Axion Quality

in Warped Extra-Dimension

Chang Sub Shin (Chungnam National University)

Based on arXiv:2604.08700 · with Kiwoon Choi (IBS-CTPU) and Chang Hyeon Lee (CNU)

July 27th, 2026 @ Axions in Seoul

Outline

1. Strong CP, the QCD axion, and the quality problem
2. Extra-dimensional axion on a circle — automatic quality
3. Why an orbifold, why warping — three new ingredients
4. Selection rule · warped loops · brane operators
5. Results and discussion — one scaling law at the end

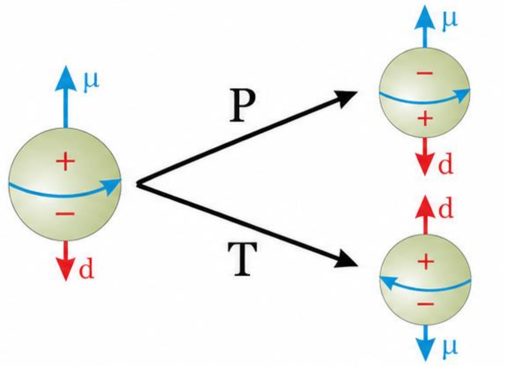
The Strong CP Problem and the QCD Axion

Why is CP violation from QCD unobservably small?

THE PUZZLE

$$\Delta\mathcal{L}_{\rm QCD} = \frac{\bar\theta}{32\pi^2} G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}, \qquad |\bar\theta| \lesssim 10^{-10} \quad (\text{nEDM})$$

C. Abel et al., PRL 124 (2020) 081803

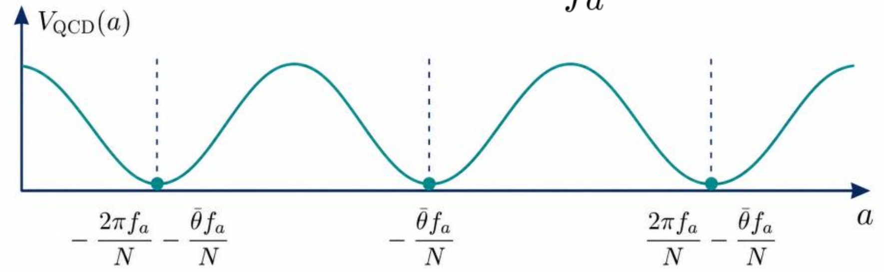


The neutron EDM bounds — but the Standard Model offers no symmetry reason for this tiny number.

PQ SOLUTION

Promote $\bar\theta$ to a dynamical field — the axion $a(x)$.

$$\bar\theta \longrightarrow \bar\theta_{\rm eff} = \bar\theta + \frac{N a(x)}{f_a}$$



$$V_{\rm QCD}(a) \propto -\cos\left(\bar\theta + \frac{Na}{f_a}\right) \quad\Longrightarrow\quad \langle\bar\theta_{\rm eff}\rangle = 0$$

QCD relaxes the axion to the CP-conserving minimum — and the same field is a compelling dark-matter candidate.

R. D. Peccei & H. R. Quinn, PRL 38 (1977) 1440 · S. Weinberg, PRL 40 (1978) 223 · F. Wilczek, PRL 40 (1978) 279
J. E. Kim (1979) · Shifman, Vainshtein & Zakharov (1980) · Dine, Fischler & Srednicki (1981) · Zhitnitsky (1980)

The axion solves strong CP only if no other potential shifts the minimum.

The Axion Quality Problem

A tiny non-QCD potential can destroy the Peccei–Quinn solution

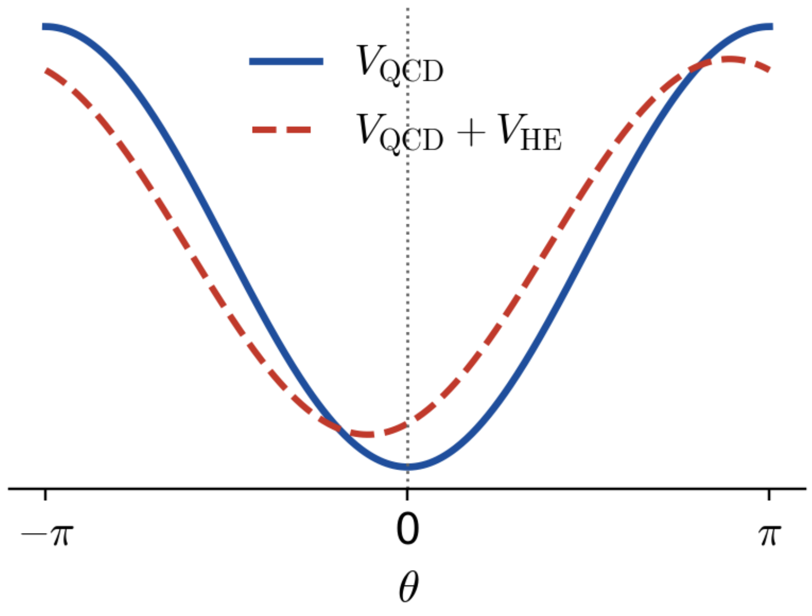
QUALITY BOUND

Any non-QCD (High Energetic) contribution V_{HE} shifts the minimum away from the QCD CP-conserving point $\theta = 0$.

The axion solution survives only if

$$V_{\mathrm{HE}} \lesssim 10^{-10}\, m_\pi^2 f_\pi^2$$

Quantum gravity is expected to violate ordinary global symmetries — so V_{HE} is generically dangerous.



R. Holman et al., PLB 282 (1992) 132 · M. Kamionkowski & J. March-Russell, PLB 282 (1992) 137 · D. Harlow & H. Ooguri, CMP 383 (2021) 1669

We need a protection mechanism that is structural — not a numerical accident.

Field-Theoretic Axions: Generic Threat, UV-Sensitive Breaking

Axion quality in axion solution to the strong CP problem

- **PQ is an ordinary global symmetry:** quantum gravity is not expected to preserve it exactly
- **But the low-energy theory does not determine how it is broken:**

  the leading process, its suppression, and its CP phase are UV sensitive
- **Concrete diagnostics include**
  - monopoles and dyon loops, often tied to non-QCD non-Abelian instanton sectors

