Parametric-resonance production of QCD axion DM

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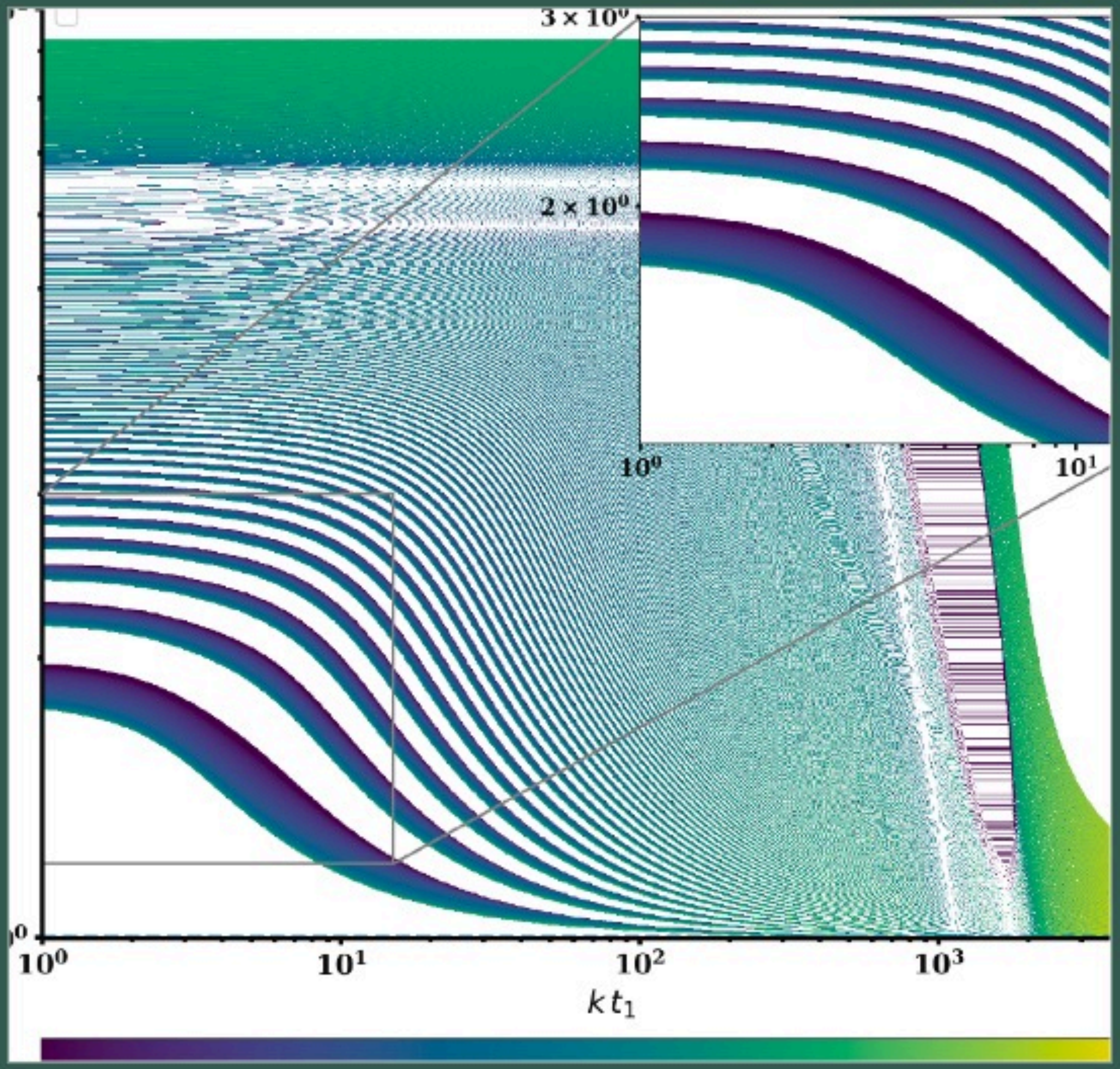
Axions in Seoul
July, 2026

Phys. Lett. B **879** (2026) 140680
Companion work: Chin. Phys. C **49** (2025) 125108



Resonance band-map from the full equation of motion — recurring growth episodes of axion modes

Summary

1 Primordial temperature perturbations periodically modulate the axion mass during the QCD transition — the axion field mass undergoes Mathieu-type parametric resonance.

2 Treated fully numerically, modes meet recurrent instability bands; the resonance component is numerically converged ($< 1\%$).

3 The resulting relic abundance shifts the 100%-DM axion mass window to **39 μeV -192 μeV**.

Summary

4 The resonance-generated component weakens the dependence on conventional isocurvature and defect production, and the mechanism generalizes to ALPs and other light bosonic dark matter. The post-inflationary case requires further study.

5 Outlook: sharper plasma modeling, the small-scale primordial spectrum, and the high-frequency haloscope programs that can test this window.

The new mass window

Pre-inflation scenario; the post-inflation scenario could need a dedicated treatment including strings and domain walls.

For the first time, the preferred QCD-axion dark-matter mass leaves the classical decade.

