Symmetry Perspective on Exotic Axion-Photon Coupling

Work in collaboration with T. Gherghetta and J. Terning
arXiv: 2509.14395 (Phys. Rev. D **112**, 095023)

References:
Sokolov, Ringwald arXiv: 2205.02605, 2303.10170
Heindenreich, McNamara, Reece: arXiv: 2309.07957

Gongjun Choi (Institute for Basic Science)

Axions in Seoul 2026, Seoul National University
Jul/27/2026

Outline

- Review of coupling btw dyon-philic axion and photon
  (proposed by Sokolov & Ringwald)
  - why (phenomenologically) interesting?

- Existence of non-invertible chiral symmetry
  (given the exotic ABJ anomaly)

- Conclude if dyon-philic "QCD axion" is viable

Maxwell with matter fields

- Action

$$S = \int -\frac{1}{2e^2} F \wedge \star F + \frac{\theta}{8\pi^2} F \wedge F - A \wedge \star J + \int d^4x \sqrt{-g} \sum_{j=1}^{N_f} i\bar{\Psi}_j \partial_\mu \gamma^\mu \Psi_j \,,$$

- Manifestly Lorentz invariant Lagrangian with a single A that couples to both electric *J* and magnetic current *K* → not possible

- But, can still study dynamics of *J* and *K* with e.o.m

e.o.m of A

$$\boxed{\mathrm{d}\left(\frac{1}{e^2} \star F - \frac{\theta}{2\pi}\frac{F}{2\pi}\right) = \star J}$$

$$\boxed{\mathrm{d}\left(\frac{F}{2\pi}\right) = \star K}$$

Maxwell equations

- Maxwell equations

$$\boxed{\mathrm{d}\left(\frac{1}{e^2}\star F\right) = \star J}$$

$$\boxed{\mathrm{d}\left(\frac{F}{2\pi}\right) = \star K}$$

$$\begin{aligned}
\nabla \cdot \vec{E} &= \rho_E \\
\nabla \cdot \vec{B} &= \rho_M \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} - \vec{K} \\
\nabla \times \vec{B} &= \frac{\partial \vec{E}}{\partial t} + \vec{J}
\end{aligned}$$

Modified Maxwell equations in axion-Maxwell

- Modified Maxwell equations

$$g_{a\gamma\gamma} = \frac{(q_e e)^2}{4\pi^2 f}$$

$$\begin{aligned}
\nabla \cdot \vec{E} &= \rho_E - g_{a\gamma\gamma} \vec{B} \cdot \nabla a \\
\nabla \cdot \vec{B} &= \rho_M \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} - \vec{K} \\
\nabla \times \vec{B} &= \frac{\partial \vec{E}}{\partial t} + \vec{J} - g_{a\gamma\gamma} \left(-\frac{\partial a}{\partial t} \vec{B} + \vec{E} \times \nabla a \right)
\end{aligned}$$

$$\vec{J}_{\mathrm{eff,a}}$$

Modified Maxwell equations in axion-Maxwell

- Modified Maxwell equations

$$g_{a\gamma\gamma} = \frac{(q_e e)^2}{4\pi^2 f}$$

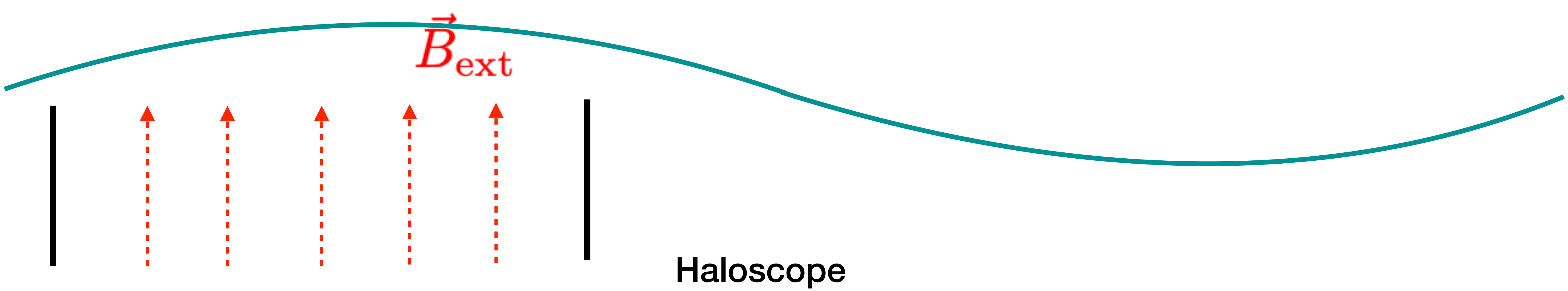
$$\nabla \cdot \vec{E} = \rho_E - g_{a\gamma\gamma}\vec{B} \cdot \nabla a$$

$$\nabla \cdot \vec{B} = \rho_M$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{K}$$

$$\vec{J}_{\mathrm{eff,a}}$$

$$\nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J} - g_{a\gamma\gamma}\left(-\frac{\partial a}{\partial t}\vec{B} + \vec{E} \times \nabla a\right)$$



Exotic axion-photon coupling

- What if a heavy quark in Kim-Shifman-Vainstein-Zakharov is dyonic? Ringwald, Sokolov 22',23'

- Modified axion coupling induced by dyonic fermion (q_e,q_m)

$$S_{\text{eff}} \supset \int_M \left(q_e^2 - \left(q_m \frac{2\pi}{e^2} \right)^2 \right) \frac{\theta(x)}{8\pi^2} F \wedge F - \left(2 q_e q_m \frac{2\pi}{e^2} \right) \frac{\theta(x)}{8\pi^2} F \wedge \star F$$

Modified Maxwell equations in axion-Maxwell

- Modified Maxwell equations

$$
\begin{aligned}
\nabla \cdot \vec{E} &= \rho_E - g_{aEE} \vec{B} \cdot \nabla a \\
\nabla \cdot \vec{B} &= \rho_M \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} - \vec{K} \\
\nabla \times \vec{B} &= \frac{\partial \vec{E}}{\partial t} + \vec{J} - g_{aEE} \left(-\frac{\partial a}{\partial t} \vec{B} + \vec{E} \times \nabla a \right)
\end{aligned}
$$

$$g_{aEE} = \frac{(q_e e)^2}{4\pi^2 f}$$

Modified Maxwell equations in axion-Maxwell

- Modified Maxwell equations w/ dyonic fermion (q_e,q_m)

