Extra-Dimensional Axions

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Axions in Seoul

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AT HARVARD UNIVERSITY

Summary

- Extra-dimensional axions are well-motivated from both bottom-up and top-down viewpoints
- **"Quantum Gravity Cutoff from Axions":** Measuring the decay constant of an extra-dimensional axion can offer a window into the fundamental scale:

$$\Lambda_{\mathrm{QG}} \lesssim 2\pi\sqrt{S_{\mathrm{inst}}}f$$

- Extra-dimensional axions may have distinctive cosmological histories

references: MR, 2406.08543; MR, Tom Rudelius, and Christopher Tudball, 2505.22713, 2606.21650; Priyesh Chakraborty, Junyi Cheng, MR, and Zekai Wang, 2507.12519; more work in progress

Axion QFT Basics

An axion is a *periodic* scalar field: *gauge redundancy* $\theta \cong \theta + 2\pi$.

Symmetries of a free axion theory:

“electric” shift symmetry

$$\mathrm{d}(\star \mathrm{d}\theta) = 0 : \quad j_\mu = \partial_\mu \theta$$

ordinary (0-form) global symmetry, nonlinearly realized

explicitly broken by *instantons* (anything generating an $\mathrm{e}^{iq\theta}$ effect) or *Chern-Simons terms* $\theta F \wedge F$

“magnetic” winding symmetry

$$\mathrm{d}(\mathrm{d}\theta) = 0 : \quad j_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda}\partial^\lambda \theta$$

higher (2-form) global symmetry

explicitly broken by *axion strings*, vortices for which $\oint \mathrm{d}\theta = 2\pi w$

Axion Quality

Applications of axions—e.g., a QCD axion that solves the Strong CP problem, or a light axion-like particle that plays a role in cosmological dynamics—generally require "high quality."

This means **explicit shift-symmetry breaking effects are small** and under control.

Any breaking of the shift symmetry generates an axion mass.

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Small subtlety: for U(1), $\theta F \wedge F$ breaks shift symmetry but does not produce a mass. This is due to a *non-invertible shift symmetry* that survives (Córdova, Ohmori 2205.06243). It is broken by the existence of *magnetic monopoles* for U(1). Monopole loops generate a mass (Fan, Fraser, MR, Stout 2105.09950).

4d Axions vs Extra-Dimensional Axions

Types of Axions: 4d Axions

To explain where a high-quality axion comes from, we need to explain the origin of the approximate shift symmetry. Two main categories. The first:

4d axions: The shift symmetry comes from spontaneous breaking of a ***linearly*** realized approximate global symmetry in 4d.

$\theta(x)$ is the phase of a 4d operator $\Phi(x)$, **where $\Phi(x) \to 0$ makes sense**.

Quality explained by ***accidental*** symmetry (enforced by new gauge symmetries), ***locality of 4d fields in extra dimensions***, or some other mechanism.

Slogan: **"Axions are pseudo-Nambu-Goldstone bosons."**

(Peccei, Quinn 1977; Weinberg, Wilczek 1978; KSVZ 1979-80; DFSZ 1980-81; …)

Types of Axions: Extra-Dimensional Axions

The second category:

Extra-dimensional axions: The shift symmetry comes from an electric higher-form symmetry that is ***nonlinearly realized*** already in $d > 4$ dimensions.

$\theta(x)$ is $\oint_\Sigma A$ for a U(1) p-form gauge field A and a p-cycle Σ.

Quality explained by ***large mass*** of electrically charged fields or ***large volume*** of extra dimensions.

Slogan: **"Axions are Wilson loops."**

(Witten, 1984; … Choi, 2003; … emphasis on higher-form symmetries: MR 2304.08512, Craig & Kongsore 2408.10295)

Limiting Non-Derivative Interactions

Extra dimensional models—first found in string theory, not engineered (Witten 1984)—are a good bottom-up starting point for axion quality.

Key insight: interactions involving $\partial_\mu \theta$ don't lead to a potential; only those involving θ **without derivatives** do.

Gauge theory: **most terms** involve $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ (i.e., $\vec{E}, \vec{B}$). Only **very special terms** involve A_μ without a derivative acting on it:

- **Charge** of particles (covariant derivatives)
- **Chern-Simons terms** like $\int A \wedge \mathrm{tr}(G \wedge G)$ in 5d

Axions, Instantons, Strings

Given n extra dimensions and a general p-form gauge field A (with $p \leq n$):

- Axions come from integrating the gauge field over any p-cycle Σ, $\theta = \oint_\Sigma A$.
- **Instantons** come from Euclidean charged $(p-1)$-branes wrapped on Σ. In particular $S_{\text{inst}} = \mathcal{T}_e \text{Vol}(\Sigma)$. Potentials $V \propto \exp(-S_{\text{inst}})$, **relatively safe!**
- **Axion strings** come from magnetically charged $(d-p-3)$-branes wrapped on a $(d-p-2)$-cycle Γ intersecting Σ. In particular $\mu_{\text{string}} = \mathcal{T}_m \text{Vol}(\Gamma)$.
- **Chern-Simons terms** match: $\int_{(4+n)d} A \wedge \text{tr}(G \wedge G) \to \int_{4d} \theta \, \text{tr}(G \wedge G)$.

Axion Discovery: A New Energy Scale

A discovery of an axion (e.g., as dark matter) would measure the ***axion decay constant*** f, expected to lie in the range $10^9\,\mathrm{GeV} \lesssim f \lesssim 10^{16}\,\mathrm{GeV}$.

This is a dimensionful scale much **larger** than collider scales but much **smaller** than the Planck scale. We have few signposts in this range. **What would we learn about fundamental physics?**

For a conventional 4d axion interpretation, physics at the scale f is ordinary 4d physics.

For an extra-dimensional axion, we expect to learn something more fundamental.

Warmup: 5d Axion and the KK Scale

$\theta = \oint A$. What about the key experimental observable f?

$$\int \frac{1}{e_5^2} |F|^2 \Rightarrow \int f^2 |\mathrm{d}\theta|^2, \quad \theta \sim A_5 R, \quad f^2 \sim \frac{1}{e_5^2 R}.$$

units of energy, cutoff on gauge theory, need $1/e_5^2 \gtrsim 1/R$ for consistency

