda_f^2 \, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\,\text{TeV}$$

$\boxed{\Rightarrow \text{SUSY at TeV scale provides attractive solution of hierarchy problem}}$

Supersymmetry (SUSY) : Symmetry between

$$\text{Bosons} \leftrightarrow \text{Fermions}$$

$$Q\ |\text{Fermion}\rangle \rightarrow |\text{Boson}\rangle$$

$$Q\ |\text{Boson}\rangle \rightarrow |\text{Fermion}\rangle$$

Simplified examples:

$$Q\ |\text{top},\ t\rangle \rightarrow |\text{scalar top},\ \tilde{t}\rangle$$

$$Q\ |\text{gluon},\ g\rangle \rightarrow |\text{gluino},\ \tilde{g}\rangle$$

$\Rightarrow$ each SM multiplet is enlarged to its double size

Unbroken SUSY: All particles in a multiplet have the same mass

Reality: $m_e \neq m_{\tilde{e}}$ $\Rightarrow$ SUSY is broken ...

...via soft SUSY-breaking terms in the Lagrangian (added by hand)

SUSY particles are made heavy: $\boxed{M_{\text{SUSY}} = \mathcal{O}(1\ \text{TeV})}$

## The Higgs sector of the MSSM

Comparison with SM case:

$$\mathcal{L}_{\text{SM}} = \underbrace{m_d \bar{Q}_L \Phi d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \Phi_c u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \Phi_c = i\sigma_2 \Phi^*, \quad \Phi \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \Phi_c \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L \Phi^*$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on $\varphi_i$, not on $\varphi_i^*$

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ $H_d (\equiv H_1)$ and $H_u (\equiv H_2)$ needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

# The MSSM Higgs sector:

Enlarged Higgs sector: Two Higgs doublets

$$\begin{aligned} H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\ H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ &\quad + \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2 \end{aligned}$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$ Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$t_\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (t_\beta + \cot\beta)$$

Tree-level result for $m_h$, $m_H$:

$$m_{H,h}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_h \le M_Z$ at tree level

Tree-level result for $m_h$, $m_H$:

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**Q: What are the implications?**

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Measurement of $m_h$, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

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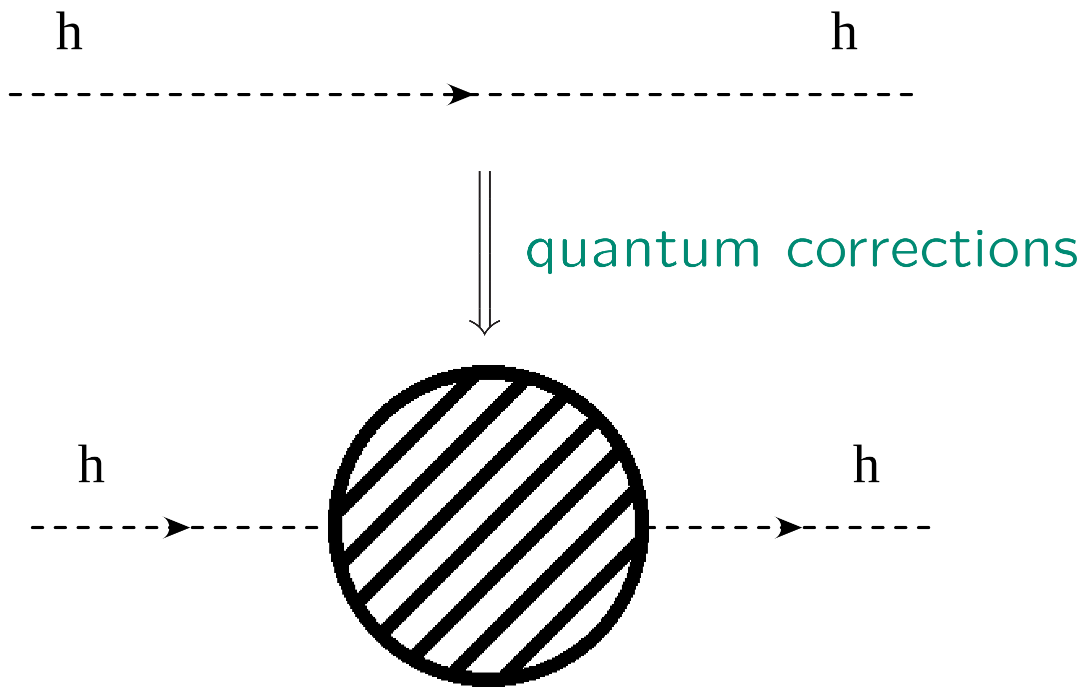
**LHC measurements: $M_h \sim 125$ GeV**

**Is this a problem?**

# Quantum corrections to the MSSM Higgs mass

Higgs propagator:



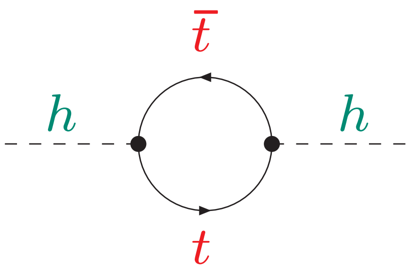

Inverse propagator:

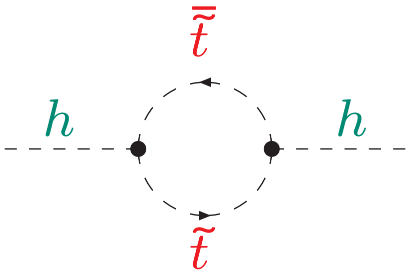
$$-i(q^2 - m^2) \quad \longrightarrow \quad -i\left(q^2 - m^2 + \hat{\Sigma}_h(q^2)\right)$$

$\hat{\Sigma}_h(q^2)$: renormalized Higgs self-energy

Loop corrected mass$^2$: $M^2 = m^2 - \hat{\Sigma}_h(m^2)$

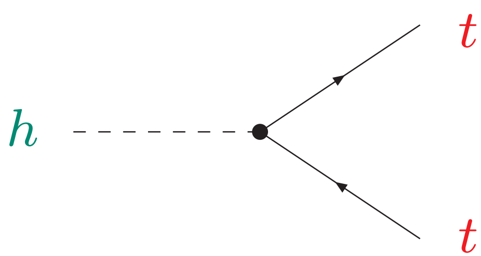
## 1-Loop: Feynman diagrams:

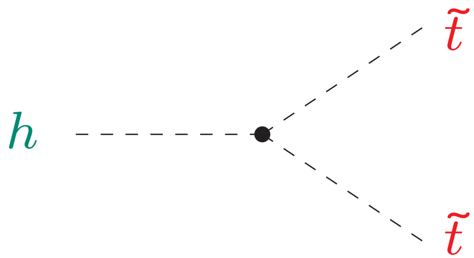



$\Rightarrow$ let's estimate their size!

## Feynman rules:




$$\sim \frac{i\, e\, m_t}{2\, M_W s_W \sin\beta} \qquad\qquad \sim \frac{i\, e\, m_t^2}{2\, M_W s_W \sin\beta}$$

Consider:

- $\Delta M_h^2 = -\hat{\Sigma}_h$, dim = $[m^2]$
- SUSY is constructed in a way that corrections go with $\sim \log(m_{\text{SUSY}}^2)$, (never like $\sim m_{\text{SUSY}}$ or $\sim m_{\text{SUSY}}^2$)
- logs must not have a dimensionful argument
- top diagram: $m_t^2/M_W^2$ from the couplings, only other mass scale available: $m_t$
- stop diagram: $m_t^4/M_W^2$ from the couplings
- color factor for tops or stops: 3
- fermion loops yield a relative factor of (-1)
- loop factor: $1/(4\,\pi^2)$

$\Rightarrow$ **estimate the formula for $\Delta M_h^2$**

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$\Rightarrow$

$$\Delta M_h^2 \sim \frac{3\,e^2\,m_t^4}{16\,\pi^2\,M_W^2 s_{\text{W}}^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

## Consider:

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$\Rightarrow$ real calculation:

$$\Delta M_h^2 \sim \frac{3\,e^2\,m_t^4}{8\,\pi^2\,M_W^2 s_{\mathrm{W}}^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

$$\text{dominant 1-loop}: \Delta M_h^2 \sim \frac{3\, e^2\, m_t^4}{8\, \pi^2\, M_W^2 s_\mathrm{W}^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Numerical estimate:

$$\begin{aligned} e^2/(4\pi) &= \alpha = 1/137 \\ m_t &= 175\ \text{GeV} \\ M_W &= 80\ \text{GeV}\ (M_Z = 90\ \text{GeV}) \\ s_\mathrm{W} &= 1/2 \\ \sin\beta &= 1 \\ m_{\tilde{t}} &= 1000\ \text{GeV} \end{aligned}$$

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$$\begin{aligned} \text{dominant 1-loop}: \Delta M_h^2 &\approx 7000 \text{ GeV}^2 \\ M_h^2 &= m_{h,\text{tree}}^2 + \Delta M_h^2 \\ m_{h,\text{tree}} &\approx M_Z \end{aligned}$$

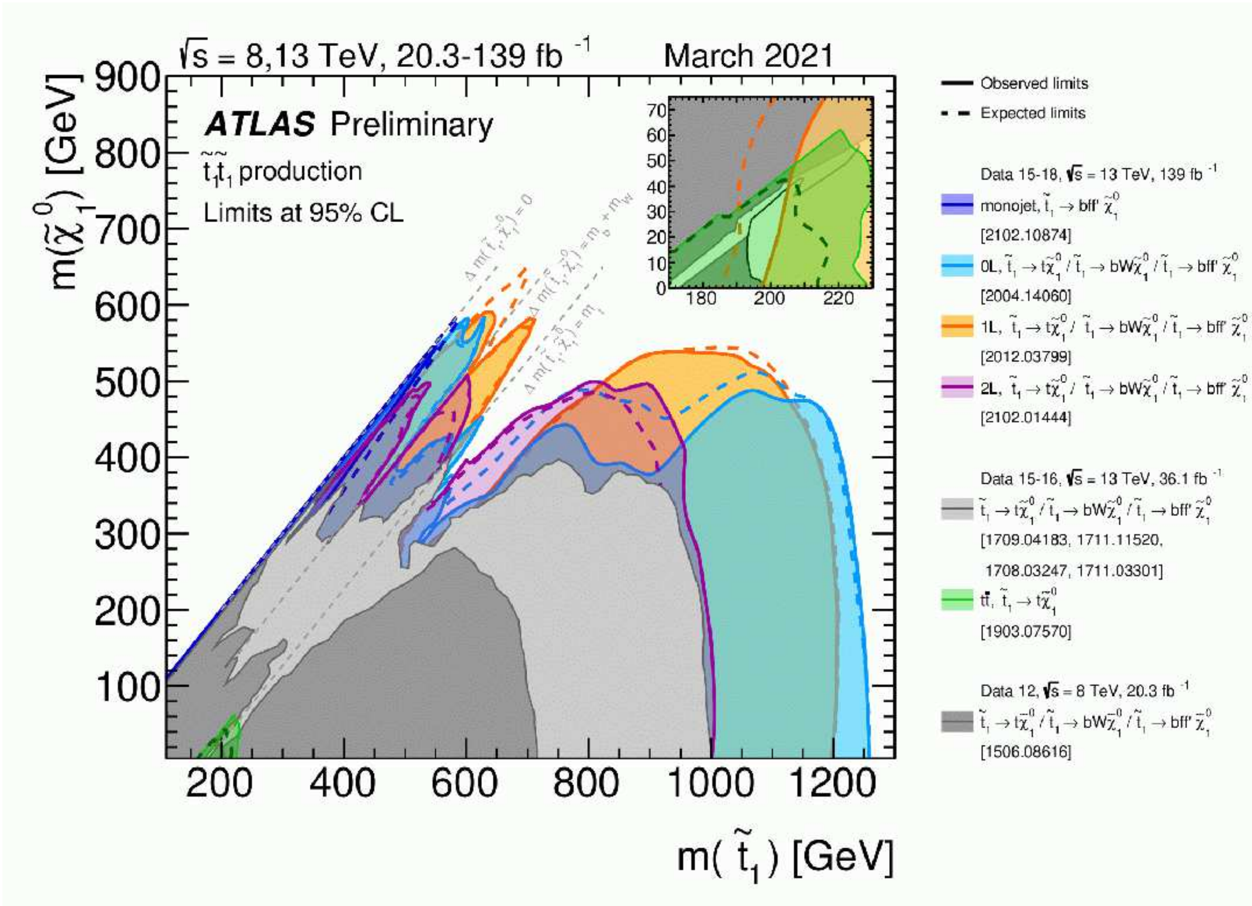
$$\text{dominant 1-loop}: M_h \approx 123 \text{ GeV}$$

dominant 1 – loop : $M_h \approx 123$ GeV

$\Rightarrow$ many, many, many, . . . more refinements have been calculated

$\Rightarrow$ stops above the TeV scale naturally yield $M_h \sim 125$ GeV

Comparison with LHC searches:




$\Rightarrow$ $M_h = 125$ GeV is in perfect agreement with the non-observation of stops

## Upper bound on $M_h$ in the MSSM:

"Unconstrained MSSM":

$M_A$, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, $\mu$, $m_{\tilde{g}}$, $M_2$

$$\boxed{M_h \lesssim 135 \text{ GeV}}$$

for $m_t = 173.2 \pm 0.9\,\text{GeV}$ and $m_{\tilde{t}} \lesssim 2$ TeV
(including theoretical uncertainties from unknown higher orders)
$\Rightarrow$ in agreement with all LHC Higgs measurements and stop searches

Obtained with:

*FeynHiggs*

`www.feynhiggs.de`

[*H. Bahl, T. Hahn, S.H., W. Hollik, S. Passehr, H. Rzehak, G. Weiglein '98 – '25*]

$\to$ all Higgs masses, couplings, BRs, XSs (easy to link, easy to use :-)

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$\boxed{M_h \lesssim 135\ \text{GeV}}$ Note : $125 < 135!$

