Domain Walls

And their Gravitational Waves

Fabrizio Rompineve

Universitat Autònoma de Barcelona & IFAE

Axions in Seoul, 28/07/2026

with KimFest in Honor of Professor Jihn E. Kim

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AGENCIA ESTATAL DE INVESTIGACIÓN

Barbini, Notari, Pujolàs, F. Torrenti 26/Notari, F. Torrenti 25/ Caprini, Pujolàs, Quelquejay-Leclere, Steer 24/ R.Z. Ferreira, A. Notari, O. Pujolàs, '22, 24'

Outline

Domain Walls (and the strong CP problem)

Transient cosmological sources of GWs

Domain Wall Annihilation

Conclusions

Motivation I:

Domain Walls, Axions and the Strong CP problem

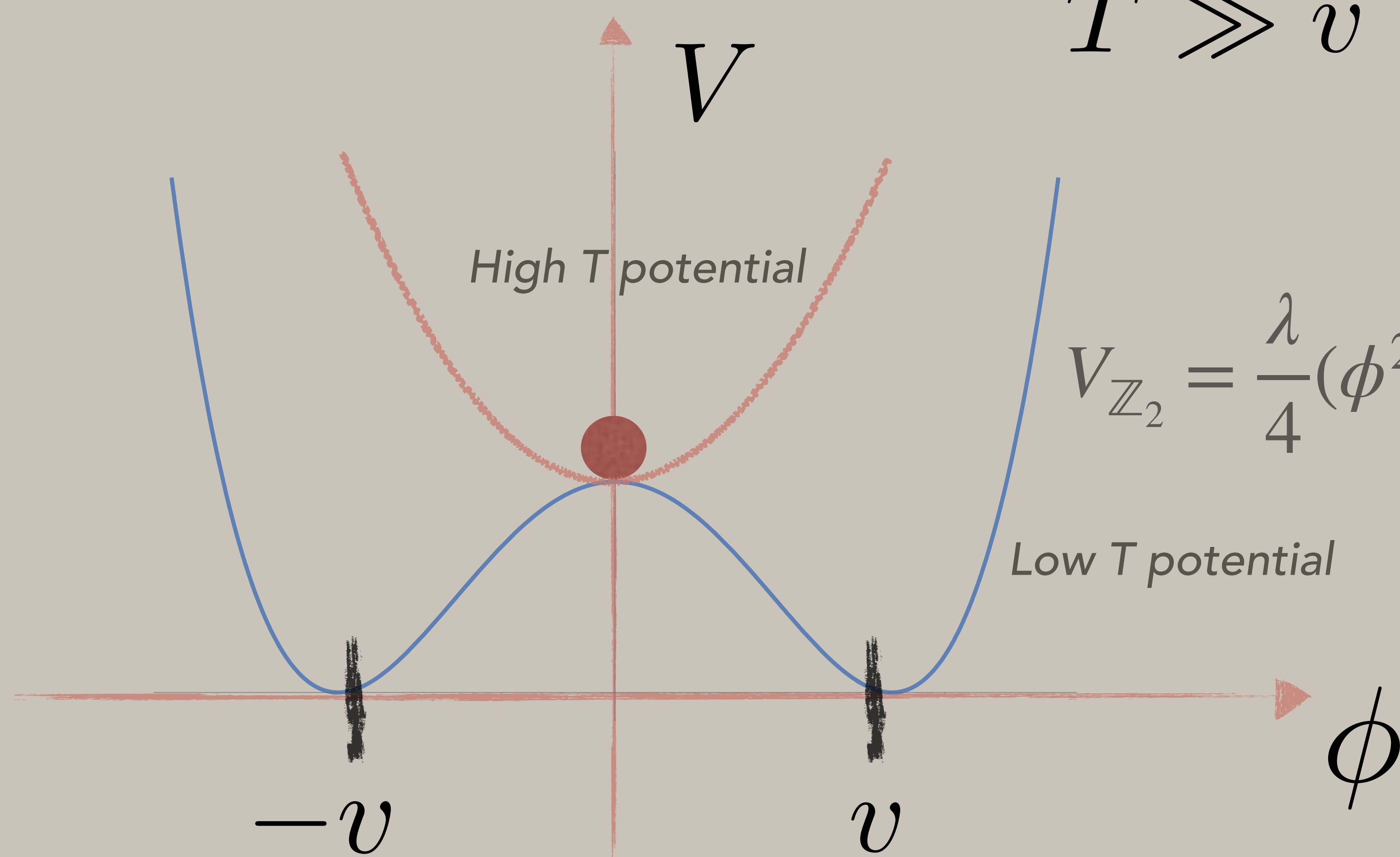
See also talks by:
Di Luzio, Gherghetta,
Hong, Villadoro

Domain Walls

Simplest realization:
real scalar field with $\mathbb{Z}_2$ *symmetric potential*

$T \gg v$

Post-inflationary Universe



$$V_{\mathbb{Z}_2} = \frac{\lambda}{4}(\phi^2 - v^2)^2$$

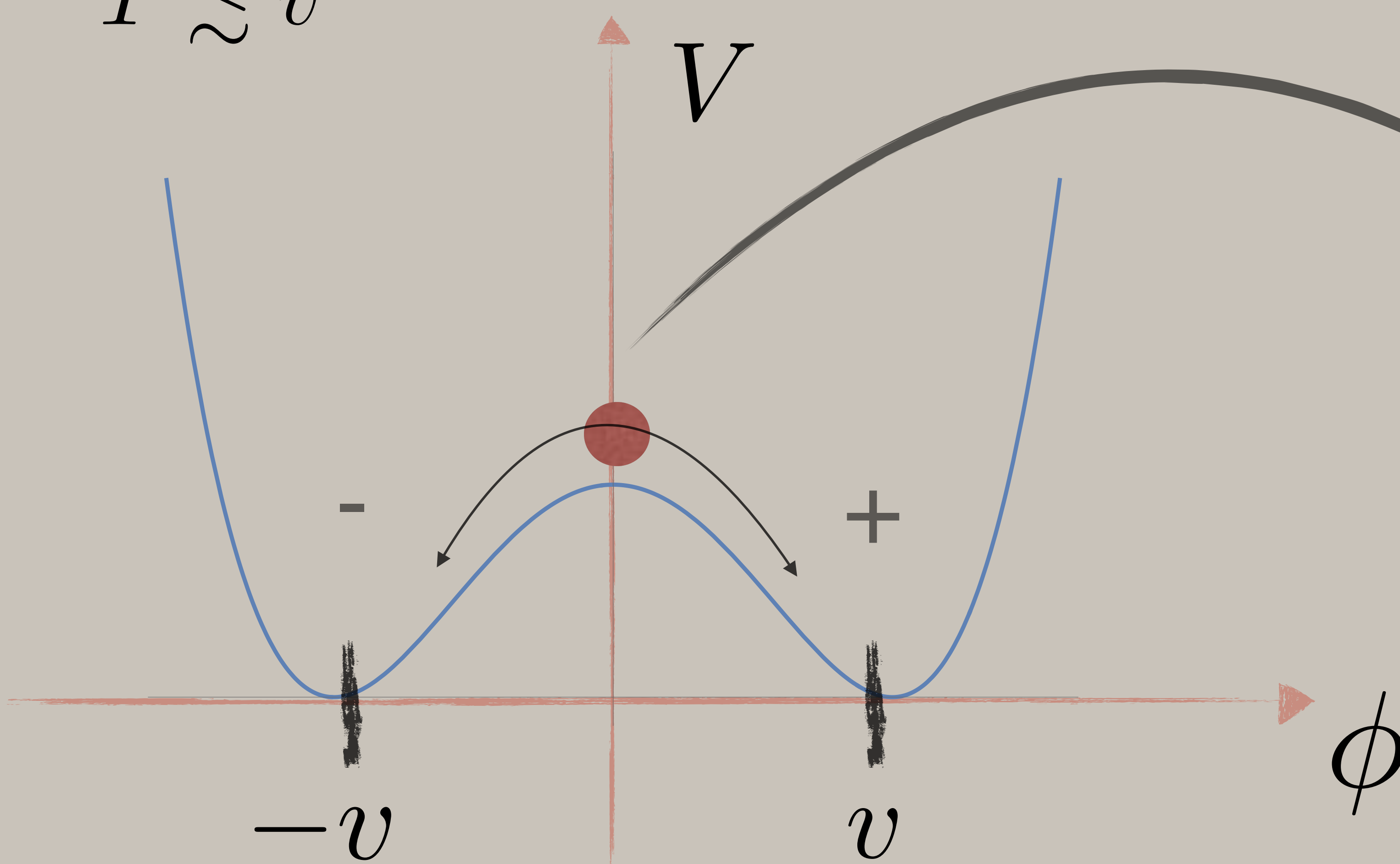
*See Linde+90/…/Garriga+15,16/Faria, Ferreira, **FR** 25 for impact of inflationary DWs*

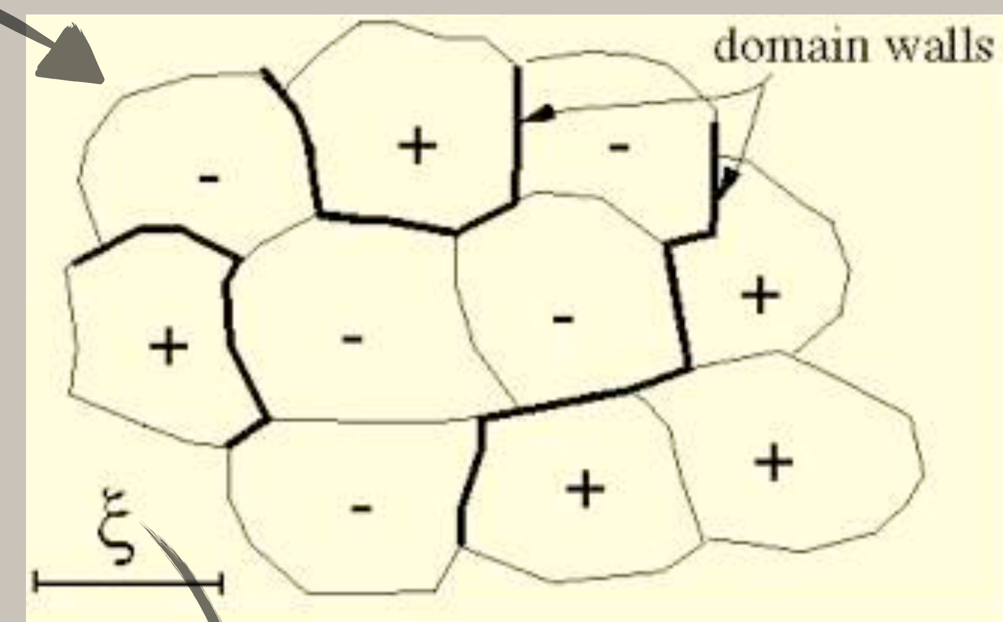
Different formation mechanisms
also possible
(Inflationary fluctuations, ...)

Kibble Mechanism

Different vacua are populated **randomly** in different regions of the Universe due to (thermal) fluctuations

$T \lesssim v$



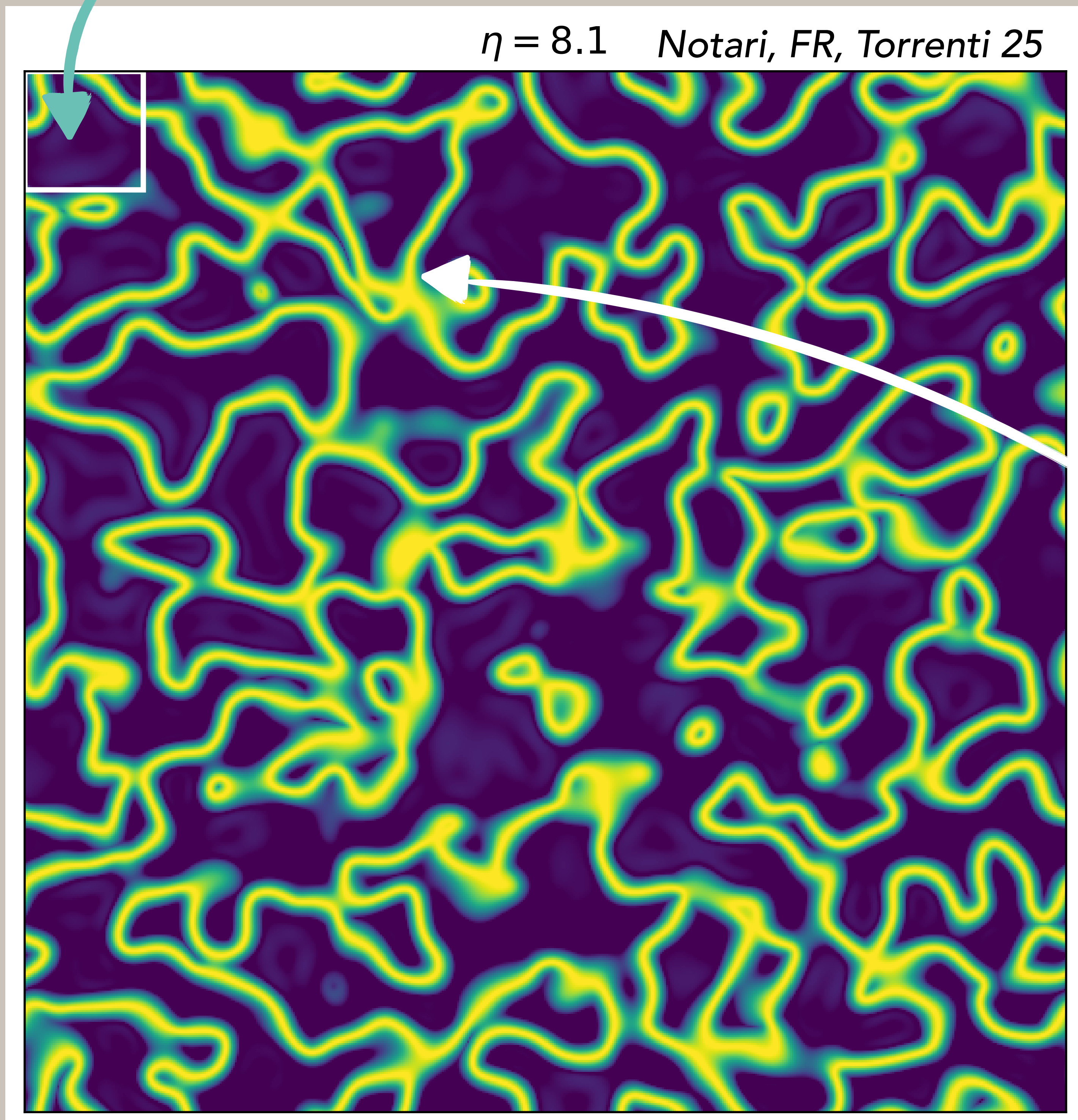


Credit: CTP

Initial "Correlation Length" $\zeta(t) \leq H^{-1}$ By causality

DW parameters

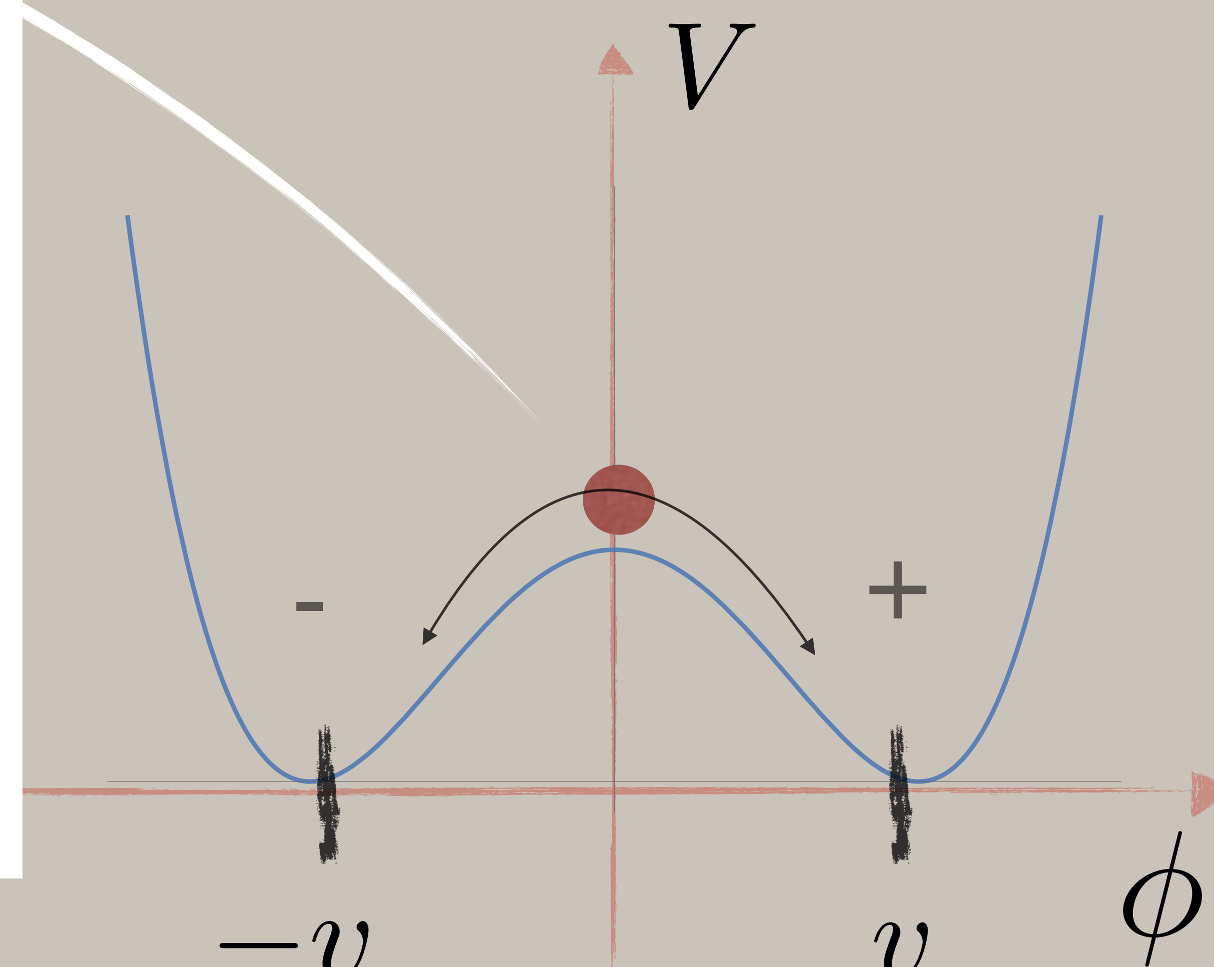
Size of the Hubble patch



Slices of gradient energy density
From our 3D **lattice field theory simulations**
(With CosmoLattice Figueroa+20)