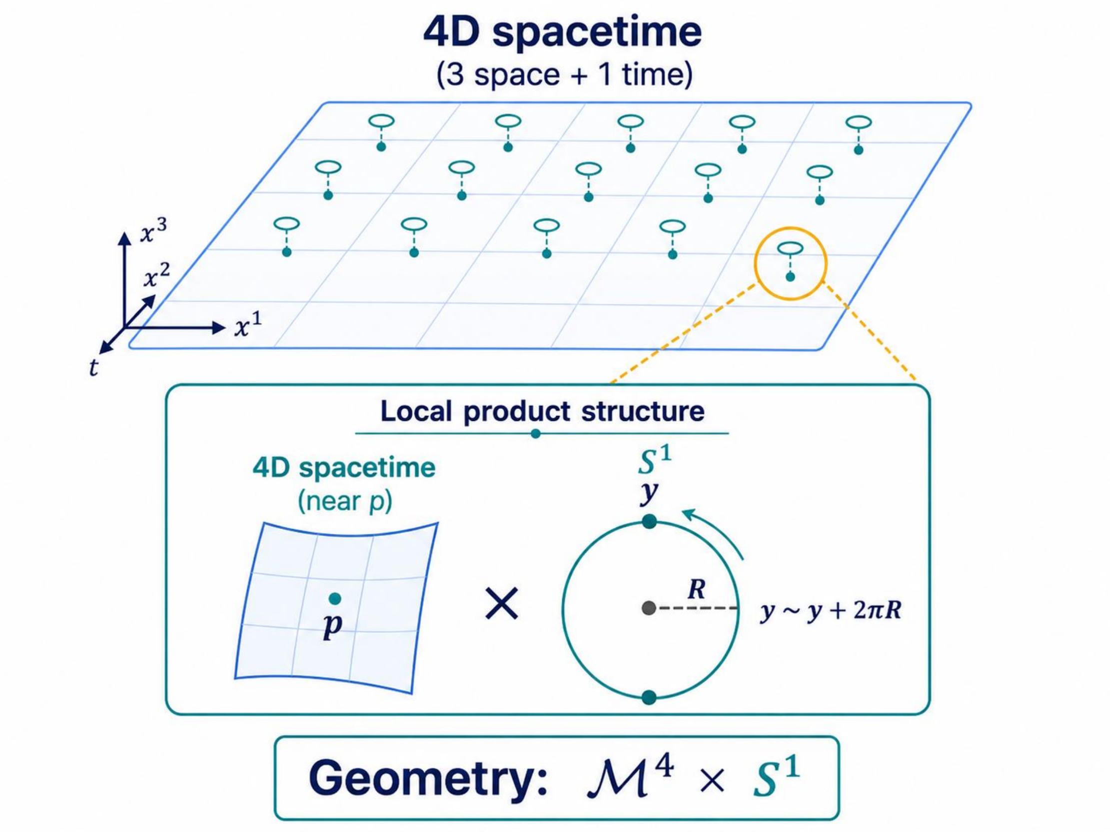
    J. Fan et al., PRL 127 (2021) 131602 · G. Ghoi et al., PRD 112 (2025) 095023 · I. Garcia Garcia et al., JHEP 05 (2026) 244
  - Euclidean wormholes, whose action depends on the axion–gravity completion.

    S. Giddings & A. Strominger, NPB 306 (1988) 890 · K.-M. Lee, PRL 61 (1988) 263 · D. Y. Cheong, S. C. Park & C. S. Shin, JHEP 07 (2024) 039, …
- **Controlling these effects requires additional UV structure, usually assessed case by case**

Higher-dimensional gauge invariance organizes PQ breaking to identifiable, calculable nonlocal processes.

Extra-Dimensional Axion on a Circle

A 5D U(1) gauge field C_M — the axion is its Wilson line



5D GAUGE FIELD

$$S_C = -\int \mathrm{d}^5x\sqrt{|g|}\,\frac{1}{4g_C^2}C_{MN}C^{MN}$$

PQ descends from 5D gauge invariance — not from an ordinary global symmetry.

WILSON-LINE AXION

$$\frac{a(x)}{f_a} \equiv \oint_{S^1} \mathrm{d}y\, C_5(x,y)$$

Periodicity $\theta \equiv \theta + 2\pi$ is guaranteed by large gauge transformations — the axion is an angle.

K. Choi, PRL 92 (2004) 101602 · M. Reece, JHEP 07 (2025) 130 · N. Craig & M. Kongsore, PRD 111 (2025) 015047

Gauge invariance in 5D — a shift symmetry with a topological pedigree in 4D.

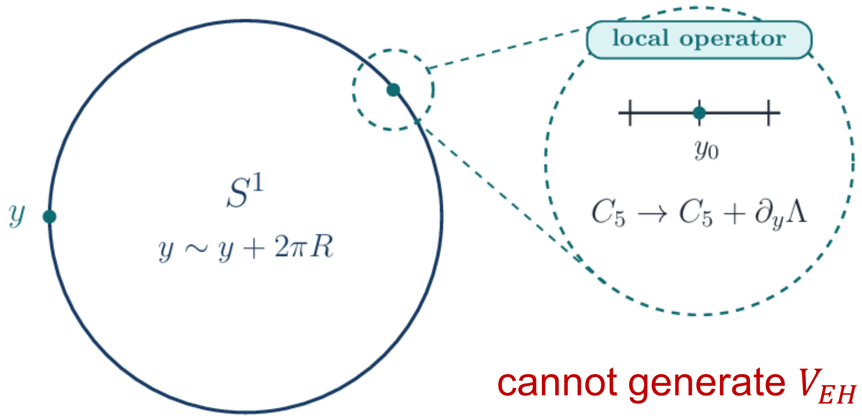
PQ: a Gauge-Protected Shift Symmetry

A global 1-form symmetry generated by the harmonic form dy

$$U(1)_{\rm PQ}: \quad C \longrightarrow C + \frac{\alpha}{2\pi R}\mathrm{d}y, \qquad \theta(x) \longrightarrow \theta(x) + \alpha$$

LOCALLY

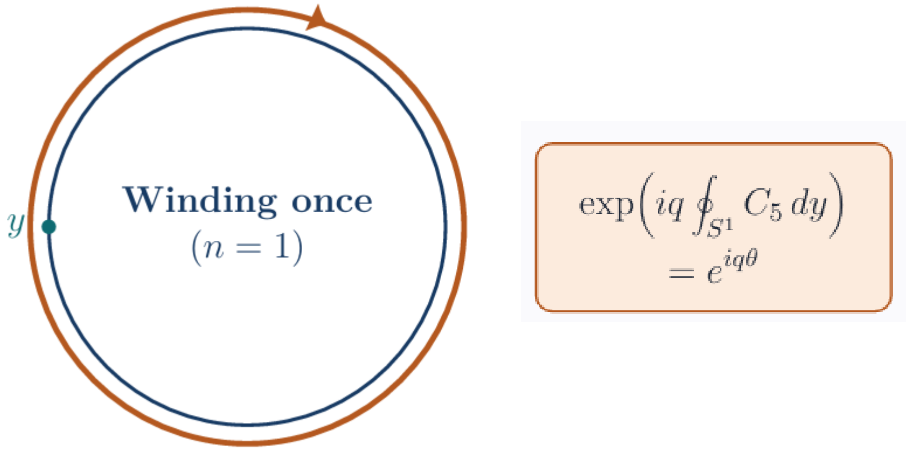
The shift is a gauge transformation with $\Lambda = \alpha y/(2\pi R)$



cannot generate $V_{EH}(\theta)$

GLOBALLY

Single-valued on S^1 only for $\alpha = 2\pi n$, so a continuous global shift remains in the four-dimensional theory.



K. Choi, PRL 92 (2004) 101602 · M. Reece, JHEP 07 (2025) 130 · N. Craig & M. Kongsore, PRD 111 (2025) 015047

Any local operator is blind to θ. Breaking θ-dependence requires physics that winds around the circle.

Charged Matter and the Winding Phase

Weak gravity requires U(1)-charged states — they cannot be decoupled

WEAK GRAVITY

$$M \lesssim \mathcal{O}\left(g_C M_5^{3/2}\right)$$

Charged messengers exist below the 5D gravitational scale — candidates for PQ breaking.

$$\mathcal{L} = |D_M \Phi|^2 - M^2 |\Phi|^2$$

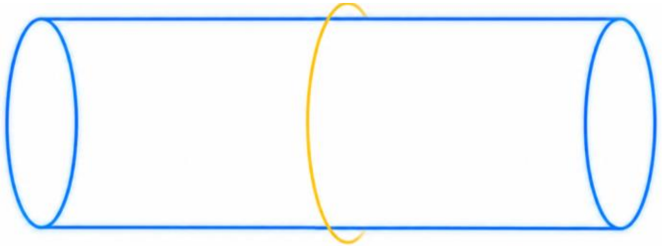
$$\Phi(x, y) = \Phi(x, y + 2\pi R)$$

$$U(1)_{\mathrm{PQ}} : \Phi \to e^{i \frac{q\alpha}{2\pi R} y} \Phi$$

WINDING PHASE

$$nq \oint_{S^1} \mathrm{d}y \, C_5 = nq\theta \implies \text{phase } e^{inq\theta} \quad (n : \text{ winding number})$$

A worldline of winding number n picks up the Aharonov–Bohm phase $e^{inq\theta}$.



y direction = Euclidean time

$(y \equiv y + 2\pi R)$

N. Arkani-Hamed et al., JHEP 06 (2007) 060 · N. Arkani-Hamed et al., PRL 90 (2003) 221302

The Casimir energy of winding worldlines depends on θ — the only channel of PQ breaking on the circle.