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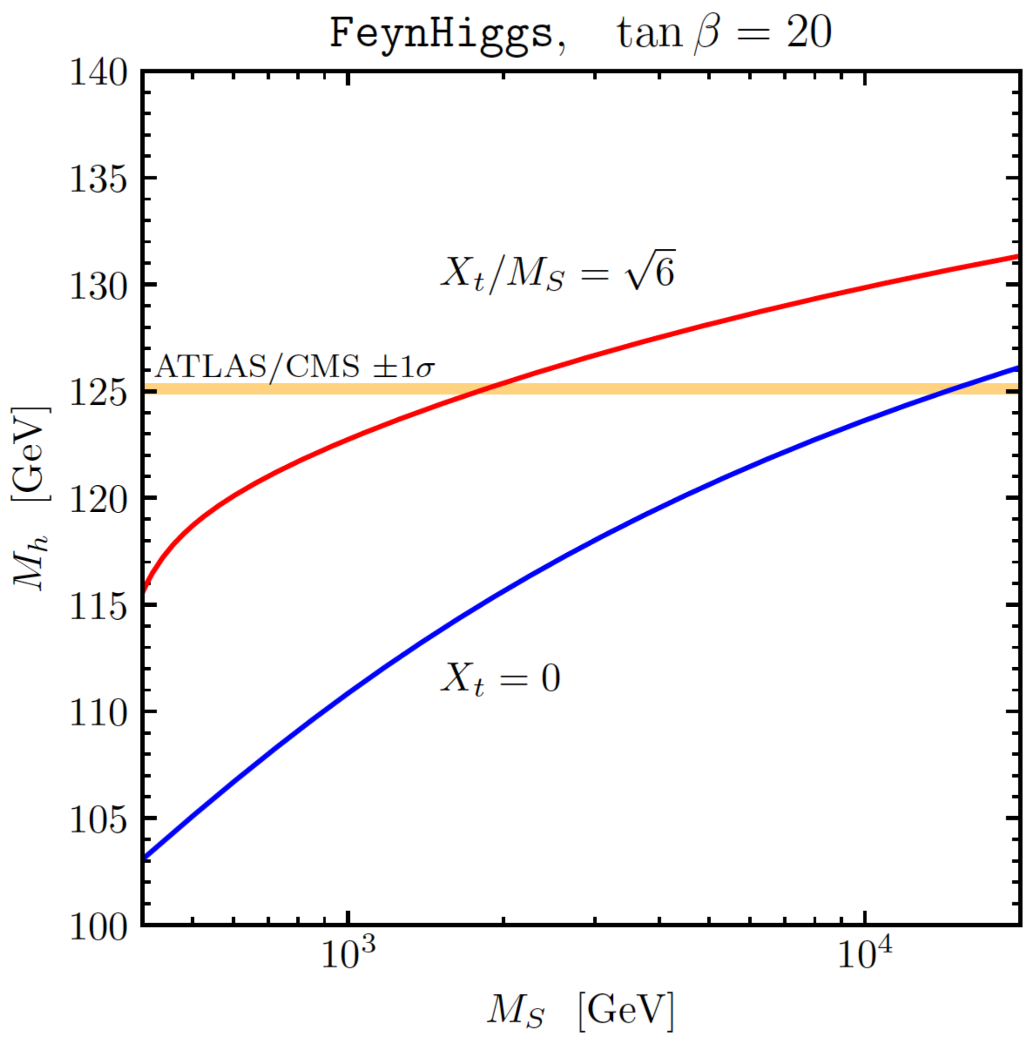
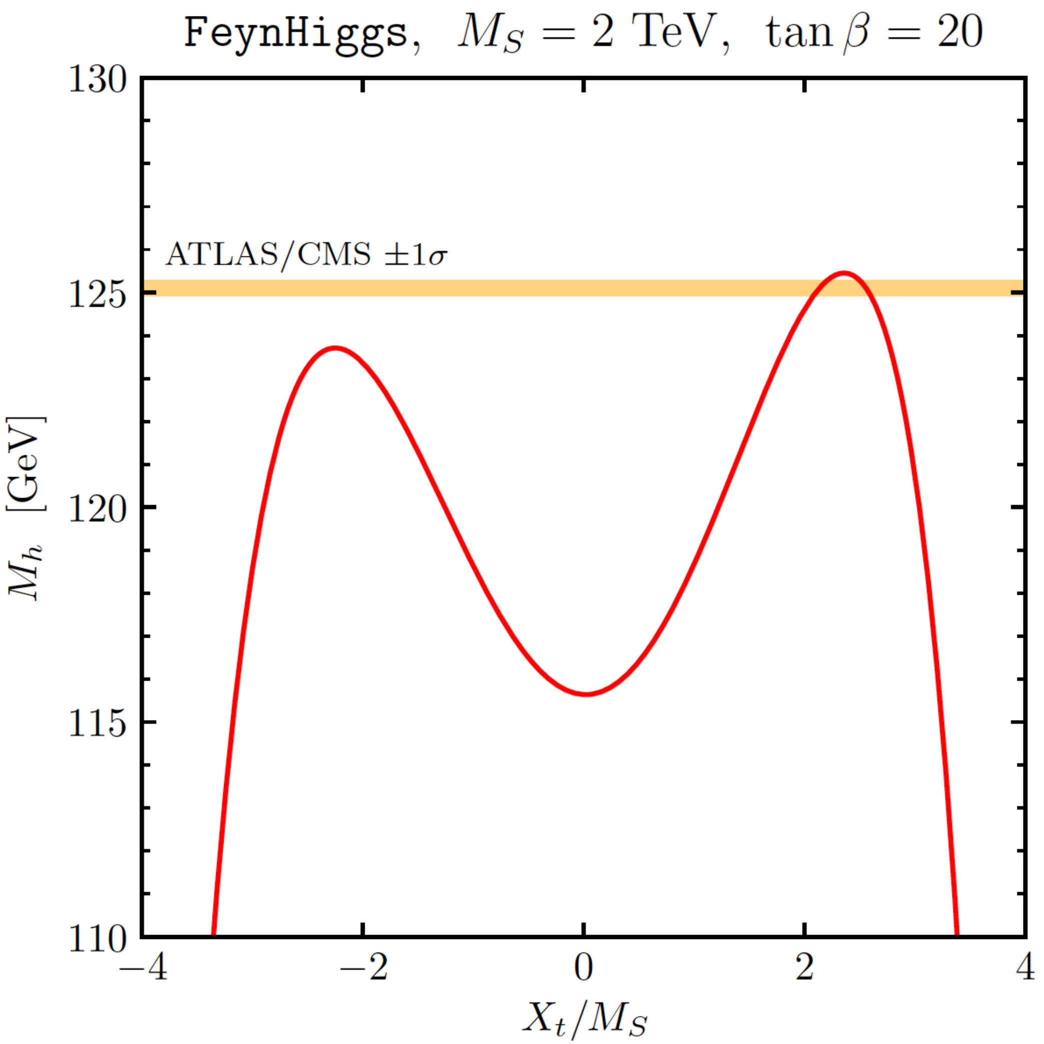
# Example for $M_h$ prediction in a simple MSSM scenario (I): [KUTS report '20]




$\Rightarrow M_h \sim 125$ GeV can easily be reached

## Example for $M_h$ prediction in a simple MSSM scenario (II): [*KUTS report '20*]




$\Rightarrow M_h \sim 125$ GeV "easier" with mixing in the scalar top sector

## A simple exercise on stop masses:

$\Rightarrow$ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

> The MSSM Higgs sector is connected to all other sector via loop corrections
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⇒ only certain combinations of stop parameters are compatible with the Higgs discovery.

**Q: clear prediction for the LHC?**

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| |
|---|
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For this exercise make sure:
$\Rightarrow$ use the best available Higgs mass calculation!

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For this exercise make sure:
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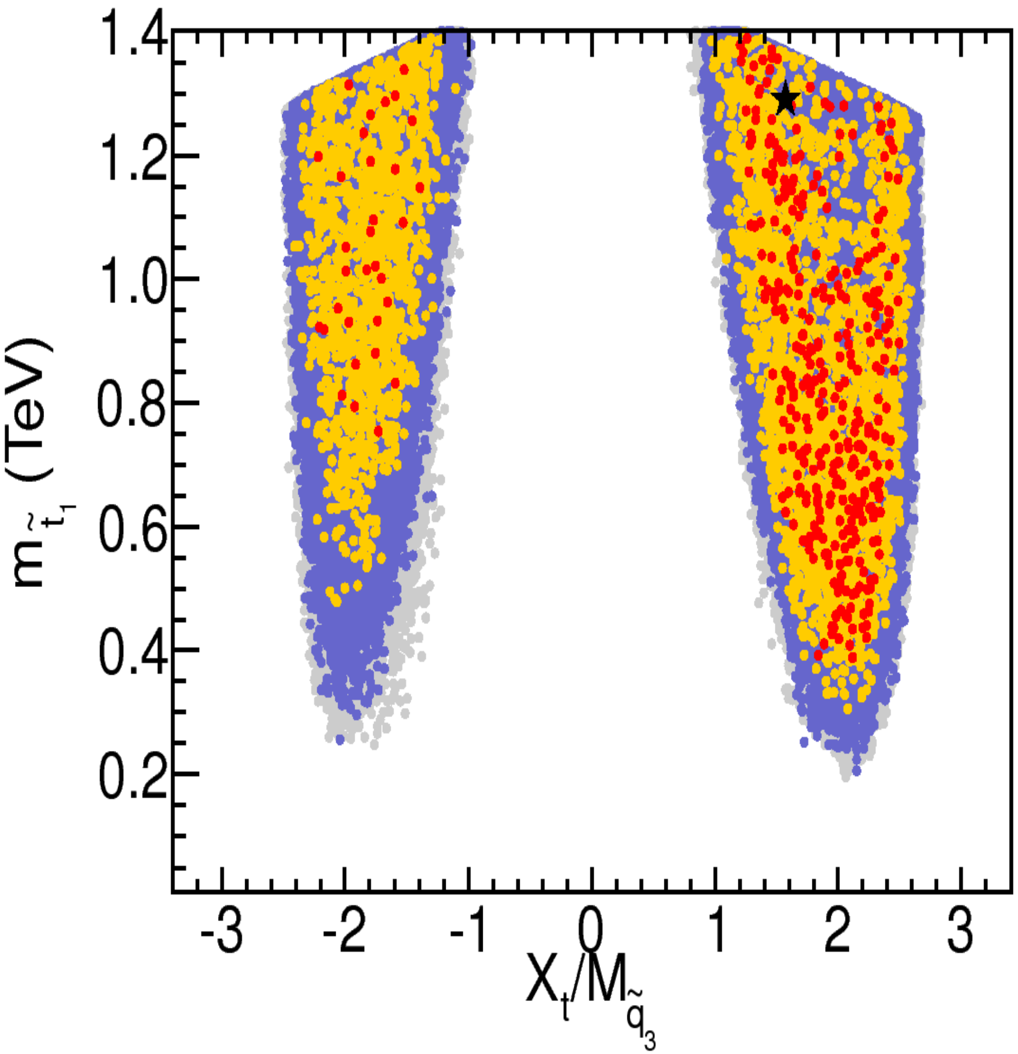
## Stop masses:

[P. Bechtle, S.H., O. Stål, T. Stefaniak, G. Weiglein, L. Zeune '16]




$M_h = 125 \pm 3$ GeV

$\star$: best-fit point

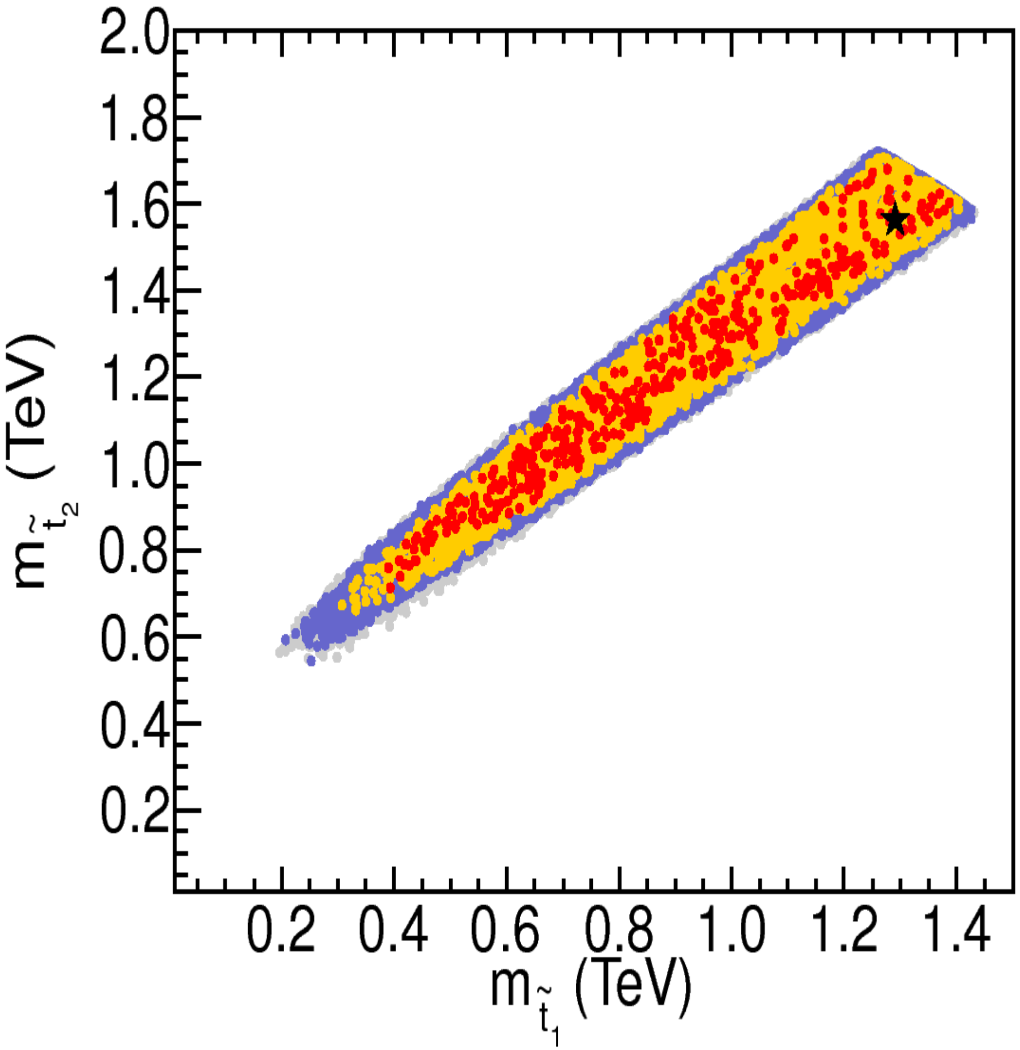
red: $\Delta\chi^2 < 2.3$

orange: $\Delta\chi^2 < 5.99$

blue: all points HiggsBounds allowed

gray: all scan points

$\Rightarrow M_h \sim 125$ GeV requires large $X_t$ and/or large $M_{\text{SUSY}}$

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gray: all scan points

$\Rightarrow$ light and heavy stops compatible with $M_h \simeq 125$ GeV

# The Higgs mass accuracy: experiment vs. theory:

Experiment:

ATLAS: $M_h^{\text{exp}} = 125.36 \pm 0.37 \pm 0.18$ GeV

CMS: $M_h^{\text{exp}} = 125.03 \pm 0.27 \pm 0.15$ GeV

combined: $M_h^{\text{exp}} = 125.09 \pm 0.21 \pm 0.11$ GeV

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## MSSM theory:

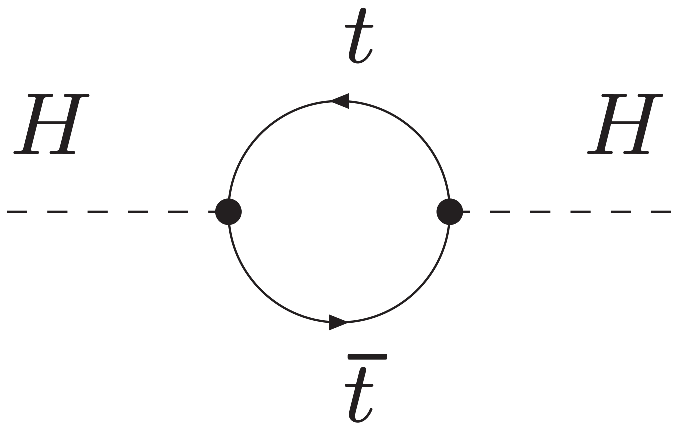
LHCHWG adopted FeynHiggs for the prediction of MSSM Higgs boson masses and mixings (considered to be the code containing the most complete implementation of higher-order corrections)

FeynHiggs: $\delta M_h^{\text{theo}} \sim 0.5 - 1$ GeV

$\to$ rough estimate, FeynHiggs contains algorithm to evaluate uncertainty, depending on parameter point

## This is not a SUSY specific feature!

Nearly any model: large coupling of the Higgs to the top quark:




$\Rightarrow$ one-loop corrections $\Delta M_H^2 \sim G_\mu m_t^4$

$\Rightarrow$ $M_H$ depends sensitively on $m_t$ in all models where $M_H$ can be predicted (SM: $M_H$ is free parameter)

SUSY as an example: $\Delta m_t \approx \pm 1$ GeV $\Rightarrow$ $\Delta M_h \approx \pm 1$ GeV

$\Rightarrow$ Precision Higgs physics needs $e^+e^-$ precision of $\mathcal{O}$(50 MeV) in $m_t$!