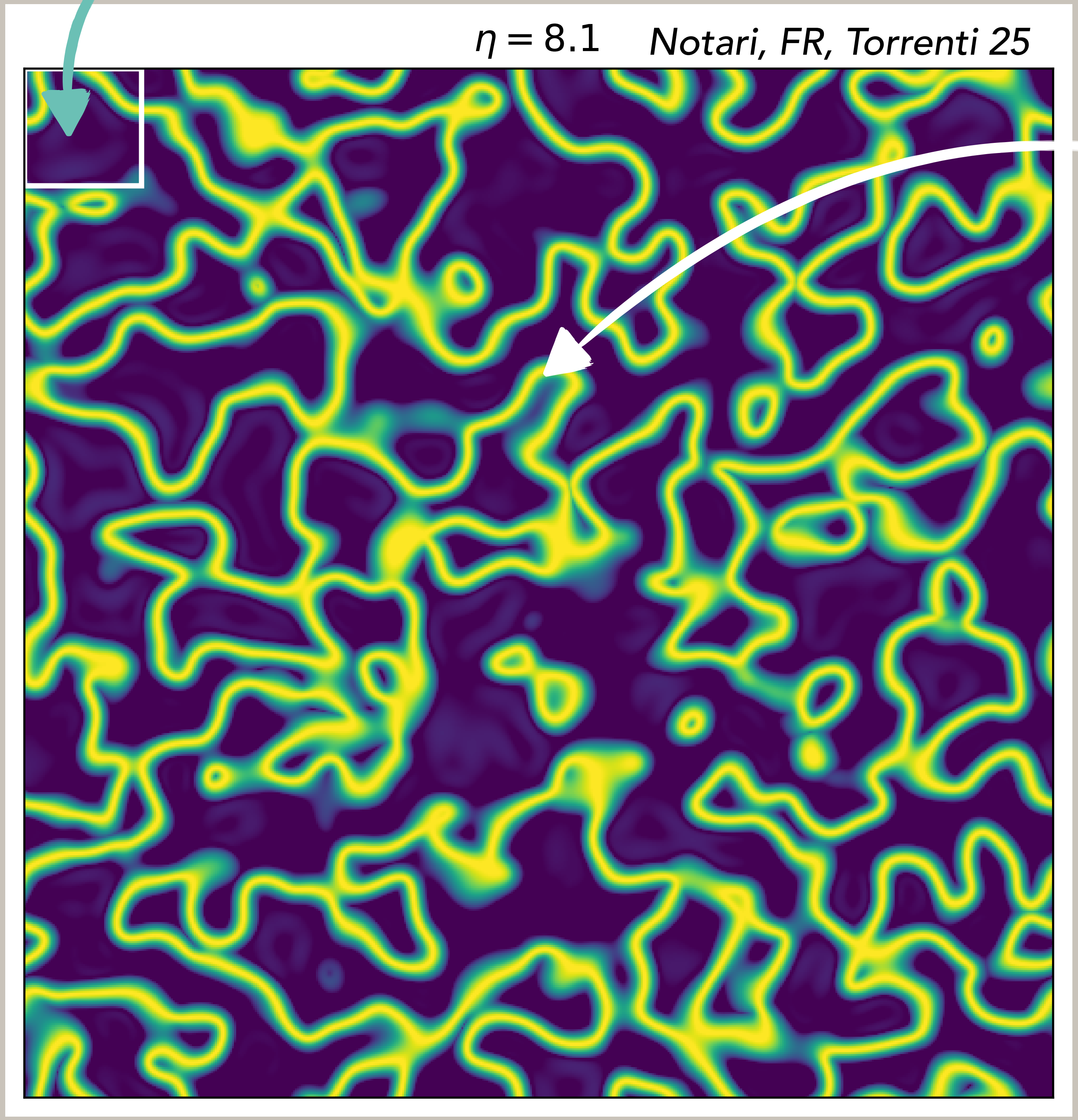
DW mass per unit surface
(Tension)

$$\sigma = \int_{-v}^{v} \sqrt{2V(\phi)}\, d\phi \sim m v^2$$



Size of the Hubble patch

DW parameters



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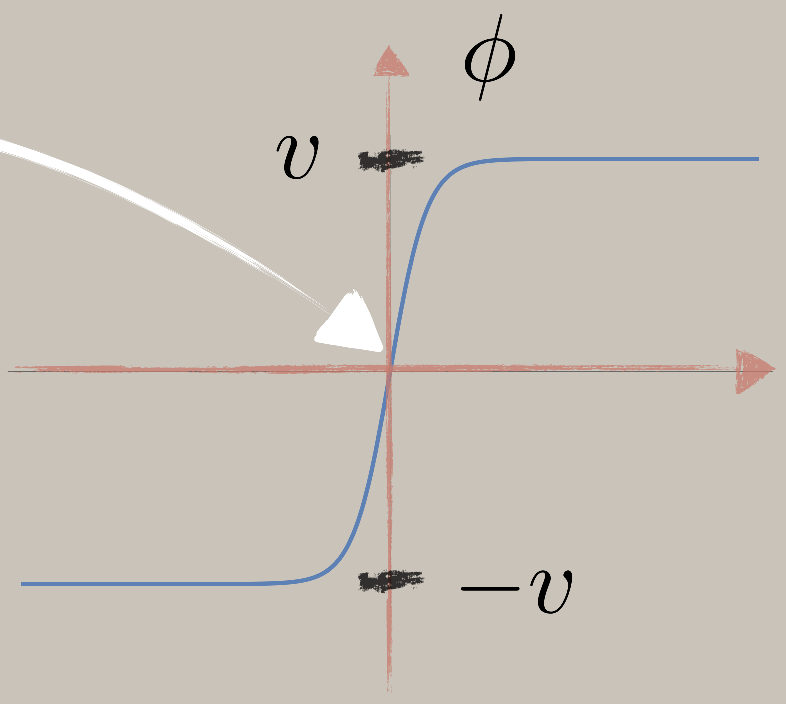


Domain Wall width

$$\delta_w \sim m^{-1}$$

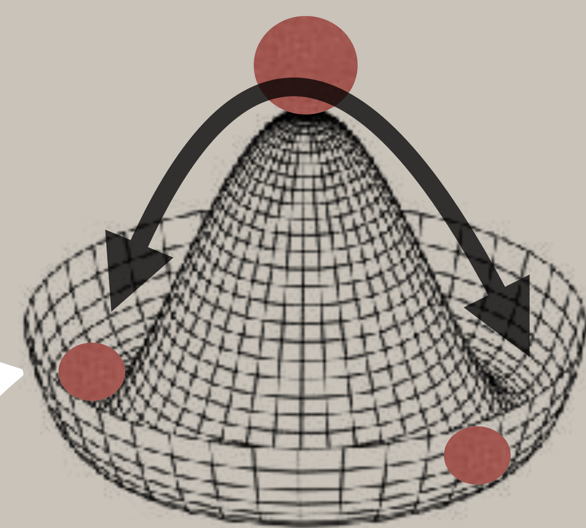
"Axionic" Defects

See Giovanni's talk

$$V(\Phi) = \lambda(|\Phi|^2 - v_a^2/2)^2$$

$U(1)$

breaking



$T \lesssim v_a$

$$\Phi \sim \varphi e^{-\frac{a}{v_a}}$$

Goldstone=Axion

Loop in
physical space

Cosmic string defect

Associated to massive
radial mode

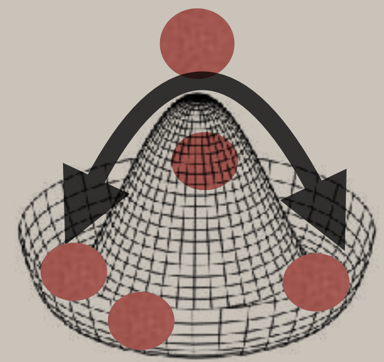
φ

"Axionic" Domain Walls

T

$T \lesssim v_a$

$U(1)$



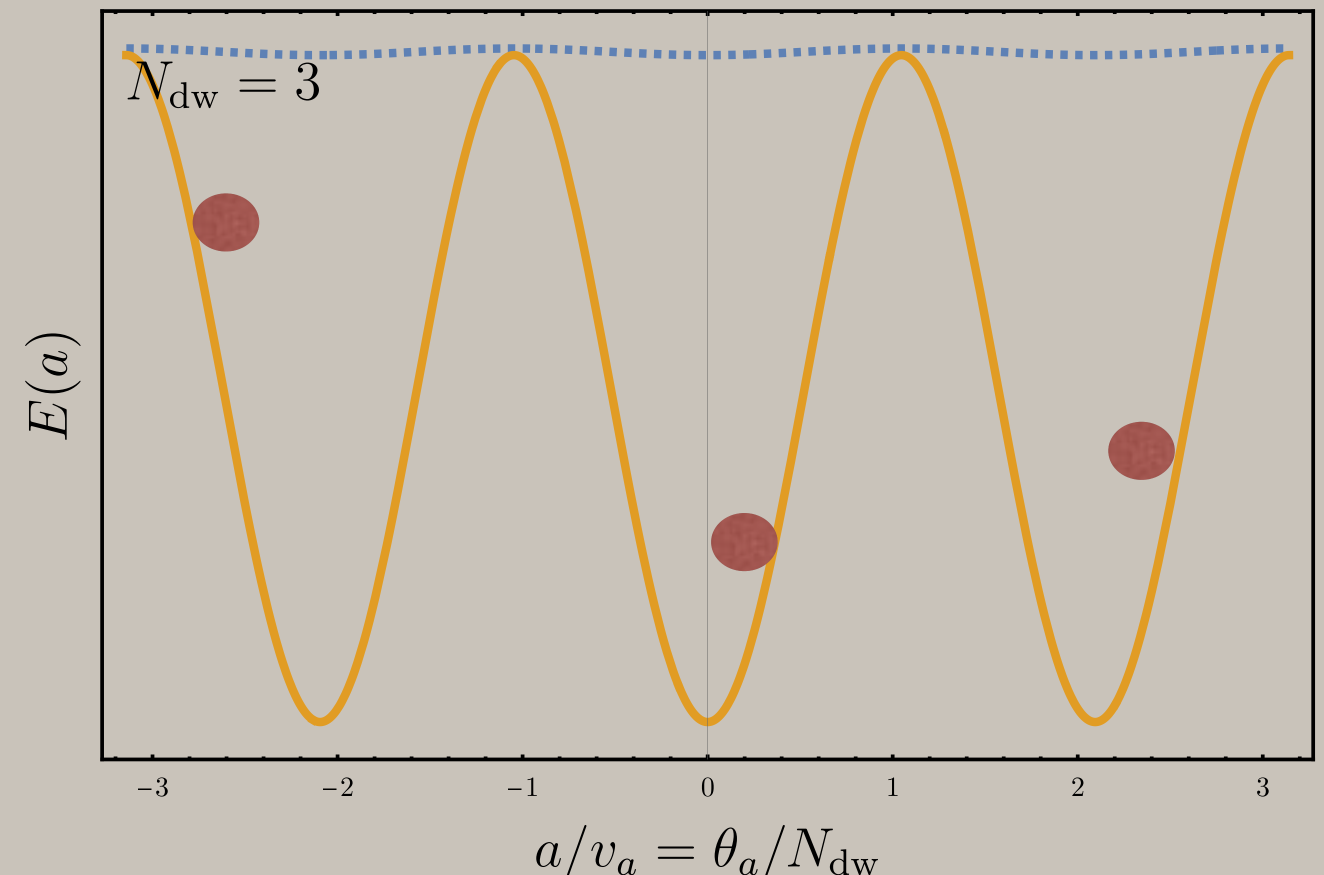
$$\Phi \sim \varphi e^{-\frac{a}{v_a}}$$

$\mathbb{Z}_{N_{\rm dw}}$

Explicit breaking (from non-perturbative effects) may still respect discrete shift symmetry

$$V(a) \sim \kappa\, \Lambda^4 \cos(a/f_a)$$

$$m_a \propto \Lambda^2/f_a, \quad f_a \equiv v_a/N_{\rm dw}$$



Domain Wall Number

Depends on the microphysics

e.g. in QCD axion KSVZ, DFSZ models

$$\psi, q \to e^{-i\gamma_5 \frac{a}{2v_a}} \psi, q \Rightarrow \delta\mathcal{L} = N_{\rm dw} \frac{a}{v_a} \frac{\alpha_s}{8\pi} G\tilde{G}$$

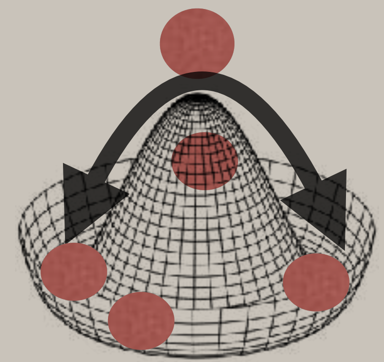
Generally $N_{\rm dw} > 1$ *(e.g. more than one fermion charged under PQ)*

"Axionic" Domain Walls

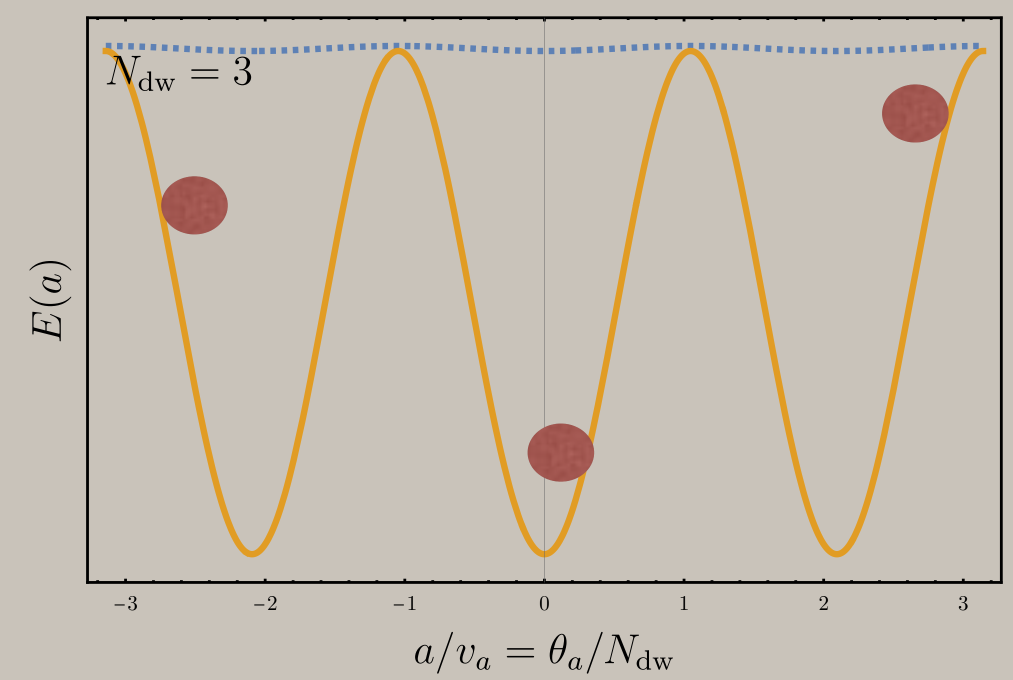
T

$T \lesssim v_a$

$U(1)$



$$\Phi \sim \varphi e^{-\frac{a}{v_a}}$$

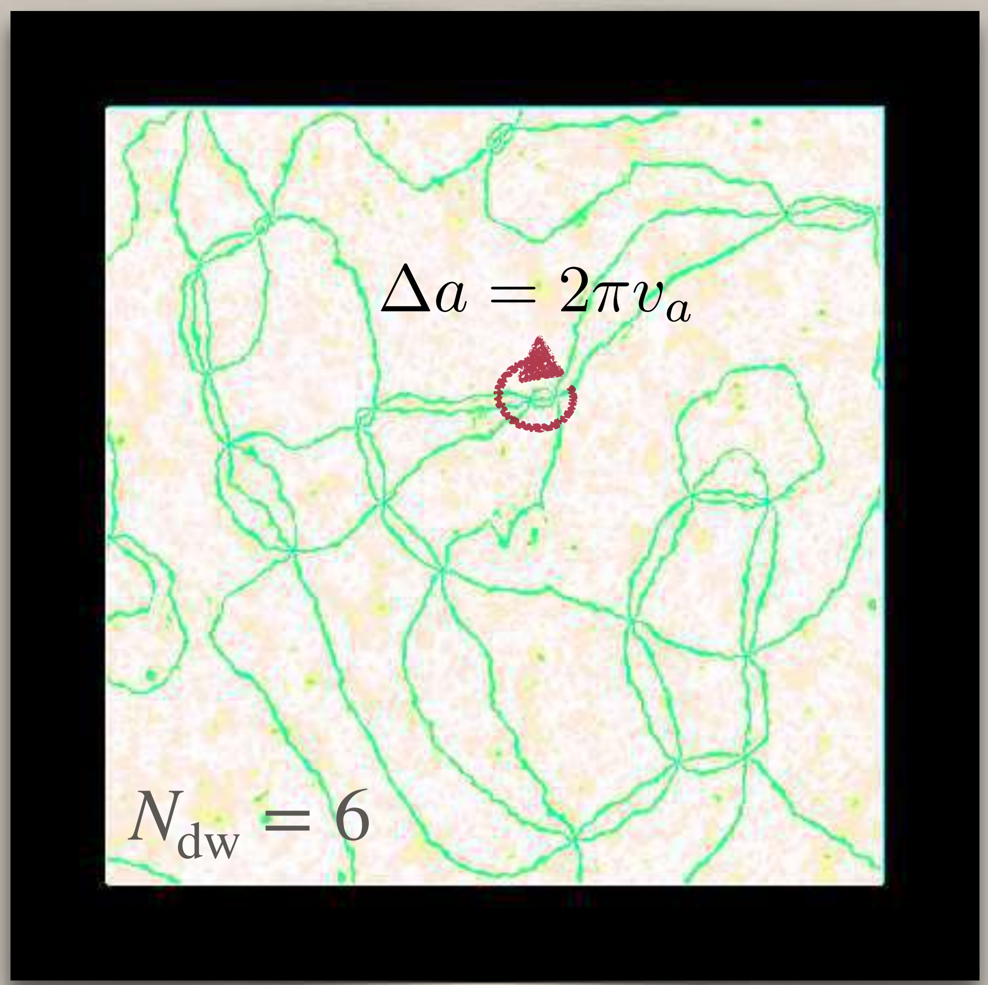


Explicit breaking (from non-perturbative effects) may still respect discrete shift symmetry