Ringwald, Sokolov 22',23'

$$\nabla \cdot \vec{E} = \rho_E - g_{aEE}\vec{B}\cdot\nabla a - g_{aEM}\vec{E}\cdot\nabla a$$

$$\nabla \cdot \vec{B} = \rho_M - g_{aMM}\vec{E}\cdot\nabla a - g_{aEM}\vec{B}\cdot\nabla a$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{K} + g_{aMM}(\vec{B}\times\nabla a + \frac{\partial a}{\partial t}\vec{E}) + g_{aEM}(\vec{E}\times\nabla a - \frac{\partial a}{\partial t}\vec{B})$$

$$\nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J} - g_{aEE}\left(-\frac{\partial a}{\partial t}\vec{B} + \vec{E}\times\nabla a\right) - g_{aEM}\left(\frac{\partial a}{\partial t}\vec{E} + \vec{B}\times\nabla a\right)$$

$$g_{aEE} = \frac{(q_e e)^2}{4\pi^2 f}, \quad g_{aEM} = \frac{q_e q_m}{4\pi^2 f}, \quad g_{aMM} = \frac{q_m^2}{4\pi^2 f}\left(\frac{2\pi}{e}\right)^2$$

Modified Maxwell equations in axion-Maxwell

- Modified Maxwell equations w/ dyonic fermion (q_e,q_m)

Ringwald, Sokolov 22',23'

$$\nabla \cdot \vec{E} = \rho_E - g_{aEE}\vec{B}\cdot\nabla a - g_{aEM}\vec{E}\cdot\nabla a$$

$$\nabla \cdot \vec{B} = \rho_M - g_{aMM}\vec{E}\cdot\nabla a - g_{aEM}\vec{B}\cdot\nabla a$$

Enhanced signal!

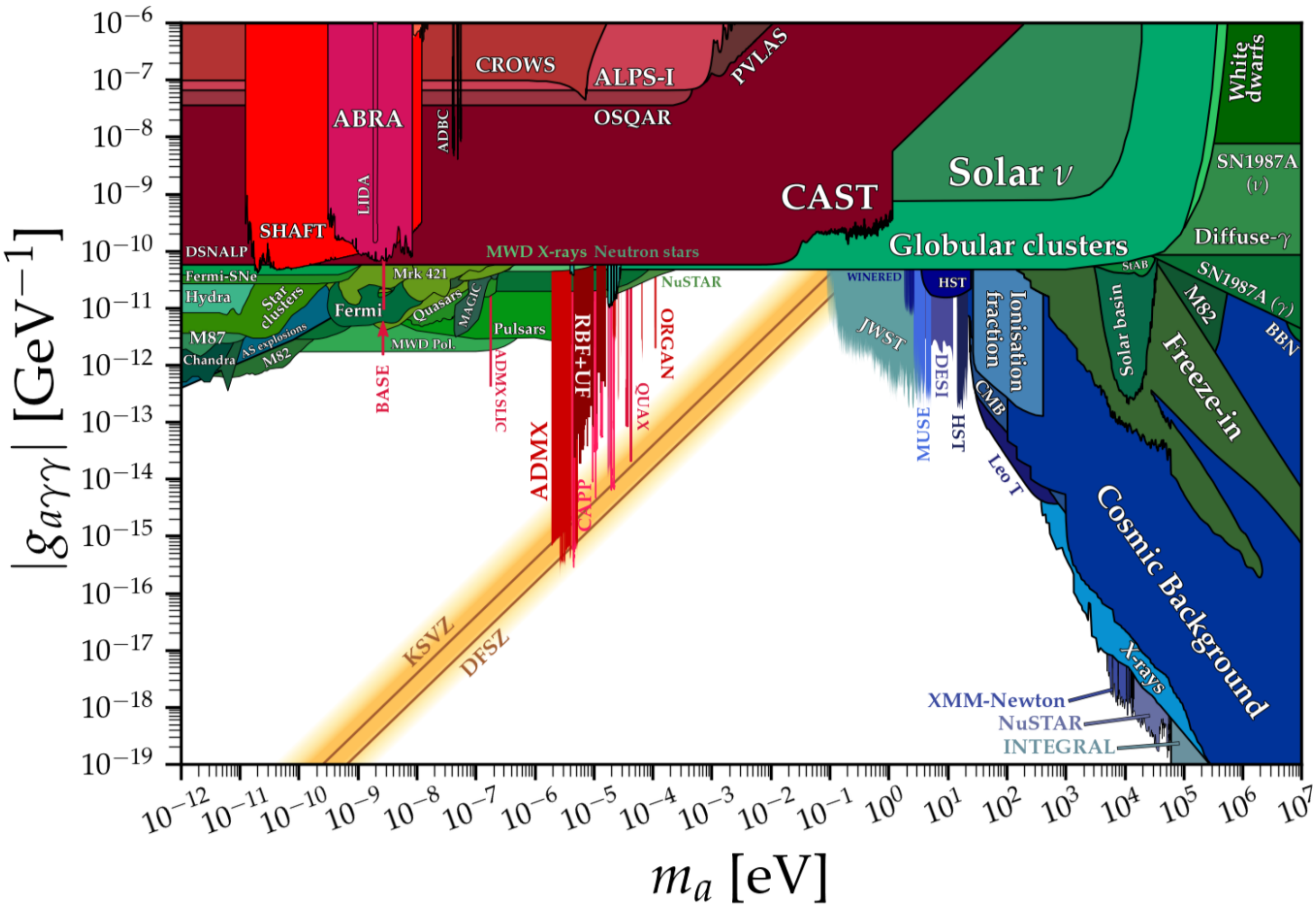
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{K} + g_{aMM}(\vec{B}\times\nabla a \boxed{+ \frac{\partial a}{\partial t}\vec{E})} + g_{aEM}(\vec{E}\times\nabla a - \frac{\partial a}{\partial t}\vec{B})$$

$$\nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J} - g_{aEE}\left(-\frac{\partial a}{\partial t}\vec{B} + \vec{E}\times\nabla a\right) - g_{aEM}\left(\frac{\partial a}{\partial t}\vec{E} + \vec{B}\times\nabla a\right)$$

$$g_{aEE} = \frac{(q_e e)^2}{4\pi^2 f}, \quad g_{aEM} = \frac{q_e q_m}{4\pi^2 f}, \quad g_{aMM} = \frac{q_m^2}{4\pi^2 f}\left(\frac{2\pi}{e}\right)^2$$