# Some NMSSM Higgs theory ($Z_3$ invariant NMSSM)

MSSM Higgs sector: Two Higgs doublets

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
V &= (\tilde{m}_1^2 + |\mu \quad|^2) H_1 \bar{H}_1 + (\tilde{m}_2^2 + |\mu \quad|^2) H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\
&\quad + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2
\end{aligned}
$$

## Some NMSSM Higgs theory ($Z_3$ invariant NMSSM)

NMSSM Higgs sector: Two Higgs doublets + one Higgs singlet

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} \\
S &= v_s + S_R + I S_I
\end{aligned}
$$

$$
\begin{aligned}
V = {} & (\tilde{m}_1^2 + |\cancel{\mu}\lambda S|^2) H_1 \bar{H}_1 + (\tilde{m}_2^2 + |\cancel{\mu}\lambda S|^2) H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\
& + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \\
& + |\lambda(\epsilon_{ab} H_1^a H_2^b) + \kappa S^2|^2 + m_S^2 |S|^2 + (\lambda A_\lambda (\epsilon_{ab} H_1^a H_2^b) S + \frac{\kappa}{3} A_\kappa S^3 + \text{h.c.})
\end{aligned}
$$

Free parameters:

$$\lambda,\ \kappa,\ A_\kappa,\ M_{H^\pm},\ \tan\beta,\ \mu_{\text{eff}} = \lambda v_s$$

## Higgs spectrum:

$$\begin{aligned} \mathcal{CP}\text{−even} &: h_1, h_2, h_3 \\ \mathcal{CP}\text{−odd} &: a_1, a_2 \\ \text{charged} &: H^+, H^- \\ \text{Goldstones} &: G^0, G^+, G^- \end{aligned}$$

## Neutralinos:

$$\mu \to \mu_{\text{eff}}$$

compared to the MSSM: one singlino more

$$\to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0$$

## Mass of the lightest $\mathcal{CP}$-even Higgs:

$$m^2_{h,\text{tree},\text{NMSSM}} = m^2_{h,\text{tree},\text{MSSM}} + M_Z^2 \frac{\lambda^2}{g^2} \sin^2 2\beta$$

## Mass of the $\mathcal{CP}$-odd Higgs:

$$\text{MSSM}: M_A^2 = -m_{12}^2 (t_\beta + \cot\beta) = \mu B (t_\beta + \cot\beta)$$

$$\text{NMSSM}: \text{"} M_A^2 \text{"} = \mu_{\text{eff}} B_{\text{eff}} (t_\beta + \cot\beta)$$

$$\text{with } B_{\text{eff}} = A_\lambda + \kappa\, s,\ \mu_{\text{eff}} = \lambda\, s \qquad \Rightarrow \ \text{one very light } a_1$$

## Mass of the charged Higgs:

$$\text{MSSM}: M^2_{H^\pm} = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2$$

$$\text{NMSSM}: M^2_{H^\pm} = M_A^2 + v^2 \left( \frac{g^2}{2} - \lambda^2 \right)$$

## Mass of the lightest $\mathcal{CP}$-even Higgs:

$$m^2_{h,\text{tree},\text{NMSSM}} = m^2_{h,\text{tree},\text{MSSM}} + M_Z^2 \frac{\lambda^2}{g^2} \sin^2 2\beta$$

## Mass of the $\mathcal{CP}$-odd Higgs:

$$\text{MSSM}: M_A^2 = -m_{12}^2 (t_\beta + \cot\beta) = \mu B (t_\beta + \cot\beta)$$

$$\text{NMSSM}: \text{"}M_A^2\text{"} = \mu_{\text{eff}} B_{\text{eff}} (t_\beta + \cot\beta)$$

with $B_{\text{eff}} = A_\lambda + \kappa\, s,\ \mu_{\text{eff}} = \lambda\, s$ $\Rightarrow$ one very light $a_1$

## Mass of the charged Higgs:

$$\text{MSSM}: M_{H^\pm}^2 = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2$$

$$\text{NMSSM}: M_{H^\pm}^2 = M_A^2 + v^2 \left( \frac{g^2}{2} - \lambda^2 \right)$$

$\Rightarrow M_{h_1}^{\text{MSSM,tree}} \le M_{h_1}^{\text{NMSSM,tree}}$, one light $a_1$, $M_{H^\pm}^{\text{MSSM,tree}} \ge M_{H^\pm}^{\text{NMSSM,tree}}$

## 4. How to distinguish BSM Higgs sectors experimentally

Remember:

We have a discovery!

The SM cannot be the ultimate theory!

**Conclusion: It cannot be "the SM Higgs"!**

**Q:** Does the BSM physics have any (relevant) impact on the Higgs?
**Q':** Which model?

**A1:** check changed properties of the $h_{125}$

**A2:** check for additional Higgs bosons
**A2':** check for additional Higgs bosons above and below 125 GeV

⇒ let's take a look at A1

## Recommendation of the ESPPU:

(European Strategy for Particle Physics Update)

The next large facility after the (HL-)LHC
for particle physics should be an $e^+e^-$ collider.

– to study the Higgs at $\sim$ 125 GeV
– top/EW physics
– BSM searches
– . . .

$\Rightarrow$ This new $e^+e^-$ collider will come after,
or in the end phase of the HL-LHC

$\Rightarrow$ physics potential of the new $e^+e^-$ collider must be viewed
in the context of HL-LHC results

$\Rightarrow$ often $e^+e^-$ expectations are shown in comparison to HL-LHC

## Overview of current at possible future collider experiments:

LHC (Large Hadron Collider): running

$pp$ collisions at 13(14) TeV

HL-LHC final high-luminosity phase: approved

HE-LHC new magnets $\Rightarrow$ 27 TeV (possible?)

ILC (International Linear Collider) considered in Japan

$e^+e^-$ collisions at 250 GeV (final stage 1000 GeV)

CLIC (Compact LInear Collider)

$e^+e^-$ collisions at 380 GeV (final stage 3000 GeV)

LCF (Linear Collider Facillity (at CERN))

$e^+e^-$ collisions at $250 - 550$ GeV (final stage open)

FCC-ee (Future Circular Collider $e^+e^-$)

$e^+e^-$ collisions up to 365 GeV

CEPC (Circular $e^-e^+$ Collider)

$e^+e^-$ collisions up to 365 GeV

FCC-hh (Future Circular Collider had-had)

$pp$ collisions at 100 TeV (possible?)

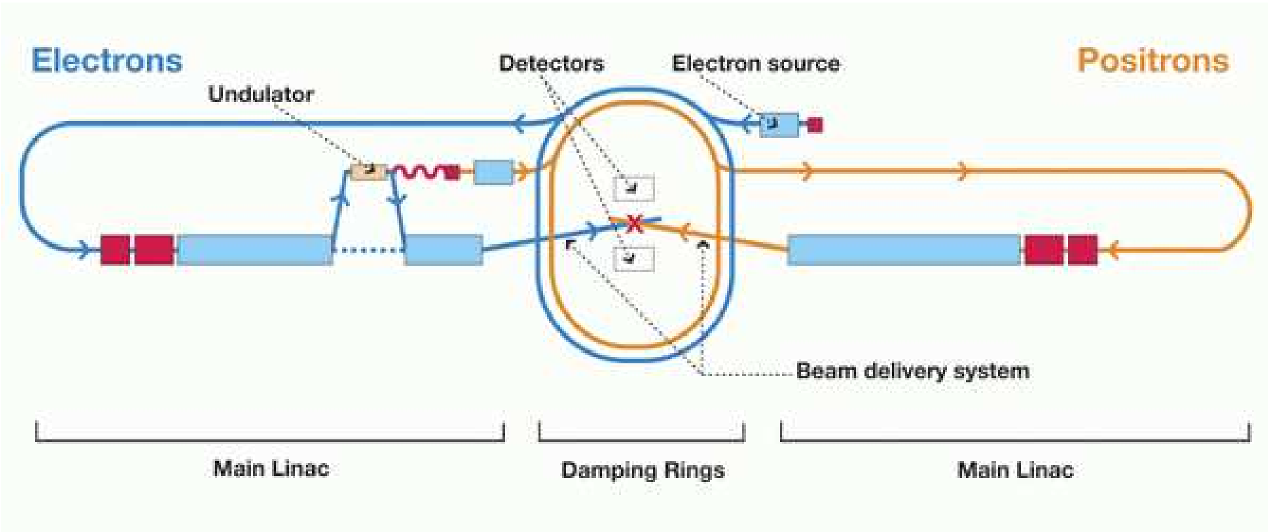
# Mini overview of the International Linear Collider (ILC)

# Mini overview of the International Linear Collider (ILC)

Linear $e^+e^-$ collider, $\sqrt{s} = 250 - 1000$ GeV

based on superconducting cavities (cold technology) (ITRP decision 2004)

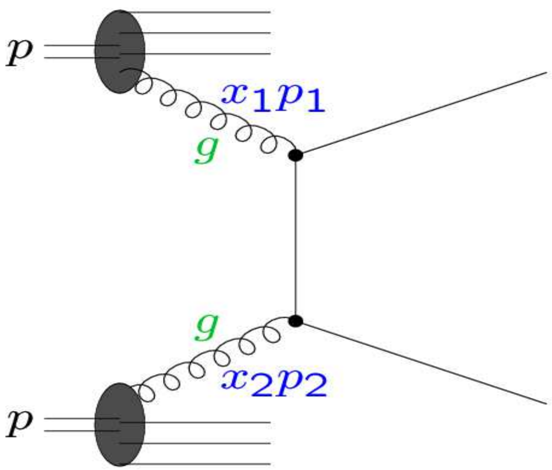
Schematic:




– two detectors in one interaction region (push-pull)
– undulator based $e^+$ source
– polarized beams for $e^-$ and $e^+$ ($P_{e^-} = 80\%$, $P_{e^+} = 60\%$)
– tunable energy

# Mini-comparison of LHC and ILC:

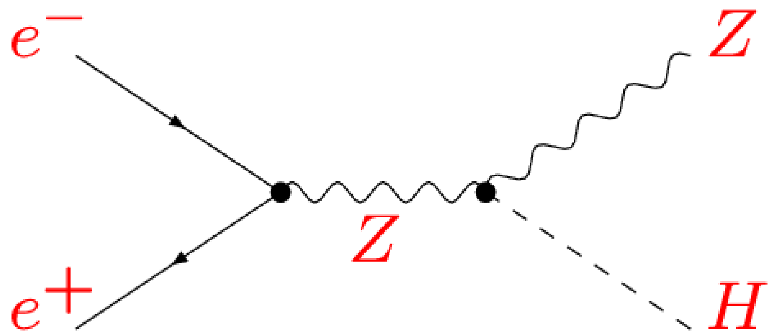
LHC: $pp$ scattering at 14 TeV




Scattering process of proton constituents with energy up to several TeV,
strongly interacting

$\Rightarrow$ huge QCD backgrounds,
low signal–to–background ratios

ILC: $e^+e^-$ scattering
at $\approx$0.25–1 TeV

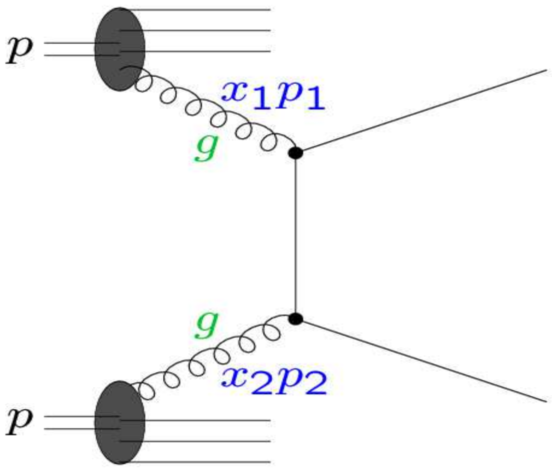



Clean exp. environment:
well-defined initial state,
tunable energy,
beam polarization, GigaZ,
$\gamma\gamma$, $e\gamma$, $e^-e^-$ options, ...

$\Rightarrow$ rel. small backgrounds
high-precision physics

## Mini-comparison of LHC and ILC:

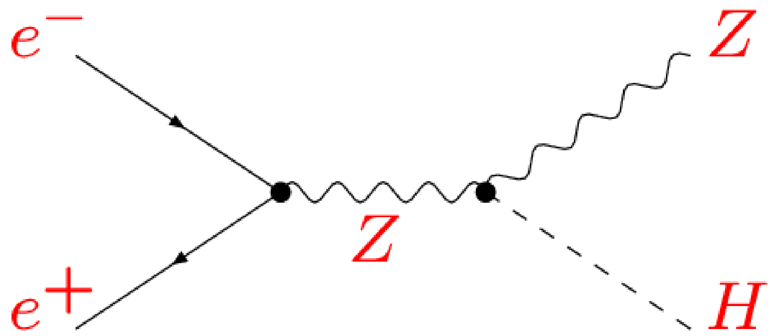
LHC: $pp$ scattering at 14 TeV




interaction rate of $10^9$ events$/s$

$\Rightarrow$ can trigger on only
1 event in $10^7$

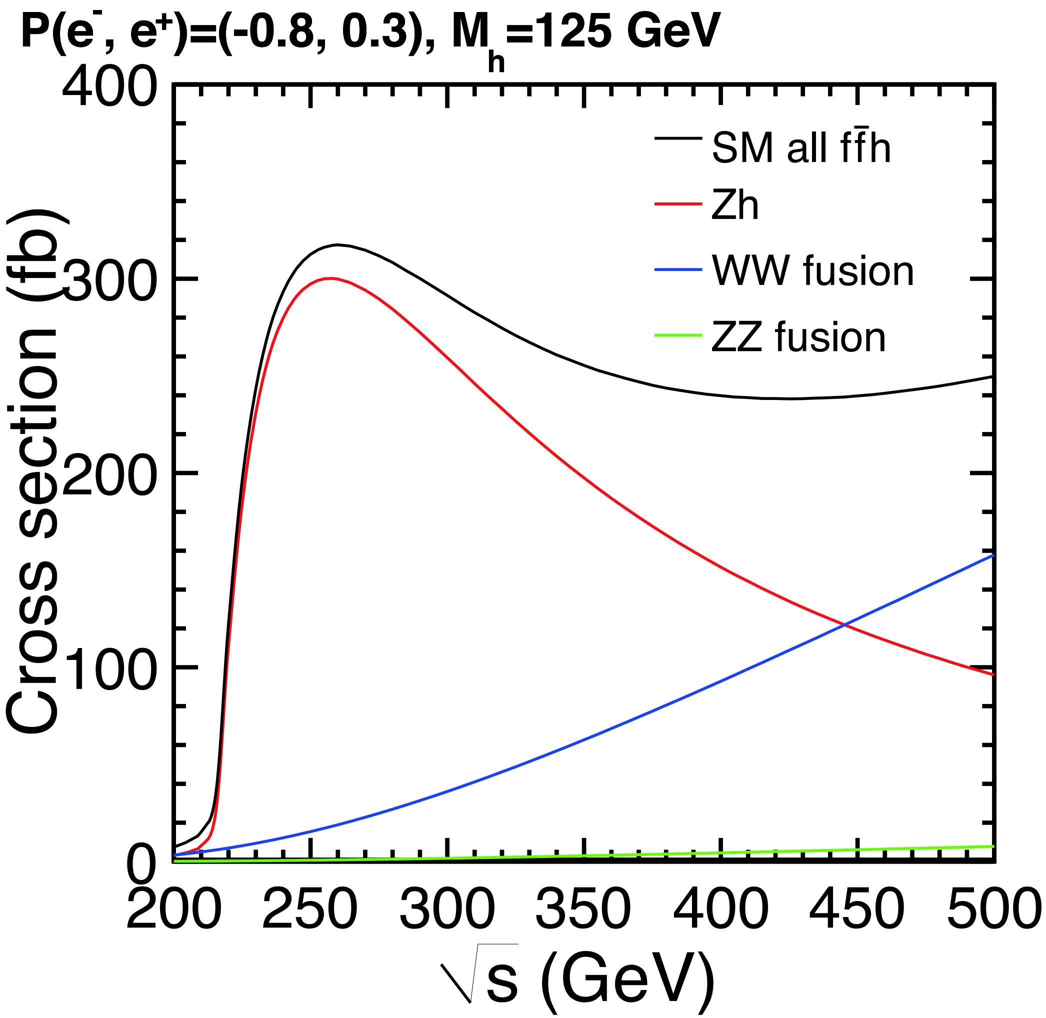
ILC: $e^+e^-$ scattering
at $\approx$0.25–1 TeV




untriggered operation

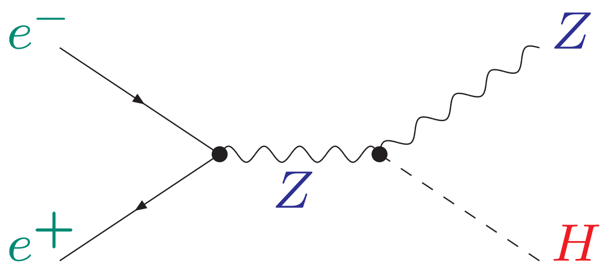
$\Rightarrow$ can find signals of unexpected new physics (direct production + large indirect reach) that manifests itself in events that are not selected by the LHC trigger strategies

# Higgs production cross sections at $e^+e^-$ colliders:



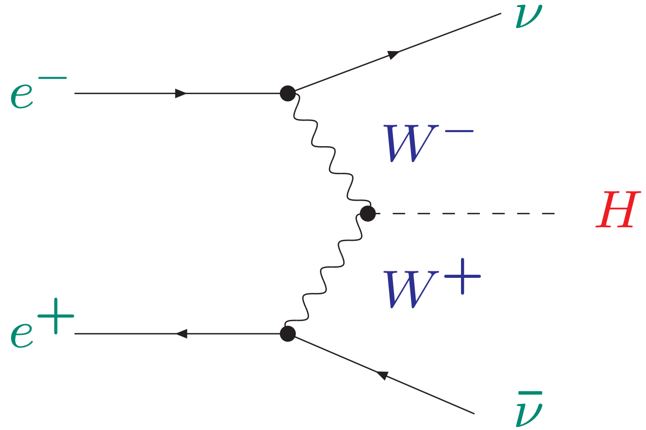

Higgs-strahlung:

$e^+e^- \to Z^* \to ZH$




weak boson fusion (WBF):

$e + e- \to \nu\bar{\nu}H$




$\sqrt{s} \sim 250$ GeV, Higgs-strahlung dominated

## Measurement of Higgs couplings

LHCHWG recommendation: Higgs coupling strength scale factors: $\kappa_i$

Assumptions for $\kappa$-framework:

1. Signal corresponds to only one state, no overlapping signal etc.
2. Zero-width approximation
3. Only modification of coupling strength (absolute values of couplings) but not of tensore structure wrt. to SM
4. Use state-of-the-art predictions in the SM and rescale the predictions with "leading order inspired" scale factors $\kappa_i$
   ($\kappa_i = 1$ corresponds to the SM case)

LHC always measures $\sigma \times$ BR

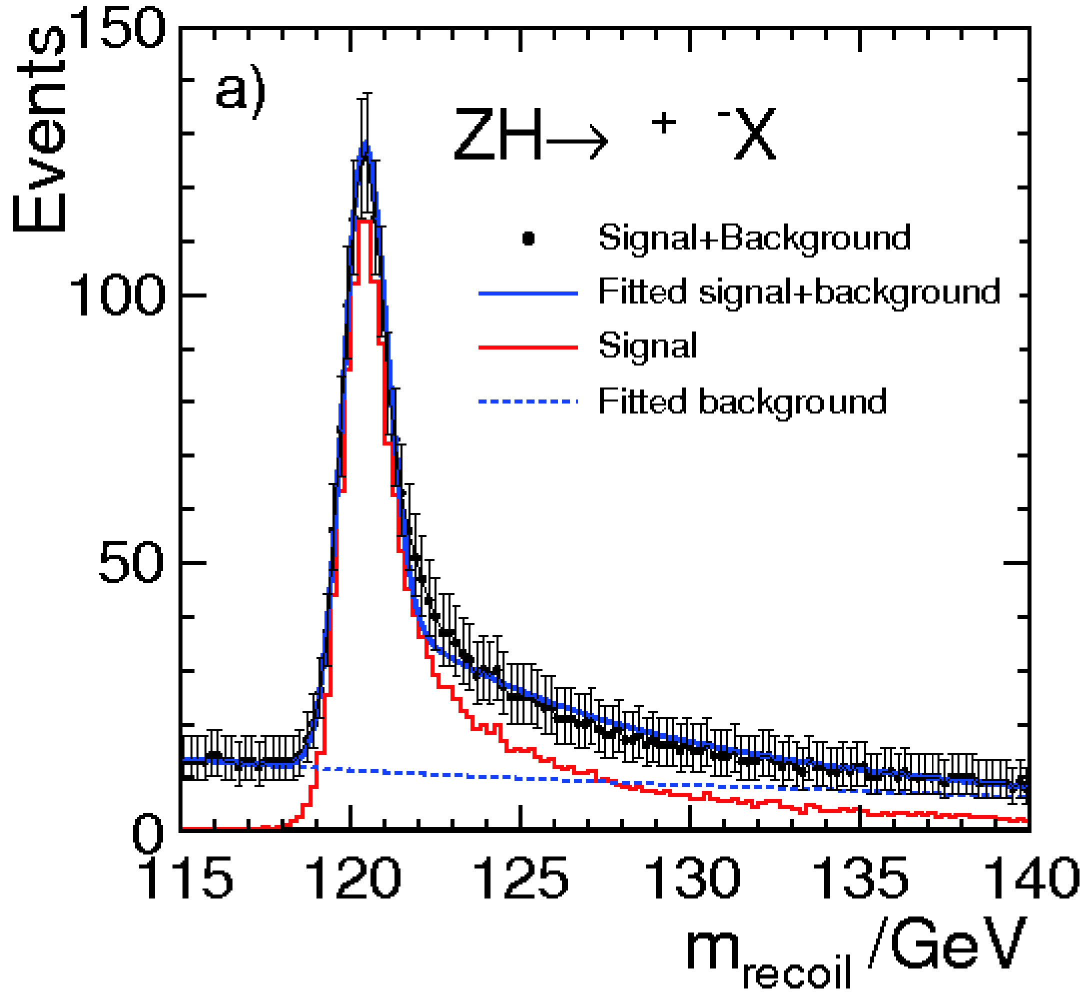
– no additional theory assumptions on your model:

$\Rightarrow$ Determination of ratios of scaling factors, e.g. $\kappa_i \kappa_j / \kappa_H$

– additional theory assumptions (on $\Gamma_{H,\mathrm{tot}}$ or $\kappa_{W,Z}$ or $H \to \mathrm{NP}$)

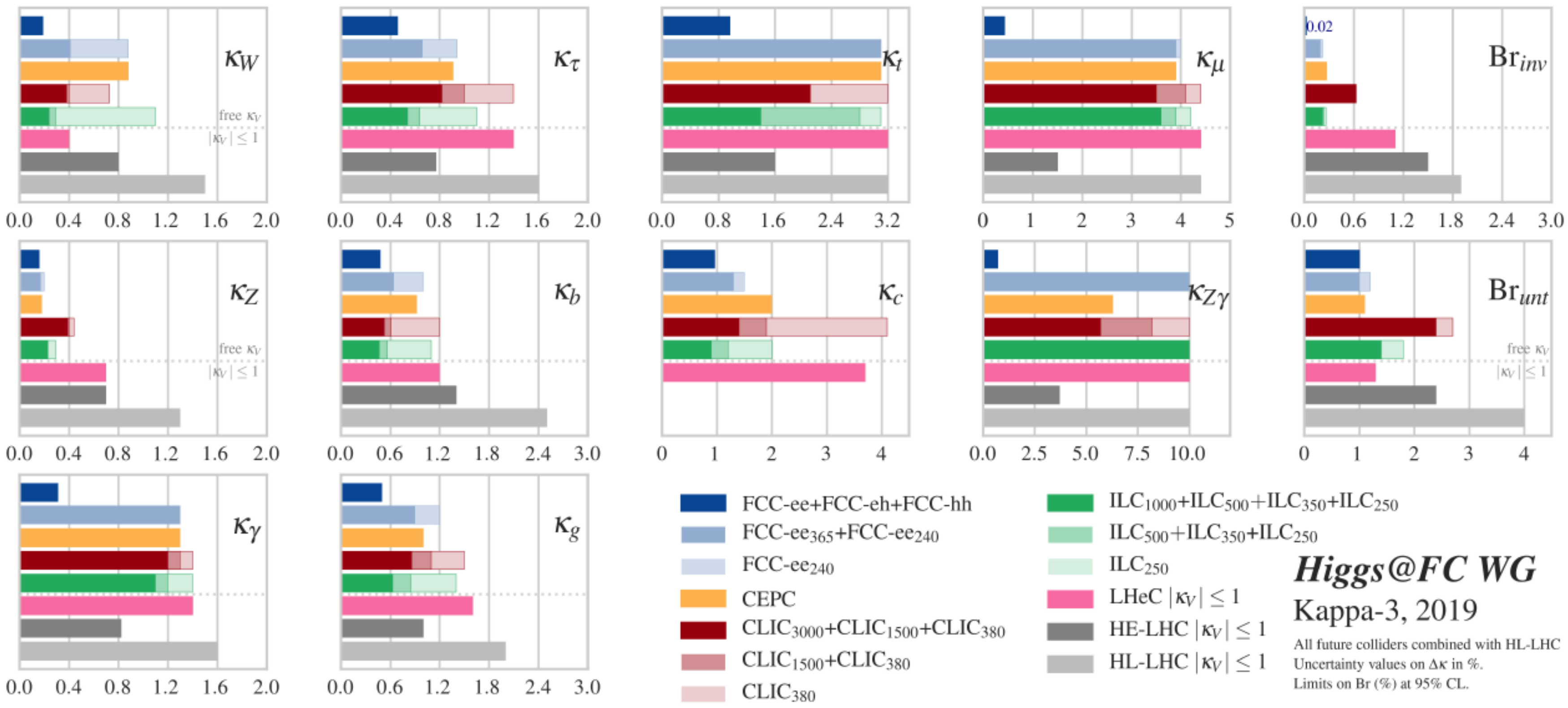
$\Rightarrow$ Determination of $\kappa_i$ (evaluated to NLO QCD accuracy)

$Z$-recoil method: $e^+e^- \to ZH \to \mu^+\mu^- X$ $\Rightarrow$ "pure" cross section




$\Rightarrow$ crucial for a model independent coupling measurement! $\delta M_H^{\rm exp} \lesssim 0.05$ GeV

# Future expectations for $\kappa$ (kappa-3 framework)

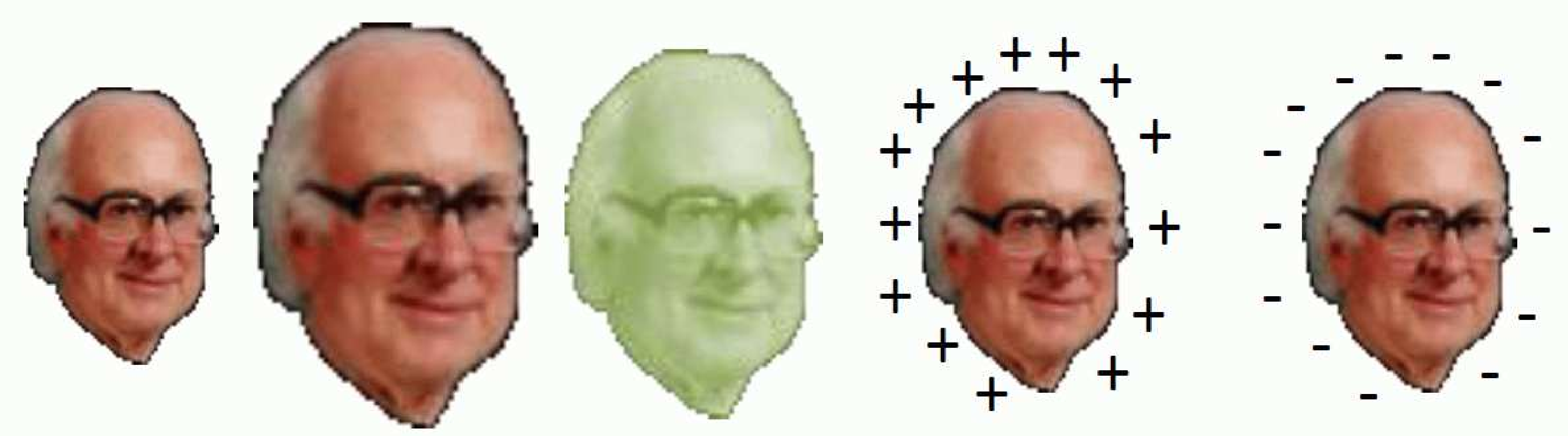



$\Rightarrow$ all $e^+e^-$ options yield roughly similar results

$\Rightarrow$ FCC-hh/-he/-ee appears better
(FCC-hh uses different theory assumptions, uncertainties $\lesssim 1\%$)

$\Rightarrow$ also remember different time scales!

# BSM Higgs: what can we learn?




– let's assume that we do see a deviation in the measurements of the $h_{125}$ couplings

– **What do we learn from that?** – "Higgs inverse problem"

# Required precision for Higgs couplings?

MSSM example:

$$\kappa_V \approx 1 - 0.5\% \left(\frac{400\ \text{GeV}}{M_A}\right)^4$$

$$\kappa_t = \kappa_c \approx 1 - \mathcal{O}(10\%) \left(\frac{400\ \text{GeV}}{M_A}\right)^2 \cot^2\beta$$

$$\kappa_b = \kappa_\tau \approx 1 + \mathcal{O}(10\%) \left(\frac{400\ \text{GeV}}{M_A}\right)^2$$

Composite Higgs example:

$$\kappa_V \approx 1 - 3\% \left(\frac{1\ \text{TeV}}{f}\right)^2$$

$$\kappa_F \approx 1 - (3-9)\% \left(\frac{1\ \text{TeV}}{f}\right)^2$$

$\Rightarrow$ couplings to bosons in the per mille range

$\Rightarrow$ couplings to fermions in the per cent range

$\Rightarrow$ theory/experimental match?

## Let us assume that we do see a deviation

**What do we learn from that?**
**How do we learn something from that?**

⇒ We have to compare the observed deviation with predicted deviations

⇒ Preferrably with the predicted deviations in a concrete models
(A comparison with an EFT result subsequently requires the mapping to concrete models anyway . . . )

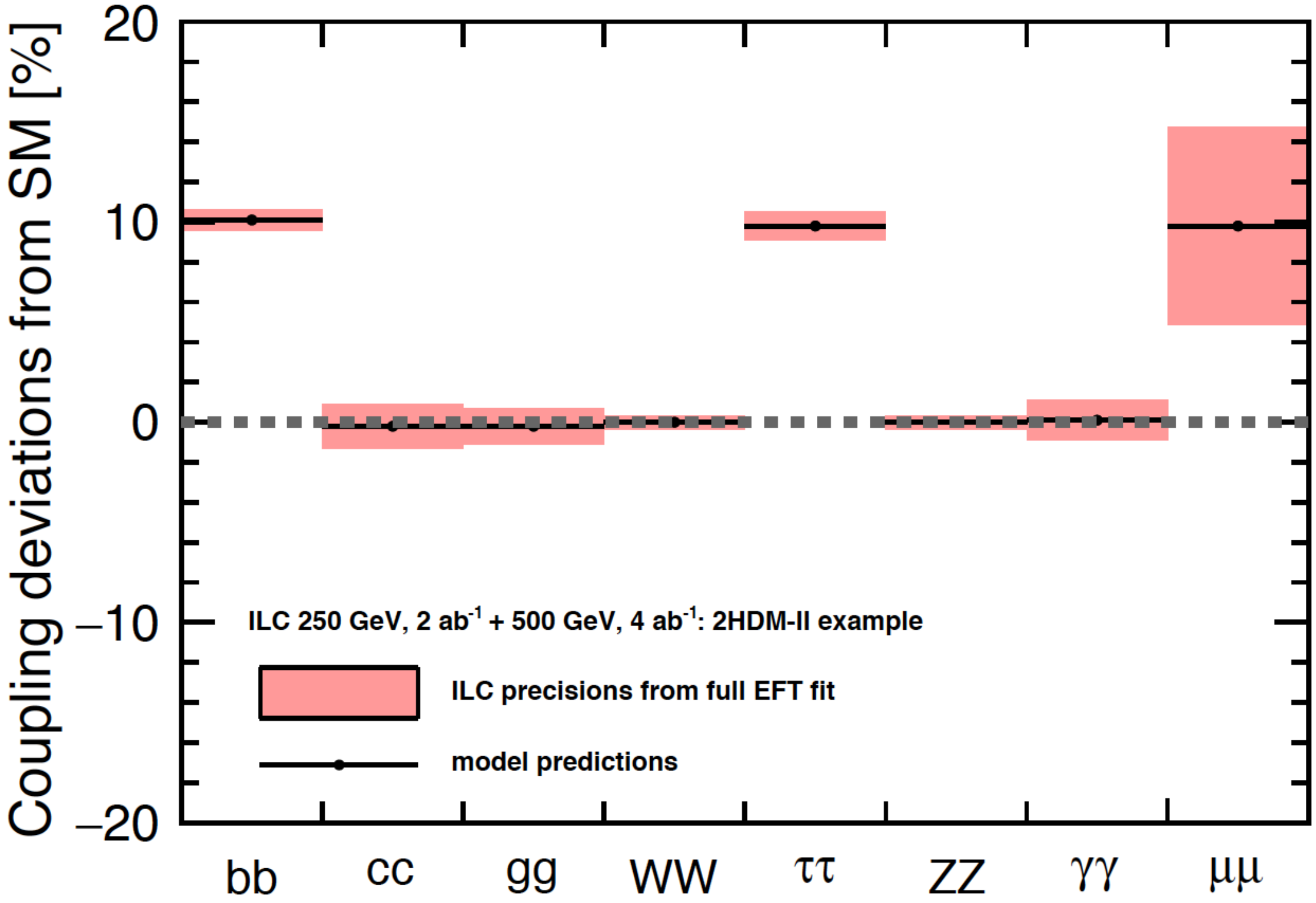
⇒ Needed: sufficiently precise predictions in BSM model
close to ready: MSSM, NMSSM
(I am not aware of uncertainty estimates in other models)