$\mathbb{Z}_{N_{\rm dw}}$

$$V(a) \sim \kappa\, \Lambda^4 \cos(a/f_a)$$

Kawasaki+ 14



(a)$\Xi = 0,\ \tau = 42$

Spontaneous breaking

I

$$m_a(T) \gtrsim H(T)$$

Hybrid string-wall network forms

$$\sigma \sim m_a f_a^2$$

Domain Walls and the Strong CP problem

Peccei-Quinn Mechanism

Peccei,Quinn/Weinberg/Wilczek 77

*Breaking of global U(1) provided by **QCD (IR) instantons***

Additional breakings also possible, constrained by strong CP (UV instantons, additional IR sectors, …)

*String-wall network arises around QCD crossover, contributes to total **axion (DM) abundance***

Sikivie/Vilenkin, Everett 82/…/ Hiramatsu+12/…/Kawasaki+14/…/ Gorghetto+Villadoro 18,20/…/Choi, Hong+26

Spacetime symmetries

(Nelson-Barr, parity)

Nelson 84, Barr 84, Babu+89,90, Barr 91

Spontaneous breaking of (gauged) CP (or parity) provided by additional scalar sector

$$\Lambda_{\rm CP} \lesssim 10^8 \text{ GeV}$$

Dine+92/Choi+92/…

$\mathbb{Z}_N$ *Domain wall network Forms at high scales*

Craig+20/ …/McNamara, Reece 22/Asadi+Reece 22

Domain Wall Problem

Balance between

More walls entering the Hubble volume

Energy density increases

VS

More interactions between walls

Energy density decreases

Scaling Regime:

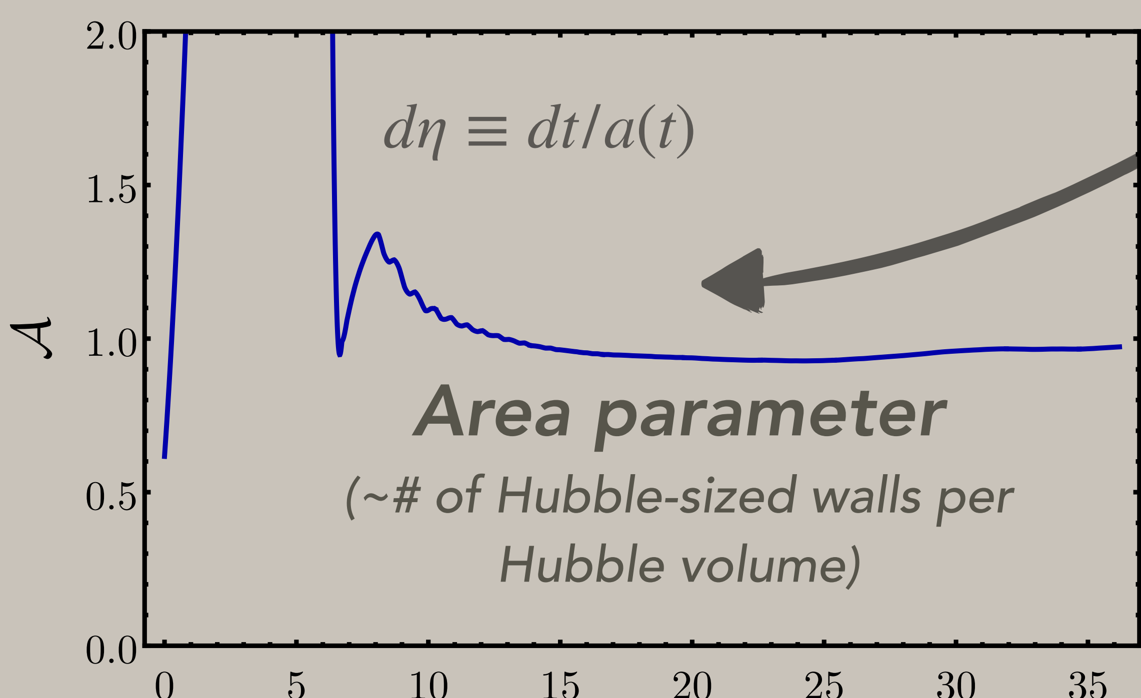
fixed number of Hubble-sized walls per Hubble volume

$$\rho_{\rm dw} = \frac{\sigma A_{\rm dw} a^2}{V a^3} = 2\mathscr{A}\sigma H \propto a^{-2}$$

See also talks by Di Luzio, Gherghetta, Hong

Notari, **FR**, Torrenti 25

$\lambda\phi^4$



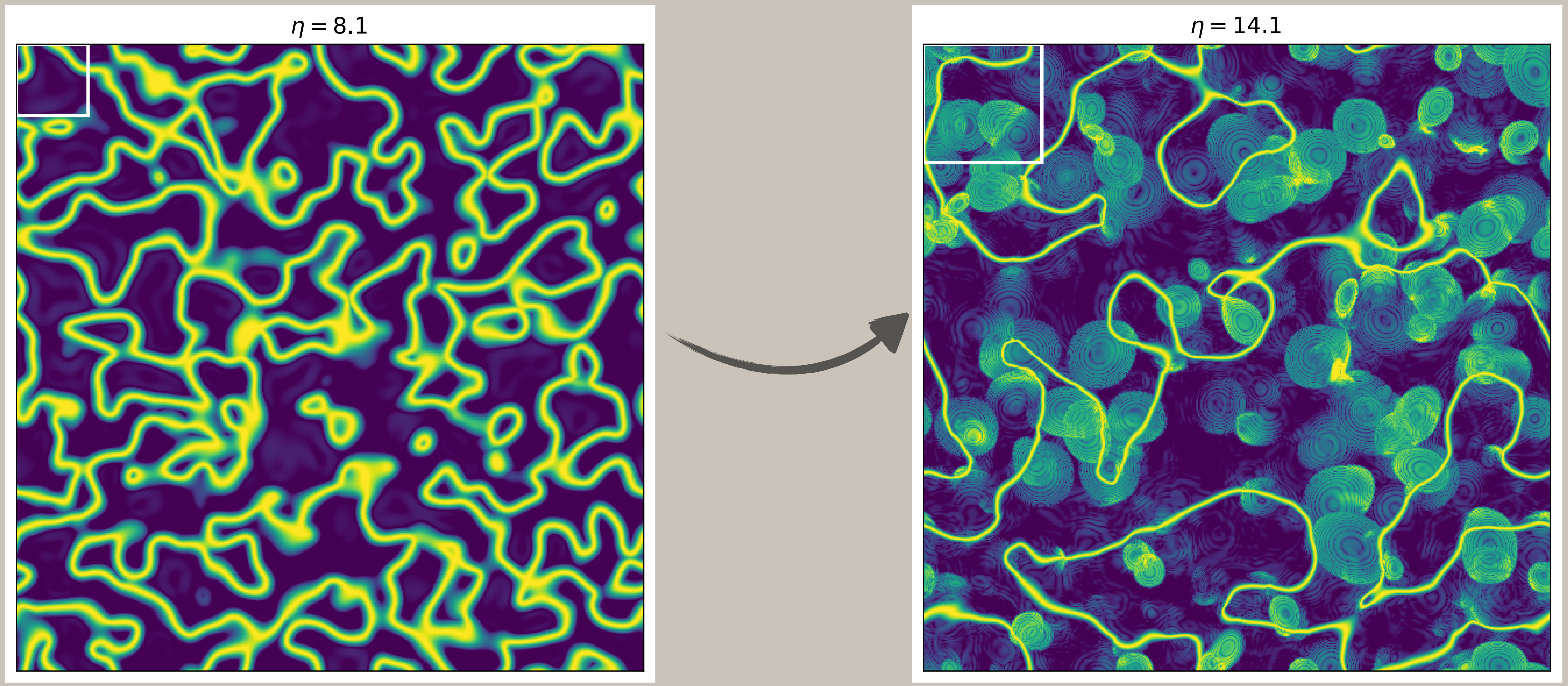
Zeldovich+ 74

Spontaneous breaking of (exact) discrete symmetries after inflation is incompatible with cosmology

Exception: string-wall with N=1, unstable (Nucleation of strings typically strongly suppressed)

Scaling Regime

Notari, **FR**, Torrenti 25



2D slice of gradient energy density (log scale)

Viability of discrete symmetries
requires understanding of possible
annihilation mechanisms *for domain wall networks*

Phenomenological implications
require detailed description of annihilation phase,
To understand abundance of relics

(Axion) Dark Matter

Hiramatsu+11/Kawasaki,Saikawa 14/
Gorghetto+20/Chang+23/Saikawa+24/
Benabou+24

Dark Radiation

Primordial Black Holes

Linde+90/…/Garriga+15,16/
Ferrer+FR 18*/Gouttenoire+24/*
Ferreira+FR 24/Faria+FR 25

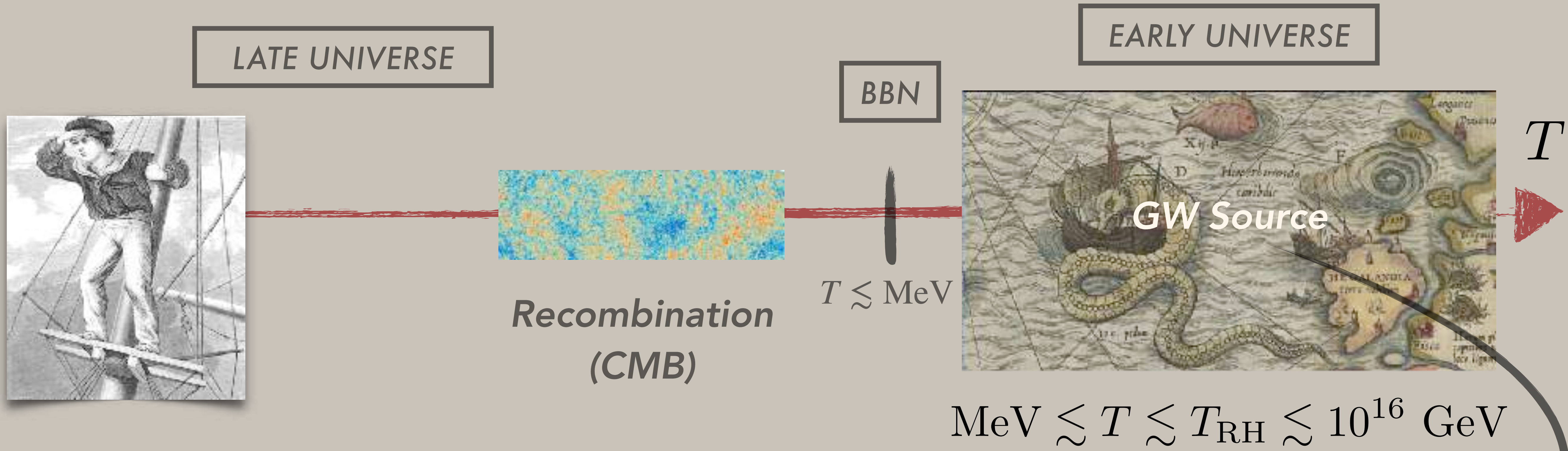
Gravitational Waves

In this talk, results that apply to general post-inflationary axions and other fields with discrete symmetry, not necessarily the QCD axion

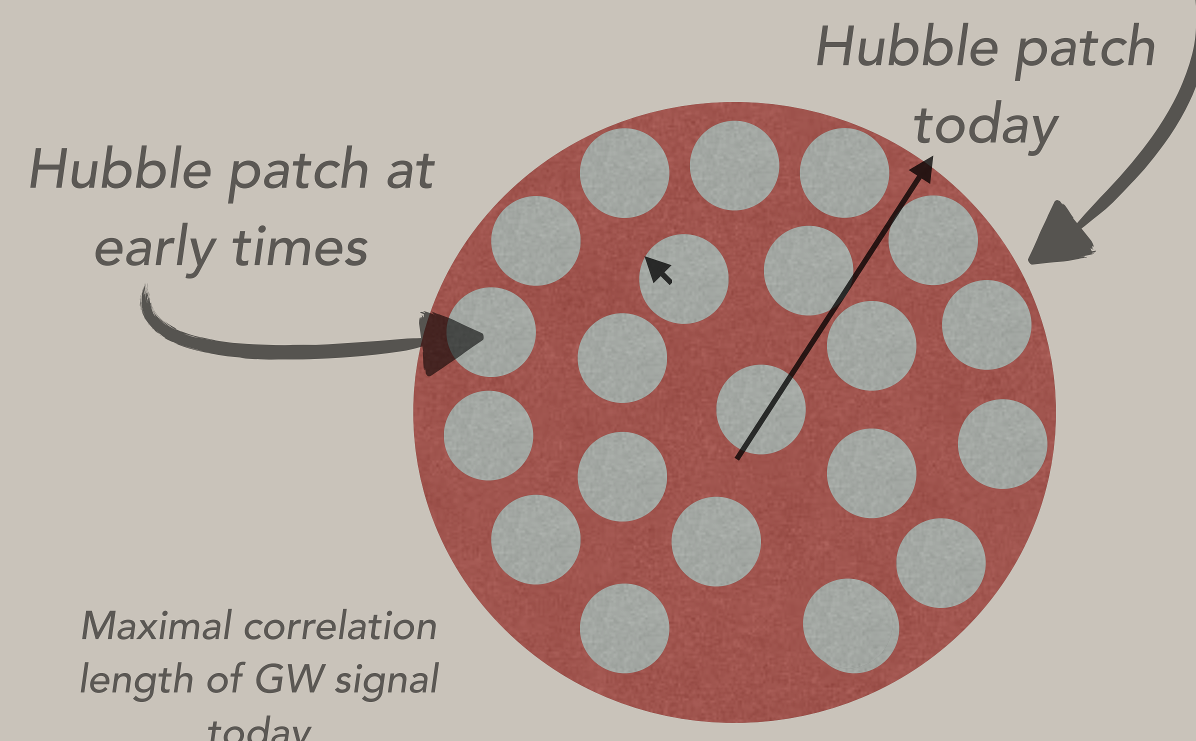
Motivation II:

Opportunity from Gravitational Wave Observations

Gravitational Waves as a probe of the early Universe



Due to causality, signal today would be a ***Isotropic Stochastic Background,*** *permanent "noise" in the detector (Similar to CMB, but for gravitons)*



$$\ell_0/H_0^{-1} \lesssim 10^{-11}(\mathrm{GeV}/T)$$

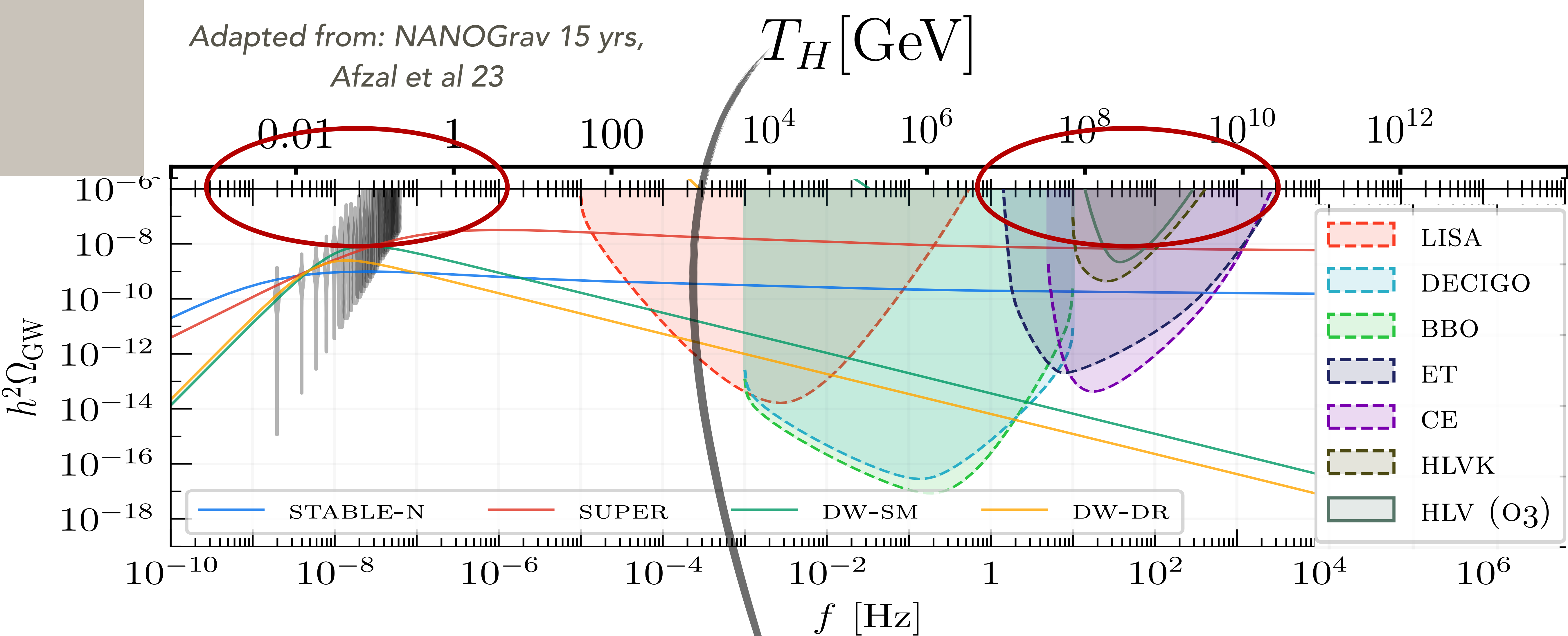
Sensitivity to stochastic Gravitational Waves

