Axion Theory & Model Building
(two routes through the PQ-quality problem)

Axions in Seoul

27 July 2026 - Seoul National University

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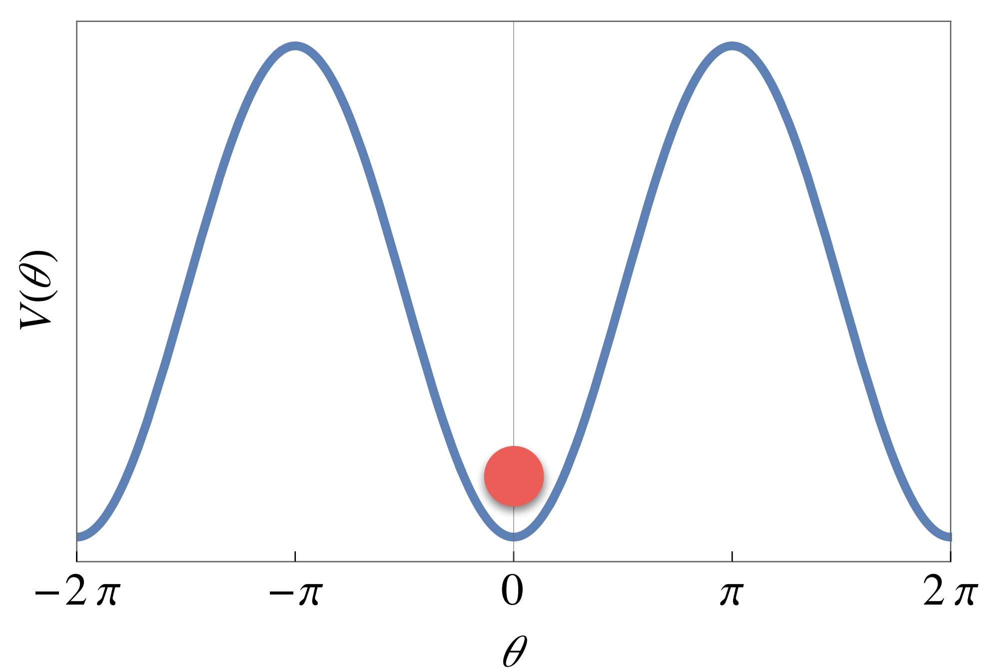
INFN PADOVA

The QCD axion

- Introduced to address the strong CP problem [Peccei, Quinn '77, Weinberg '78, Wilczek '78]

$$\delta\mathcal{L}_{\rm QCD} = \theta \frac{g_s^2}{32\pi^2} G\tilde{G} \qquad |\theta| \lesssim 10^{-10} \qquad \text{(bound from nEDM)}$$

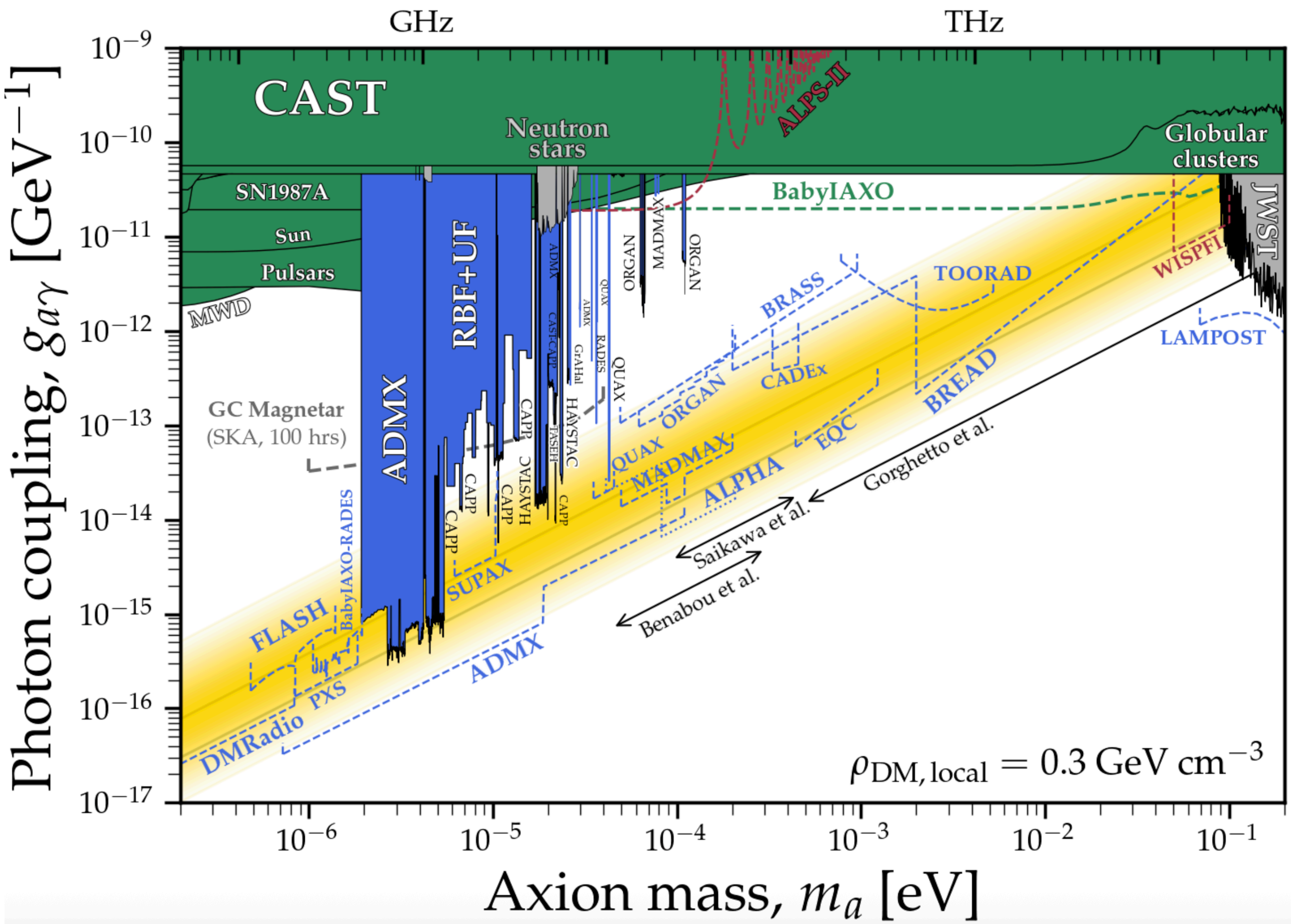
- promote θ to a dynamical field (axion): $\theta \to \dfrac{a}{f_a}$
- acquires a QCD potential and relaxes dynamically to zero
- axion oscillations in the early universe "behave" as cold dark matter



The axion hunt

- May you live in interesting times…

[https://cajohare.github.io/AxionLimits]



Back to PQ mechanism

- PQ as an approximate shift symmetry broken only by QCD

$$\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G} \longrightarrow E(0) \leq E(\langle a \rangle)$$

[Vafa, Witten PRL 53 (1984)]

Back to PQ mechanism

- PQ as an approximate shift symmetry broken only by QCD

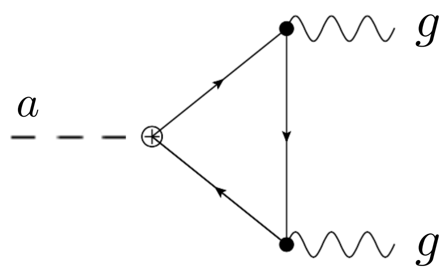
$$\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G} \longrightarrow E(0) \leq E(\langle a \rangle)$$

[Vafa, Witten PRL 53 (1984)]

- its origin* can be traced back to a global $U(1)_{PQ}$ [Peccei, Quinn '77, Weinberg '78, Wilczek '78]

1. spontaneously broken (the axion is the associated pNGB)

2. QCD anomalous



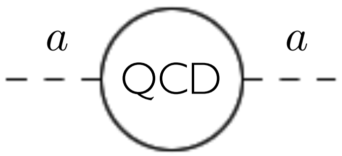
$$\partial^\mu J_\mu^{PQ} = \frac{N\alpha_s}{4\pi} G \cdot \tilde{G}$$

**axions can also arise as zero modes from string theory compactification* [See talk by M. Reece]

Axion properties

- Consequences of $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G}$

Axion mass

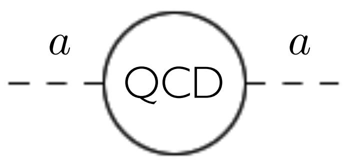


$\sim \frac{\Lambda_{\rm QCD}^4}{f_a^2} \longrightarrow m_a \sim \Lambda_{\rm QCD}^2 / f_a$

Axion properties

- Consequences of $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G}$

Axion mass



$\sim \frac{\Lambda^4_{\mathrm{QCD}}}{f_a^2}$

$$m_a = 5.691(51)\,\text{meV}\left(\frac{10^9\,\text{GeV}}{f_a}\right)$$

[Gorghetto, Villadoro 1812.01008 (NNLO chiPT)
Bonati et al, 1512.06746 + Borsanyi et al 1606.07494 (lattice)]