~10 μeV → **39 μeV** | **192 μeV**

misalignment only ($\theta_0 \sim O(1)$)

$\theta_0 = 1.3$ · 100% DM

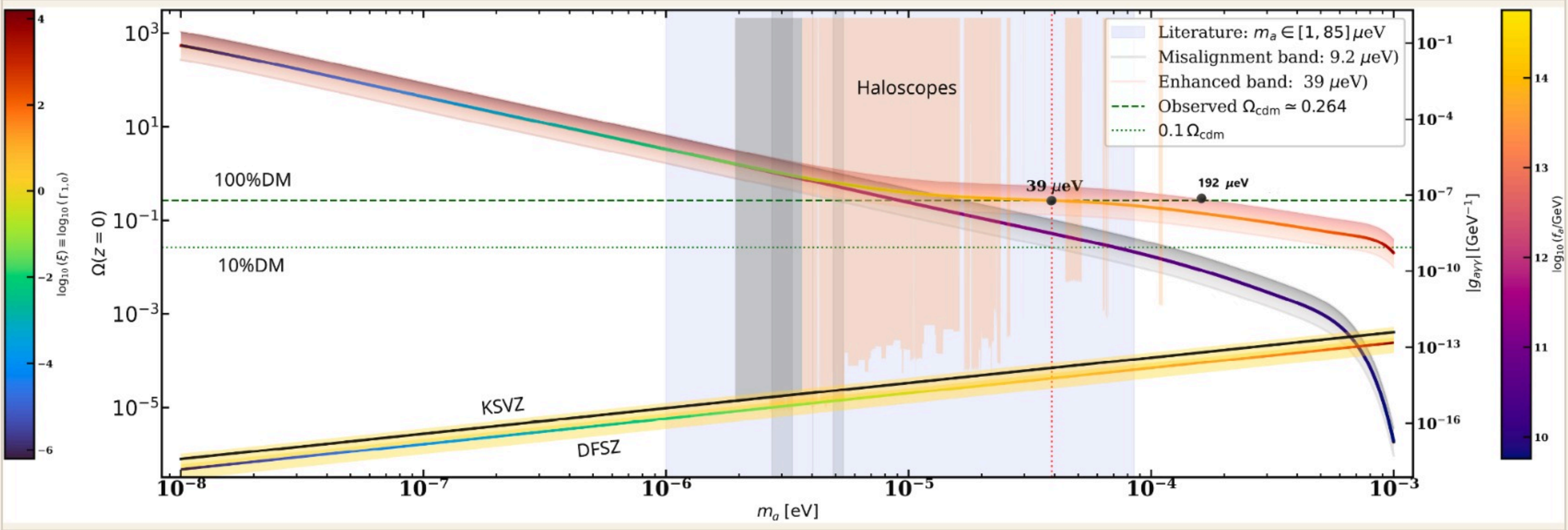
$\theta_0 = 2.0$ · 100% DM

- The preferred decade shifts — largely unexplored by running haloscopes.
- If misalignment underproduces (between the 10% and 100% lines), resonance still supplies the observed density.
- The ratio of resonance-to-misalignment abundance is θ_0 -independent — the shift is a robust relative statement.

Theory now points at the high-frequency frontier.

the axion DM preferred mass moves up

Resonance-enhanced production reaches 100% of dark matter at much higher mass.



Relic abundance vs. axion mass: homogeneous misalignment (grey band) and resonance-enhanced (red band); dashed green: observed $\Omega_{cdm} \simeq 0.264$; dotted: $0.1\Omega_{cdm}$. Lower band: KSVZ/DFSZ mass–coupling relation; shaded: haloscope constraints

The strong CP problem and the axion

QCD secretly allows a CP-violating term — but Nature keeps it absurdly small.

The problem

$$L \supset \theta \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

- Neutron-EDM experiments bound $|\bar{\theta}| \lesssim 10^{-10}$ — essentially zero
- Why is a free parameter of the theory tuned below 10^{-10} ? Pure fine-tuning

The Peccei–Quinn mechanism (1977)

- Promote $\bar{\theta}$ to a dynamical field $a(x)/f_a$ via a spontaneously broken global $U(1)_{PQ}$
- The QCD potential relaxes $\bar{\theta} \to 0$ dynamically
- The pseudo-Nambu–Goldstone boson of the breaking is the axion (Weinberg, Wilczek)

Benchmark realizations

KSVZ — a heavy quark carries the PQ charge

DFSZ — two Higgs doublets couple to the PQ current

The QCD axion cold-dark-matter

$$m_a \simeq 5.7\mu\text{eV}\left(\frac{10^{12}\text{GeV}}{f_a}\right)$$
$$L \supset -g_{a\gamma\gamma} a E \cdot B$$

- Produced non-thermally in the early Universe → born cold
- Extremely weak couplings, all $\propto 1/f_a$ → collisionless
- Effectively stable on cosmological timescales

✓ **Pressureless**

born non-relativistic, $v \approx 0$ today

✓ **Collisionless**

all couplings suppressed by $f_a \gtrsim 10^8\text{GeV}$

✓ **Stable**

lifetime far exceeds the age of the Universe

What remains is to pin down WHERE in $(m_a, g_{a\gamma\gamma})$ the QCD axion lives — that is the theory question this talk addresses.

The misalignment mechanism

Pre-inflation version; the post-inflation version needs to add evolutions of topological defects etc.