Result on the Circle

The winding sum can be exactly evaluated

LEADING HARMONIC

$$V^{\mathrm{flat}}(\theta) \simeq -\frac{M^2}{16\pi^4 R^2}\, e^{-2M\pi R} \cos(q\theta)$$

$$S_{\mathrm{inst}} = (\text{worldline length}) \times M = 2\pi R M$$

Scalars and fermions differ only by a spin factor.

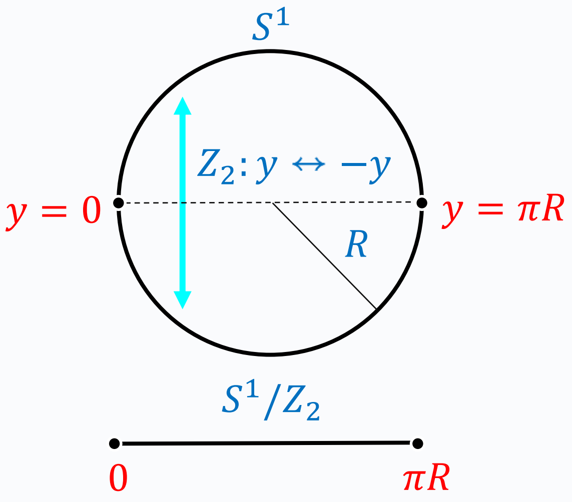
Y. Hosotani, PLB 126 (1983) 309 · N. Arkani-Hamed et al., PRL 90 (2003) 221302

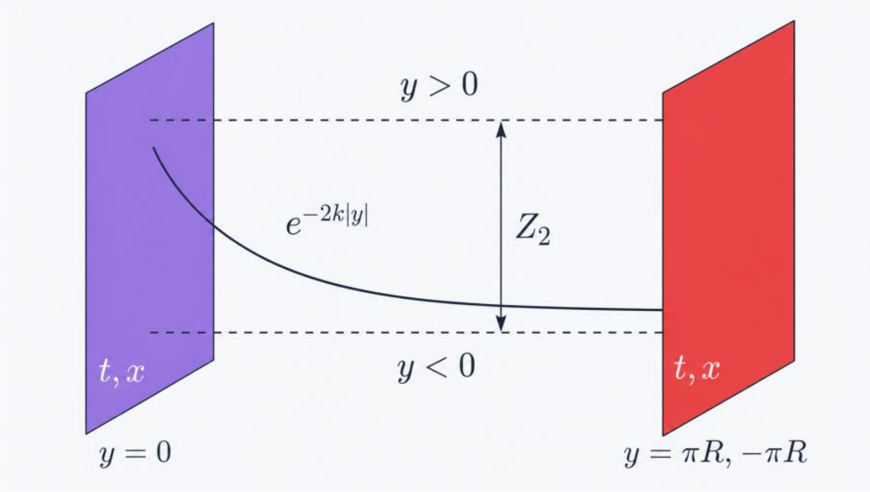
Once $M\pi R \gg 1$, the quality problem seems well addressed. So why go further?

Realistic chirality and an intermediate f_a require an orbifold and warping.

Why an Orbifold, Why Warping

Two requirements push beyond the flat circle — at a price





01 Chiral fermions

A circle gives no chirality; realistic matter asks for S^1/Z_2.

02 Intermediate f_a

Warping red-shifts the decay constant without tuned parameters.

03 The price: fixed points

Branes appear — and localized operators are allowed there.

L. Randall & R. Sundrum, PRL 83 (1999) 3370 · K. Choi, PRL 92 (2004) 101602 · T. Flacke et al., JHEP 01 (2007) 061 ,…

This work: how do the orbifold, the warping, and the branes affect the axion quality?

Three New Ingredients

Everything that follows organizes around what S^1/Z_2 + warping introduce

1 **Parity assignments**

boundary conditions select who can break PQ

SELECTION RULE

2 **Warped geometry**

the loop computation becomes subtle — three independent methods

WARPED LOOPS

3 **Fixed points**

brane-localized operators open new channels

BRANE OPERATORS

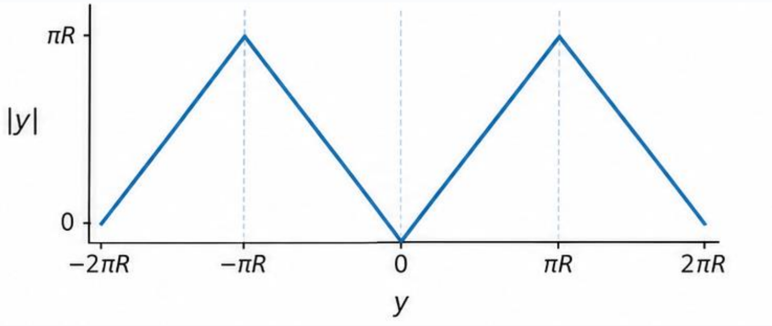
One simple answer at the end:

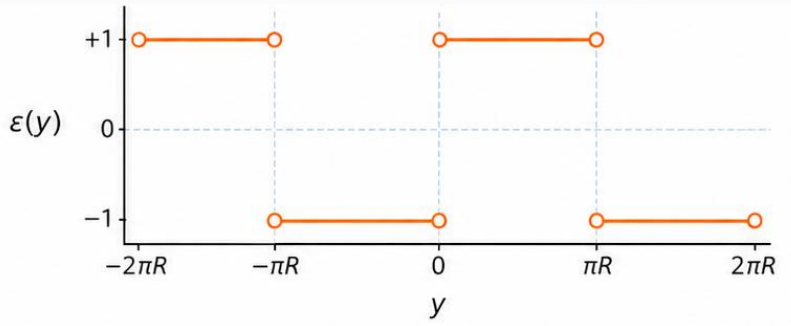
$$V_{\mathrm{HE}}(\theta) \sim \left(e^{-k\pi R}\right)^{n\Delta} \cos\left(\frac{nq}{2}\theta\right), \qquad n = \text{number of traversals between } y=0,\, y=\pi R$$

Parameters at a Glance

Geometry, Matter contents, Axion

$\boldsymbol{k, R}$	AdS curvature, orbifold radius; strong warping $k\pi R \gg 1$
$\boldsymbol{M_5, M_P}$	5D and 4D (reduced) Planck scales
$\boldsymbol{\lvert y \rvert}$	Z_2-even kink function
$\boldsymbol{\varepsilon(y)}$	Z_2-odd sign function





$\boldsymbol{M, J, b}$	bulk masses, brane couplings at the fixed points for the charged fields
$\boldsymbol{M_{eff}, \Delta}$	Scales for instanton interpretation, $\Delta = 2 + M_{eff}/k$: the dual scaling dimension
$\boldsymbol{g_C, q \in Z}$	5D U(1) coupling and integer charge
$\boldsymbol{\theta = a / f_a}$	Wilson-line axion; $f_a \sim e^{-k\pi R} M_P$