$f_a \gtrsim 10^9$ GeV from astrophysics

very light & weakly coupled

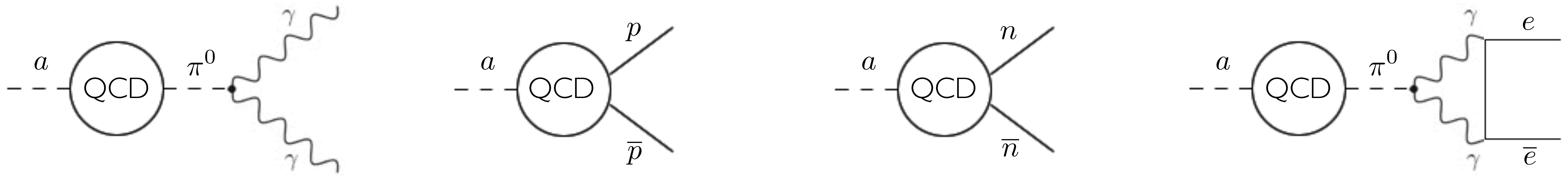
experimentally, more useful to think about the axion as a "wave"

$$\lambda_a = \frac{h}{m_a c} \sim 20\ \text{cm}\ \frac{\mu\text{eV}}{m_a}$$

Axion properties

- Consequences of $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G}$

Axion couplings to photons, nucleons, electrons, …



$C_\gamma = -1.92(4)$ $\quad$ $C_p = -0.47(3)$ $\quad$ $C_n = -0.02(3)$ $\quad$ $C_e = -7.8(2)\times 10^{-6}\log\left(\frac{f_a}{m_e}\right)$

$$\mathcal{L}_a \supset \frac{\alpha}{8\pi}\frac{C_\gamma}{f_a} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{C_f}{2f_a}\partial_\mu a \overline{f}\gamma^\mu\gamma_5 f \qquad (f = p, n, e)$$

[Grilli di Cortona, Hardy, Vega, Villadoro 1511.02867 (NLO chiPT)
Lu, Du, Guo, Meißner, Vonk 2003.01625 (NNLO chiPT)]

Axion properties

- Consequences of $\frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G}$

Axion couplings to photons, nucleons, electrons, …

→ A motivated target for experiments,
although UV completion can drastically affect low-energy axion properties

[see e.g. LDL, Giannotti, Nardi, Visinelli 2003.01100 (Phys. Rept.)]

$$C_\gamma = -1.92(4) \qquad C_p = -0.47(3) \qquad C_n = -0.02(3) \qquad C_e = -7.8(2)\times 10^{-6}\log\left(\frac{f_a}{m_e}\right)$$

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Benchmark axion models

- global U(1)$_{PQ}$ (QCD anomalous + spontaneously broken)

$$U(1)_{\rm PQ} \times SU(3)_c^2$$

SM fermions

2Higgs

PQWW

[Peccei, Quinn '77,
Weinberg '78, Wilczek '78]

$f_a \sim v$ ruled out

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- global $U(1)_{PQ}$ (QCD anomalous + spontaneously broken)

VOLUME 43, NUMBER 2 PHYSICAL REVIEW LETTERS 9 JULY 1979

Weak-Interaction Singlet and Strong *CP* Invariance

Jihn E. Kim
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 16 February 1979)

Strong *CP* invariance is *automatically* preserved by a spontaneously broken chiral $U(1)_4$ symmetry. A weak-interaction singlet heavy quark Q, a new scalar meson σ^0, and *a very light axion* are predicted. Phenomenological implications are also included.

here is the birth of the "invisible" axion !

Benchmark axion models

- global $U(1)_{PQ}$ (QCD anomalous + spontaneously broken)

$$U(1)_{\rm PQ} \times SU(3)_c^2$$

SM fermions | BSM fermions

2Higgs | 2Higgs+Singlet | Higgs+Singlet

PQWW | DFSZ | KSVZ

[Peccei, Quinn '77, Weinberg '78, Wilczek '78]

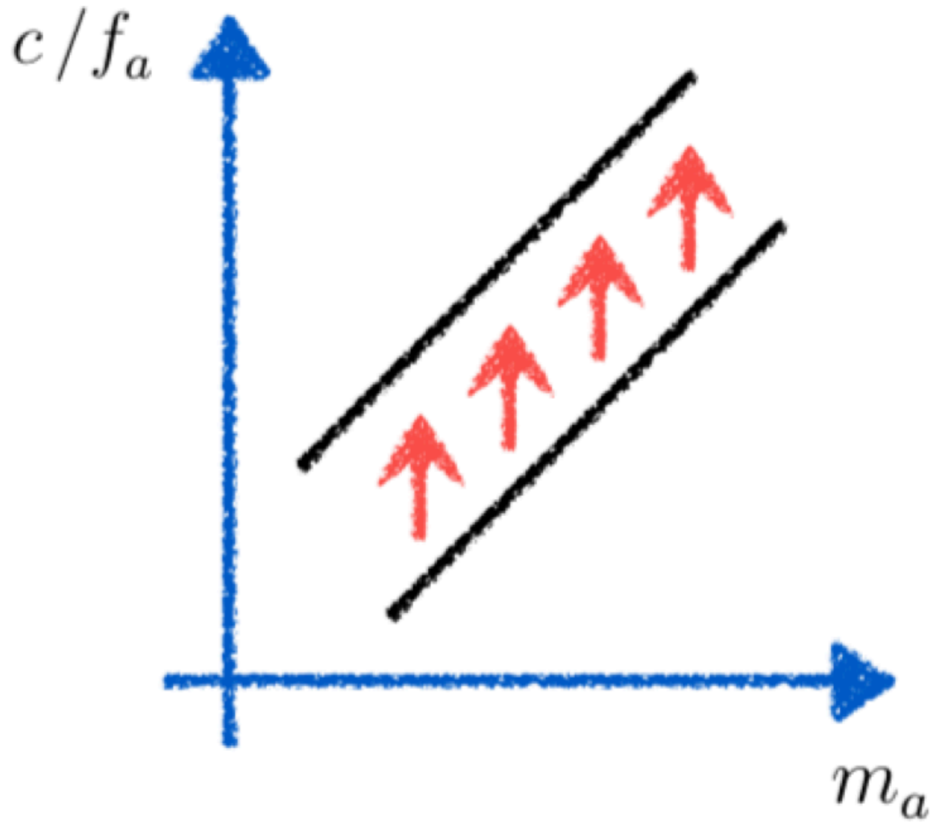
[Zhitnitsky '80, Dine, Fischler, Srednicki '81]

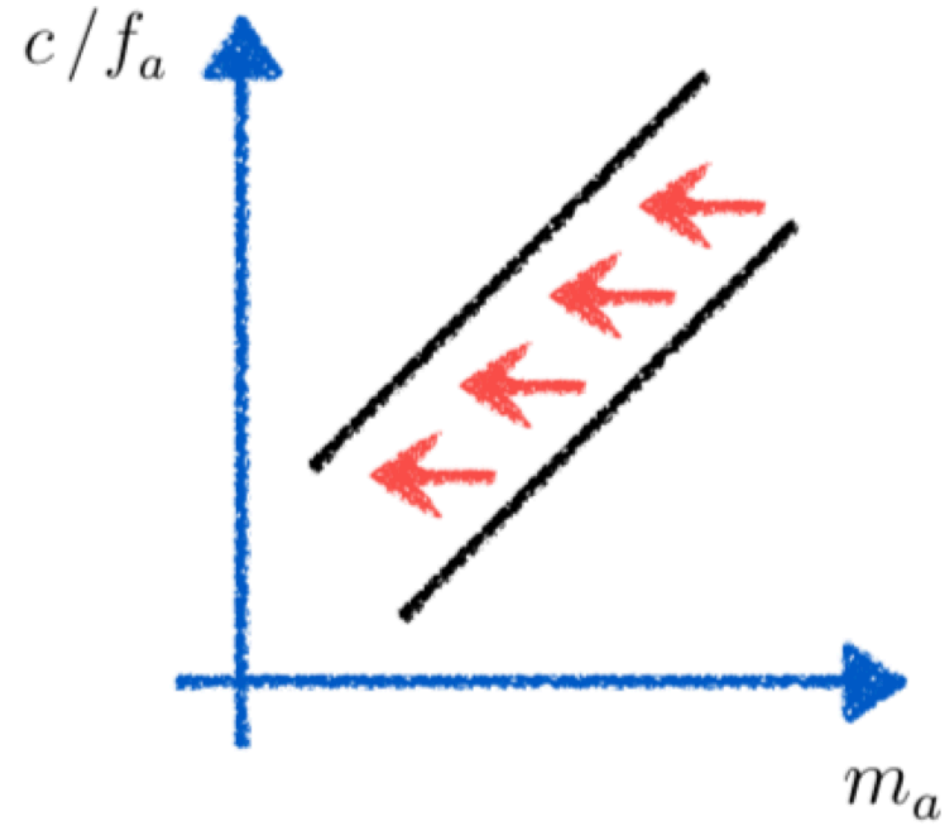
[Kim '79, Shifman, Vainshtein, Zakharov '80]

$f_a \sim v$ ruled out

$f_a \gg v$ "Invisible" axion (phase of singlet field, mostly)

KSVZ & DFSZ are landmarks, not boundaries





enhance Wilson coefficient for fixed m_a

[LDL, Mescia, Nardi 1610.07593 + 1705.05370
Farina, Pappadopulo, Rompineve, Tesi 1611.09855
Agrawal, Fan, Reece, Wang 1709.06085
Darme', LDL, Giannotti, Nardi 2010.15846
Ringwald, Sokolov 2104.02574, …]

suppress axion mass for fixed f_a

[Hook 1802.10093,
LDL, Gavela, Quilez, Ringwald 2102.00012 + 2102.01082, …]

