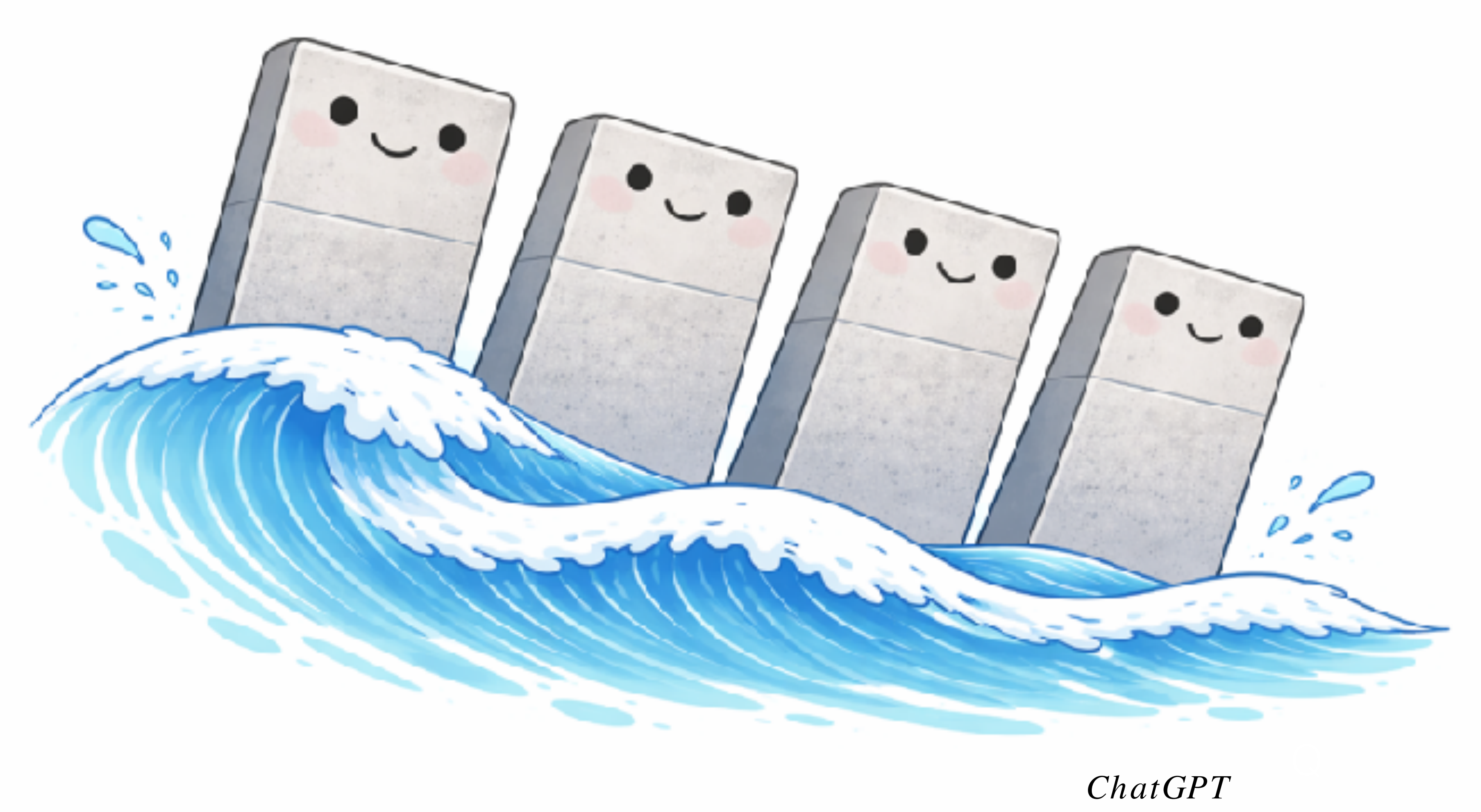
Revisiting Axion Dark Matter with Nonlinear Transition : Caveats and Implications



TOKYO METROPOLITAN UNIVERSITY
東京都立大学

Based on Narita and WY, 2507.03157,
Miyazaki, Yaginuma, Song, Narita, and WY, 2509.13292

Wen Yin (Tokyo Metropolitan University)

@2026/7/29, Axions in Seoul.

Generic limit of ALP DM from misalignment

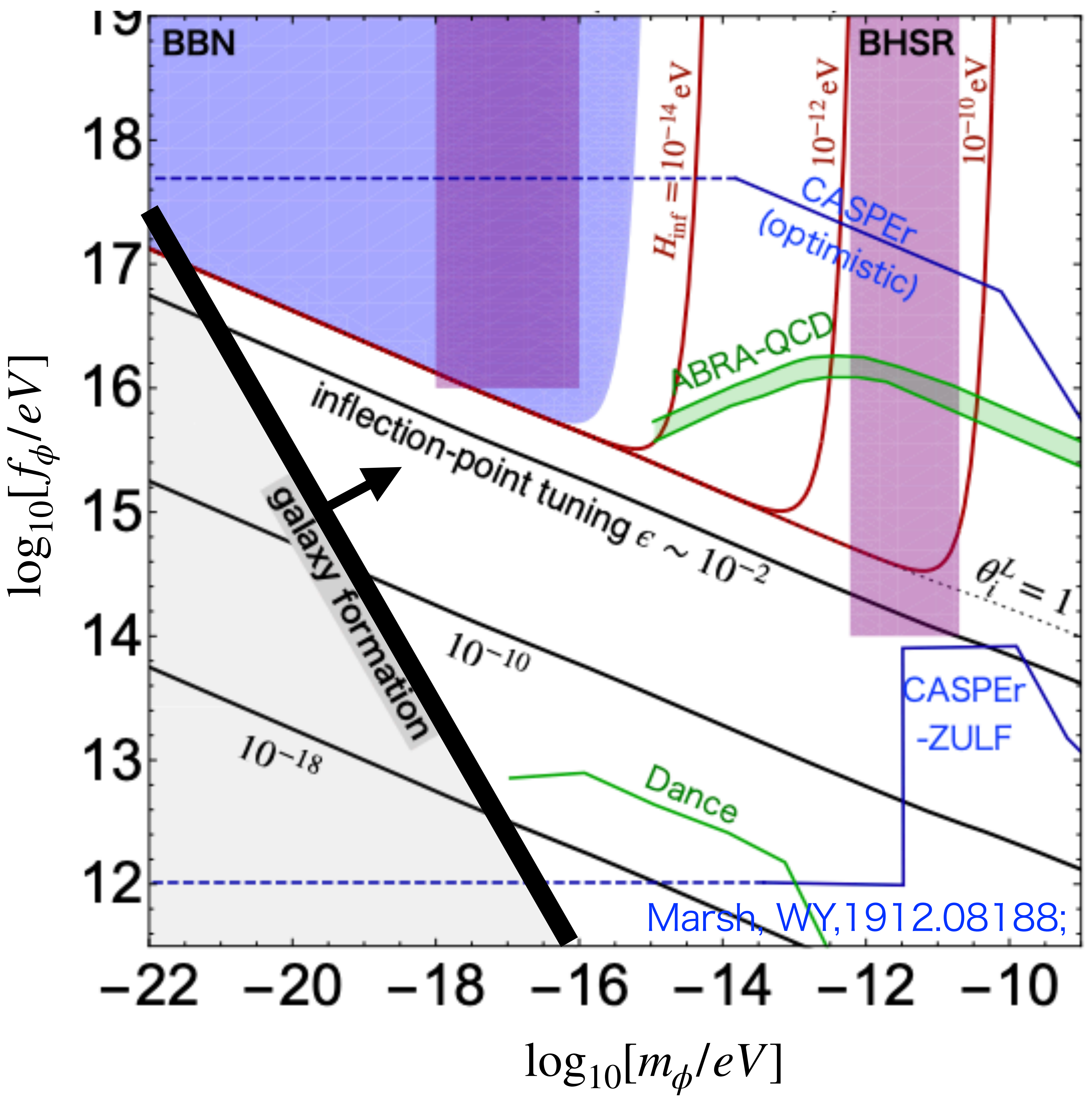
Marsh, WY,1912.08188;

A follow-up of Jihn E. Kim and David J.E. Marsh 1510.01701

See also Dror, Leedom, 2008.02279

- Let's take the onset of oscillation period, $z_{\rm osc}$, free.
- $z_{\rm osc} \gtrsim 10^6$, due to the Lyman-α limit: $\rho_\phi \leq m_\phi^2 f_\phi^2$ at z = 10^6
- Taking $\rho_\phi = \rho_{\rm DM}^0 (1+z)^3$,

$$f_\phi \gtrsim 4\times 10^{13} GeV \frac{10^{-18}{\rm eV}}{m_\phi}$$



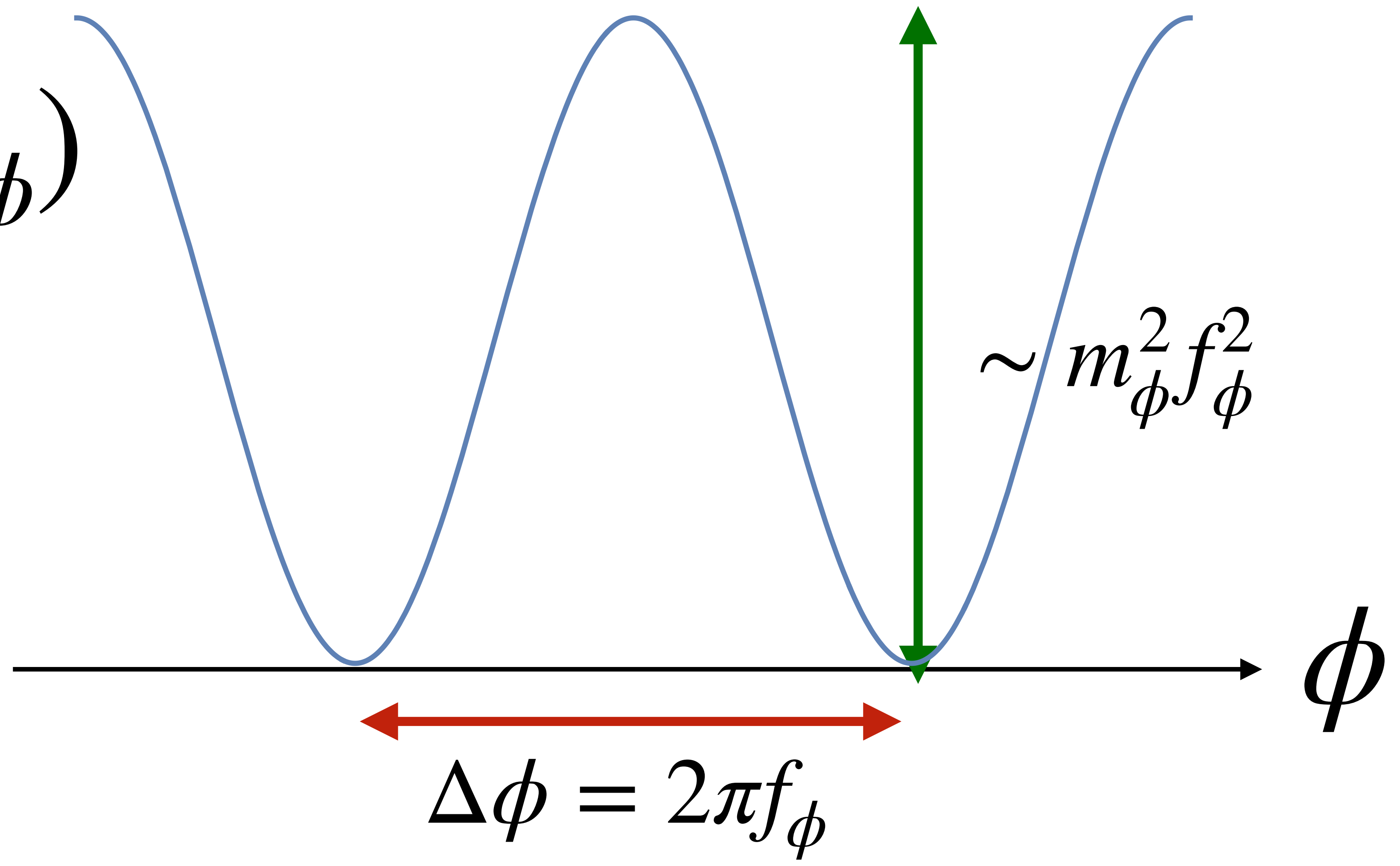
Roadmap

- 1. Introduction
- 2. A nonlinear transition
- 3. Implications of the nonlinear transition
- 4. Conclusions

Definition of axion/axion-like particle(ALP)

Axion has a periodic field space satisfying $\phi \leftrightarrow \phi + 2\pi f_\phi$,
and an approximate shift symmetry, $\phi \leftrightarrow \phi + C$.

$$V(\phi) = V(\phi + 2\pi f_\phi)$$



- Axion $\neq$ QCD axion
- $\phi \leftrightarrow \phi + 2\pi f_\phi$ can be realized by multi-branch potential from the hidden QCD.

Tunneling and 1st order phase transition of zero temperature hidden QCD can be induced. Sugeno, WY, 2511.19429

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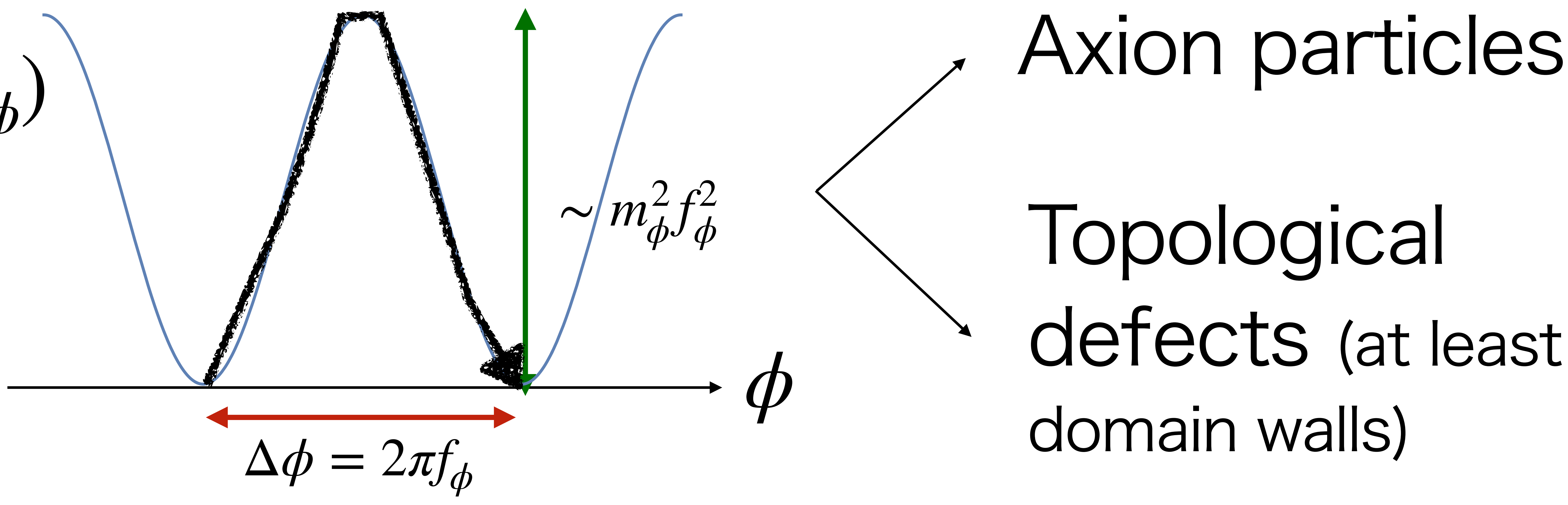

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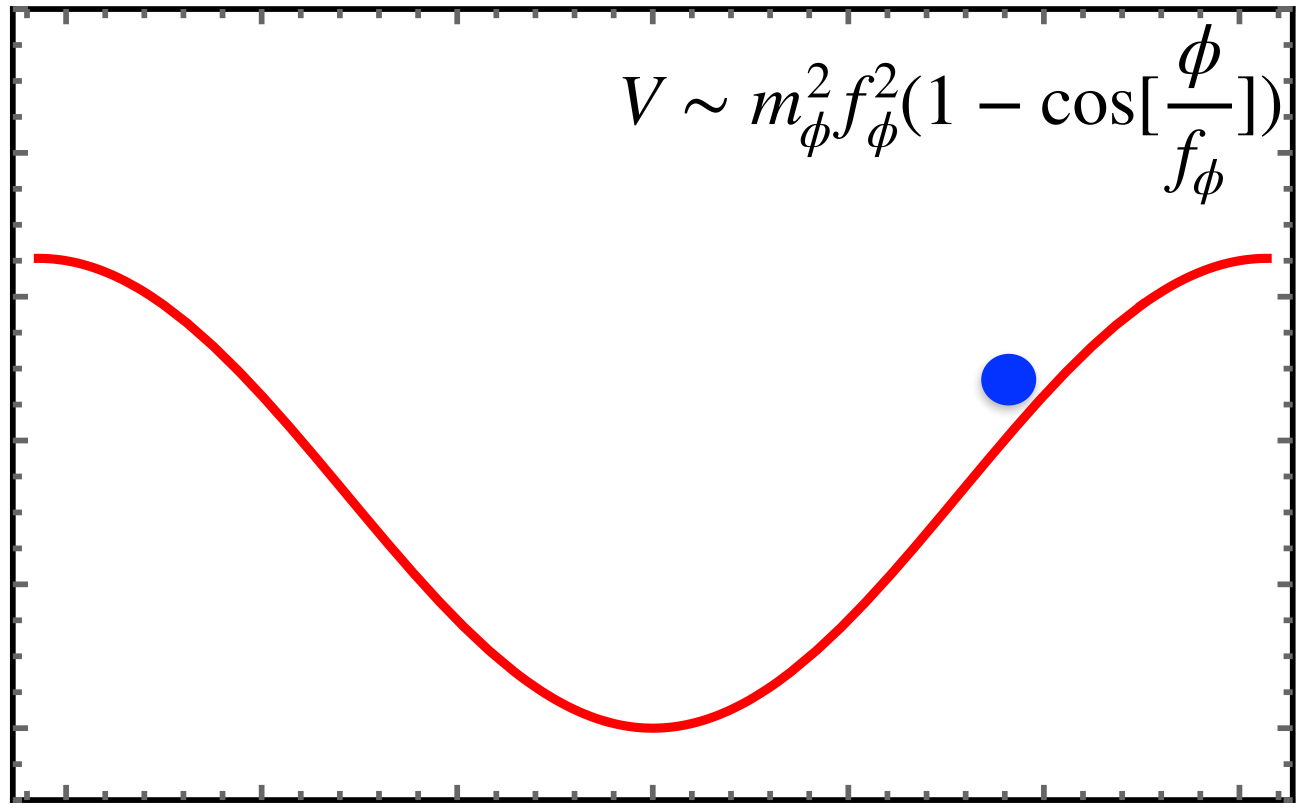
Axion dark matter can be produced through many mechanisms

Produced by itself,

(misalignment)



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etc

By others (nonzero mode production)

See talks by Giovanni and Fabrizio

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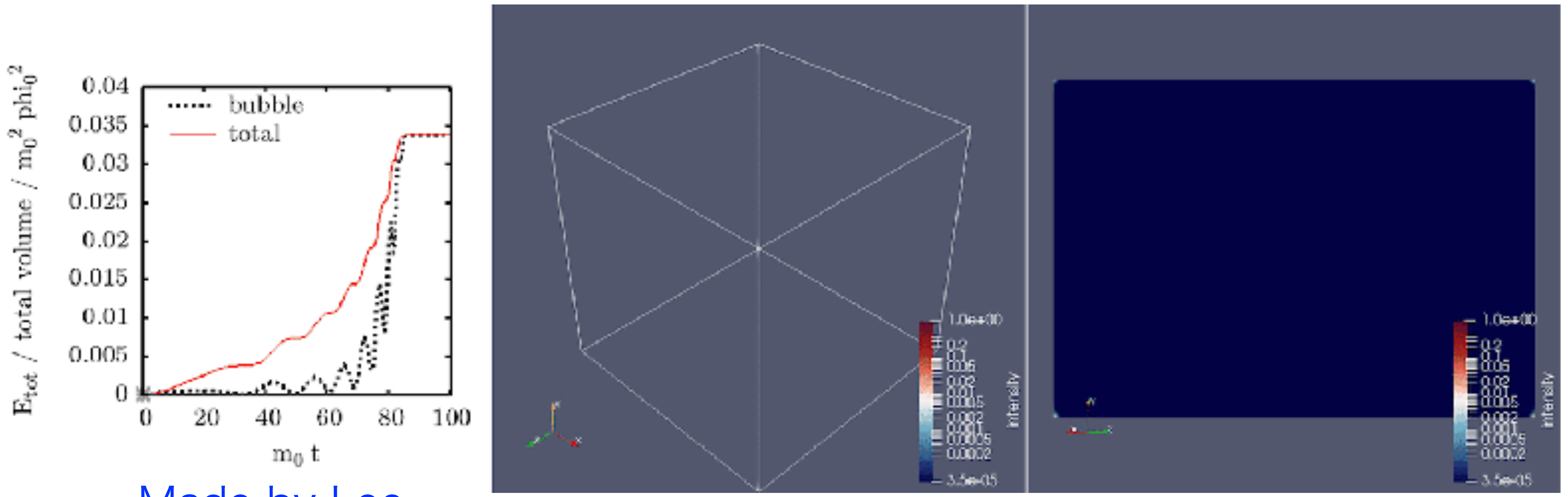

