Precision Cosmology Beyond Linear Theory

Sanity checks on second-order observables descriptions

Matteo Magi
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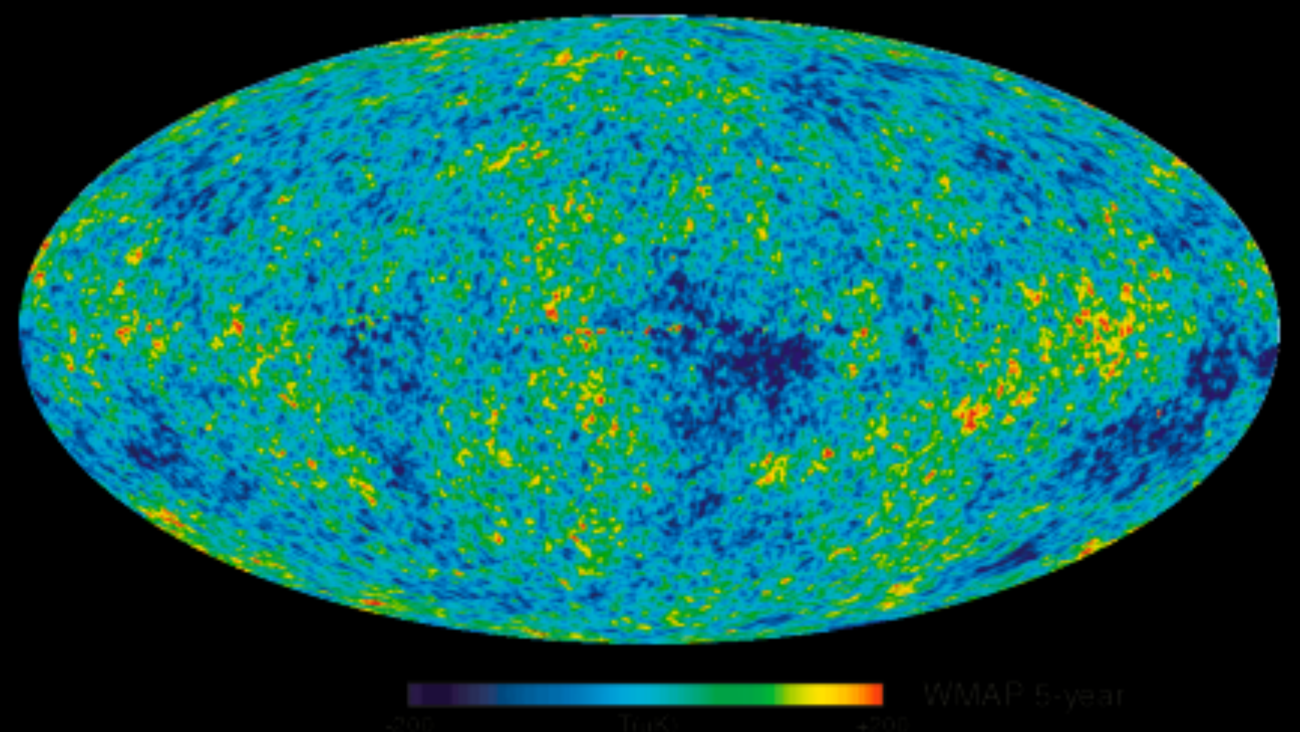
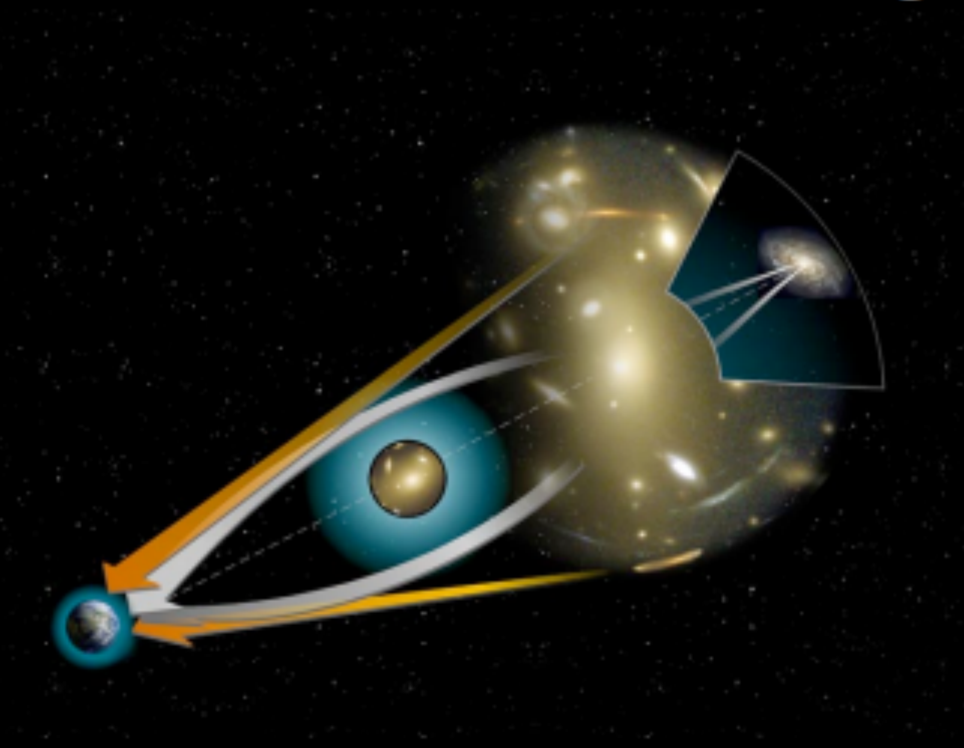
Cosmological Probes

Temperature
Anisotropies
$T(\hat{n})$

Cosmic Microwave Background



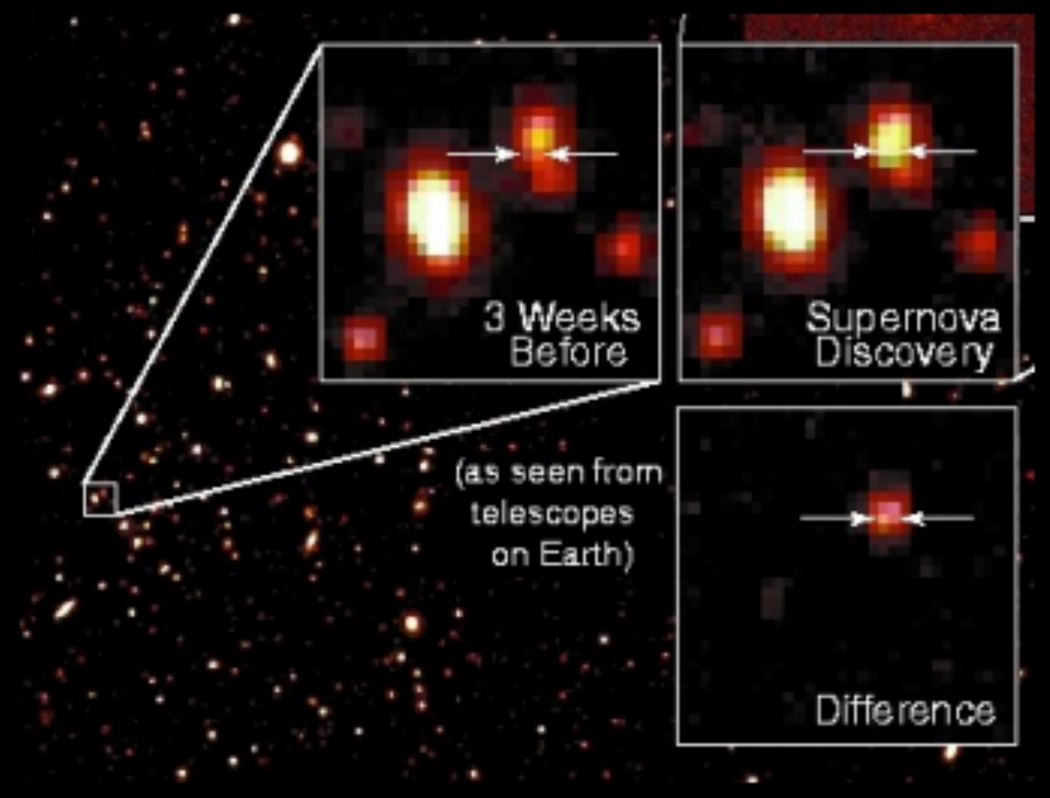
Gravitational Lensing



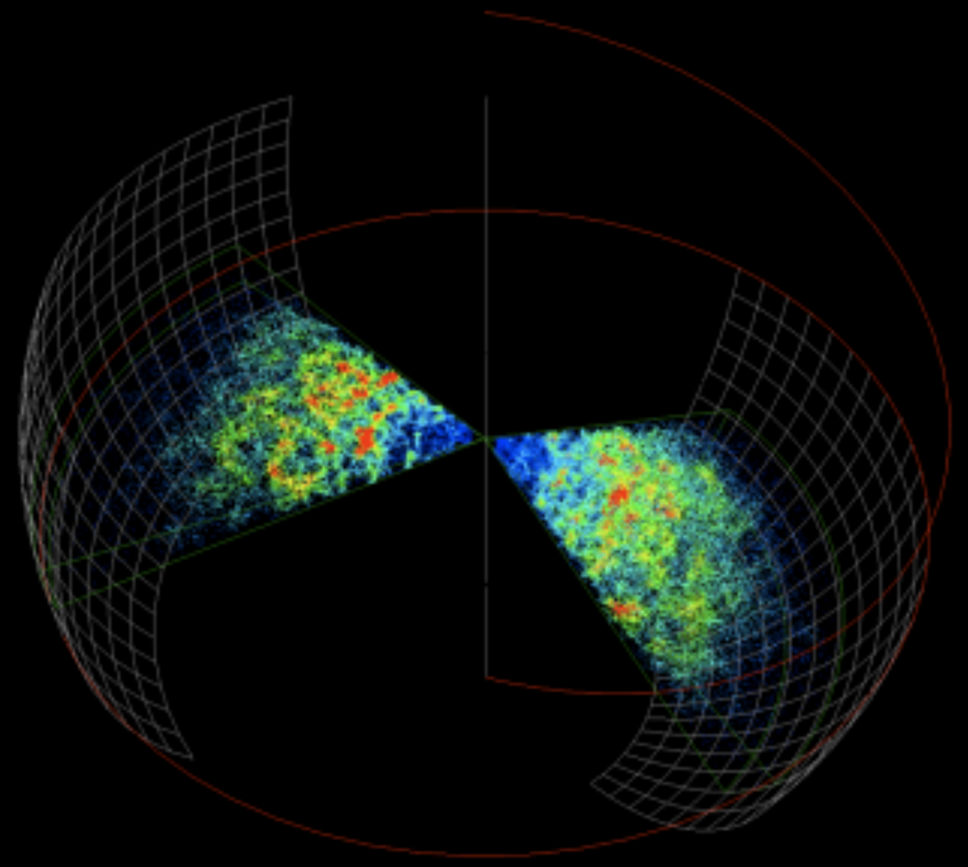
Galaxies
Ellipticity
$\varepsilon(\hat{n}, z)$