→ QCD axion parameter space <u>much larger</u> than what traditionally thought

The dirty side of the axion

- U(1)$_{PQ}$ is not expected to be exact (being a global symmetry)

$$\theta_{\text{eff}} \equiv \frac{\langle a \rangle}{f_a} \lesssim 10^{-10}$$

$$\mathcal{O}_{\text{PQ}-\text{break}} = \frac{\phi^d}{\Lambda_{\text{UV}}^{d-4}} \quad \longrightarrow \quad \left(\frac{f_a}{\Lambda_{\text{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\text{QCD}}^4$$

$$\phi \sim f_a e^{i\frac{a}{f_a}}$$

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[Georgi, Hall, Wise NPB 192 (1981)
Barr, Seckel PRD 46 (1992)
Kamionkowski, March-Russell PLB 282 (1992)
Holman+ PLB 282 (1992), …]

$\longrightarrow$ $d_{\rm PQ} \gtrsim 9$ (for $\Lambda_{\rm UV} \sim M_{\rm Pl}$ and $f_a \sim 10^9$ GeV) PQ-quality problem

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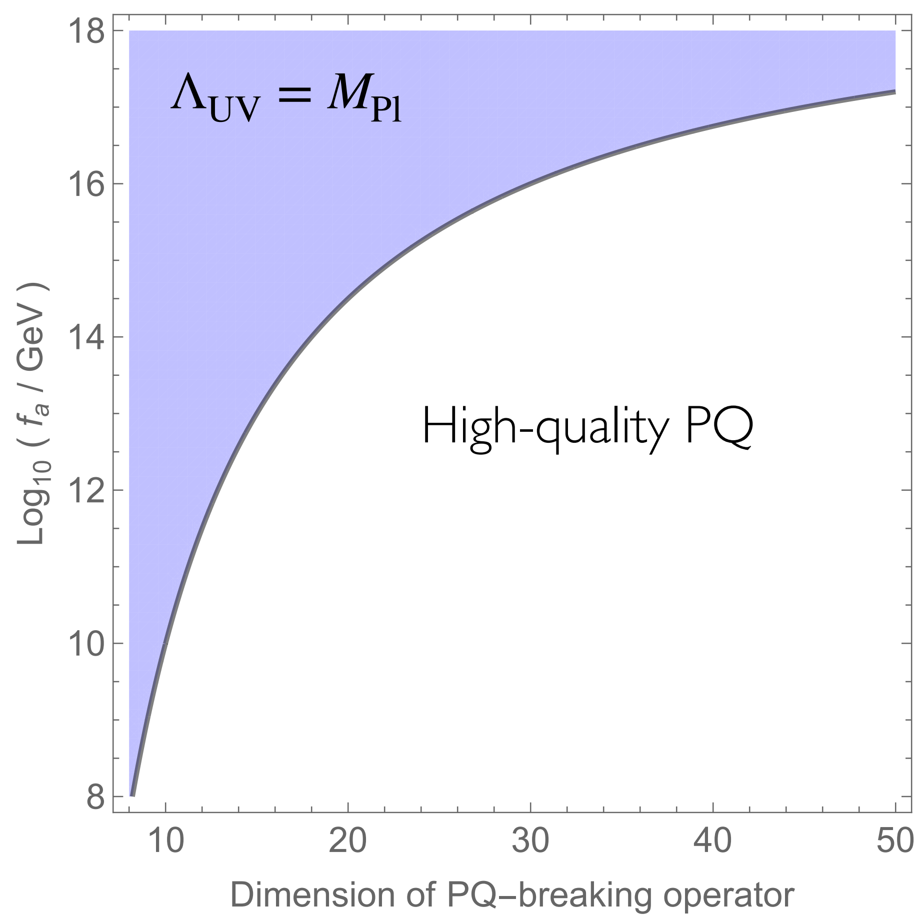

$$\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4 \lesssim 10^{-10}\Lambda_{\rm QCD}^4$$

[Georgi, Hall, Wise NPB 192 (1981)
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→ $d_{\rm PQ} \gtrsim 9$ (for $\Lambda_{\rm UV} \sim M_{\rm Pl}$ and $f_a \sim 10^9$ GeV)

PQ-quality problem

This talk

- Two recent model-building attempts, related to the PQ quality problem*

Lower f_a

can the PQ symmetry be broken at the QCD scale, and the axion be hidden among the known pseudo-scalar resonances ?

[Murayama 2601.02465
LDL, Redi Strumia, Tesi, Titov 2602.10057]

Raise $d_{\rm PQ}$

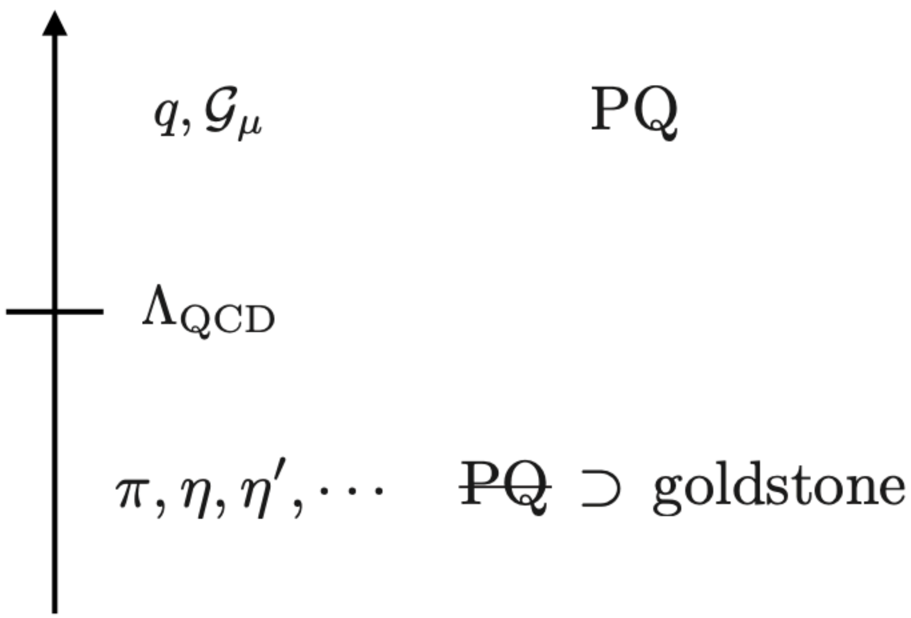
models where the PQ arises accidentally, face a quality/cosmology tension in the post-inflationary regime

[Lu, Reece, Sun 2312.07650
LDL, Di Valeriano, Nardecchia, Nardi (in preparation)]

*other routes: extra-dimensional axions or a QCD-aligned PQ-breaking sector raising m_a

Part I - Lower f_a

- The QCD condensate spontaneously breaks the PQ symmetry [Murayama 2601.02465]



How to realise this in practice ?

PQ needs anomaly under color

New light colored fermions *<u>not</u>* *allowed*

Part I - Lower f_a

- The QCD condensate spontaneously breaks the PQ symmetry [Murayama 2601.02465]

$q, \mathcal{G}_\mu$	$\varphi \bar{u} u$		
$\Lambda_{\rm QCD}$	$\langle \bar{u} u \rangle \sim B_0 f_\pi^2$	$\longrightarrow$	$\langle \varphi \rangle \neq 0$
$\pi, \eta, \eta', \cdots$	$\varphi \propto \sigma e^{ia}$		

[akin to bosonic technicolor with elementary Higgs; Samuel NPB 347 (1990)]

- The only way to have PQ broken at the QCD scale:

  - is to deform the massless up-quark solution …

  - … rendering the up-quark massless prior to chiral SSB

  - to mitigate the impact of lattice results, effective quark masses induced by a PQ scalar

$\longrightarrow$ *very different from usual QCD axion, where PQ is broken at high energies !*

Part I - Lower f_a

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- Interesting (but critical) consequences:

  - new scalar/pseudo-scalar around GeV scale (hidden among QCD resonances)
  - no astrophysical limits, lab constraints (flavor & collider) seem OK
  - solves PQ quality problem, but no dark matter since unstable
  - sizeable coupling to light quarks, alters χPT predictions

Model and generalizations

- Renormalizable Lagrangian below the electroweak scale* [LDL, Redi Strumia, Tesi, Titov 2602.10057]

$$\mathscr{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} + i \sum_{q=u,d,s} \bar{q} \not{D} q$$
$$+ f_\varphi^2 \left[|\partial_\mu \varphi|^2 - m_\varphi^2 |\varphi|^2 - \lambda f_\varphi^2 |\varphi|^4 \right] - \sum_{q=\{u,d,s\}} m_q (\sqrt{2}\varphi)^{n_q} \bar{q}_L q_R + \text{h.c.} \qquad m_q = \kappa_q \frac{f_\varphi}{\sqrt{2}}$$

- PQ complex field φ (dimensionless), no quadratic instability $m_\varphi^2 > 0$

- if you prefer, work with $\varphi_{\text{can}} \equiv f_\varphi \varphi$

**UV completion at the EW scale discussed by Murayama (see backup slides)*

Model and generalizations

- Renormalizable Lagrangian below the electroweak scale [LDL, Redi Strumia, Tesi, Titov 2602.10057]

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- PQ complex field φ (dimensionless), no quadratic instability $m_\varphi^2 > 0$

- Light quarks can have PQ charges: $\frac{E}{N} \equiv \frac{2}{3} \frac{(4n_u + n_d + n_s)}{\sum_q n_q}, \quad N_{\mathrm{DW}} = \left| \sum_q n_q \right|$