Setup: a Randall–Sundrum Slice

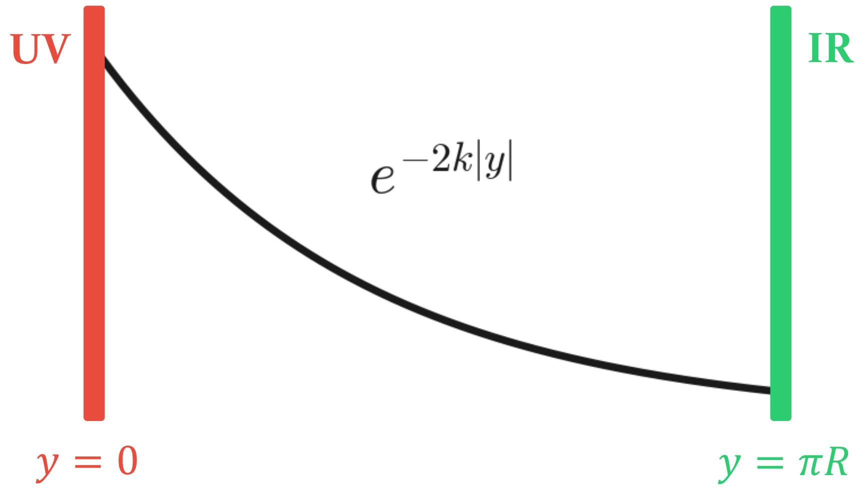
Two orbifold fixed points — the UV and IR branes

BACKGROUND GEOMETRY

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}y^2$$

$$y \in [0, \pi R]$$

Strong warping: $k\pi R \gg 1$. Bulk fields live on the covering circle S^1, subject to Z_2 boundary conditions.



L. Randall & R. Sundrum, PRL 83 (1999) 3370

$$C_\mu(x,-y) = -C_\mu(x,y), \qquad C_5(x,-y) = +C_5(x,y)$$

→ **the axion zero mode in C_5 survives.**

The geometry supplies the hierarchy; the orbifold supplies the boundary data.

The Axion Kinetic Term Is IR-Weighted

Solving for the PQ current fixes the profile and the decay constant

PROFILE

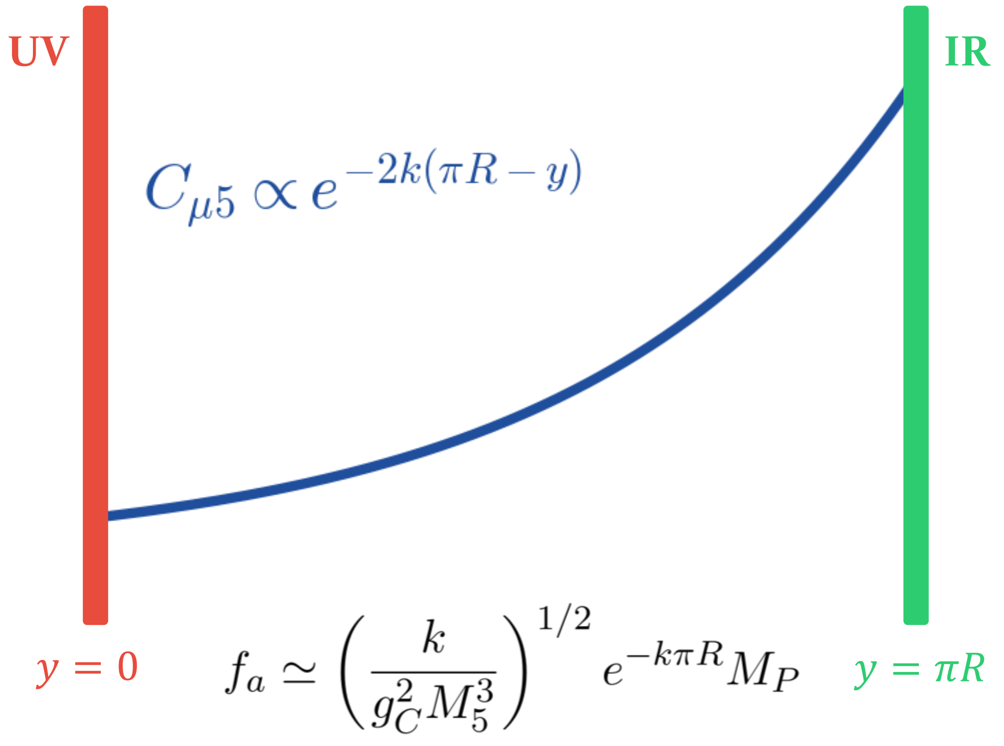
$$C_{\mu 5}(x,y) = \frac{k\, e^{-2k(\pi R - y)}}{1 - e^{-2k\pi R}}\, \partial_\mu \theta(x)$$

Kinetic term is dynamically localized near IR brane

DECAY CONSTANT

$$f_a^2 = \frac{1}{g_C^2} \frac{k}{e^{2k\pi R} - 1}$$

Geometric redshift by $e^{-k\pi R}$



K. Choi, PRL 92 (2004) 101602

Intermediate f_a is easily achieved with a moderate size of Ex-dim

Coupling to QCD: a Chern–Simons Term

The SM can live in the bulk. The required PQ breaking by the QCD anomaly, with integer coefficient N

5D CHERN–SIMONS

$$S_{\rm CS} = \frac{N}{16\pi^2}\int C \wedge G^a \wedge G^a$$

$$\Longrightarrow \quad \mathcal{L}_{\rm eff} \supset \frac{N}{32\pi^2}\frac{a}{f_a} G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}$$

Quantized and gauge-invariant — it generates V_{QCD} without spoiling the protection.

THE QUESTION

What else generates a potential?

V_{HE} — the unwanted, non-QCD part.

From here on, “quality” means controlling the non-QCD part of the axion potential.

K. Choi, PRL 92 (2004) 101602

Two Classes of Boundary Conditions

SELECTION RULE

The orbifold identification fixes the Z_2 parity for 5D charged matter fields K. Choi, C. H. Lee, C. S. Shin, 2026.08700

P-TYPE

$$\widetilde{\phi}(x,-y) = \pm\widetilde{\phi}(x,y),$$

$$\widetilde{\psi}(x,-y) = \pm\gamma^5\widetilde{\psi}(x,y)$$

$$D_M\widetilde{\Phi} = \big(\nabla_M - iq\,\epsilon(y)\,C_M\big)\widetilde{\Phi}$$

coupling $q\varepsilon(y)$: Z_2-odd

C-TWISTED

$$\phi(x,-y) = \pm\phi^*(x,y),$$

$$\psi(x,-y) = \pm\psi^c(x,y)$$

$$D_M\Phi = \big(\nabla_M - iq\,C_M\big)\Phi$$

coupling q: constant

The Z_2-odd kink charges are forced by orbifold symmetry — the first hint that P-type and C-twisted fields behave differently.

$$S_{\text{bulk}} \supset -\int d^5x\sqrt{|g|}\left[D_M\phi^* D^M\phi - M_\phi^2|\phi|^2 + (\phi\to\widetilde{\phi})\right]$$

$$+\int d^5x\sqrt{|g|}\left[\frac{i}{2}\left(\bar{\psi}\Gamma^M D_M\psi - \text{h.c.}\right) - M_\psi\bar{\psi}\psi + (\psi\to\widetilde{\psi})\right]$$

$$M_\phi = M_{\tilde{\phi}} = M_\psi = M \quad (\text{constant}), \quad M_{\tilde{\psi}} = M\epsilon(y) \quad (\mathbb{Z}_2\text{-odd kink})$$