See also talk by Kim

$$\Omega_{\rm gw} \equiv (d\rho_{\rm gw}/d\ln f)/\rho_c$$

$$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij}(\mathbf{x},t)\,\dot{h}_{ij}(\mathbf{x},t)\rangle}{32\pi G}$$

Adapted from: NANOGrav 15 yrs, Afzal et al 23

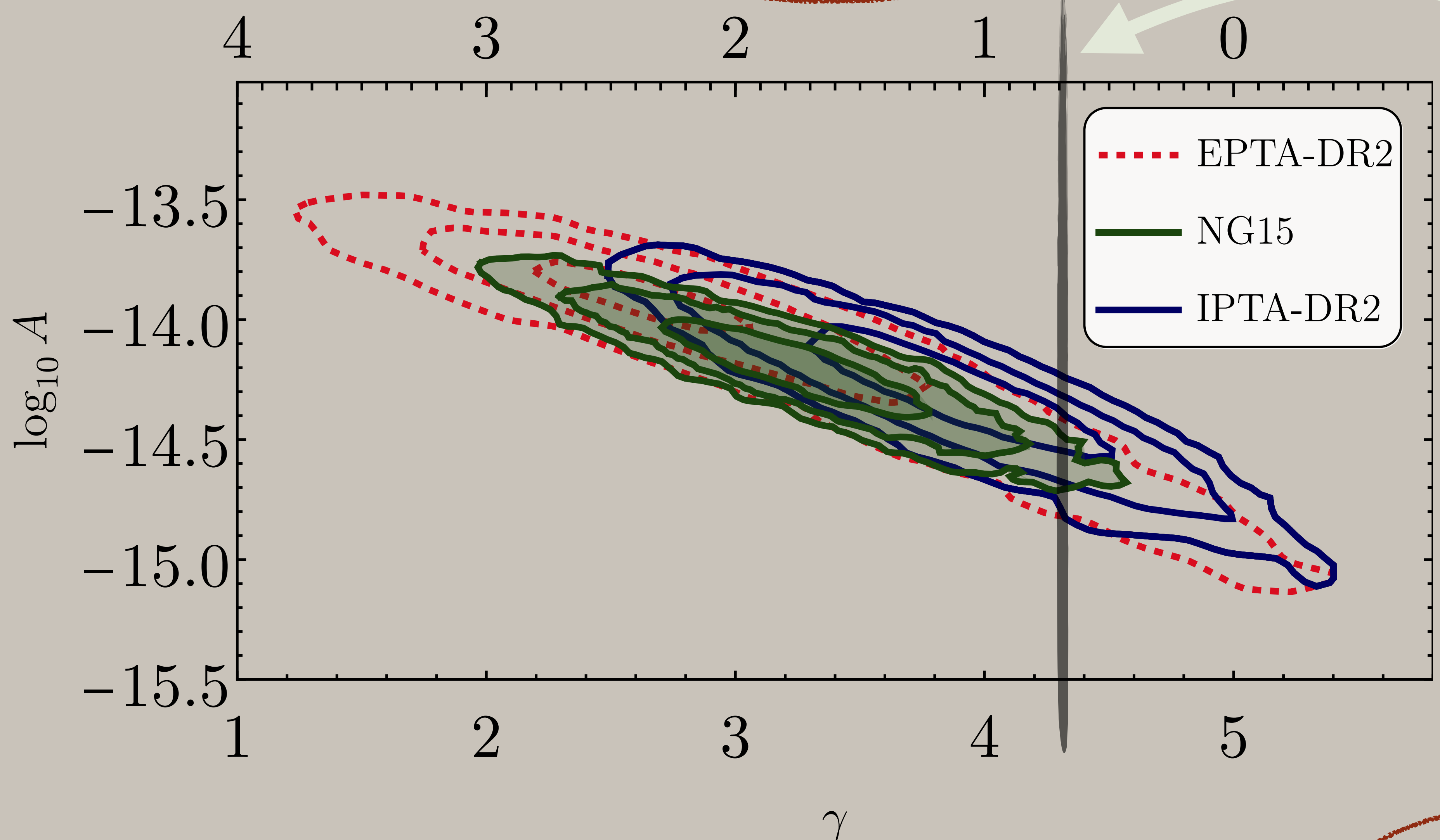


For a broad class of sources $f(\text{emission}) \propto H(T)$

On the verge of a detection at Pulsar Timing Arrays (PTAs)?

From Franciolini, Racco, FR 23

EPTA-DR2 Antoniadis+ 23
NANOGrav 15 yrs, Agazie+ 23
IPTA-DR2 Antoniadis+ 22



"Vanilla" Astro prediction (SMBHBs in circular orbits w/ evolution driven by GWs)

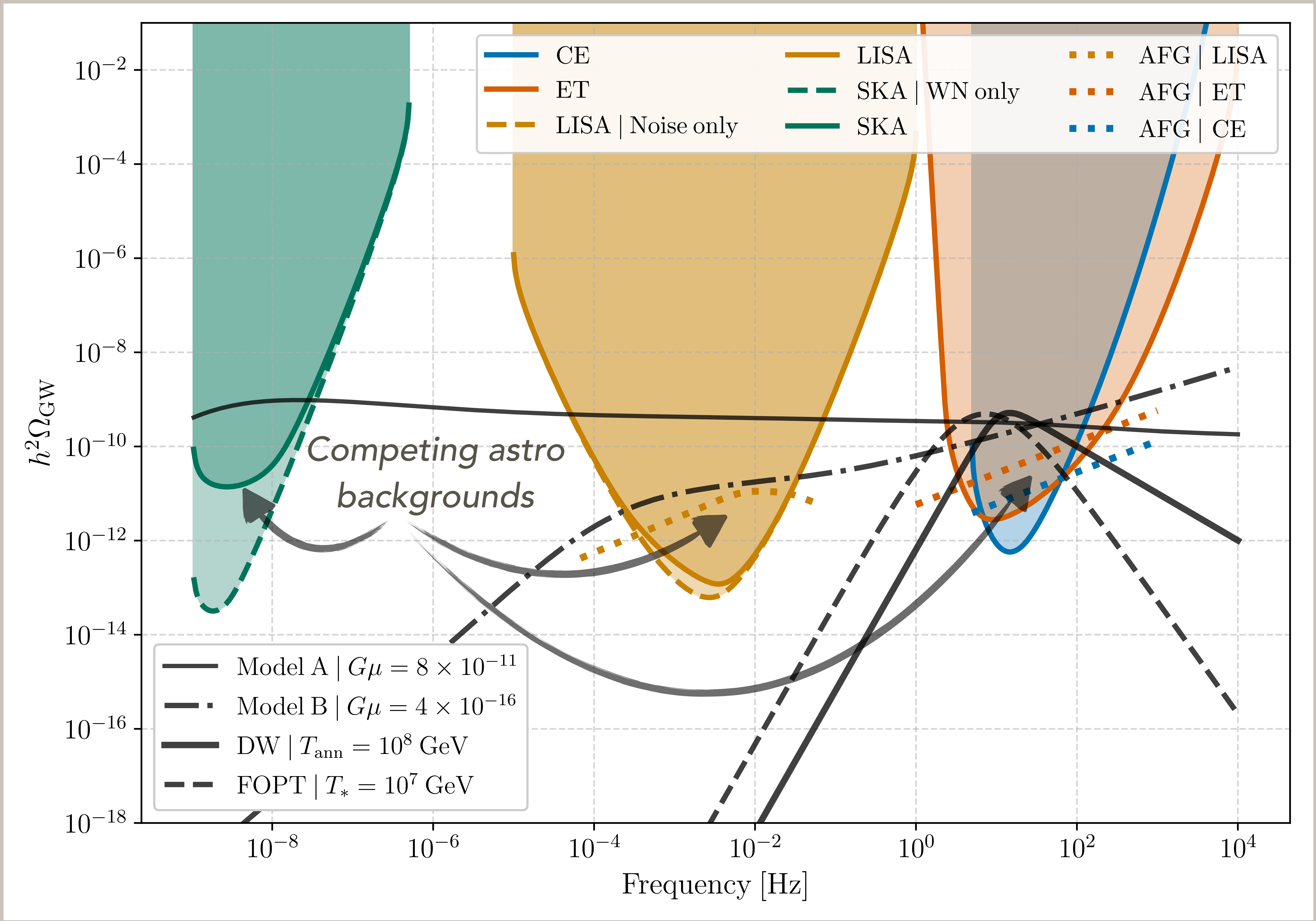
$$\Omega_{\rm gw} h^2 \simeq 10^{-8} \left(\frac{A}{5\cdot 10^{-15}}\right)^2 \left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma \;\equiv\; n_T}$$

$f_{\rm yr} \approx 32$ nHz

Caprini, Pujolàs, Quelquejay-Leclere, Steer, FR 24

The future (>2035)

See also talk by Kim



What sources can we see in the near future?

Transients in cosmological history

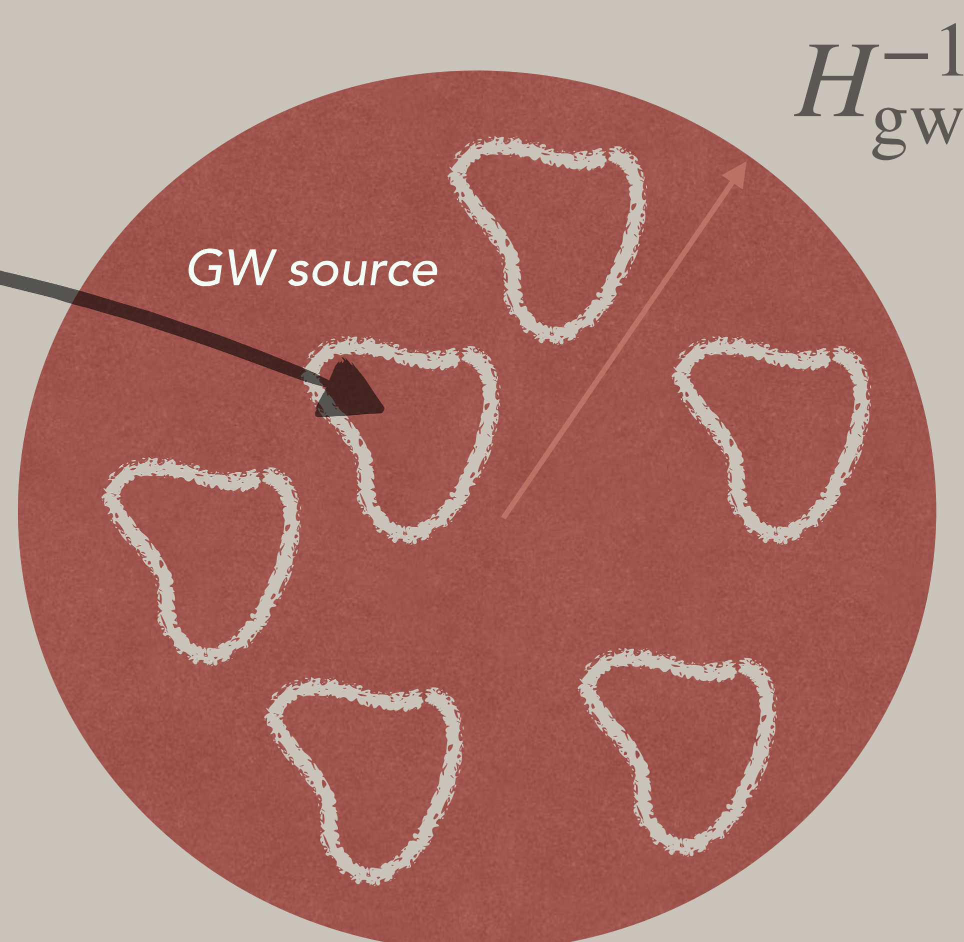
Domain Walls belong to a broad class of sources

active at definite epoch/temperature, on a short time scale

$$t_{\rm gw} \lesssim H(T_{\rm gw})^{-1}$$

Length (correlation) scale of the source

$$l_{\rm gw} \lesssim H_{\rm gw}^{-1}$$



Hubble sphere

Long-lived sources also potentially observable (Cosmic strings, inflation, ...)

With an energy density fraction $\alpha_{\rm gw} \equiv \rho_s/(3H^2M_p^2)|_{T=T_{\rm gw}}$

Amplitude/Relic Abundance

$$\boxed{\Omega_{\rm gw}h^2 \simeq 10^{-8}\epsilon_{\rm gw}\left(\frac{g_*(T_{\rm gw})}{74}\right)^{-1/3}\left(\frac{\alpha_{\rm gw}/(1+\alpha_{\rm gw})}{0.14}\right)^2}$$

$$\epsilon_{\rm gw} \propto (t_{\rm gw}/H_{\rm gw}^{-1})^p \lesssim 1$$

Observability now and in the near future (5 to 10 yrs) requires

$$\alpha_{\rm gw} \gtrsim 0.05 \qquad t_{\rm gw}/H_{\rm gw}^{-1} \gtrsim 0.1$$

Amplitude for Domain Walls

Domain walls easily satisfy both conditions, because of scaling regime!

Vilenkin 81/…/Hiramatsu+13/…/Ferreira+FR 21, 22, 24/…/Notari+FR 25/Hong, Lee+25/…Barbini+FR 26

$$\alpha_{\rm gw} = \frac{2\mathscr{A}\sigma H}{3H^2M_p^2} \propto T^2$$

Easy to get large fractions

$$t_{\rm gw} \simeq H^{-1}$$

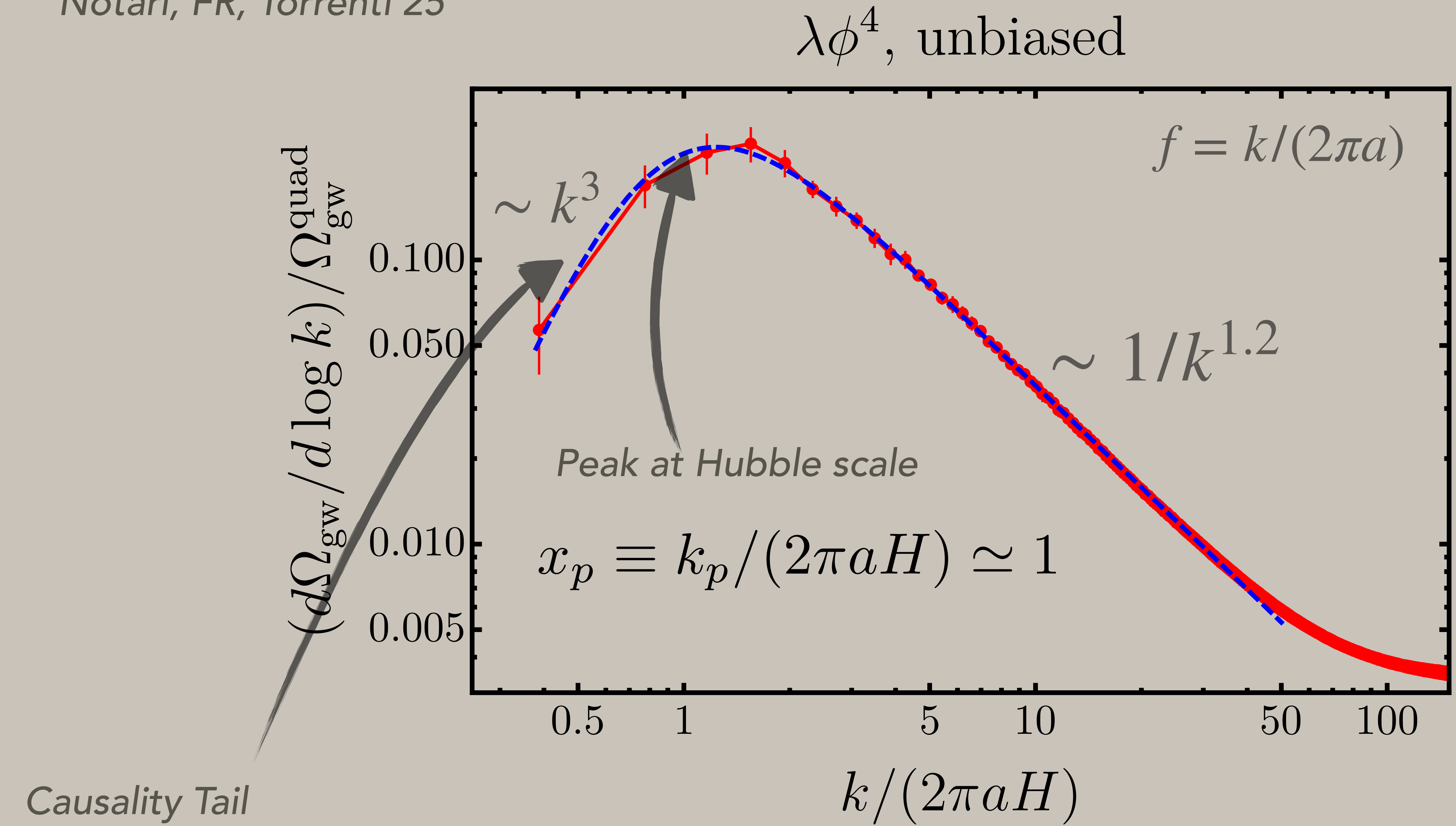
Because of O(1) number of Hubble-sized walls