1. After PQ breaking, the potential is nearly flat — the field rests at a random initial angle $\theta_0 = \varphi_0/f_a \sim O(1)$, frozen by Hubble friction

3. Approaching the QCD scale: instanton effects switch on the potential $V(\varphi)$

5. Once $m_a(T) \gtrsim 3H$: the field rolls and oscillates coherently — a classical condensate

7. Oscillation energy redshifts as pressureless matter, $\rho_a \propto a^{-3}$ $\rightarrow$ cold dark matter

The relic density is set by an initial condition

$$\ddot{\varphi} + 3H\dot{\varphi} + m_a^2(T)\varphi = 0$$

The mass turns on at the QCD transition

- Lattice QCD: the topological susceptibility — and hence $m_a(T)$ — rises steeply through the QCD crossover: $m_a(T) \propto T^{-n}, n \simeq 3$ (Borsányi et al. 2016)

- Oscillations begin at t_1 defined by $m_a(t_1) \sim 3H(t_1)$ ($T \gtrsim 1\mathrm{GeV}$)

- The mass saturates by the end of the transition: $t_2 \sim 10^{-5} s$, $T \sim 100\mathrm{MeV}$

- While $m_a(T)$ evolves, ANY temperature perturbation modulates the axion mass — the seed of parametric resonance

The standard prediction: $m_a \sim \mu\text{eV}$

Requiring the axion to be 100% of dark matter fixes a remarkably sharp target.

- With $\theta_0 \sim O(1)$: misalignment gives $m_a \approx 10\mu\text{eV}$ as the 100%-DM point
- The literature window spans roughly 1–85 μeV depending on $\theta_0 \in [0.5, \pi]$, cosmology, and the mass-turn-on index n
- Lighter masses overproduce dark matter unless $\theta_0 \ll 1$ (tuned); heavier masses underproduce — the axion becomes a sub-dominant component
- Haloscopes (ADMX and successors) have been built around this classical window — so far without a signal
- Post-inflation scenario suggests a heavier mass (see talk before)

$m_a \sim$ μeV

the classical 100%-DM target for $\theta_0 \sim O(1)$

1 – 85 μeV

literature window

Preskill, Wise & Wilczek PLB 120 (1983) 127; Abbott & Sikivie PLB 120 (1983) 133; Dine & Fischler PLB 120 (1983) 137

Two horns of the standard picture

Axion exists BEFORE inflation

- Inflationary quantum fluctuations imprint $\delta\theta = H_I/(2\pi f_a)$ on the axion field
- These are isocurvature perturbations — uncorrelated with the adiabatic CMB spectrum
- CMB bounds force H_I far below f_a → the axion must be very light, or inflation very cold — a sharp constraint

Axion appears AFTER inflation

- θ_0 is random from horizon to horizon
- The random field network leaves cosmic strings and domain walls (topological defects)
- Defects decay

Both horns trace back to relics of initial conditions — and current haloscope null results add pressure from below.

The key idea

1

Inflation seeds primordial metric perturbations Φ_p — amplitude $A \sim 10^{-9}$, nearly scale-invariant.

2

The radiation fluid oscillates acoustically → periodic temperature perturbations $\delta T/T$ at each wavenumber k.

3

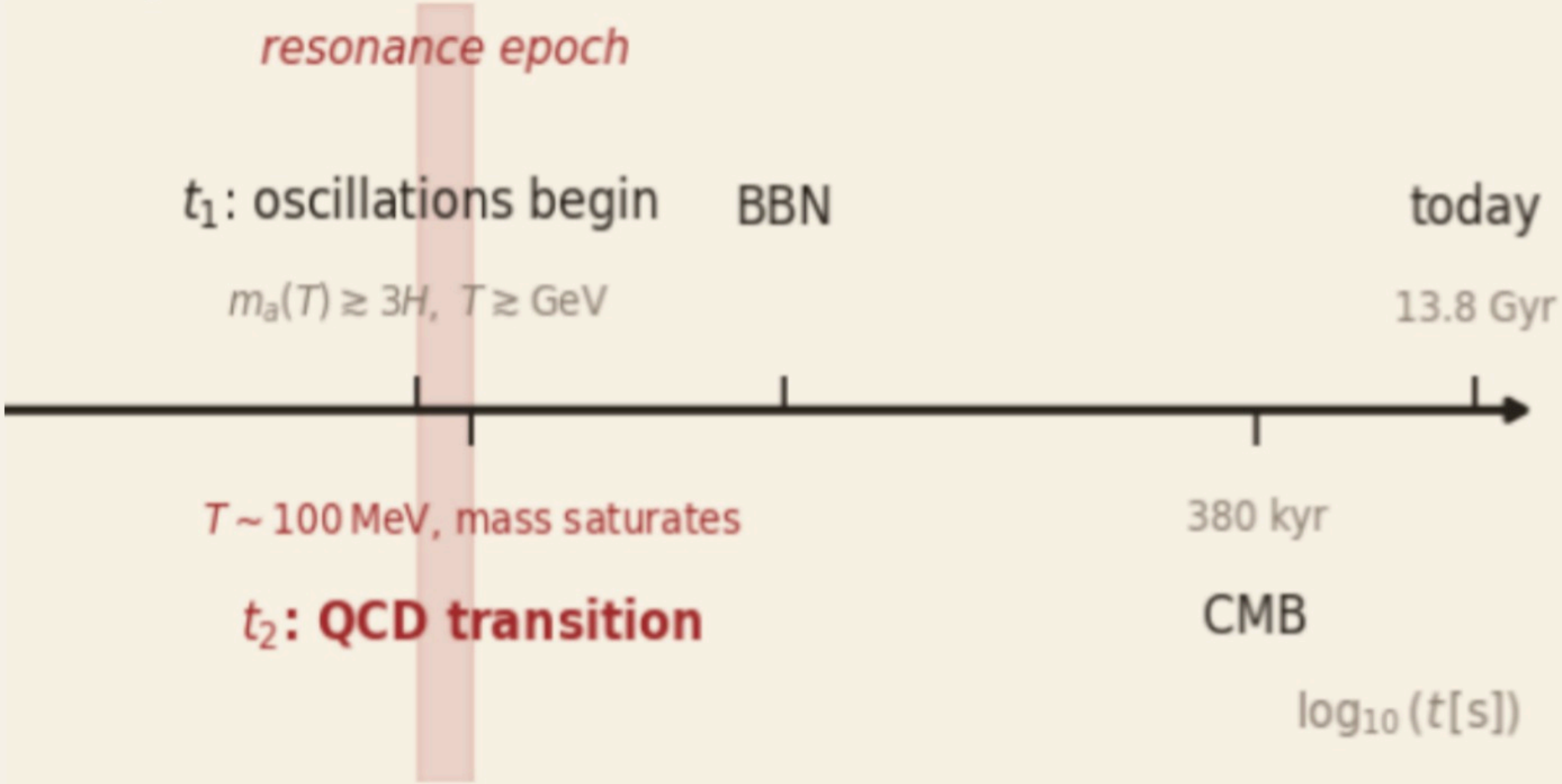
During the QCD transition, $m_a^2(T)$ is modulated periodically — a parametric drive on the axion field.