Luminosity
Distance
$D_L(\hat{n}, z)$

Supernovae

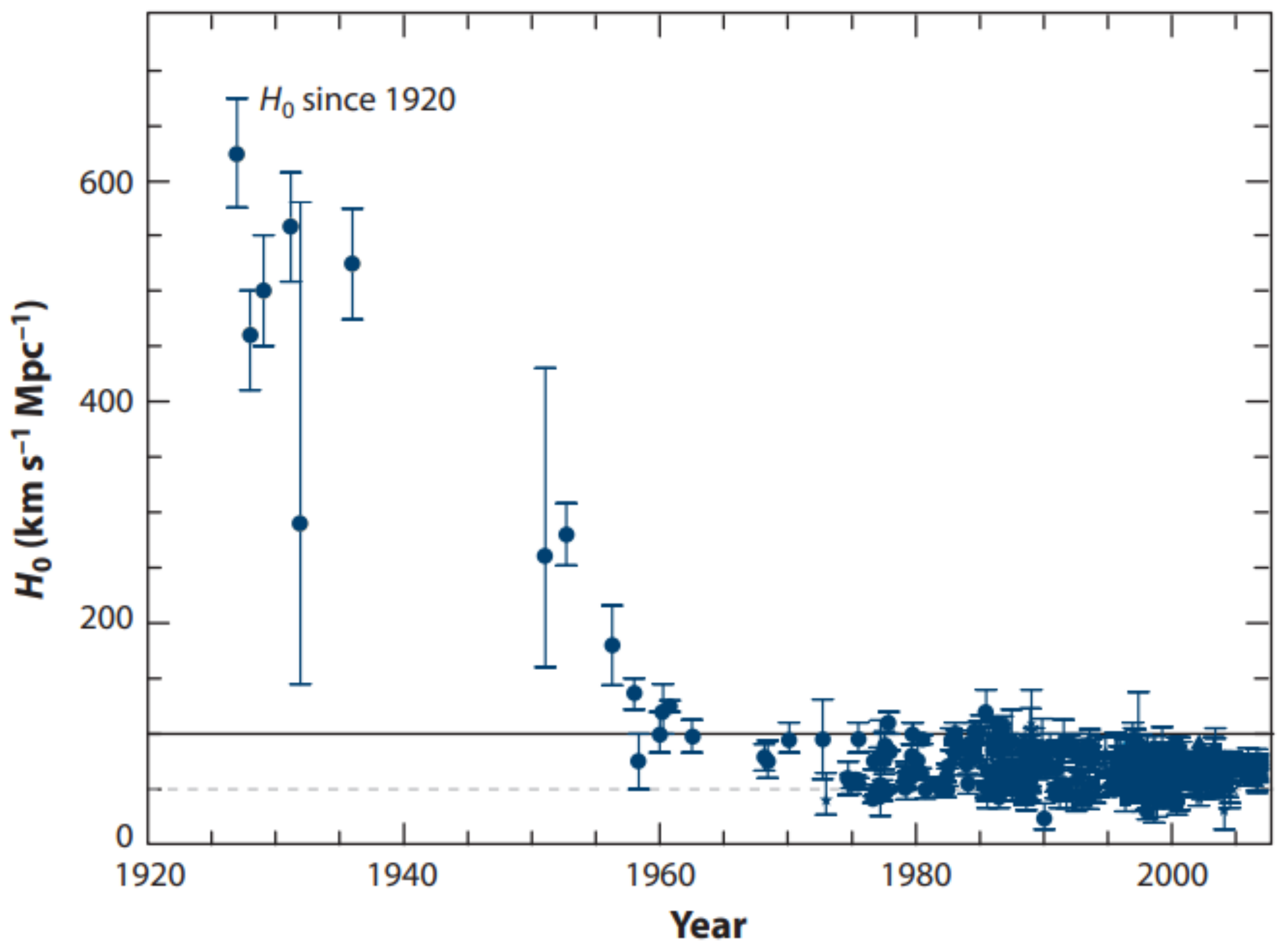


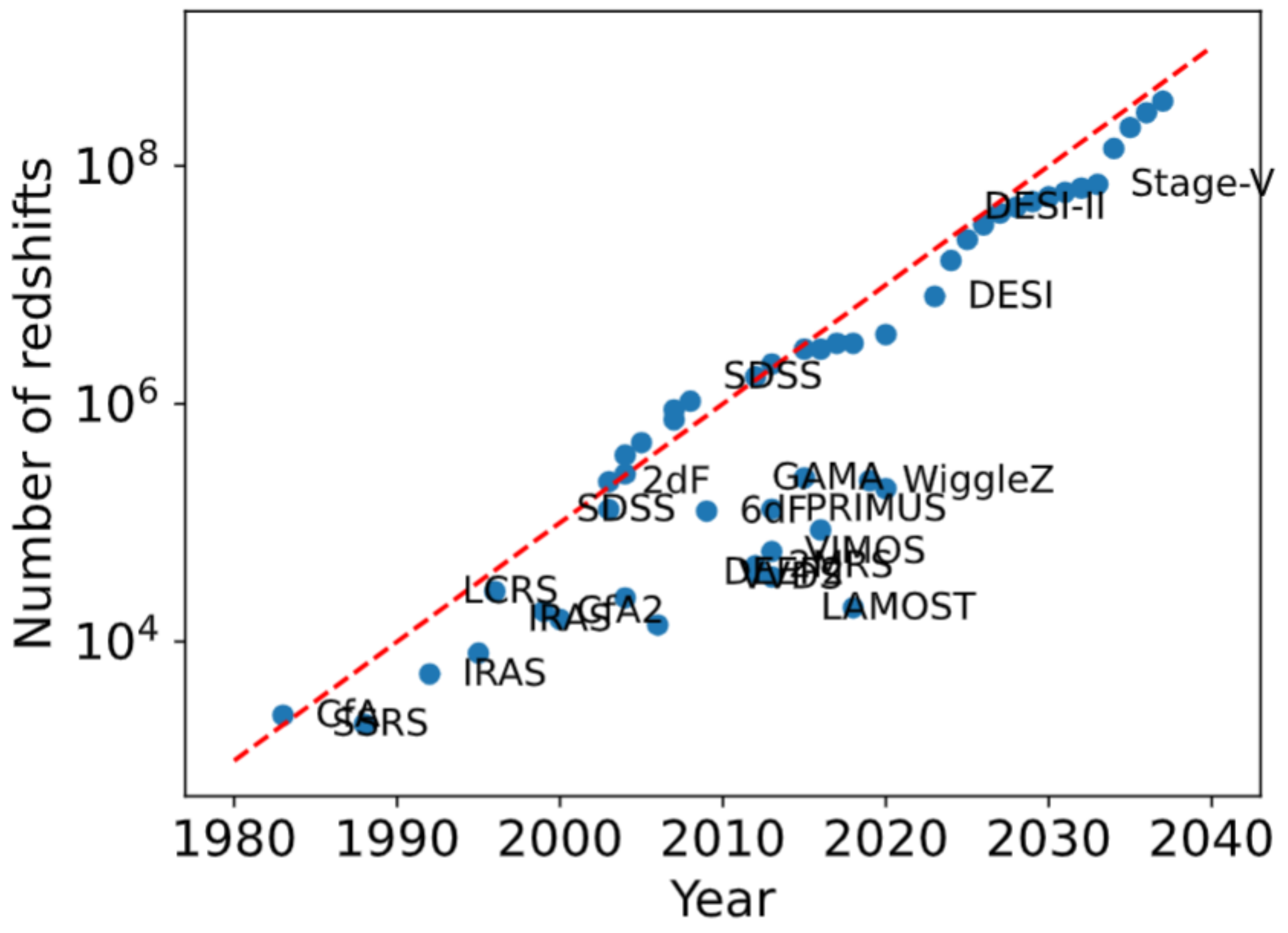
Galaxy Clustering

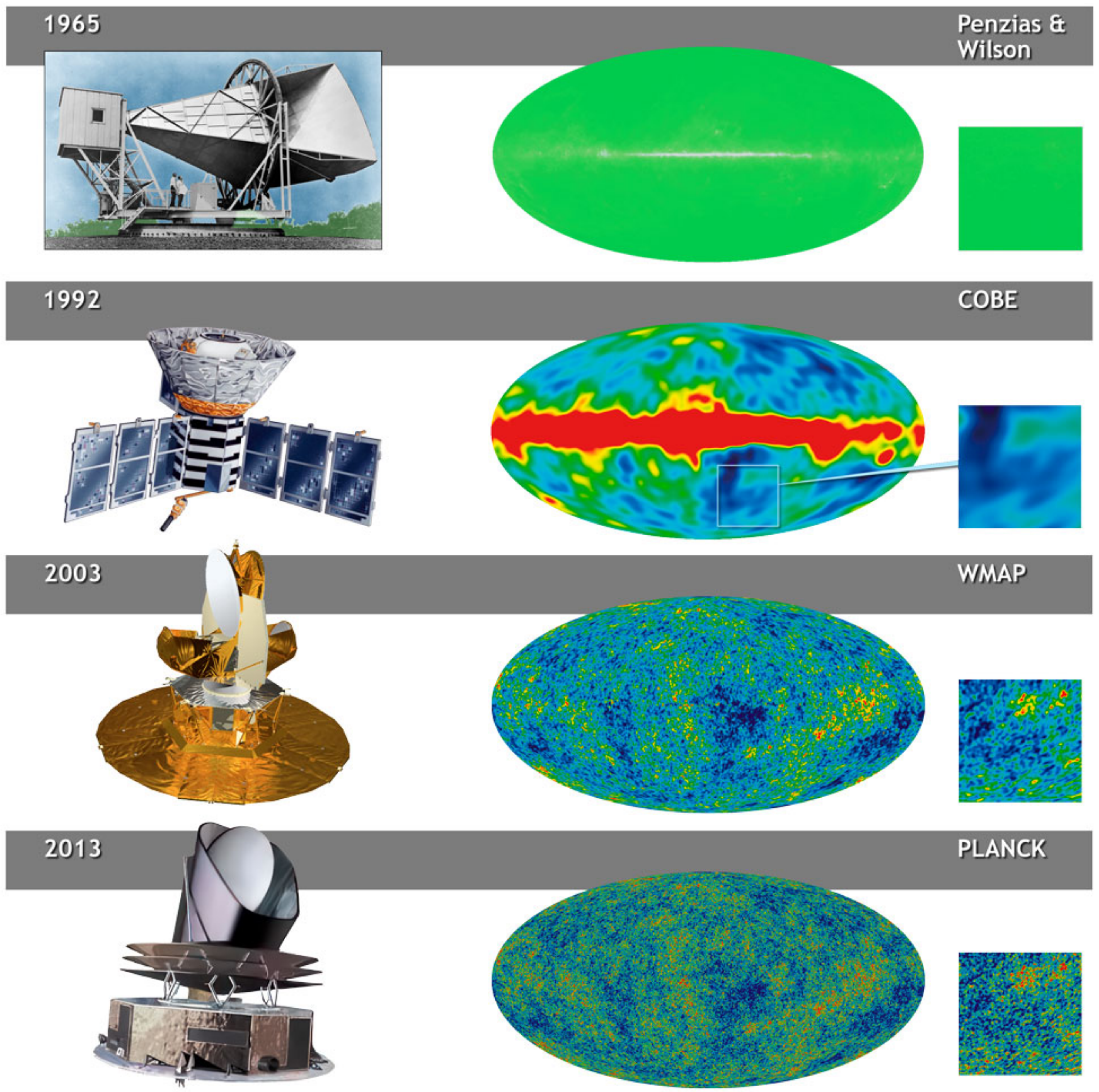


Galaxies
Number Density
$\delta_g(\hat{n}, z)$

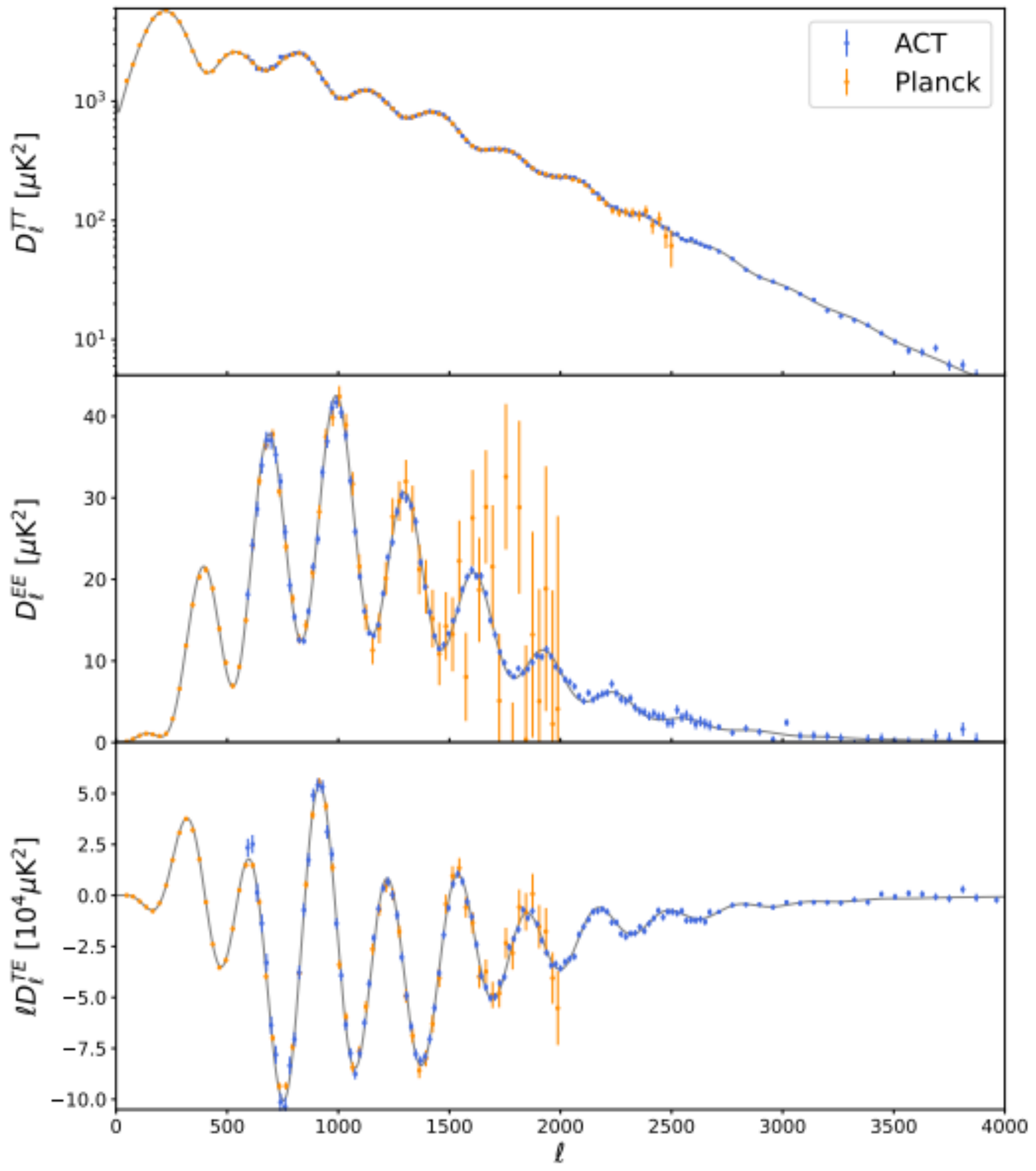
Advent of Precision Cosmology





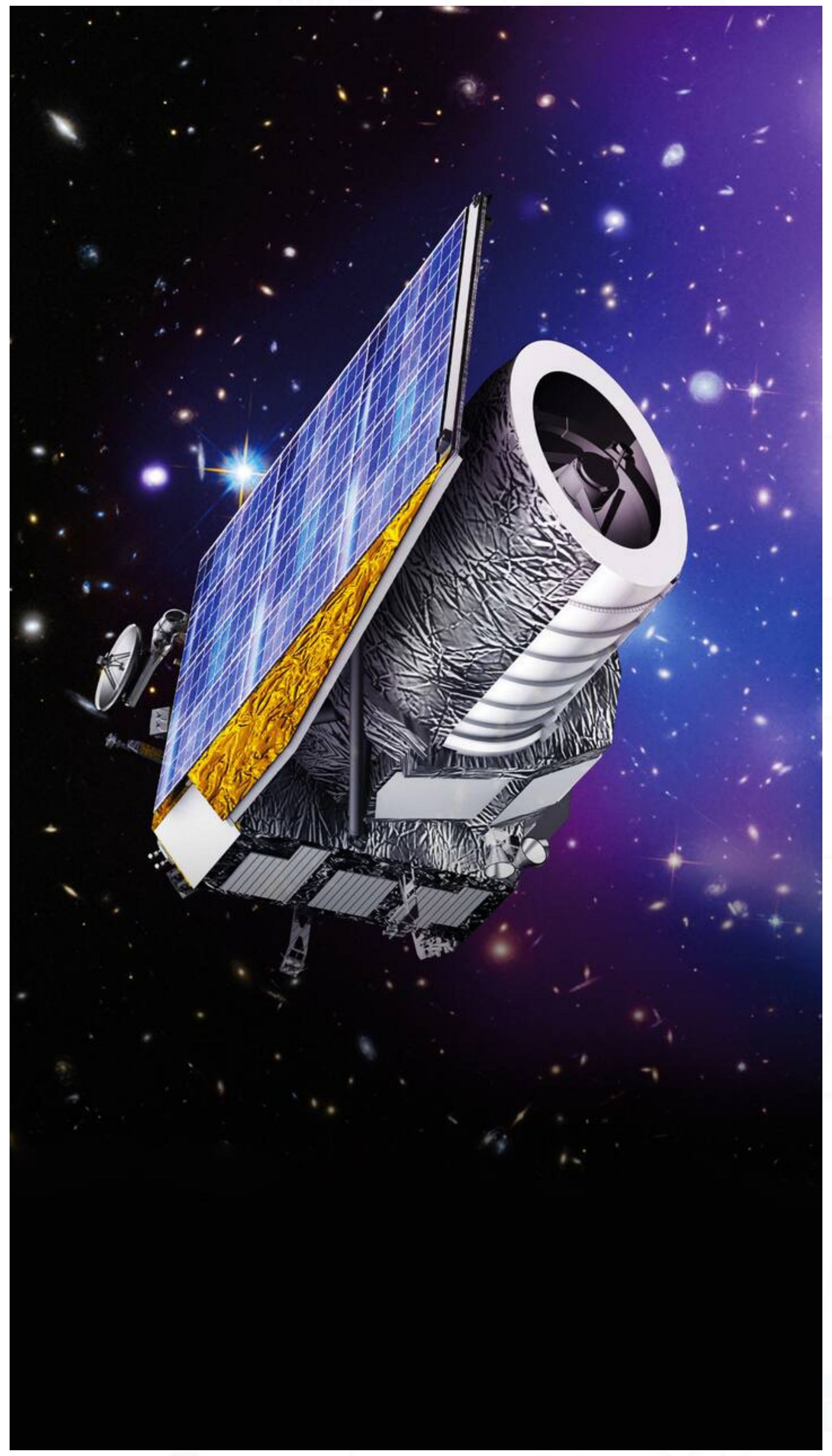


Advent of Precision Cosmology

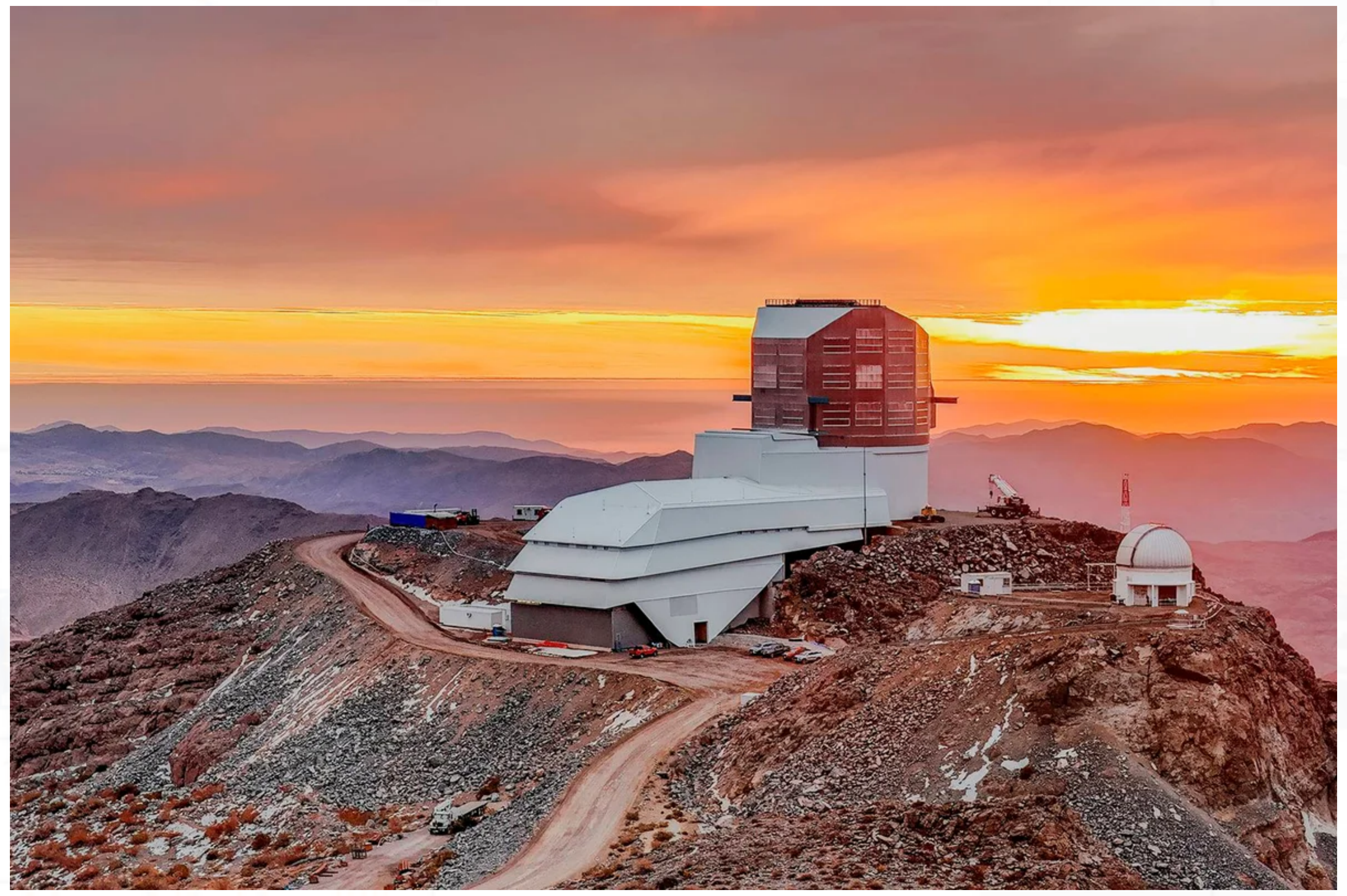


ACT DR6 and Planck PR3 (Planck Collaboration 2020b) combined TT (top), EE (middle), and TE (bottom) power spectra. The gray lines show the joint ACT and Planck (P-ACT) ΛCDM best-fit power spectra.

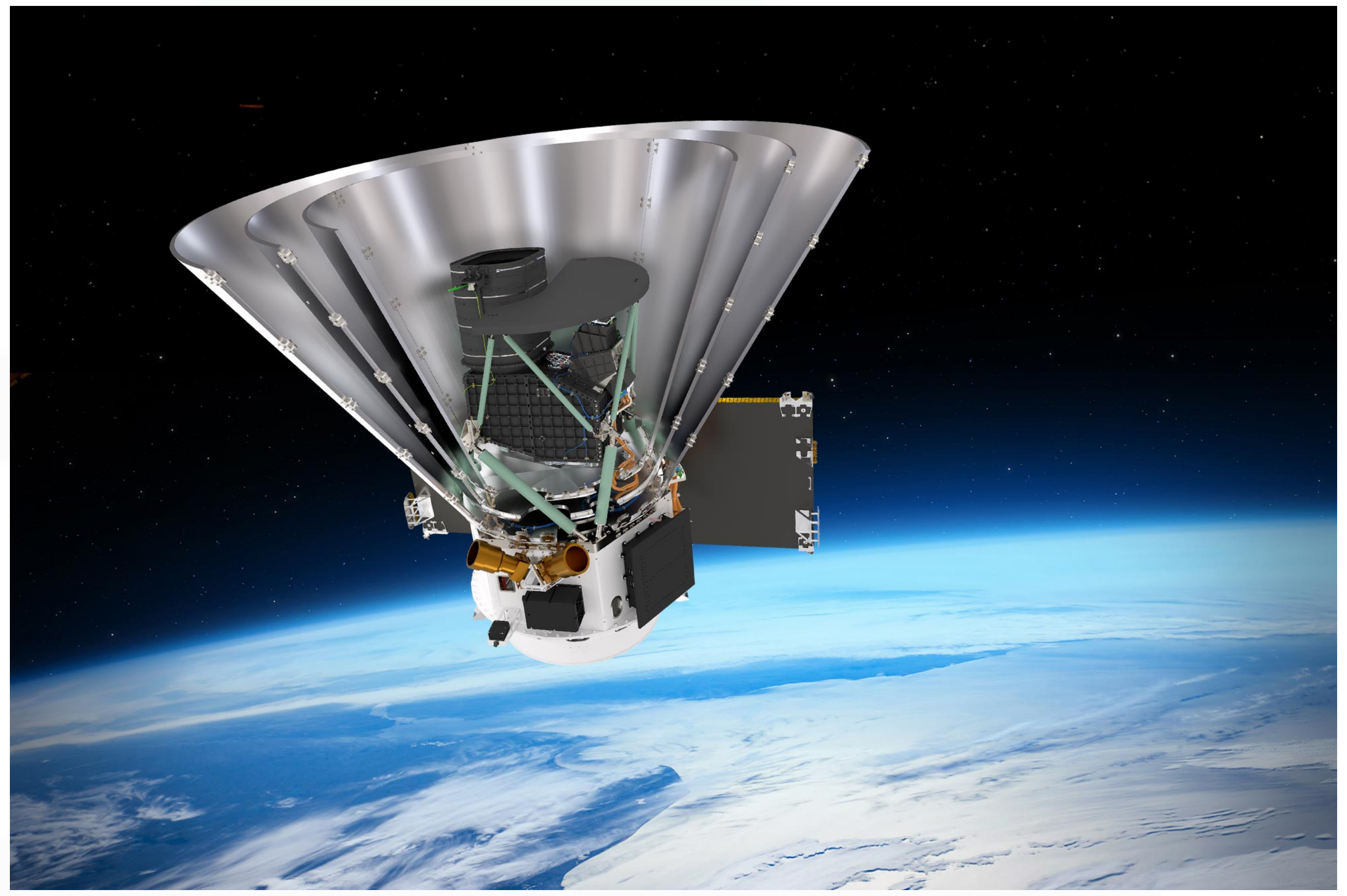
Advent of Precision Cosmology



Euclid satellite



Vera C. Rubin Observatory