Enhanced experimental signal

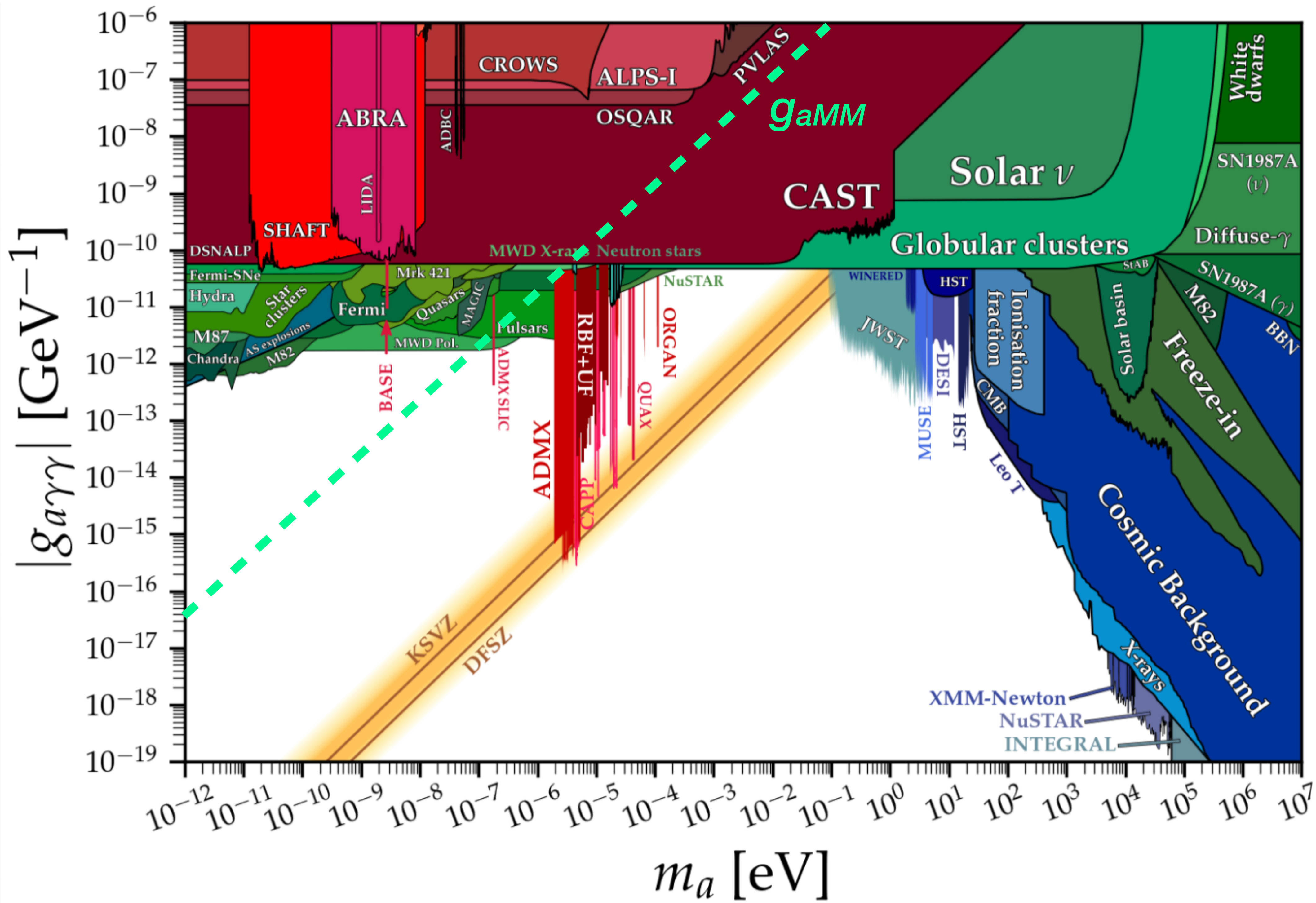
Ringwald, Sokolov 22',23'



$$g_{aMM} = g_{aEE} \times \frac{1}{4\alpha_{em}^2} \simeq 4700\, g_{aEE}$$

Enhanced experimental signal

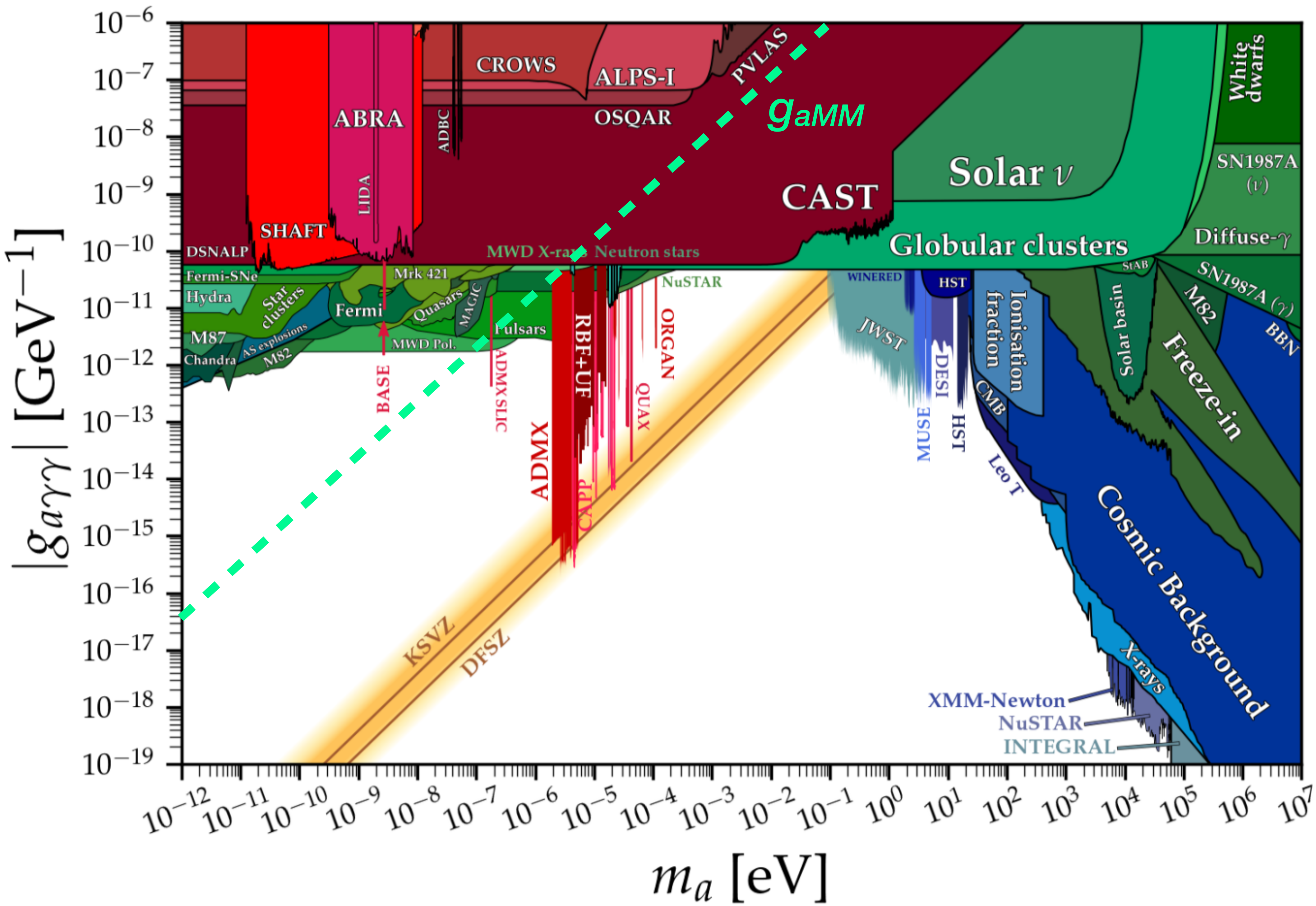
Ringwald, Sokolov 22',23'