4

Mathieu-type resonance grows the resonant modes exponentially → an extra relic population.

— Sourced by temperature fluctuations, not by the initial displacement. does not introduce an independent isocurvature mode.

— axions are created parametrically even as θ_0 is small.


resonance epoch
t_1: oscillations begin
BBN
today
$m_a(T) \gtrsim 3H,\ T \gtrsim$ GeV
13.8 Gyr
$T \sim 100$ MeV, mass saturates
380 kyr
t_2: QCD transition
CMB
$\log_{10}(t[\mathrm{s}])$

Parametric resonance

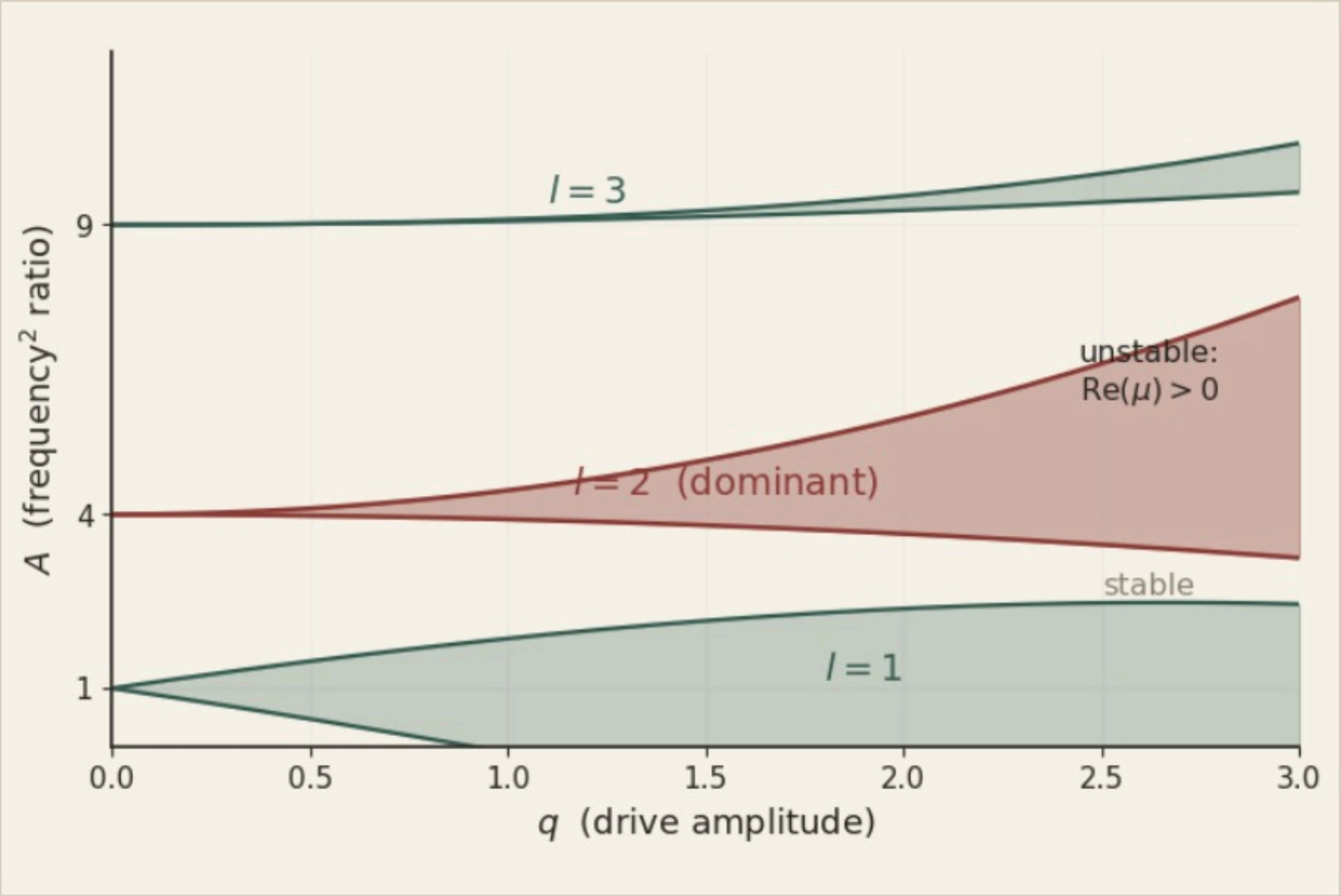
$$\frac{d^2\varphi}{dz^2} + (A - 2q\cos 2z)\varphi = 0$$

— A = (natural frequency / drive frequency)^2; q = dimensionless drive amplitude.