SPHEREx

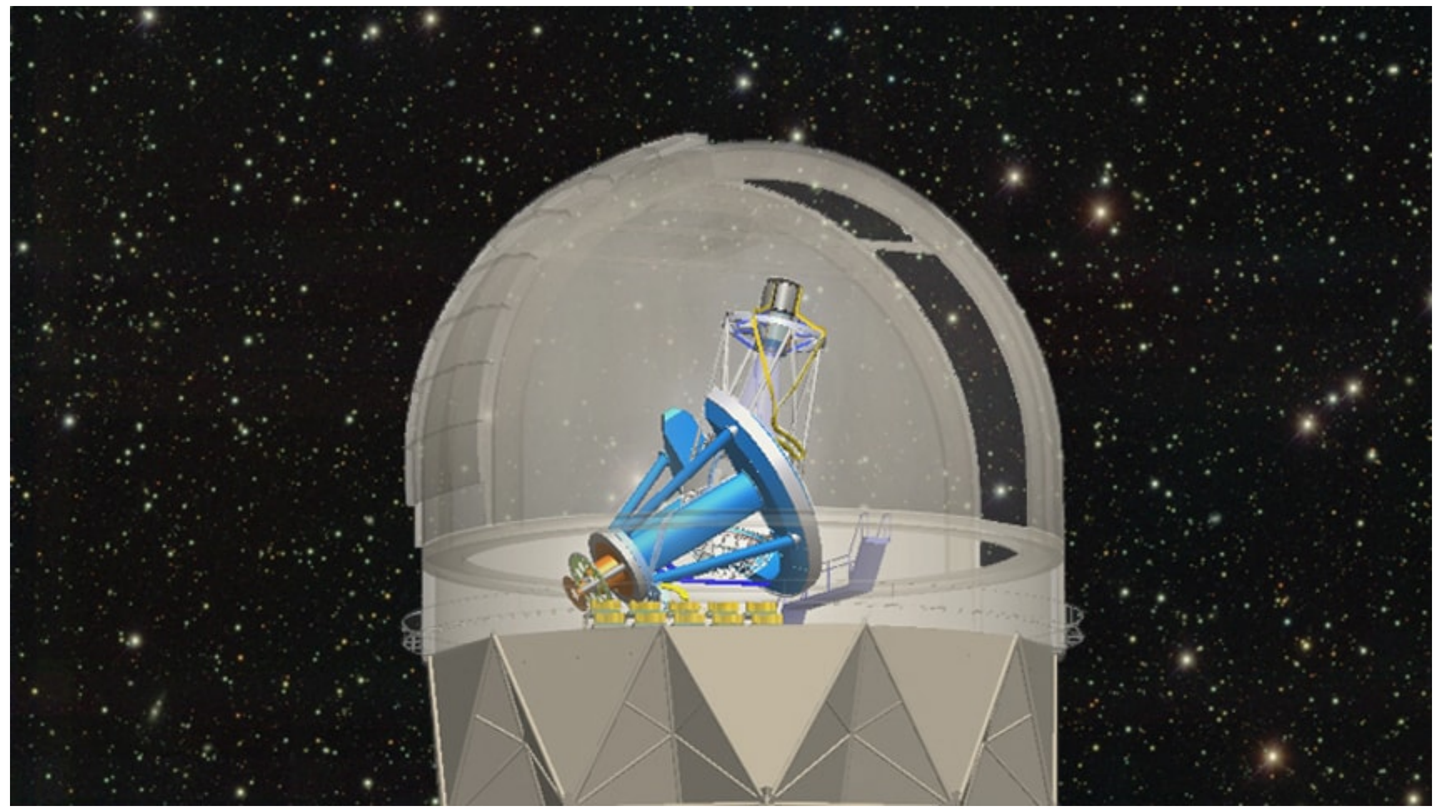
Probe large 3D volume (large survey area and depth) with great accuracy. More statistical power than CMB, a 2D surface

Are current theoretical descriptions adequate to match experimental precision?

What do we really measure in Cosmology?

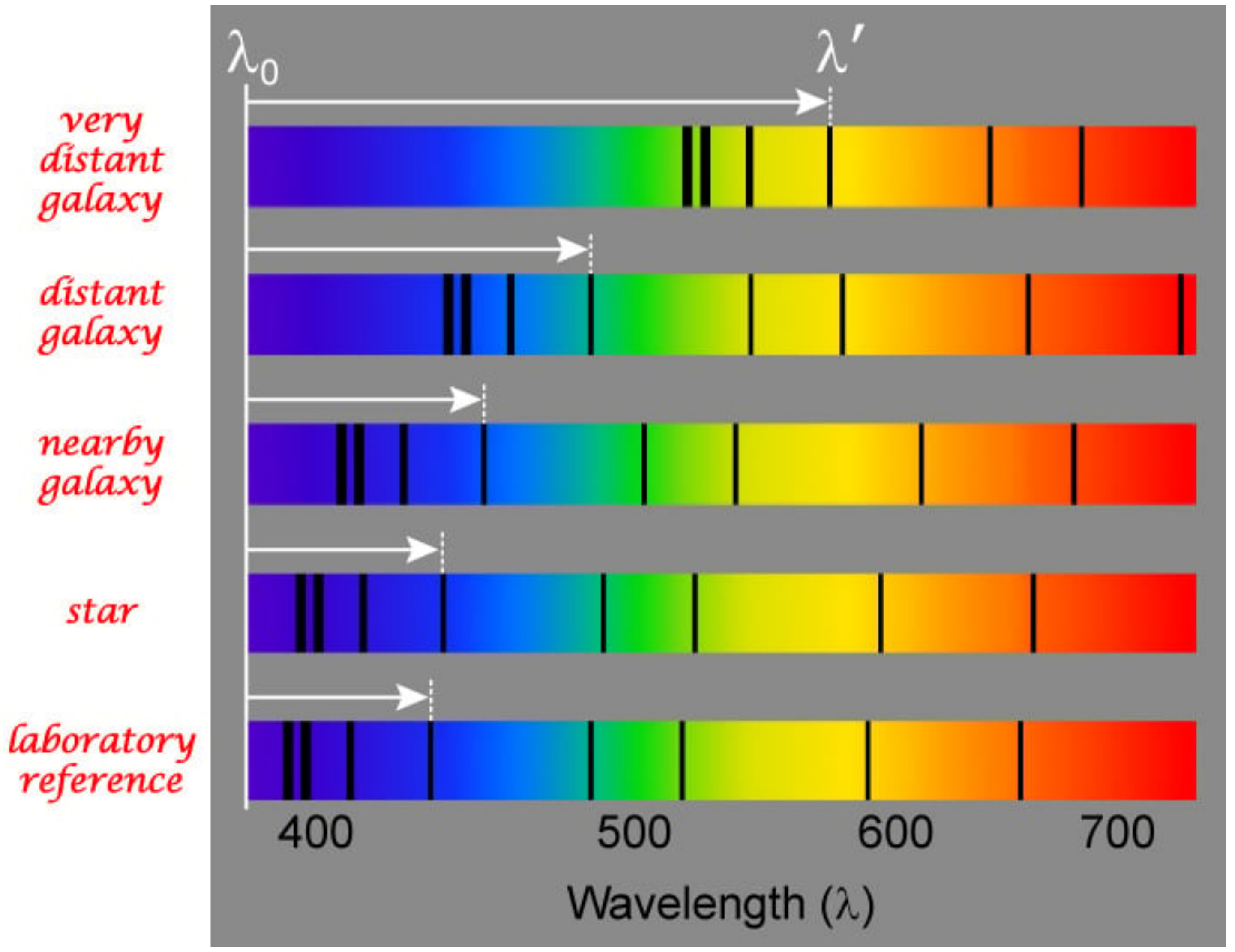
- **Functional relations** between observables: temperature of radiation coming from a certain direction $T(\hat{n})$, number of galaxies per unit solid angle and redshift bin $\dfrac{dN}{d\Omega dz}$

- Fundamental set of observables: redshift, angles and flux



Dark Energy Spectroscopic Instrument (DESI)

Cosmic Origins Spectrograph



Redshift of spectral lines

What do we really measure in Cosmology?