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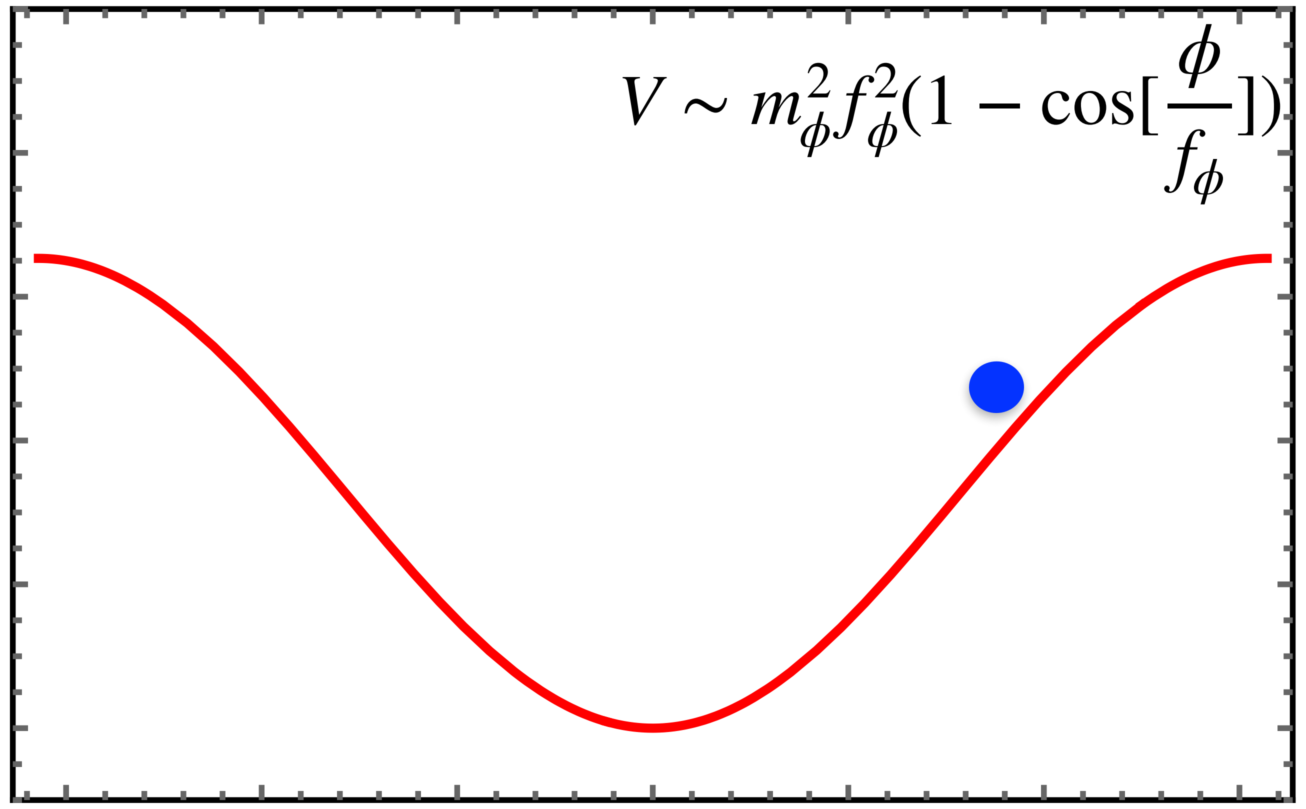

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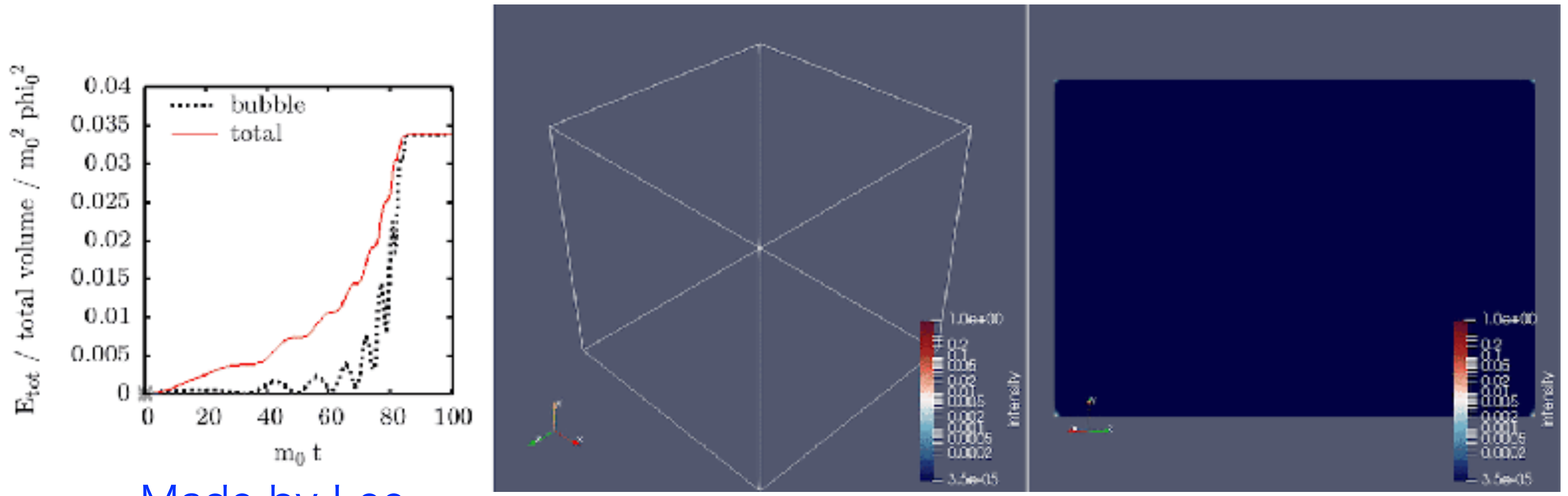

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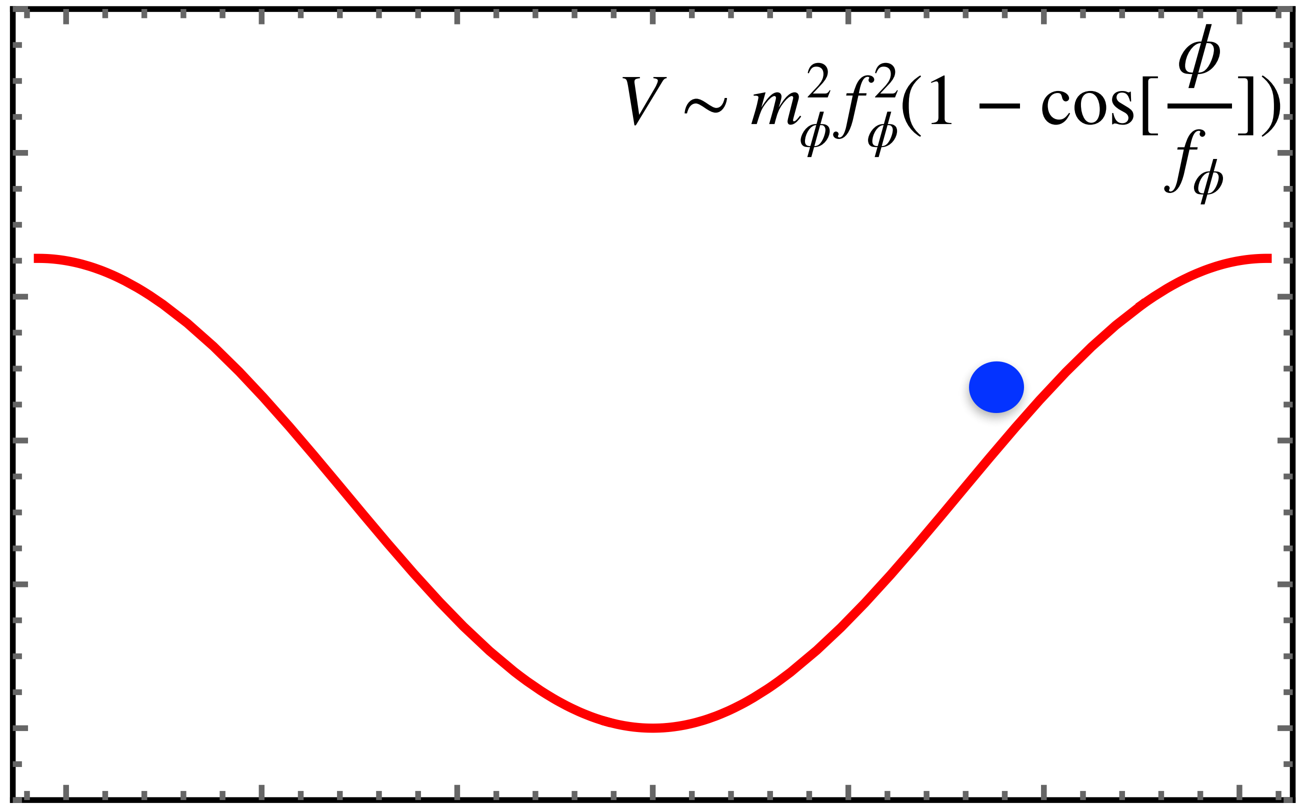

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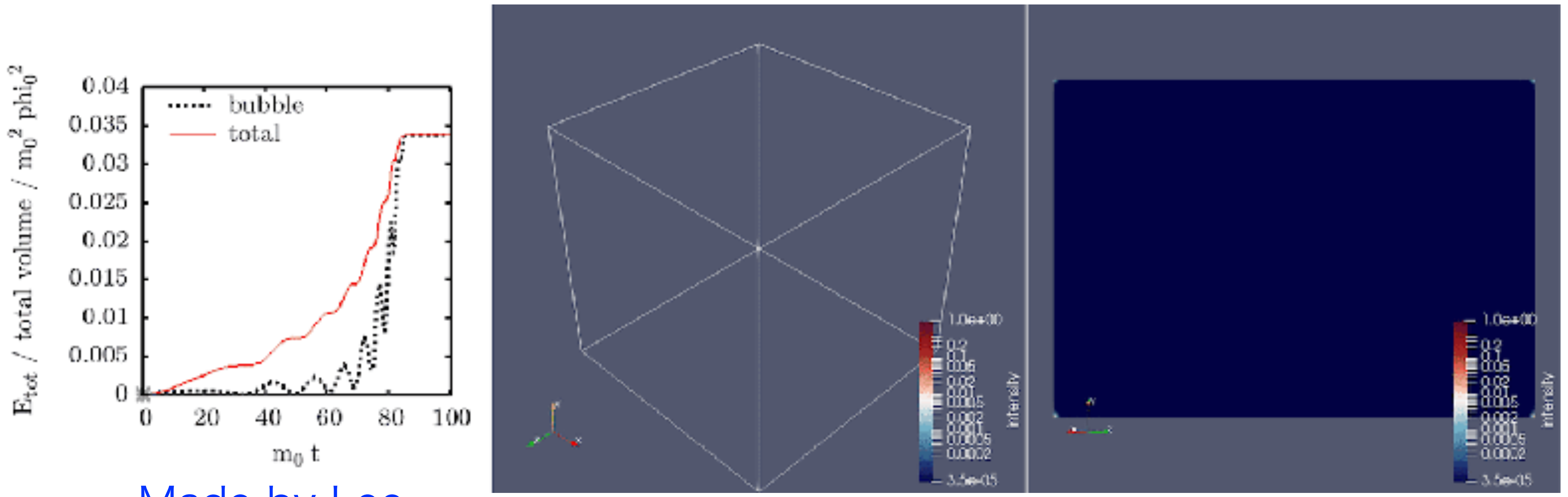

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Generic description of dark matter evolution

(in radiation dominated universe):

@$t \gtrsim t_{\rm prod}$

Number conservation: $$\frac{n_\phi^{\rm prod}}{s_{\rm prod}}$$

Momentum conservation: $$\langle p_\phi^{\rm prod}\rangle \times a_{\rm prod}$$

→

@late time

$$\Omega_\phi = \frac{s_0}{\rho_c} m_\phi \frac{n_\phi^{\rm prod}}{s_{\rm prod}}$$

$$\frac{\langle p_\phi^{\rm prod}\rangle}{a_{Ly\alpha}} \times a_{\rm prod} < m_\phi$$

$a_{Ly\alpha} \sim 10^{-6}$,

For light axion DM, @$t_{\rm prod}$ we need large $n_\phi^{\rm prod}$ and small $p_\phi^{\rm prod}$

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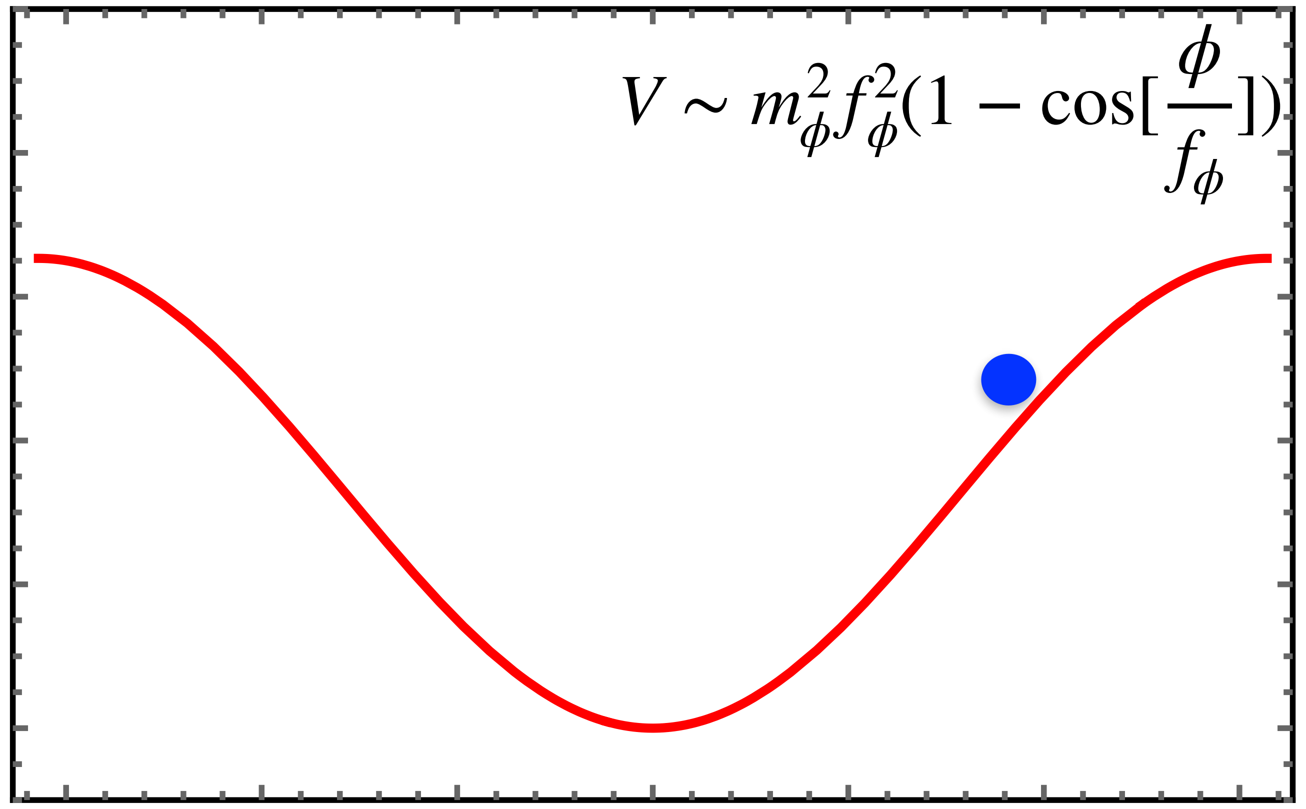

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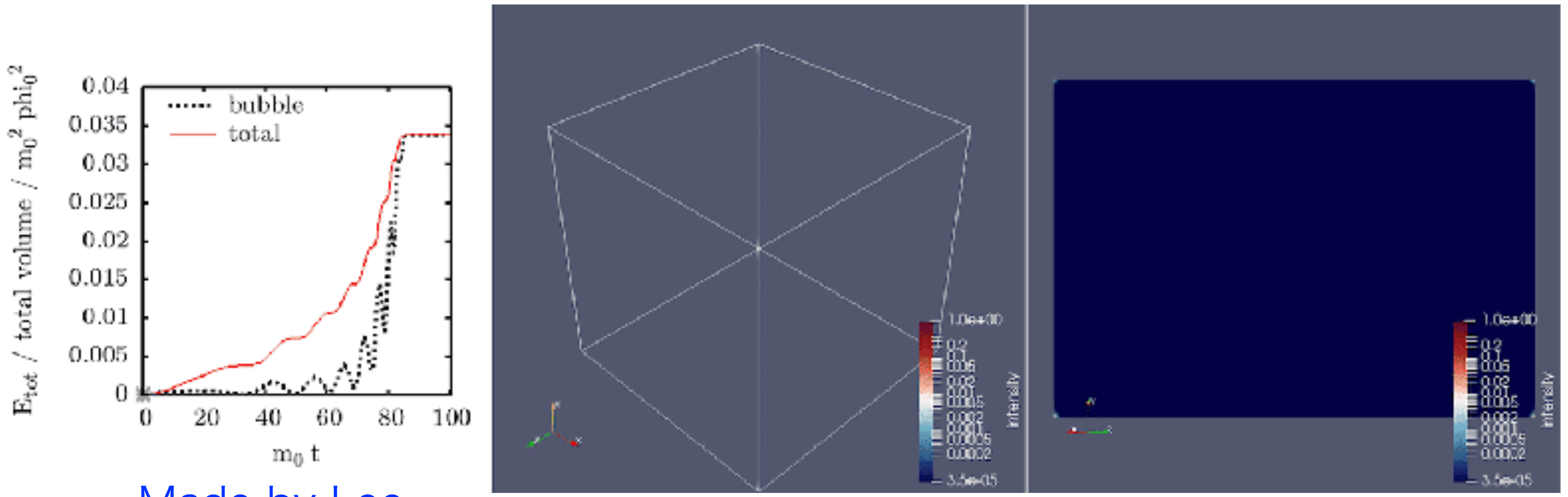

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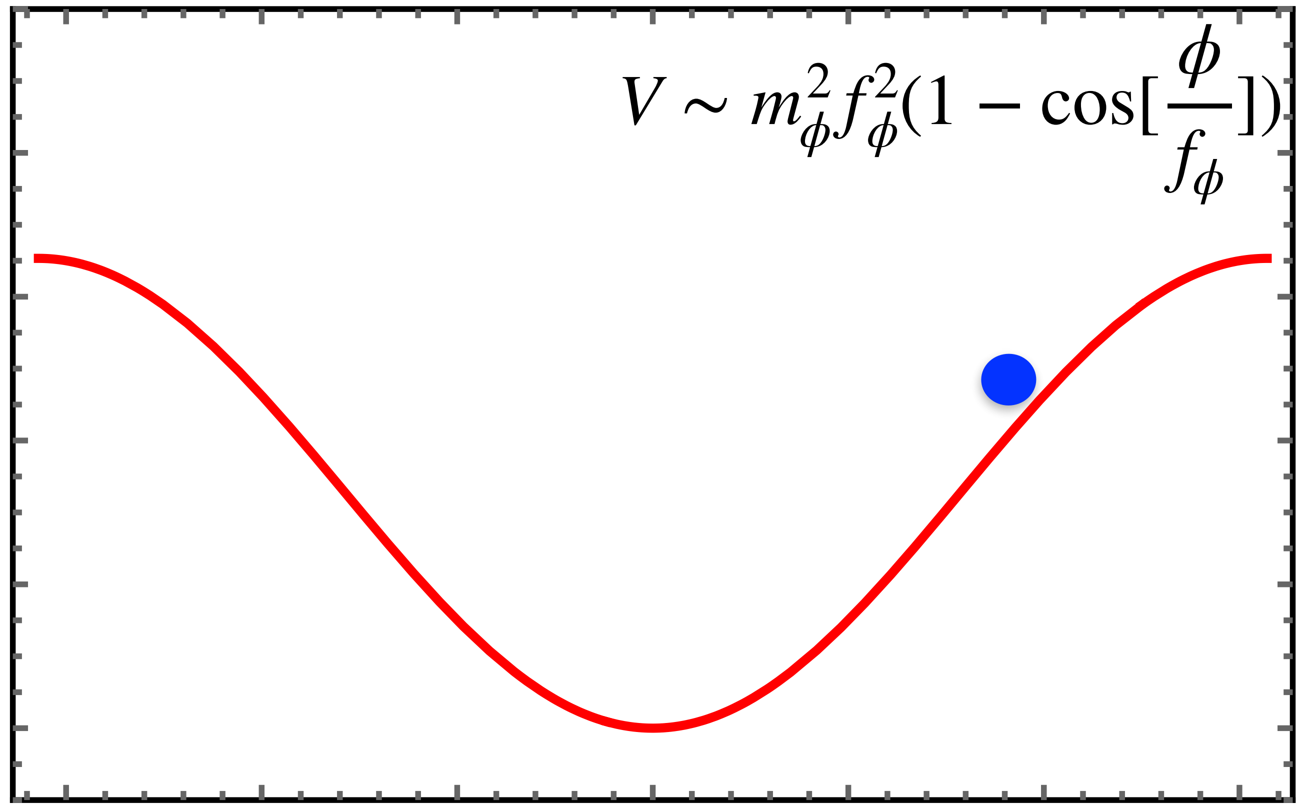

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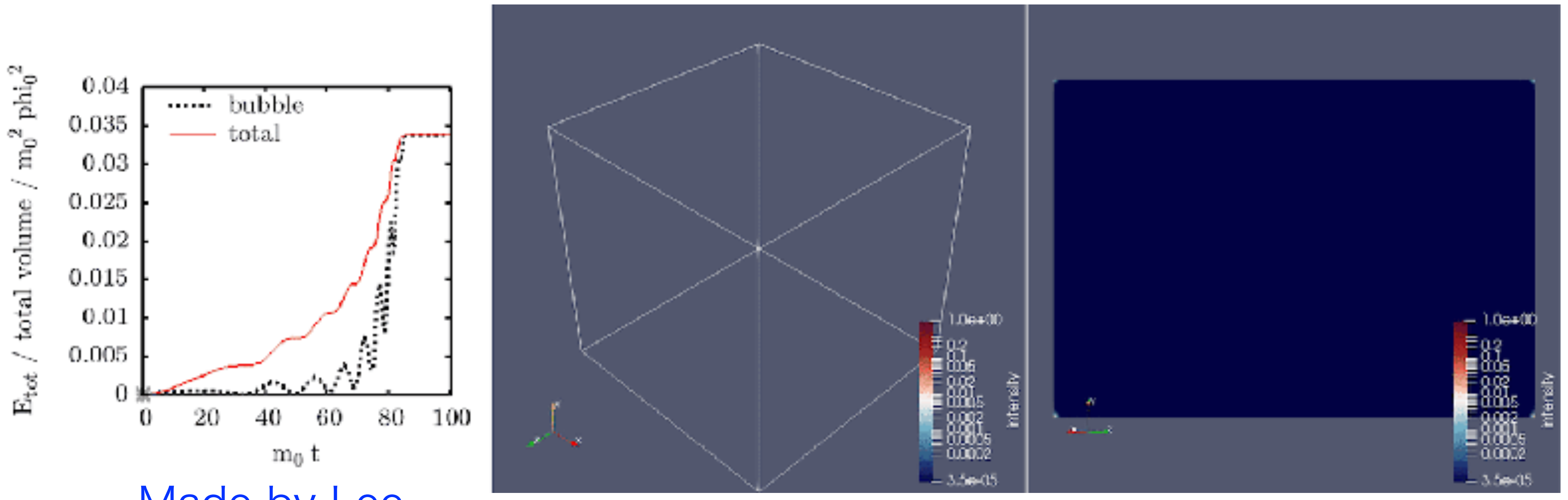