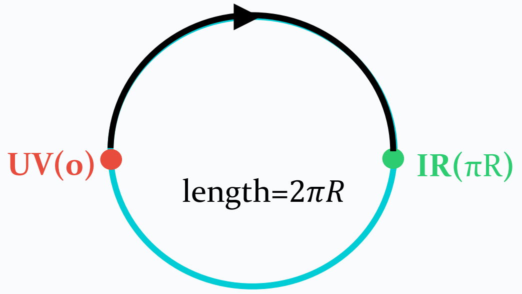
The Phase of One Traversal

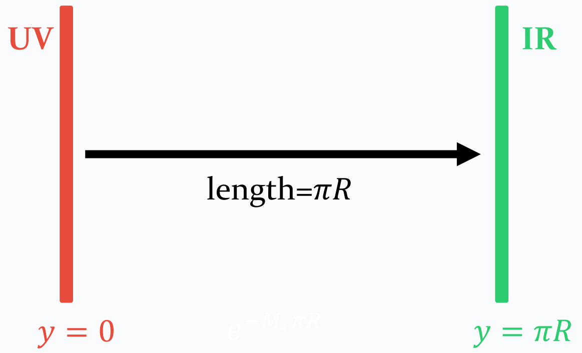
SELECTION RULE

A charged particle crossing the interval feels the axion background

For a single UV-to-IR traversal,

$$\exp\left(iq\int_0^{\pi R} \mathrm{d}y\, C_5\right) = e^{iq\theta/2}, \qquad C_5 = \frac{\theta}{2\pi R}$$



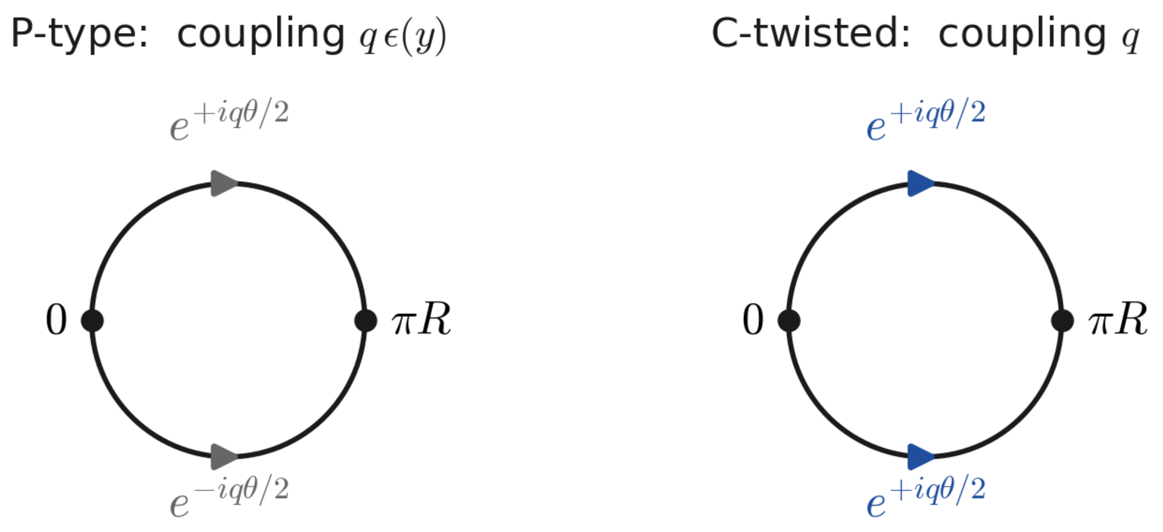


A full winding around the covering circle consists of two traversals.
— whether their phases add or cancel is fixed by the Z_2 parity of the coupling.

An Aharonov–Bohm Argument

SELECTION RULE

Two traversals — add or cancel



$$e^{iq\theta/2} \cdot e^{-iq\theta/2} = 1$$

P-type: no potential

$$e^{iq\theta/2} \cdot e^{iq\theta/2} = e^{iq\theta}$$

C-twisted: $V \propto \cos(q\theta)$

This is a bulk selection rule. Brane-localized operators can reopen P-type channels, which is why the fixed points must be studied separately.

What Warping Changes

AdS curvature entangles momentum with position

Momentum no longer separates from the position in the 5th dim:

$$S_E = \int_0^1 \mathrm{d}\tau \left[\frac{\dot{y}^2}{4T} + T\left(M_{\text{eff}}^2 + e^{2k|y|} p^2 \right) \right]$$

Spin connection → kink fermion mass:

$$M_\psi(y) = M + \frac{k}{2}\,\epsilon(y)$$

KK spectra become Bessel functions:

T. Gherghetta & A. Pomarol, NPB 586 (2000) 141

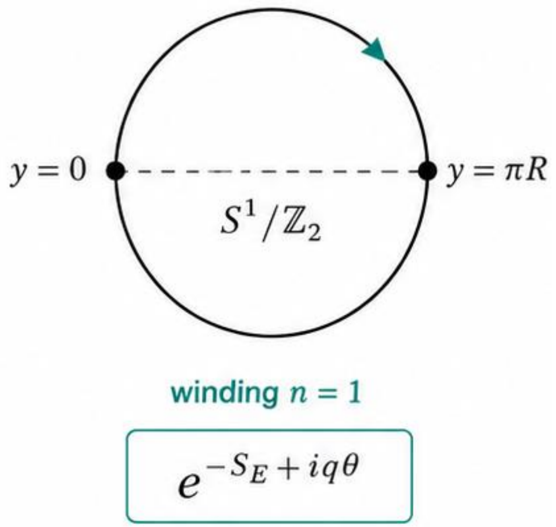
$$\sin^2\!\left(\pi R\sqrt{z^2 - M^2}\right) \longrightarrow J_\alpha,\ Y_\alpha\,, \qquad \alpha = \sqrt{4 + M^2/k^2}$$

We use three independent approaches, and cross-check them.

Three Independent Methods

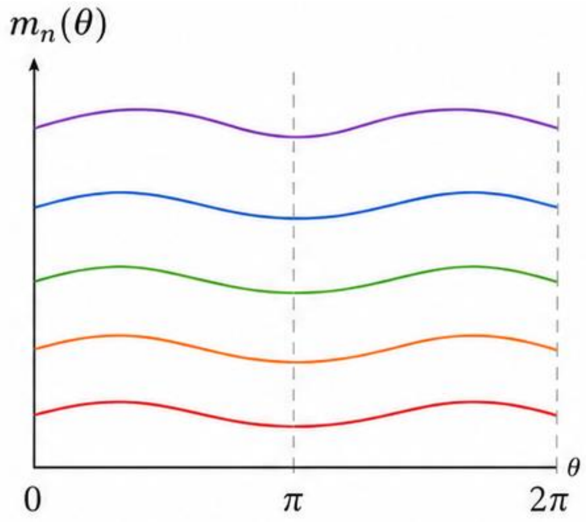
Each method closes a different logical gap in the warped calculation

WORLDLINE



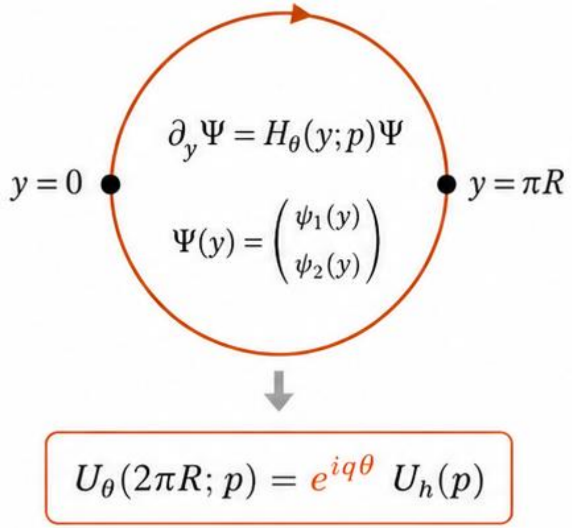
Winding instanton —makes nonlocality and exponential suppression manifest; semiclassical, valid for $M \gtrsim k/2$

KK SPECTRAL FUNCTION



Exact θ dependent KK spectrum — valid for all M, k; systematically incorporates brane operators

MONODROMY MATRIX



First-order fermion transport — isolates θ in one phase in a 2×2 matrix; extends the result to $M < k/2$

Each confirms the leading exponential; prefactors agree at leading order in k/M.