Theoretical descriptions of observables relations must not depend on coordinates:
<u>not currently achieved</u>

Second-order gauge-invariant formalism for the cosmological observables: complete verification of their gauge-invariance

Matteo Magi, Jaiyul Yoo. [JCAP 09 (2022) 071]

Disclaimer:

Consider galaxy number density as an example:

$$\delta_g(z,\hat{n}) = b\,\delta_m + \frac{1}{\mathcal{H}}\partial_r^2 v - \left(1-\frac{t}{2}\right)\int_0^{\bar{r}_z} d\bar{r}\,\left(\frac{\bar{r}_z-\bar{r}}{\bar{r}_z\bar{r}}\right)\Delta_\Omega\,(\Psi-\Phi)$$

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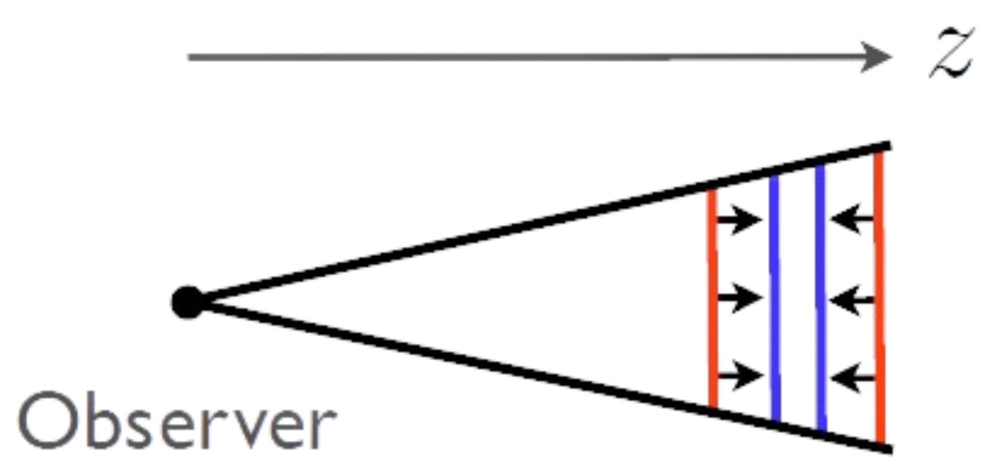
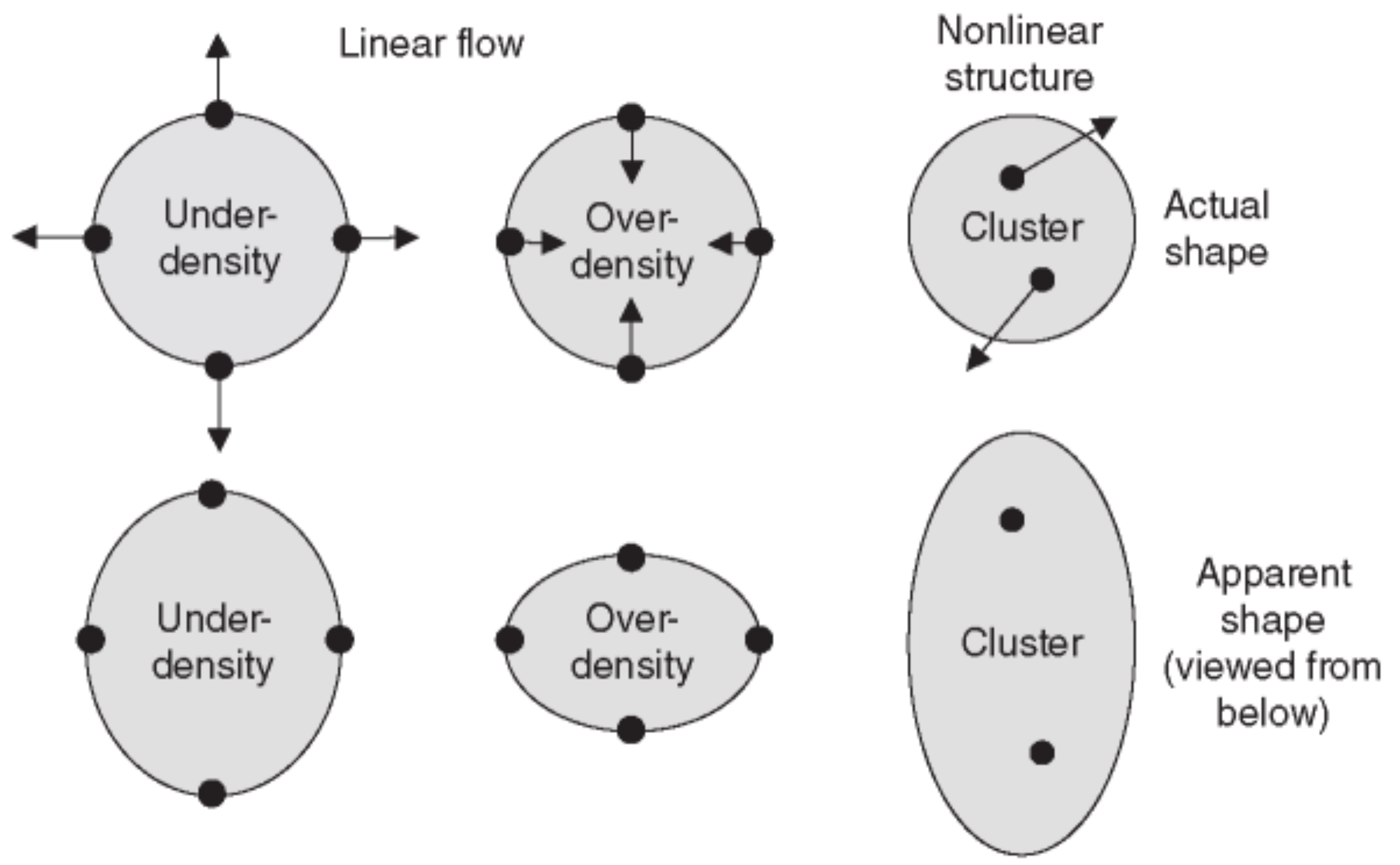
Change in the bin size:
Redshift distortions





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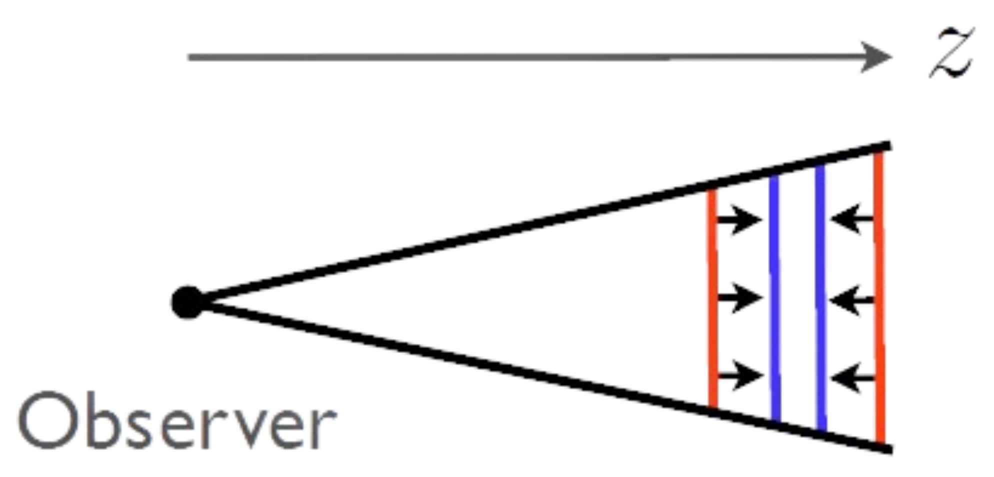
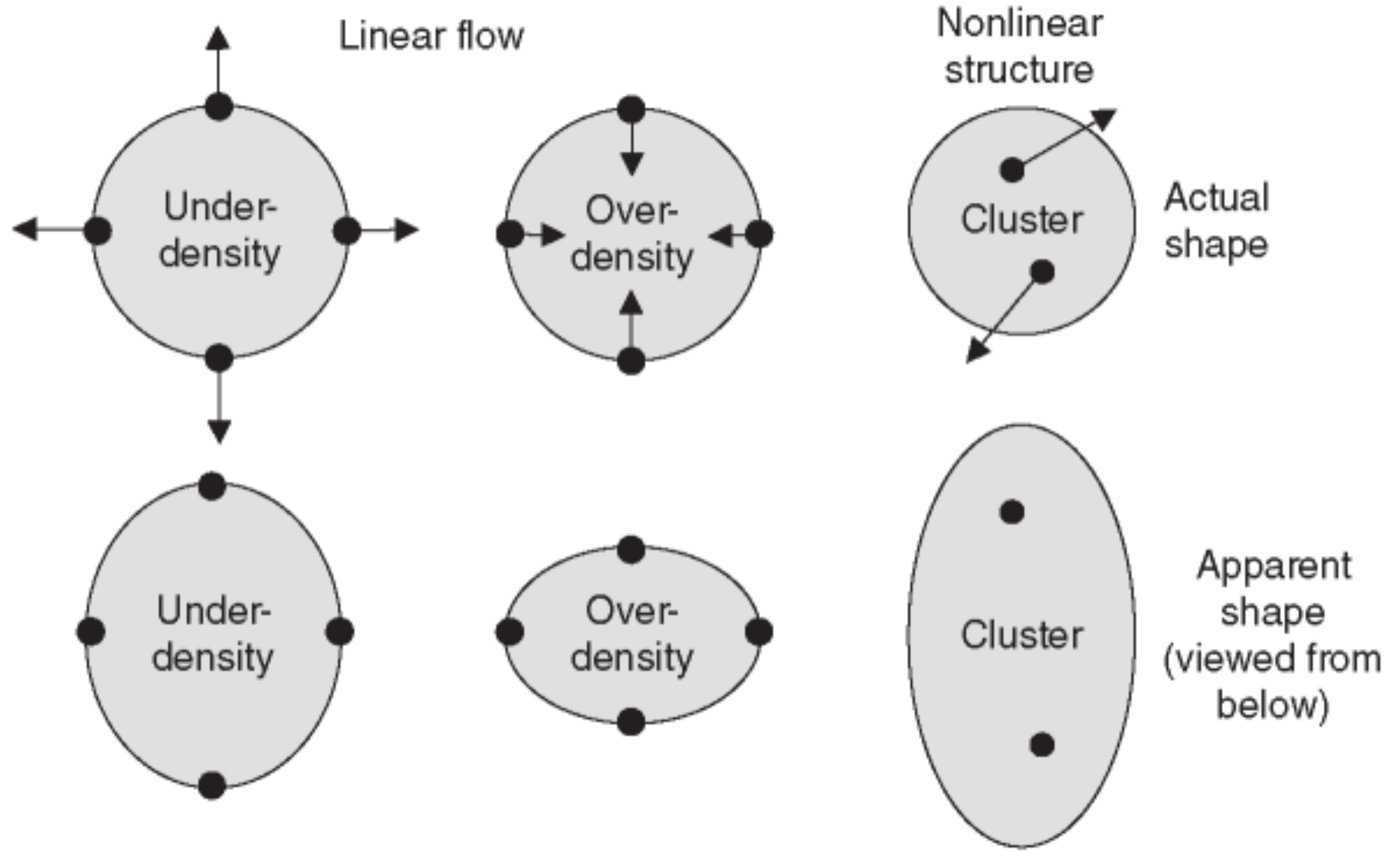
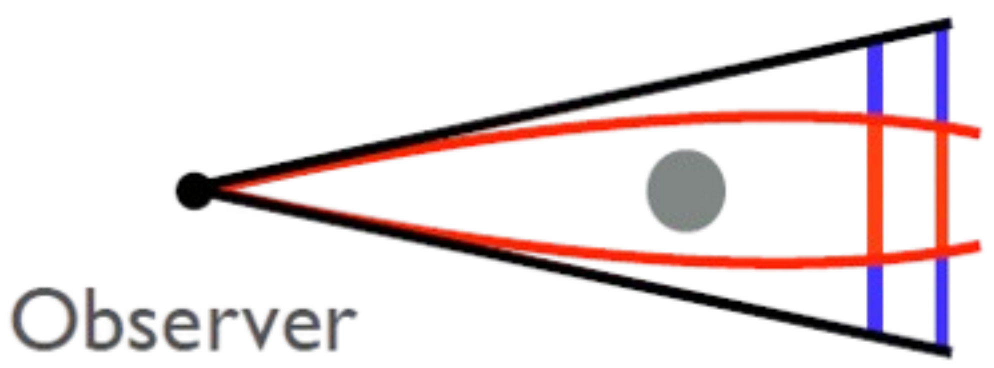




Change in the solid angle



Change in the flux

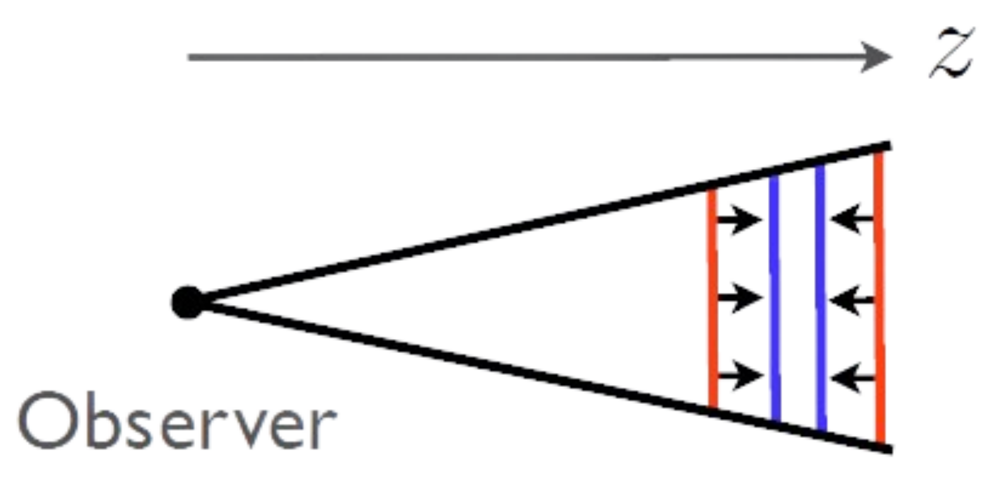
Disclaimer:

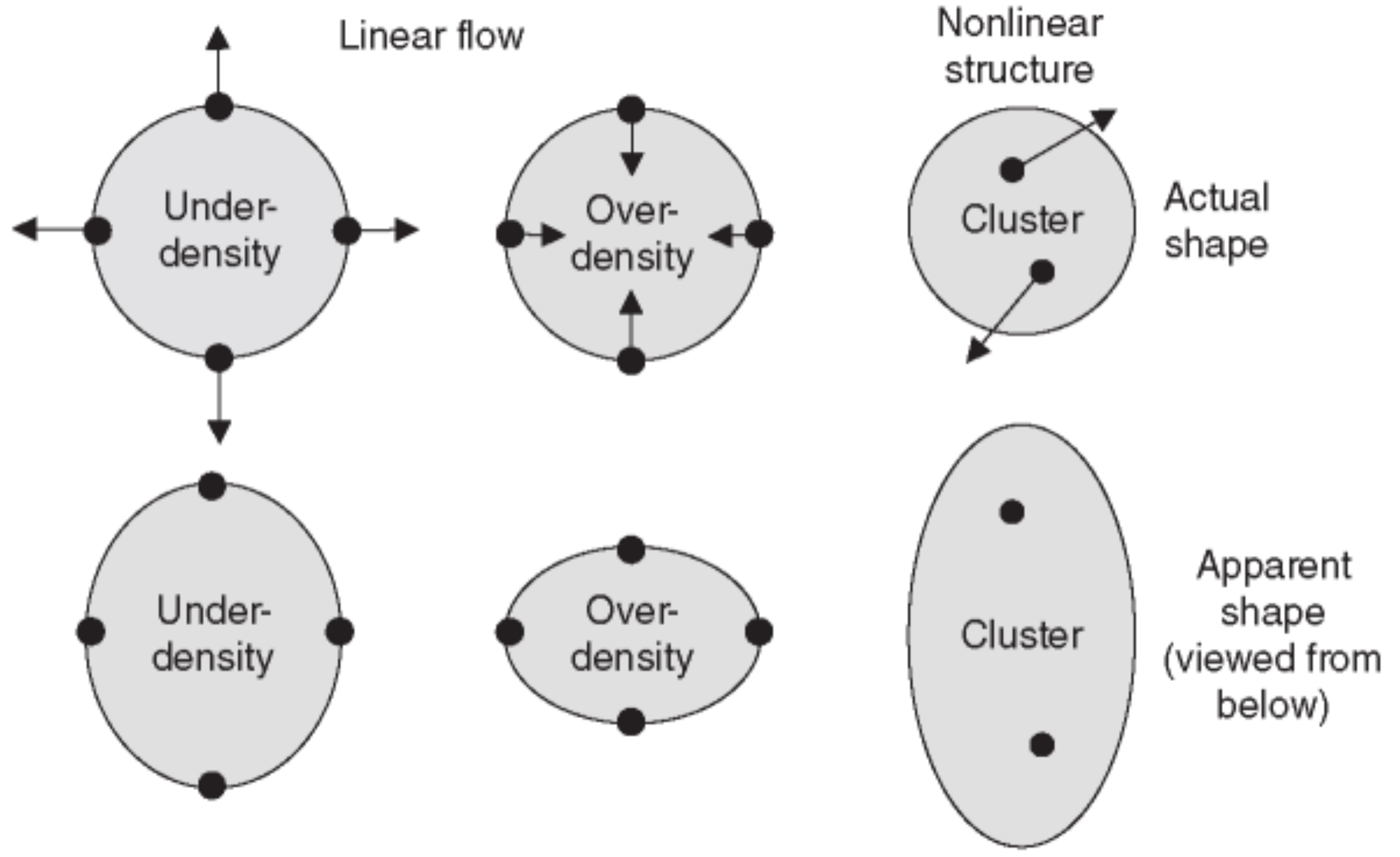
Newtonian description accurate but incomplete:
ignores contributions relevant close to **horizon scale**