$\{n_u, n_d, n_s\} = \{1, 0, 0\}$ [massless up] **[Murayama]**

$\{n_u, n_d, n_s\} = \{0, 1, 0\}$ [massless down]

$\{n_u, n_d, n_s\} = \{1, 1, 0\}$ [massless up-down]

$\{n_u, n_d, n_s\} = \{0, 0, 1\}$ [massless strange :)]

$T_{\mathrm{PQ}} = \mathrm{diag}[n_u, n_d, n_s]$

PQ symmetry generator, needs to have non-zero trace

Matching to chiral Lagrangian

- PQ scalar φ as an external field, keep track in the mass matrix spurion $M_q(\varphi)$

$$\mathcal{L}_{\pi\varphi} = f_\varphi^2 \left[|\partial_\mu \varphi|^2 - m_\varphi^2 |\varphi|^2 - \lambda f_\varphi^2 |\varphi|^4 \right] + \frac{f_\pi^2}{4} \mathrm{tr}[D_\mu U D^\mu U^\dagger] + \frac{f_\pi^2 B_0}{2} \mathrm{tr}[M_q U^\dagger + M_q^\dagger U] - \frac{m_1^2 f_\pi^2}{12} \left[\left(-\frac{i}{2} \ln\det U + h.c. \right) - \theta \right]^2$$

- $U = \exp(i\sqrt{2}\Pi/f_\pi)$ transforms as $U \to L U R^\dagger$ under $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$

$$\frac{\Pi}{f_\pi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} \end{pmatrix}$$

Matching to chiral Lagrangian

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- quark mass matrix depends on PQ scalar

$$M_q = \mathrm{diag}\left[m_u (\sqrt{2}\varphi)^{n_u},\, m_d (\sqrt{2}\varphi)^{n_d},\, m_s (\sqrt{2}\varphi)^{n_s} \right]$$

- SSB of PQ controlled by chiral condensate, e.g. for $\{n_u, n_d, n_s\} = \{1,0,0\}$ model

$$\frac{f_\pi^2 B_0}{2} \mathrm{tr}[M_q + h.c.] = \sqrt{2} f_\pi^2 B_0 m_u \varphi + \cdots$$

- VEV from tadpole condition

$$f_\varphi^2 = \frac{f_\pi^2 B_0 m_u}{m_\varphi^2}$$

Spectrum

- Polar decomposition $\varphi = \frac{1+\sigma}{\sqrt{2}} e^{ia}$

  - tadpole condition from minimum in $\sigma = 0$ and $a = 0$ $\longrightarrow$ $\langle \varphi \rangle = \frac{1}{\sqrt{2}}$

$$m_\varphi^2 = \frac{B_0 f_\pi^2}{f_\varphi^2} \sum_q m_q n_q - \lambda f_\varphi^2$$

  - $m_\varphi^2 > 0$ for $f_\varphi \lesssim \mathrm{GeV} \left(\frac{5 \times 10^{-5}}{\lambda} \right)^{1/4}$ $\longrightarrow$ *upper bound on quartic for fixed f_φ*

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$$m_\varphi^2 = \frac{B_0 f_\pi^2}{f_\varphi^2} \sum_q m_q n_q - \lambda f_\varphi^2 \approx \frac{B_0 f_\pi^2}{\sqrt{2} f_\varphi} \sum_q \kappa_q n_q = \frac{B_0 f_\pi^2}{2} \sum_q \frac{n_q \kappa_q^2}{m_q}$$

neglect quartic + $m_q = \frac{\kappa_q}{\sqrt{2}} f_\varphi$

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  - e.g. in Murayama's model $\{n_u, n_d, n_s\} = \{1,0,0\}$

$$m_\varphi \approx 2.3 \kappa_u \mathrm{GeV} \qquad\qquad f_\varphi \approx 3 \kappa_u^{-1} \mathrm{MeV}$$

  - CP-even radial mode: $m_\sigma^2 = m_\varphi^2 + 3\lambda f_\varphi^2$
  - CP-odd Goldstone mode mixes with π, η, η' (next slides): $m_a^2 \approx m_\varphi^2 - 2\lambda f_\varphi^2$

$\rightarrow$ *does not look like the usual axion since* $f_\varphi \ll f_\pi \approx 92$ *MeV*

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- Polar decomposition $\varphi = \frac{1+\sigma}{\sqrt{2}} e^{ia}$

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  - actual numbers depend on model (which quark is massless prior to chiral SSB)

Where is φ ?

- New state coupled to QCD, hidden among heavy iso-singlet resonances

State	J^{PC}	Mass [MeV]	Width [MeV]
$\eta(547)$	0^{-+}	547.9 ± 0.0	$(1.31 \pm 0.05) \times 10^{-3}$
$\eta'(958)$	0^{-+}	957.8 ± 0.1	0.198 ± 0.009
$\eta(1295)$	0^{-+}	1294 ± 4	55 ± 5
$\eta(1405)$	0^{-+}	1409 ± 2	51 ± 7
$\eta(1475)$	0^{-+}	1476 ± 4	85 ± 9
$\eta(1760)$	0^{-+}	1756 ± 9	96 ± 70
$\eta(2225)$	0^{-+}	2221 ± 8	185 ± 40
$f_0(500)$	0^{++}	400–550	400–700
$f_0(980)$	0^{++}	990 ± 20	40–100
$f_0(1370)$	0^{++}	1200–1500	200–500
$f_0(1500)$	0^{++}	1505 ± 6	109 ± 7
$f_0(1710)$	0^{++}	1723 ± 6	135 ± 8
$f_0(2020)$	0^{++}	1992 ± 16	442 ± 60
$f_0(2200)$	0^{++}	2189 ± 13	238 ± 50
$f_0(2330)$	0^{++}	2325 ± 35	200 ± 70

[PDG]

- main decay channels hadronic: $\Gamma(a \to 3\pi) \approx 0.1\kappa_u^2 m_a, \;\; \Gamma(\sigma \to \pi\pi) \approx \frac{\kappa_u^2}{8\pi} m_\sigma.$

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$\kappa_u \sim 0.6$

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[PDG]

[Whether one or two different states - η(1405) and η(1475) - exist is an open question, in which case the η(1405) would be supernumerary, i.e. do not fit 0^{-+} nonet (Sect. 63.4 in PDG)]

[One of the three iso-scalars $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ appears to be supernumerary, i.e. do not fit 0^{++} nonet (Sect. 63.3.1 in PDG)]

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Main issue

- New state a mixes with pseudo-scalar mesons: (a, π^0, η, η')

$$\mathcal{M}^2 = B_0 \begin{pmatrix} \frac{f_\pi^2}{f_\varphi^2}\sum_q m_q n_q^2 & \frac{f_\pi}{f_\varphi}(m_d n_d - m_u n_u) & \frac{-f_\pi}{\sqrt{3}\, f_\varphi}(m_u n_u + m_d n_d - 2 m_s n_s) & \frac{-\sqrt{2} f_\pi}{\sqrt{3}\, f_\varphi}\sum_q m_q n_q \\ - & m_u + m_d & \frac{(m_u - m_d)}{\sqrt{3}} & \sqrt{\frac{2}{3}}(m_u - m_d) \\ - & - & \frac{1}{3}(m_d + m_u + 4 m_s) & \frac{\sqrt{2}}{3}(m_d + m_u - 2 m_s) \\ - & - & - & m_1^2 + \frac{2}{3}(m_u + m_d + m_s) \end{pmatrix}$$

Main issue

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$f_\varphi \ll f_\pi \approx 92\,\text{MeV}$

$$\mathcal{M}^2 = B_0 \begin{pmatrix} \boxed{\begin{matrix} \frac{f_\pi^2}{f_\varphi^2}\sum_q m_q n_q^2 & \frac{f_\pi}{f_\varphi}(m_d n_d - m_u n_u) \\ - & m_u + m_d \end{matrix}} & \begin{matrix} \frac{-f_\pi}{\sqrt{3}\,f_\varphi}(m_u n_u + m_d n_d - 2m_s n_s) \\ \frac{(m_u - m_d)}{\sqrt{3}} \end{matrix} & \begin{matrix} \frac{-\sqrt{2}f_\pi}{\sqrt{3}\,f_\varphi}\sum_q m_q n_q \\ \sqrt{\frac{2}{3}}(m_u - m_d) \end{matrix} \\ \begin{matrix} - & \quad - \end{matrix} & \frac{1}{3}(m_d + m_u + 4m_s) & \frac{\sqrt{2}}{3}(m_d + m_u - 2m_s) \\ \begin{matrix} - & \quad - \end{matrix} & - & m_1^2 + \frac{2}{3}(m_u + m_d + m_s) \end{pmatrix}$$

- π^0 gets negative shift at tree-level (while $\pi^\pm$ unaffected)

$$\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi} \approx -\frac{1}{2}\frac{(m_u n_u - m_d n_d)^2}{(m_d n_d^2 + m_u n_u^2)(m_u + m_d)}$$

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$$\left.\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi}\right|_{\text{exp}} \approx -0.03 \approx \left.\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi}\right|_{\text{EM}}$$

[$(m_{\pi^\pm} - m_{\pi^0})_{\text{EM}}$ = 4.622(95) MeV in good agreement (2%) with $(m_{\pi^\pm} - m_{\pi^0})_{\text{exp}}$ = 4.5936(5) MeV
e.g. from Frezzotti, Gagliardi, Lubicz, Martinelli, Sanfilippo, Simula, Phys. Rev. D 106 (2022)]