(In contrast to first order phase transition, where typically $t_{\rm gw} \ll H^{-1}$ because of percolation)

Gravitational Waves from Scaling DW Networks

also computed on the lattice

Notari, FR, Torrenti 25



Roughly confirmed previous results from Hiramatsu+13

(see also Dankovsky +24)

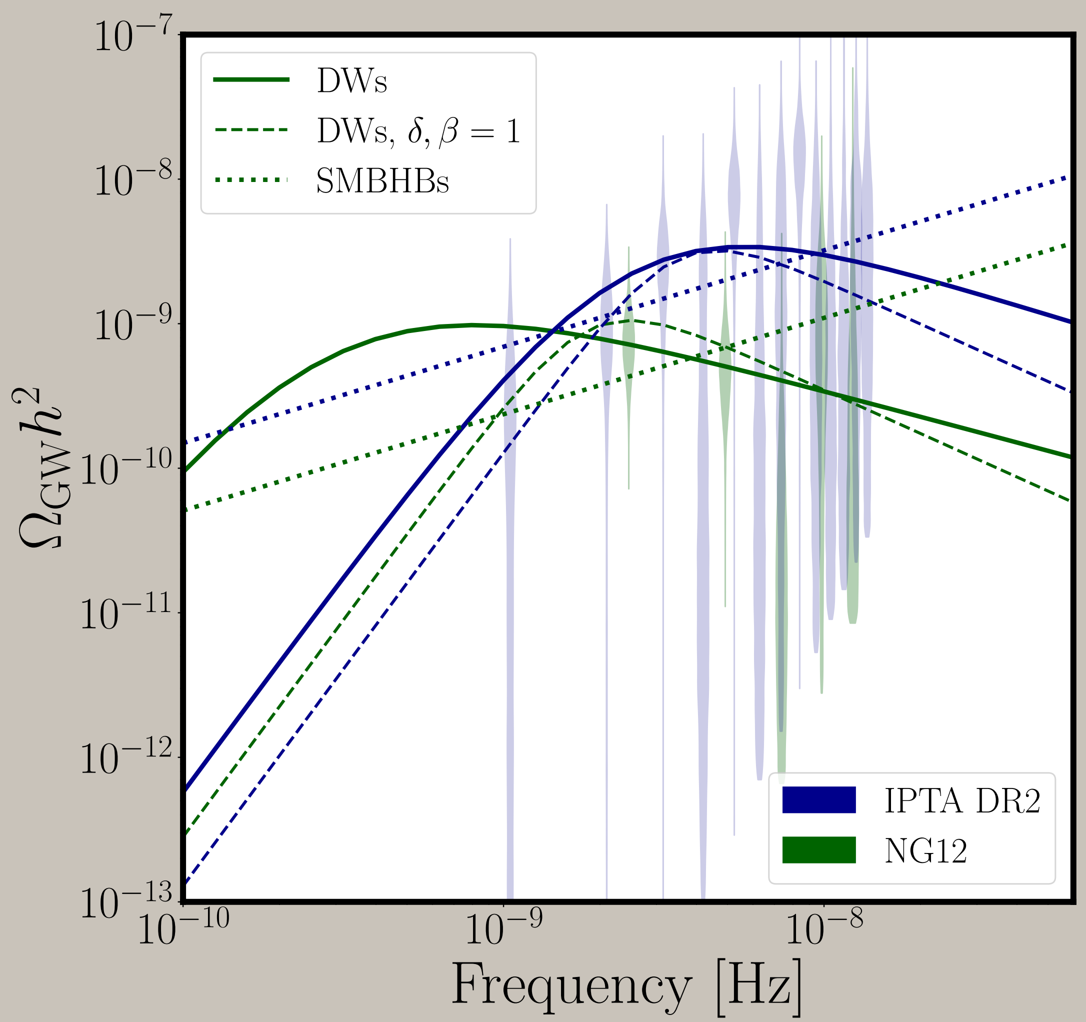
$$\Omega_{\rm gw}^{\rm quad,s} \equiv \frac{3}{32\pi}\left(\frac{2\sigma H(\eta)}{\rho_c(\eta)}\right)^2$$

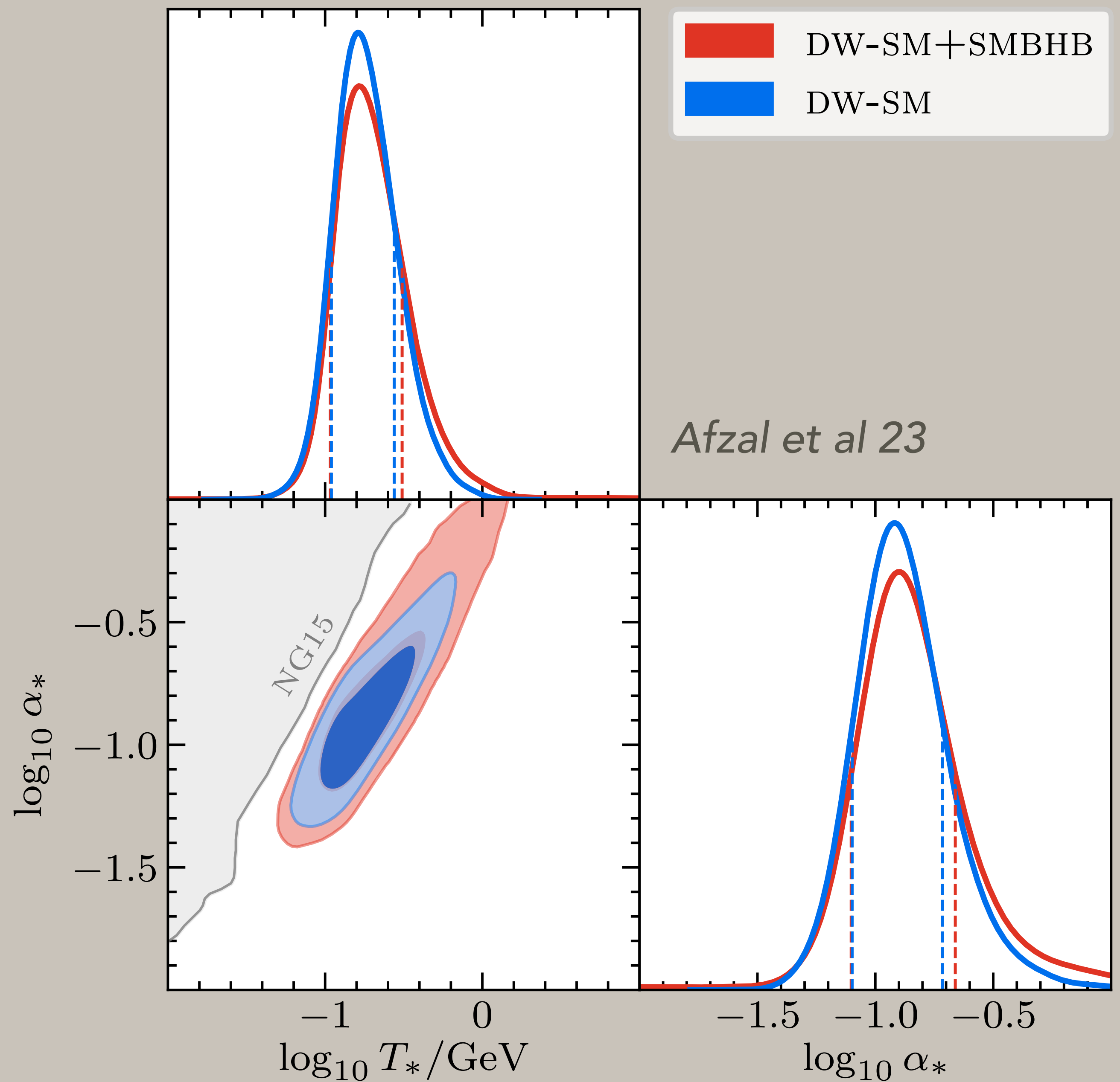
$$f_p \simeq 24\ x_p\ \mathrm{nHz}\left(\frac{g_*(T_{\rm gw})}{100}\right)^{1/6}\left(\frac{T_{\rm gw}}{150\ \mathrm{MeV}}\right)$$

Gravitational Waves from DW networks

Interpretation of PTA data,
Using scaling regime GW spectrum

*R.Z. Ferreira, A. Notari, O. Pujolàs, **FR** 22'*





Afzal et al 23

Cannot be standard QCD axion DWs, since $\alpha_{\rm dw}^{\rm QCD} \lesssim 10^{-7}$

NANOGrav 15 yrs results

The need for an analysis of the annihilation phase

In a viable model, domain walls annihilate around some temperature

$$T_{\rm gw}$$

Amplitude of GW signal *is highly sensitive to duration of collapse!*

$$\Omega_{\rm gw} \propto \alpha_{\rm gw}^2 = \frac{2\sigma H}{3H^2 M_P^2}\Big|_{T_{\rm gw}} \propto T_{\rm gw}^4$$

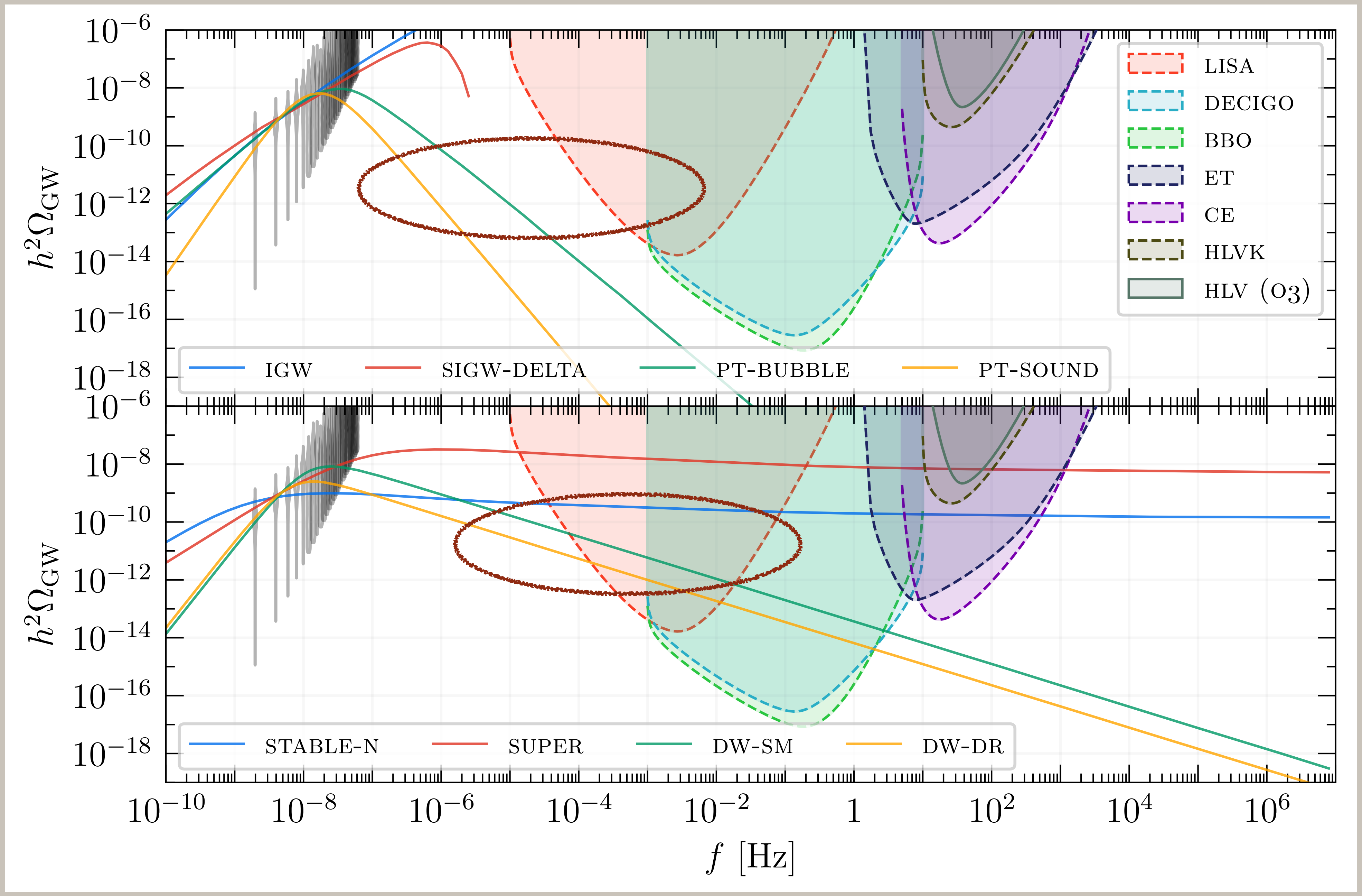
Annihilation temperature depends on microphysics, accurate estimate is required

The need for an analysis of the annihilation phase

<u>*Collapse expected to affect spectral shape,*</u>

required to fit (PTA) data

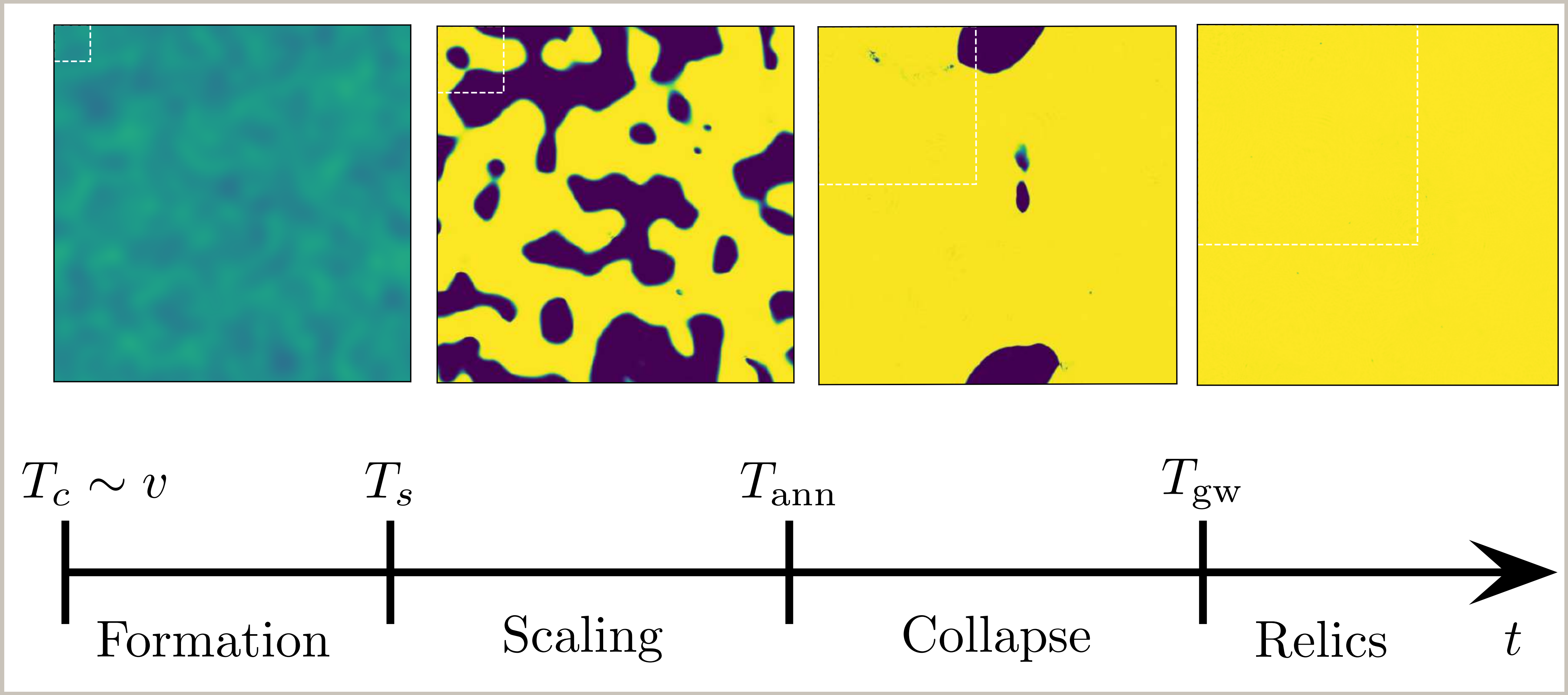
+ understand correlated signature at other observatories (e.g. LISA)



From: NANOGrav 15 yrs, Afzal et al 23

Domain Wall Annihilation

Ferreira, Notari, Pujolàs, FR 24/Notari, FR, Torrenti 25/Barbini, Notari, Pujolàs, FR, Torrenti 26