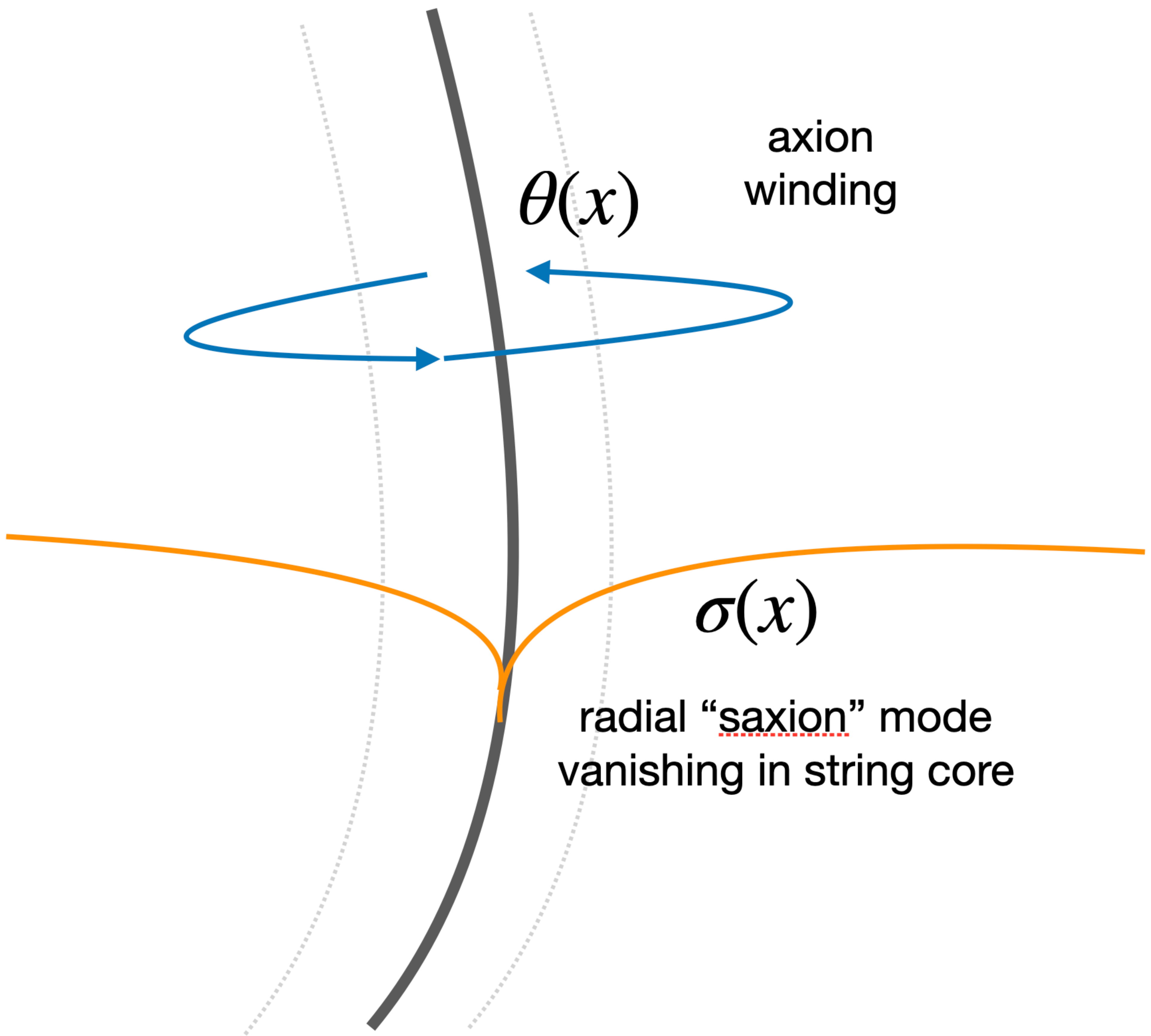
$$f \gtrsim 1/R$$

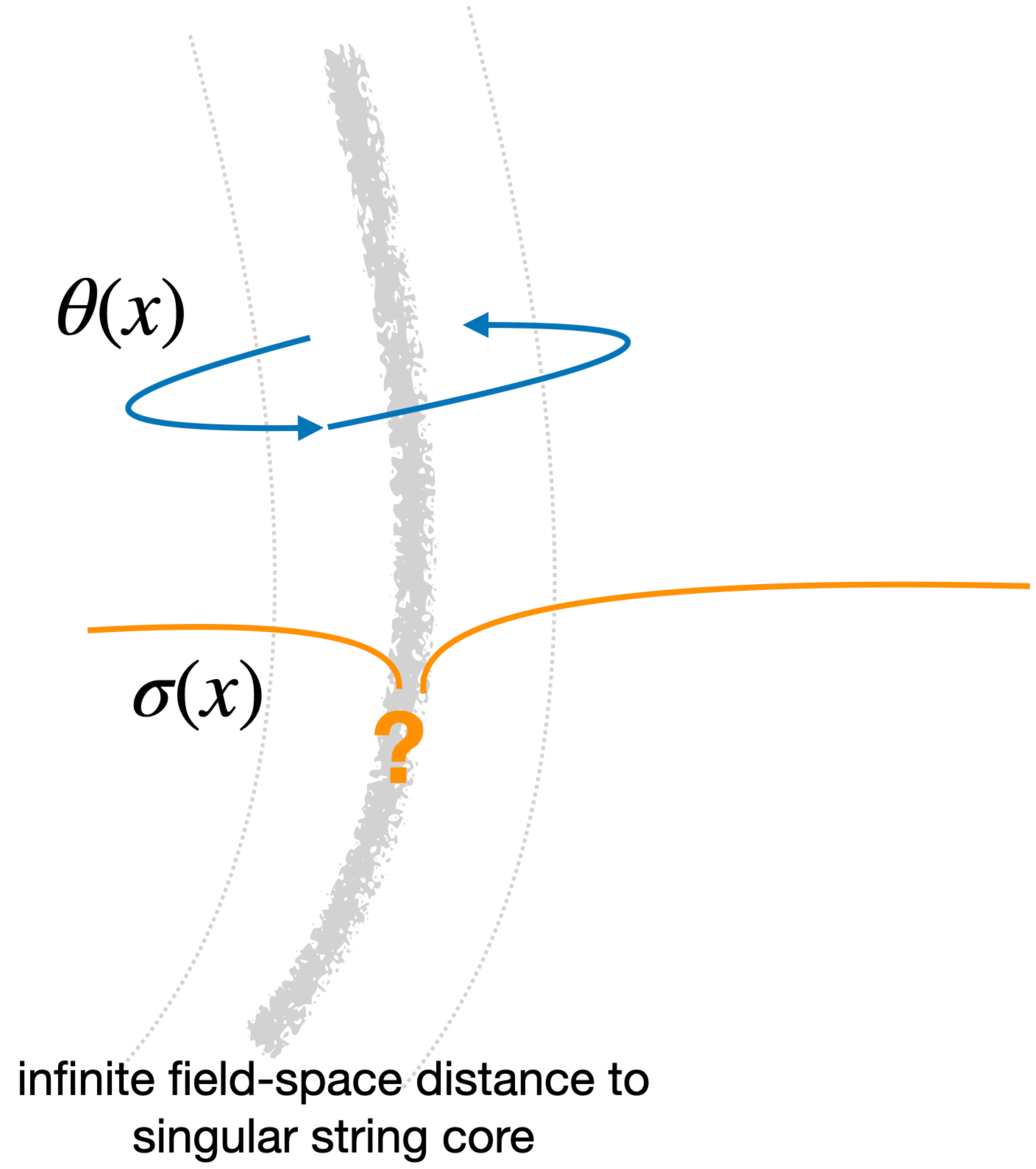
4d physics breaks down at or before the scale f. Measure f, learn lower bound on how big extra dimensions are!

What happens for more than one extra dimension? More complicated. We'll come back to this.

“Fundamental Axions”: Axion Strings and Infinite Distances

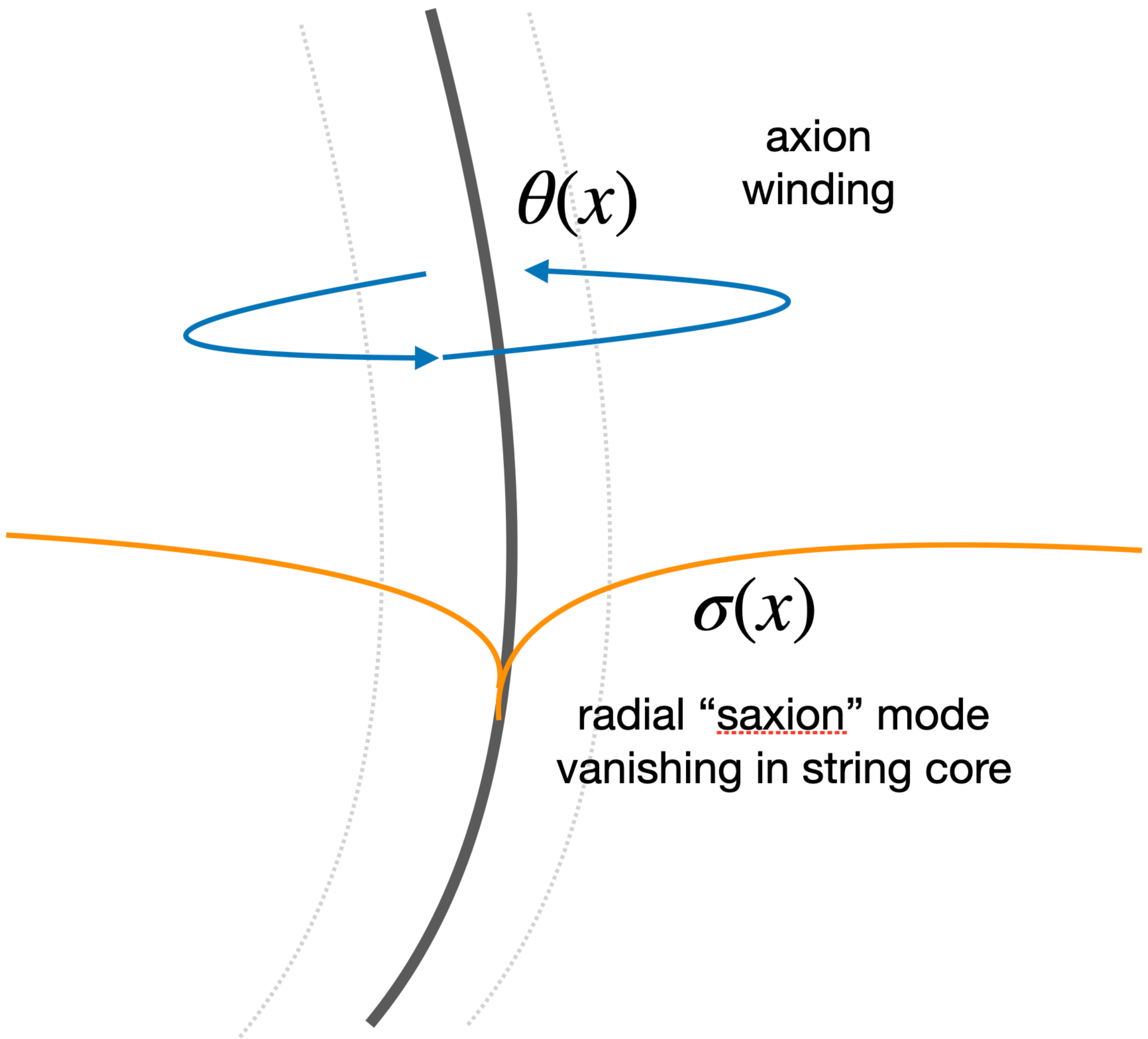
Axion String: Soliton or Fundamental?