$$g_{aMM} = g_{aEE} \times \frac{1}{4\alpha_{em}^2} \simeq 4700\, g_{aEE}$$

Enhanced experimental signal

Ringwald, Sokolov 22',23'



$$g_{aMM} = g_{aEE} \times \frac{1}{4\alpha_{em}^2} \simeq 4700\, g_{aEE}$$

Is this consistent with Standard Model??

Strategy

Is this consistent with Standard Model??

- Non-invertible subgroup of axion shift symmetry
  → Useful!

- Derive the exotic axon-photon coupling using SL(2,Z)
- Is there non-invertible chiral symmetry?
- Implication for the variant of KSVZ

Dual gauge field

- Action

$$S = \int -\frac{1}{2e^2} F \wedge \star F + \frac{\theta}{8\pi^2} F \wedge F - A \wedge \star J$$
$$+ \int d^4x \sqrt{-g} \sum_{j=1}^{N_f} i\bar{\Psi}_j \partial_\mu \gamma^\mu \Psi_j \,,$$

- Away from electric charge, define dual gauge field $\tilde{A}$

From e.o.m of A

$$\mathrm{d}\left(\frac{1}{e^2} \star F - \frac{\theta}{2\pi}\frac{F}{2\pi}\right) = 0 \quad \longrightarrow \quad \frac{\mathrm{d}\tilde{A}}{2\pi} = -\frac{1}{e^2} \star F + \frac{\theta}{2\pi}\frac{F}{2\pi}$$

Dual frame and SL(2,Z)

- Complexified gauge coupling

$$\tau \equiv \frac{\theta_0}{2\pi} + \frac{2\pi}{e^2} i$$

- Under the transformation

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

- $(A, \tilde{A})$ and $(\star J, \star K)$ transform as

$$\begin{bmatrix} d & c \\ b & a \end{bmatrix} \begin{bmatrix} A \\ \tilde{A} \end{bmatrix} = \begin{bmatrix} A' \\ \tilde{A}' \end{bmatrix} \qquad \begin{bmatrix} a & -b \\ -c & d \end{bmatrix} \begin{bmatrix} \star J \\ \star K \end{bmatrix} = \begin{bmatrix} \star J' \\ \star K' \end{bmatrix}$$

Dual frame and SL(2,Z)

- From now on, we define the QED frame with

$$\mathrm{d} \star j_{\mathrm{chiral}} = \frac{\mathcal{A}}{8\pi^2} F \wedge F$$

and everything will be "primed" in QED frame.

- Everything in a dual frame will be "unprimed".

ABJ (chiral) anomaly

- ABJ anomaly in U(1) gauge theory w/ m=0 dyonic fermions?
  → don't know how dyon measure changes under $U(1)_{\text{chiral}}$
  → But, know how electron measure changes under $U(1)_{\text{chiral}}$

Primed frame

$$\mathrm{d} \star j_{\text{chiral}} = \frac{\mathcal{A}}{8\pi^2} F' \wedge F'$$

Unprimed frame

?

τ

ABJ (chiral) anomaly

- ABJ anomaly in U(1) gauge theory w/ m=0 dyonic fermions?
  → don't know how dyon measure changes under $U(1)_{\mathrm{chiral}}$
  → But, know how electron measure changes under $U(1)_{\mathrm{chiral}}$

$$\tau \equiv \frac{\theta_0}{2\pi} + \frac{2\pi}{e^2} i$$

Primed frame

$$\mathrm{d} \star j_{\mathrm{chiral}} = \frac{\mathcal{A}}{8\pi^2} F' \wedge F'$$

$$\tau'$$

SL(2,Z)

Unprimed frame

?

$$\tau$$

ABJ (chiral) anomaly

- ABJ anomaly in U(1) gauge theory w/ m=0 dyonic fermions?
  → don't know how dyon measure changes under $U(1)_{\text{chiral}}$
  → But, know how electron measure changes under $U(1)_{\text{chiral}}$

Primed frame

$$\mathrm{d} \star j_{\text{chiral}} = \frac{\mathcal{A}}{8\pi^2} F' \wedge F'$$

$$\tau' \xrightarrow[U(1)_{\text{chiral}}]{}$$

Unprimed frame

?

$$\tau$$

ABJ (chiral) anomaly

- ABJ anomaly in U(1) gauge theory w/ m=0 dyonic fermions?
  → don't know how dyon measure changes under $U(1)_{\mathrm{chiral}}$
  → But, know how electron measure changes under $U(1)_{\mathrm{chiral}}$

Primed frame

$$\mathrm{d} \star j_{\mathrm{chiral}} = \frac{\mathcal{A}}{8\pi^2} F' \wedge F'$$

$$\tau' \xrightarrow{U(1)_{\mathrm{chiral}}} \tau' - \frac{\mathcal{A}\alpha}{2\pi}$$

Unprimed frame

?

$$\tau$$

ABJ (chiral) anomaly

- ABJ anomaly in U(1) gauge theory w/ m=0 dyonic fermions?
  → don't know how dyon measure changes under $U(1)_{\text{chiral}}$
  → But, know how electron measure changes under $U(1)_{\text{chiral}}$

Primed frame

$$\mathrm{d}\star j_{\text{chiral}} = \frac{\mathcal{A}}{8\pi^2} F' \wedge F' \qquad \tau' \qquad \tau' - \frac{\mathcal{A}\alpha}{2\pi}$$

SL(2,Z)$^{-1}$

Unprimed frame

? $\qquad \tau$

ABJ (chiral) anomaly

- ABJ anomaly in U(1) gauge theory w/ m=0 dyonic fermions?
  → don't know how dyon measure changes under U(1)$_{\text{chiral}}$
  → But, know how electron measure changes under U(1)$_{\text{chiral}}$

Primed frame

$$\mathrm{d} \star j_{\text{chiral}} = \frac{\mathcal{A}}{8\pi^2} F' \wedge F' \qquad \tau' \qquad \tau' - \frac{\mathcal{A}\alpha}{2\pi}$$