Consider galaxy number density as an example:

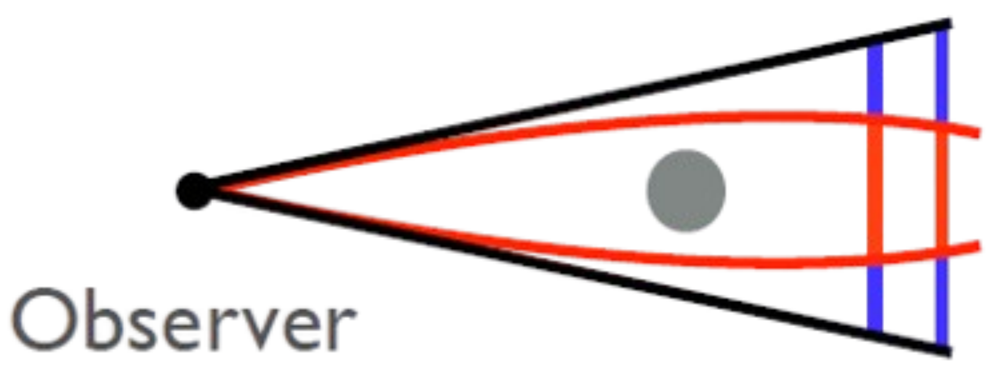
$$\delta_g(z,\hat{n}) = b\,\delta_m + \frac{1}{\mathcal{H}}\partial_r^2 v - \left(1-\frac{t}{2}\right)\int_0^{\bar{r}_z} d\bar{r}\left(\frac{\bar{r}_z-\bar{r}}{\bar{r}_z\bar{r}}\right)\Delta_\Omega\left(\Psi-\Phi\right)$$

Change in the bin size:
Redshift distortions





Change in the solid angle



Change in the flux

Disclaimer:

Yoo 2009, Yoo 2010,
Bonvin & Durrer 2011,
Challinor & Lewis 2011,
Schmidt & Jeong 2012

Consider galaxy number density as an example:

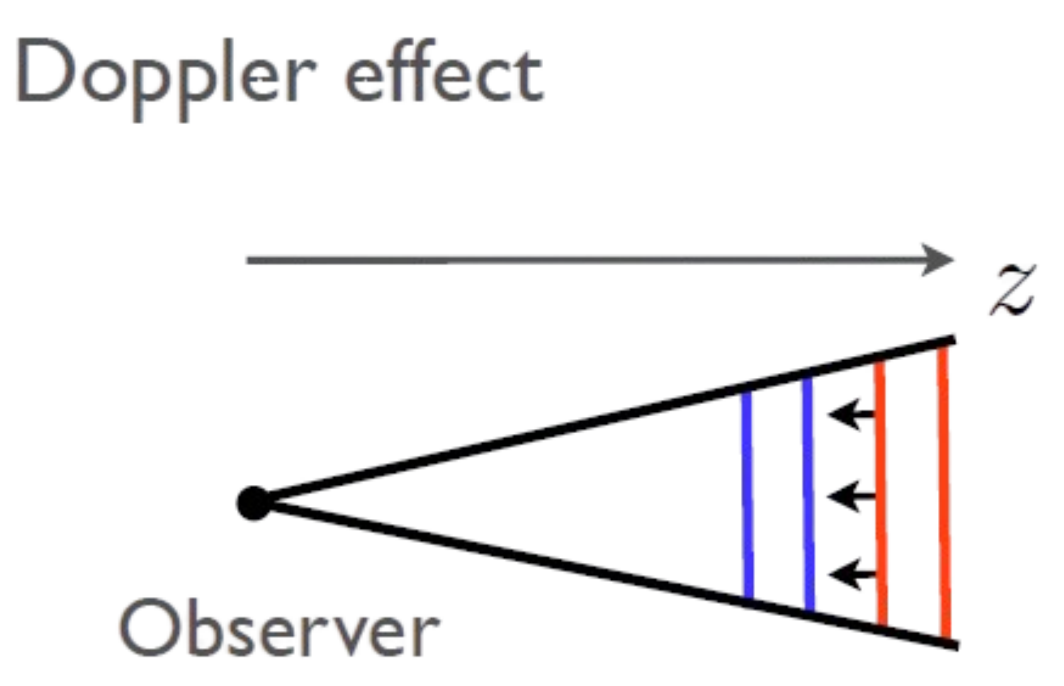
$$\delta_g(z,\hat{n}) = b\,\delta_m + \frac{1}{\mathcal{H}}\partial_r^2 v - \left(1-\frac{t}{2}\right)\int_0^{\bar{r}_z} d\bar{r}\left(\frac{\bar{r}_z-\bar{r}}{\bar{r}_z\bar{r}}\right)\Delta_\Omega\left(\Psi-\Phi\right)$$

"Relativistic effects" or "Projection effects"

$$-\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\partial_r v\Big|_o^z - \partial_r v - (2-t)\partial_r v\Big|_o + \frac{1}{\mathcal{H}}\left(\partial_r\Psi - \partial_r v'\right)$$
$$+\Psi + (2-t)\left[\Phi - \frac{v_o}{\bar{r}_z} + \frac{1}{\bar{r}_z}\int_0^{\bar{r}_z} d\bar{r}\,\left(\Psi-\Phi\right)\right] - \frac{1}{\mathcal{H}}\Phi' - e_z\mathcal{H}v$$
$$-\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left[\mathcal{H}_o v_o + \Psi - \Psi_o + \int_0^{\bar{r}_z} d\bar{r}\left(\Psi'-\Phi'\right)\right]$$

Complete linear-order description of galaxy clustering valid on all scales = well established

Disclaimer:



Consider galaxy number density as an example:

"Relativistic effects" or "Projection effects"

$$\delta_g(z,\hat{n}) = b\,\delta_m + \frac{1}{\mathcal{H}}\partial_r^2 v - \left(1-\frac{t}{2}\right)\int_0^{\bar{r}_z} d\bar{r}\,\left(\frac{\bar{r}_z-\bar{r}}{\bar{r}_z\bar{r}}\right)\Delta_\Omega\left(\Psi-\Phi\right)$$
$$-\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\partial_r v\big|_o^z - \partial_r v - (2-t)\partial_r v\big|_o + \frac{1}{\mathcal{H}}\left(\partial_r\Psi - \partial_r v'\right)$$
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$$-\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left[\mathcal{H}_o v_o + \Psi - \Psi_o + \int_0^{\bar{r}_z} d\bar{r}\left(\Psi'-\Phi'\right)\right]$$

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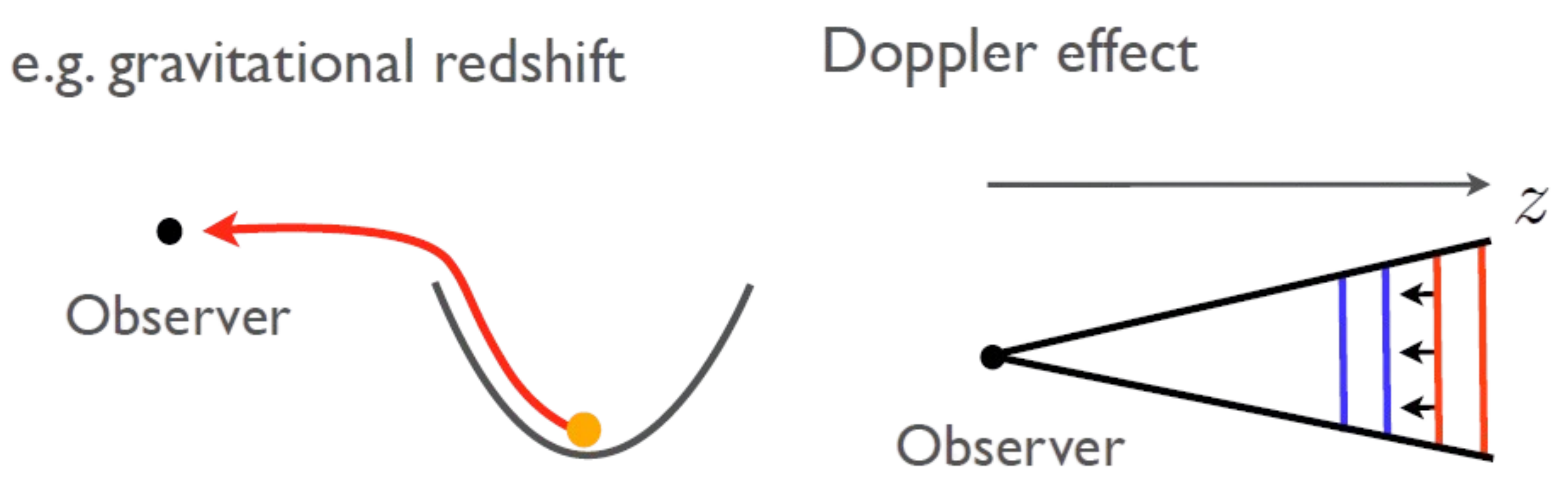

Consider galaxy number density as an example:

$$
\begin{aligned}
\delta_g(z,\hat{n}) = {} & b\,\delta_m + \frac{1}{\mathcal{H}}\partial_r^2 v - \left(1-\frac{t}{2}\right)\int_0^{\bar{r}_z} d\bar{r}\left(\frac{\bar{r}_z-\bar{r}}{\bar{r}_z\bar{r}}\right)\Delta_\Omega\left(\Psi-\Phi\right)\\
& - \left(3 - e_z - t - \frac{2-t}{\mathcal{H}\bar{r}_z} + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\partial_r v\big|_o^z - \partial_r v - (2-t)\partial_r v\big|_o + \frac{1}{\mathcal{H}}\left(\partial_r\Psi - \partial_r v'\right)\\
& + \Psi + (2-t)\left[\Phi - \frac{v_o}{\bar{r}_z} + \frac{1}{\bar{r}_z}\int_0^{r_z} d\bar{r}\,\left(\Psi-\Phi\right)\right] - \frac{1}{\mathcal{H}}\Phi' - e_z\mathcal{H}v\\
& - \left(3 - e_z - t - \frac{2-t}{\mathcal{H}\bar{r}_z} + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left[\mathcal{H}_o v_o + \Psi - \Psi_o + \int_0^{\bar{r}_z} d\bar{r}\left(\Psi'-\Phi'\right)\right]
\end{aligned}
$$

"Relativistic effects" or "Projection effects"

Complete linear-order description of galaxy clustering valid on all scales = well established

Disclaimer:

No agreement beyond linear order: nobody had ever checked consistency of second order expression!!

Consider galaxy number density as an example:

$$
\begin{aligned}
\delta_g(z,\hat{n}) = {} & b\,\delta_m + \frac{1}{\mathcal{H}}\partial_r^2 v - \left(1-\frac{t}{2}\right)\int_0^{\bar{r}_z} d\bar{r}\,\left(\frac{\bar{r}_z-\bar{r}}{\bar{r}_z\bar{r}}\right)\Delta_\Omega\left(\Psi-\Phi\right) \\
& -\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\partial_r v\big|_o^z - \partial_r v - (2-t)\partial_r v\big|_o + \frac{1}{\mathcal{H}}\left(\partial_r\Psi - \partial_r v'\right) \\
& +\Psi + (2-t)\left[\Phi - \frac{v_o}{\bar{r}_z} + \frac{1}{\bar{r}_z}\int_0^{\bar{r}_z} d\bar{r}\,\left(\Psi-\Phi\right)\right] - \frac{1}{\mathcal{H}}\Phi' - e_z\mathcal{H}v \\
& -\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left[\mathcal{H}_o v_o + \Psi - \Psi_o + \int_0^{\bar{r}_z} d\bar{r}\left(\Psi'-\Phi'\right)\right]
\end{aligned}
$$

+ ''products of perturbations''

Literature on second-order observables

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Galaxy clustering

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Cosmological distances and weak lensing

Why Going Beyond Linear Theory?