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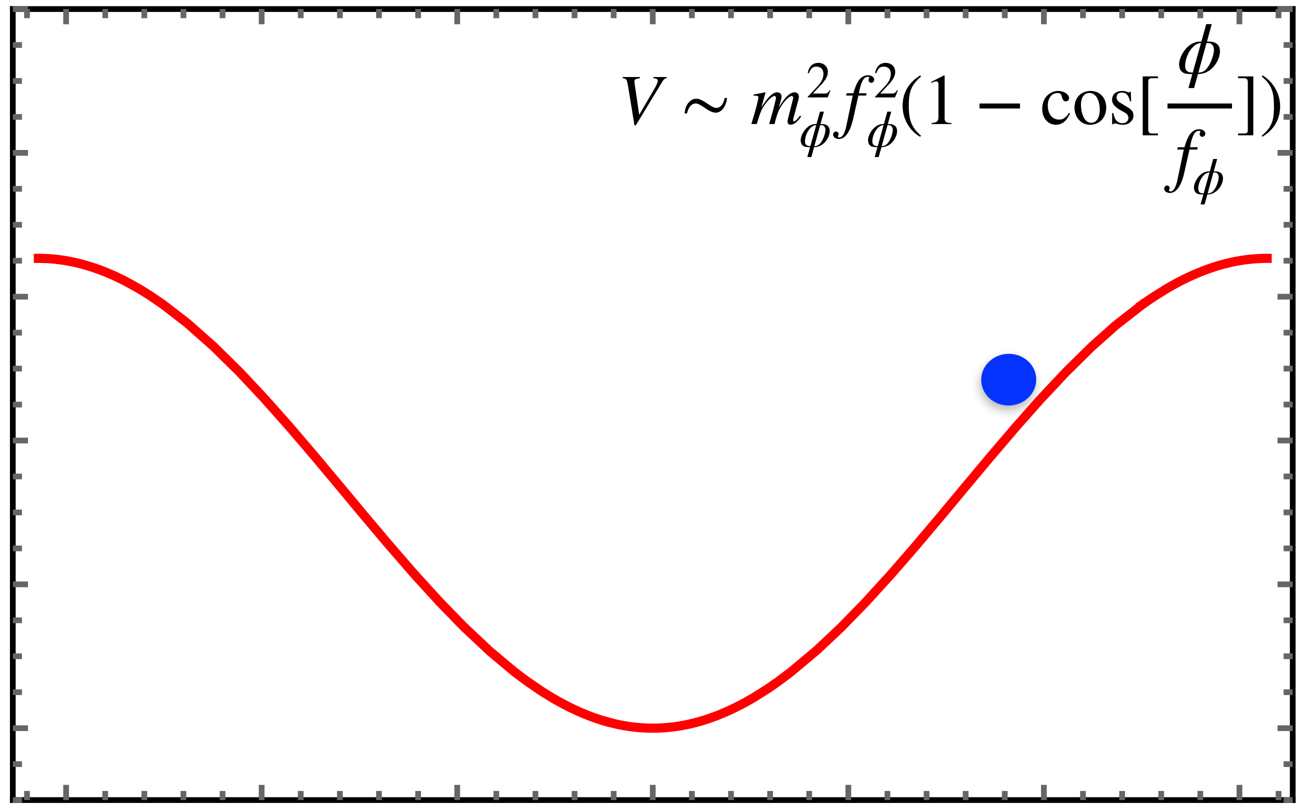

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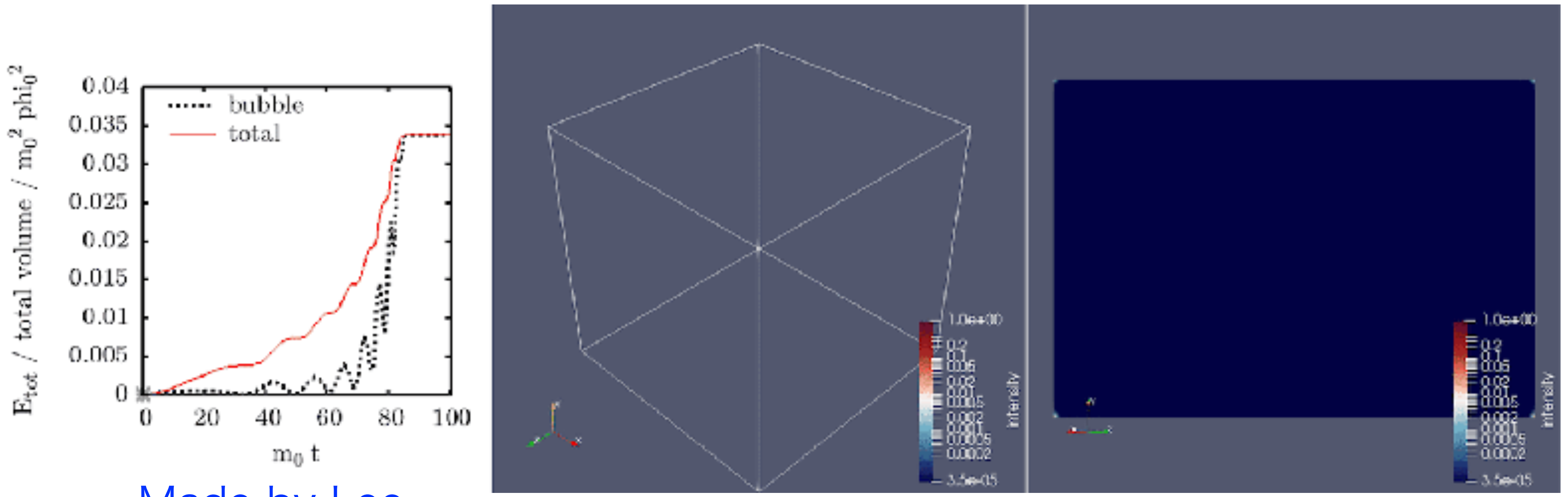

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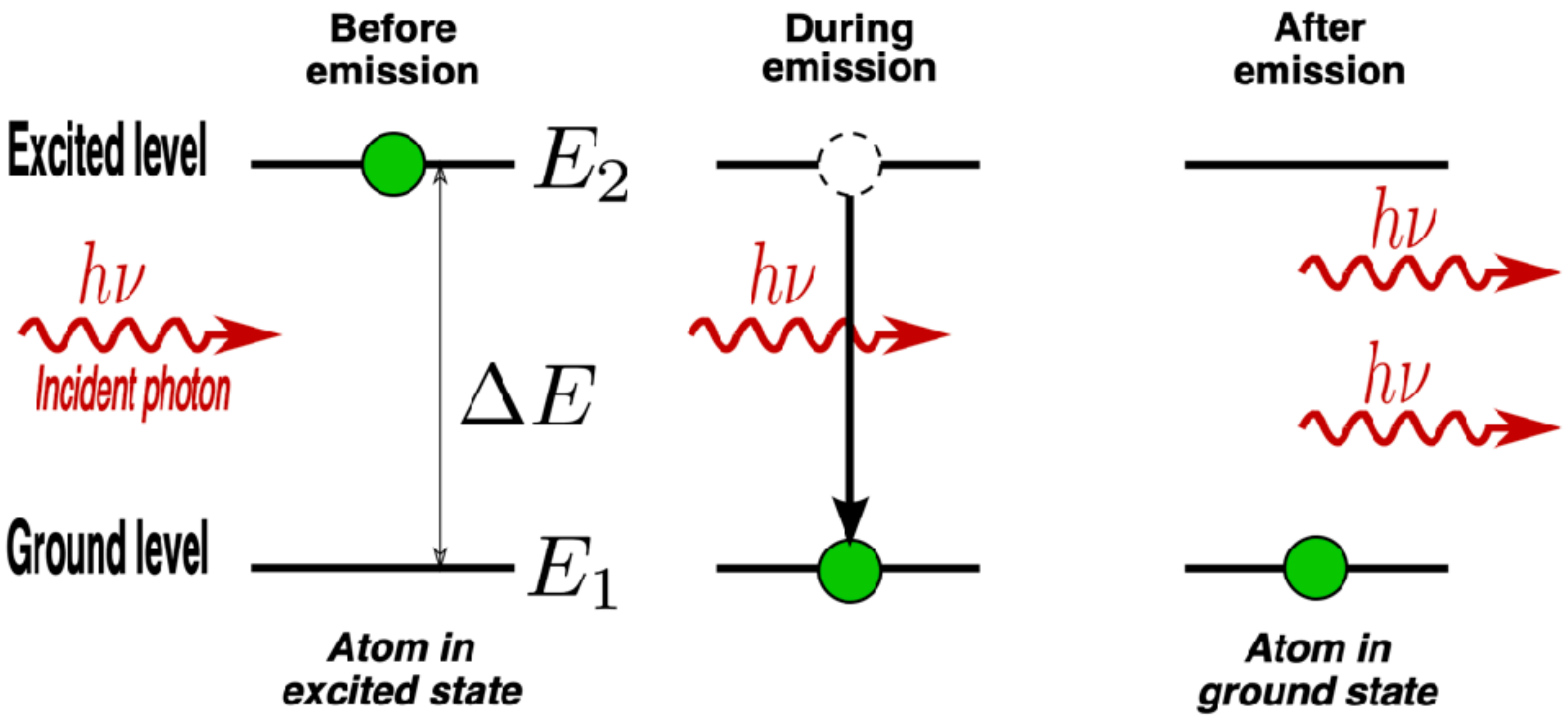
Example: Stimulated decay can produce bosonic dark matter

Moroi, WY, 2011.09475 ,
See also 2011.12285 for the relation to narrow parametric resonance

$$\Phi \to \mathrm{DMDM}$$

Laser, wikipedia



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$$\bar{f} \equiv \left. \frac{4\pi^2 \Gamma n_\Phi}{H P_{\mathrm{DM}}^3} \right|_{t=t_{k=P_{\mathrm{DM}}}}$$

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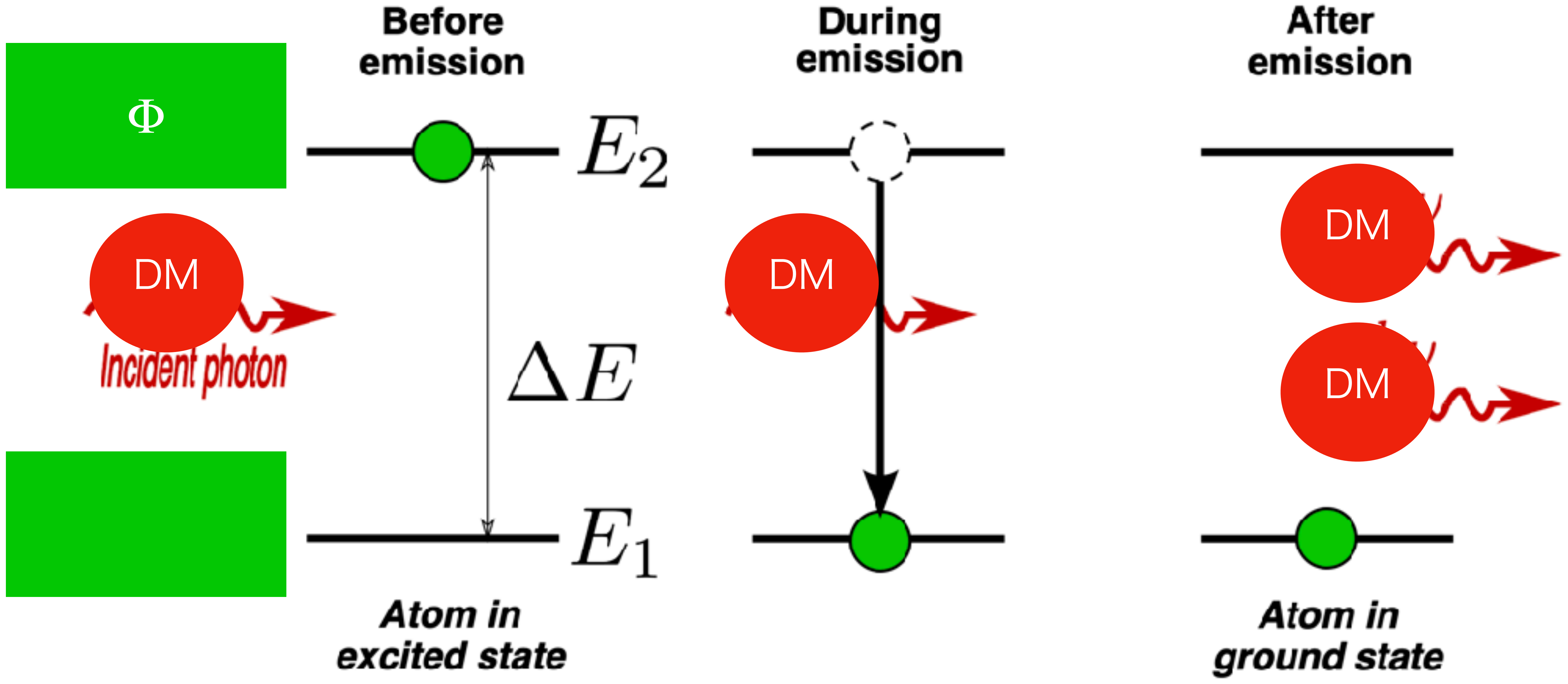

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Inflaton decay produces light dark matter

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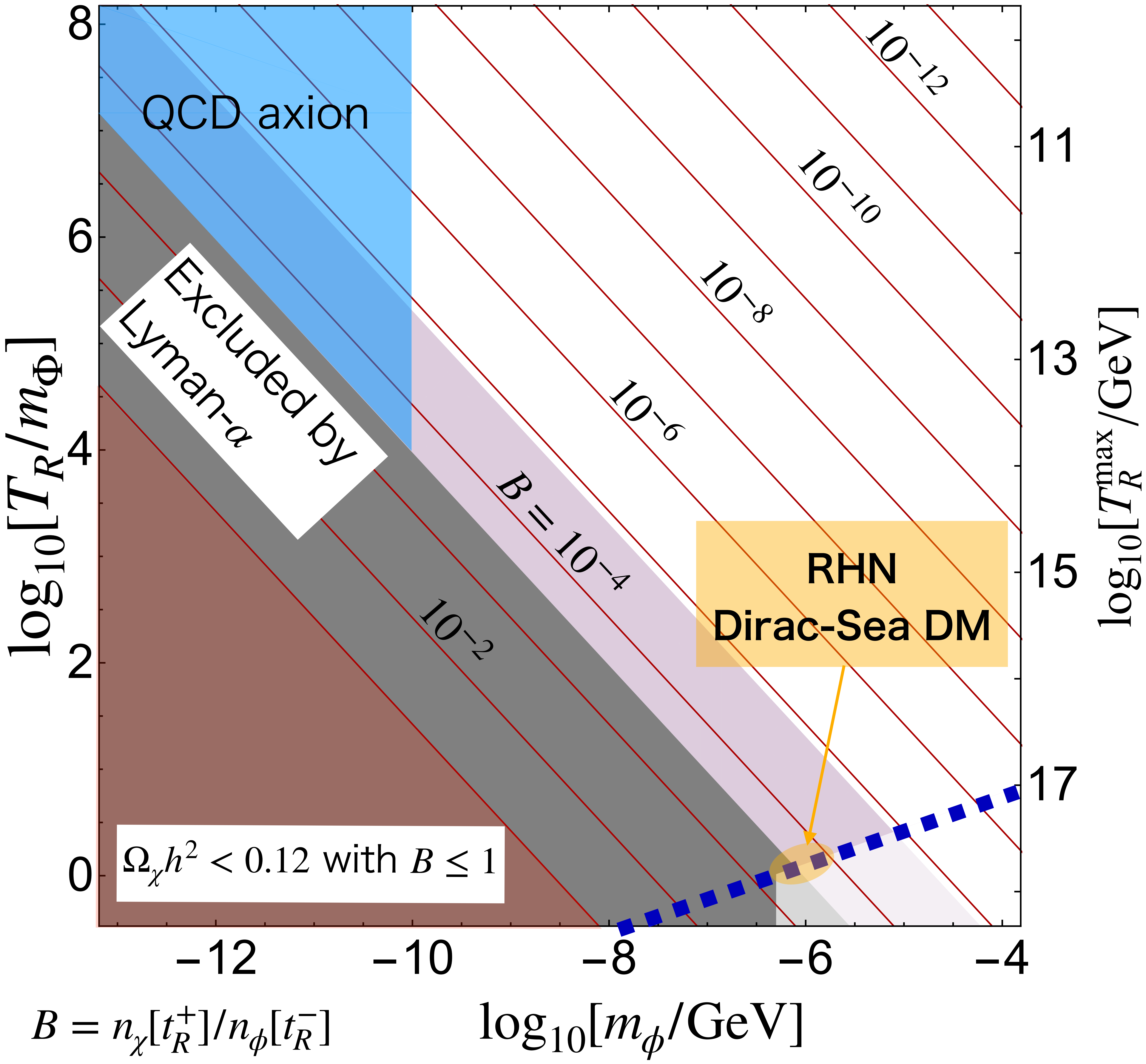

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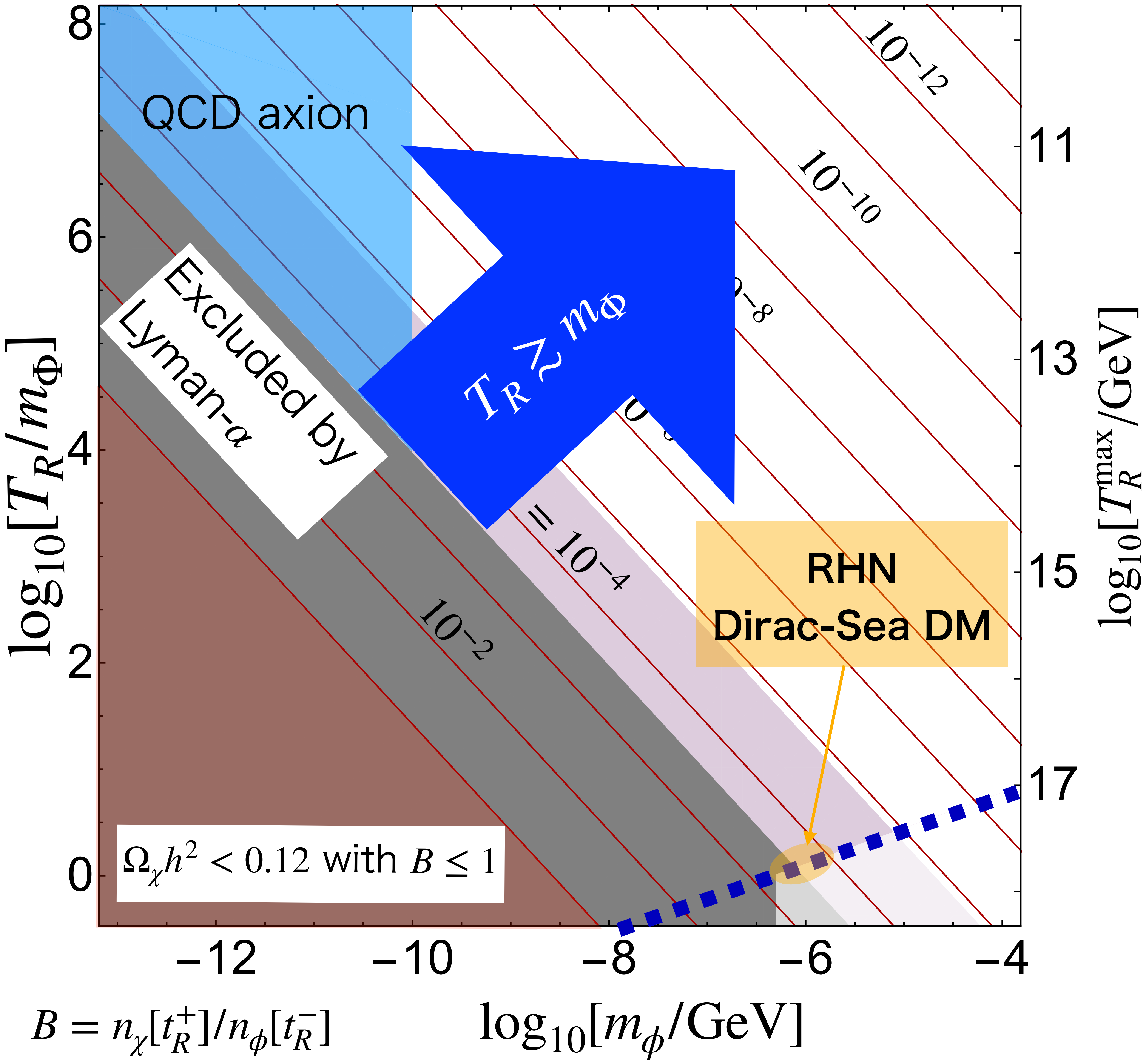

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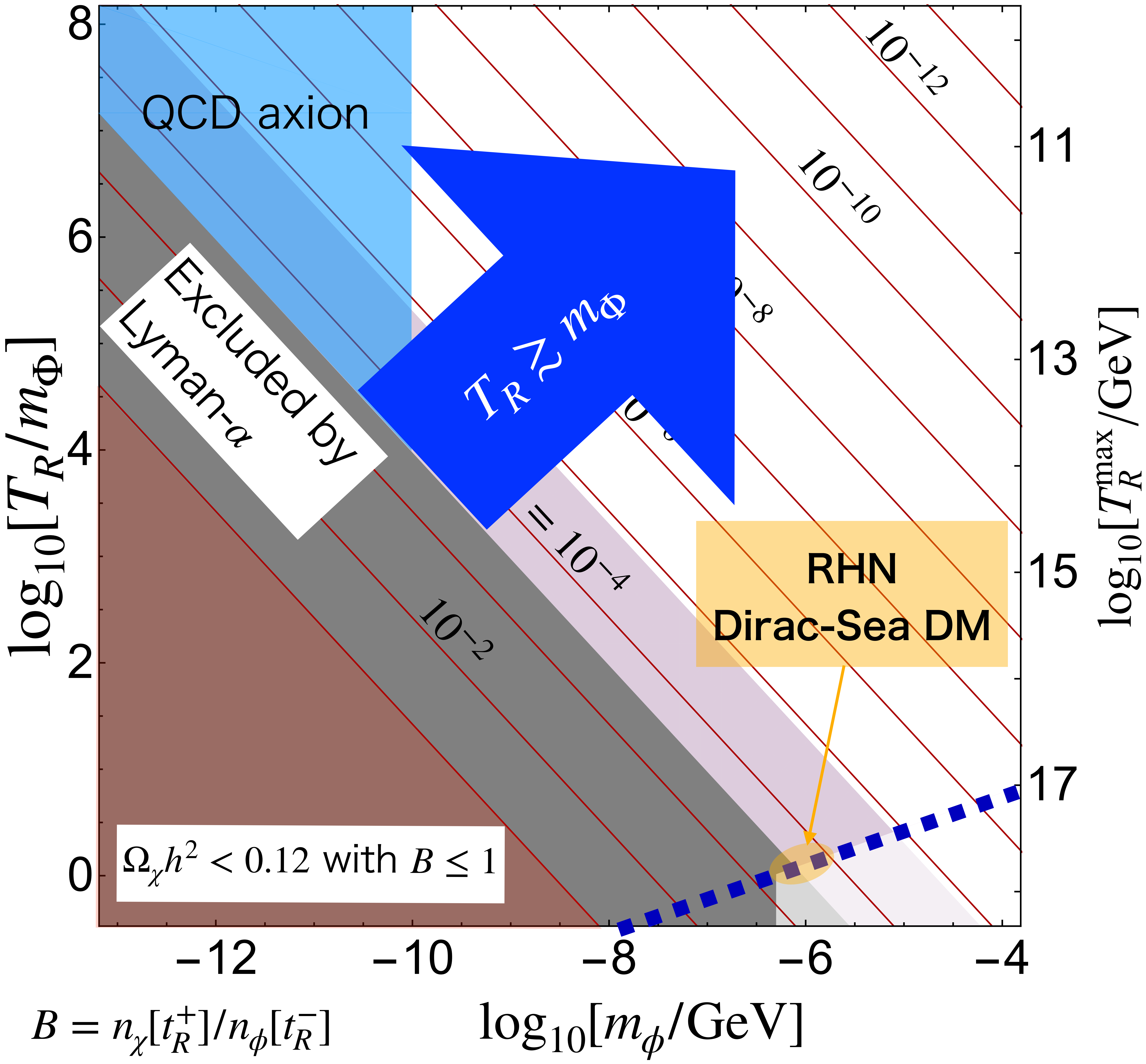

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- @ $t_{\rm prod}$ $\rho_\phi^{\rm prod} \gg m_\phi^2 f_\phi^2$.
- $\langle p_\phi^{\rm prod}\rangle \ll T_{\rm prod}$
- $f_\phi^4 \gg \rho_\phi^{\rm prod}$

(i.e. axion EFT is valid)

- A minimal possibility?
- Moduli decay also work with non-minimal coupling

Miyazaki, Yaginuma, Song, Narita, and WY, 2509.13292

What I will talk about:

The usual DM evolution may not be true for axions.