— Solutions grow exponentially, $\varphi \propto e^{\mu z}$, inside instability bands around $A \simeq l^2$, $l = 1, 2, \ldots$ — the drive near twice the natural frequency ($l = 2$) dominates.

— Narrow regime $|q| \ll 1$: band width $\Delta A \sim q^2$ — but growth accumulates over many cycles.

— Not a forced resonance: the spring constant itself is modulated — like a child pumping a swing, never pushing directly.



Instability tongues of the Mathieu equation around $A = l^2$. The $l = 2$ tongue ($A \simeq 4$) dominates the cosmological application (schematic).

Axion field dynamics in the perturbed Universe

Start from the Klein–Gordon equation with a temperature-dependent mass — then add the leading scalar perturbations.

$$D^\mu \partial_\mu \varphi(\vec{x},t) - m^2(T)\varphi(\vec{x},t) = 0$$

In conformal Newtonian gauge, keep terms linear in the metric potentials Φ and Ψ :

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2}{a^2}\varphi + m^2(T)\varphi + f(\vec{x},t,\varphi) = 0$$

The unperturbed backbone

— Radiation era: $a(t) = \sqrt{t/t_1}$, so $k^2/a^2(t) = k^2 t_1/t$.

— t_1 marks the onset of oscillations, $m_a(t_1) \sim 3H(t_1)$.

$$\ddot{\varphi}(\vec{k},t) + \tfrac{3}{2t}\dot{\varphi}(\vec{k},t) + \omega_k^2(t)\varphi(\vec{k},t) = 0$$

with $\omega_k^2(t) \equiv k^2 t_1/t + m^2(t)$.

The perturbation

— $f(\vec{x},t,\varphi)$ collects everything linear in Φ , Ψ and δT .

— Sourced by the primordial field $\Phi_p(\vec{k})$ — stochastic, ensemble.

— This is where the periodic drive lives.

One perturbation, three modulation channels

The perturbation functional, linear in Φ , Ψ and δT :

$$f(\vec{x},t,\varphi) = -2\Psi\ddot{\varphi} - (\dot{\Psi} - 3\dot{\Phi} + 6H\Psi)\dot{\varphi} + \frac{2\Phi}{a^2}\nabla^2\varphi$$

$$-\frac{1}{a^2}\partial_j(\Phi + \Psi)\partial_j\varphi + \frac{dm^2}{dT}\delta T\varphi$$

i

Inertia renormalization

The $\ddot{\varphi}$ term rescales the effective kinetic operator — a periodic mass–inertia.

ii

Friction modulation

The $\dot{\varphi}$ term makes the Hubble drag oscillate at the drive frequency.

iii

Mass modulation

Modulation via $(dm^2/dT)T$, sourced by δT — dominates energy injection because $m(T)$ grows steeply through the transition.

All three channels oscillate at the acoustic frequency of the radiation fluid — together they form one coherent periodic drive.

Acoustic driving sets the resonance clock

— Sub-horizon in the radiation era, the primordial potential oscillates acoustically

— The perturbation terms inherit the phase $\Theta_k(t) = 2k\sqrt{tt_1/3}$ — a clock whose rate grows with k .

— Resonance strikes when the drive hits an integer multiple of the mode frequency, the $l = 2$ band dominates.

— Because $m(T)$ keeps growing, each mode meets its $l = 2$ condition in a short window around $t_R(k)$; high- k modes resonate earlier.

— Reference scale: $k_2 \equiv m\sqrt{3t_2/t_1}$.

Consequences

— The coefficients vary slowly during one crossing a local Mathieu analysis is valid.

— But the drift lets one mode cross several bands
— full numerics are needed for the net effect.

The drive needs no tuning: as $m(T)$ grows, every mode finds its own resonance window.

Reduction to the Mathieu equation

1 · stretch time to the acoustic variable:

$$z \equiv \frac{k}{\sqrt{3}}\eta(t) = 2k\sqrt{tt_1/3}$$

2 · redefine the field to remove the friction term;

3 · near a resonance episode, neglect the slow drift:

$$\partial_z^2\varphi_k + (A_k - 2q\cos 2z)\varphi_k = 0$$

$$A_k = 3 + \frac{3m^2(t)}{k^2}, q = \frac{dm^2(t)}{dT}\,\frac{9\Phi_p(\vec{k})T(t)}{4k^2}$$

Instability bands sit at $A_k \simeq l^2$; the dominant $l = 2$ band corresponds to $A_k \simeq 4$, with band width $\Delta A_k \sim q^2$.