KM solution

- Murayama suggested Kaplan-Manohar (KM) ambiguity can help mitigate the tension

$$M_q|_{\rm KM} = \text{diag}\begin{pmatrix} m_u\sqrt{2}\varphi + x_{\rm KM} m_s m_d e^{i\sqrt{6}\eta'} \\ m_d + x_{\rm KM} m_u m_s \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \\ m_s + x_{\rm KM} m_u m_d \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \end{pmatrix} \qquad x_{\rm KM} \sim \frac{1}{4\pi f_\pi}$$

- KM does not modify tadpole condition, but allows for a reduced m_u (enters pion mixing)

- Define: $m_u = \frac{m_u|_{\rm KM}}{R}$ (where $m_u|_{\rm KM}$ identified with lattice value)

$$\boxed{\frac{\Delta m_{\pi^0}}{m_{\pi^0}} \approx -\frac{0.15}{R}, \qquad m_\varphi \approx 2.3\,\kappa_u \sqrt{R}\,\text{GeV}}$$

- remember tadpole condition:

$$m_\varphi^2 = \frac{B_0 f_\pi^2}{2}\sum_q \frac{n_q \kappa_q^2}{m_q}$$

KM solution

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$$\frac{\Delta m_{\pi^0}}{m_{\pi^0}} \approx -\frac{0.15}{R}, \qquad m_\varphi \approx 2.3\,\kappa_u\sqrt{R}\,{\rm GeV}$$

- $R \sim 5$ to match exp. value

[Murayama's point of view: "we conservatively require the mass shift to be at most the observed value"]

- $R \sim 200$ to match lattice error of EM contribution

- $R = \infty$ corresponds to massless up-quark solution

→ *caveat: lattice results assume SM (decoupling regime for heavy φ = QCD)*

We tried to improve…

- Model $\{n_u, n_d, n_s\} = \{1,1,0\}$ [LDL, Redi Strumia, Tesi, Titov 2602.10057]

$$\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi} \approx -\frac{1}{2}\frac{(m_u n_u - m_d n_d)^2}{(m_d n_d^2 + m_u n_u^2)(m_u + m_d)} \approx -0.07$$

- splitting down to 7 % (need $< 3\,\%$ to match $(\Delta m_\pi / m_\pi)_{\rm exp}$)

- KM does not help here, since φ enters also KM correction

$$M_q|_{\rm KM} = {\rm diag}\begin{pmatrix} m_u\sqrt{2}\varphi + x_{\rm KM} m_s m_d \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \\ m_d\sqrt{2}\varphi + x_{\rm KM} m_u m_s \sqrt{2}\varphi^* e^{i\sqrt{6}\eta'} \\ m_s + x_{\rm KM} m_u m_d (\sqrt{2}\varphi^*)^2 e^{i\sqrt{6}\eta'} \end{pmatrix}$$

We tried to improve…

- Model $\{n_u, n_d, n_s\} = \{2,1,0\}$ [LDL, Redi Strumia, Tesi, Titov 2602.10057]

$$\frac{\Delta(m_{\pi^0} - m_{\pi^\pm})}{m_\pi} \approx -\frac{1}{2}\frac{(m_u n_u - m_d n_d)^2}{(m_d n_d^2 + m_u n_u^2)(m_u + m_d)} \approx -0.0005$$

- splitting down to $0.05\,\%$ (need $< 0.07\,\%$ to match lattice error on $(\Delta m_\pi / m_\pi)_{\mathrm{EM}}$)

- accident, due to $m_d/m_u \sim 2$

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- splitting down to $0.05\ \%$ (need $< 0.07\ \%$ to match lattice error on $(\Delta m_\pi / m_\pi)_{\rm EM}$)

- accident, due to $m_d / m_u \sim 2$

- unfortunately, the cut-off of this effective theory is around m_u

$$\mathcal{L} \supset -\kappa_u \frac{\varphi^2}{\Lambda}\overline{u}_L u_R - \kappa_d \varphi \overline{d}_L d_R + \text{h.c.} \qquad \langle \overline{q_L} q_R \rangle = -\frac{B f_\pi^2}{2} \equiv -\mu^3$$

$$\langle \varphi \rangle = \frac{\kappa_d \mu^3}{m_\varphi^2 - 2\kappa_u \mu^3 / \Lambda}$$

$$m_u = \kappa_u \frac{\langle \varphi \rangle^2}{\Lambda}$$

$$m_d = \kappa_d \langle \varphi \rangle$$

$$\longrightarrow$$

$$m_\varphi = \sqrt{\frac{\kappa_d^2 \mu^3}{m_d}\left(2\frac{m_u}{m_d} + 1\right)} = 2.1\ \text{GeV}\ \kappa_d$$

$$\Lambda = \frac{\kappa_u}{\kappa_d^2}\frac{m_d^2}{m_u} = 10.2\ \text{MeV}\ \frac{\kappa_u}{\kappa_d^2}$$

We tried to improve…

- Model $\{n_u, n_d, n_s\} = \{0,0,1\}$ [LDL, Redi Strumia, Tesi, Titov 2602.10057]

  - preserves SU(2) symmetry of u-d sector → pion mass splitting OK

  - new problems arise:

    1) too light mass parameter (CP-odd component even lighter upon diagonalization)

$$m_\varphi = \left(\frac{B_0 f_\pi^2}{2m_s}\right)^{1/2} \kappa_s \approx 0.36 \text{ GeV}\, \kappa_s$$

    2) pion mass issue effectively shifted to eta sector, i.e. $\Delta m_\eta / m_\eta \approx +15\%$

We tried to improve…

- Non-minimal models

[LDL, Redi Strumia, Tesi, Titov 2602.10057]

  - models with multiple PQ scalars

  - add a PQ-neutral scalar just to make the π^0 heavier

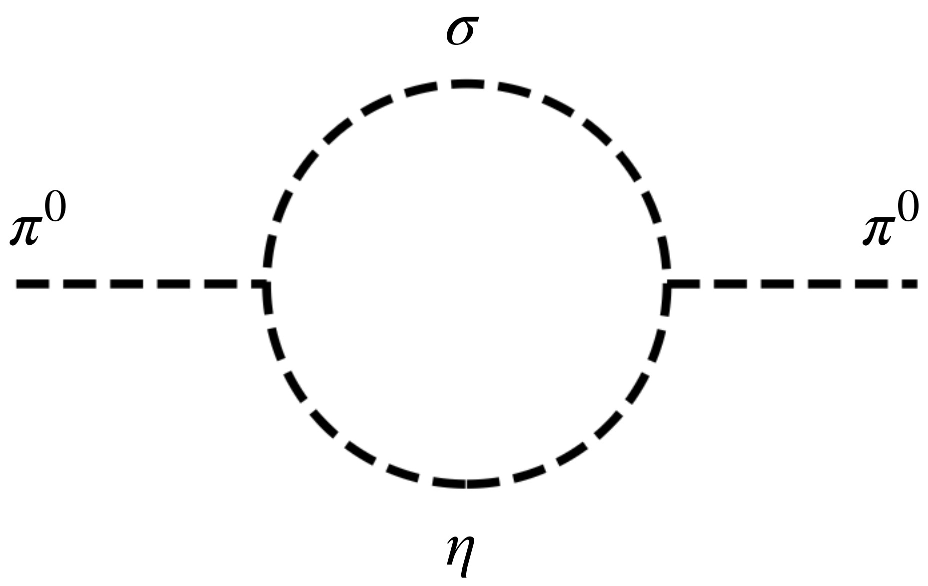
→ *configurations which can fix the pion mass problem systematically require new light states (consistent with pattern emerging for* $\{n_u, n_d\} = \{2,1\}$ *and* $\{n_s\} = \{1\}$ *models)*

We tried to improve…

- Loops [LDL, Redi Strumia, Tesi, Titov 2602.10057]

  - κ is sizeable, so one should worry about loop corrections

  - e.g. $\pi^0\sigma\eta$ vertex $\propto B_0(\kappa_u - \kappa_d)$ affect only π^0 mass



We tried to improve…

- Loops

[LDL, Redi Strumia, Tesi, Titov 2602.10057]

- κ is sizeable, so one should worry about loop corrections

- in general:

$$\Delta M^2_{ij} = -(\partial_\varphi M^2_{ik}) \left[\int \frac{d^4Q}{(2\pi)^4} \frac{1}{Q^2+m^2_\varphi} \frac{1}{Q^2+m^2} \right]_{kl} (\partial_\varphi M^2_{lj}),$$

- attempt an estimate by taking a cut-off $Q \lesssim \Lambda = 1$ GeV and expand for $m_\varphi \gtrsim \Lambda$

$$\Delta M^2_{ij} \approx -\frac{(\partial_\varphi M^2_{ik})}{16\pi^2 m^2_\varphi} \left[\Lambda^2 - M^2_{kk} \ln \frac{\Lambda^2 + M^2_{kk}}{M^2_{kk}} \right] (\partial_\varphi M^2_{kj})$$

- e.g. for $n_u = 1$ model

$$\Delta(m^2_{\pi^0} - m^2_{\pi^\pm}) \approx -\frac{\kappa^2 B_0^2 \Lambda^2}{(4\pi m_\varphi)^2}$$

→ *unlikely a cancellation with tree-level (cf. size and sign)*

Why is it difficult to make it work ?