@$t_{\rm prod}$

Some Production Mechanism

@late time

Conserved: $\dfrac{n_\phi^{\rm prod}}{s_{\rm prod}}$

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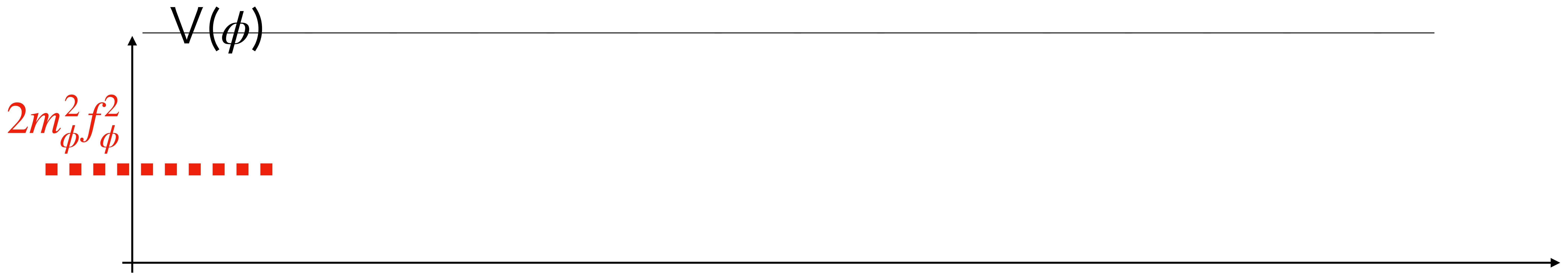
- 2. A Nonlinear Transition

Key point: axion potential is periodic and bounded



Question 1:

Narita and WY, 2507.03157

Are axion particles radiation or matter with

$$\bar{p}_\phi \equiv \langle p_\phi \rangle < m_\phi \text{ and } \rho_\phi > 2m_\phi^2 f_\phi^2 ?$$

$$(\langle \phi \rangle = 0, \ \ \langle \dot{\phi} \rangle = 0)$$

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$$\rho_\phi = \frac{1}{2}\left(\dot{\phi}^2 + (\vec{\nabla}\phi)^2\right) + \cancel{V_\phi}$$

$$(< 2m_\phi^2 f_\phi^2)$$

Radiation

See also similar phenomenon in "axion roulette" Daido, Kitajima, Takahashi, 1505.07670, and Giovanni's talk for QCD axion from cosmic string.

Answer 1: Axions behave as radiation.

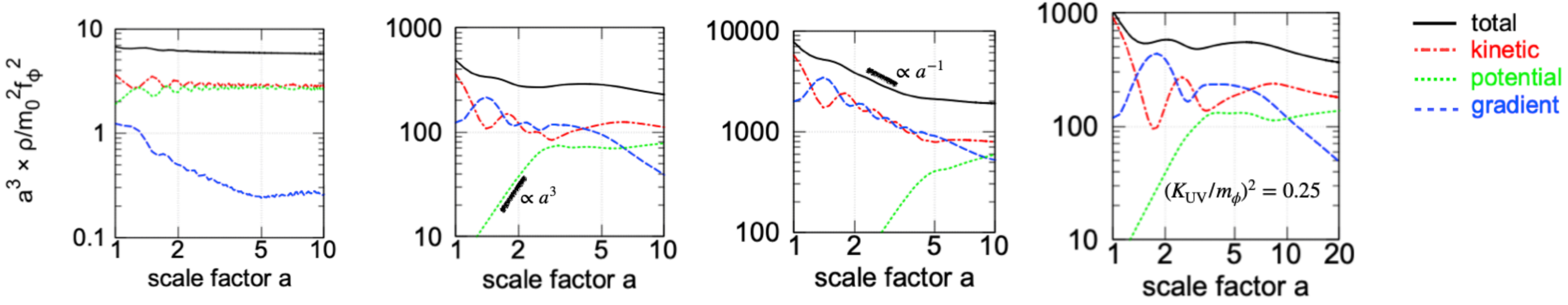
It becomes matter at $\rho_\phi \sim m_\phi^2 f_\phi^2$, through a nonlinear transition.

Narita and WY, 2507.03157

$$V = m_\phi^2 f_\phi^2 (1 - \cos\phi/f_\phi)$$

$$w_k^i = \sqrt{k^2 + m_\phi^2}.$$

$\{0.25 m_\phi^2 f_\phi^2, 20 H_i^2\}$ $\{25 m_\phi^2 f_\phi^2, 20 H_i^2\}$ $\{400 m_\phi^2 f_\phi^2, 20 H_i^2\}$ ★ $\{25 m_\phi^2 f_\phi^2, 20 H_i^2\}$ $\{\langle(\nabla\phi_i)^2\rangle, m_\phi^2\}$



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$k^3 < \phi_k \phi_{-k} > \propto k^q, q = 2$ for plots in this slide

Oscillation of kinetic, gradient energy disappear if we take $w = k$.

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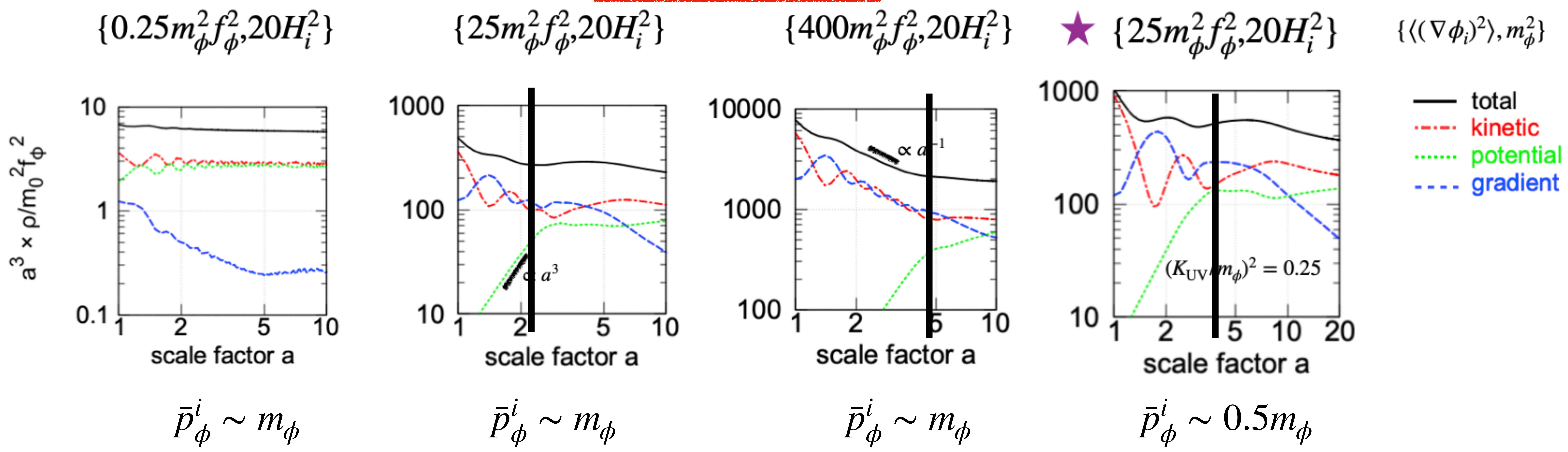

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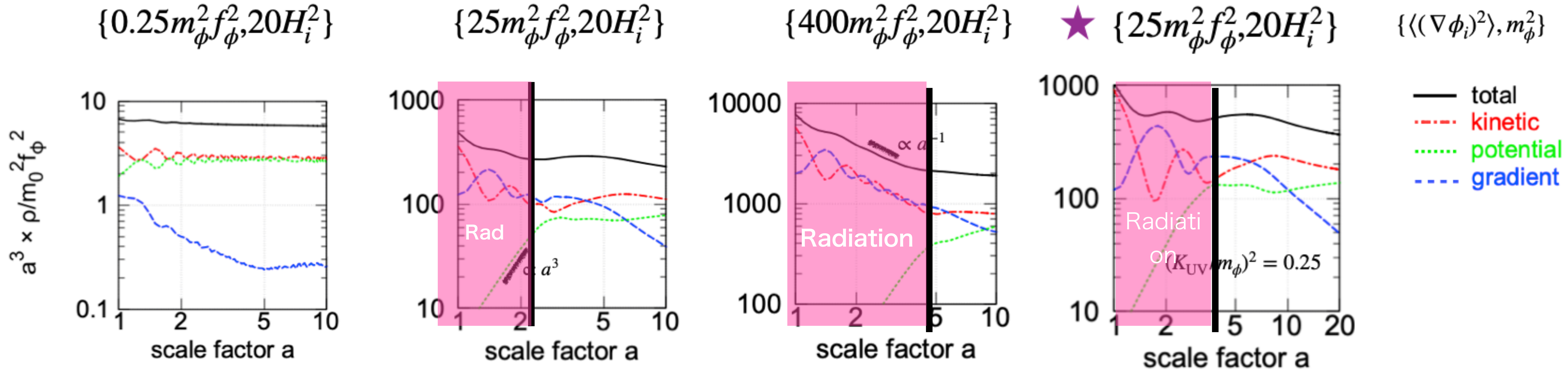

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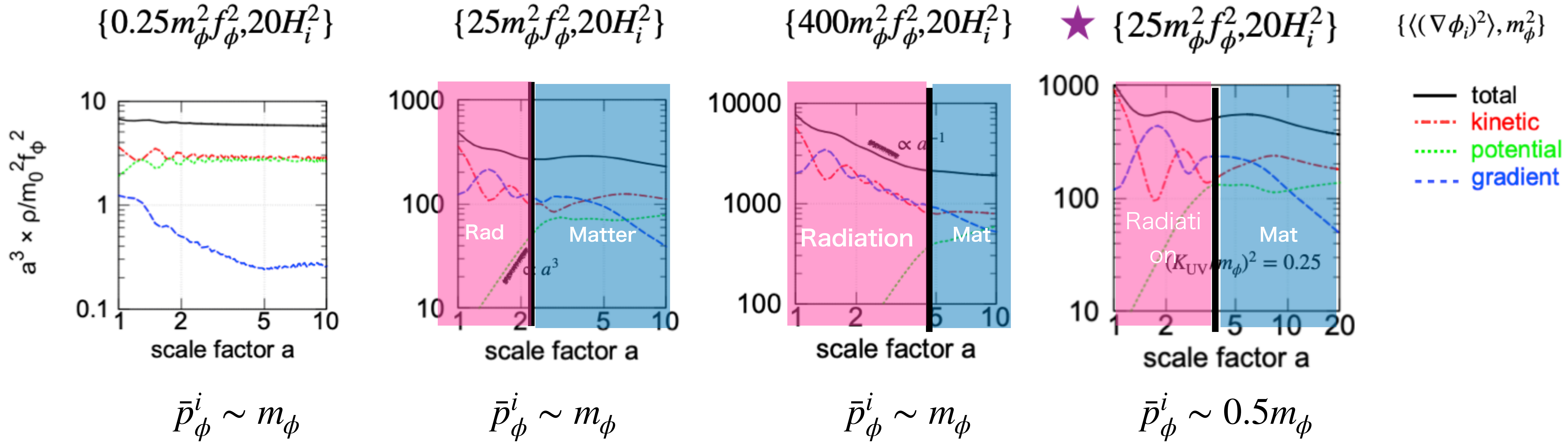

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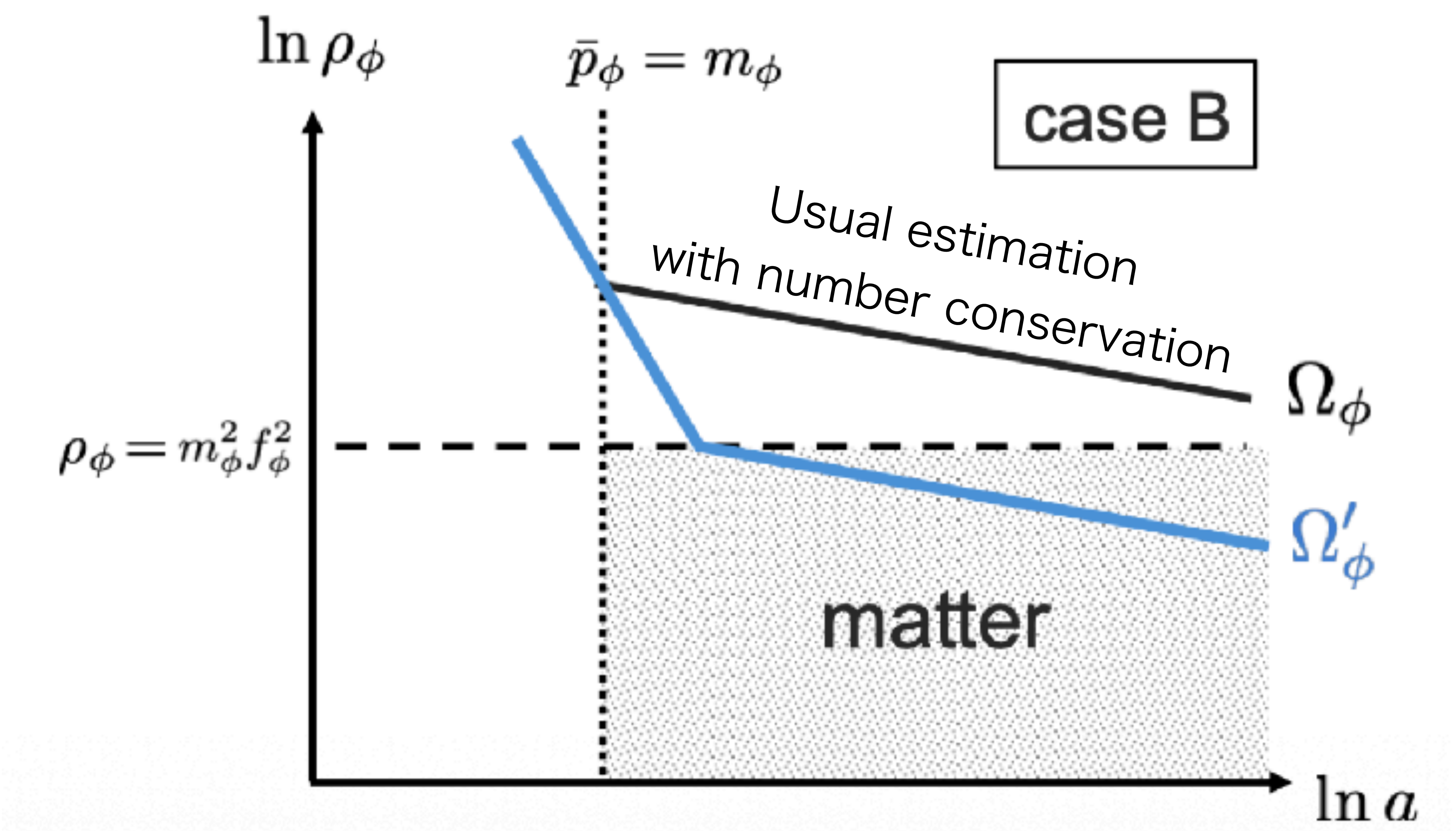
What drives the nonlinear transition?



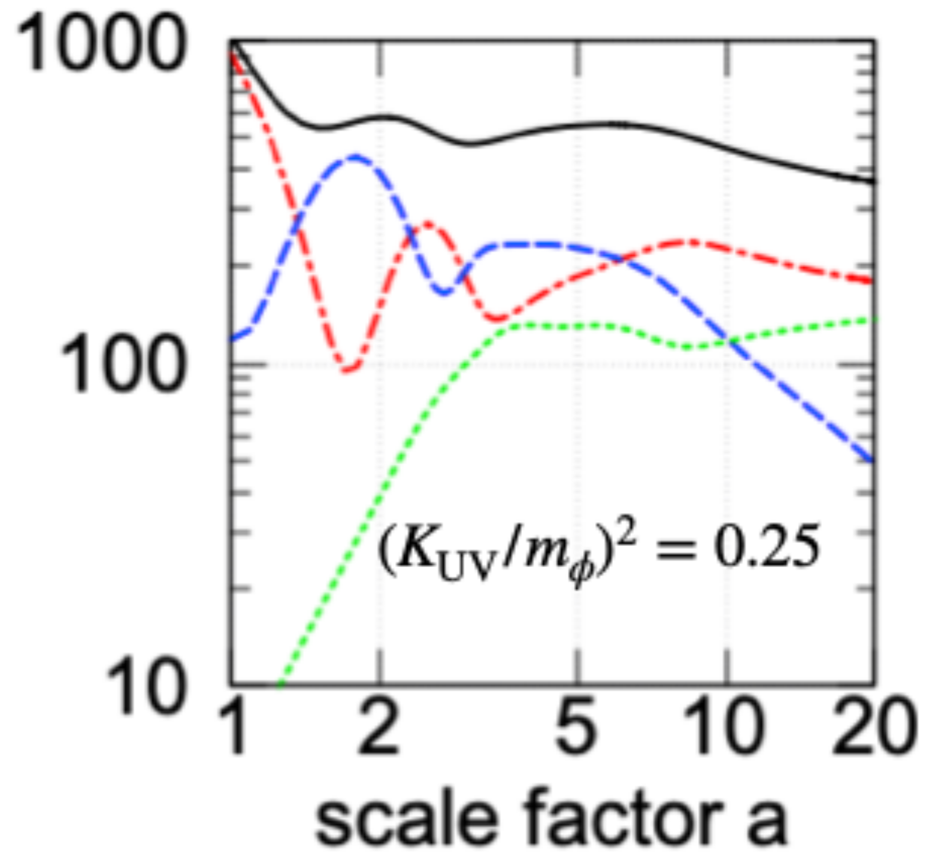
From Nemin Yaginuma's slide

Number must not be conserved.