$\downarrow$ SL(2,Z)$^{-1}$

Unprimed frame

? $\qquad \tau \qquad \boxed{\tau_{\{\alpha\}}}$

ABJ (chiral) anomaly in Dual frame

- Complexified gauge coupling after $U(1)_{\text{chiral}}$-rotating phases of dyonic fermions

$$
\begin{aligned}
\tau_{\{\alpha\}} &= \frac{d\tau'_{\{\alpha\}} - b}{-c\tau'_{\{\alpha\}} + a} = \frac{\tau - \mathcal{A}d(c\tau + d)\frac{\alpha}{2\pi}}{1 + \mathcal{A}c(c\tau + d)\frac{\alpha}{2\pi}} \\
&= \tau - \mathcal{A}(c\tau + d)^2 \frac{\alpha}{2\pi} + \mathcal{O}\left[\left(\frac{\alpha}{2\pi}\right)^2\right]
\end{aligned}
$$

ABJ (chiral) anomaly in Dual frame

- Complexified gauge coupling after $U(1)_{\text{chiral}}$-rotating phases of dyonic fermions

$$\boxed{\tau_{\{\alpha\}}} = \frac{d\tau'_{\{\alpha\}} - b}{-c\tau'_{\{\alpha\}} + a} = \frac{\tau - \mathcal{A}d(c\tau + d)\frac{\alpha}{2\pi}}{1 + \mathcal{A}c(c\tau + d)\frac{\alpha}{2\pi}}$$

$$\boxed{= \tau} - \mathcal{A}(c\tau + d)^2 \frac{\alpha}{2\pi} + \mathcal{O}\left[\left(\frac{\alpha}{2\pi}\right)^2\right]$$

ABJ (chiral) anomaly in Dual frame

- Complexified gauge coupling after $U(1)_{\text{chiral}}$-rotating phases of dyonic fermions

$$\begin{aligned}
\mathrm{Re}[\Delta\tau] &= \mathcal{A}\left\{-\left(c\frac{\theta_0}{2\pi}+d\right)^2+\left(c\frac{2\pi}{e^2}\right)^2\right\}\frac{\alpha}{2\pi} \\
&\quad +\mathcal{O}\left[\left(\frac{\alpha}{2\pi}\right)^2\right], \\
\mathrm{Im}[\Delta\tau] &= \mathcal{A}\left\{-2\left(c\frac{\theta_0}{2\pi}+d\right)\left(c\frac{2\pi}{e^2}\right)\right\}\frac{\alpha}{2\pi} \\
&\quad +\mathcal{O}\left[\left(\frac{\alpha}{2\pi}\right)^2\right].
\end{aligned}$$

ABJ (chiral) anomaly in Dual frame

- Complexified gauge coupling after $U(1)_{\text{chiral}}$-rotating phases of dyonic fermions

$$\int \prod_{i=1}^{N_f} \mathcal{D}\Psi_i\, \mathcal{D}\bar{\Psi}_i \mathcal{D}A\; e^{iS - i\alpha \int \mathrm{d}\star j_{\text{chiral}}}$$
$$= \int \prod_{i=1}^{N_f} \mathcal{D}\Psi_i\, \mathcal{D}\bar{\Psi}_i \mathcal{D}A\; e^{iS_{\{\alpha\}}}\,,$$

$$\longrightarrow \quad \exp\left[-i\alpha \int \mathrm{d}\star j_{\text{chiral}}\right]$$
$$= \exp\left[i \int -\frac{\mathrm{Im}[\Delta\tau]}{4\pi} F \wedge \star F + \frac{\mathrm{Re}[\Delta\tau]}{4\pi} F \wedge F\right]$$

ABJ (chiral) anomaly in Dual frame

- ABJ chiral anomaly in a unprimed frame

$$
\begin{aligned}
\mathrm{d}\star j_{\text{chiral}} &= -\frac{\mathrm{Re}[\Delta\tau]}{4\pi\alpha} F\wedge F + \frac{\mathrm{Im}[\Delta\tau]}{4\pi\alpha} F\wedge\star F\,, \\
&= \frac{\mathcal{A}}{8\pi^2}\left\{\left(c\frac{\theta_0}{2\pi}+d\right)^2 - \left(c\frac{2\pi}{e^2}\right)^2\right\} F\wedge F \\
&\quad -\frac{\mathcal{A}}{8\pi^2}\left\{2\left(c\frac{\theta_0}{2\pi}+d\right)\left(c\frac{2\pi}{e^2}\right)\right\} F\wedge\star F
\end{aligned}
$$

ABJ (chiral) anomaly in Dual frame

- ABJ chiral anomaly in a unprimed frame

$$
\begin{aligned}
\mathrm{d}\star j_{\mathrm{chiral}} &= -\frac{\mathrm{Re}[\Delta\tau]}{4\pi\alpha} F\wedge F + \frac{\mathrm{Im}[\Delta\tau]}{4\pi\alpha} F\wedge\star F\,, \\
&= \frac{\mathcal{A}}{8\pi^2}\left\{\left(c\frac{\theta_0}{2\pi}+d\right)^2 - \left(c\frac{2\pi}{e^2}\right)^2\right\} F\wedge F \\
&\quad -\frac{\mathcal{A}}{8\pi^2}\left\{2\left(c\frac{\theta_0}{2\pi}+d\right)\left(c\frac{2\pi}{e^2}\right)\right\} F\wedge\star F
\end{aligned}
$$

- Dyon with (q_e,q_m) in unprimed → Electron' with (-1,0) in primed for

$$
\boxed{\begin{array}{l} c=-q_m, d=-q_e \\ -aq_e+bq_m=1 \end{array}}
$$

ABJ (chiral) anomaly in Dual frame

- ABJ chiral anomaly in a unprimed frame

$$\mathrm{d} \star j_{\text{chiral}} = \left(q_e^2 - \left(q_m \frac{2\pi}{e^2} \right)^2 \right) \frac{1}{8\pi^2} F \wedge F - \left(2 q_e q_m \frac{2\pi}{e^2} \right) \frac{1}{8\pi^2} F \wedge \star F$$

- Indeed, the exotic axion-photon coupling arises

$$S_{\text{eff}} \supset \int_M \left(q_e^2 - \left(q_m \frac{2\pi}{e^2} \right)^2 \right) \frac{\theta(x)}{8\pi^2} F \wedge F - \left(2 q_e q_m \frac{2\pi}{e^2} \right) \frac{\theta(x)}{8\pi^2} F \wedge \star F$$

ABJ (chiral) anomaly in Dual frame

- ABJ chiral anomaly in a unprimed frame

$$\mathrm{d}\star j_{\text{chiral}} = \left(q_e^2 - \left(q_m \frac{2\pi}{e^2}\right)^2\right)\frac{1}{8\pi^2}F\wedge F - \left(2q_e q_m \frac{2\pi}{e^2}\right)\frac{1}{8\pi^2}F\wedge\star F$$

- Indeed, the exotic axion-photon coupling arises

$$S_{\text{eff}} \supset \int_M \left(q_e^2 - \left(q_m \frac{2\pi}{e^2}\right)^2\right)\frac{\theta(x)}{8\pi^2}F\wedge F - \left(2q_e q_m \frac{2\pi}{e^2}\right)\frac{\theta(x)}{8\pi^2}F\wedge\star F$$

If θ(x) were QCD axion,………

Is this consistent with Standard Model??