The Worldline Representation

Winding sectors and their fluctuating trajectories on the covering circle

$$V_\phi(\theta) = -\pi R \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \int_0^\infty \frac{\mathrm{d}T}{T} \sum_{n\in\mathbb{Z}} e^{inq\theta} \int_{y(1)=y(0)+2\pi R n} \mathcal{D}y\, e^{-S_E}$$

$$S_E = \int_0^1 \mathrm{d}\tau \left[\frac{\dot{y}^2}{4T} + T\left(M_{\text{eff}}^2 + e^{2k|y|} p^2 \right) \right]$$

$M_{eff}^2 = M^2 + 4k^2$ (scalar)

$M_{eff}(y) = M + \frac{k}{2}\epsilon(y)$ (fermion)

$$y(\tau) = 2\pi R n\, \tau + \eta(\tau), \qquad \eta(0) = \eta(1) = 0$$

Classical winding + all fluctuations within the same topological sector

J. Schwinger, PR 82 (1951) 664 · N. Arkani-Hamed et al., PRL 90 (2003) 221302

Each sector contains an ensemble of paths;
the Aharonov–Bohm phase $e^{inq\theta}$ tags its winding number.

The worldline integral makes the exponential suppression and the nonlocal origin visible before the integral is evaluated

The Warped Path Average

From an ensemble of winding paths to the classical trajectory

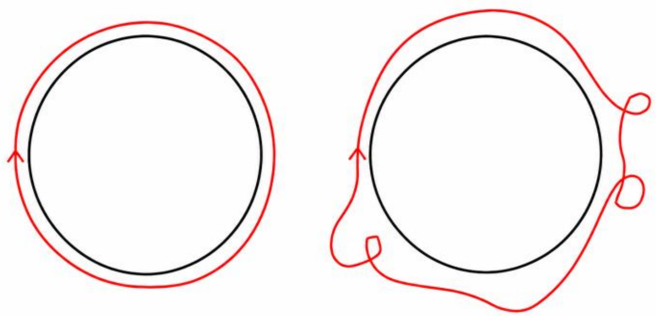
MASTER FORMULA

$$V_\phi^{\text{warped}}(\theta) = -2\pi R\, e^{-4k\pi R} \cos(q\theta) \int_0^\infty \frac{\mathrm{d}T}{T(4\pi T)^2} \int_{n=1} \mathcal{D}y\, \frac{1}{I[y]^2}\, e^{-S_0[y;T]}$$

$$S_0[y;T] = \int_0^1 d\tau \left(\frac{\dot{y}^{\,2}}{4T} + T M_{\text{eff}}^2 \right), \quad I[y] = \int_0^1 d\tau\, e^{-2k(\pi R - |y(\tau)|)}$$

PATH ENSEMBLE

$$y_{\text{cl}}(\tau) = 2\pi R \tau, \qquad I_0 \equiv I[y_{\text{cl}}] = \frac{1 - e^{-2k\pi R}}{2k\pi R}$$



$$\left\langle \frac{1}{I[y]^2} \right\rangle_0 \equiv \frac{\int_{n=1} \mathcal{D}y\, I[y]^{-2} e^{-S_0[y;T]}}{\int_{n=1} \mathcal{D}y\, e^{-S_0[y;T]}} = \frac{1}{I_0^2} \left(1 + \mathcal{O}\left(\frac{k}{M_{\text{eff}}} \right) \right)$$

Corrections are systematic in k/M_{eff} and vanish in the flat limit.

The KK Spectral Function

Casimir energy of the θ-dependent Kaluza–Klein spectrum

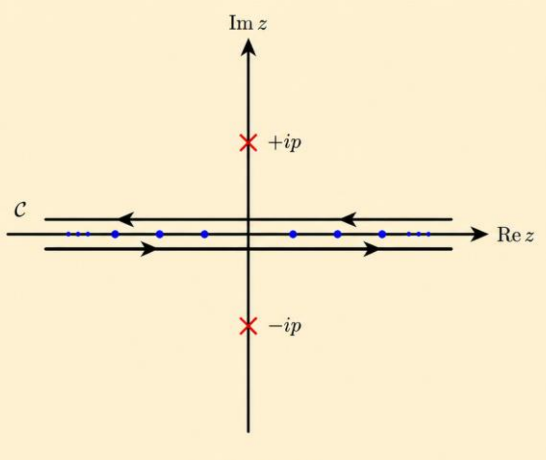
The one-loop vacuum energy as a sum over the θ dependent KK spectrum

$$V_\Phi(\theta) = \frac{(-1)^{n_F}}{2} \sum_n \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln\left[p^2 + m_n^2(\theta)\right]$$

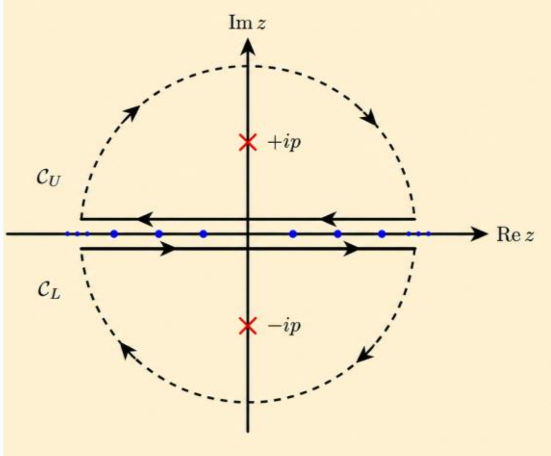
The KK masses as zeros of a holomorphic spectral function,

$$N_\Phi(m_n(\theta);\theta) = 0, \quad N_\Phi(z;\theta) = (N_\Phi(z) + A_\Phi(\theta))^{g_\Phi}$$

A. Falkowski, PRD 75 (2007) 025017 · K. Choi, I.-W. Kim, C. S. Shin, New J. Phys. 12 (2010) 075014



(a) Contour $\mathcal{C}$.



(b) Deformed contours $\mathcal{C}_U$ and $\mathcal{C}_L$.