MR, 1808.09966; also see Dolan, Draper, Kozaczuk, Patel 1701.05572

Axion String as Soliton



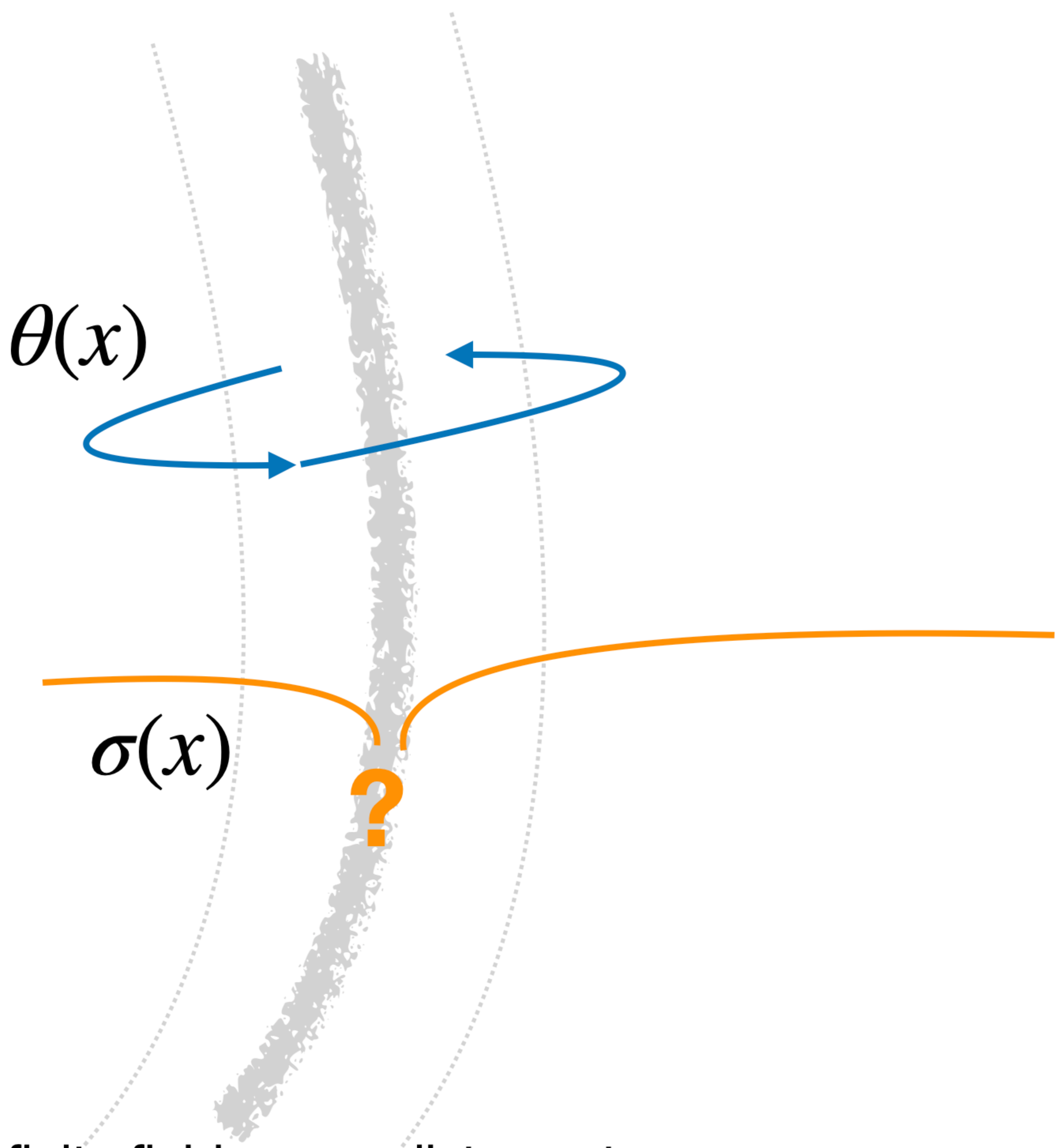
Realized for 4d axions:
$\Phi(x) = \sigma(x)e^{i\theta(x)}$, $\sigma \to 0$ at finite distance in field space.

e.g.,

$$\mathscr{L} = \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}\sigma^2(\partial\theta)^2$$

MR, 1808.09966; also see Dolan, Draper, Kozaczuk, Patel 1701.05572

Axion String as Fundamental Object



infinite field-space distance to
singular string core

This time $\Phi(x) = \sigma(x)e^{i\theta(x)}$, but $\sigma \to 0$ at *infinite* distance in field space.

e.g.,

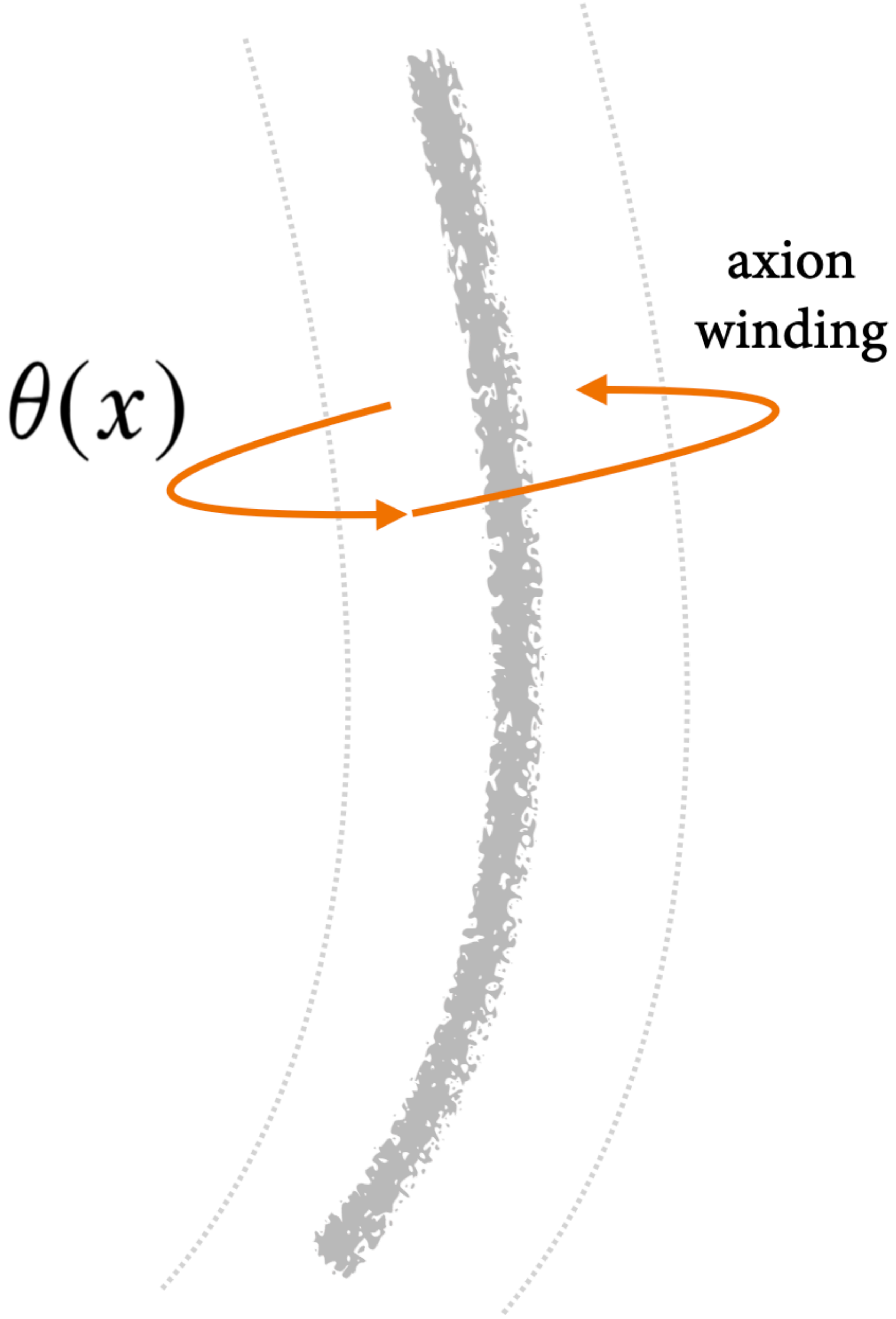
$$\mathscr{L} = \frac{M_*^2}{2\sigma^2}(\partial\sigma)^2 + \frac{1}{2}\sigma^2(\partial\theta)^2$$

Realized in simple extra-dimensional cases.