Primordial Non-Gaussianity:

- Linear theory preserves Gaussian statistics $\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle$
- Second-order effects generate non-Gaussian correlations providing a window into inflationary physics

Test nonlinear predictions of General Relativity:

- Second-order perturbations include self-coupling of first-order modes, generating new effects like mode-mode interactions
- Generate Vector and Tensor Modes from Scalars

Plan for the remainder of the talk:

I) How to derive complete expressions for cosmological observables order by order: light propagation & observation

II) Flaws of standard practice at second order and how to resolve them:

- Geometrical sanity check: diffeomorphism invariance
- Dynamical sanity check: infrared insensitivity

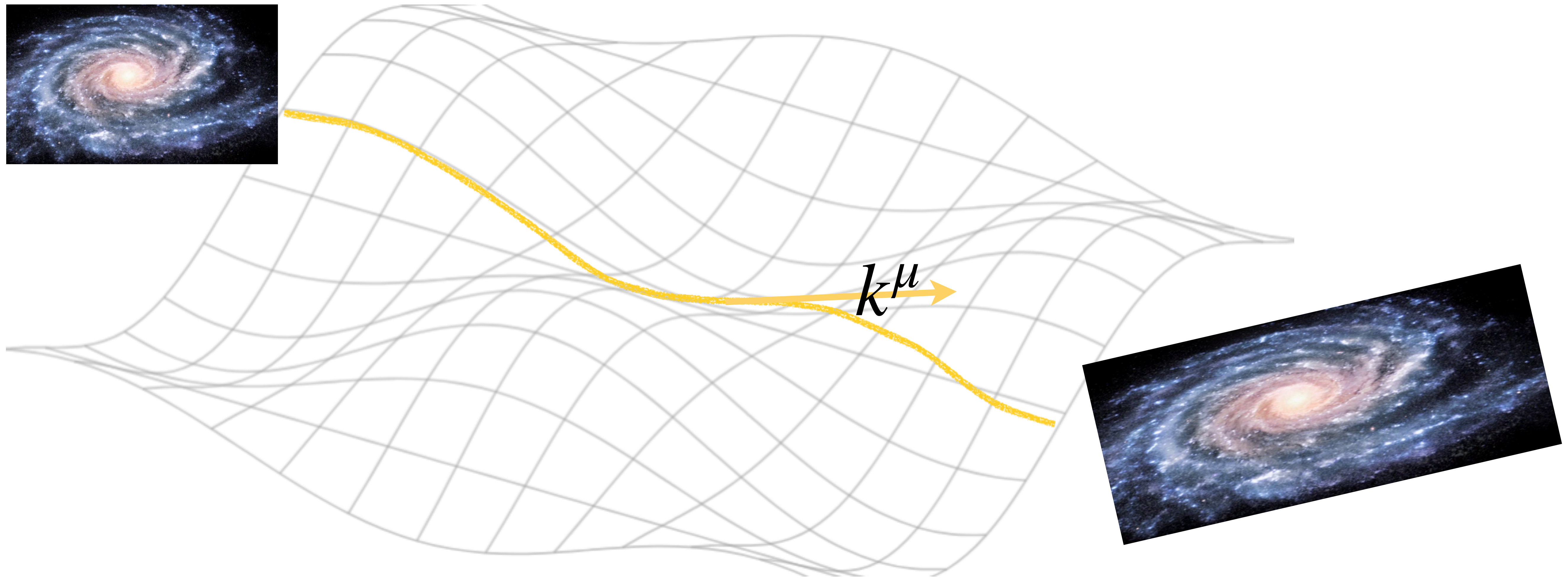
Relativistic Perturbation Theory for Cosmological Observables

Light Propagation in curved spacetime

In Einstein (and similar) theory, light feels gravity by propagating in a curved spacetime

$$g_{\mu\nu}dx^{\nu}dx^{\nu} = -a^2(\eta)\Big[(1+2\mathscr{A})d\eta^2 + 2\mathscr{B}_{\alpha}dx^{\alpha}d\eta - (\delta_{\alpha\beta}+2\mathscr{C}_{\alpha\beta})dx^{\alpha}dx^{\beta}\Big]$$

on specific trajectories called (null) geodesics $k^{\nu}\nabla_{\nu}k^{\mu}=0$, $k^{\mu}k_{\mu}=0$



Observations in curved spacetime

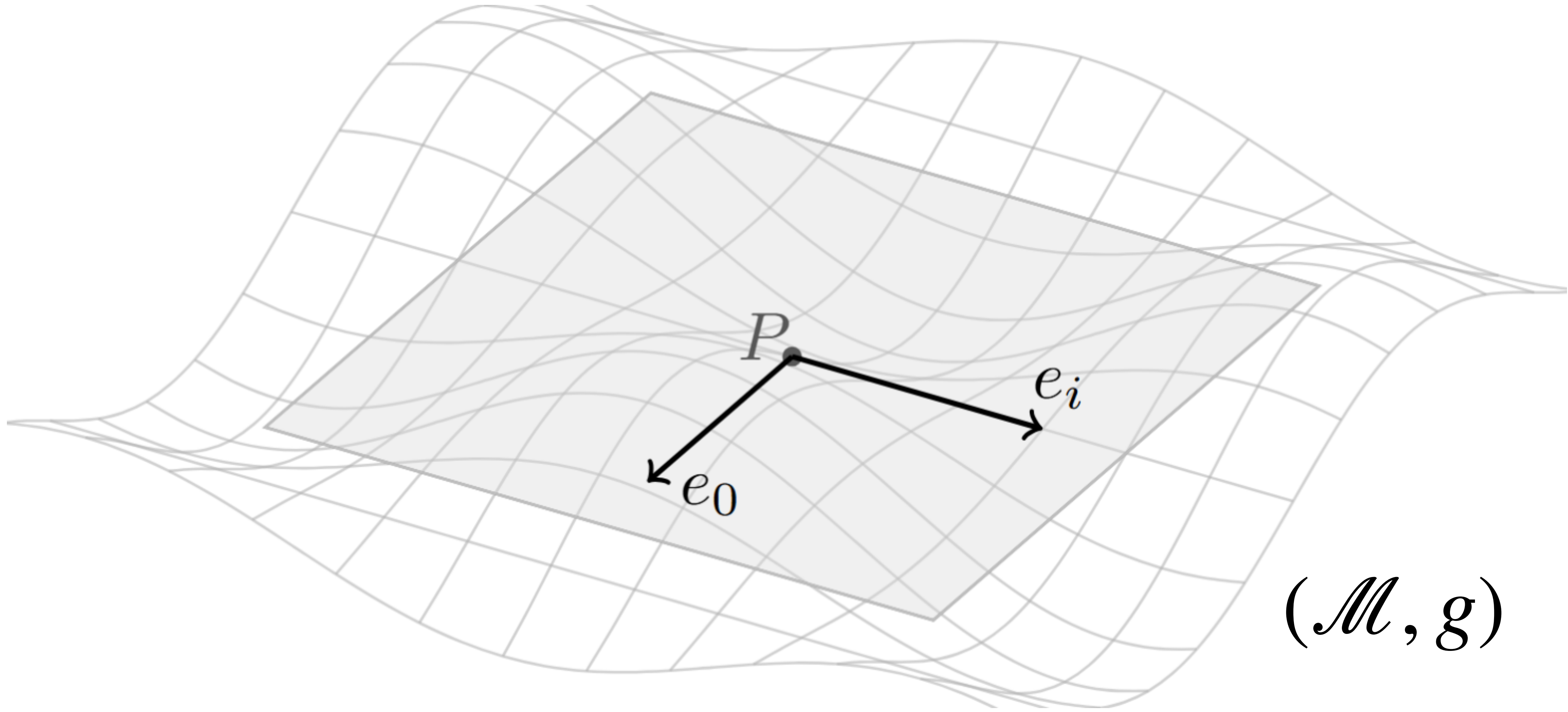
Measurements are performed in rest frame of the laboratory, where the metric is Minkowski

$$\{e_0 \equiv u, e_1, e_2, e_3\} \qquad\qquad g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$$

At any point we can choose a tetrad independently of the coordinates used in that point

E.g. observed light wave vector

$$\begin{cases} k = k^a e_a \\ k^a = (\omega, -\hat{n}^i)^a \end{cases}$$



$(\mathscr{M}, g)$

Examples

Observed redshift = ratio light frequency in the source rest frame and observer rest frame

Physical volume element = volume element in the rest frame of the source

$$1 + z = \frac{(u_\mu k^\mu)_s}{(u_\mu k^\mu)_o}$$

$$\mathrm{d}V = \sqrt{-g}\, \varepsilon_{\rho\mu\nu\sigma}\, u_s^\rho \frac{\partial x_s^\mu}{\partial z} \frac{\partial x_s^\nu}{\partial \theta} \frac{\partial x_s^\sigma}{\partial \phi}\, \mathrm{d}z\, \mathrm{d}\theta\, \mathrm{d}\phi$$

Express observables in ANY coordinates where we solve geodesics

$\Longrightarrow$ Right-hand-side depends only on metric and velocity components

Gauge Freedom

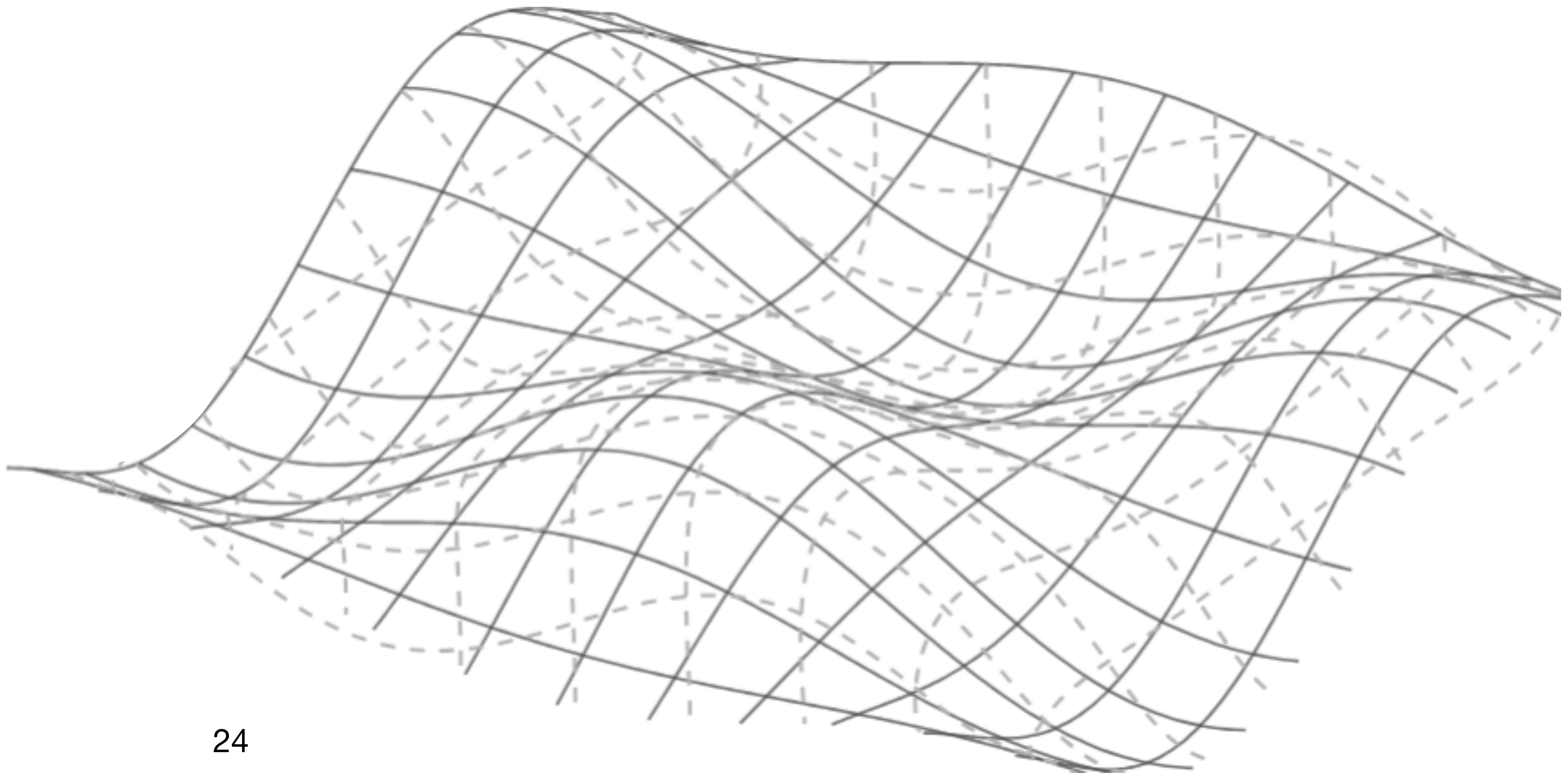
Physics is independent of the choice of coordinates (principle of general covariance)

$$x^\mu \to x^\mu + \xi^\mu(x)$$

Freedom to choose 4 functions

No unique way to define perturbations around the background ("gauge" choice). Different gauges amount to different spacetime coordinates set by the choice of perturbations

E.g. different time slicing: $v \to 0$ comoving gauge, $\delta\rho \to 0$ uniform density gauge



Going Non-Linear: Second-Order Cosmological Observables

Technical complication:

- Lengthy expressions prone to mistakes

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- Lengthy expressions prone to mistakes