- Some general lessons:

  - playing little with KM, otherwise one ends up with the SM massless up "solution"

  - loop has no reason to cancel tree

  - find models where isospin is accidentally restored, at finite "m_u"

  - despite some partial improvement, we did not find a good model

  - integrating out $m_\varphi \gtrsim$ GeV allows for a <u>PQ-invariant chiral expansion</u> to gain further insight (e.g. $\pi\pi$ scattering, see backup slides)

  - invoke a new lattice simulation (QCD with massless up + new scalar) ?

Part 2 - Raise d_{PQ}

- $U(1)_{PQ}$ often imposed 'by hand', while a proper PQ theory should:

1. realise the PQ as an accidental symmetry
2. protect the $U(1)_{PQ}$ against UV sources of PQ breaking (PQ-quality problem)

Part 2 - Raise d_{PQ}

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  1. realise the PQ as an accidental symmetry

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- 4D approaches to PQ quality:

  - discrete gauge symmetries: e.g. $\mathbf{Z}_N$ acting on ϕ allows only ϕ^N terms and higher

  - gauged U(1) constructions: PQ survives as an accidental global combination
    [e.g. Barr, Seckel PRD 46 (1992) 539; Fukuda et al. 1703.01112]

  - product groups / clockwork: PQ breaking requires operators connecting multiple sectors
    [e.g. Redi, Sato 1602.05427; Lillard, Tait 1811.03089]

  - composite axions: confinement yields an accidental PQ, broken only by fermionic operators
    [e.g. Choi, Kim PRD 32 (1985), Randall PLB 284 (1992) 77; Contino, Podo, Revello 2112.09635]

  - large-N non-abelian groups: determinant operator first break PQ at $d_{\mathrm{PQ}} = N$
    [e.g. LDL, Nardi, Ubaldi 1704.01122; Ardu et al. 2007.12663]

Cosmology/Quality tension

- Known models "suffer" in the post-inflationary QCD axion regime [Lu, Reece, Sun 2312.07650]

  1. DW problem, 2. Fractionally charged relics, 3. Low-scale Landau poles ($\Lambda_{\rm LP} < \Lambda_{\rm UV}$)

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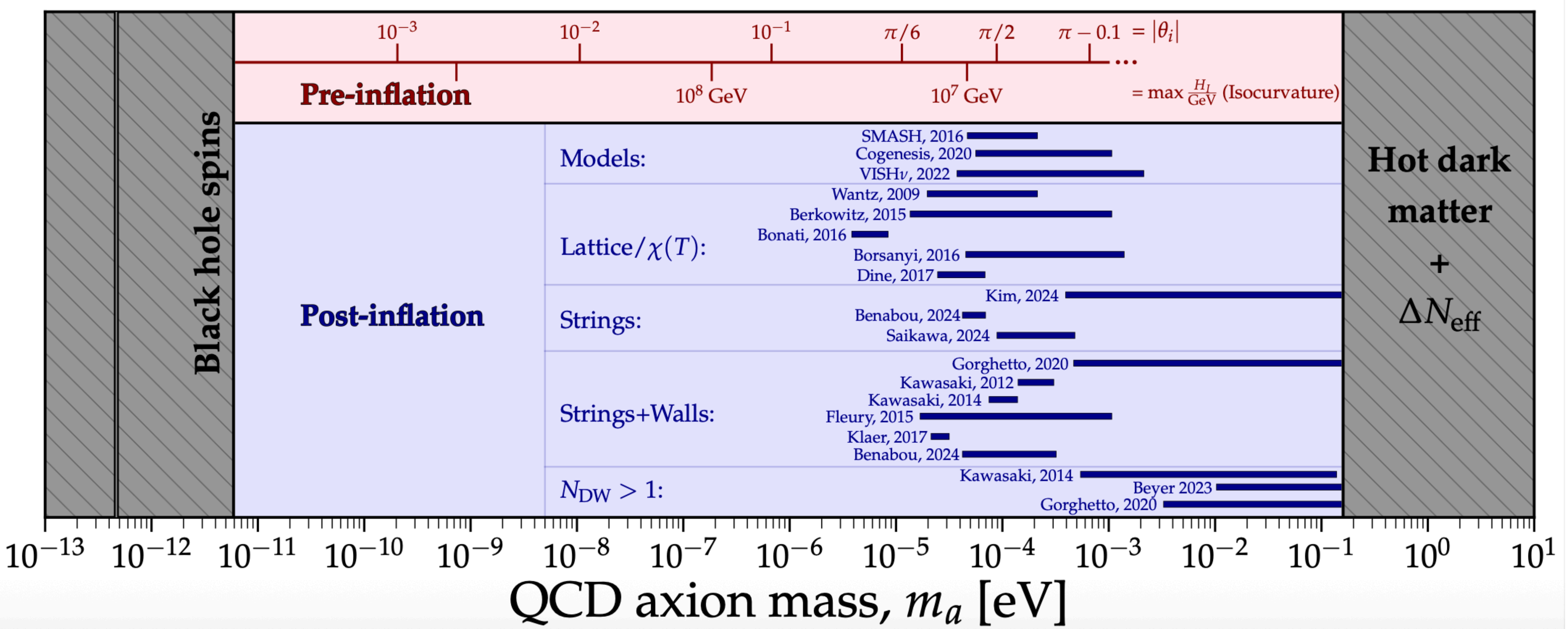
- Why important ?

  - a consistent post-inflationary history makes the axion DM mass predictive (in principle)

    [See talk by G. Villadoro]

[https://cajohare.github.io/AxionLimits]



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- Barr Seckel model [Barr, Seckel PRD 46 (1992)]

	spin	$SU(3)_C$	$U(1)_Y$	$U(1)'$	copies	$U(1)_{PQ}$
Q_0	$1/2$	$\mathbf{3}$	0	0	$(p+q)$	c
$\widetilde{Q}_p$	$1/2$	$\overline{\mathbf{3}}$	0	p	q	$a-c$
$\widetilde{Q}_{-q}$	$1/2$	$\overline{\mathbf{3}}$	0	$-q$	p	$-b-c$
Φ_p	0	$\mathbf{1}$	0	p	1	a
Φ_q	0	$\mathbf{1}$	0	q	1	b

We will assume that p and q are co-prime integers, such that the lowest order operator that breaks $U(1)_{PQ}$ symmetry is $(\Phi_p^\dagger)^q(\Phi_q)^p$. The correction to the axion potential induced by this operator is proportional to $v_p^q v_q^p/\Lambda^{p+q-4}$, where Λ is the UV scale. The Barr-Seckel model ensures a high quality $U(1)_{PQ}$ symmetry when $p+q$ is sufficiently large. For example, for $v_p \sim v_q \sim f_a$ and the UV scale set to the Planck mass $\Lambda = M_{Pl}$, we need $p+q \geq 10$ for 10^{10} GeV $< f_a < 10^{12}$ GeV so that $\bar{\theta}$ will come out sufficiently small.

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  - Composite axions [Choi, Kim PRD 32 (1985), Randall PLB 284 (1992), Dobrescu hep-ph/9609221, …]

B. Randall's model

There is only one axion model in the literature which does not involve scalars and avoids large Planck scale effects [12]. To achieve this, Randall's model includes another gauge interaction, which is weak, in addition to the confining axicolor. The left-handed fermions transform under the $SU(N) \times SU(m) \times SU(3)_C$ gauge group as

$$\psi:(N,m,3), \quad \phi_i:(N,\bar{m},1), \quad \chi_j:(\bar{N},1,\bar{3}), \quad \omega_k:(\bar{N},1,1), \tag{3.12}$$

where $i=1,2,3$, $j=1,\ldots,m$, and $k=1,\ldots,3m$ are flavor indices. Axicolor $SU(N)$ becomes strong at the Λ_a scale and the fermions condense. If the $SU(m)$ gauge coupling g_m is turned off, the vacuum will align as in Kim's model and will preserve color. When g_m is nonzero, the $SU(m)$ gauge interaction will tend to change the vacuum alignment and break the $SU(N)$ gauge symmetry. However, since g_m is small, this will not happen, as we know from QCD where the weak interactions of the quarks do not affect the quark condensates. Therefore, the

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- large-N non-abelian groups [LDL, Nardi, Ubaldi 1704.01122]

$$SU(N)_L \times SU(N)_R \xrightarrow{V} SU(N)_{L+R} \qquad Y \sim (N, \bar{N})$$ scalar in the bi-fundamental

Invariants: $\mathrm{Tr}\,[YY^\dagger]$ $(\mathrm{Tr}\,[YY^\dagger])^2$ $\mathrm{Tr}\,[YY^\dagger YY^\dagger]$ $\ldots$ $\mathrm{Det}\,Y$

Accidental U(1): $Y \to e^{i\alpha} Y$

broken at dim = N

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broken at dim = N

+ colored fermions to make U(1) anomalous under QCD $f_a = V/N$ $\longrightarrow$ $N_{\rm DW}^{\rm naive} = N$

action of $\mathbb{Z}_N \subset SU(N)_L \times SU(N)_R$ makes axion vacua gauge equivalent $\longrightarrow$ $N_{\rm DW} = 1$

[Lazarides Shafi PLB 115 (1982)]

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  - large-N non-abelian groups [LDL, Nardi, Ubaldi 1704.01122]