“Fundamental axion”

MR, 1808.09966; also see Dolan, Draper, Kozaczuk, Patel 1701.05572; fundamental axion strings are also called “EFT strings,” see Lanza, Marchesano, Martucci, Valenzuela 2104.05726

Fundamental Axion String and QG Scale



Fundamental axion strings come from fundamental QG objects: strings or branes wrapped around internal dimensions, with tension

$$\mathscr{T} \gtrsim \Lambda_{\mathrm{QG}}^2/(2\pi).$$

(Martucci, Risso, Valenti, Vecchi 2404.14489; MR 2406.08543)

$\mathscr{T}$ is hard to measure directly, but in examples it is related to f, as we will discuss.

Comparing Axion Classifications

"Electric" classifications (source of shift symmetry)

"Magnetic" classifications (core of axion string)

4d axion $\xrightarrow{\text{always}}$ axion string is soliton

sometimes*

extra-dimensional axion $\xleftrightarrow{\text{often}}$ fundamental axion: string core at infinite distance

* examples of solitonic strings for extra-dimensional axions: C_4 axion at conifold point; U(1) from SU(2) higgsing, see Petrossian-Byrne, Villadoro 2606.15957, Craig, Madden 2606.16445

Fundamental Axions are Different

4d axions are conventional (pseudo-)Nambu-Goldstone bosons, arising from a symmetry breaking phase transition at the scale f. In particular, axion strings can be produced cosmologically by the **Kibble-Zurek mechanism** at the phase transition.

Fundamental axions are *not PNGBs* in the traditional sense. There is no restoration of PQ symmetry at high T. The core of the string probes asymptotic UV physics, not just large-T physics.
Always "pre-inflation cosmology."

For *typical* extra-dimensional axions, we do not expect cosmological production of axion strings in any significant abundance.

Extra-Dimensional Axions and the UV Cutoff

The Quantum Gravity Cutoff from Axions

Suppose that we have an **extra-dimensional axion** with decay constant f, as might arise from string theory. Then we expect that the **quantum gravity cutoff (species scale, string scale) is bounded above:**

(arXiv:2406.08543 MR;
arXiv:2505.22713 + 2606.21650 MR, Tom Rudelius, Christopher Tudball;
also arXiv:2505.15884 Josh Benabou, Katie Fraser, Mario Reig, Ben Safdi)

$$\Lambda_{\mathrm{QG}} \lesssim 2\pi\sqrt{S_{\mathrm{inst}}}\,f$$

$$\sim 100\,f \quad \text{(for a QCD axion)}$$

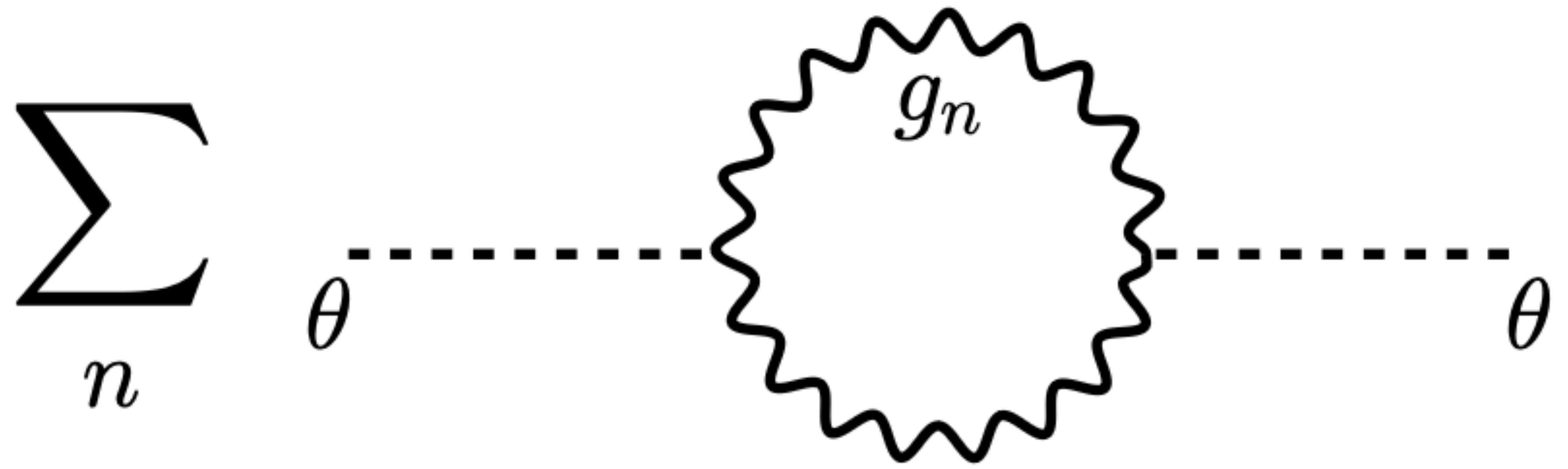
The most persuasive arguments are for **fundamental** extra-dimensional axions, but in examples in string theory it holds for extra-dimensional axions even when the axion strings are solitonic (at conifold points).

Relating f to the UV Cutoff

Non-renormalizable axion-gauge field interaction: $\Lambda_{\mathrm{UV}} \lesssim \sqrt{\pi} S_{\mathrm{inst}} f$ by standard unitarity arguments.

Extra-dimensions: stronger bound from coupling to **a tower of KK modes of gauge fields**, with number ~ volume ~ $1/g^2$.

E.g., naturalness argument (MR '24):