Non-invertible chiral symmetry

- For an anomalous $U(1)_{\mathrm{chiral}}$ under Abelian gauge group, ABJ anomaly reads

Choi, Lam, Shao 22'
Cordova, Ohmori 22'

$$\mathrm{d}\star j_{\mathrm{chiral}} = \frac{1}{8\pi^2} F' \wedge F'$$

- If there is $U(1)^{[1]}$ magnetic symmetry,
  → ∃ gauge invariant & topological symmetry defect operator

$$\mathcal{D}_{\frac{1}{N}}(F'/N) = \int [Da] \exp\left[\oint_M \frac{2\pi}{N} \star j_{\mathrm{chiral}} + \frac{iN}{4\pi} a \wedge da + \frac{i}{2\pi} a \wedge F'\right]$$

- This is why we have selection rules imposed by a full $U(1)_{\mathrm{chiral}}$ in QED in spite of ABJ anomaly.

Non-invertible chiral symmetry

- For an anomalous $U(1)_{\mathrm{chiral}}$ under Abelian gauge group, ABJ anomaly reads

Choi, Lam, Shao 22'
Cordova, Ohmori 22'

$$\mathrm{d}\star j_{\mathrm{chiral}} = \frac{1}{8\pi^2} F' \wedge \boxed{F'}$$

$$\boxed{F'/N = B^{[2]}}$$

- If there is $U(1)^{[1]}$ magnetic symmetry,
→ ∃ gauge invariant & topological symmetry defect operator

$$\mathcal{D}_{\frac{1}{N}}(B^{[2]}) = \int [Da] \exp\left[\oint_M \frac{2\pi}{N} \star j_{\mathrm{chiral}} + \frac{iN}{4\pi} a \wedge da + \frac{iN}{2\pi} a \wedge \boxed{B^{[2]}}\right]$$

Non-invertible chiral symmetry

GC, Gherghetta, Terning 25'

- Expectation: there must exist non-invertible $U(1)_{\text{chiral}}$

- Given ABJ anomaly induced by dyonic fermion,

$$\mathrm{d} \star j_{\text{chiral}} = \left(q_e^2 - \left(q_m \frac{2\pi}{e^2} \right)^2 \right) \frac{1}{8\pi^2} F \wedge F - \left(2 q_e q_m \frac{2\pi}{e^2} \right) \frac{1}{8\pi^2} F \wedge \star F$$

Non-invertible chiral symmetry

GC, Gherghetta, Terning 25'

- Expectation: there must exist non-invertible $U(1)_{\text{chiral}}$

- Given ABJ anomaly induced by dyonic fermion,

$$\mathrm{d}\star j_{\text{chiral}} = \boxed{\left(q_e^2 - \left(q_m \frac{2\pi}{e^2}\right)^2\right) \frac{1}{8\pi^2} F \wedge F - \left(2 q_e q_m \frac{2\pi}{e^2}\right) \frac{1}{8\pi^2} F \wedge \star F}$$

- Should find 3d TQFT s.t.

Anomaly inflow = RHS

Non-invertible chiral symmetry

GC, Gherghetta, Terning 25'

- Expectation: there must exist non-invertible $U(1)_{\text{chiral}}$

- If ABJ anomaly can be written as

$$\mathrm{d} \star j_{\text{chiral}} = \frac{1}{2} \star \hat{j}^{[2]} \wedge \star \hat{j}^{[2]}$$

and

$$\mathrm{d} \star \hat{j}^{[2]} = 0$$

- Then, can construct gauge invariant & topological symmetry defect operator (for α=2π/N)

$$\mathcal{D}_{\frac{1}{N}}(B^{[2]}) = \int [Da] \exp\left[\oint_M \frac{2\pi}{N} \star j_{\text{chiral}} + \frac{iN}{4\pi} a \wedge da + \frac{iN}{2\pi} a \wedge B^{[2]}\right]$$

Non-invertible chiral symmetry

GC, Gherghetta, Terning 25'

- Expectation: there must exist non-invertible $U(1)_{\text{chiral}}$

- If ABJ anomaly can be written as

$$d \star j_{\text{chiral}} = \frac{1}{2} \star \hat{j}^{[2]} \wedge \boxed{\star \hat{j}^{[2]}}$$

and

$$d \star \hat{j}^{[2]} = 0$$

$$\boxed{\star \hat{j}^{[2]} / N = B^{[2]}}$$

- Then, can construct gauge invariant & topological symmetry defect operator (for α=2π/N)

$$\mathcal{D}_{\frac{1}{N}}(B^{[2]}) = \int [Da] \exp\left[\oint_M \frac{2\pi}{N} \star j_{\text{chiral}} + \frac{iN}{4\pi} a \wedge da + \frac{iN}{2\pi} a \wedge \boxed{B^{[2]}}\right]$$

Non-invertible chiral symmetry

GC, Gherghetta, Terning 25’