⇒ in the following: $\boxed{\text{model prediction (w/o TH unc.)} \Leftrightarrow e^+e^- \text{ precision}}$

⇒ “Wäscheleinen-Plots” (concrete: ILC500)

## Wäscheleine I: $e^+e^-$ precision vs. 2HDM type II prediction:

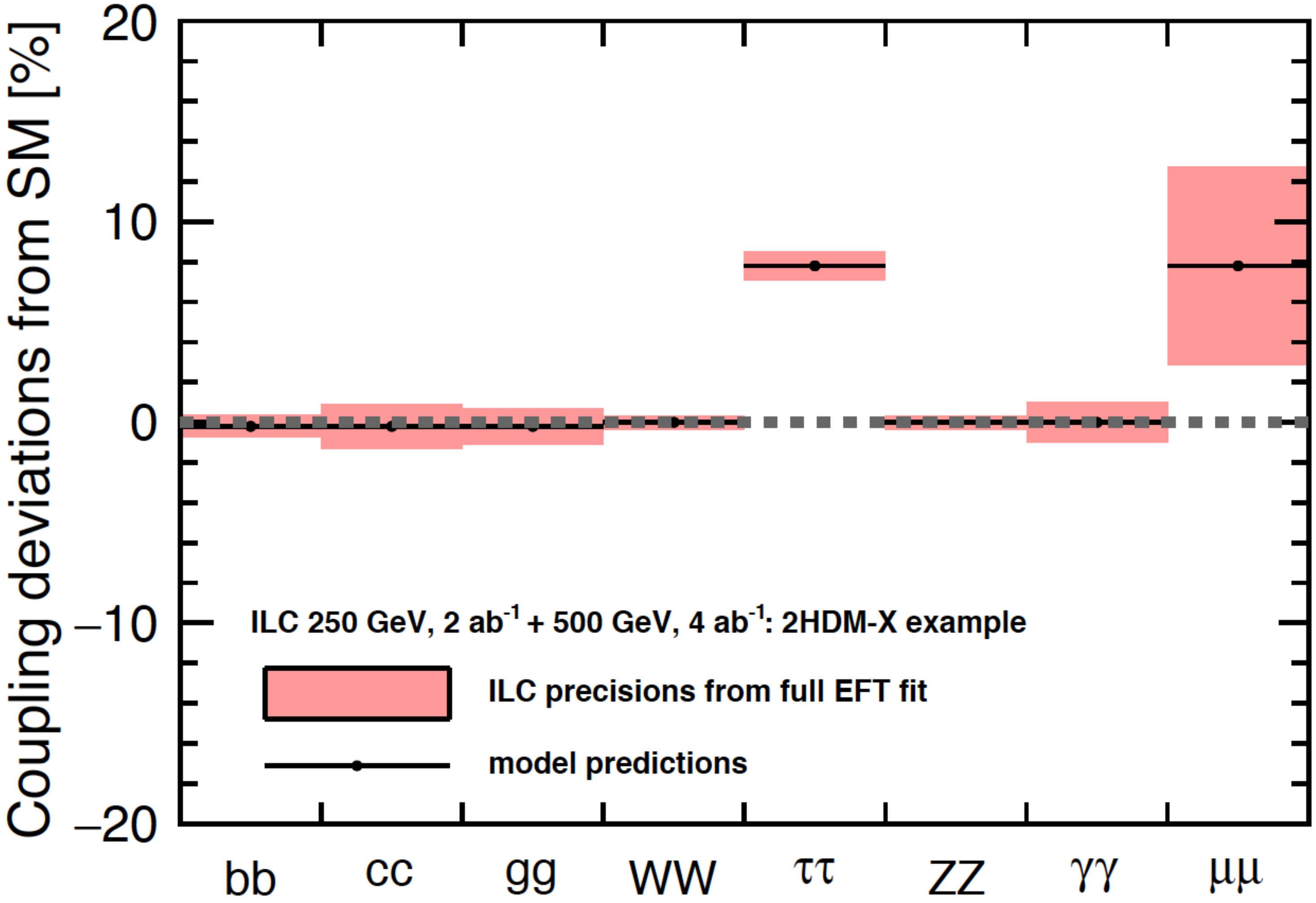
[*T. Barklow et al., '17*]




⇒ clear pattern, distinctive for 2HDM type II?!

## Wäscheleine II: $e^+e^-$ precision vs. 2HDM type X prediction:

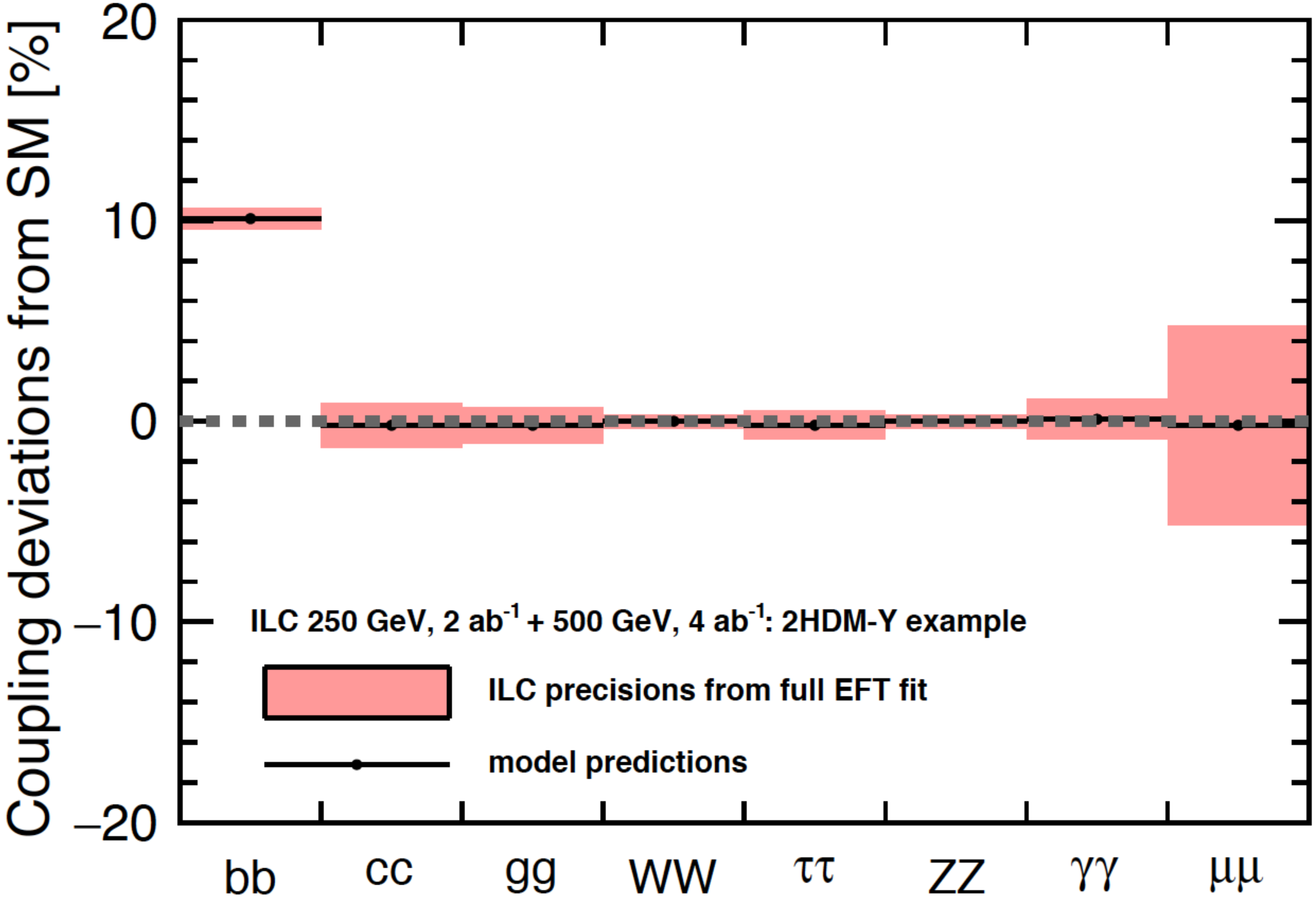
[*T. Barklow et al., '17*]




⇒ clear pattern, distinctive for 2HDM type X?!

## Wäscheleine III: $e^+e^-$ precision vs. 2HDM type Y prediction:

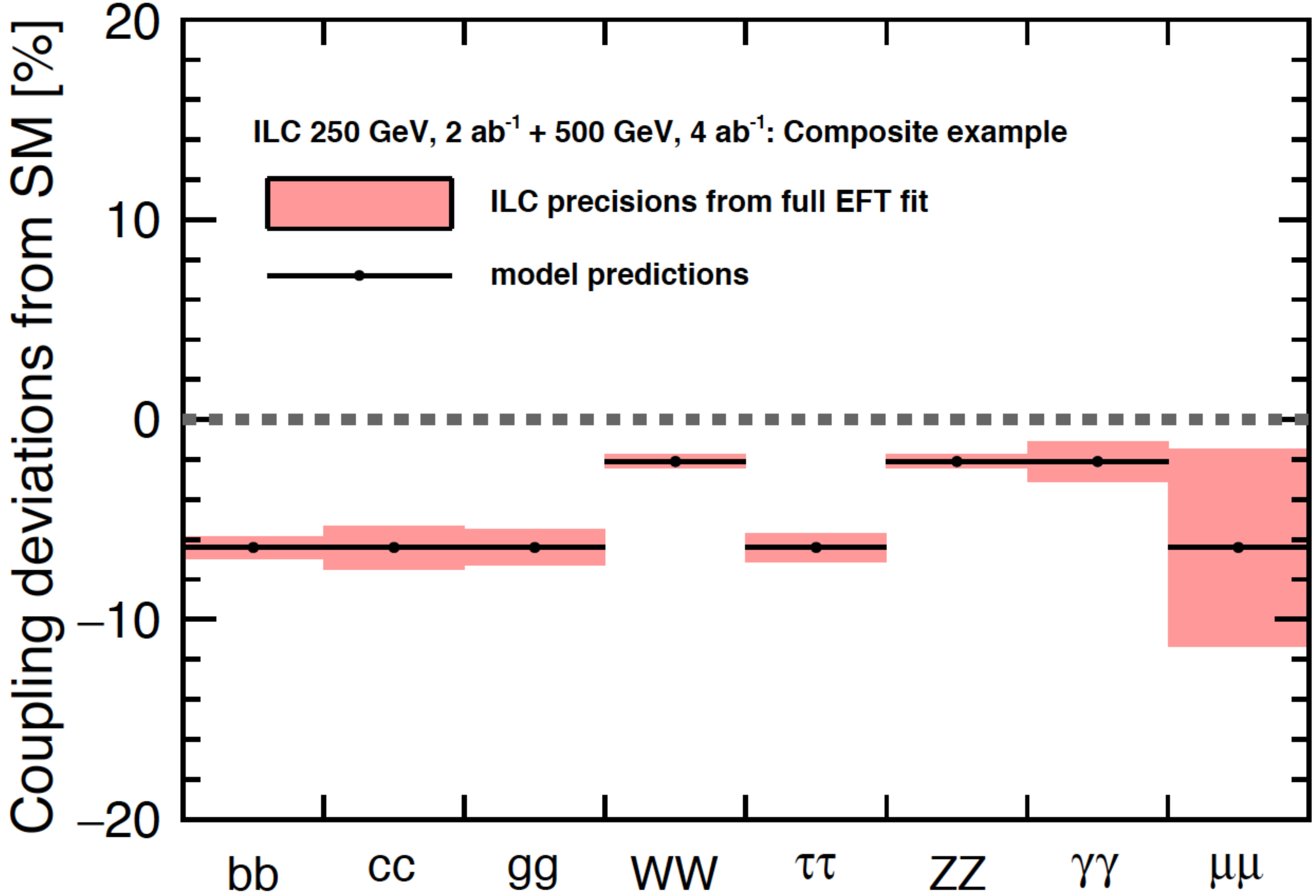
[*T. Barklow et al., '17*]




$\Rightarrow$ clear pattern, distinctive for 2HDM type Y?!

# Wäscheleine IV: $e^+e^-$ precision vs. Composite Higgs prediction:

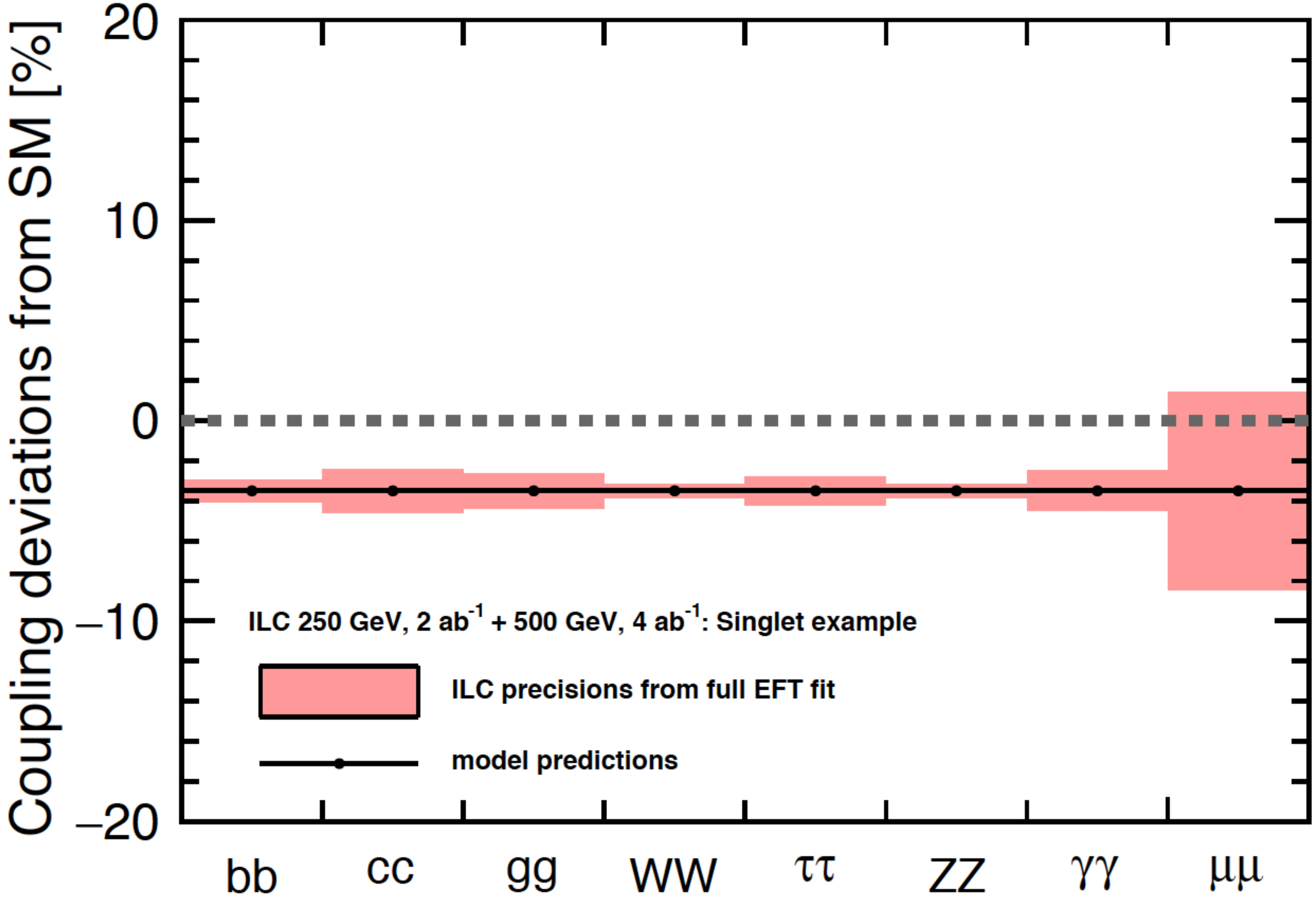
[*T. Barklow et al., '17*]




⇒ clear pattern, distinctive for Composite Higgs?!