Two couplings $\sim g^2/f$

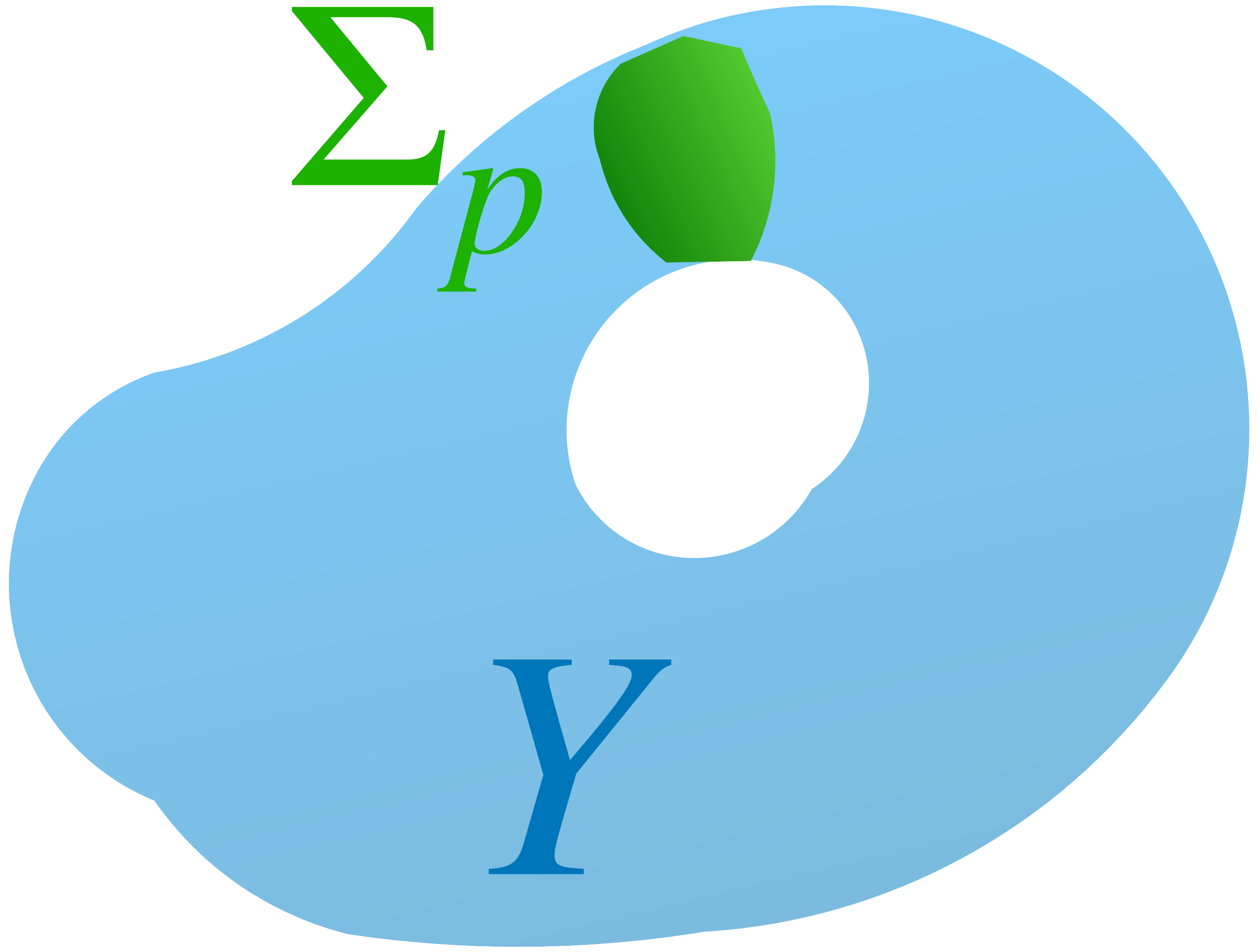
Kaluza-Klein modes $\sim 1/g^2$

$$\Rightarrow \quad \Lambda_{\mathrm{UV}} \lesssim O(1) f/g$$

(naturalness argument: arXiv:2406.08543 MR; similar unitarity argument: arXiv:2505.15884 Josh Benabou, Katie Fraser, Mario Reig, Ben Safdi; also see Seo, arXiv:2407.16156)

Extra-Dimensional Axion Decay Constant

Ansatz: $C_p(x, y) = \theta(x)\hat{\omega}_p(y)$, where $\theta(x)$ = 4d axion field, $\hat{\omega}_p(y)$ = harmonic p-form in the extra dimensions, normalized to $\displaystyle\int_{\Sigma_p} \hat{\omega}_p = 1.$



Decay constant: $\displaystyle f_a^2 = \frac{1}{e_p^2}\int_Y \hat{\omega}_p \wedge \star_Y \hat{\omega}_p$

Simple estimate:

$$f_a^2 \sim \frac{1}{e_p^2}\frac{\mathrm{Vol}(\Gamma_{n-p})}{\mathrm{Vol}(\Sigma_p)}$$

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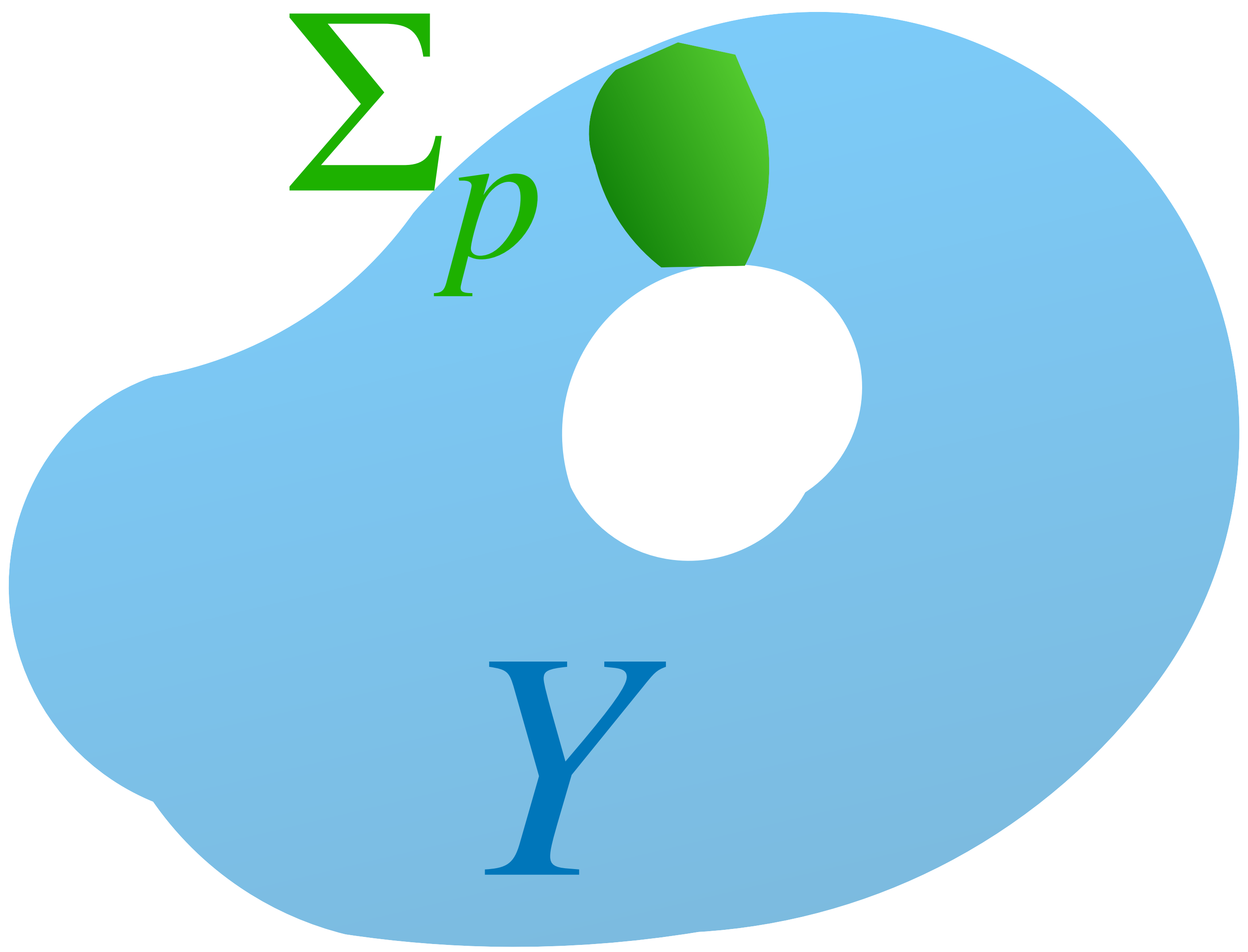

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Typically of order cutoff (string scale)

(MR, arXiv:2406.08543)