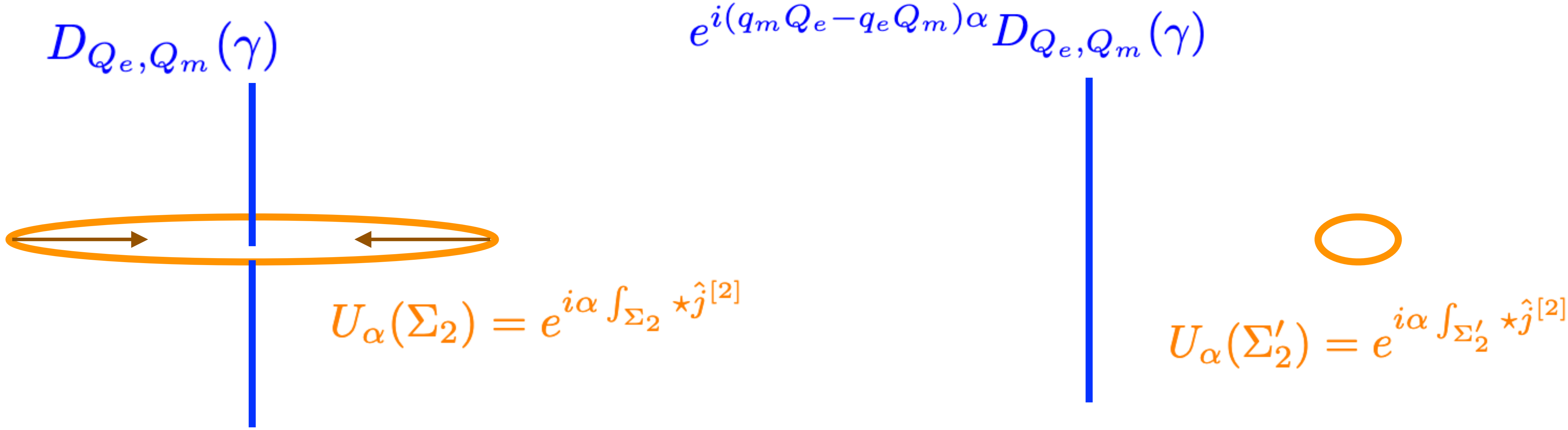
- Indeed, one can find

$$\star\hat{j}^{[2]} = -c\frac{\star F}{e^2} + d\frac{F}{2\pi}$$

- This generates 1-form dyonic symmetry.

- For (composite) line operator $D_{Qe,Qm}(\gamma) = W_{Qe}(\gamma)T_{Qm}(\gamma)$,

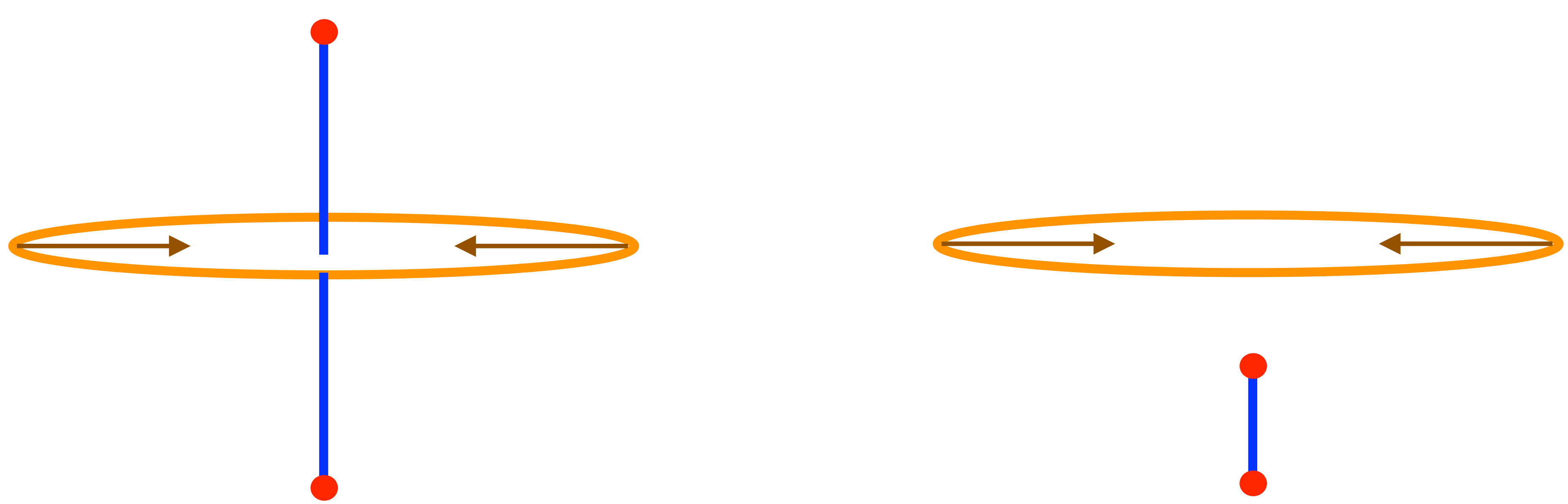
1-form charge $= q_m Q_e - q_e Q_m$



Non-invertible chiral symmetry

GC, Gherghetta, Terning 25'

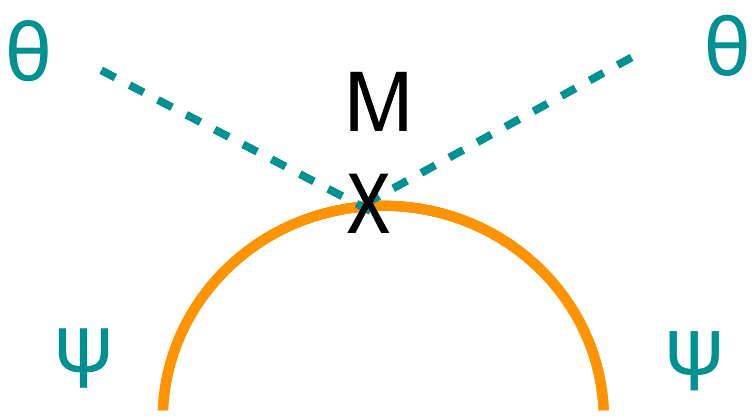
- In SM, ∄ such line operator
  because it could end on dynamical light leptons, quarks in SM
  → action of $U_\alpha(\Sigma_2)$ on $D_{Q_e,Q_m}(\gamma)$ is not well-defined
  →1-form dyonic symmetry is broken
  → non-invertible chiral symmetry is broken
  → Expectation:
  Light leptons, quarks lift up the axion's flat direction



Axion potential generation by SM lepton

Heindenreich, McNamara, Reece 23'

- Leptons obtain fractional "magnetic" charge under shift of θ
- If the following vertex (monodromic mass) is admitted due to dual Witten effect,