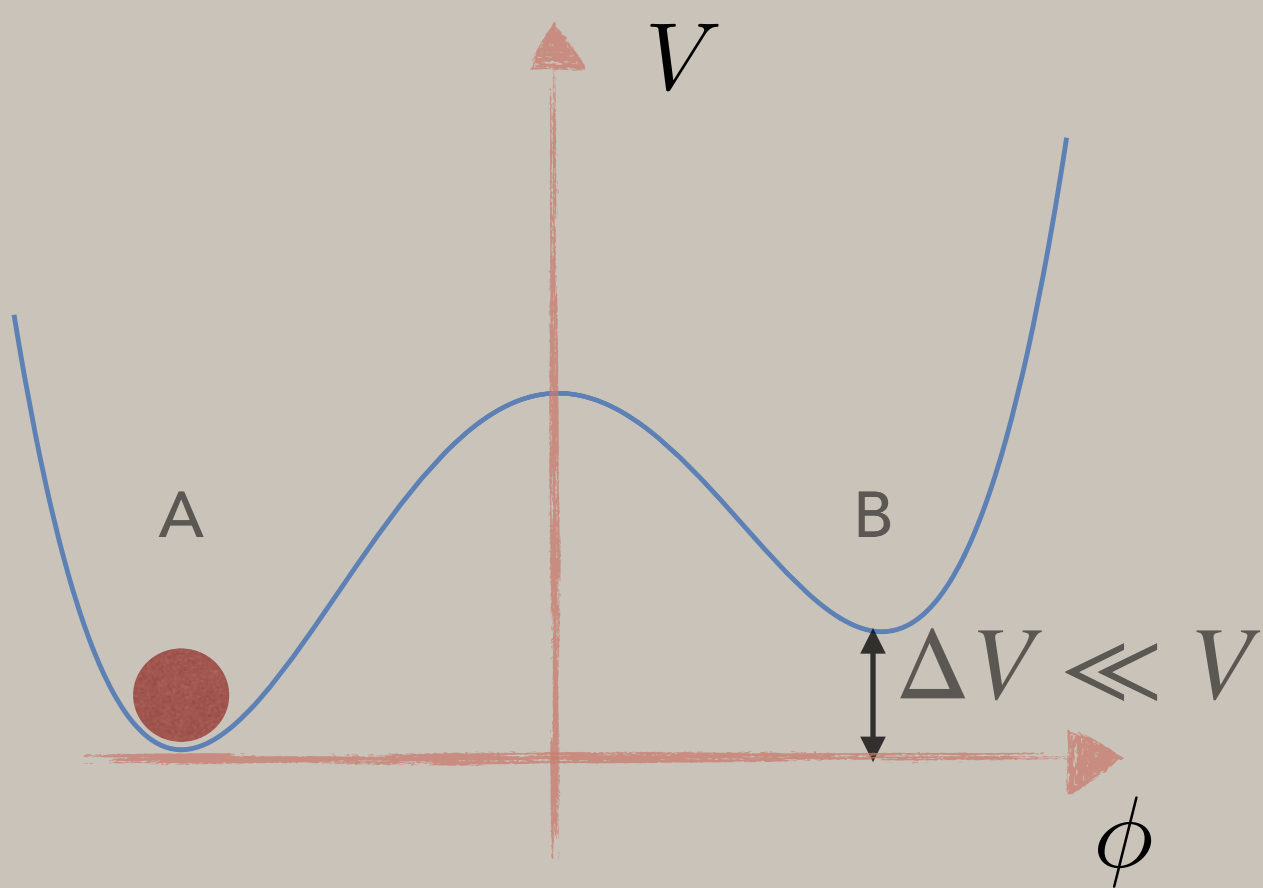


Focus here on theory with <u>one real scalar for simplicity</u>,
many implications apply to axion (string-)walls as well

Biased Domain Wall Networks

Explicit breaking of discrete symmetry

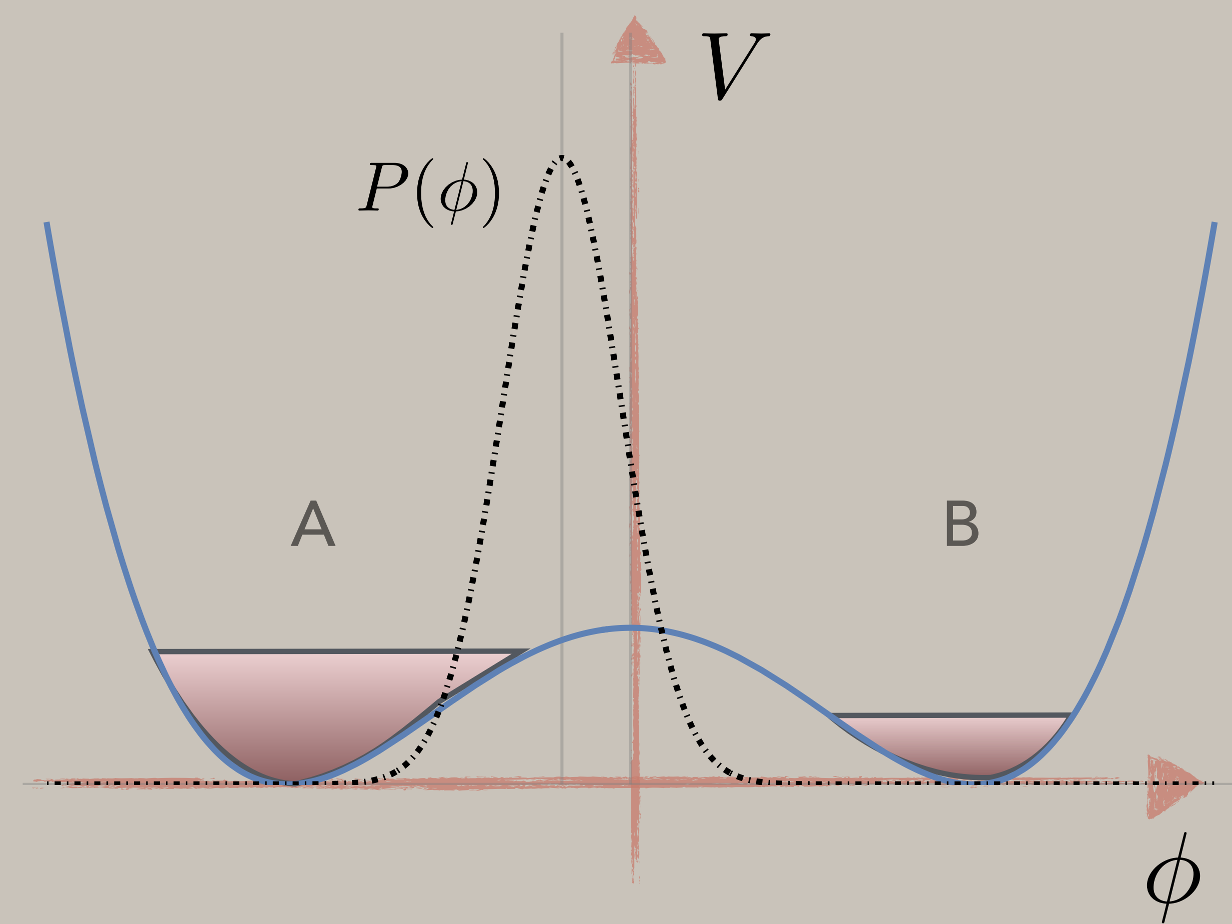
Zeldovich+ 74, Sikivie 82/…



Potential *(pressure)* **bias**

Breaking by Initial Condition

Lalak+93, Coulson+96, Hindmarsh 96, Larsson+97/ …



Population bias

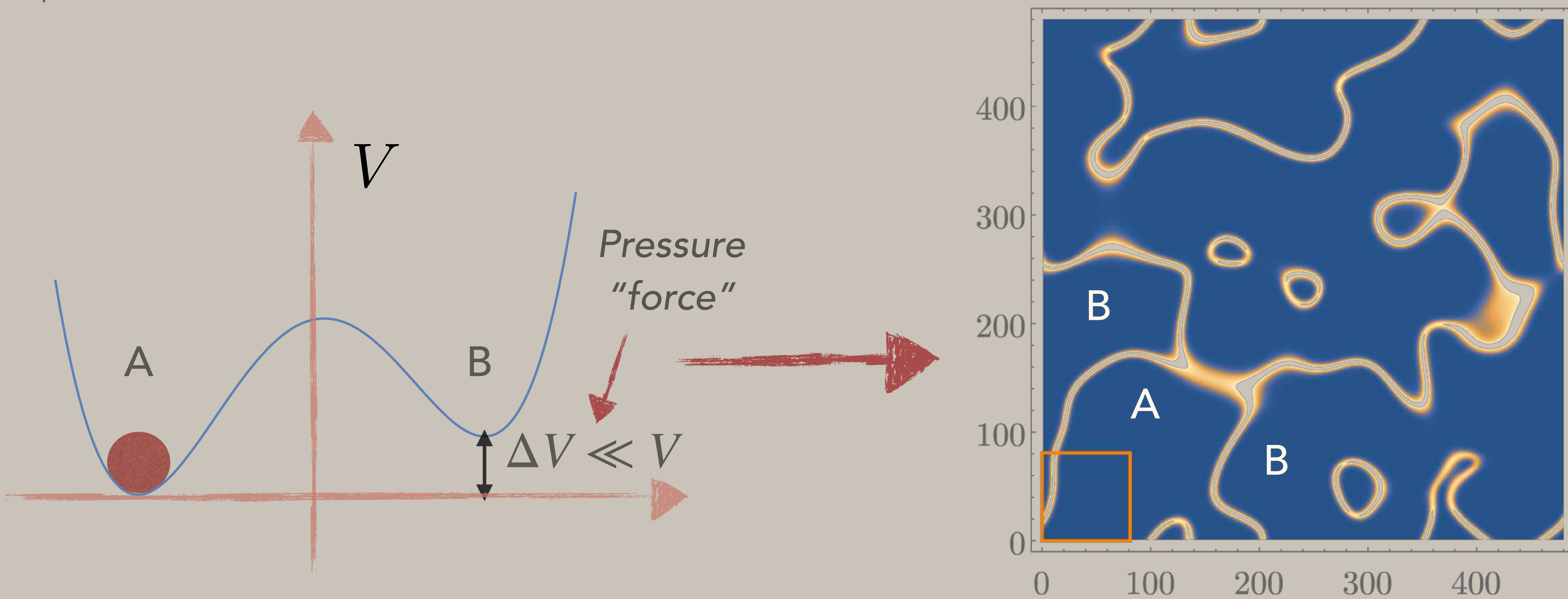
Other possibilities: restoration (Vilenkin 81/…/ Babichev+23), (quantum) nucleation (strongly suppressed)

Potential Bias

Explicit breaking expected for global symmetries

Dine, Seiberg 86/…/ Kamionkowski, March-Russel 92/…/ Svrcek, Witten 06/

For (QCD) axions, can be additional periodic term with larger periodicity due to non-perturbative contributions, UV or IR



*Network annihilates **roughly** once acceleration beats Hubble friction*

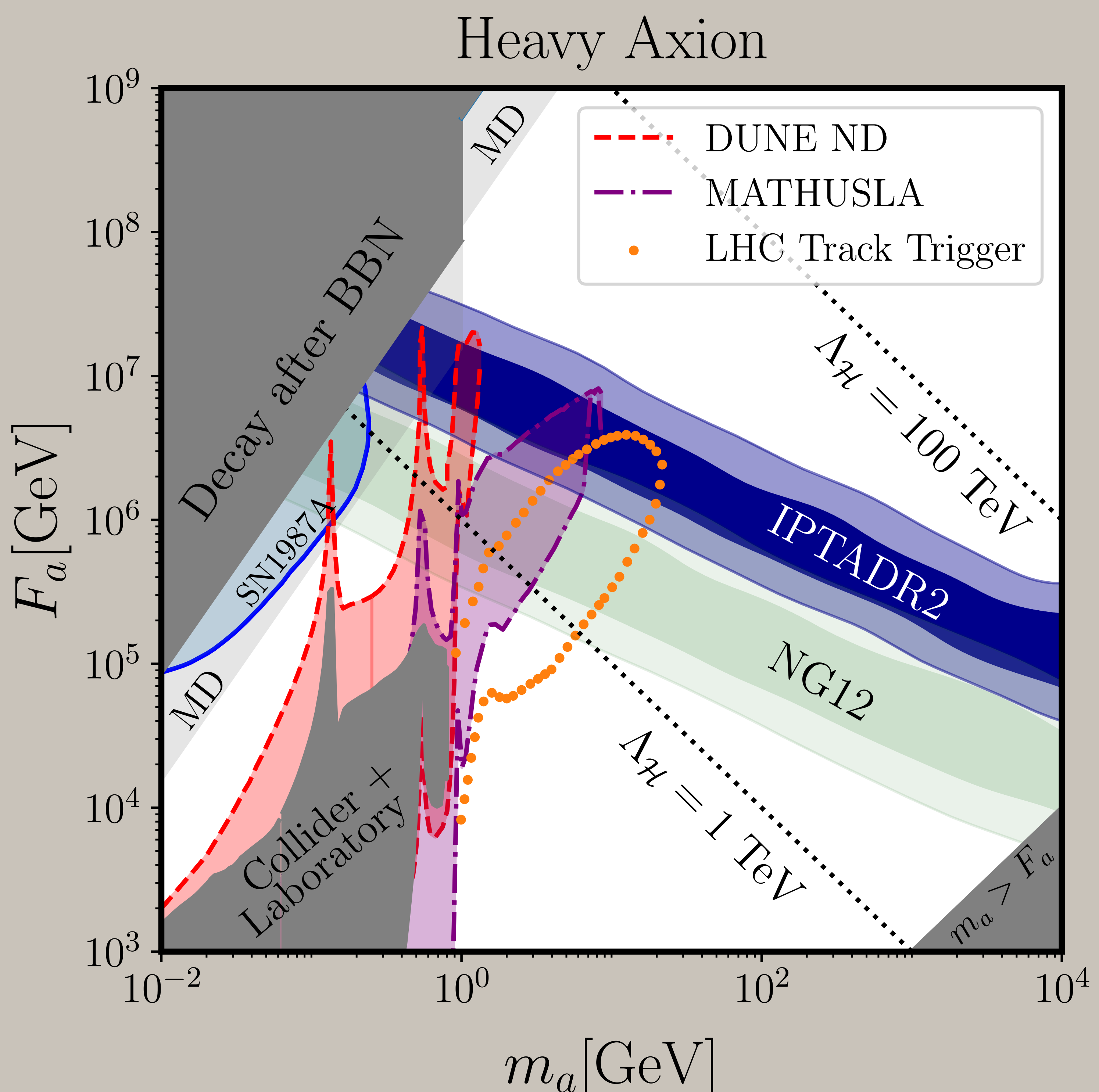
$$\ddot{x} \sim \Delta V/\sigma \gtrsim H(T)$$

***For QCD axion**, generic potential bias causes shift:* $\Delta\theta \sim (\Delta V/m_a^2 f_a^2) < 10^{-10}$

Example: Heavy Axions

Ferreira, Notari, Pujolàs, FR 22

Assuming potential bias,
but using scaling regime spectrum



$$m_a = \frac{\Lambda_H^2}{F_a} \gg \frac{\Lambda_{QCD}^2}{F_a}$$

If coupled to QCD, need model building to align the two contributions

Mirror Dark Sector

Rubakov 97.../ Hook et al 19/...

Small Instantons

Holdom 85/Choi, Kim 98/.../Csáki+ 19
Gherghetta et al 20/...

Heavy confining gauge sector Unified with QCD

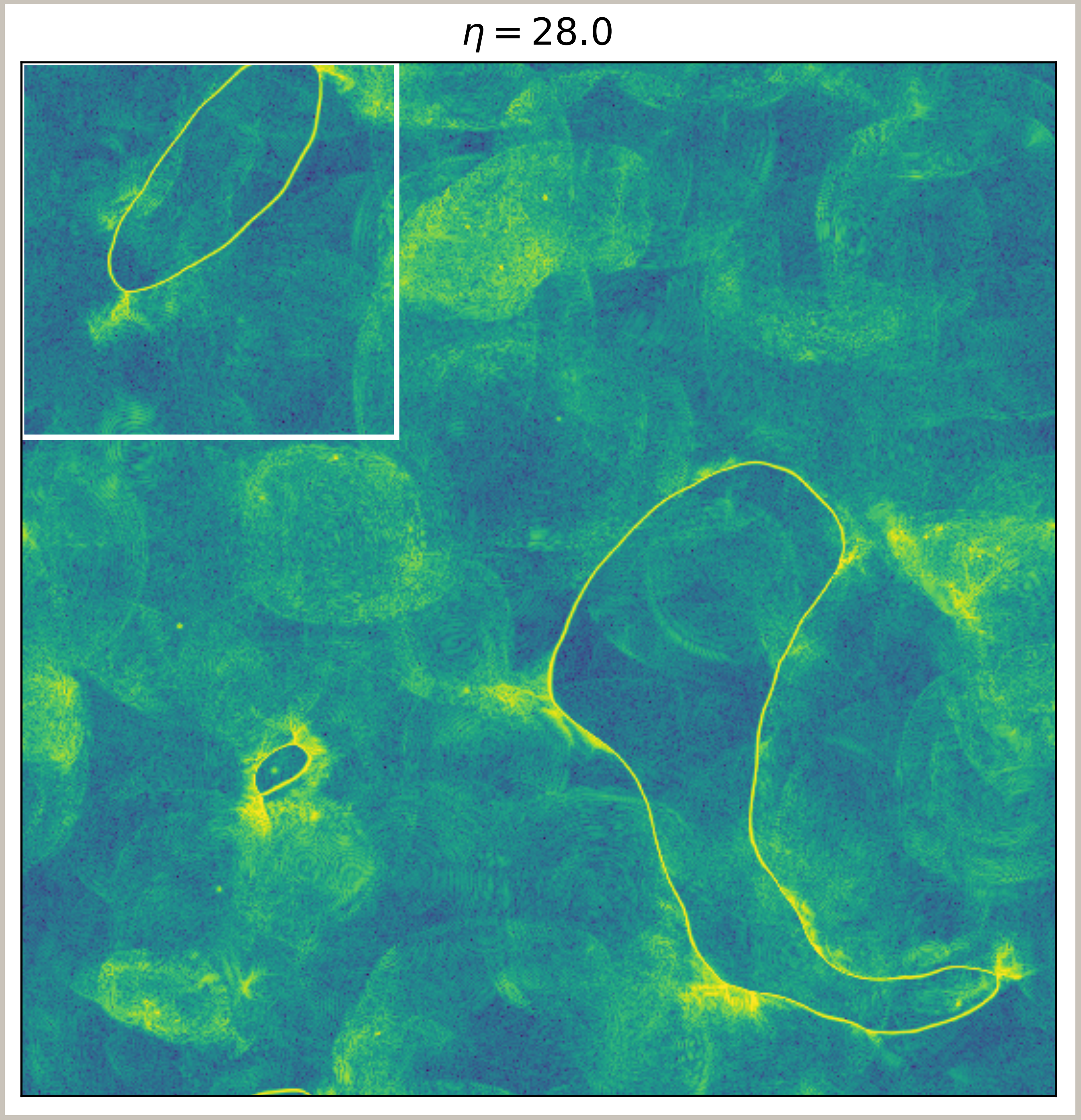
Gherghetta et al 16/...