# Wäscheleine V: $e^+e^-$ precision vs. HxSM prediction:

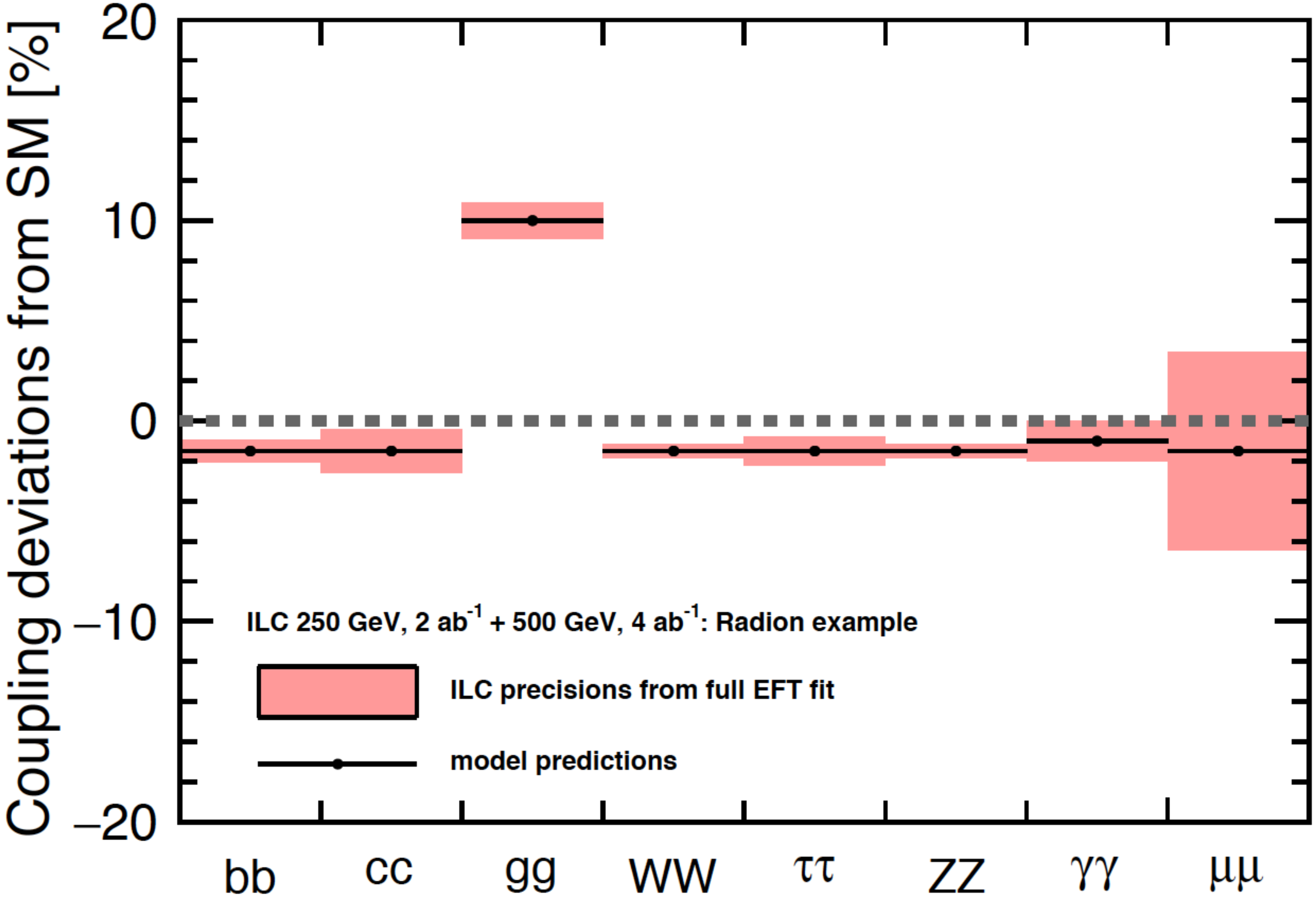
[*T. Barklow et al., '17*]




⇒ clear pattern, distinctive for HxSM?!

# Wäscheleine VI: $e^+e^-$ precision vs. Higgs-Radion prediction:

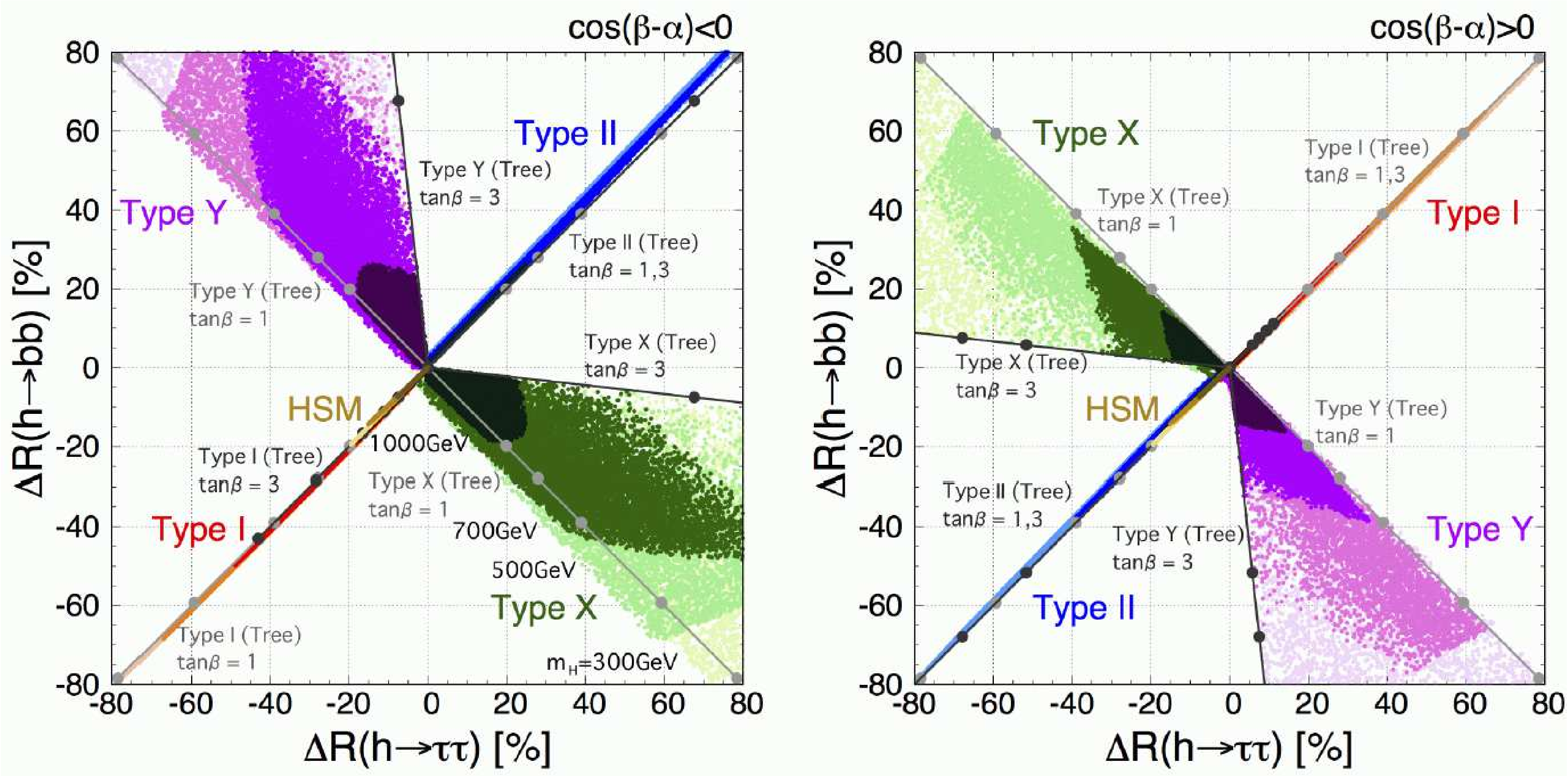
[*T. Barklow et al., '17*]




⇒ clear pattern, distinctive for Higgs Radion?!

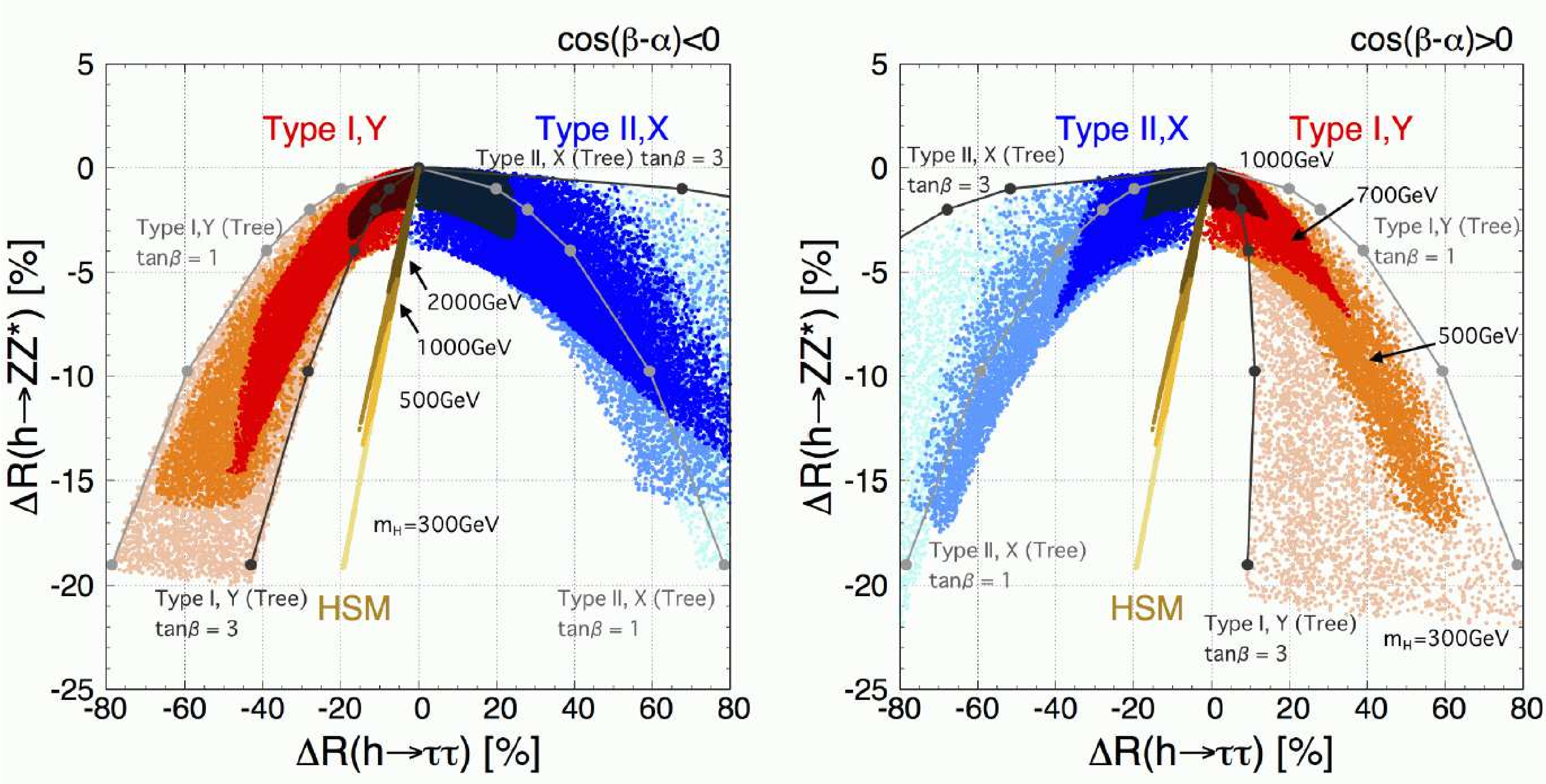
## 2HDM example (I):

[*S. Kanemura et al. '18*]



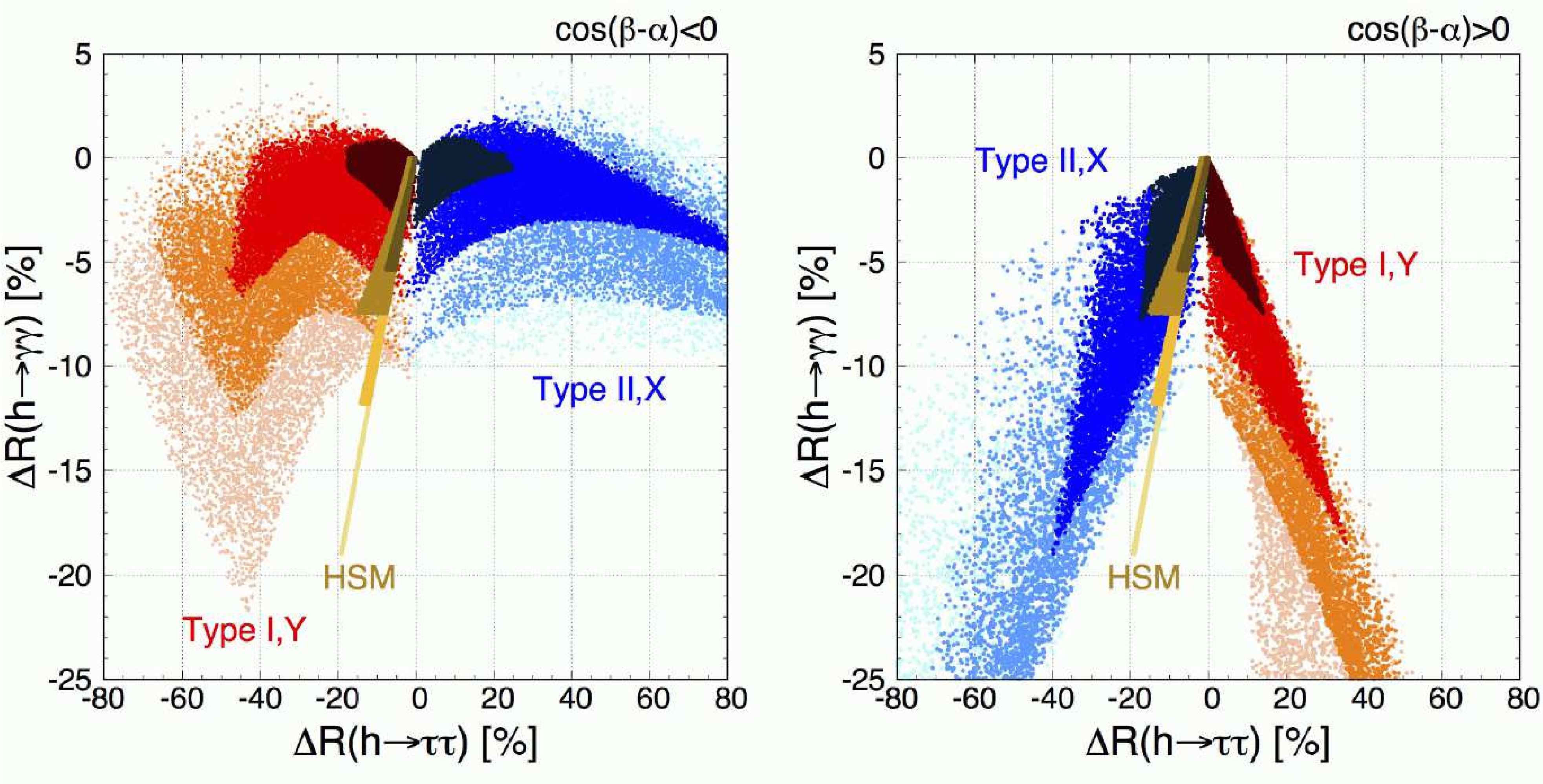

⇒ LC precision has a great potential to discriminate the models!

# 2HDM example (II): [S. Kanemura et al. '18]




⇒ LC precision has a great potential to discriminate the models!