Why this stays perturbative

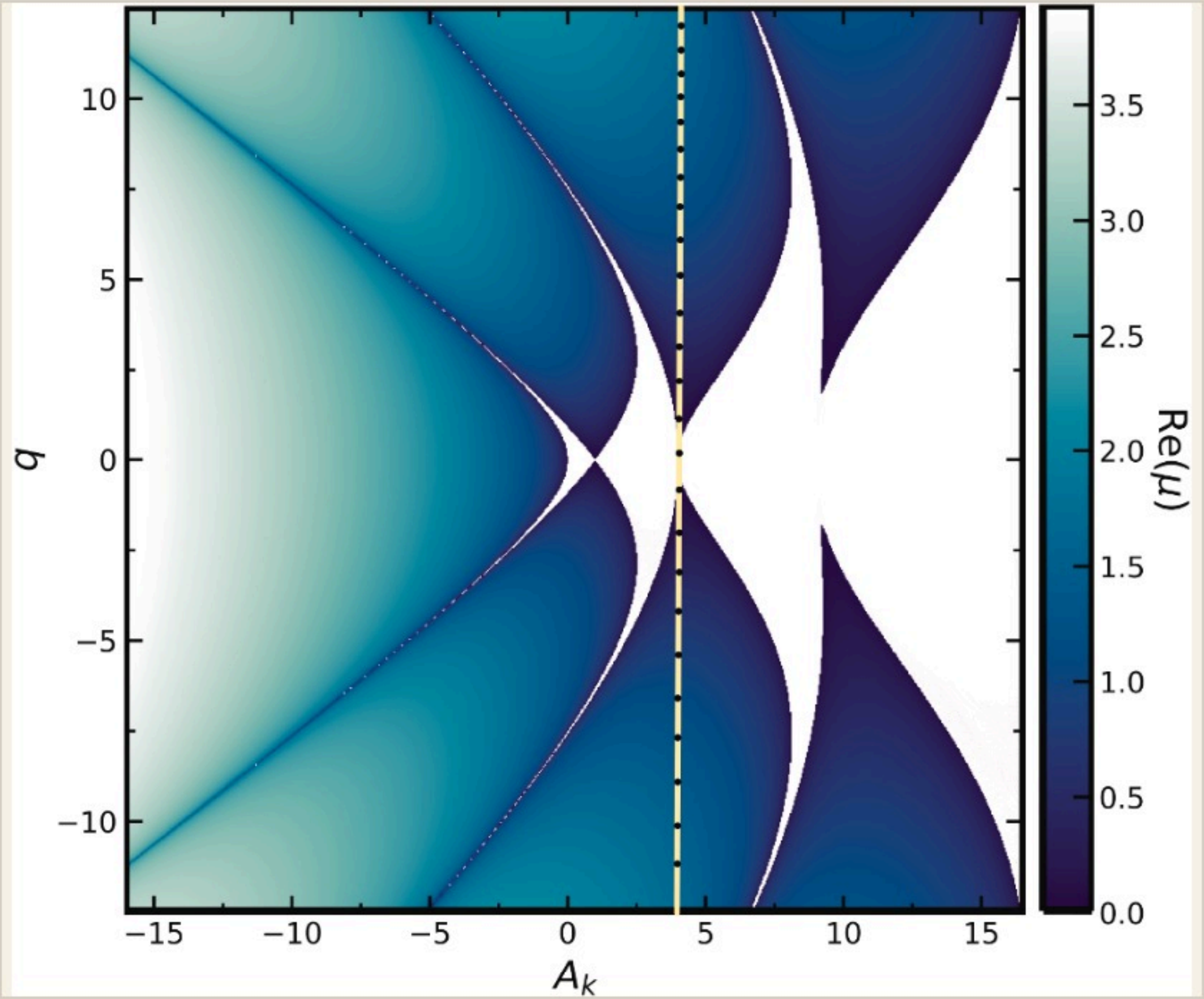
— $q \propto \Phi_p$ ($\sim 10^{-4} - 10^{-5}$, times mass factors) narrow resonance, $|q| \ll 1$.

— Each crossing is brief → dynamics stay linear; backreaction and mode coupling negligible.

— $q \propto dm^2/dT$ → the pump is ON only while the mass turns on; $q \propto n$ → the lattice index $n = 3$ sets the benchmark efficiency.

— The $A_k \simeq 4$ condition ($l = 2$) picks out the resonant modes.

The instability chart



Largest Floquet exponent $\mathrm{Re}(\mu)$ in the (A_k, q) plane; colored regions grow. Black contour: instability boundary; vertical dash-dot line: $l = 2$ resonance center at $A_k = 4$

— White wedges: stability; colored tongues: exponential growth — darkest = fastest.

— Our cosmological drive is weak ($|q| \ll 1$) — the physics hugs the band centers $A_k \simeq l^2$.

— The dash-dot line $A_k = 4$ marks the dominant $l = 2$ band.

— Because $A_k(t)$ and $q(t)$ drift as $m(T)$ grows, a given mode sweeps through this chart — recurrent growth episodes, not one clean crossing.

A mode's trajectory through this chart — not a single crossing — sets its total growth