$$\delta V = 3\Big[\mathcal{H}_{\bar o}\Big(\delta x^\mu_o(\partial_\mu\delta t)\big|_{\bar o} - \frac{1}{2}\mathcal{H}_{\bar o}\delta t_o^2 + \int_0^{\bar t_o} dt\,\Big(-\mathcal{A}+\frac12\mathcal{A}^2+\frac12\mathcal{U}^\alpha\mathcal{U}_\alpha-\mathcal{U}^\alpha\mathcal{B}_\alpha-\frac1a\mathcal{U}^\alpha\delta t_{,\alpha}\Big)\Big|_{\bar x_t}$$
$$+\Big(-\mathcal{A}-\mathcal{U}_\|+\mathcal{B}_\|+\frac32\mathcal{A}^2-\mathcal{C}_{\alpha\|}(\mathcal{U}^\alpha+\mathcal{B}^\alpha)+\mathcal{A}(\mathcal{U}_\|-2\mathcal{B}_\|)-\mathcal{U}^\alpha\mathcal{B}_\alpha$$
$$+\frac12\mathcal{U}_\alpha\mathcal{U}^\alpha+(\mathcal{U}^\alpha-\mathcal{B}^\alpha)C_{[\alpha,\|]}+\varepsilon_{\alpha ij}(\mathcal{U}^\alpha-\mathcal{B}^\alpha)n^i\Omega^j\Big)_{\bar o}+\int_0^{\bar r_z}d\bar r\Big[\delta\nu\delta\nu'-\delta n^\alpha\delta\nu_{,\alpha}$$
$$+\mathcal{A}'-2\mathcal{A}_{,\|}+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}+2\delta\nu(\mathcal{A}'-\mathcal{A}_{,\|})-2\mathcal{A}\Big(\mathcal{A}'-2\mathcal{A}_{,\|}+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}\Big)$$
$$-2\delta n^\alpha\Big(\mathcal{A}_{,\alpha}-\mathcal{B}_{(\alpha,\|)}-\mathcal{C}'_{\alpha\|}\Big)-\mathcal{B}^\alpha\Big(\mathcal{A}_{,\alpha}-\mathcal{B}'_\alpha+2\mathcal{B}_{[\alpha,\|]}-2\mathcal{C}'_{\alpha\|}+2\mathcal{C}_{\alpha\|,\|}-\mathcal{C}_{\|\|,\alpha}\Big)\Big]\Big|_{\bar x_{\bar r}}$$
$$+\mathcal{A}+\mathcal{U}_\|-\mathcal{B}_\|+\delta\nu\mathcal{A}-\frac12\mathcal{A}^2+\mathcal{A}\mathcal{B}_\|+\delta n_\alpha(\mathcal{U}^\alpha-\mathcal{B}^\alpha)+\frac12\mathcal{U}^\alpha\mathcal{U}_\alpha+2\mathcal{C}_{\alpha\|}\mathcal{U}^\alpha$$
$$+\frac12(\mathcal{H}_{\bar o}^2+\mathcal{H}'_{\bar o})\delta\eta_o^2+\mathcal{H}_{\bar o}\delta\eta_o\widehat{\Delta\nu}+\Delta x^\mu_s(\partial_\mu\widehat{\Delta\nu})\big|_{\bar x_z}\Big]+\mathcal{A}+\mathcal{C}^\alpha_\alpha-\frac12\mathcal{A}^2+\frac12\mathcal{B}^\alpha\mathcal{B}_\alpha+\mathcal{A}\,\mathcal{C}^\alpha_\alpha$$
$$+\frac12\mathcal{C}^\alpha_\alpha\mathcal{C}^\beta_\beta-\mathcal{C}^{\alpha\beta}\mathcal{C}_{\alpha\beta}+H_z\frac{\bar r_z}{2}\frac{\partial}{\partial z}(\delta\theta^2+\sin^2\theta\,\delta\phi^2)+\frac32(\delta\theta^2+\sin^2\theta\,\delta\phi^2)$$
$$+\Big(\frac{2}{\bar r_z}+H_z\frac{\partial}{\partial z}\Big)\Big\{-\bar r_z\Big[\mathcal{U}_\|+n^in_j\delta^j_\alpha\Big(A^\alpha{}_i+\mathcal{C}^\alpha_i+\frac12\mathcal{B}_i\mathcal{B}^\alpha-\frac12\mathcal{U}_i\mathcal{U}^\alpha-\frac12\Omega_i\Omega^\alpha$$
$$+\frac12\delta^\alpha_i\Omega^k\Omega_k-\frac32\mathcal{C}^\beta_i\mathcal{C}^\alpha_\beta-\mathcal{C}^\beta_{(i}\varepsilon^{\alpha)}{}_{k\beta}\Omega^k-\frac12C_{\beta,(i}\mathcal{C}^{\alpha)\beta}+\frac12C^\beta{}_{,(i}\varepsilon^{\alpha)}{}_{k\beta}\Omega^k-\frac12C_{(i,\beta}\varepsilon^{\alpha)}{}_k{}^\beta\Omega^k$$
$$+\frac18C^\beta{}_{,i}C_\beta{}^{,\alpha}+\frac18C_{i,}{}^\beta C^\alpha{}_{,\beta}+\frac12\mathcal{C}^\beta_{(i}C^{\alpha)}{}_{,\beta}-\frac14C^\beta{}_{,(i}C^{\alpha)}{}_{,\beta}\Big)-\mathcal{A}-\mathcal{U}_\|+\mathcal{B}_\|+\frac32\mathcal{A}^2$$
$$-\mathcal{C}_{\alpha\|}(\mathcal{U}^\alpha+\mathcal{B}^\alpha)+\mathcal{A}(\mathcal{U}_\|-2\mathcal{B}_\|)-\mathcal{U}^\alpha\mathcal{B}_\alpha+\frac12\mathcal{U}_\alpha\mathcal{U}^\alpha+(\mathcal{U}^\alpha-\mathcal{B}^\alpha)C_{[\alpha,\|]}$$
$$+\varepsilon_{\alpha ij}(\mathcal{U}^\alpha-\mathcal{B}^\alpha)n^i\Omega^j\Big]\Big|_{\bar o}+n_\alpha\delta x^\mu_o(\partial_\mu\delta x^\alpha)\big|_{\bar o}+n_\alpha\int_0^{\bar t_o}dt\,\frac1a\Big(\mathcal{U}^\alpha+\mathcal{A}\mathcal{U}^\alpha-\mathcal{U}^\beta\delta x^\alpha{}_{,\beta}\Big)\Big|_{\bar x_t}$$
$$-\frac{1}{\mathcal{H}_z}\Big[\Big(-\mathcal{A}-\mathcal{U}_\|+\mathcal{B}_\|+\frac32\mathcal{A}^2-\mathcal{C}_{\alpha\|}(\mathcal{U}^\alpha+\mathcal{B}^\alpha)+\mathcal{A}(\mathcal{U}_\|-2\mathcal{B}_\|)-\mathcal{U}^\alpha\mathcal{B}_\alpha$$
$$+\frac12\mathcal{U}_\alpha\mathcal{U}^\alpha+(\mathcal{U}^\alpha-\mathcal{B}^\alpha)C_{[\alpha,\|]}+\varepsilon_{\alpha ij}(\mathcal{U}^\alpha-\mathcal{B}^\alpha)n^i\Omega^j\Big)_{\bar o}+\int_0^{\bar r_z}d\bar r\Big(\delta\nu\delta\nu'-\delta n^\alpha\delta\nu_{,\alpha}$$
$$+\mathcal{A}'-2\mathcal{A}_{,\|}+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}+2\delta\nu(\mathcal{A}'-\mathcal{A}_{,\|})-2\mathcal{A}\Big(\mathcal{A}'-2\mathcal{A}_{,\|}+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}\Big)$$
$$-2\delta n^\alpha\Big(\mathcal{A}_{,\alpha}-\mathcal{B}_{(\alpha,\|)}-\mathcal{C}'_{\alpha\|}\Big)-\mathcal{B}^\alpha\Big(\mathcal{A}_{,\alpha}-\mathcal{B}'_\alpha+2\mathcal{B}_{[\alpha,\|]}-2\mathcal{C}'_{\alpha\|}+2\mathcal{C}_{\alpha\|,\|}-\mathcal{C}_{\|\|,\alpha}\Big)\Big)\Big|_{\bar x_{\bar r}}$$
$$+\mathcal{A}+\mathcal{U}_\|-\mathcal{B}_\|+\delta\nu\mathcal{A}-\frac12\mathcal{A}^2+\mathcal{A}\mathcal{B}_\|+\delta n_\alpha(\mathcal{U}^\alpha-\mathcal{B}^\alpha)+\frac12\mathcal{U}^\alpha\mathcal{U}_\alpha+2\mathcal{C}_{\alpha\|}\mathcal{U}^\alpha$$
$$+\frac12(\mathcal{H}_{\bar o}^2+\mathcal{H}'_{\bar o})\delta\eta_o^2+\mathcal{H}_{\bar o}\delta\eta_o\widehat{\Delta\nu}+\Delta x^\mu_s(\partial_\mu\widehat{\Delta\nu})\big|_{\bar x_z}\Big]+\frac12\frac{\mathcal{H}_z^2+\mathcal{H}'_z}{\mathcal{H}_z}\Big(\frac{\delta z}{\mathcal{H}_z}\Big)^2$$
$$-\Big(\frac{\mathcal{H}_{\bar o}}{\mathcal{H}_z}-1\Big)\Big[\delta x^\mu_o(\partial_\mu\delta t)\big|_{\bar o}+\int_0^{\bar t_o}dt\,\Big(-\mathcal{A}+\frac12\mathcal{A}^2+\frac12\mathcal{U}^\alpha\mathcal{U}_\alpha-\mathcal{U}^\alpha\mathcal{B}_\alpha-\frac1a\mathcal{U}^\alpha\delta t_{,\alpha}\Big)\Big|_{\bar x_t}\Big]$$
$$+\frac12\mathcal{H}_{\bar o}\delta t_o^2+n_\alpha\Big[\Delta x^\mu_{\bar r}\,\partial_\mu(\Delta x^\alpha+n^\alpha\Delta\eta)\big|_{\bar x_{\bar r}}\Big]^z_{\bar o}+n_\alpha\int_0^{\bar r_z}d\bar r\Big[-\delta n^\beta(\Delta x^\alpha{}_{,\beta}+n^\alpha\Delta\eta_{,\beta})$$

$$+\delta\nu(\Delta x'^\alpha+n^\alpha\Delta\eta')-(\bar r_z-\bar r)\Big(-\delta\nu\delta n'^\alpha+\delta n^\beta\delta n^\alpha{}_{,\beta}+\mathcal{A}^{,\alpha}-\mathcal{B}'^\alpha-\mathcal{B}_\|{}^{,\alpha}+\mathcal{B}^\alpha{}_{,\|}$$
$$-2\mathcal{C}'^\alpha_\|+2\mathcal{C}^\alpha_{\|,\|}-\mathcal{C}_{\|\|}{}^{,\alpha}+\delta\nu\Big(2\mathcal{A}^{,\alpha}-2\mathcal{B}'^\alpha-\mathcal{B}_\|{}^{,\alpha}+\mathcal{B}^\alpha{}_{,\|}-2\mathcal{C}'^\alpha_\|\Big)$$
$$-\delta n^\gamma\Big(\mathcal{B}_\gamma{}^{,\alpha}-\mathcal{B}^\alpha{}_{,\gamma}+2\mathcal{C}'^\alpha_\gamma-4\mathcal{C}^\alpha_{(\|,\gamma)}+2\mathcal{C}_{\|\gamma}{}^{,\alpha}\Big)+\mathcal{B}^\alpha\Big(\mathcal{A}'+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}-2\mathcal{A}_{,\|}\Big)$$
$$-2\mathcal{C}^{\alpha\gamma}\Big(2\mathcal{C}_{\gamma\|,\|}-\mathcal{C}_{\|\|,\gamma}+\mathcal{A}_{,\gamma}-\mathcal{B}'_\gamma-2\mathcal{B}_{[\|,\gamma]}-2\mathcal{C}'_{\|\gamma}\Big)+n^\alpha\Big(\delta\nu\delta\nu'-\delta n^\alpha\delta\nu_{,\alpha}$$
$$+\mathcal{A}'-2\mathcal{A}_{,\|}+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}+2\delta\nu(\mathcal{A}'-\mathcal{A}_{,\|})-2\mathcal{A}\Big(\mathcal{A}'-2\mathcal{A}_{,\|}+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}\Big)$$
$$-2\delta n^\alpha\Big(\mathcal{A}_{,\alpha}-\mathcal{B}_{(\alpha,\|)}-\mathcal{C}'_{\alpha\|}\Big)-\mathcal{B}^\alpha\Big(\mathcal{A}_{,\alpha}-\mathcal{B}'_\alpha+2\mathcal{B}_{[\alpha,\|]}-2\mathcal{C}'_{\alpha\|}+2\mathcal{C}_{\alpha\|,\|}-\mathcal{C}_{\|\|,\alpha}\Big)\Big)\Big]\Big|_{\bar x_{\bar r}}\Big\}$$
$$+\Big(\cot\theta+\frac{\partial}{\partial\theta}\Big)\Big(\frac{1}{\bar r_z}\theta_\alpha\Big\{-\bar r_z\Big[\mathcal{U}^\alpha+n^i\Big(A^\alpha{}_i+\mathcal{C}^\alpha_i+\frac12\mathcal{B}_i\mathcal{B}^\alpha-\frac12\mathcal{U}_i\mathcal{U}^\alpha-\frac12\Omega_i\Omega^\alpha+\frac12\delta^\alpha_i\Omega^k\Omega_k$$
$$-\frac32\mathcal{C}^\beta_i\mathcal{C}^\alpha_\beta-\mathcal{C}^\beta_{(i}\varepsilon^{\alpha)}{}_{k\beta}\Omega^k-\frac12C_{\beta,(i}\mathcal{C}^{\alpha)\beta}+\frac12C^\beta{}_{,(i}\varepsilon^{\alpha)}{}_{k\beta}\Omega^k-\frac12C_{(i,\beta}\varepsilon^{\alpha)}{}_k{}^\beta\Omega^k+\frac18C^\beta{}_{,i}C_\beta{}^{,\alpha}$$
$$+\frac18C_{i,}{}^\beta C^\alpha{}_{,\beta}+\frac12\mathcal{C}^\beta_{(i}C^{\alpha)}{}_{,\beta}-\frac14C^\beta{}_{,(i}C^{\alpha)}{}_{,\beta}\Big)\Big]\Big|_{\bar o}+\delta x^\mu_o(\partial_\mu\delta x^\alpha)\big|_{\bar o}+\Big[\Delta x^\mu_{\bar r}\,\partial_\mu(\Delta x^\alpha+n^\alpha\Delta\eta)\big|_{\bar x_{\bar r}}\Big]^z_{\bar o}$$
$$+\int_0^{\bar t_o}dt\,\frac1a\Big(\mathcal{U}^\alpha+\mathcal{A}\mathcal{U}^\alpha-\mathcal{U}^\beta\delta x^\alpha{}_{,\beta}\Big)\Big|_{\bar x_t}+\int_0^{\bar r_z}d\bar r\Big[-\delta n^\beta(\Delta x^\alpha{}_{,\beta}+n^\alpha\Delta\eta_{,\beta})$$
$$+\delta\nu(\Delta x'^\alpha+n^\alpha\Delta\eta')-(\bar r_z-\bar r)\Big(-\delta\nu\delta n'^\alpha+\delta n^\beta\delta n^\alpha{}_{,\beta}+\mathcal{A}^{,\alpha}-\mathcal{B}'^\alpha-\mathcal{B}_\|{}^{,\alpha}+\mathcal{B}^\alpha{}_{,\|}$$
$$-\delta n^\gamma\Big(\mathcal{B}_\gamma{}^{,\alpha}-\mathcal{B}^\alpha{}_{,\gamma}+2\mathcal{C}'^\alpha_\gamma-4\mathcal{C}^\alpha_{(\|,\gamma)}+2\mathcal{C}_{\|\gamma}{}^{,\alpha}\Big)+\mathcal{B}^\alpha\Big(\mathcal{A}'+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}-2\mathcal{A}_{,\|}\Big)$$
$$-2\mathcal{C}^{\alpha\gamma}\Big(2\mathcal{C}_{\gamma\|,\|}-\mathcal{C}_{\|\|,\gamma}+\mathcal{A}_{,\gamma}-\mathcal{B}'_\gamma-2\mathcal{B}_{[\|,\gamma]}-2\mathcal{C}'_{\|\gamma}\Big)\Big]\Big|_{\bar x_{\bar r}}\Big\}\Big)$$
$$+\frac{\partial}{\partial\phi}\Big(\frac{1}{\bar r_z\sin\theta}\phi_\alpha\Big\{-\bar r_z\Big[\mathcal{U}^\alpha+n^i\Big(A^\alpha{}_i+\mathcal{C}^\alpha_i+\frac12\mathcal{B}_i\mathcal{B}^\alpha-\frac12\mathcal{U}_i\mathcal{U}^\alpha-\frac12\Omega_i\Omega^\alpha+\frac12\delta^\alpha_i\Omega^k\Omega_k-\frac32\mathcal{C}^\beta_i\mathcal{C}^\alpha_\beta$$
$$-\mathcal{C}^\beta_{(i}\varepsilon^{\alpha)}{}_{k\beta}\Omega^k-\frac12C_{\beta,(i}\mathcal{C}^{\alpha)\beta}+\frac12C^\beta{}_{,(i}\varepsilon^{\alpha)}{}_{k\beta}\Omega^k-\frac12C_{(i,\beta}\varepsilon^{\alpha)}{}_k{}^\beta\Omega^k+\frac18C^\beta{}_{,i}C_\beta{}^{,\alpha}$$
$$+\frac18C_{i,}{}^\beta C^\alpha{}_{,\beta}+\frac12\mathcal{C}^\beta_{(i}C^{\alpha)}{}_{,\beta}-\frac14C^\beta{}_{,(i}C^{\alpha)}{}_{,\beta}\Big)\Big]\Big|_{\bar o}+\delta x^\mu_o(\partial_\mu\delta x^\alpha)\big|_{\bar o}+\Big[\Delta x^\mu_{\bar r}\,\partial_\mu(\Delta x^\alpha+n^\alpha\Delta\eta)\big|_{\bar x_{\bar r}}\Big]^z_{\bar o}$$
$$+\int_0^{\bar t_o}dt\,\frac1a\Big(\mathcal{U}^\alpha+\mathcal{A}\mathcal{U}^\alpha-\mathcal{U}^\beta\delta x^\alpha{}_{,\beta}\Big)\Big|_{\bar x_t}+\int_0^{\bar r_z}d\bar r\Big[-\delta n^\beta(\Delta x^\alpha{}_{,\beta}+n^\alpha\Delta\eta_{,\beta})$$
$$+\delta\nu(\Delta x'^\alpha+n^\alpha\Delta\eta')-(\bar r_z-\bar r)\Big(-\delta\nu\delta n'^\alpha+\delta n^\beta\delta n^\alpha{}_{,\beta}+\mathcal{A}^{,\alpha}-\mathcal{B}'^\alpha-\mathcal{B}_\|{}^{,\alpha}+\mathcal{B}^\alpha{}_{,\|}$$
$$-2\mathcal{C}'^\alpha_\|+2\mathcal{C}^\alpha_{\|,\|}-\mathcal{C}_{\|\|}{}^{,\alpha}+\delta\nu\Big(2\mathcal{A}^{,\alpha}-2\mathcal{B}'^\alpha-\mathcal{B}_\|{}^{,\alpha}+\mathcal{B}^\alpha{}_{,\|}-2\mathcal{C}'^\alpha_\|\Big)$$
$$-\delta n^\gamma\Big(\mathcal{B}_\gamma{}^{,\alpha}-\mathcal{B}^\alpha{}_{,\gamma}+2\mathcal{C}'^\alpha_\gamma-4\mathcal{C}^\alpha_{(\|,\gamma)}+2\mathcal{C}_{\|\gamma}{}^{,\alpha}\Big)+\mathcal{B}^\alpha\Big(\mathcal{A}'+\mathcal{B}_{\|,\|}+\mathcal{C}'_{\|\|}-2\mathcal{A}_{,\|}\Big)$$
$$-2\mathcal{C}^{\alpha\gamma}\Big(2\mathcal{C}_{\gamma\|,\|}-\mathcal{C}_{\|\|,\gamma}+\mathcal{A}_{,\gamma}-\mathcal{B}'_\gamma-2\mathcal{B}_{[\|,\gamma]}-2\mathcal{C}'_{\|\gamma}\Big)\Big]\Big|_{\bar x_{\bar r}}\Big\}\Big)$$