See also similar dynamics in "axion roulette" Daido, Kitajima, Takahashi, 1505.07670
and Giovanni's talk for QCD axion from cosmic string

Answer 2: Multiple closed domain walls drive the transition

$a_{tr} \sim 2-3$ ★ $\{25 m_\phi^2 f_\phi^2, 20 H_i^2\}$



$$\because \rho_\phi \sim \bar{p}_\phi^2 \langle \delta\phi^2 \rangle \sim f_\phi^2 m_\phi^2, \bar{p}_\phi < m_\phi$$

$$\therefore \sqrt{\langle \delta\phi^2 \rangle} > f_\phi$$

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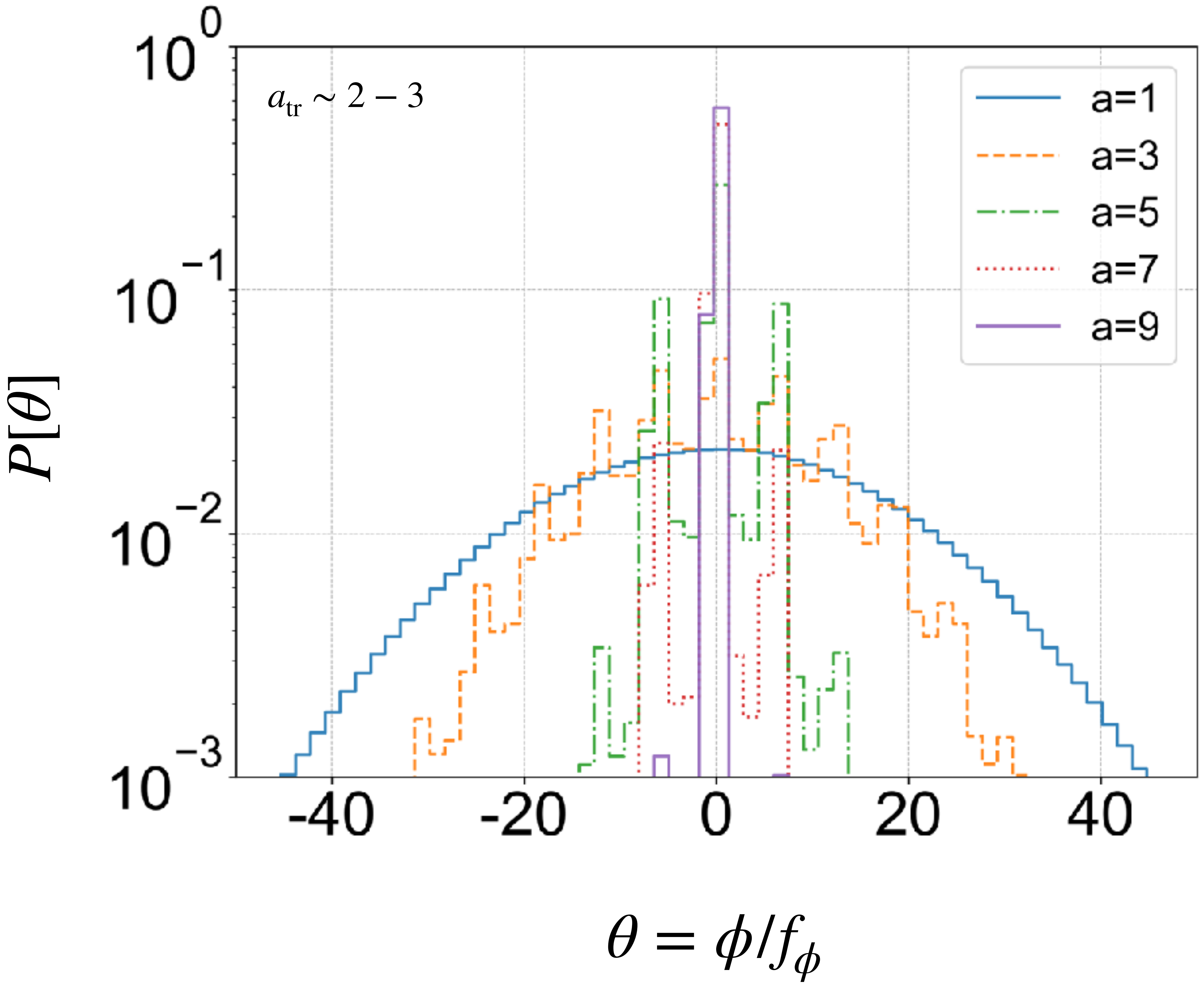

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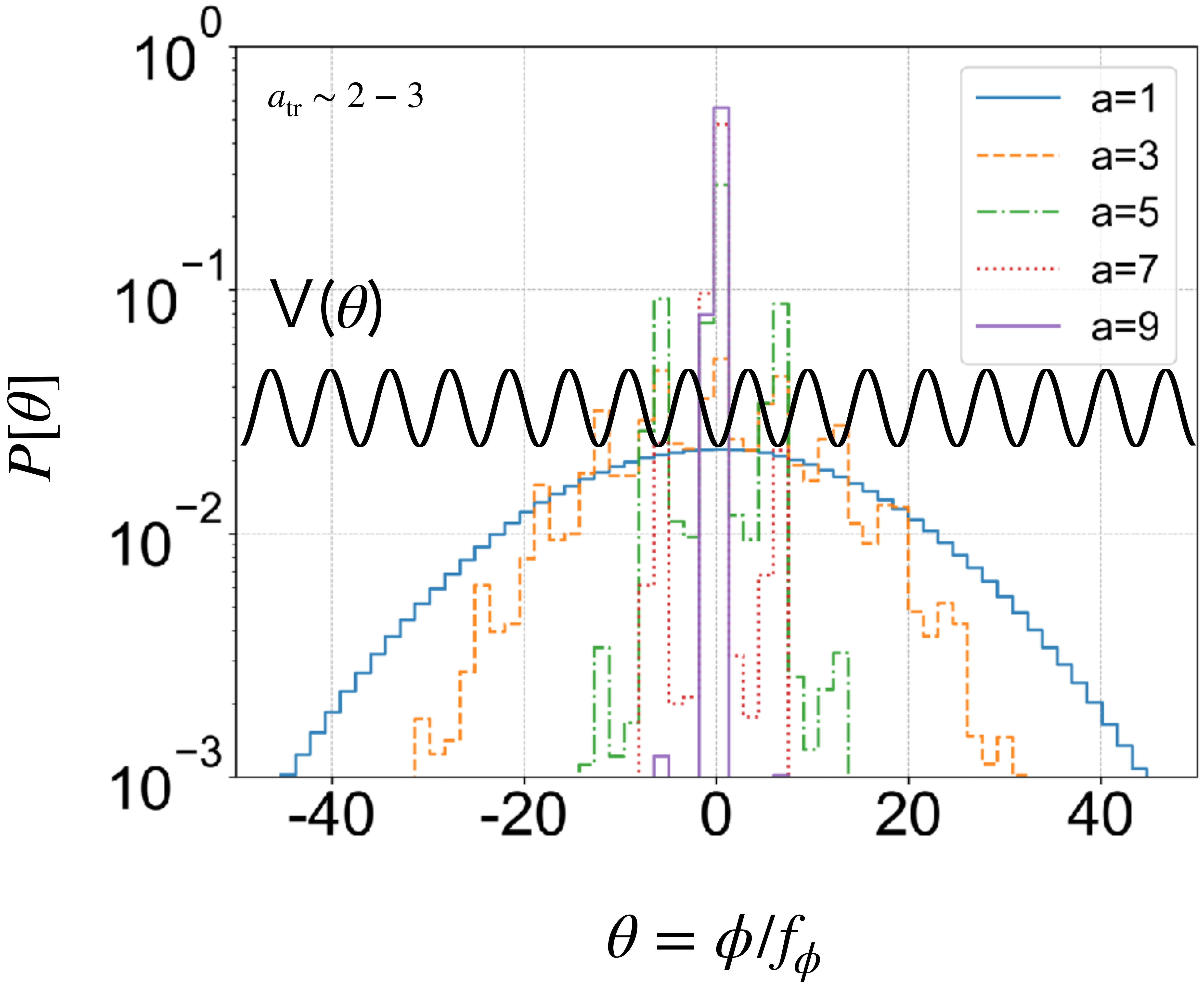

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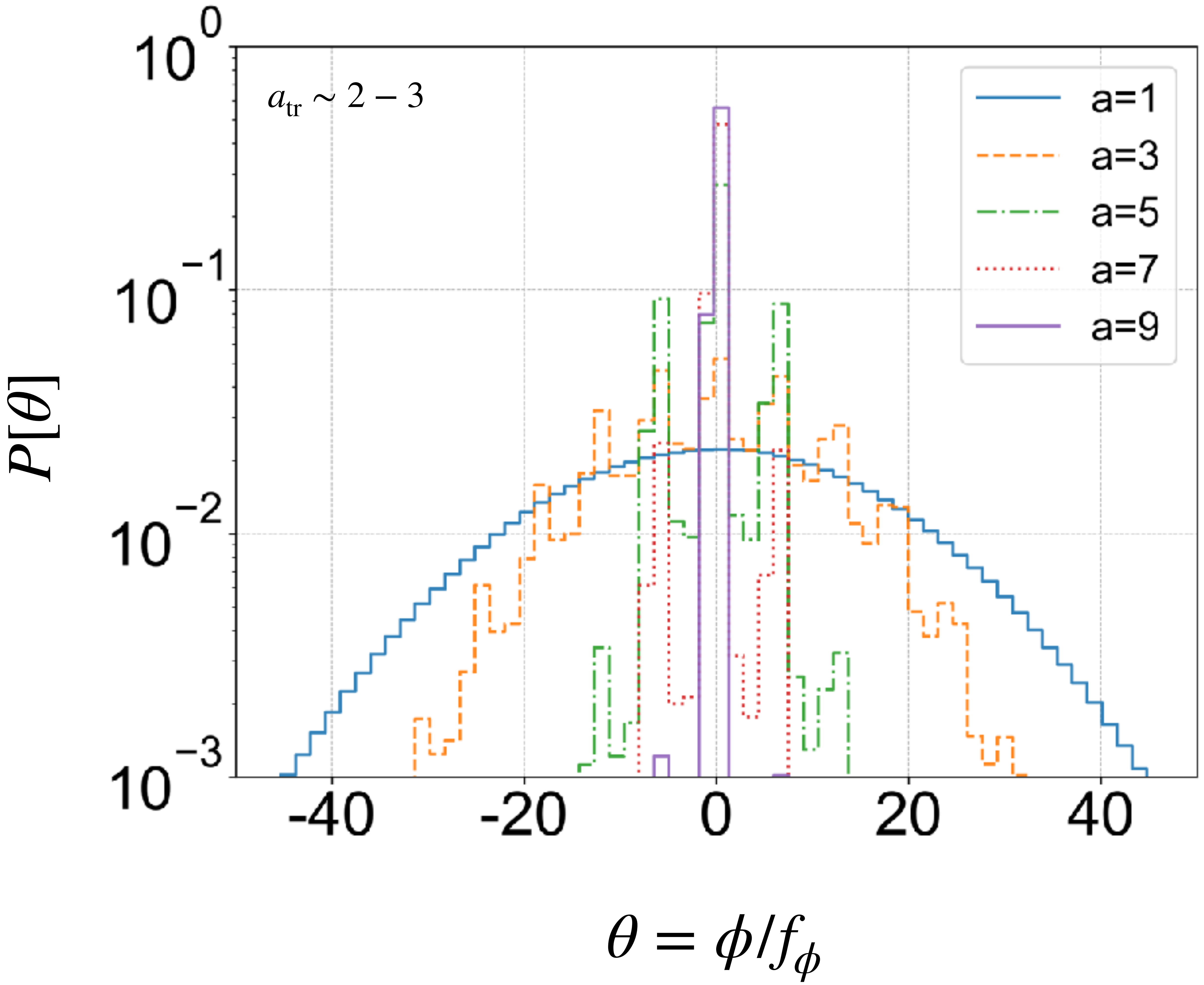

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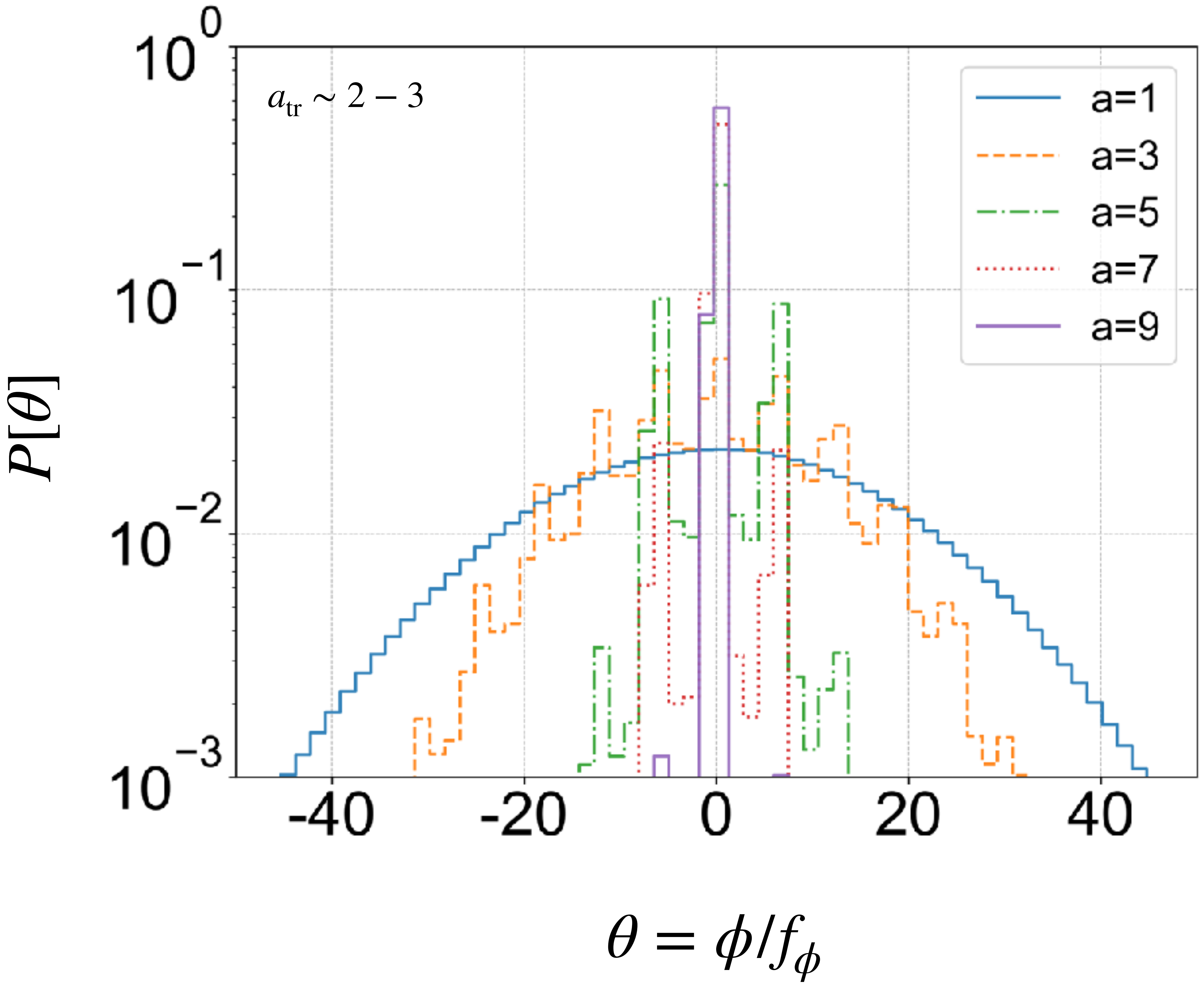

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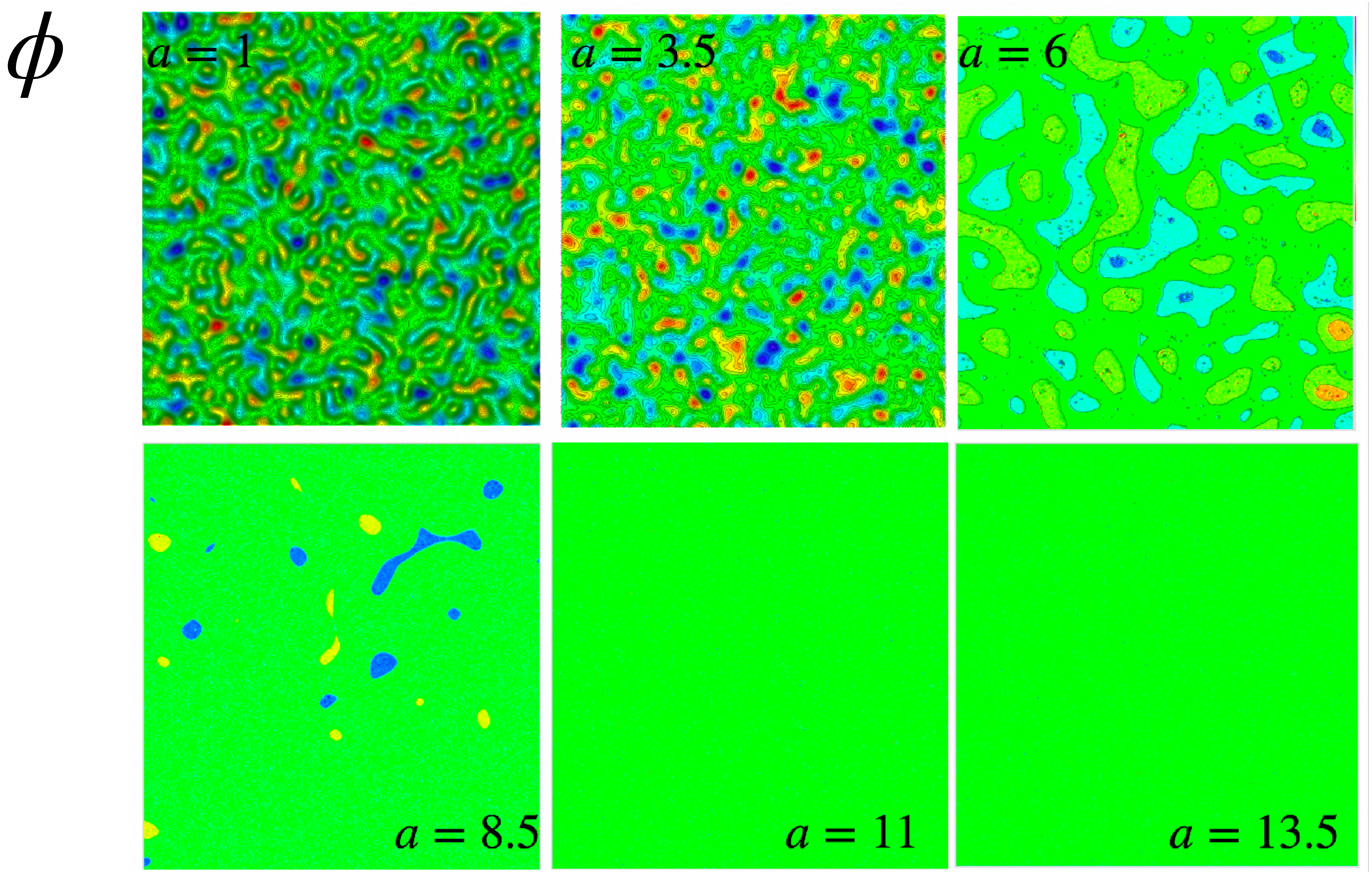
$\therefore$ Domain wall forms but collapses soon.

(for white noise. Scale invariant fluc. gives long-lived domain wall)

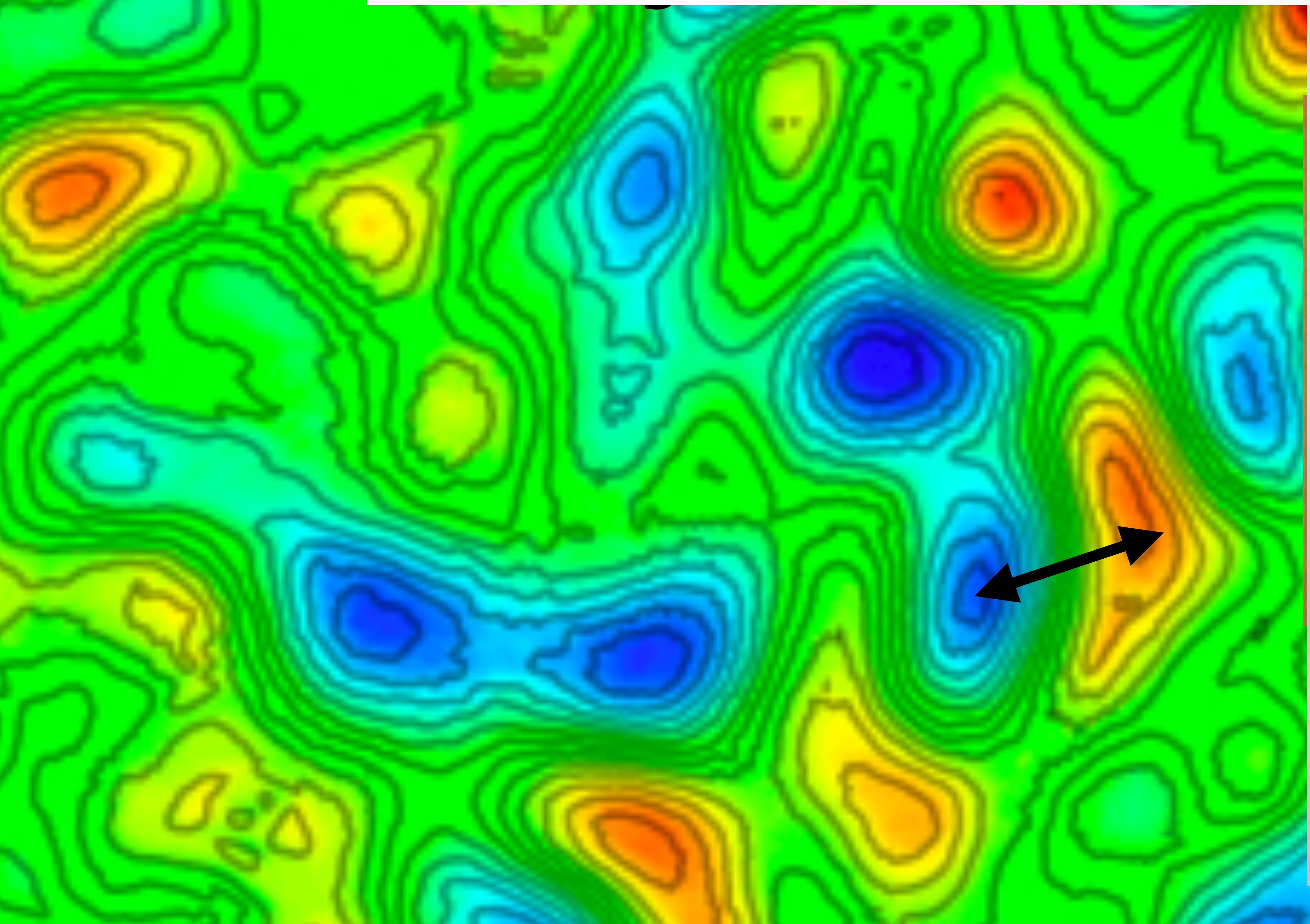
Gonzalez, Takahashi, WY, 2211.06849



ϕ
$a = 1$
$a = 3.5$
$a = 6$
$a = 8.5$
$a = 11$
$a = 13.5$

Many Baumkuchens!

Many Bossam!