Beyond instantaneous annihilation

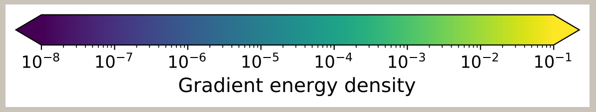
We simulated (for technical simplicity)

$$V = V_{\mathbb{Z}_2} + q\phi^3$$

Notari, FR, Torrenti 25



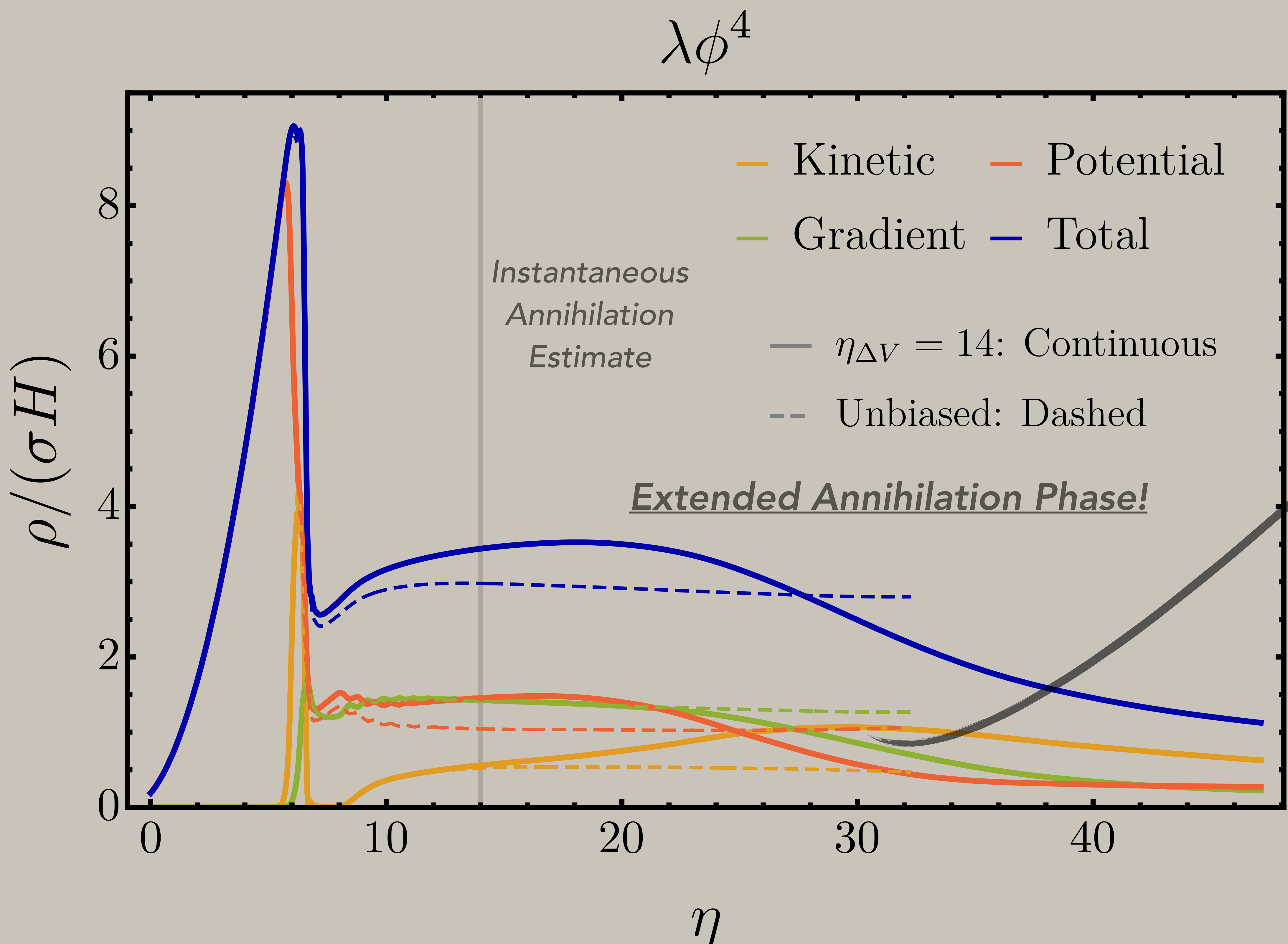
2d slice of gradient energy density (Log scale)



Diagnostics

With bias s.t. annihilation condition is satisfied at $\eta_{\Delta \mathrm{V}} = 14$

In units of inverse mass

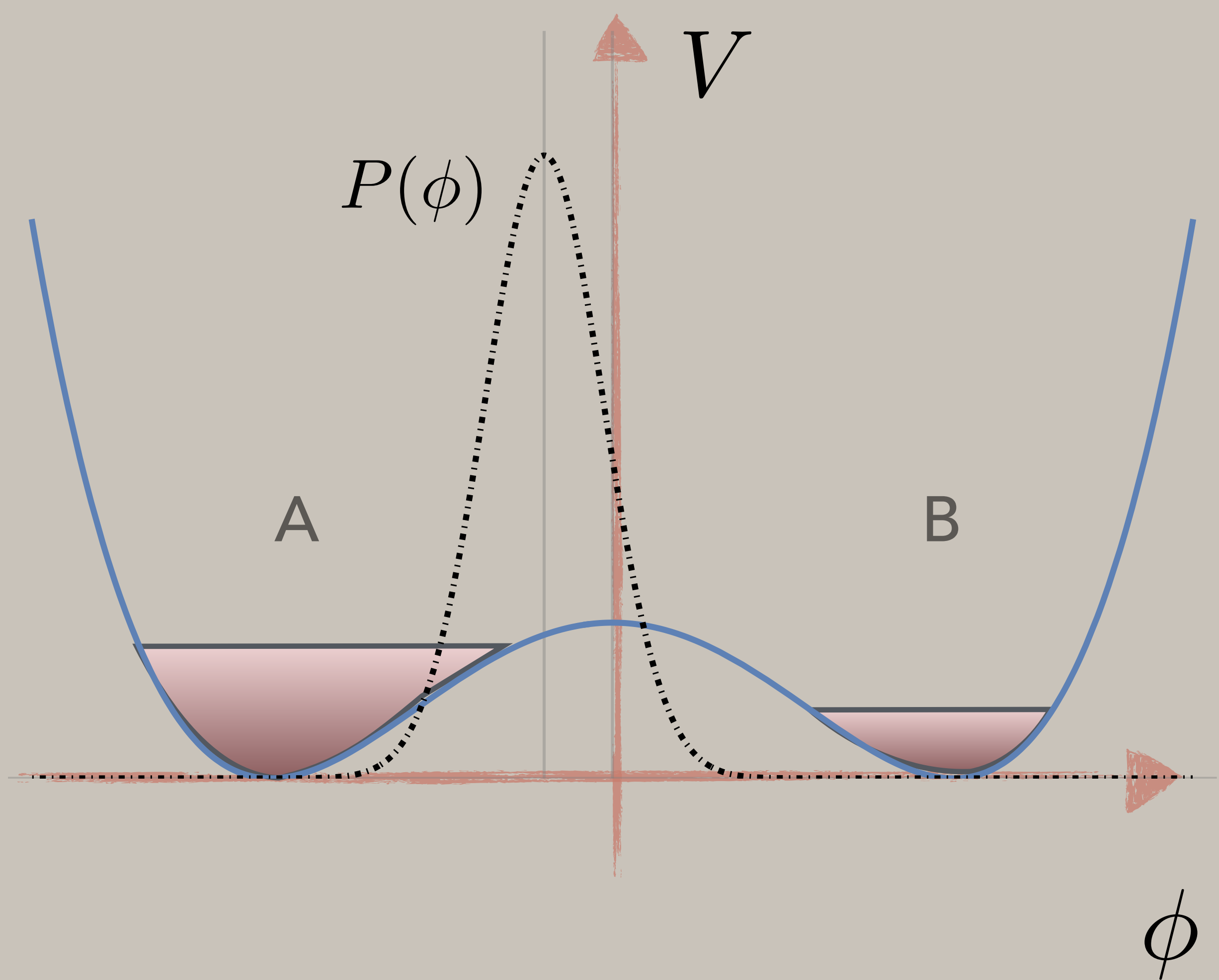


Gradients decrease, but kinetic energy increases!

Barbini, Notari, Pujolàs, FR, Torrenti 26

Population Bias

Alternative when potential bias is too small/forbidden



"False Vacuum" fraction

$$\mathcal{F} \equiv \frac{\mathrm{Vol}_B}{\mathrm{Vol}_A + \mathrm{Vol}_B}$$

Bias parameter

$$\mathcal{B} \equiv \frac{1}{2} - \mathcal{F}$$

In 3D, percolation of "false vacuum" occurs even when bias is relatively large (threshold is away from 0.5, not true in 2D, that most previous works are based on)

Correia+14,18/Krajewski+21/ Gonzalez+22/Kitajima+23

Heuristic picture

In Nambu-Goto picture (thin wall) for simple wall shapes characterized by one length scale

$$R'' + \left(\frac{n}{R} + \frac{3a'}{a}R'\right)\left(1 - R'^2\right) = 0$$

n=0, 1, 2
planar, spherical, cylindrical

Only force acting on walls is due to extrinsic curvature K

For wall networks, reasonable to expect

$\langle K \rangle \approx 0$ *when network is unbiased*

But consider e.g. two spherical shapes

$$R_\pm = R_0 \pm \delta \Rightarrow \mathcal{B} \sim (\Delta K)/(K_+ + K_-) \sim \delta/R_0$$

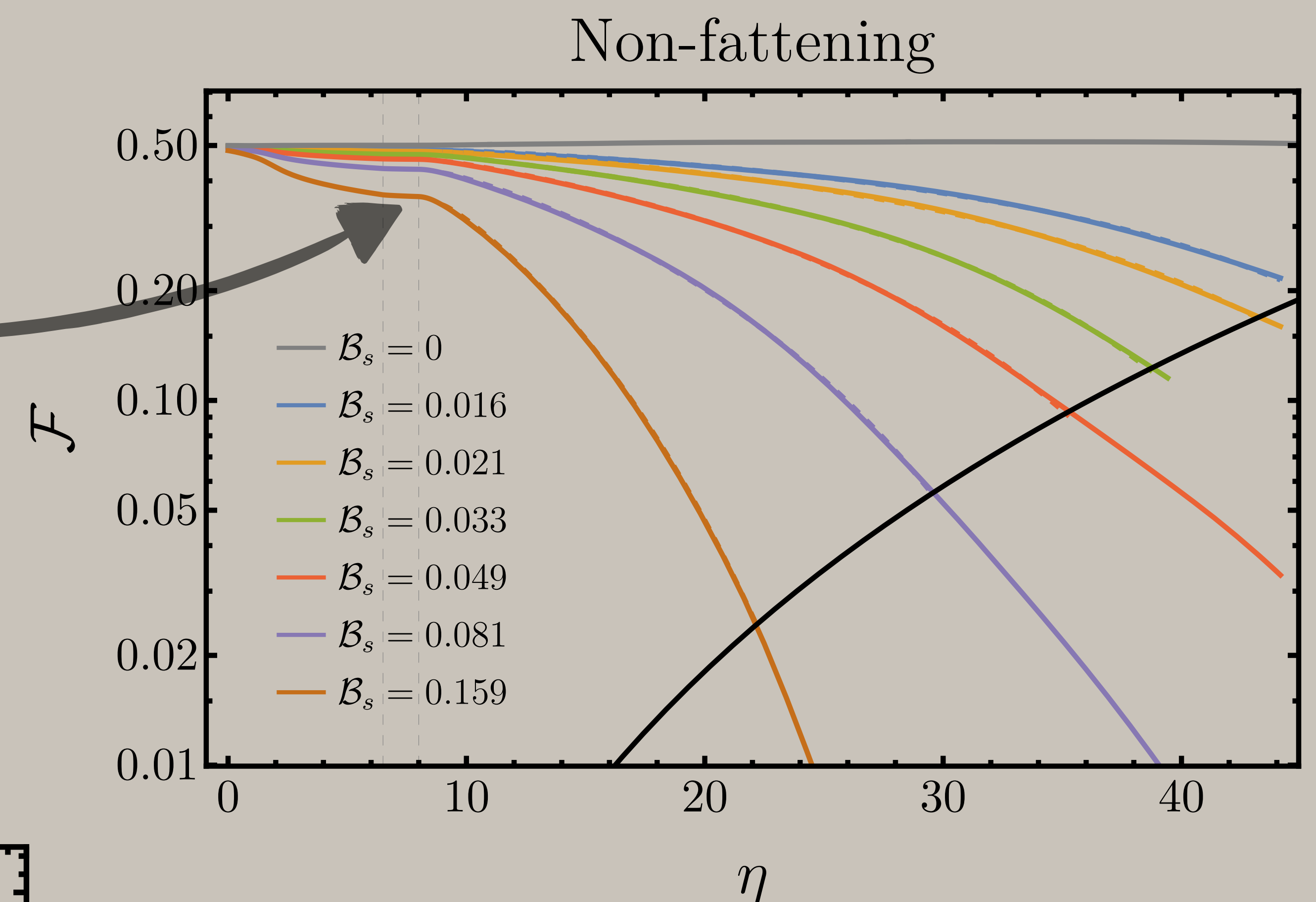
→ *Bias will grow with time* $\mathcal{B} \equiv \mathcal{B}(t)$

Population Bias

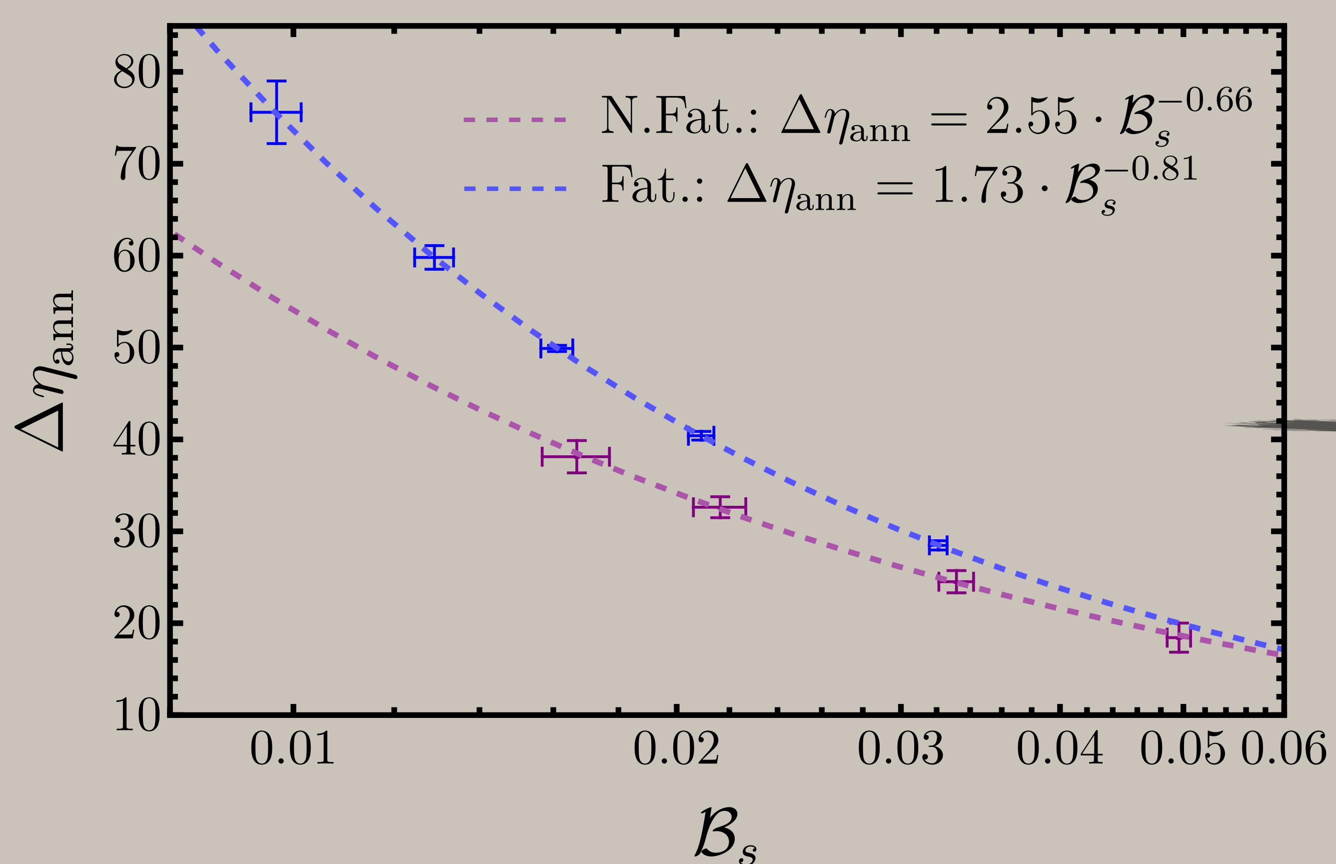
$$\mathcal{B}_s$$

= Initial value of Bias, at the onset of scaling

$$\eta_s$$



$$\mathcal{F} = \mathcal{F}_s \exp\left[-\left(\frac{\Delta\eta}{\Delta\eta_{\mathrm{ann}}}\right)^{\tilde{p}}\right]$$