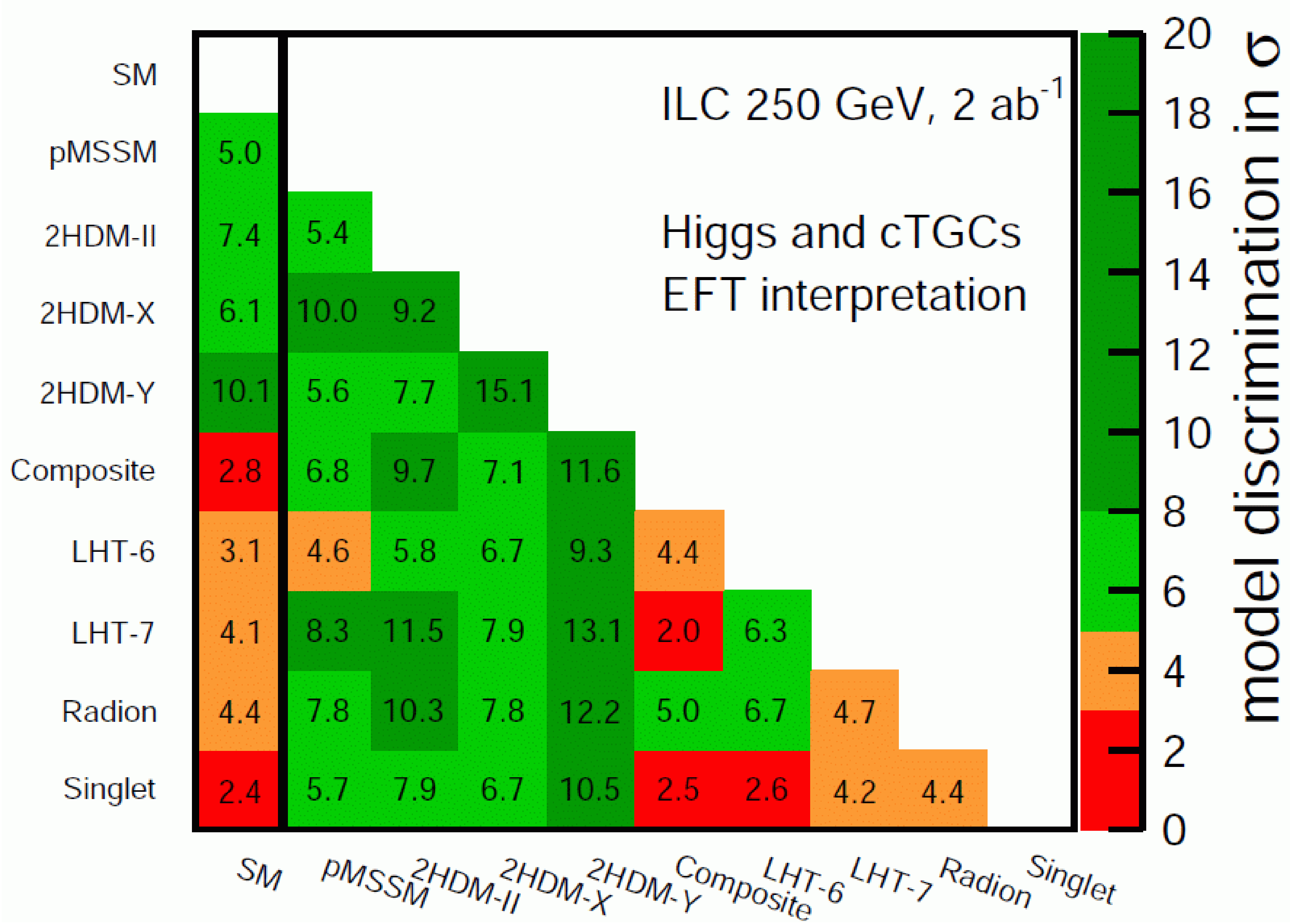
[S. Kanemura et al. '18]




⇒ LC precision has a great potential to discriminate the models!

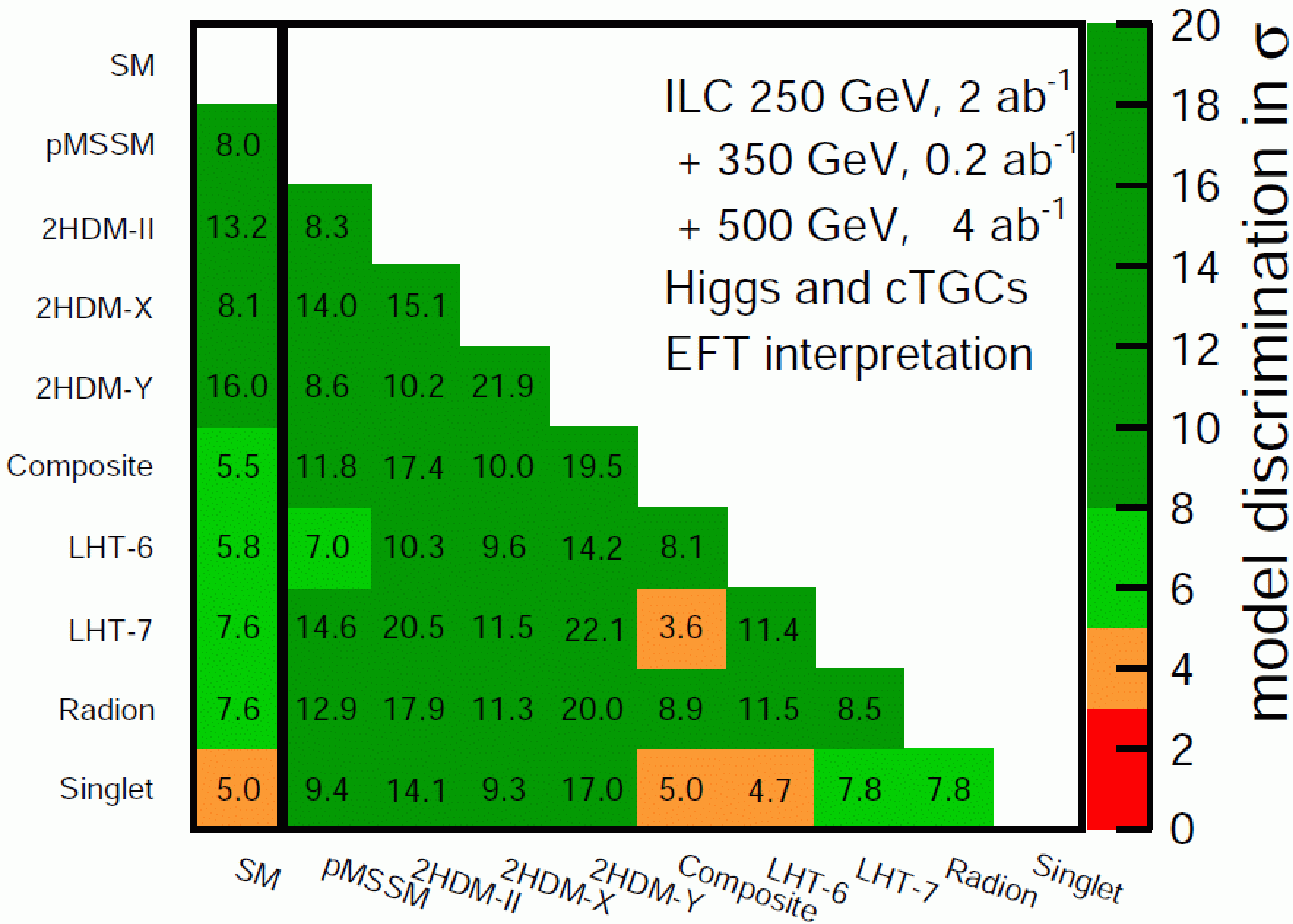
# Model discrimination at ILC250:

[*K. Fujii et al. '17*]



## Model discrimination at ILC500:

[K. Fujii et al. '17]



## Compare future colliders:

$\Rightarrow$ focus on Higgs searches and measurements

**HL-LHC:**

– will improve direct search limits

– will improve rate measurements (production $\times$ decay)
  systematic/theory uncertainties: S2 scenario

[*M. Cepeda et al. '19 – YR18*]

**ILC:** (similar for FCC-ee/CEPC/CLIC)

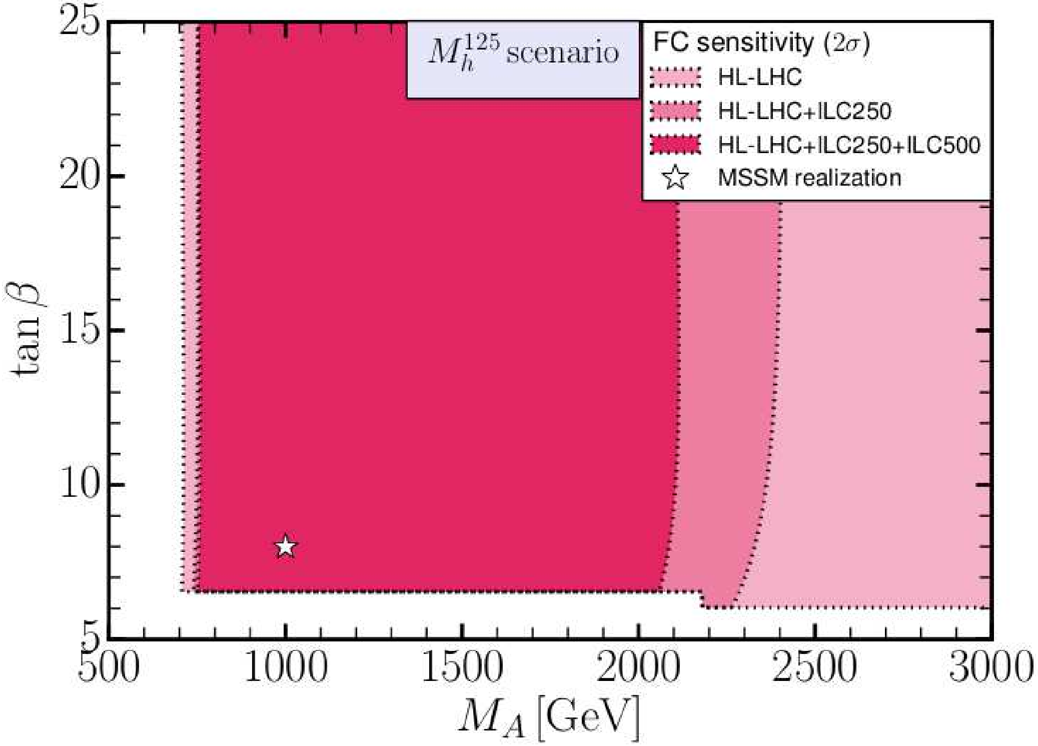
– will improve rate measurements (no theory assumptions!)
  – $250\,\text{fb}^{-1}$ at ILC250 $\oplus$ $500\,\text{fb}^{-1}$ at ILC500
  – polarization: $P(e^-, e^+) = (-80\%, +30\%)$

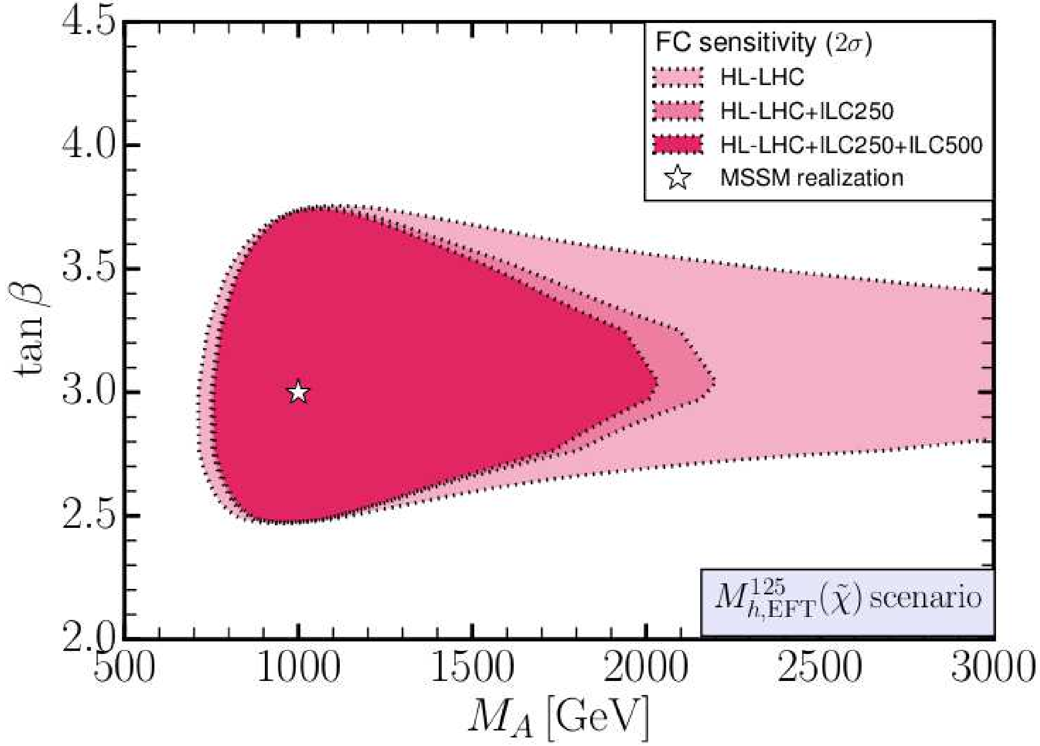
[*T. Barklow et al. '17, '19*]

## Relevance of ILC measurements:

[*H. Bahl et al., '20*]

– Assume a realization of an MSSM point: $M_A = 1$ TeV, $\tan\beta = 7/3$
– What limits can be set from rate/coupling measurements?