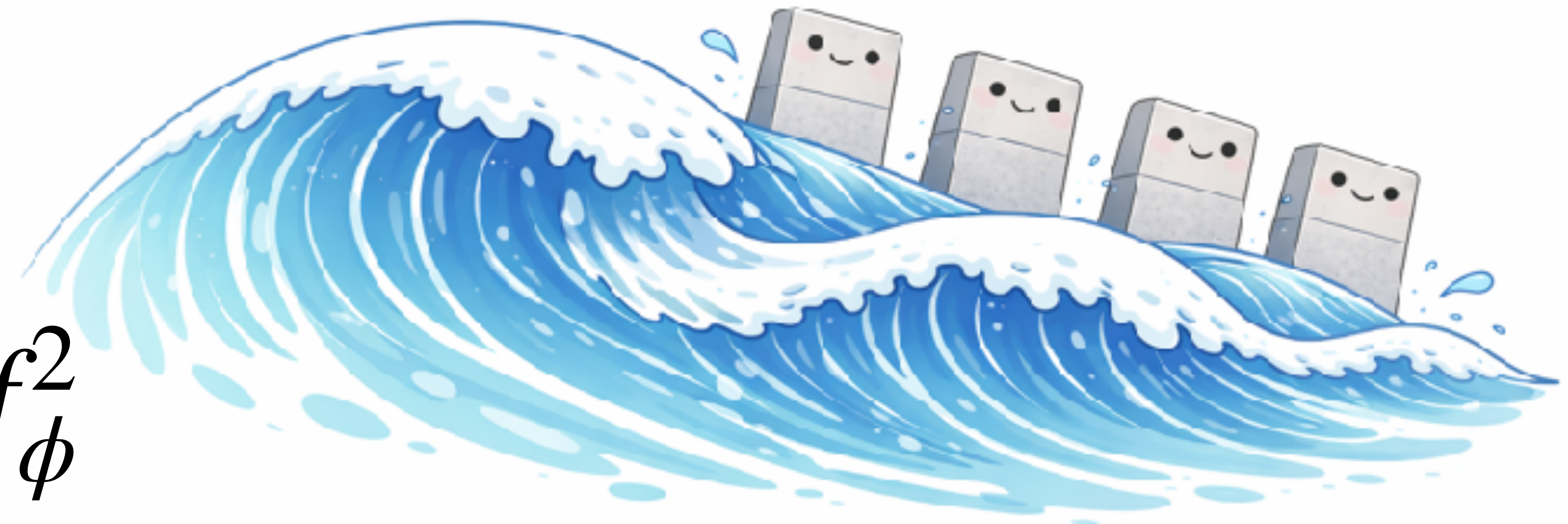


Baumkuchen domain-wall (BDW) properties

Before nonlinear transition $(\bar{p}_\phi^2\langle\delta\phi^2\rangle \gg m_\phi^2 f_\phi^2)$

The number of domain walls within half wave length, $\pi/\bar{p}_\phi$, i.e. within a **BDW**:

$$N_{\rm BDW} \equiv \sqrt{\langle\delta\phi^2\rangle}/(\pi f_\phi)$$



The tension:

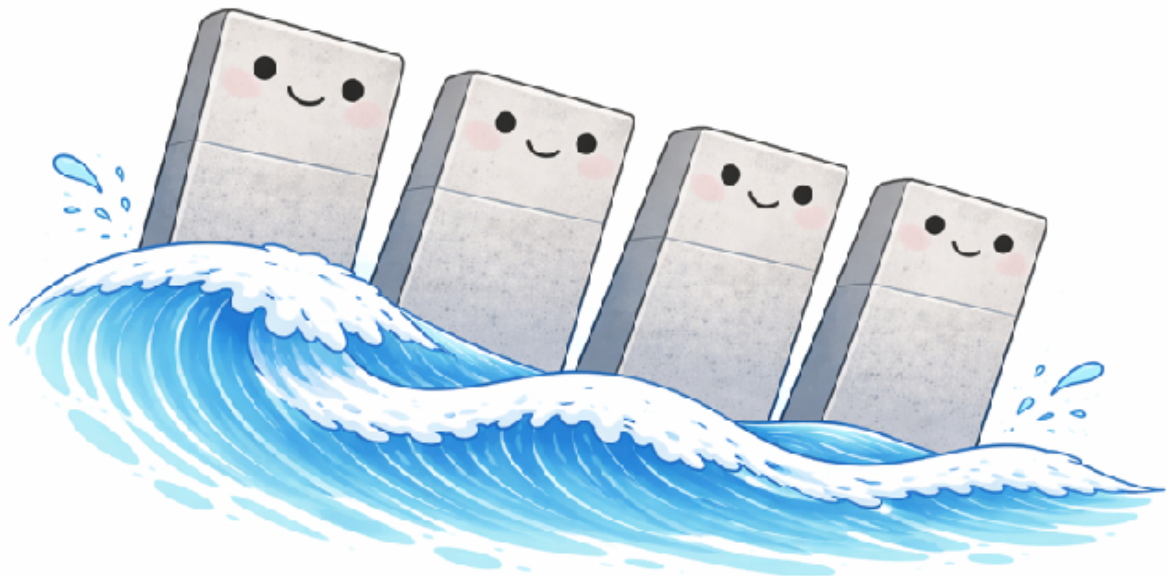
$$\sigma_{\rm BDW} \approx N_{\rm BDW} \times 8 m_\phi f_\phi^2$$

The energy density:

$$\rho_{\rm BDW} \approx \frac{\bar{p}_\phi}{\pi} \times N_{\rm BDW} 8 m_\phi f_\phi^2 \sim \bar{p}_\phi \sqrt{\langle\delta\phi^2\rangle} m_\phi f_\phi$$

At nonlinear transition $(\bar{p}_\phi^2\langle\delta\phi^2\rangle \sim m_\phi^2 f_\phi^2)$

$$\rho_\phi \sim \rho_{\rm BDW} \sim m_\phi^2 f_\phi^2$$



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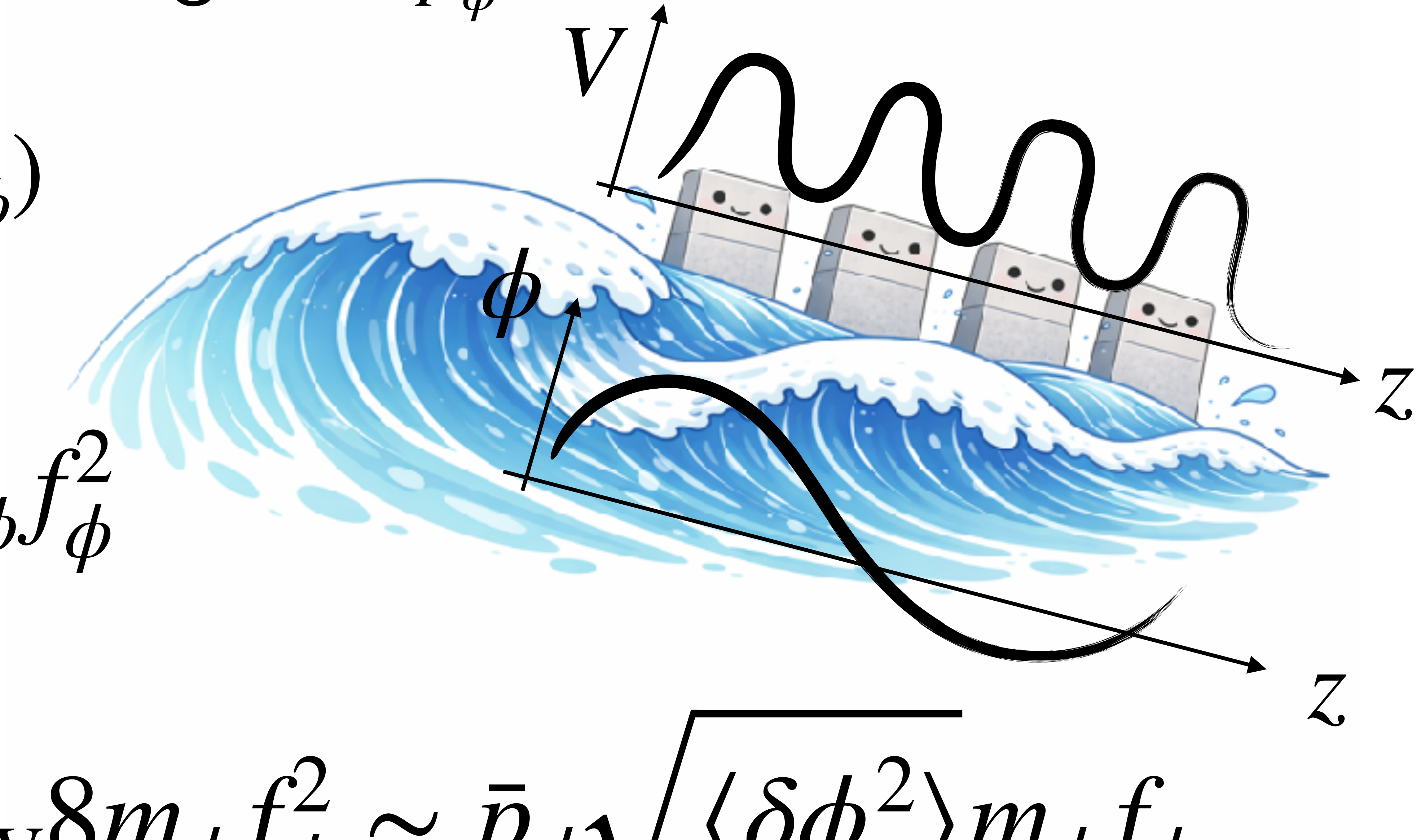

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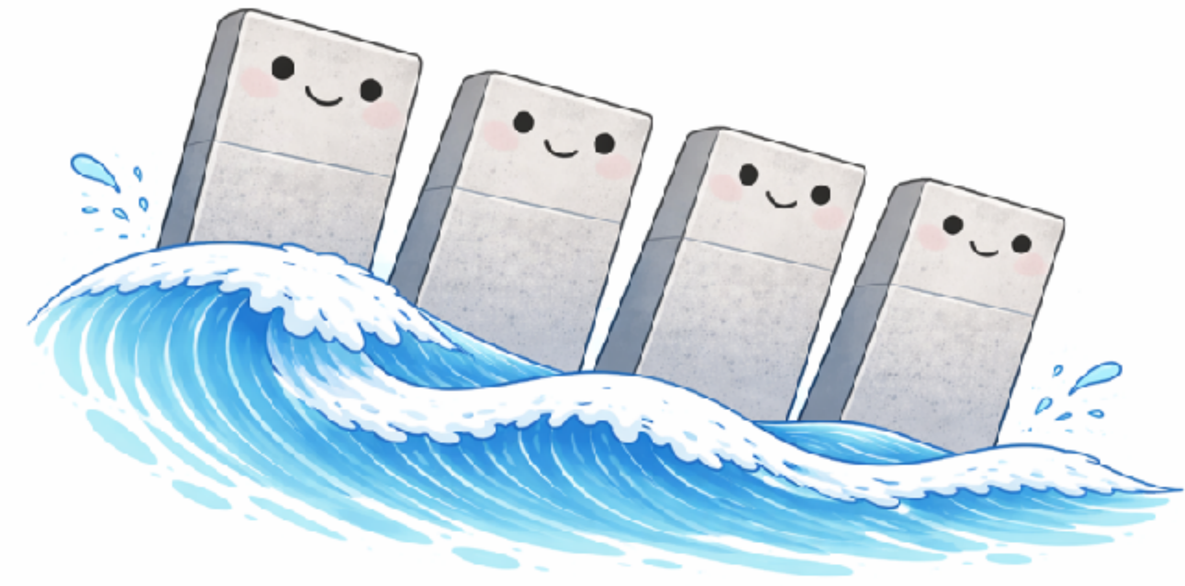
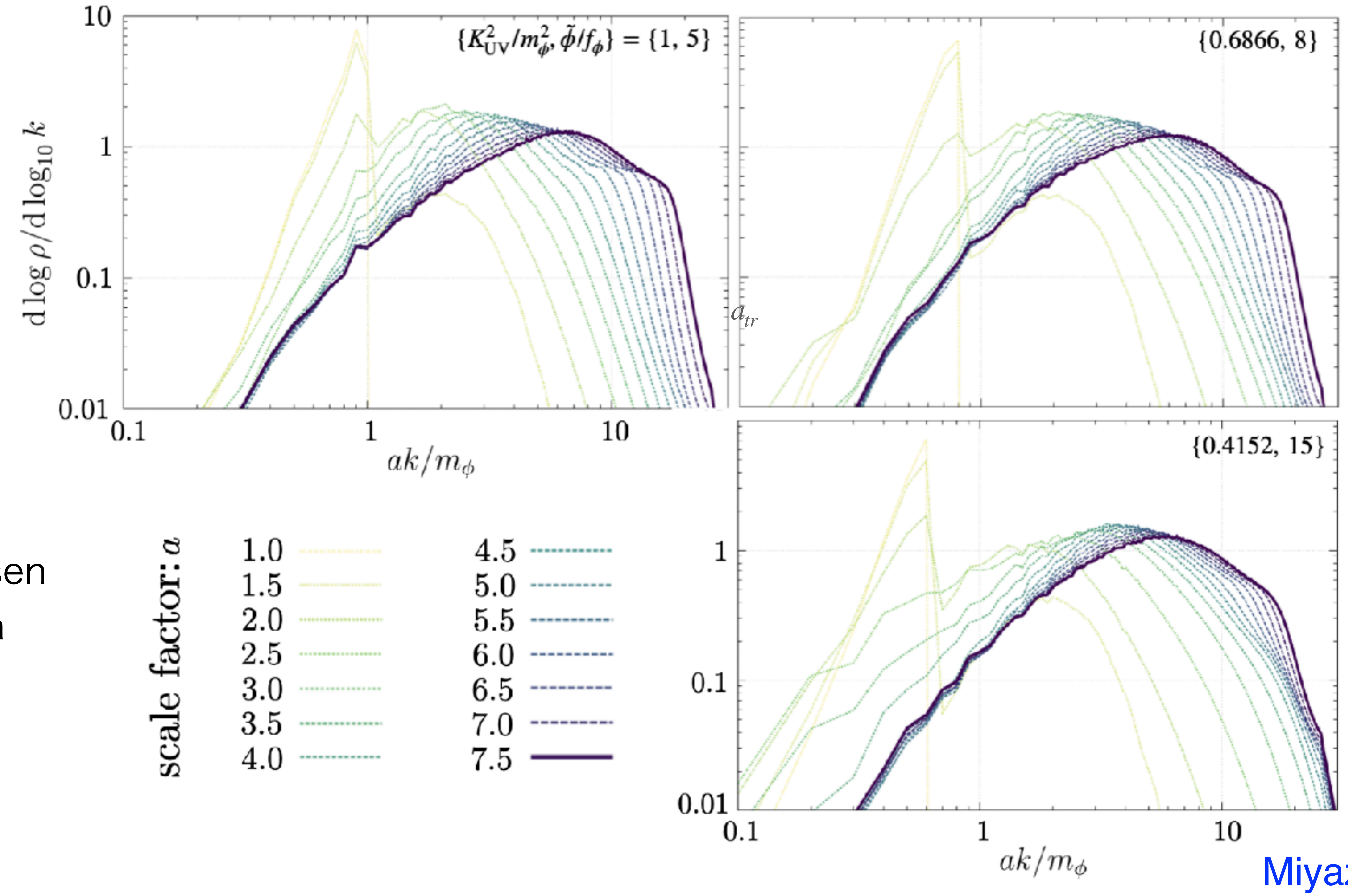


Gradient -> BDW -> ALP particles



Parameters chosen so that transition periods are the same.

$w = k$ is used

Miyazaki, Yaginuma, Song, Narita, and WY, 2509.13292

A short summary:

- $\rho_\phi > m_\phi^2 f_\phi^2$, axion is not "matter" even if $\bar{p}_\phi < m_\phi$.
- the number conservation is violated.
- the late time axions are from the BDM collapse irrelevant to the original production mechanism $\rho_\phi \sim m_\phi^2 f_\phi^2, \bar{p}_\phi \sim m_\phi$, @ $t_{\rm tr}$

- 3. **Implications of the nonlinear transition**

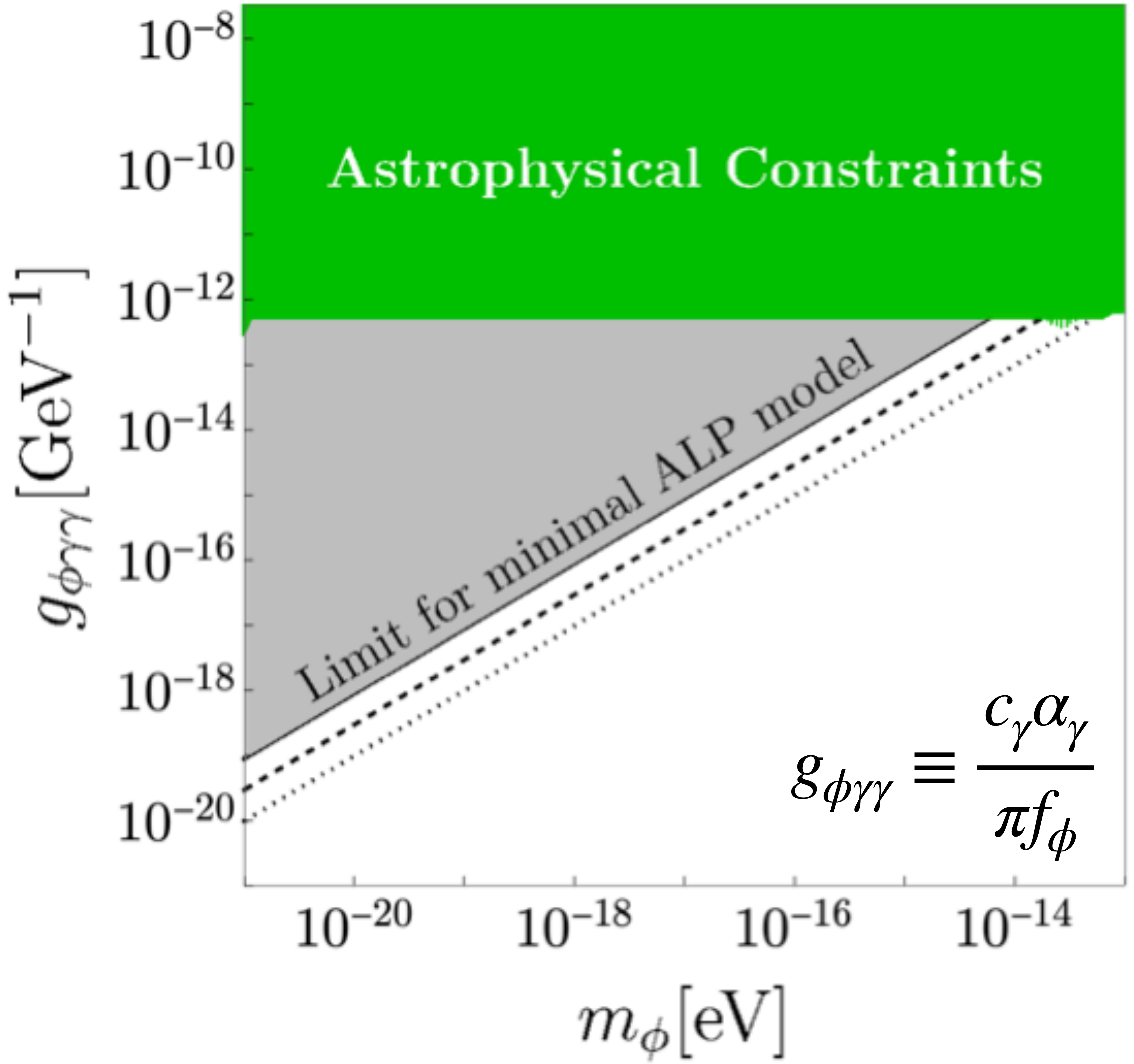
Narita and WY, 2507.03157, Miyazaki, Yaginuma, Song, Narita, and WY, 2509.13292

- A generic lower bound on the ALP decay constant:

Narita and WY, 2507.03157
See also Marsh, WY,1912.08188; Dror, Leedom, 2008.02279

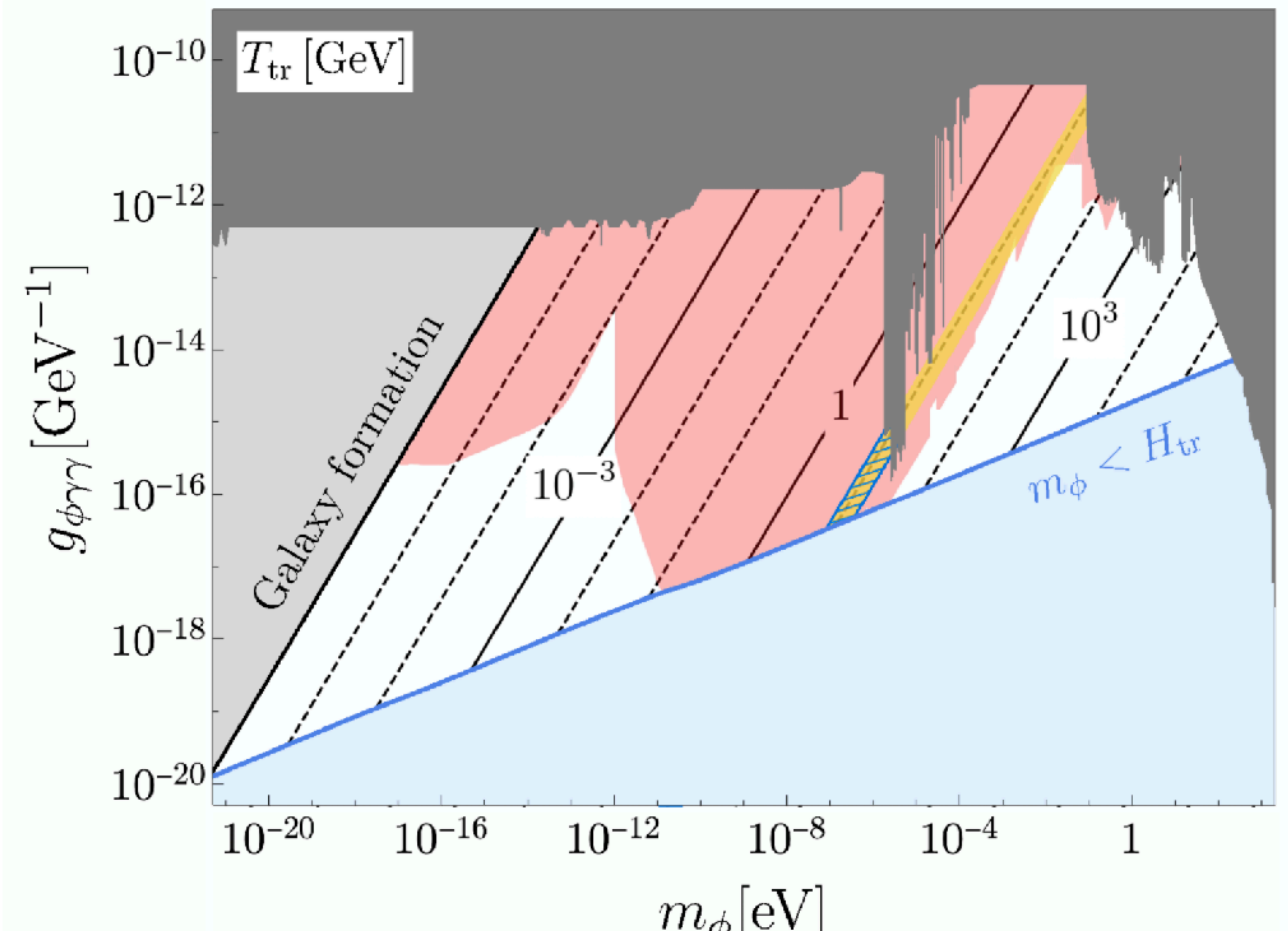
$\because$ we need $\rho_\phi < m_\phi^2 f_\phi^2$, $at\ a_{\mathrm{Ly}\alpha} \sim 10^{-6}$.