$$\eta \sim T^{-1}$$

$$T_{\mathrm{ann}} \simeq T_s\ \mathcal{B}_s^{0.8}$$

Gravitational Waves from Annihilating Walls

Signal is dominated by collapsing phase

*See also Kitajima+23/ Ferreira, Pujolàs, Notari, **FR** 24/Cyr+25/Babichev+25*

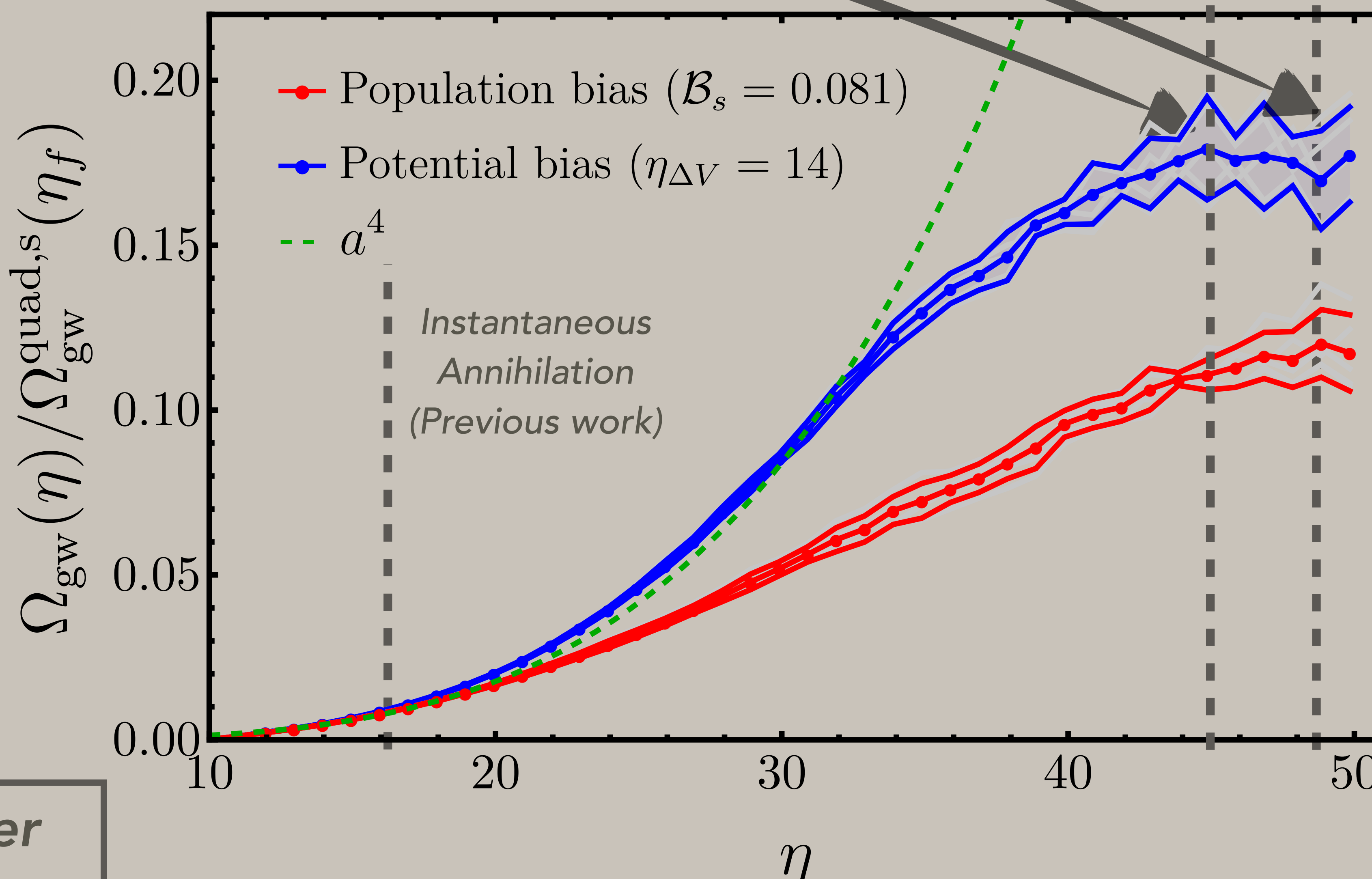
GW emission stops at

$$\eta_{\mathrm{gw}} \approx 3\eta_{\mathrm{ann}}$$

$$\Rightarrow T_{\mathrm{gw}} \approx 3T_{\mathrm{ann}}$$

$$\downarrow$$

$$\Omega_{\mathrm{gw}} \propto \alpha_{\mathrm{gw}}^2 \propto T_{\mathrm{gw}}^4$$



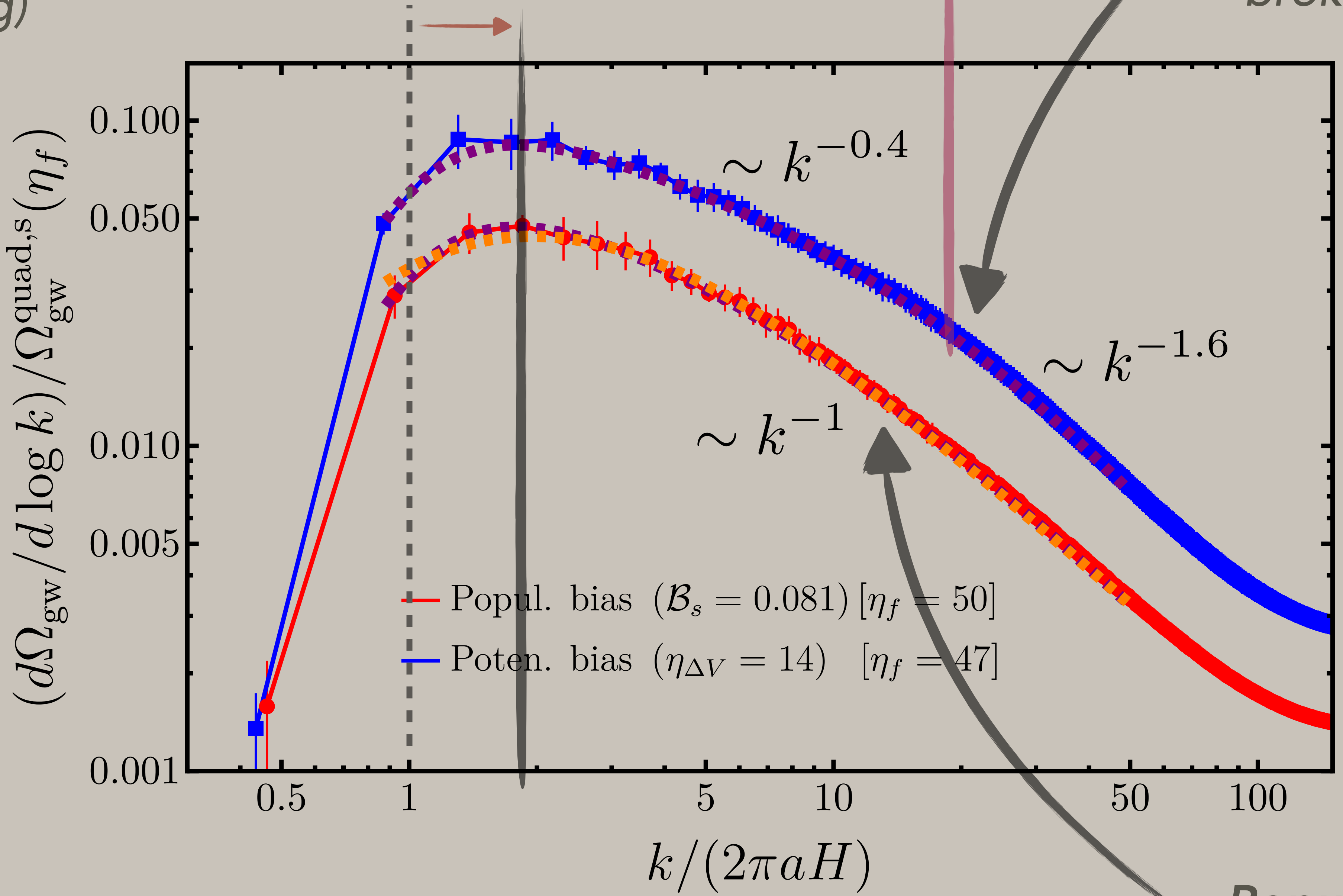
$$\Omega_{\mathrm{gw}}^{\mathrm{quad,s}} \equiv \frac{3}{32\pi}\left(\frac{2\sigma H(\eta)}{\rho_c(\eta)}\right)^2$$

Amplitude is ~80 times larger Than in instantaneous approx

Spectral Shape

Shift of the peak (Roughly twice as large as in scaling)

Potential Bias: *Change of UV shape, double broken power law*

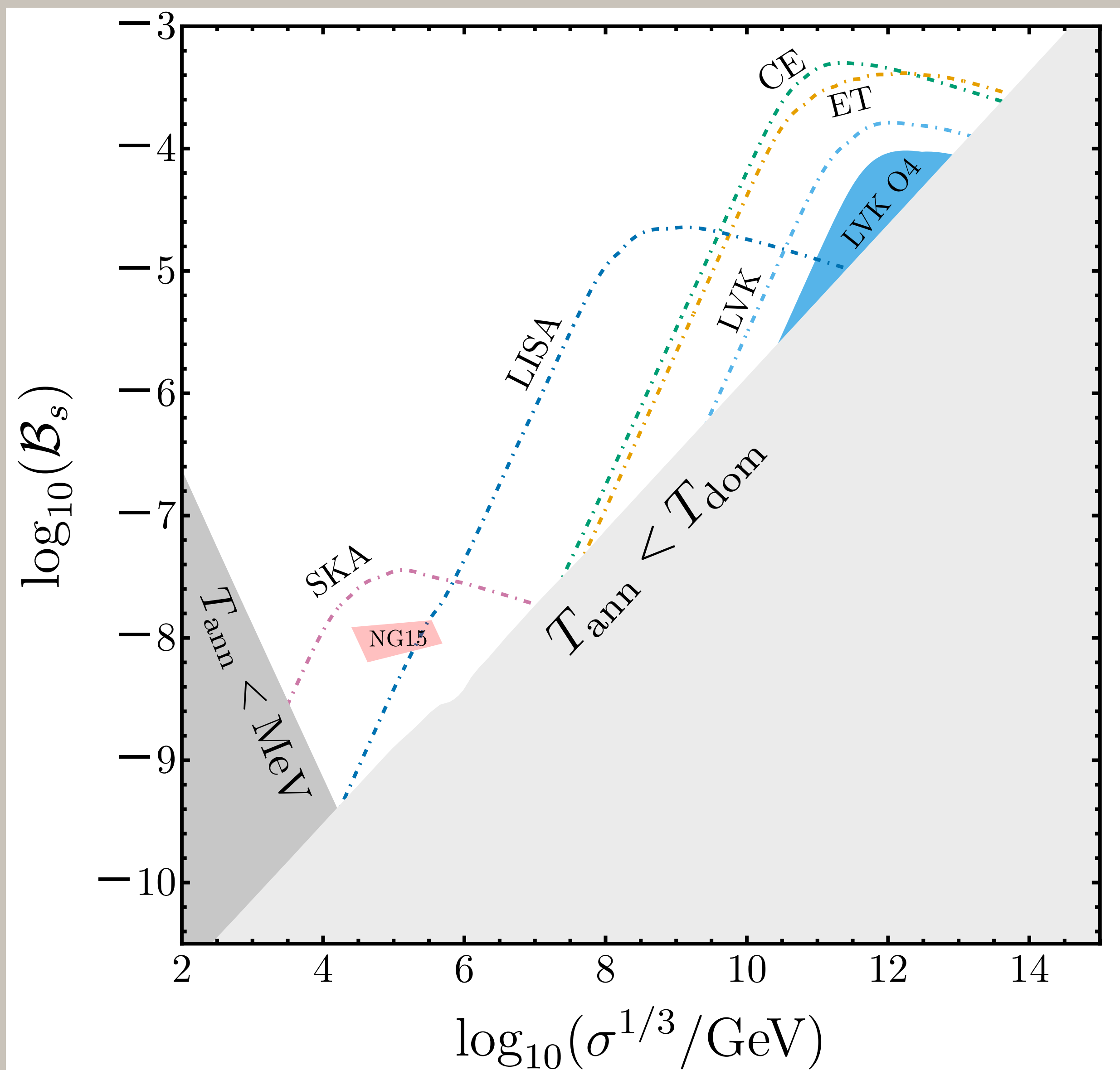


Population Bias: *Single broken power law*

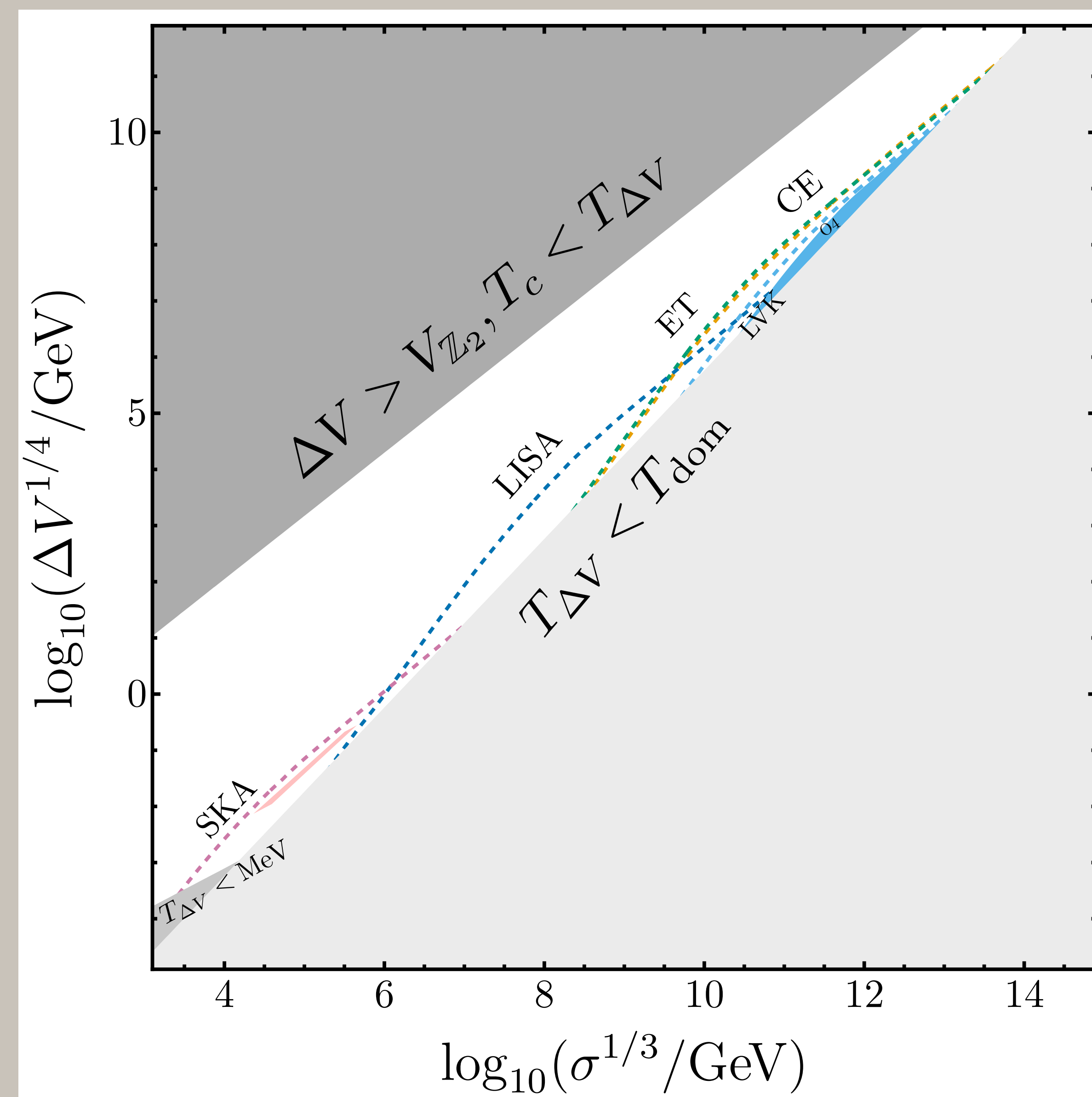
Similar results for cosine potential, time-dependent bias

Implication for GW observations

Expected to be (roughly) valid also for (axionic) string-wall networks



Population Bias



Potential Bias

Conclusions

Domain walls are closely tied to solutions to the strong CP problem (Both PQ and CP, parity) and to more general axions

They are also among the loudest sources of cosmological GWs, thanks to scaling regime

Detailed studies of possible annihilation mechanisms are crucial to:

Assess viability of (spontaneously broken) discrete symmetries

Infer abundance of relics from annihilation, e.g. Gravitational Waves

Conclusions

Population bias effective in annihilating networks where potential bias is too small or forbidden (gauge)!

Lattice field theory allows to obtain detailed description of annihilation from highly non-linear evolution, and GW signal

In both mechanisms, GW emission extends well beyond naive estimates, amplitude is more than one order of magnitude larger and spectrum is distorted

In progress

Origin of population bias (from e.g. inflation)

Other relics: Dark Matter, Primordial Black Holes from DW collapse

Thank you!