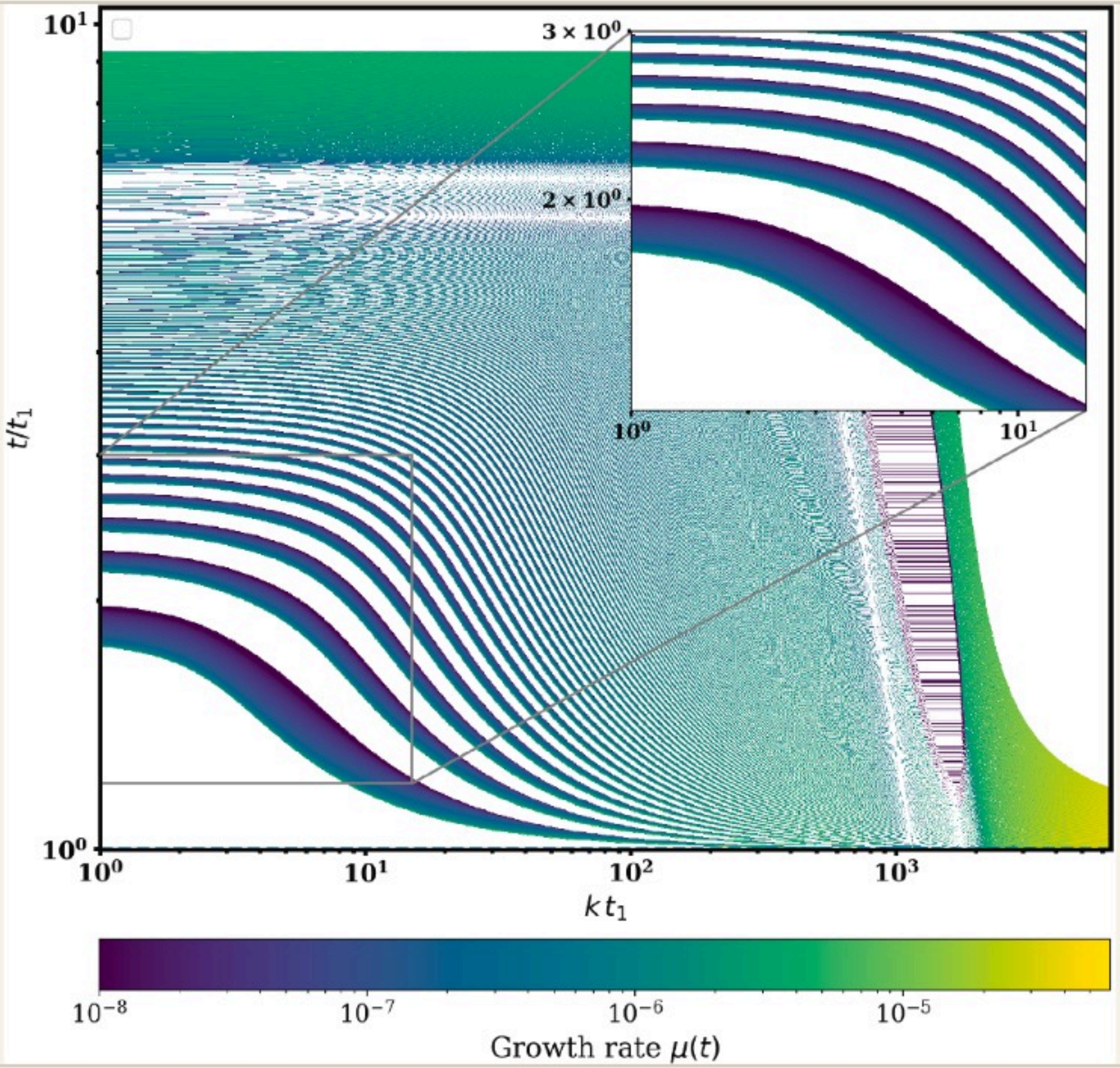
Band map from the full equation of motion

— Dark bands: epochs where the plasma drive efficiently produces axions.

— Bands drift and cluster at large kt_1 — one mode meets several episodes.

— High kt_1 modes resonate earlier → their energy is more redshifted by t_2 .

— Broken texture near $kt_1 \sim 2 \times 10^3$: numerical precision limit.



Numerical procedure: an ensemble over the primordial field

Ensemble average. The primordial potential is drawn Gaussian, $\langle \Phi_p \rangle = 0$, $\mathrm{Var}(\Phi_p) = A \simeq O(10^{-9})$, with a nearly scale-invariant spectrum

Mode energy yields the spectral density.

Numerical validation. Linear response: $\varphi_1 \propto \Phi_p$, $\rho_1 \propto \langle \Phi_p^2 \rangle = A$ — the correction is physical, not a numerical transient.

Varying time-step, integration interval, and band resolution changes the relic result by $< 1\%$.

Integrating over the resonant modes $k \in [t_1^{-1}, k_2]$ gives a conservative upper bound.

From resonant modes to today's relic density

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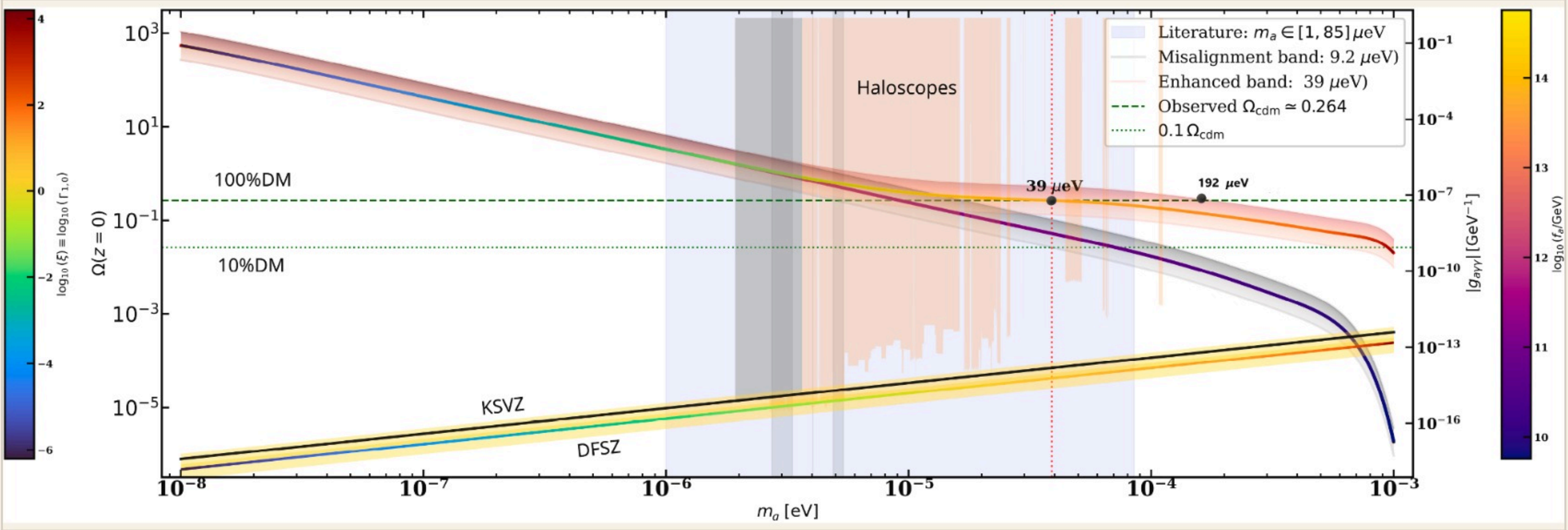
- Resonance therefore enters as a **multiplicative boost** of the standard misalignment density.