MASTER FORMULA

KK sum into a contour integral with a deformed contour

$$V_\Phi(\theta) = (-1)^{n_F} \frac{g_\Phi}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln\left[1 + \frac{A_\Phi(\theta)}{N_\Phi(ip)}\right], \qquad A_\Phi(\theta) = 2\sin^2\frac{q\theta}{2}$$

The Monodromy Matrix

First-order fermion transport around one covering circle

Functional determinant of the Dirac operator $(\partial_y - H_\theta(y;p))$ reduces to ordinary 2 by 2 matrix determinant

$$H_\theta(y;p) = \left(2k\epsilon(y) + \frac{iq\theta}{2\pi R}\right)\mathbf{1}_2 + h(y;p), \quad h(y;p) = \begin{pmatrix} -M & e^{k|y|}p \\ e^{k|y|}p & M \end{pmatrix}$$

$$V_\psi(\theta) \propto \ln\det(\partial_y - H_\theta(y;p)) \propto \ln\det(\mathbf{1}_2 - U_\theta(2\pi R;p))$$

$$U_\theta(2\pi R;p) \equiv \mathcal{P}\exp\left(\int_0^{2\pi R} dy' H_\theta(y';p)\right) = e^{iq\theta}\mathcal{P}\exp\left(\int_0^{2\pi R} dy' h(y';p)\right) \equiv e^{iq\theta}U_h(p)$$

Wilson-line dependence is isolated in one phase. Since $\det U_h = 1$, write its eigenvalues as $e^{\ell(p)}, e^{-\ell(p)}$. Then

$$V_\psi(\theta) = \sum_{n=1}^{\infty} \frac{2\cos(nq\theta)}{n} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} e^{-n\ell(p)}$$

$\ell(p) = 2M\pi R + O(p^2)$

Result: the C-twisted Scalar

Large mass and strong warping, $M\pi R \gg 1,\ k\pi R \gg 1$

$$V_\phi(\theta) \simeq -\frac{(M^2+4k^2)\,k^2}{4\pi^2}\, e^{-4k\pi R}\, e^{-2\pi R\sqrt{M^2+4k^2}}\cos(q\theta)$$

$e^{-4k\pi R}$: IR redshift of the axion

Supplied by the AdS curvature. The induced V_{HE} is localized near the IR brane, like the axion kinetic term itself.

$e^{-2M_{\rm eff}\pi R}$: winding instanton, $M_{\rm eff} = \sqrt{M^2+4k^2}$

Penalty for a closed worldline winding the full covering circle (length $2\pi R$) with the curvature enhanced M_{eff}

$$S_0 = \int_0^1 \mathrm{d}\tau \left(\frac{\dot{y}^2}{4T} + T M_{\rm eff}^2(y)\right) \geq \oint dy\ M_{\rm eff}(y)$$

Result: the C-twisted Fermion

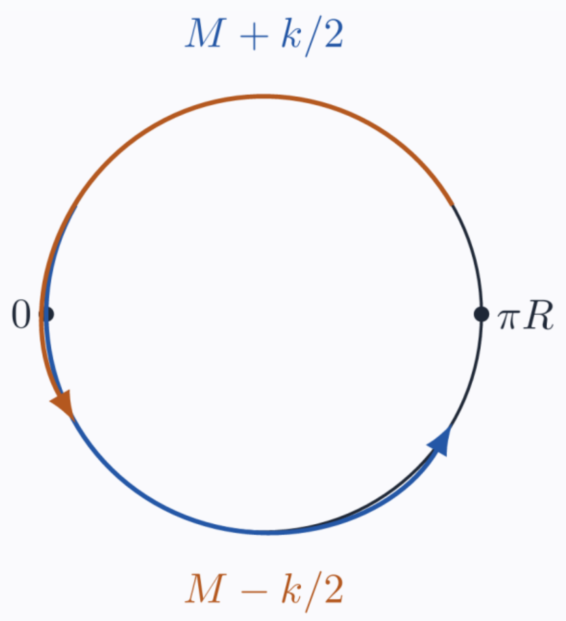
A piecewise winding action on the two half-circles

The squared Dirac operator contains the kink-like effective mass

$$M_\psi(y) = M + \frac{k}{2}\epsilon(y)$$

The semiclassical winding action for $M > k/2$ is therefore

$$\oint \mathrm{d}y\, M_\psi(y) = \left(M + \frac{k}{2}\right)\pi R + \left(M - \frac{k}{2}\right)\pi R = 2M\pi R$$



$$V_\psi(\theta) \propto e^{-4k\pi R}\, e^{-2M\pi R} \times \cos(q\theta)$$

The worldline saddle explains the exponent for $M > k/2$; the exact methods will show that $e^{-2M\pi R}$ persists beyond this regime.

The KK spectral function and monodromy matrix independently reproduce $e^{-2M\pi R}$ including $M < k/2$, where the explicit piecewise worldline saddle is unavailable.

Hypermultiplet and the SUSY Check

Piecewise-constant masses — the action is a sum of two half-circles

HYPERMULTIPLET

For the C-twisted supersymmetric hypermultiplet $\{H_1, H_2, \psi\}$, the scalar effective masses are piecewise

$$M_{\text{eff}}^{(1)}(y) = \left|M + \frac{k}{2}\epsilon(y)\right|, \qquad M_{\text{eff}}^{(2)}(y) = \left|M - \frac{k}{2}\epsilon(y)\right|$$

SUSY LIMIT

When $M > k/2$, each scalar has the same instanton action as the Dirac fermion

$$S_{H_1} = S_{H_2} = \pi R\left(M + \frac{k}{2} + M - \frac{k}{2}\right) = 2\pi RM = S_\psi$$

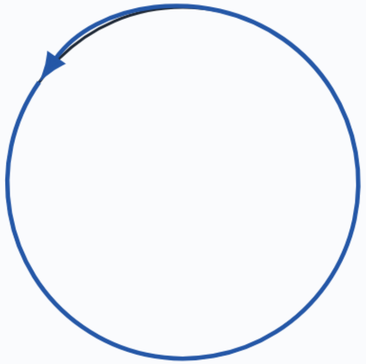
$$V_{H_1} = V_{H_2} = -\frac{1}{2}V_\psi, \qquad V_{H_1} + V_{H_2} + V_\psi = 0$$

The supersymmetric cancellation tests more than the exponent: it cross-checks the piecewise trajectories,spin multiplicities, and leading prefactors

Loop versus Tree

The number of traversals fixes both exponential suppressions

LOOP (closed worldline)



two traversals
length $2\pi R$

TREE (brane-to-brane)

one traversal
length πR

THE POINT

The fixed-point-to-fixed-point profile pays only one interval crossing. Both the mass exponent and the geometric red-shift are therefore halved.

$$V_{\text{loop}} \sim e^{-4k\pi R}\, e^{-2M_{\text{eff}}\pi R}, \quad V_{\text{tree}} \sim e^{-2k\pi R}\, e^{-M_{\text{eff}}\pi R}$$

Tree exponent = ½ × loop exponent → potentially the most dangerous channel.

The Allowed Fixed-Point Terms

BRANE OPERATORS

U(1) is broken to its Z_2 subgroup at the fixed points

RESIDUAL GAUGE TRANSFORMATION

$$\mathbb{Z}_C : \qquad \Phi \longrightarrow (-1)^{q_\Phi}\Phi \quad \text{at} \quad y = 0,\ \pi R$$

ALLOWED OPERATORS

The scalar operators explicitly analyzed in the paper are

$$\Delta\mathcal{L}_i^{\text{scalar}} = J_i\phi + \widetilde{J}_i\widetilde{\phi} + \frac{1}{2}b_i\phi^2 + \frac{1}{2}\widetilde{b}_i\widetilde{\phi}^2 + \text{h.c.}, \qquad i = 0, \pi.$$

$$J_i,\ \widetilde{J}_i \neq 0 \ \text{ only for } \ q \in 2\mathbb{Z}$$

Fermion bilinears and additional scalar or fermion mixing terms can also be allowed by the residual symmetry

$$U(1)_{\text{PQ}}\ (\theta \to \theta + \alpha) : C_5 \to C_5 + \frac{\alpha}{2\pi R}, \quad \Phi \to e^{i\frac{q\alpha}{2\pi R}(y-y_0)}\Phi \quad y_0 \text{ arbitrary}$$

$\Phi(x, y_0) \to \Phi(x, y_0)$. No single brane can break PQ physically: only the relative phase between operators at $y = 0$ and $y = \pi R$, communicated by bulk propagation, is gauge invariant and can generate $V(\theta)$.