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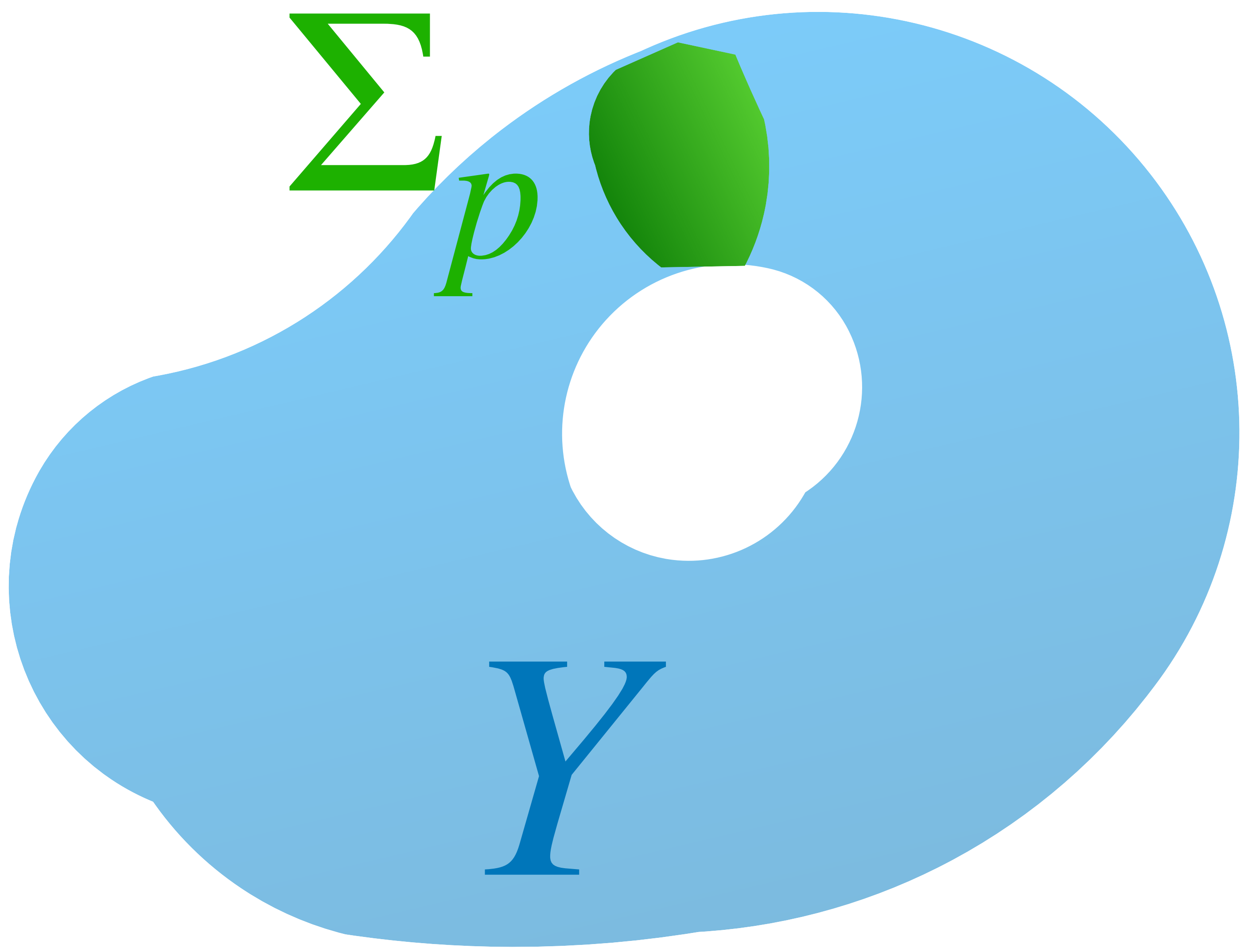

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Determines instanton action, proportional to SM $1/g^2$

(MR, arXiv:2406.08543)

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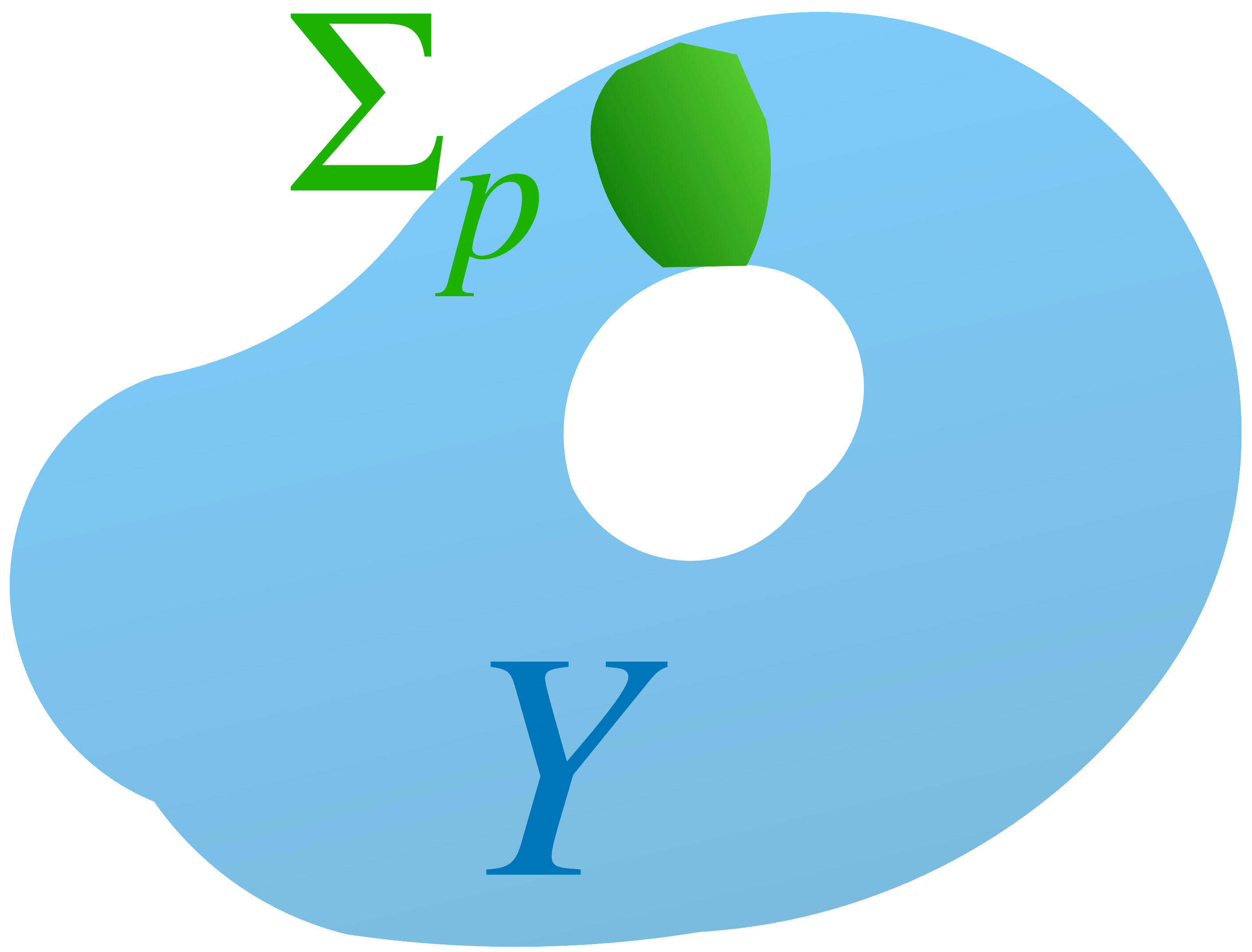

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Simple estimate:

$$f_a^2 \sim \frac{1}{e_p^2}\frac{\mathrm{Vol}(\Gamma_{n-p})}{\mathrm{Vol}(\Sigma_p)}$$

Intersecting cycle volume, determines axion string tension

(MR, arXiv:2406.08543)

Co-Scaling and Axion Strings

When such a simple estimate works, there is an axion string with tension

$$\mathcal{T} \sim 2\pi S_{\mathrm{inst}} f^2.$$

At weak coupling / large volume $2\pi\mathcal{T} \gtrsim \Lambda_{\mathrm{QG}}^2$ (cf Martucci, Risso, Valenti, Vecchi ’24).

This is an instance of “electric/magnetic co-scaling”:

$$\mathcal{T}_{\mathrm{mag}}/\mathcal{T}_{\mathrm{el}} \approx g_{\mathrm{mag}}/g_{\mathrm{el}} = 2\pi/g_{\mathrm{el}}^2\,.$$

This scaling is **very common** but not universal. For example, it fails near flop transitions where an electric object becomes light but no magnetic object becomes light (MR, Rudelius, Tudball ’25). However, in such examples we **still** find

$$\Lambda_{\mathrm{QG}} \lesssim 2\pi\sqrt{S_{\mathrm{inst}}}\,f.$$

The GUT-scale axion

If $\mathrm{Vol}(Y) \sim \mathrm{Vol}(\Gamma_{n-p}) \times \mathrm{Vol}(\Sigma_p)$, $\mathscr{T} \sim 2\pi S_{\mathrm{inst}} f_a^2$ holds via the even stronger statements

$$f_a S_{\mathrm{inst}} \sim M_{\mathrm{Pl}}, \qquad \mathscr{T} \sim 2\pi f_a M_{\mathrm{Pl}}$$ ("naive WGC saturation")