- Estimate for $n = 3:$ $\Omega_1 \propto A(m_0 t_2)^{7/4}$ — the tiny primordial amplitude is compensated by the huge number of oscillations across the transition, $m_0 t_2 \gg 1$.

the relic curve moves up

Resonance-enhanced production reaches 100% of dark matter at much higher mass.



Red band: 100% DM at **39 μeV** ($\theta_0 = 1.3$) and **192 μeV** ($\theta_0 = 2$) — versus 9.2 μeV for misalignment alone.

How robust is the shift?

Systematics

- Dominant uncertainty: the mass-turn-on index n . In linear response $\delta\rho/\rho \approx 2\delta n/n$ — $n = 2.5$ instead of 3 moves the resonance density ≈ 34%.
- Preferred masses (39 / 192 μeV) are therefore benchmark values at $n = 3$, not point predictions.
- Low-mass suppression: $q \propto dm^2/dT \propto m^2$ → pumping efficiency falls quadratically for $m_a \ll 10$ μeV — resonance is a high-mass phenomenon.
- Residuals: plasma modeling and the small-scale primordial spectrum.
- This work considered pre-inflation scenario; Post-inflation scenario

  is more complicated and should be studied in the future.

Validity

✓ $m_a \leq 10^{-3}\mathrm{eV}$ keeps q small — the perturbative expansion holds.

✓ Brief band crossings → linear regime throughout; nonlinear backreaction neglected, consistently.

✓ Numerical stability < 1% against step-size, interval, and band-resolution changes.

✓ Majority of resonance-boosted modes non-relativistic by t_2 → dilution treatment consistent.

What this means for the experimental search

Current haloscopes were optimized for the classical window — the resonance shift argues for higher frequencies.

- Running haloscope sensitivity concentrates around ~1–40 μeV; the new 100%-DM points (39, 192 μeV) sit at and beyond the upper edge.
- Because $g_{a\gamma\gamma} \propto m_a$ on the QCD line, heavier targets are not hopeless — couplings grow with mass along the KSVZ/DFSZ band.
- A confirmed high-mass signal would itself discriminate production histories.

Probing the high-frequency frontier

- **MADMAX** — dielectric haloscope, tens to hundreds of μeV
- **Millimeter-wave dielectric haloscope** — dark-photon/axion constraints (PRL 2026)
- **BRASS-p** — permanently magnetized conversion surface
- **BREAD** — broadband dish antenna (44–52 μeV demonstrated)

The mechanism converts a cosmological observation — primordial perturbations — into a concrete experimental target.

What the mechanism buys us

Reduced sensitivity to pre-inflationary isocurvature constraints — production is sourced by temperature fluctuations of the plasma, not by an initial field displacement;.

An enlarged, heavier window — the preferred mass moves from μeV into 10– $100\mu\mathrm{eV}$, opening unexplored territory.

Topological defects — resonance provides an additional plasma-sourced abundance; its interplay with strings and domain walls remains to be calculated...

A generic phenomenon — temperature perturbations are intrinsic to the cosmological plasma; the same mechanism applies to ALPs and other light bosonic dark matter (companion work: CPC 49 (2025) 125108).

Summary

Primordial perturbations periodically modulate the axion mass during the QCD transition — the axion field undergoes Mathieu-type parametric resonance.

Treated fully numerically, modes meet recurrent instability bands; the resonance component obeys $\rho_1 \propto A$ and is numerically converged ($< 1\%$).

The relic abundance is boosted: the 100%-DM mass moves from ~10 μeV to **39 μeV** or **192 μeV** (Pre-inflation scenario) and a dedicated treatment including strings and domain walls. (Post-inflation scenario).

The channel relieves isocurvature bounds, relieves the topological-defect issue, and generalizes to ALPs and other light bosonic dark matter.

Outlook: From mechanism to discovery

Establish robustness:
improved QCD and plasma modelling;
independent numerical reproduction;
nonlinear lattice evolution and backreaction...

Map the physical target:
robust mass envelope;
phase-space and coherence properties;
small-scale density structure and CMB consequences...

Outlook: From mechanism to discovery

Connect to experiments:

high-frequency haloscopes;detector-facing signal predictions; complementary atomic and interferometric possibilities...

The next stage will benefit from broader expertise in finite-temperature QCD, nonlinear field simulations, and high-frequency axion detection. I would welcome discussions and collaborations.

THANK YOU

Questions & discussion

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companion: Chin. Phys. C 49 (2025) 125108