What Each Term Does

Three operators, three different fates

- **Linear terms** $J_i\phi,\ \tilde{J}_i\tilde{\phi}\ \ (q \in 2Z)$ — open the tree channel — a classical brane-to-brane profile
- **P-type masses** $\tilde{b}_i\tilde{\phi}^2$ — reopen the loop the selection rule had closed: $V \propto \tilde{b}_0\tilde{b}_\pi^*$
- **C-twisted masses** $b_i\phi^2$ — modify only the prefactor of the existing loop

The genuinely new PQ-breaking channels require both fixed points: for linear and P-type quadratic terms, θ enters only through a two-brane product, and hence through brane-to-brane propagation.

A Classical Brane-to-Brane Bridge

BRANE OPERATORS

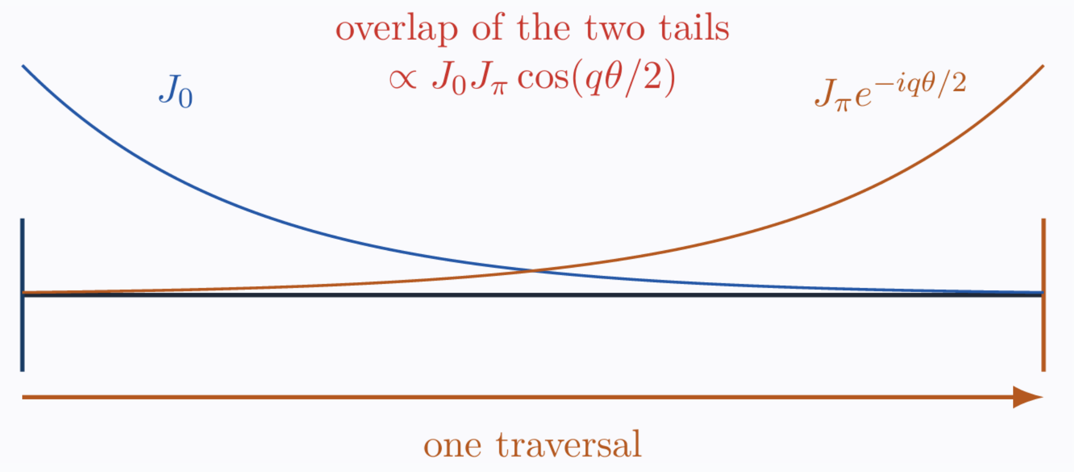
Brane linear terms source a θ-dependent vacuum profile

PROFILE ANSATZ

$$\langle\phi(x,y)\rangle = e^{2k|y|}e^{iq\theta y/(2\pi R)}v(y)$$

$$(\partial_y^2 - M_{\text{eff}}^2)v(y) = \delta(y)\big[J_0 - 4kv(0)\big] + \delta(y-\pi R)\big[e^{-2k\pi R}e^{-iq\theta/2}J_\pi + 4kv(\pi R)\big]$$

PROFILE



The potential is generated by the overlap of two brane-sourced tails; the IR source carries precisely the phase of one traversal.

Tree Result: a Single Traversal

Even-charge scalars sourced on both branes: Both C-twisted and P-type linear sources share the same exponent

$$V_{\phi}^{\rm tree}(\theta) \simeq \frac{2M_{\rm eff}}{M^2} J_0 J_\pi \, e^{-2k\pi R} e^{-M_{\rm eff}\pi R} \cos\left(\frac{q\theta}{2}\right),$$
$$V_{\widetilde{\phi}}^{\rm tree}(\theta) \simeq -\frac{2M_{\rm eff}}{M^2} |\widetilde{J}_0 \widetilde{J}_\pi| \, e^{-2k\pi R} e^{-M_{\rm eff}\pi R} \cos\left(\frac{q\theta}{2} - \delta_J\right),$$
$$\delta_J \equiv \mathrm{Arg}(\widetilde{J}_0 \widetilde{J}_\pi^*), \qquad M_{\rm eff} = \sqrt{M^2 + 4k^2}.$$

- One traversal gives half of both loop exponents
- cos(qθ/2): consistent with 2π periodicity because linear terms require even q
- Vanishes if only one brane is sourced — no nonlocal process, no potential

The dangerous tree channel exists only when an even-charge scalar communicates between both fixed points.

Summary of Channels

Strong warping and large mass — and one pattern

Loop, C-twisted (baseline)	$e^{-4k\pi R} e^{-2M_{\text{eff}}\pi R} \cos(q\theta)$
Loop, P-type × brane masses	$\lvert\tilde{b}_0\tilde{b}_\pi\rvert\, e^{-4k\pi R} e^{-2M_{\text{eff}}\pi R} \cos(q\theta - \delta_b)$
Tree, brane linear terms (scalar)	$\lvert J_0 J_\pi\rvert\, e^{-2k\pi R} e^{-M_{\text{eff}}\pi R} \cos(q\theta/2)$

$M_{eff} = \sqrt{M^2 + 4k^2}$ (scalar), $M_{eff} = M$ (fermion)

A Holographic Dictionary

PQ-breaking operator of dimension Δ in the dual CFT — The traversal number becomes the number of operator insertions

DIMENSION

$$\Delta = 2 + \frac{M_{\rm eff}}{k}$$

$M_{eff} = \sqrt{M^2 + 4k^2}$ (scalar)

$M_{eff} = M$ (fermion)

E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253 · S. Gubser, I. Klebanov & A. Polyakov, PLB 428 (1998) 105 · P. Cox, T. Gherghetta & M. D. Nguyen, JHEP 01 (2020) 188

SCALING

The explicit 5D suppressions reorganize

$$\frac{\mu_{\rm IR}}{\mu_{\rm UV}} = e^{-k\pi R}$$

$$e^{-4k\pi R}e^{-2M_{\rm eff}\pi R} = \left(\frac{\mu_{\rm IR}}{\mu_{\rm UV}}\right)^{2\Delta},$$

$$e^{-2k\pi R}e^{-M_{\rm eff}\pi R} = \left(\frac{\mu_{\rm IR}}{\mu_{\rm UV}}\right)^{\Delta}.$$

$$V_n \propto \left(\frac{\mu_{\rm IR}}{\mu_{\rm UV}}\right)^{n\Delta}, \qquad n = \text{number of traversals.}$$

In the dual language, the loop behaves parametrically like two insertions of the PQ-breaking operator, while the tree bridge behaves like one. This is an interpretation of the 5D scaling; an explicit dual CFT calculation remains open.

Conclusions

Orbifolding separates P-type and C-twisted matter: bulk phases cancel for P-type fields but survive for C-twisted fields, while branes can reopen P-type channels.

Warping generates intermediate f_a and suppresses unwanted PQ breaking; once f_a is fixed, quality constrains M_{eff} or the dual CFT dimension.

In the regimes studied, the propagation length gives the characteristic scaling

$$V_{loop} \sim f_a^4\, e^{-2M_{eff}\pi R} \cos(q\theta), \quad V_{tree} \sim f_a^2\, e^{-M_{eff}\pi R} \cos(q\theta/2),$$

so an allowed brane-to-brane tree channel is generally more restrictive.

Worldline, KK spectral-function, and monodromy methods agree in complementary regimes; extensions include SUSY, general warped backgrounds, and mixing with dual 5D axions.