  - Extension with non-zero hypercharges [Lu, Reece, Sun 2312.07650]

	spin	$SU(n)_L$	$SU(n)_R$	$SU(3)_C$	$U(1)_Y$	copies	$U(1)_{PQ}$
Y	0	$\square$	$\overline{\square}$	$\mathbf{1}$	0	1	$1/n$
$D^{(i)}$	1/2	$\square$	$\mathbf{1}$	$\mathbf{3}$	$-1/3$	2	$1/n$
$\widetilde{D}^{(i)}$	1/2	$\mathbf{1}$	$\overline{\square}$	$\overline{\mathbf{3}}$	$+1/3$	2	0
U	1/2	$\overline{\square}$	$\mathbf{1}$	$\mathbf{3}$	$+2/3$	1	$-1/n$
$\widetilde{U}$	1/2	$\mathbf{1}$	$\square$	$\overline{\mathbf{3}}$	$-2/3$	1	0
$\Psi^{(j)}$	1/2	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	0	3	$-1/n$
$\widetilde{\Psi}^{(j)}$	1/2	$\mathbf{1}$	$\square$	$\mathbf{1}$	0	3	0

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- $SU(N) \to SO(N)$ [broken by a 2-index symmetric] [Ardu, LDL, Landini, Strumia, Teresi, Wang 2007.12663]

Field	Lorentz	Gauge symmetries				Global accidental symmetries		
name	spin	$\mathrm{U}(1)_Y$	$\mathrm{SU}(2)_L$	$\mathrm{SU}(3)_c$	$\mathrm{SU}(\mathcal{N})$	$\mathrm{U}(1)_{\mathrm{PQ}}$	$\mathrm{U}(1)_{\mathcal{Q}}$	$\mathrm{U}(1)_{\mathcal{L}}$
$\mathcal{S}$	0	0	1	1	$\mathcal{N}\mathcal{N}$	$+1$	0	0
$\mathcal{Q}_L$	1/2	$+Y_{\mathcal{Q}}$	1	3	$\mathcal{N}$	$+1/2$	$+1$	0
$\mathcal{Q}_R$	1/2	$-Y_{\mathcal{Q}}$	1	$\bar{3}$	$\mathcal{N}$	$+1/2$	-1	0
$\mathcal{L}_L^{1,2,3}$	1/2	$+Y_{\mathcal{L}}$	1	1	$\bar{\mathcal{N}}$	$-1/2$	0	$+1$
$\mathcal{L}_R^{1,2,3}$	1/2	$-Y_{\mathcal{L}}$	1	1	$\bar{\mathcal{N}}$	$-1/2$	0	-1

DWs remain problematic: $N_{\rm DW} = 2$ for even N; for odd N, the potentially unstable single-wall strings may be composite and not cosmologically produced at the PQ transition

[Lu, Reece, Sun 2312.07650]

Reconciling axion quality & cosmology

- Back to the $SU(N)_L \times SU(N)_R$ model

[LDL, Di Valeriano, Nardecchia, Nardi (in preparation)]

	Lorentz	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$G_{LR}^{(\mathcal{N})} \to \mathrm{SU}(\mathcal{N})_{L+R}$	$\mathrm{U}(1)_{\mathrm{PQ}}$	$\mathrm{U}(1)_{\mathcal{Q}-\Psi}$
Y	$(0,0)$	1	1	0	$(\mathcal{N},\bar{\mathcal{N}}) \to \mathcal{N}^2 - 1 \oplus 1$	1	0
$\mathcal{Q}_L$	$(\frac{1}{2},0)$	3	1	0	$(\mathcal{N},1) \to \mathcal{N}$	1	1
$\mathcal{Q}_R$	$(0,\frac{1}{2})$	3	1	0	$(1,\mathcal{N}) \to \mathcal{N}$	0	1
Ψ_L^a	$(\frac{1}{2},0)$	1	1	$\mathcal{Y}^a$	$(\bar{\mathcal{N}},1) \to \bar{\mathcal{N}}$	-1	-1
Ψ_R^a	$(0,\frac{1}{2})$	1	1	$\mathcal{Y}^a$	$(1,\bar{\mathcal{N}}) \to \bar{\mathcal{N}}$	0	-1

Reconciling axion quality & cosmology

- Back to the $SU(N)_L \times SU(N)_R$ model [LDL, Di Valeriano, Nardecchia, Nardi (in preparation)]

the new ingredient is a different hypercharge assignment ($\mathcal{Y}^{1,2} = -1/3$ and $\mathcal{Y}^3 = 2/3$)

✓ $N_{\rm DW} = 1$: the string-wall network is unstable

✓ Fractionally charged relics absent for N = 0 (mod 3)

✓ No Landau poles below $\Lambda_{\rm UV} \sim M_{\rm Pl}$

	Lorentz	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$G_{LR}^{(\mathcal{N})} \to \mathrm{SU}(\mathcal{N})_{L+R}$	$\mathrm{U}(1)_{\rm PQ}$	$\mathrm{U}(1)_{\mathcal{Q}-\Psi}$
Y	$(0,0)$	1	1	0	$(\mathcal{N},\bar{\mathcal{N}}) \to \mathcal{N}^2 - 1 \oplus 1$	1	0
$\mathcal{Q}_L$	$(\frac{1}{2},0)$	3	1	0	$(\mathcal{N},1) \to \mathcal{N}$	1	1
$\mathcal{Q}_R$	$(0,\frac{1}{2})$	3	1	0	$(1,\mathcal{N}) \to \mathcal{N}$	0	1
Ψ_L^a	$(\frac{1}{2},0)$	1	1	$\mathcal{Y}^a$	$(\bar{\mathcal{N}},1) \to \bar{\mathcal{N}}$	-1	-1
Ψ_R^a	$(0,\frac{1}{2})$	1	1	$\mathcal{Y}^a$	$(1,\bar{\mathcal{N}}) \to \bar{\mathcal{N}}$	0	-1

Reconciling axion quality & cosmology

- Back to the $SU(N)_L \times SU(N)_R$ model [LDL, Di Valeriano, Nardecchia, Nardi (in preparation)]

  the new ingredient is a different hypercharge assignment ($\mathcal{Y}^{1,2} = -1/3$ and $\mathcal{Y}^3 = 2/3$)

  ✓ $N_{\rm DW} = 1$: the string-wall network is unstable

  ✓ Fractionally charged relics absent for N = 0 (mod 3)

  ✓ No Landau poles below $\Lambda_{\rm UV} \sim M_{\rm Pl}$

  ✓ Cosmologically stable exotic baryon: neutral or strongly depleted by confinement

$$\boxed{N = 12, \qquad \min(m_{\mathcal{Q}}, m_\Psi) \sim 50 \text{ TeV}, \qquad f_a \sim 10^{11} \text{ GeV}, \qquad \Lambda_{\rm UV} \sim M_{\rm Pl}}$$

	Lorentz	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$G_{LR}^{(\mathcal{N})} \to \mathrm{SU}(\mathcal{N})_{L+R}$	$\mathrm{U}(1)_{\rm PQ}$	$\mathrm{U}(1)_{\mathcal{Q}-\Psi}$
Y	$(0,0)$	1	1	0	$(\mathcal{N},\bar{\mathcal{N}}) \to \mathcal{N}^2 - 1 \oplus 1$	1	0
$\mathcal{Q}_L$	$(\frac{1}{2},0)$	3	1	0	$(\mathcal{N},1) \to \mathcal{N}$	1	1
$\mathcal{Q}_R$	$(0,\frac{1}{2})$	3	1	0	$(1,\mathcal{N}) \to \mathcal{N}$	0	1
Ψ_L^a	$(\frac{1}{2},0)$	1	1	$\mathcal{Y}^a$	$(\bar{\mathcal{N}},1) \to \bar{\mathcal{N}}$	-1	-1
Ψ_R^a	$(0,\frac{1}{2})$	1	1	$\mathcal{Y}^a$	$(1,\bar{\mathcal{N}}) \to \bar{\mathcal{N}}$	0	-1

Conclusions

- Two routes to axion quality, two different bottlenecks:

Lower f_a

QCD-triggered PQ breaking improves quality, but appears to be ruled out by precision low-energy pion physics

[Murayama 2601.02465
LDL, Redi Strumia, Tesi, Titov 2602.10057]

Raise d_{PQ}

reconciling high quality with post-inflationary cosmology is challenging, but large-N PQ models offer a viable window

[Lu, Reece, Sun 2312.07650
LDL, Di Valeriano, Nardecchia, Nardi (in preparation)]

- KSVZ/DFSZ map the search, quality guides their UV completion

- axion quality links UV consistency to low-energy physics and cosmology

Backup slides

UV completion

- Need a dim = 5 operator:

$$\frac{y\kappa_u}{M_\mathcal{Q}} \bar{Q}_u \varphi H u_R + c.c. \quad \longrightarrow \quad \kappa = \frac{y\kappa_u v}{M_\mathcal{Q}}$$

- possible UV completion in terms of a vector-like $\mathcal{Q} = (\mathcal{U}, \mathcal{D})$ with PQ charge -1

$$\mathcal{L}_\mathcal{Q} = \bar{\mathcal{Q}}(i\not{D} - M_\mathcal{Q})\mathcal{Q} + (\kappa_i \bar{Q}_i \varphi \mathcal{Q} + y\bar{\mathcal{Q}} H u_R + c.c.)$$