$$f_\phi \gtrsim 4 \times 10^{13} GeV \frac{10^{-18}\mathrm{eV}}{m_\phi}$$



- **ALP DM from nonlinear transition:** many production mechanisms with abundant "cold axion" approach to the following parameter region

Miyazaki, Yaginuma, Song, Narita, and WY, 2509.13292



- **QCD axion DM with $f_\phi < 10^{12}\,\mathrm{GeV}$ can be produced $T_{tr} \sim \mathrm{GeV}$ universally.**
- **Fixing m_ϕ with smaller f_ϕ (larger $g_{\phi\gamma\gamma}$) our finding is more important. Be careful!**

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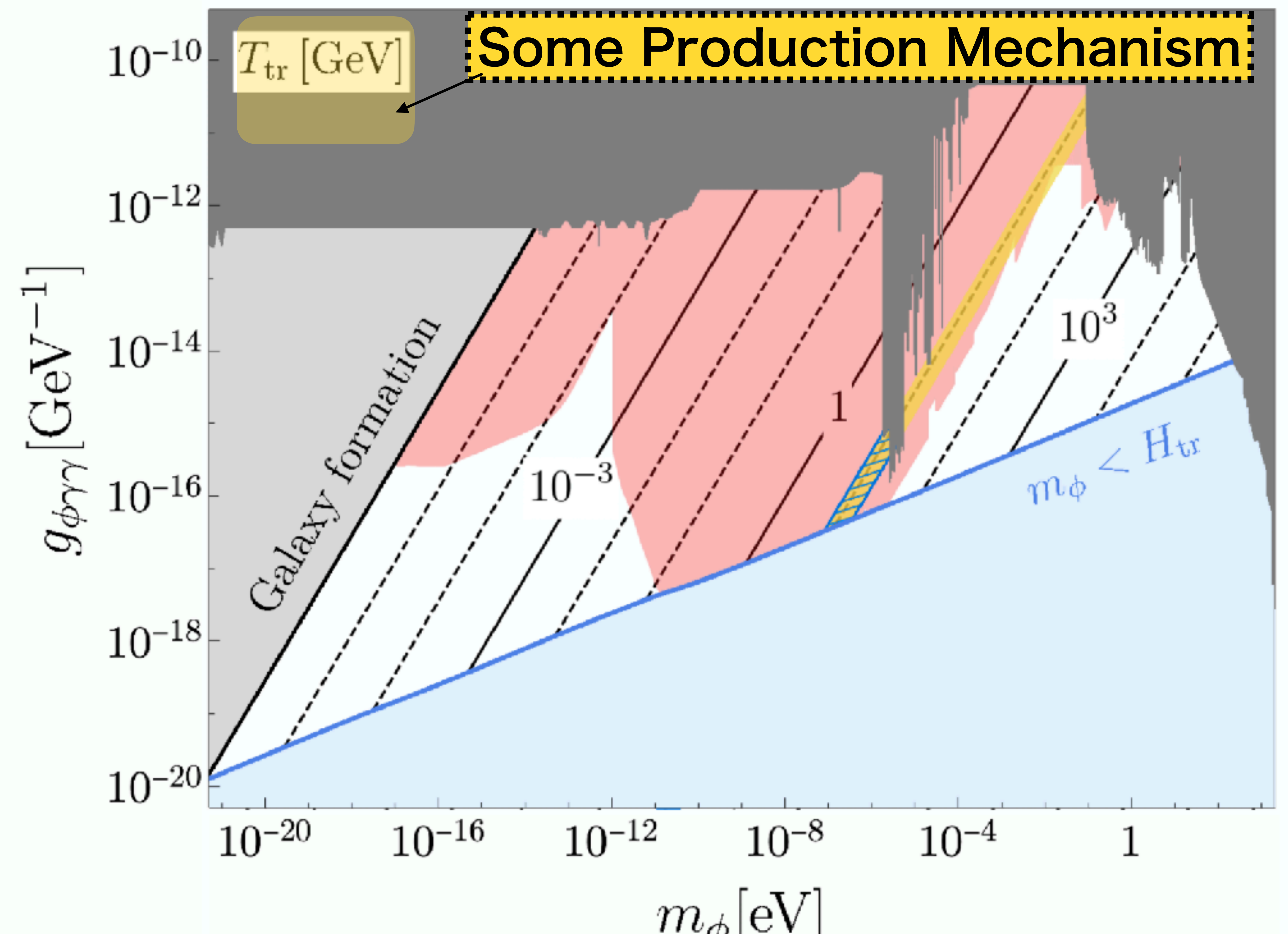

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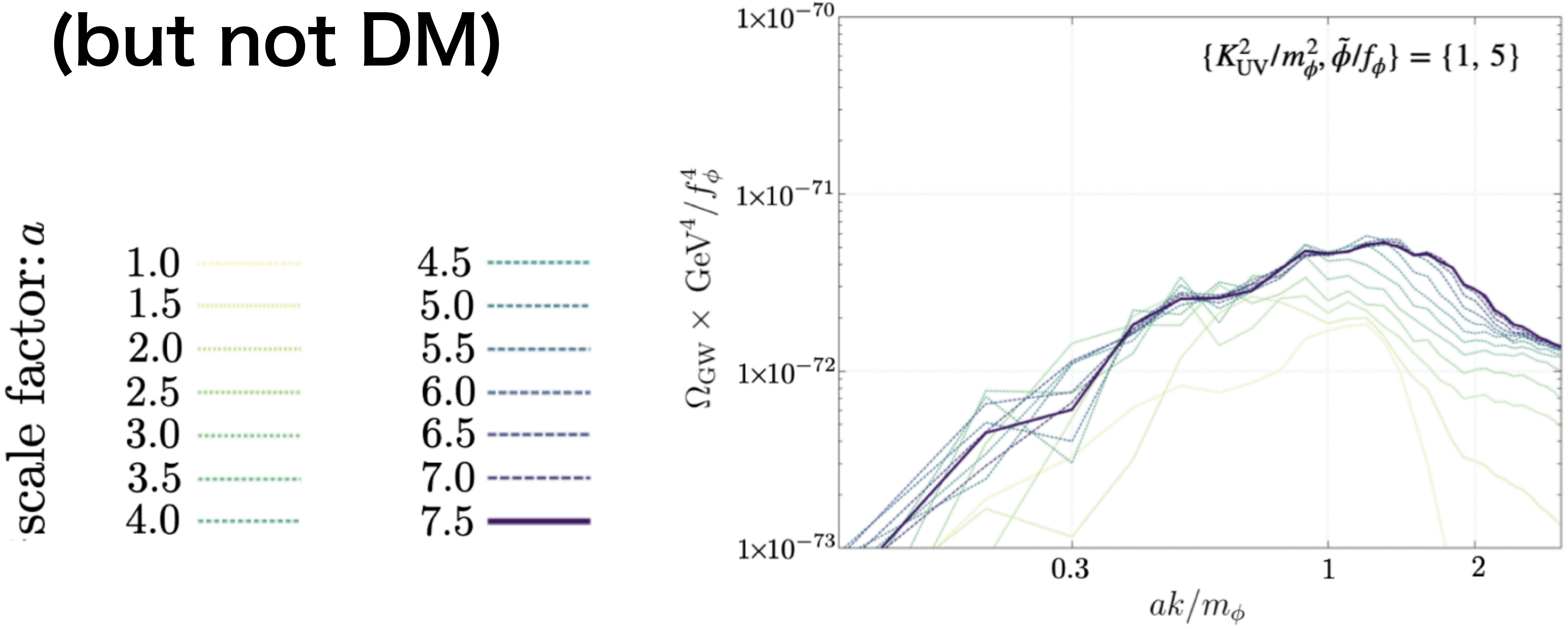
- Inhomogeneities can seed ALP clumps.

See talks by Giovanni and Jae Hyeok

- BDW collapse can source gravitational waves (but not DM)



- Inhomogeneities may seed PBHs.

Miyazaki, Yaginuma, Song, Narita, and WY, 2509.13292

Conclusions: Axion/ALP may have a nonlinear transition due to potential feature.

Narita and WY, 2507.03157, Miyazaki, Yaginuma, Song, Narita, and WY, 2509.13292

- If $\bar{p}_\phi < m_\phi$ *and* $\rho_\phi > m_\phi^2 f_\phi^2$, axions are still relativistic!
- Robust bound for DM: $f_\phi \gtrsim 4 \times 10^{13} GeV \frac{10^{-18}\mathrm{eV}}{m_\phi}$
- Once ρ_ϕ redshifts to $m_\phi^2 f_\phi^2$, we have nonlinear transition forming BDWs, to produce semi-relativistic axion particles.
- Some DM production mechanisms need revisions.

Backup

Example: Stimulated decay can produce bosonic dark matter

$$\Phi \to \mathrm{DMDM}$$

Moroi, WY, 2011.09475 , 2011.12285

$$\dot{f}_{ak}(t) = \frac{4\pi^2 n_\Phi \Gamma}{P_{\mathrm{DM}}^2}\left[(1+f_{ak})^2 - f_{ak}^2\right]\delta(k - P_{\mathrm{DM}}) \qquad P_{\mathrm{DM}} = \frac{m_\Phi}{2}\sqrt{1 - 4\frac{m_{\mathrm{DM}}^2}{m_\Phi^2}}\,.$$

Decay Inverse decay

Example: Stimulated decay can produce bosonic dark matter

$$\Phi \to \mathrm{DMDM}$$

Moroi, WY, 2011.09475 , 2011.12285

Bose enhancement factor

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Moroi, WY, 2011.09475 , 2011.12285

$$\frac{1}{2}\log(1+2f_{ak}) = \left.\frac{4\pi^2 n_\Phi \Gamma}{H P_{\rm DM}^2}\right|_{t=t_{k=P_{\rm DM}}} \qquad P_{\rm DM} = \frac{m_\Phi}{2}\sqrt{1-4\frac{m_{\rm DM}^2}{m_\Phi^2}}\,.$$

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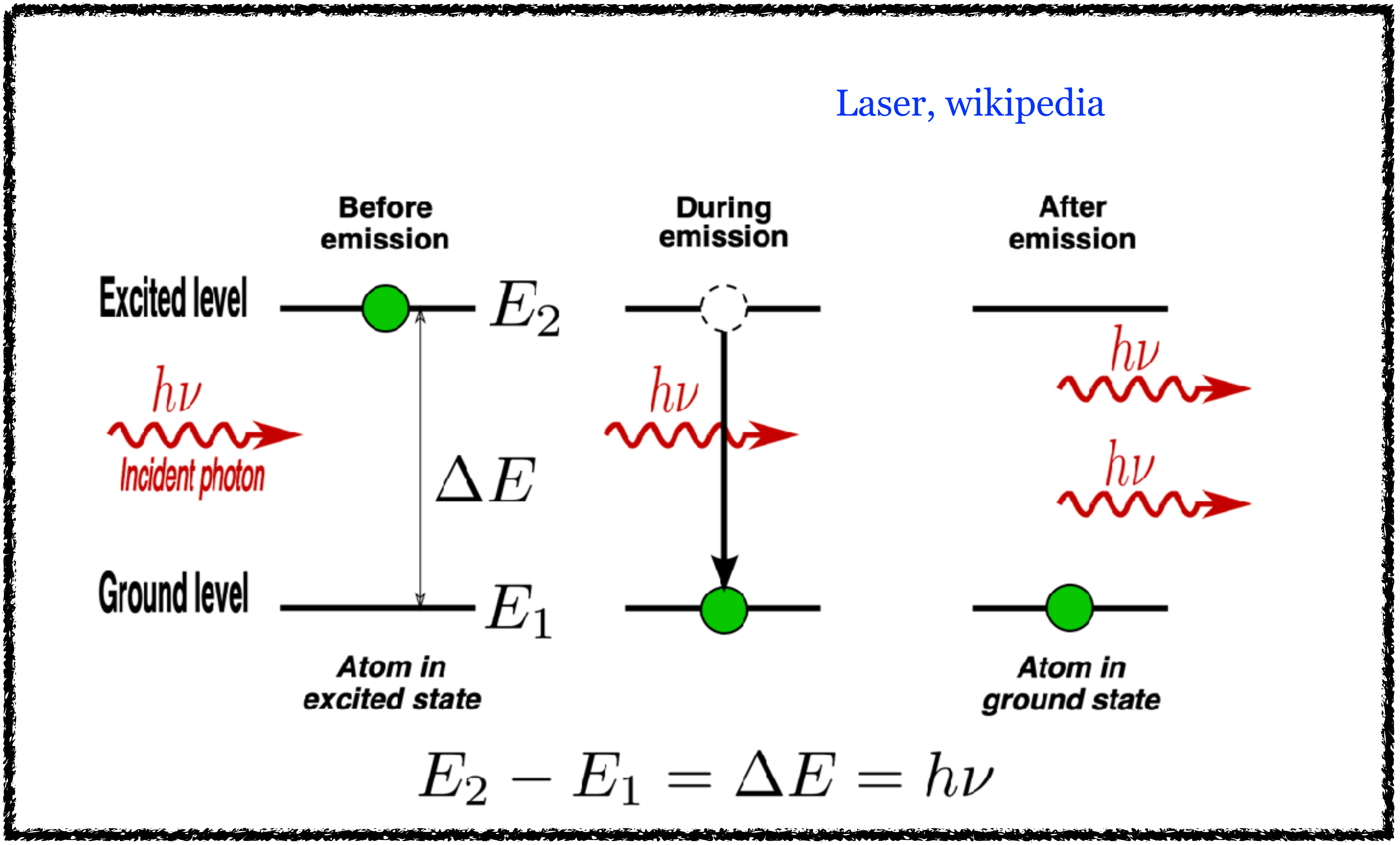
$$\to f_{ak} = \frac{1}{2}\left(e^{2\bar{f}(t_{k=P_{\mathrm{DM}}})}-1\right), \qquad \bar{f}(t_{k=P_{\mathrm{DM}}}) \equiv \left.\frac{4\pi^2 \Gamma n_\Phi}{H P_{\mathrm{DM}}^3}\right|_{t=t_{k=P_{\mathrm{DM}}}}.$$

$$a^3 n_{\mathrm{DM}} = \int \frac{d^3(ak)}{(2\pi)^3} f_{ak} \approx \begin{cases} 2\int da\, a^2 \frac{\Gamma}{H} n_\Phi \ \text{ for } \bar{f} \ll 1 \\ \int da\, a^2 P_{\mathrm{DM}}^3 e^{2\bar{f}} \sim a_{\mathrm{osc}}^3 P_{\mathrm{DM}}^3 e^{2\bar{f}(t_{\mathrm{osc}})} \ \text{ for } \bar{f} \gg 1 \end{cases}$$

Stimulated emission occurs when

$$\bar{f} \equiv \left. \frac{4\pi^2 \Gamma n_\Phi}{H P_{\rm DM}^3} \right|_{t=t_{k=P_{\rm DM}}} \gg 1$$



$$P_{\rm DM} = \frac{m_\Phi}{2}\sqrt{1 - 4\frac{m_{\rm DM}^2}{m_\Phi^2}} .$$

Moroi, WY, 2011.09475 ,
See also 2011.12285 for the
relation to narrow parametric resonance

- Various narrow resonance numerical results can be understood from the formula

For light axion DM, @$t_{\rm prod}$ we need large $n_\phi^{\rm prod}$ and small $p_\phi^{\rm prod}$.

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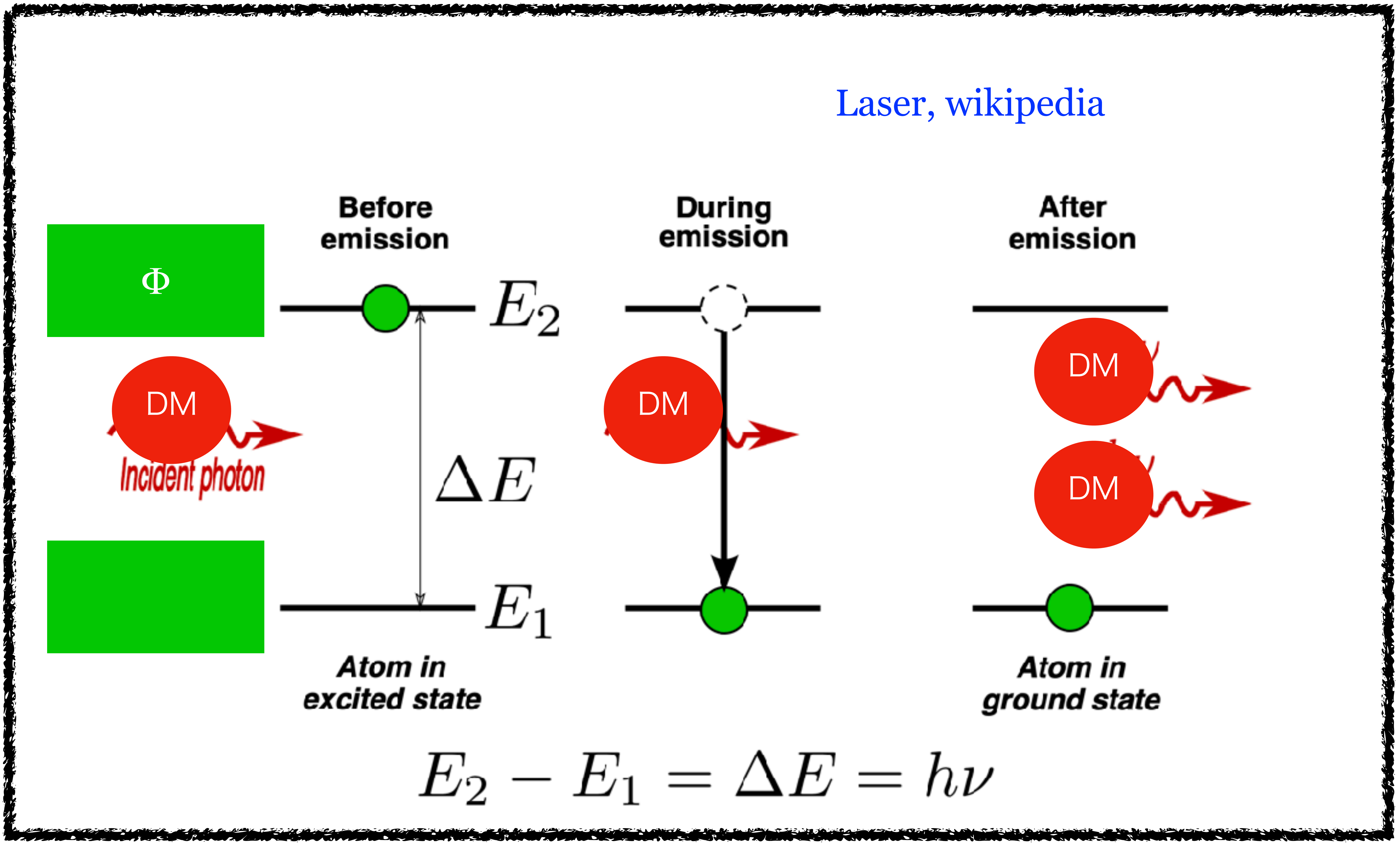

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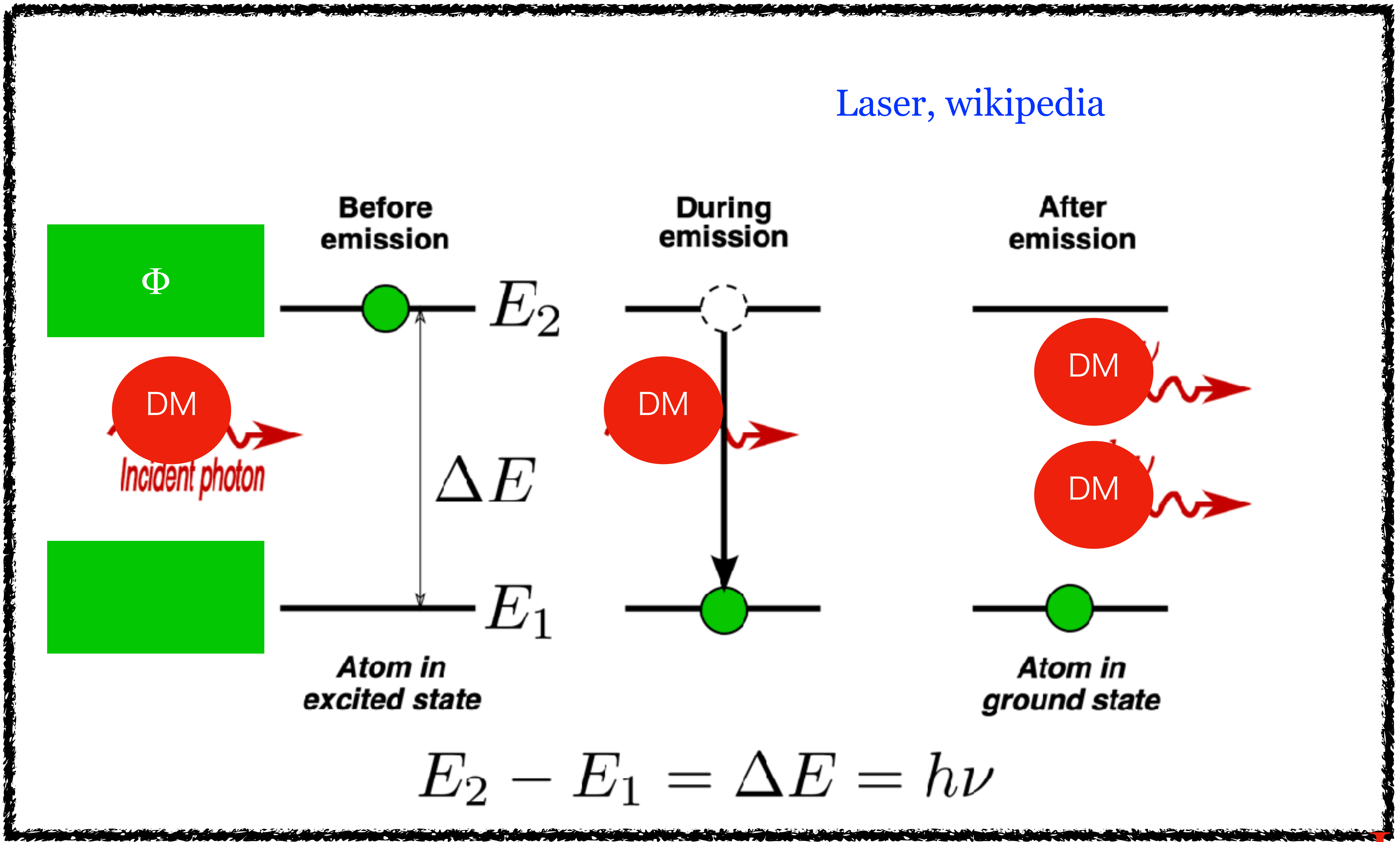