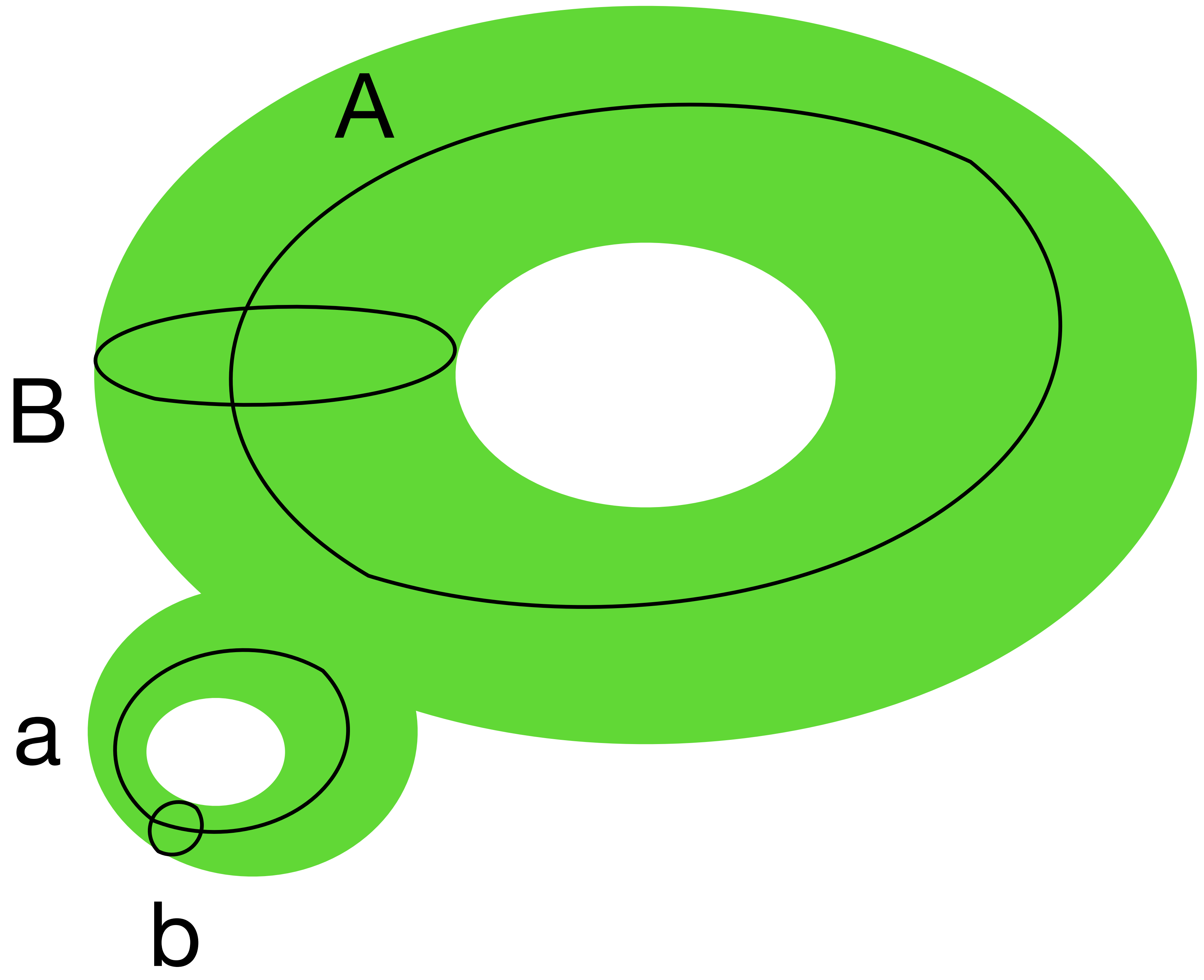
and one expects $f_a \sim \dfrac{g^2}{8\pi^2} M_{\mathrm{Pl}} \sim 2 \times 10^{16}\,\mathrm{GeV}$.

axion WGC: Arkani-Hamed, Motl, Nicolis, Vafa '06; di Ubaldo, Iliesiu, Lin, Yan '26; Maldacena, Maloney, McPeak '26

This holds in many cases (e.g., Svrcek & Witten hep-th/0605206; Benabou, Bonnefoy, Buschmann, Kumar, Safdi 2312.08425).

In particular, this is what one expects for heterotic string theory.

General geometries



The GUT estimate holds for **simple geometries.**

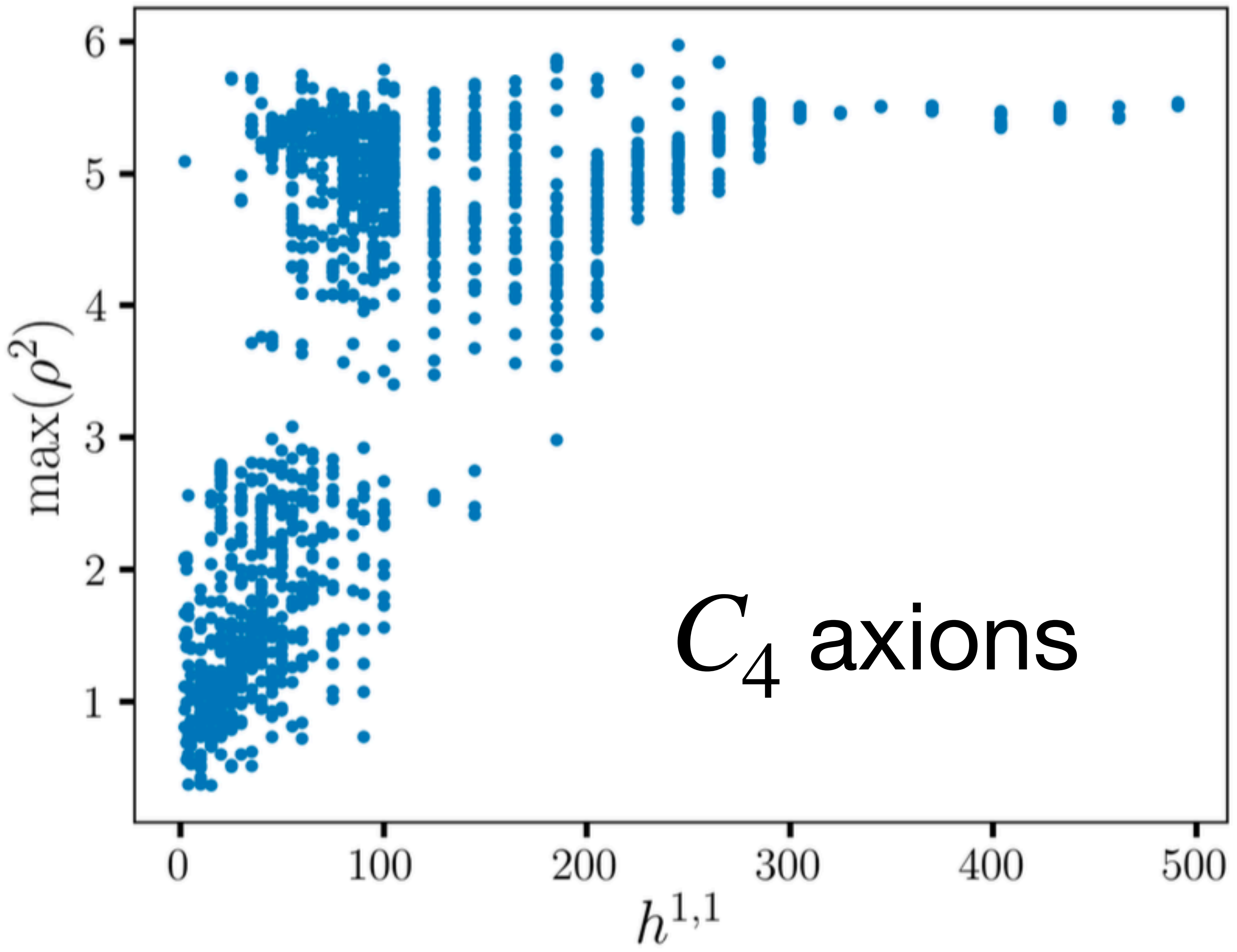
If the SM lives in a small region in a much bigger geometry (a "decoupling limit" of gravity), f **can naturally be much smaller** (e.g., Conlon hep-th/0602233, MR 2406.08543).

Cartoon: axion from $\displaystyle\int_a \theta$. Then both f and $\sqrt{\mathcal{T}}$ scale with overall volume as $M_{\mathrm{Pl}}/\sqrt{\mathcal{V}}$, but $S_{\mathrm{inst}} \sim \mathrm{Vol}(a)$ is much smaller.