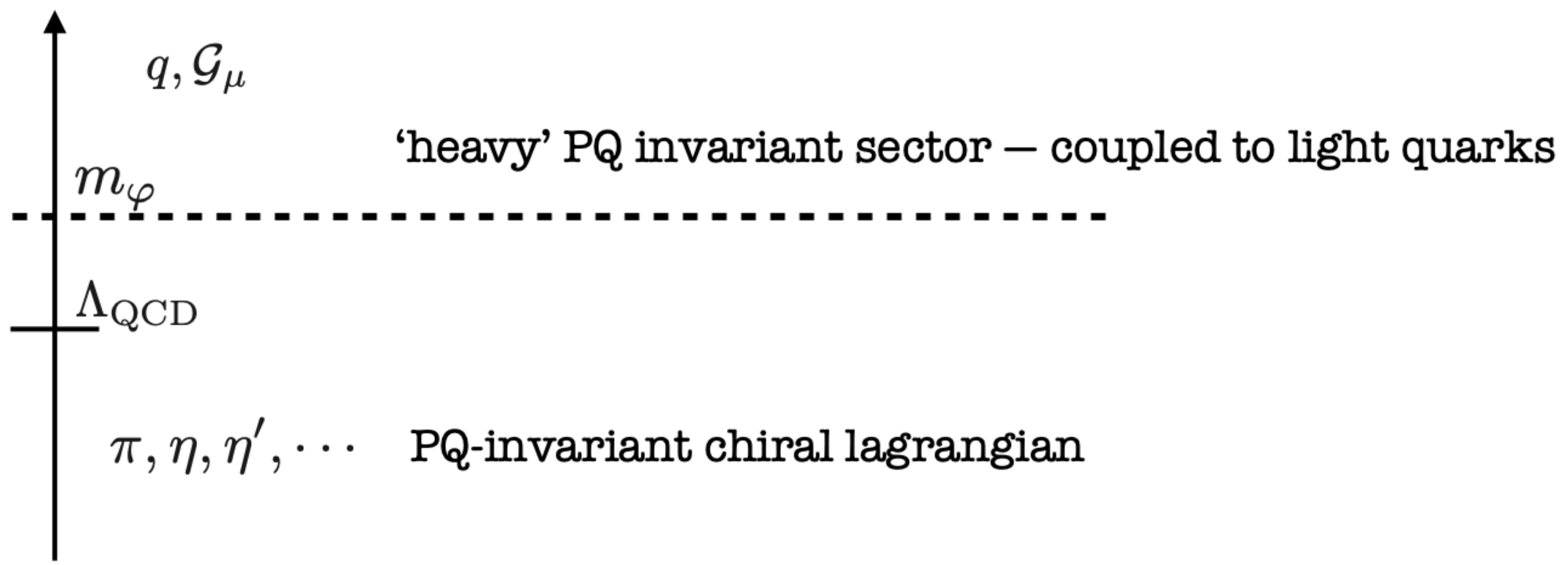
- LHC limits: $M_\mathcal{D} > 1.53$ TeV

- With $y, \kappa_u \approx 1$ and $\mathcal{Q} \approx 1.6$ TeV $\rightarrow \kappa \approx 0.11$, enhanced to $\kappa \approx 0.25$ down to QCD scale

- Peculiar LHC phenomenology of UV completion, which does not decouple (see Murayama)

EFT of heavy PQ sector

- Chiral expansion with PQ symmetry



- Two sets of spurions:

  - mass matrix with some zero eigenvalues (here $n_s = 0$ and $n_{u,d} = 0{,}1$)

$$\tilde{M}_q = \text{diag}(m_u(1-n_u), m_d(1-n_d), m_s)$$

  - couplings to light quark of the "heavy" PQ invariant sector

$$I_{\rm PQ} = \text{diag}(\kappa_u n_u, \kappa_d n_d, 0)$$

EFT of heavy PQ sector

- Chiral expansion with PQ symmetry

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}}(\tilde{M}_q) + \sum_i C_i \mathcal{O}_i^{\text{PQ}}(I_{\text{PQ}})$$

- first term: usual chiral expansion, with the new mass matrix (with zeroes)

- second term: tower of operators generated by the PQ sector at m_φ

- first non-trivial operator arises at $\mathcal{O}(p^4)$ (only one, assuming a single mediator)

$$\mathcal{O}_1^{\text{PQ}} = \text{tr}[I_{\text{PQ}}^\dagger U]\text{tr}[I_{\text{PQ}} U^\dagger]$$

- manifestly chiral and PQ invariant: $I_{\text{PQ}} \to L I_{\text{PQ}} R^\dagger$ and $I_{\text{PQ}} \to I_{\text{PQ}} e^{-i\alpha Q_{\text{PQ}}}$

$$I_{\text{PQ}} = \text{diag}(\kappa_u n_u, \kappa_d n_d, 0)$$

A main obstruction

- What is the effect of the new operator ?

$$\mathcal{O}_1^{\rm PQ} = {\rm tr}[I_{\rm PQ}^\dagger U]{\rm tr}[I_{\rm PQ} U^\dagger]$$

- it controls the pion mass splitting

$$m_{\pi^0}^2 - m_{\pi_\pm}^2 = -2\frac{C_1}{f_\pi^2}(\kappa_u n_u - \kappa_d n_d)^2$$

- only operator that breaks iso-spin while preserving PQ, others appear at $\mathcal{O}(p^6)$

- tree-level matching gives $C_1 = \frac{1}{4}\frac{f_\pi^4 B_0^2}{m_\varphi^2}$

- $\mathcal{O}(p^6)$ terms cannot cancel $\mathcal{O}(p^4)$ term, as long as chiral expansion holds

- loop corrections cannot cancel the tree-level effect, as long as one operator dominates

- $\pi\pi$ scattering affected too (see backup slides)

$\pi\pi$ scattering

- In usual QCD one has

$$\mathrm{tr}\,[M_q U^\dagger + \mathrm{h.c.}] = m_s + (m_u + m_d)\cos\pi \qquad \text{where} \qquad \pi^2 \equiv \pi^a\pi^a = (\pi^0)^2 + 2\pi^-\pi^+$$

$$\mathscr{A}(\pi_a\pi_b \to \pi_c\pi_d) = \frac{1}{f_\pi^2}\left[\delta_{ab}\delta_{cd}(s - m_\pi^2) + \delta_{ac}\delta_{bd}(t - m_\pi^2) + \delta_{ad}\delta_{bc}(u - m_\pi^2)\right]$$

- O_1^{PQ} gives a different isospin-breaking function of pions

$$\mathcal{O}_1^{\mathrm{PQ}} = (n_u\kappa_u + n_d\kappa_d)^2\cos^2\pi + (n_u\kappa_u - n_d\kappa_d)^2\frac{(\pi^0)^2}{\pi^2}\sin^2\pi$$

- tuning $n_u\kappa_u - n_d\kappa_d$ to zero allows to restore SU(2) invariance, but with a different function of π

$$\mathscr{A}(\pi^0\pi^0 \to \pi^0\pi^0) = (4 - 3X_0)\frac{m_{\pi^0}^2}{f_\pi^2}, \qquad X_0 = \begin{cases} 1 & \text{if one } n_q = 0 \\ 4 & \text{if } n_u = n_d = 1 \end{cases},$$

$$\mathscr{A}(\pi^+\pi^- \to \pi^+\pi^-) = \frac{s + t - 2X_\pm m_{\pi^\pm}^2}{f_\pi^2}, \qquad X_\pm = 1 + 3\frac{m_u n_u + m_d n_d}{m_u + m_d}$$

→ *differences w.r.t. QCD case of order one for* $X_{0,\pm} \neq 1$

Kaplan-Manohar ambiguity

- For some time (from 1986 to ~ 2000) $m_u = 0$ was not obviously ruled out

- mass ratios m_u/m_d and m_s/m_d can be extracted from fit to pseudo-scalar masses in χPT

- potential ambiguity at NLO [Kaplan Manohar PRL 56 (1986)]

$M_q\,,\ (M_q^{-1})^\dagger \det(M_q^\dagger) \longrightarrow$ transform in the same way under the chiral symmetry

- corresponds to a linear combination of $\mathcal{O}(p^4)$ terms (L_6, L_7, L_8)

$$\mathcal{O}_{\rm KM} \equiv -\frac{1}{2}\left[\mathrm{tr}(MU^\dagger MU^\dagger) - \mathrm{tr}(MU^\dagger)^2 + \mathrm{h.c.}\right] \qquad \Delta\mathscr{L} = \frac{B_0 f_\pi^2}{2} x_{\rm KM}\mathcal{O}_{\rm KM} \qquad x_{\rm KM} \sim \frac{1}{4\pi f_\pi}$$

- KM operator "dresses" LO spurion

$$M_q \to M_q|_{\rm KM} = M_q + x_{\rm KM}(M_q^{-1})^\dagger \det(M_q^\dagger) \longrightarrow \boxed{m_u|_{\rm KM} = m_u + x_{\rm KM} m_s m_d}$$

- $m_u = 0$ *could be made consistent in χPT fits thanks to NLO effects*

Cosmologically stable baryons

- $SU(N)_L \times SU(N)_R$ model [LDL, Di Valeriano, Nardecchia, Nardi (in preparation)]

$$U(1)^5 \xrightarrow{d=6} U(1)_{\rm PQ} \times U(1)_{Q-\Psi}$$

- quality of PQ directly related to quality of $U(1)_{Q-\Psi}$ → LP charged under $U(1)_{Q-\Psi}$ is stable

- upon $SU(N)_{L+R}$ confinement, spectrum depends on mass ordering

$m_Q < m_\Psi$	$m_\Psi < m_Q$
$B_Q = [Q^N]$	$B_\Psi = [\Psi^N]$
• electrically neutral; • zero QCD triality for $N = 0 \pmod 3$; • dressed into a QCD singlet by a hadronic-size gluon cloud.	• QCD singlet; • generically electrically charged; • a mixed-flavor neutral ground state is possible.