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Large n_Φ Small m_Φ

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Enhanced gravitational waves with different frequency

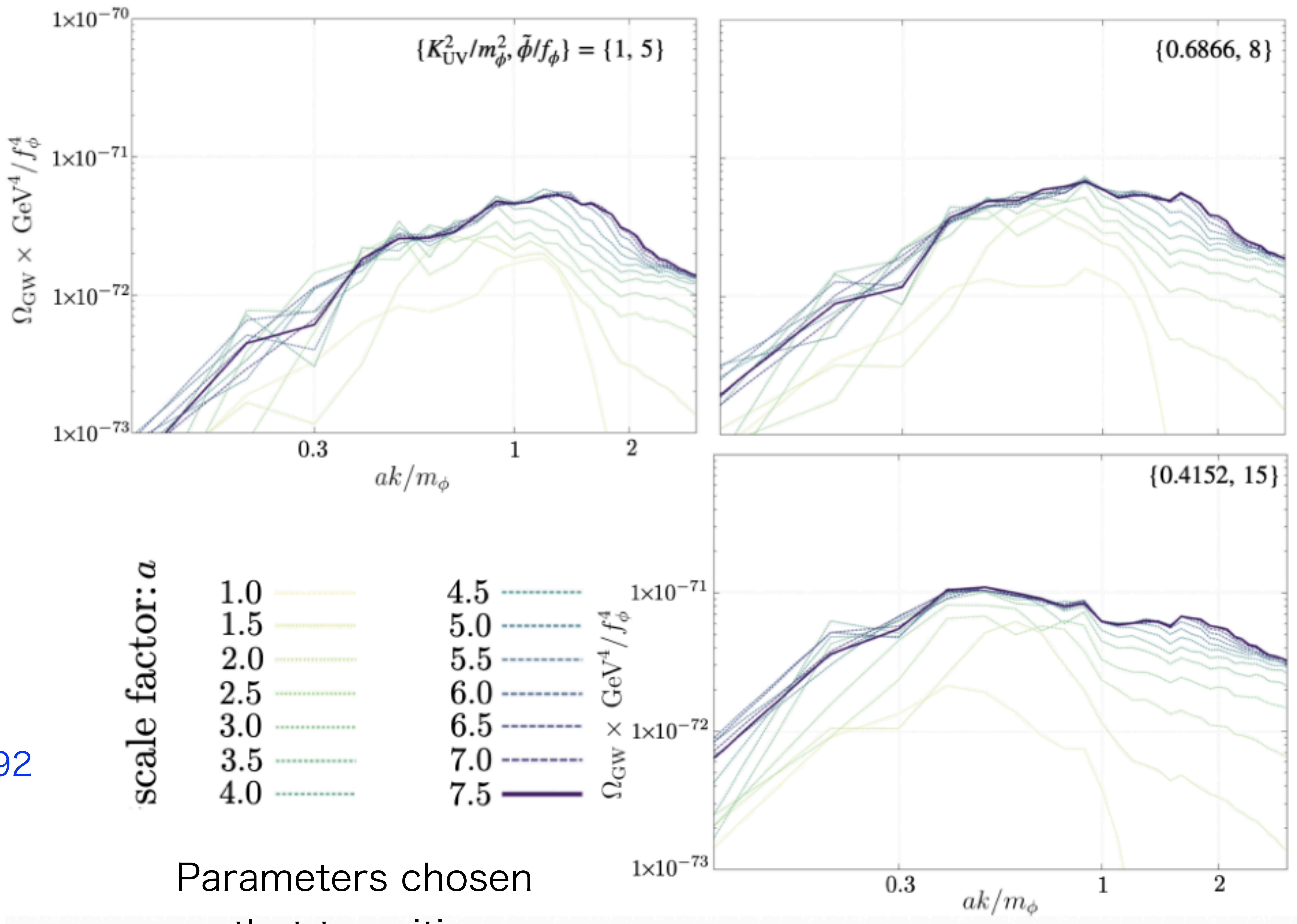
Hiramatsu et al, 1309.5001 for usual domain wall network

$$\Omega^{\rm DW}_{\rm GW,peak} \sim \frac{\sigma^2}{24\pi M_{\rm pl}^4 H_{\rm tr}^2}, \quad ak_{\rm peak} \sim aH_{\rm tr}.$$

See also Kitajima, Murai, Lee, Takahashi, WY, 2306.17146, for the precise spectra obtained from the full numerical simulation.

Collapse of Baumkuchen domain walls, Miyazaki, Yaginuma, Song, Narita, and WY,2509.13292

$$\Omega^{\rm BDW}_{\rm GW,peak} \sim \frac{N_{\rm BDW}^2 \sigma^2}{18 M_{\rm pl}^4 H_{\rm tr}^2}, \; ak_{\rm peak} \sim a\bar{p}_\phi,$$



Parameters chosen so that transition periods are the same.

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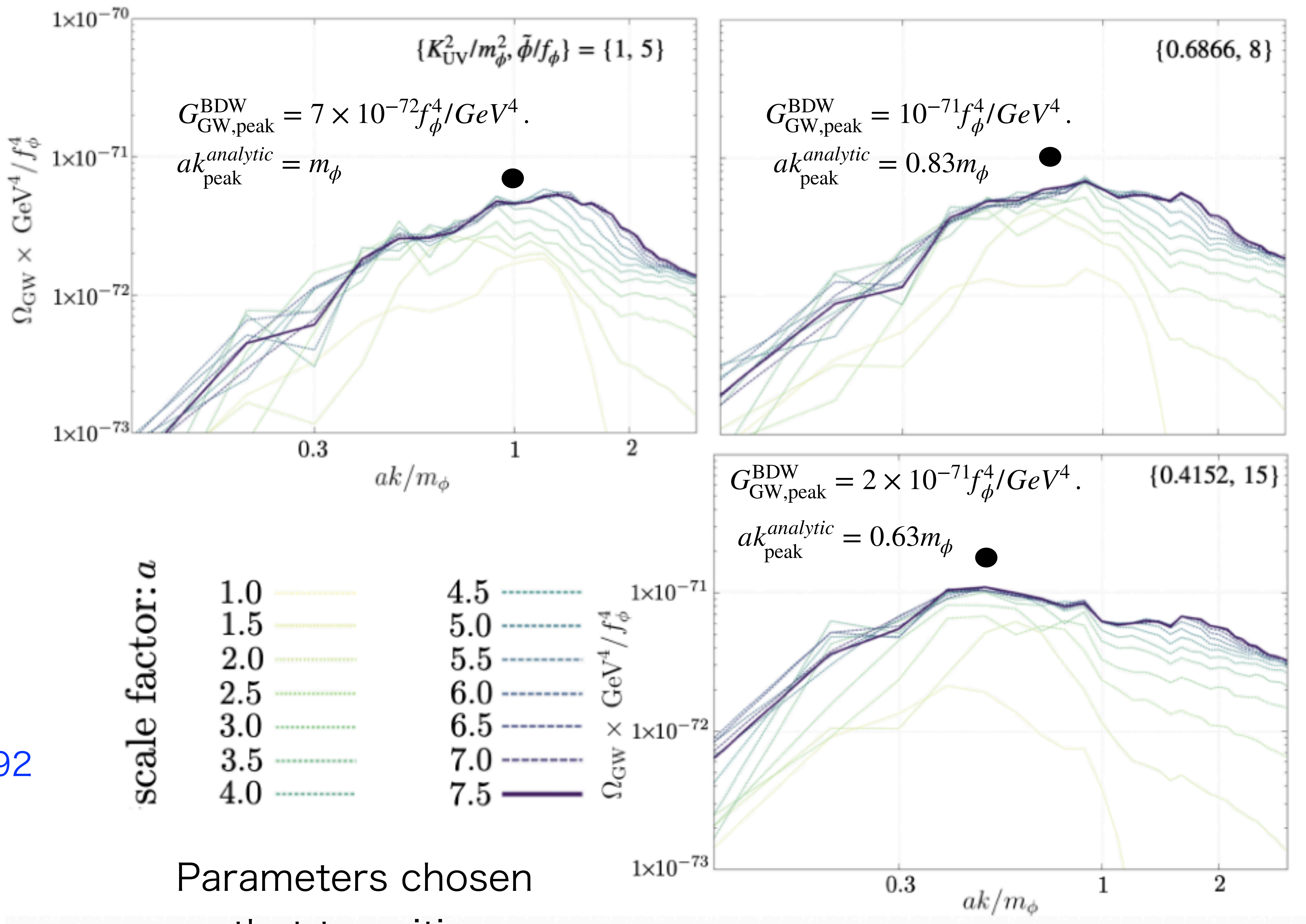

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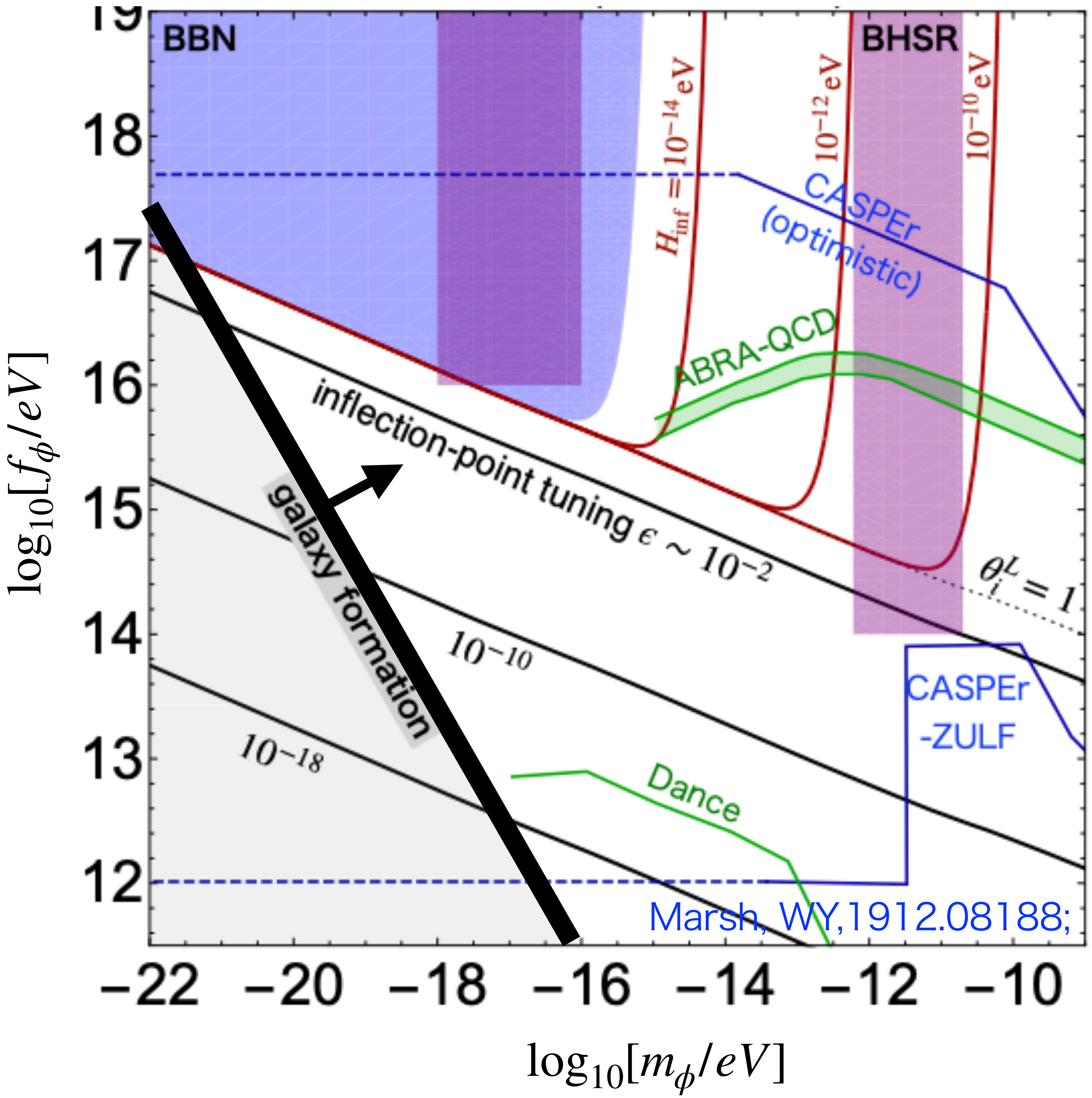
Generic limit of ALP DM from misalignment

Marsh, WY,1912.08188; Dror, Leedom, 2008.02279

- Let's take the onset of oscillation period, $z_{\rm osc}$, free.
- $z_{\rm osc} \gtrsim 10^6$, due to the Lyman-α limit: $\rho_\phi \leq m_\phi^2 f_\phi^2$ at z = 10^6
- Taking $\rho_\phi = \rho_{\rm DM}^0 (1+z)^3$,

$$f_\phi \gtrsim 4 \times 10^{13} GeV \frac{10^{-18}{\rm eV}}{m_\phi}$$



Scale invariant fluctuations

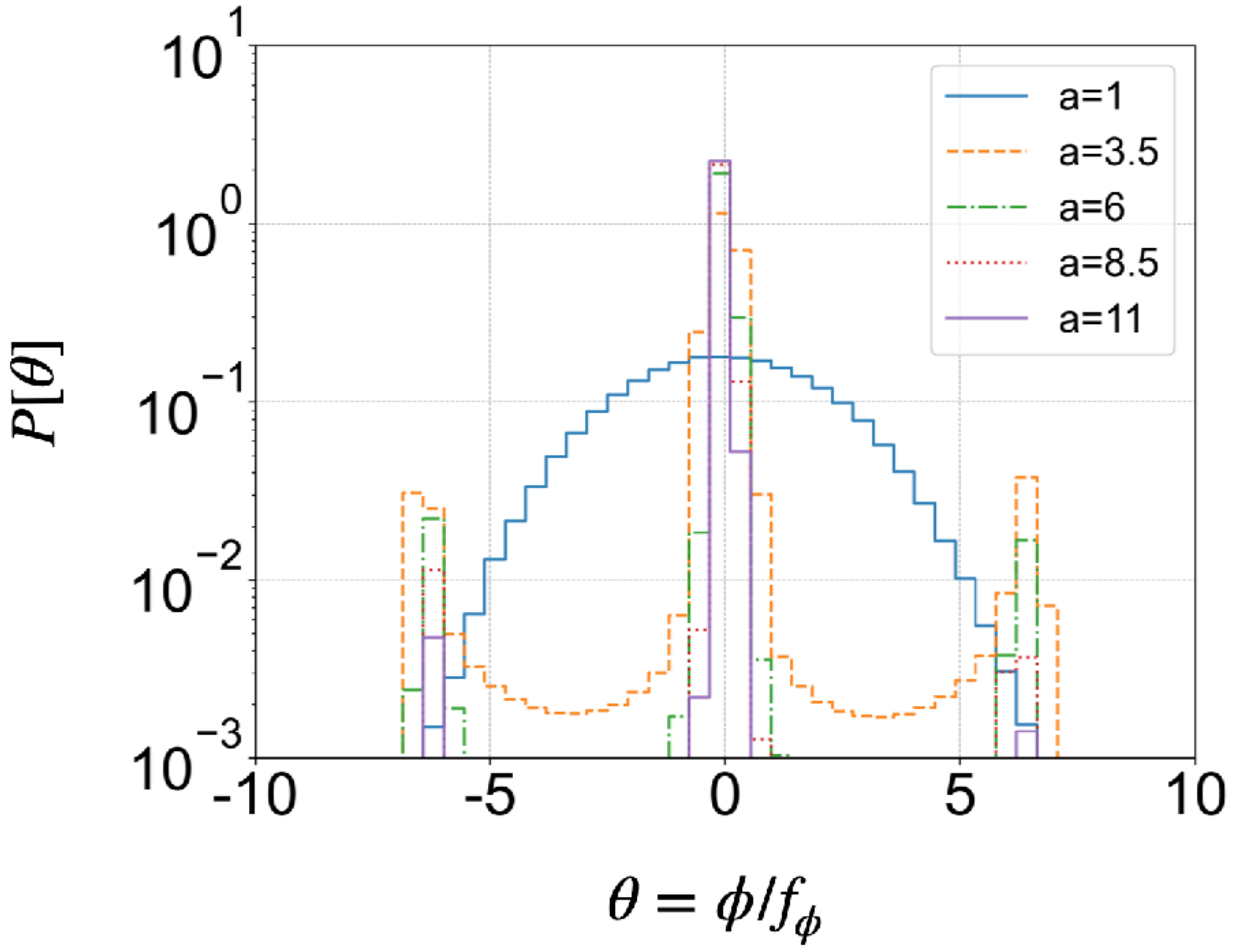


Fig. 7. Distribution of ϕ/f_ϕ for the case with scale-invariant fluctuations.

Dispersion relation of $w = k$

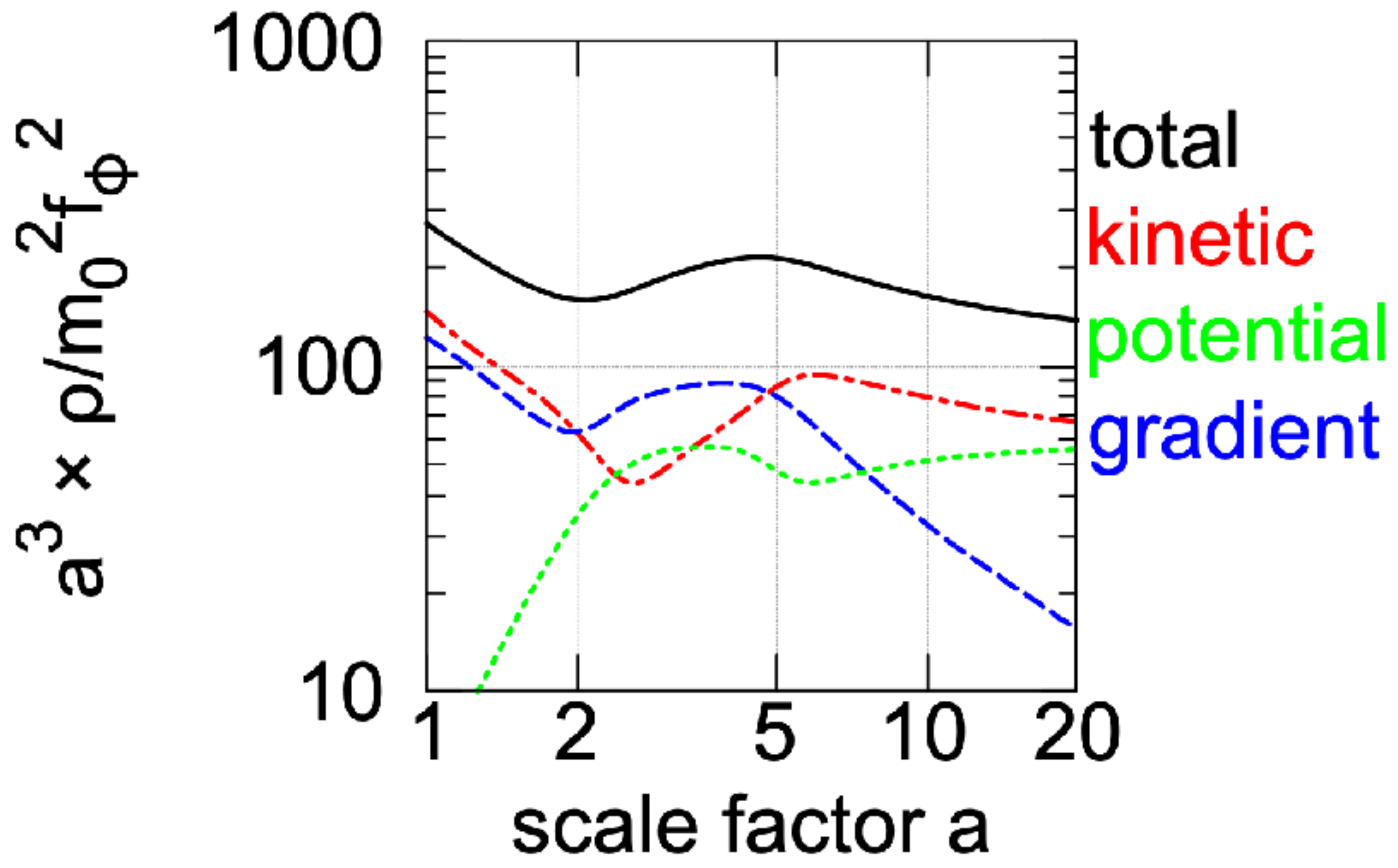


Fig. 8. Same as the bottom-left panel of Fig. 1, but adopting the initial condition $\omega = k$.