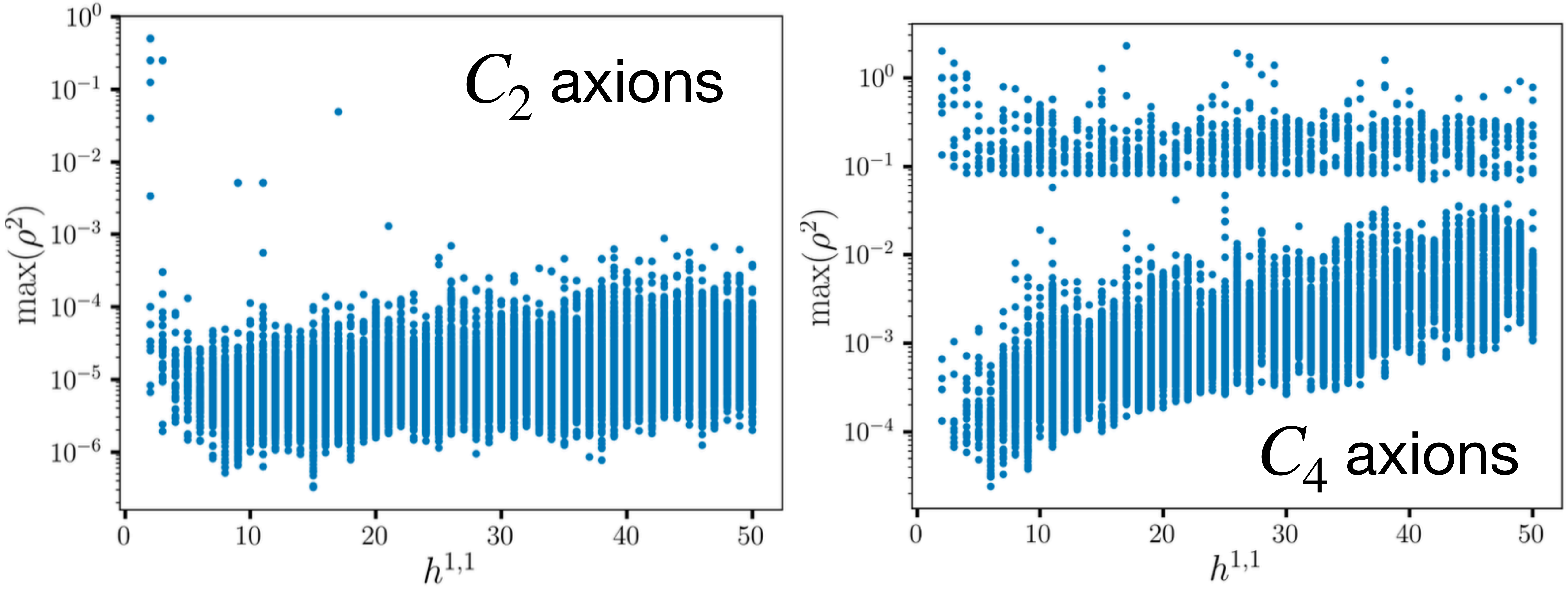
M_s vs $2\pi\sqrt{S_{\text{inst}}}f$ in the Type IIB Landscape

New results: analytic and numerical computations of $\rho = M_s/(2\pi\sqrt{S_{\text{inst}}}f)$ for Calabi-Yau compactifications of Type IIB string theory, focusing on boundaries of the Kähler cone where the most naive estimates might fail.

tip of stretched Kähler cone



near a facet of Kähler cone



MR, T. Rudelius, C. Tudball, 2606.21650

relies on CYTools (Demirtas, Rios-Tascon, McAllister ’22)

Axion Decay Constant and Λ_{QG}

Summing up: many experiments have the goal of (essentially) measuring f.

Extra-dimensional models are motivated both by the **quality problem** (from the bottom up) and by **string theory** (from the top down).

If we interpret an axion discovery in such a framework, we constrain something very fundamental: the scale Λ_{QG}.

And what we learn is **a strong constraint**: e.g., the "sweet spot" for QCD axion dark matter, $f \sim 10^{12}\,\mathrm{GeV}$, would lead to $\Lambda_{\mathrm{QG}} \lesssim 10^{14}\,\mathrm{GeV} \ll M_{\mathrm{GUT}}$, which has major implications for how we think about inflation, neutrino masses, and more.

We can try to push this lesson even further, with more assumptions….

From the Quantum Gravity Scale to the SUSY Scale

We have been discussing the **quantum gravity scale cutoff from axions:**

$$\Lambda_{\mathrm{QG}} \lesssim 2\pi\sqrt{S_{\mathrm{inst}}}f$$

Assuming a bound on the quantum gravity scale, what *else* can we learn? Suppose that at *some* scale, there is a **supersymmetric** 4d EFT description of the theory. Then we expect **the gravitino mass is bounded:**

$$m_{3/2} \lesssim O(1)\frac{\Lambda_{\mathrm{QG}}^2}{M_{\mathrm{Pl}}}$$

work in progress: Damian van de Heisteeg, MR, Tom Rudelius, Christopher Tudball

Quantum Gravity Scale to Gravitino Scale

Once we know something about $\Lambda_{\rm QG}$, we can make a statement about the scale of supersymmetry breaking, ***if we assume*** 4d supersymmetry is valid at any scale at all. Supersymmetry + gravity:

$$m_{\rm gravitino} = \underbrace{\exp\left(\frac{K}{2M_{\rm Pl}^2}\right)}_{\left(O(1)\frac{\Lambda_{\rm QG}}{M_{\rm Pl}}\right)^{\gamma}} \frac{\overbrace{\langle |W| \rangle}^{\lesssim O(1) M_{\rm Pl}}}{M_{\rm Pl}^2} \lesssim O(1)\frac{\Lambda_{\rm QG}^2}{M_{\rm Pl}}.$$

This happens from $T \gg 1$ limits where $K = -cM_{\rm Pl}^2 \log(T + T^\dagger) + \ldots$

$\gamma \geq 2$ *except* in phenomenologically unrealistic $g_s \to 0$ case.

(work in progress, Damian van de Heisteeg, MR, Tom Rudelius, Christopher Tudball)

Kähler Potential & Quantum Gravity Scale

$$\exp\left(\frac{K}{M_{\rm Pl}^2}\right) \sim \left(\frac{\Lambda_{\rm QG}}{M_{\rm Pl}}\right)^{2\gamma}$$

Scenario	$\exp(K/M_{\rm Pl}^2)$	Exponent γ
Heterotic	$\frac{1}{\mathcal{V}}\left(\frac{M_s}{M_{\rm Pl}}\right)^2 \sim \frac{1}{g_s^2}\left(\frac{M_s}{M_{\rm Pl}}\right)^4$	$\in [1,2]$
IIA	$\frac{1}{\mathcal{V}}\left(\frac{M_s}{M_{\rm Pl}}\right)^4 \sim \frac{1}{g_s^2}\left(\frac{M_s}{M_{\rm Pl}}\right)^6$	$\in [2,3]$
IIB	$\left(\frac{M_s}{M_{\rm Pl}}\right)^4$	2
Het. M-theory	$\frac{1}{\rho}\left(\frac{M_{11}}{M_{\rm Pl}}\right)^4 \sim \mathcal{V}_6\left(\frac{M_{11}}{M_{\rm Pl}}\right)^6$	$\in [2,3]$
M-theory on G_2	$\left(\frac{M_{11}}{M_{\rm Pl}}\right)^6$	3

Ongoing Research: Cosmological Implications

Reducing axion isocurvature for an extra-dimensional, pre-inflation axion: axion properties different today than during inflation.

Time-dependent mass, first-order phase transition, small-scale isocurvature, ….

w/ Prish Chakraborty, Junyi Cheng, Zekai Wang
arXiv:2507.12519 and ongoing work

Time-dependent decay constant, moduli dynamics, reheating, …

w/ Chandrika Chandrashekar (ongoing work)

Takeaway messages

- Extra-dimensional axions are elegant solutions to the quality problem (and hence the Strong CP problem).

- A variety of arguments (heuristic arguments like naturalness, detailed calculations in string theory) imply **"Quantum Gravity Cutoff from Axions"**:

$$\Lambda_{\mathrm{QG}} \lesssim 2\pi\sqrt{S_{\mathrm{inst}}}f$$

- For an extra-dimensional axion, experimental determination of the decay constant f then tells us about $\Lambda_{\mathrm{QG}} \lesssim 100 f$ *and* (if 4d SUSY exists at all) $m_{\mathrm{SUSY}} \lesssim \Lambda_{\mathrm{QG}}^2/M_{\mathrm{Pl}}$.

- The stakes are high for axion experiments!

Thank you!