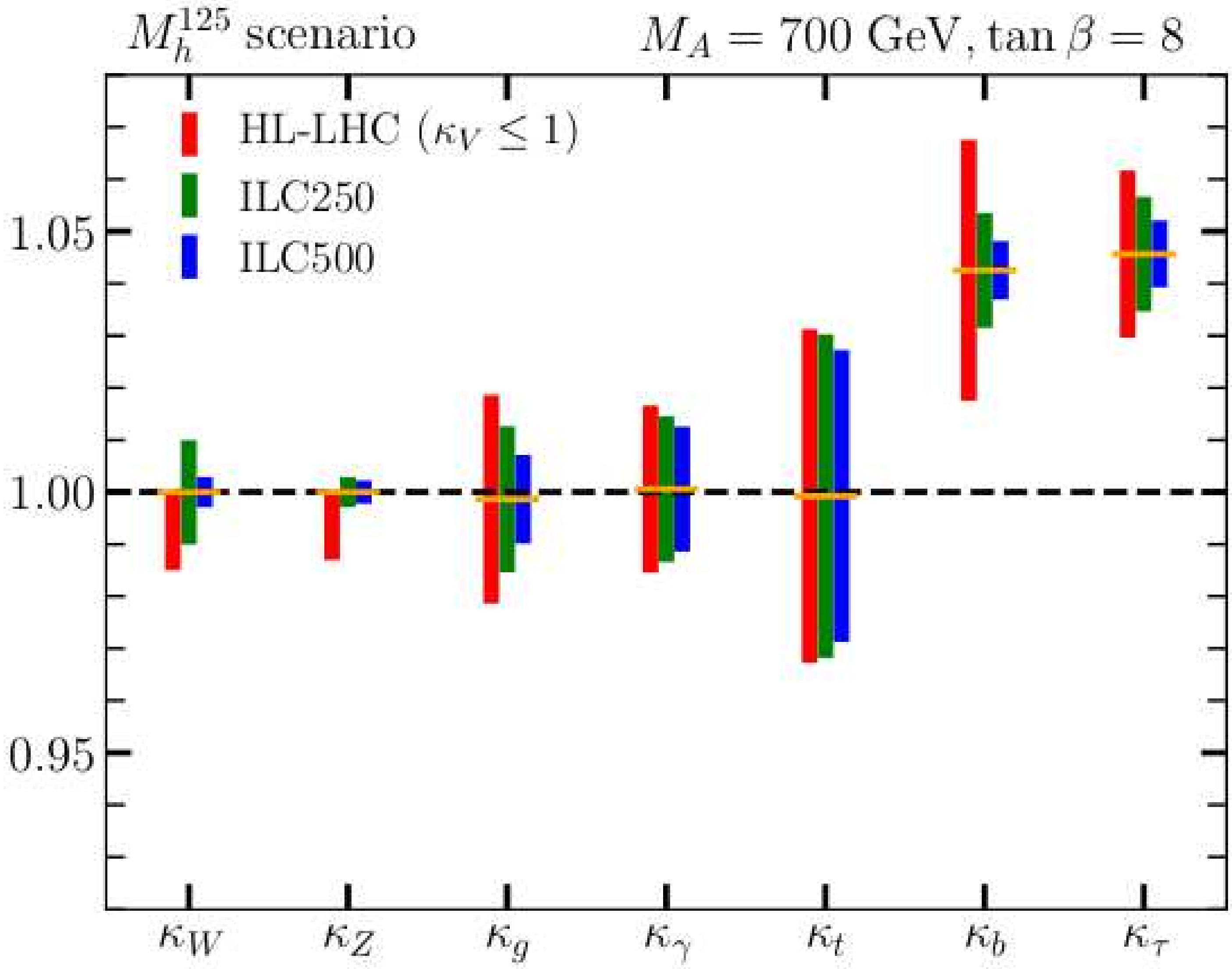




$\Rightarrow$ only ILC measurements give upper limit on $M_A$

$\Rightarrow$ limits on $\tan\beta$ only for small(er) $\tan\beta$

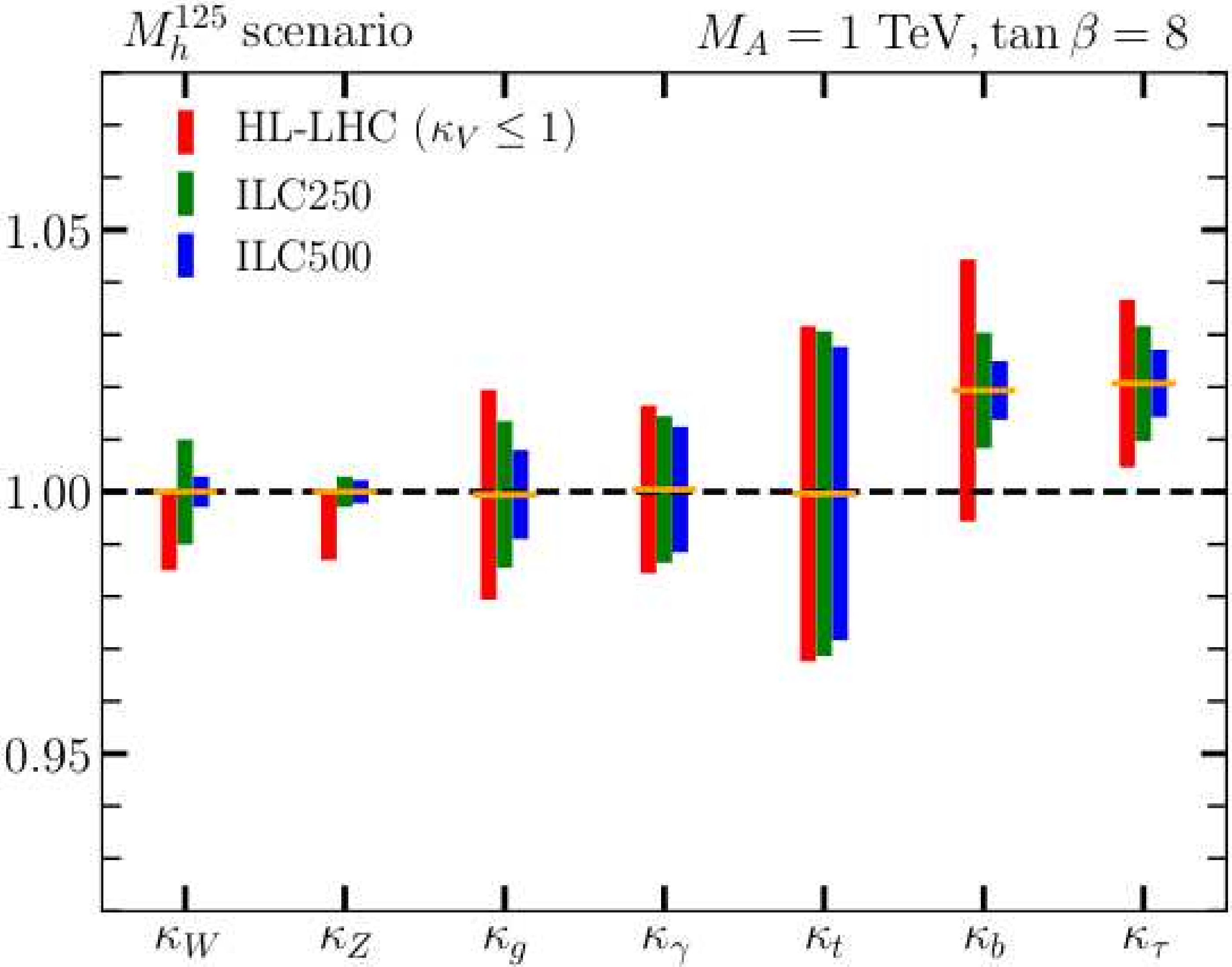
# MSSM Wäscheleine I: $e^+e^-$ precision vs. $M_h^{125}$ ($M_A = 700$ GeV, $\tan\beta = 8$)

[*H. Bahl et al. '20*]



## MSSM Wäscheleine II: $e^+e^-$ precision vs. $M_h^{125}$ ($M_A = 1000$ GeV, $\tan\beta = 8$)

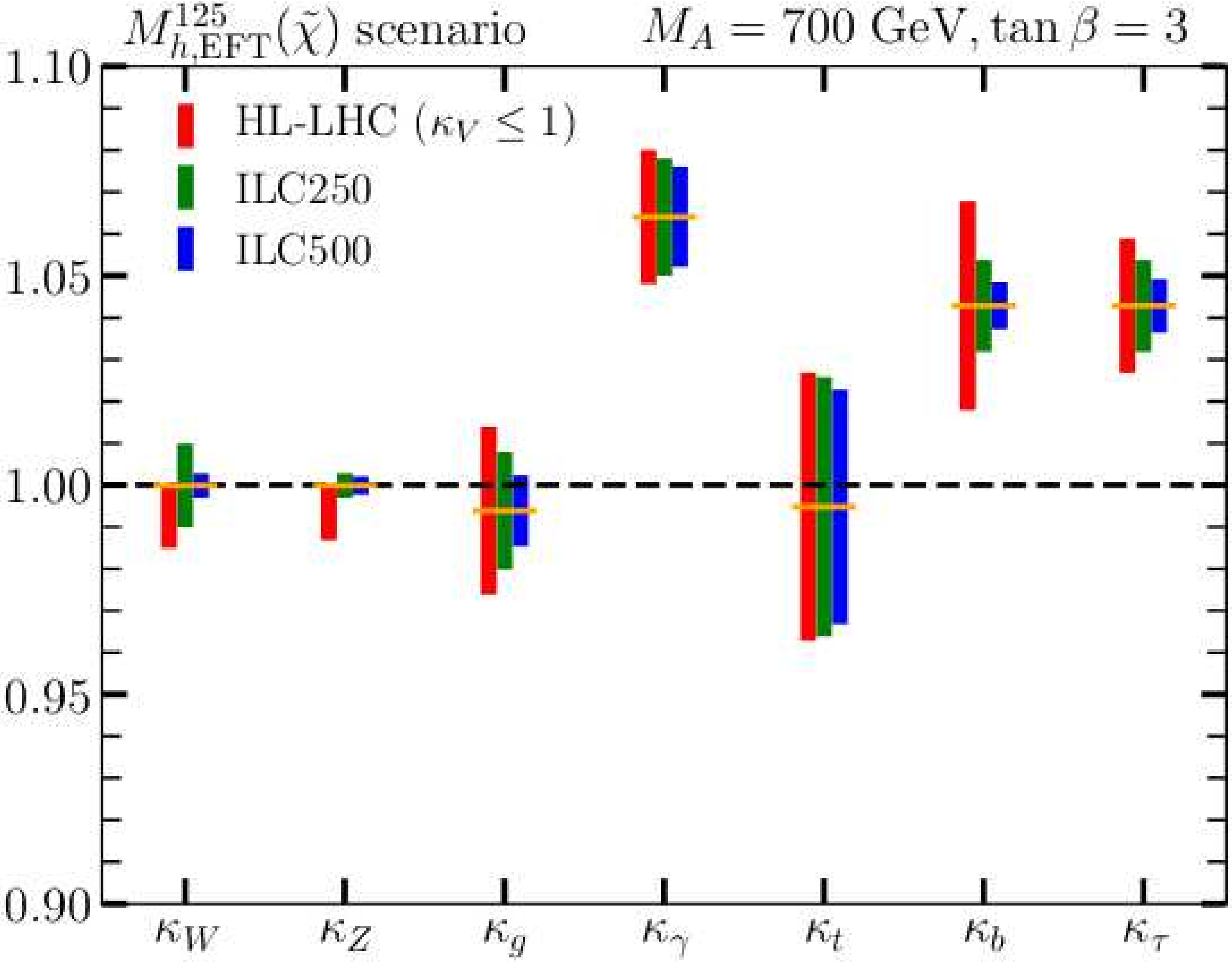
[*H. Bahl et al. '20*]




$\Rightarrow$ only $e^+e^-$ measurements allows to set upper limit on $M_A$

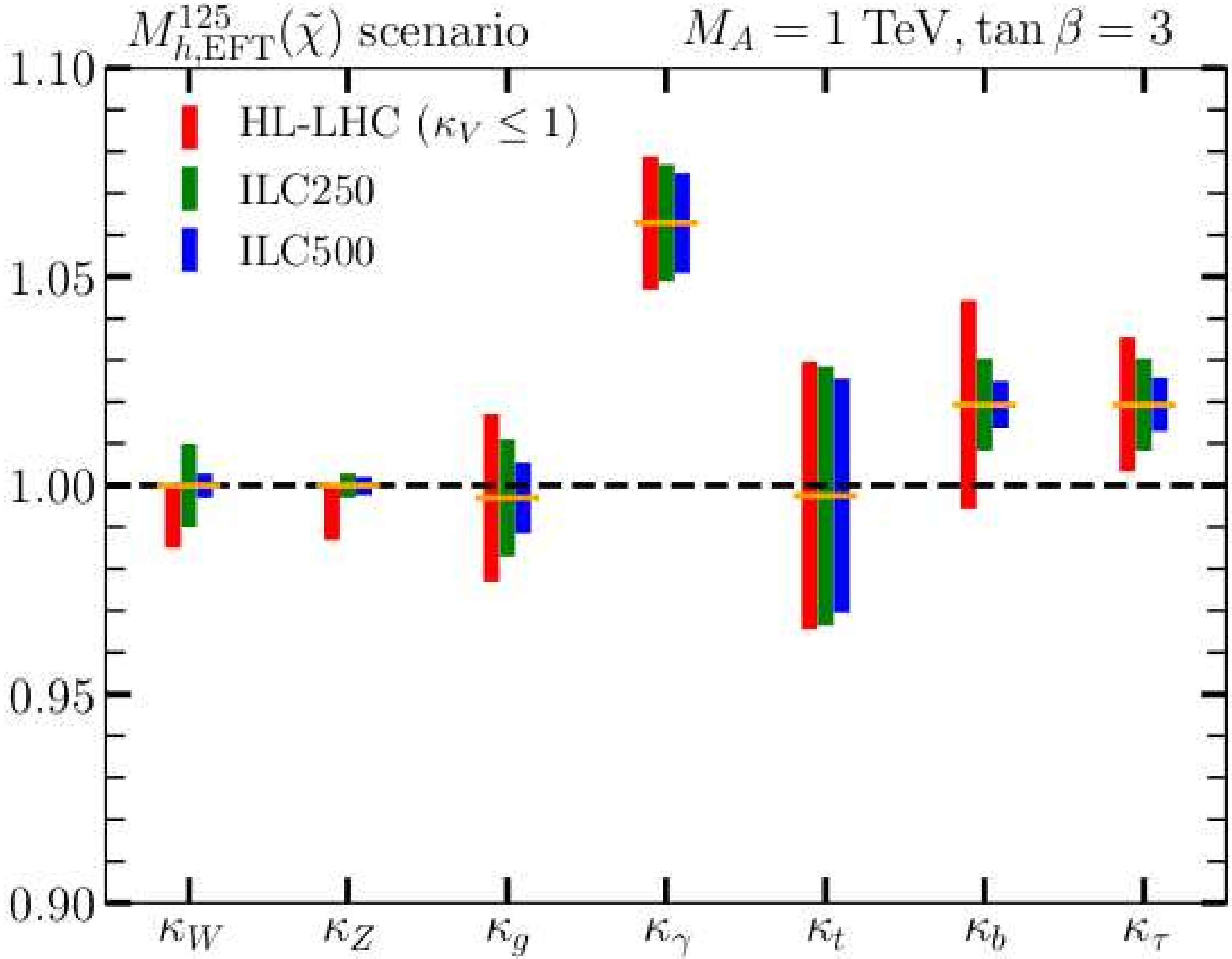
# MSSM Wäscheleine III: $e^+e^-$ vs. $M_h^{125,\text{EFT}}(\tilde{\chi})$ ($M_A = 700$ GeV, $\tan\beta = 3$)

[*H. Bahl et al. '20*]



## MSSM Wäscheleine IV: $e^+e^-$ vs. $M_h^{125,\text{EFT}}(\tilde{\chi})$ ($M_A = 1000$ GeV, $\tan\beta = 3$)

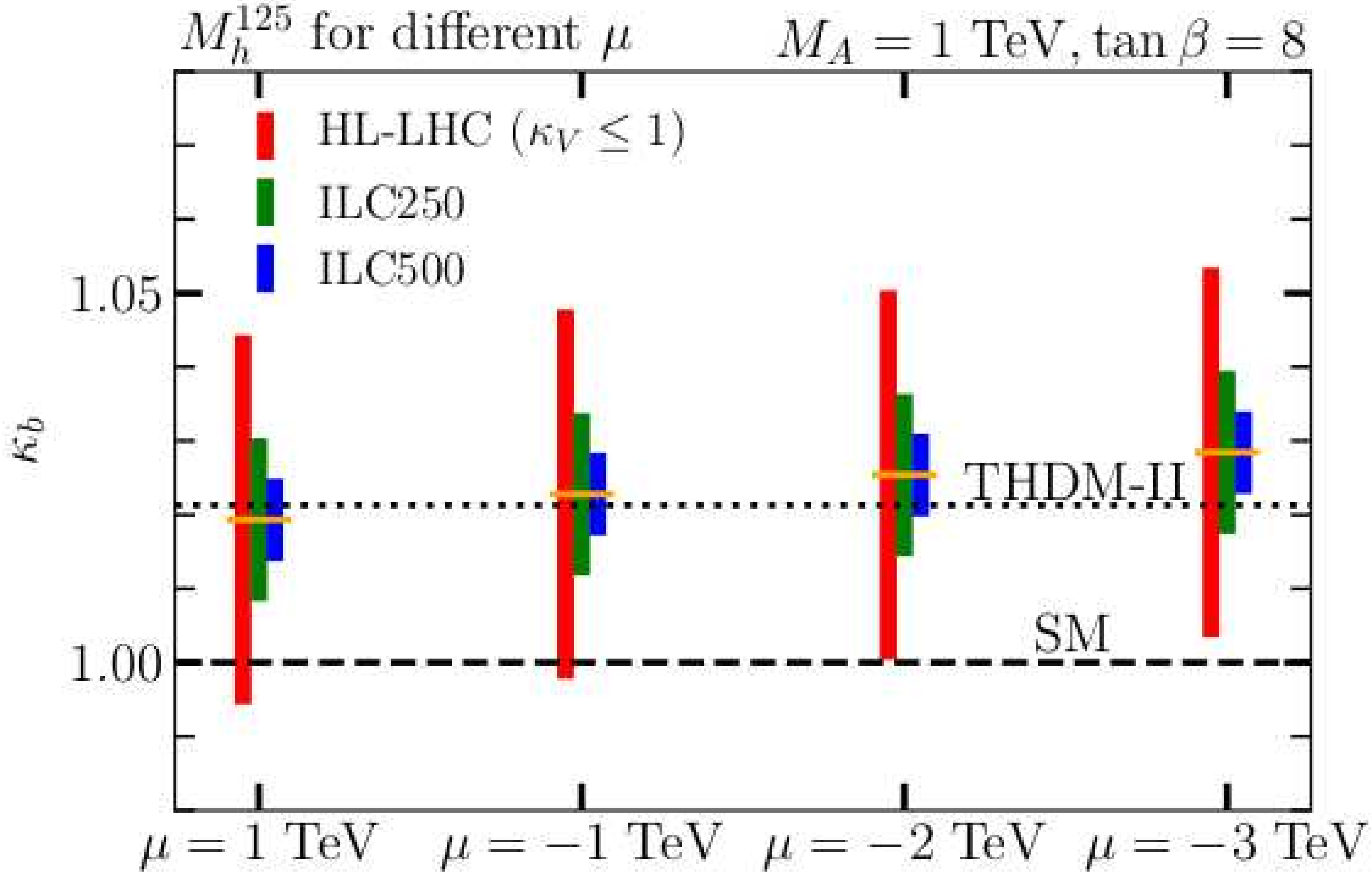
[*H. Bahl et al. '20*]




$\Rightarrow$ only $e^+e^-$ measurements allows to set upper limit on $M_A$

# MSSM Wäscheleine V: $e^+e^-$ vs. $M_h^{125}$ ($M_A = 1000$ GeV, $\tan\beta = 8$)

[*H. Bahl et al. '20*]




$\Rightarrow$ MSSM vs. 2HDM: very challenging!