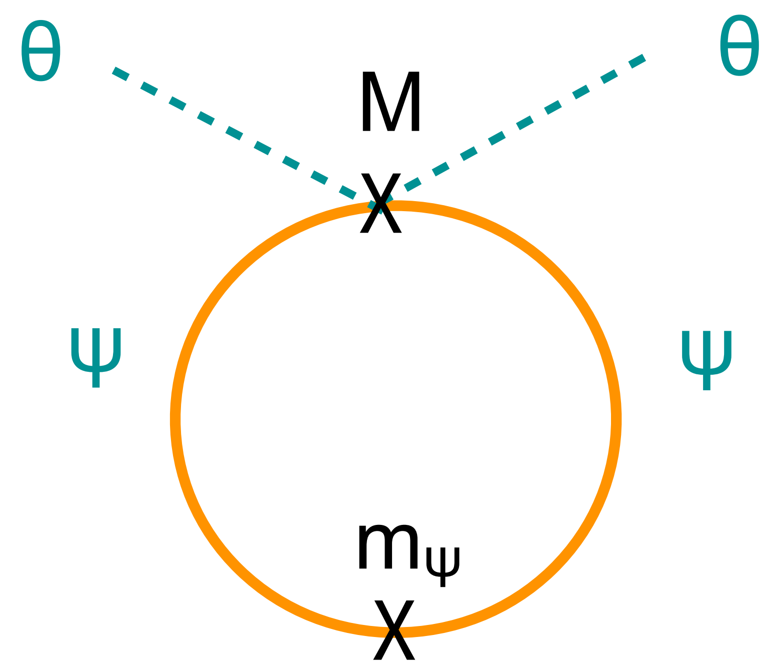
$$\mathcal{L} \supset m_\Psi \bar{\Psi}\Psi + M\left(n - \frac{\theta}{2\pi}\right)^2 \bar{\Psi}\Psi$$

Axion potential generation by SM lepton

Heindenreich, McNamara, Reece 23'

- Leptons obtain fractional "magnetic" charge under shift of θ

- If the following vertex (monodromic mass) is admitted due to dual Witten effect,



$$\mathcal{L} \supset m_\Psi \bar{\Psi}\Psi + M\left(n - \frac{\theta}{2\pi}\right)^2 \bar{\Psi}\Psi$$

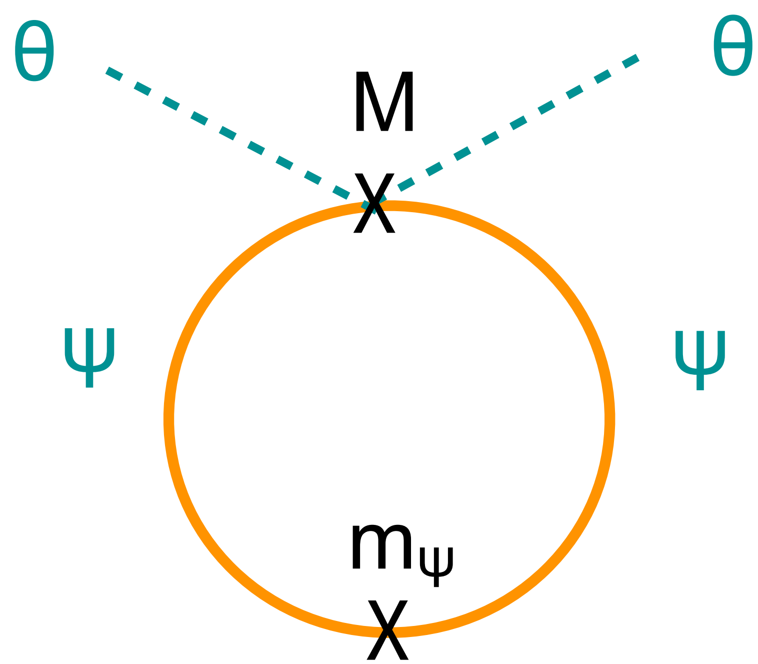
- Axion mass can be generated from dyon (electron) loop

Axion potential generation by SM lepton

Heindenreich, McNamara, Reece 23'

- Leptons obtain fractional "magnetic" charge under shift of θ

- If the following vertex (monodromic mass) is admitted due to dual Witten effect,



$$\mathcal{L} \supset m_\Psi \bar{\Psi}\Psi + M\left(n - \frac{\theta}{2\pi}\right)^2 \bar{\Psi}\Psi$$

- If Λ_{UV}~O(1)TeV, top quark contribution reads

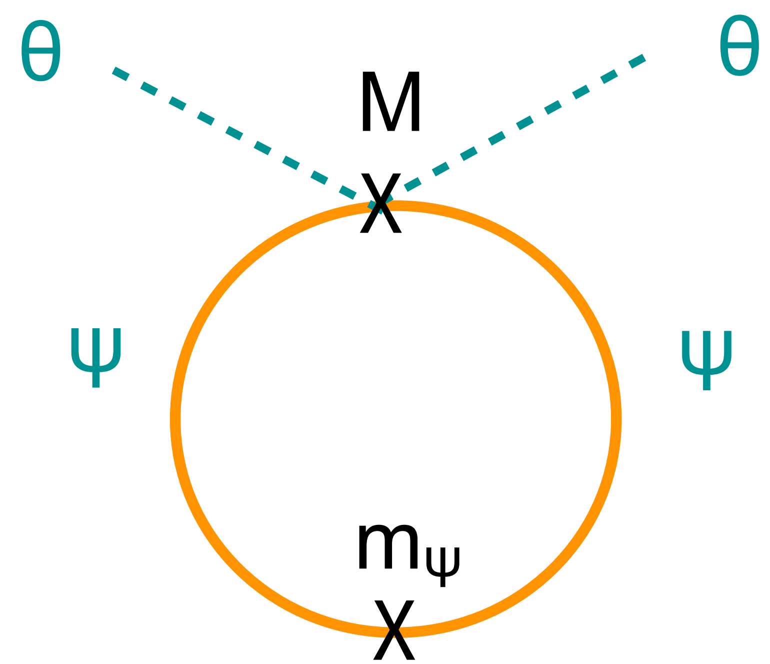
$$\Delta m_\theta^2 \sim \left(\frac{m_\psi}{200\mathrm{GeV}}\right)\left(\frac{M}{100\mathrm{GeV}}\right)\left(\frac{\Lambda^2}{\mathrm{TeV}^2}\right)\left(\frac{f}{10^{12}\mathrm{GeV}}\right)^{-2} \sim (100\mathrm{eV})^2$$

Axion potential generation by SM lepton

Heindenreich, McNamara, Reece 23'

- Leptons obtain fractional "magnetic" charge under shift of θ

- If the following vertex (monodromic mass) is admitted due to dual Witten effect,



$$\mathcal{L} \supset m_\Psi \bar{\Psi}\Psi + M\left(n - \frac{\theta}{2\pi}\right)^2 \bar{\Psi}\Psi$$

- Furthermore, as M is $SU(2)_L$ spurion,
∃ infinite amount of degenerate massless states above EWSB scale → ill-defined theory

Summary

- A variant of KSVZ model with a heavy dyonic quark can be phenomenologically interesting

- $\exists$ still non-invertible chiral symmetry despite the exotic ABJ anomaly

- Dynamical leptons, quarks in SM break dyonic 1-form symmetry
  $\rightarrow$ breaks non-invertible PQ symmetry
  $\rightarrow$ add huge correction to QCD axion mass
  $\exists$ infinitely many massless degenerate states

- This still remains interesting possibility in a dark U(1) gauge theory with dyonic dark fermion