$$+\frac{\partial}{\partial\theta}\delta\theta\frac{\partial}{\partial\phi}\delta\phi-\frac{\partial}{\partial\phi}\delta\theta\frac{\partial}{\partial\theta}\delta\phi+\cot\theta\,\delta\theta\Big(\frac{\partial}{\partial\theta}\delta\theta+\frac{\partial}{\partial\phi}\delta\phi\Big)-\frac12\delta\theta^2$$
$$+\mathcal{A}+\frac32\mathcal{A}^2+\frac12\mathcal{U}^\alpha\mathcal{U}_\alpha-\mathcal{U}_\alpha\mathcal{B}^\alpha+\mathcal{U}_\|+3\,\delta z\delta g+3\,\delta z^2-\mathcal{A}\Big(2\,\frac{\delta r}{\bar r_z}-2\kappa+H_z\frac{\partial}{\partial z}\delta r\Big)$$
$$+\frac{\delta r^2}{\bar r_z^2}+2\frac{\delta r}{\bar r_z}\Big(H_z\frac{\partial}{\partial z}\delta r-2\kappa\Big)-2\kappa H_z\frac{\partial}{\partial z}\delta r-H_z\frac{\partial}{\partial z}\delta\theta\frac{\partial}{\partial\theta}\delta r-H_z\frac{\partial}{\partial z}\delta\phi\frac{\partial}{\partial\phi}\delta r+\mathcal{U}_\theta\delta\theta$$
$$+\mathcal{U}_\|\Big(2\frac{\delta r}{\bar r_z}-2\kappa-H_z\frac{\partial}{\partial z}\Delta\eta\Big)-\frac{1}{\bar r_z}\Big(\mathcal{U}_\theta\frac{\partial}{\partial\theta}+\frac{\mathcal{U}_\phi}{\sin\theta}\frac{\partial}{\partial\phi}\Big)(\delta r+\Delta\eta)+\mathcal{U}_\phi\sin\theta\delta\phi$$
$$+(\delta g+3\,\delta z)\Big(2\frac{\delta r}{\bar r_z}-2\kappa+H_z\frac{\partial}{\partial z}\delta r-\mathcal{A}+\mathcal{U}_\|\Big)+\Delta x^\mu\partial_\mu(3\,\delta z+\delta g-\mathcal{A}+\mathcal{U}_\|)$$

Standard (dangerous) practice: fix a gauge to reduce complexity

$\Longrightarrow$ Lose possibility to verify coordinate independence

1st sanity check: Observables must not depend on coordinates

$$\mathrm{d}V = \frac{\bar{r}_z^2}{H_z(1+z)^3}\,\mathrm{d}z\,\mathrm{d}\Omega\,(1+\delta V)$$

$d\bar{V}_{obs}$

The volume we infer does not coincide with the physical volume occupied by the source because of light propagation

δV is a scalar under coordinate transformation: explicit expression must satisfy

$$x^\mu \to x^\mu + \xi^\mu(x) \iff \delta V \to \delta V$$

First geometric consistency check independent of gravity
⇒ Most robust expressions for second-order observables

Second-order gauge-invariant formalism for the cosmological observables: complete verification of their gauge-invariance

Matteo Magi, Jaiyul Yoo. [JCAP 09 (2022) 071]

Explicit checks for:

- Physical volume and area occupied by the source

$$\begin{cases} \delta V \\ \delta A \end{cases} \Longrightarrow \begin{array}{ll} \delta_g & \text{Galaxy number density} \\ \delta D_L & \text{Luminosity distance} \end{array}$$

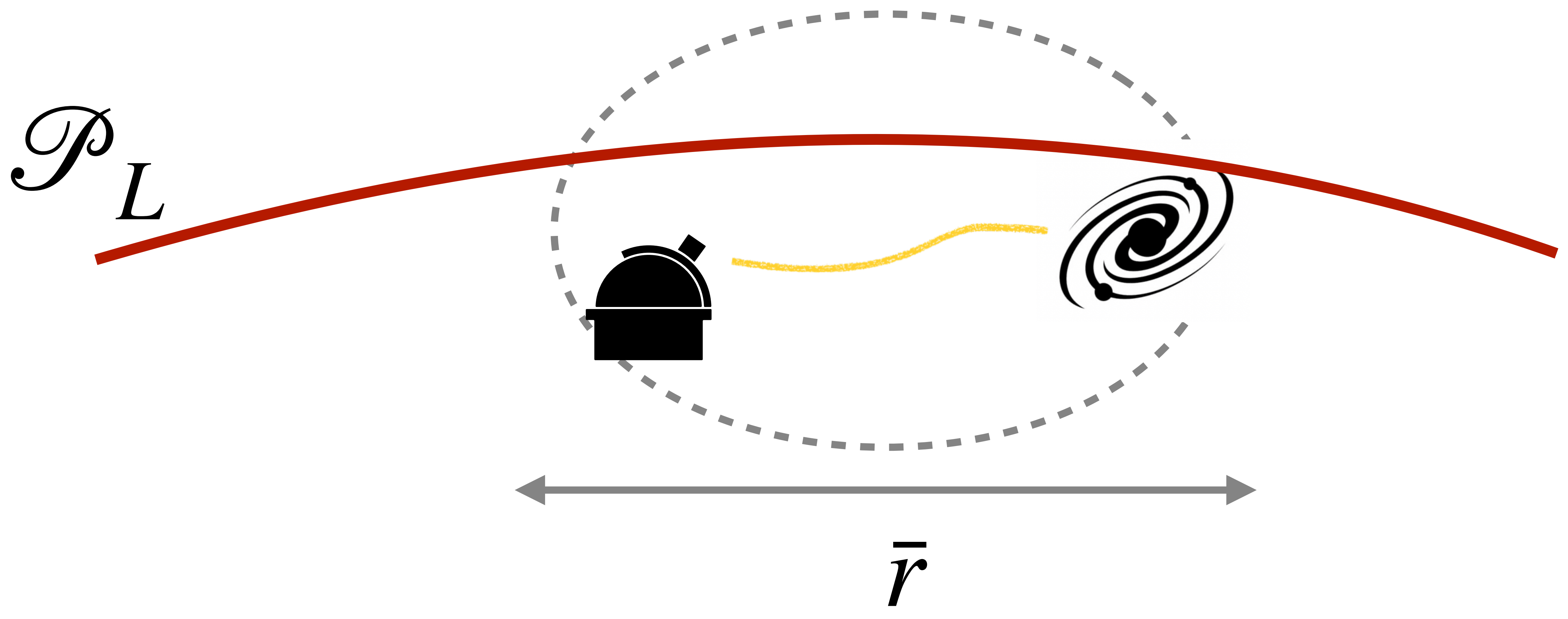
- Redshift fluctuations $\quad \delta z \Longrightarrow \Theta$ CMB temperature anisotropies

Currently checking coordinate dependence of weak lensing observables

Cosmic shear B-modes at second order in perturbation theory and their relation with the image rotation

Matteo Magi, Francesca Lepori, Julian Adamek (in preparation)

2nd sanity check: Observables in ΛCDM must be infrared insensitive



Observable quantities receive contributions from fluctuations on all scales, even $k\bar{r} \ll 1$

Long modes = fluctuations that are almost spatially constant on the scale probed

$$\mathscr{P}_L(\eta, \boldsymbol{x}) = \mathscr{P}(\eta, \boldsymbol{x}_o) + (x^\alpha - x_o^\alpha)\partial_\alpha \mathscr{P}(\eta, \boldsymbol{x}_o) + \cdots$$

Infrared sensitivity is how much an observable "feels" the presence of long modes

E.g. galaxy clustering:

$$\delta_g(z,\hat{n}) = b\,\delta_m + \frac{1}{\mathcal{H}}\partial_r^2 v - \left(1-\frac{t}{2}\right)\int_0^{\bar{r}_z} d\bar{r}\,\left(\frac{\bar{r}_z-\bar{r}}{\bar{r}_z\bar{r}}\right)\Delta_\Omega\left(\Psi-\Phi\right)$$

$$-\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\partial_r v\big|_o^z - \partial_r v - (2-t)\partial_r v\big|_o + \frac{1}{\mathcal{H}}\left(\partial_r\Psi - \partial_r v'\right)$$

$$+\Psi + (2-t)\left[\Phi - \frac{v_o}{\bar{r}_z} + \frac{1}{\bar{r}_z}\int_0^{\bar{r}_z} d\bar{r}\,\left(\Psi-\Phi\right)\right] - \frac{1}{\mathcal{H}}\Phi' - e_z\mathcal{H}v$$

$$-\left(3-e_z-t-\frac{2-t}{\mathcal{H}\bar{r}_z}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left[\mathcal{H}_o v_o + \Psi - \Psi_o + \int_0^{\bar{r}_z} d\bar{r}\,\left(\Psi'-\Phi'\right)\right]$$

- Peculiar aspect of relativistic effects: Contributions with less than two spatial derivatives

Isolate uniform potential and gradient by expanding any perturbation in the observable:

$$\mathscr{P}_L(\eta,\boldsymbol{x}) = \mathscr{P}(\eta,\boldsymbol{x}_o) + (x^\alpha - x_o^\alpha)\partial_\alpha\mathscr{P}(\eta,\boldsymbol{x}_o) + \cdots \implies \begin{cases} \delta_{g,0} = \cdots \\ \delta_{g,1} = \cdots \end{cases}$$

Explicit contribution of long modes depends on the theory of gravity and initial conditions

- In ΛCDM

$$\begin{cases} \delta_{g,0} = \cdots = 0 \\ \delta_{g,1} = \cdots = 0 \end{cases}$$

Proved at every order in perturbation theory, applies to all cosmological observables

Any perturbation in ΛCDM can be expressed in terms of curvature perturbation (adiabaticity) that is conserved on large scales (GR)

$\Longrightarrow$ Large diffeomorphism can remove all

$$\mathscr{P}(\eta, \boldsymbol{x}_o)\,, \quad \partial_\alpha \mathscr{P}(\eta, \boldsymbol{x}_o)$$

Constant potentials and uniform gradients

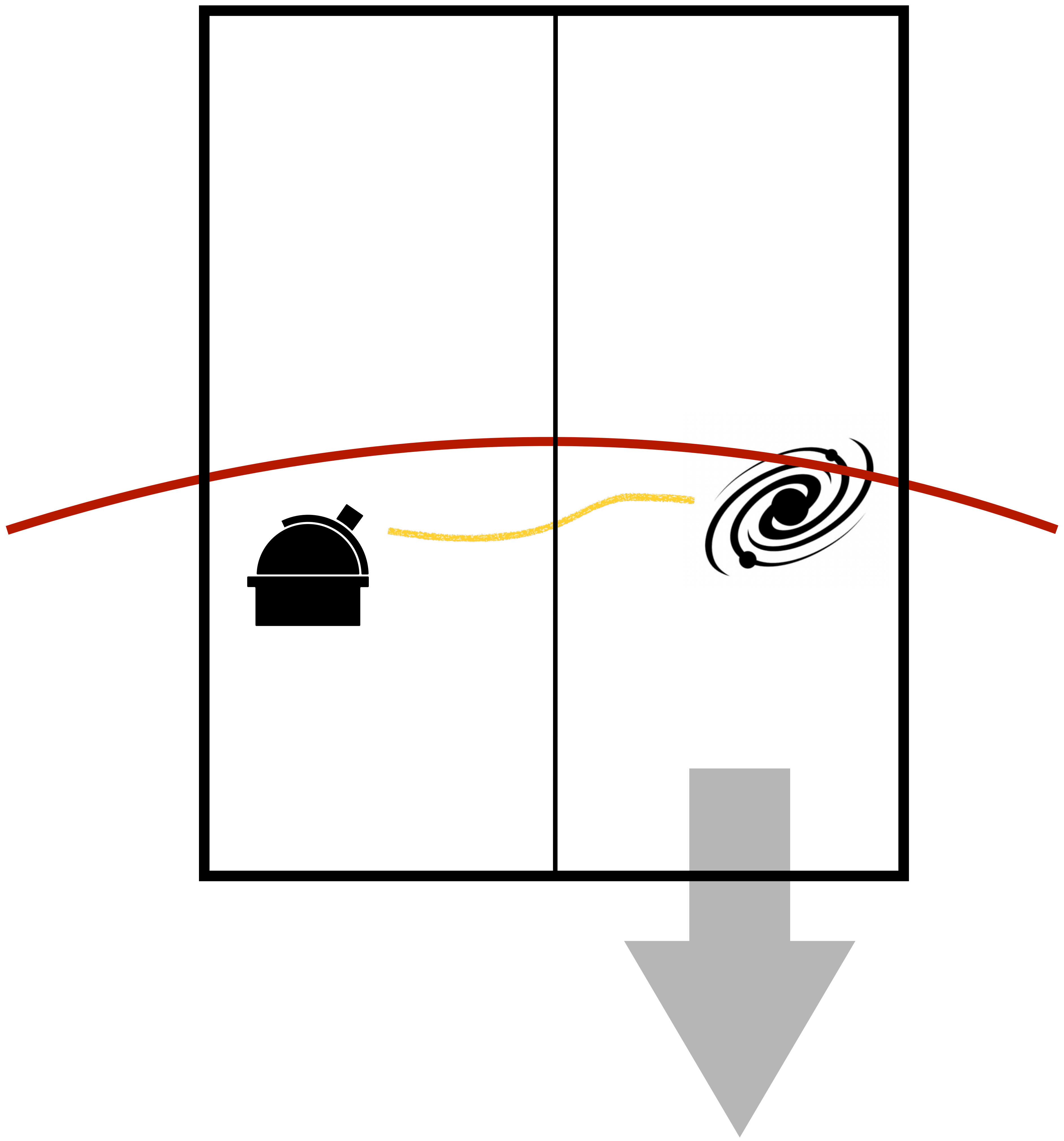
Conditions for the absence of infrared sensitivity in cosmological probes in any gravity theories

Matteo Magi, Jaiyul Yoo. [Phys.Lett.B 846 (2023) 138204]

Infrared (in)sensitivity of relativistic effects in cosmological observable statistics

Ermis Mitsou, Jaiyul Yoo, **Matteo Magi**. [Phys.Lett.B 843 (2023) 137998]

- In ΛCDM it is possible to find a non-local elevator for the observer-source system that free falls at the same rate under the modulation of long modes (cosmological equivalence principle)



2nd sanity check: Observables in ΛCDM must be infrared insensitive

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- Infrared cancellations need to occur as a consequence of cosmological EP dynamical consistency check

- Gradient expansion of perturbations in the observable
  $\mathscr{P}_L(\eta, \boldsymbol{x}) = \mathscr{P}(\eta, \boldsymbol{x}_o) + (x^\alpha - x_o^\alpha)\partial_\alpha \mathscr{P}(\eta, \boldsymbol{x}_o) + \cdots$

- Check cancellations e.g. coefficients of $\mathscr{P}(\eta, \boldsymbol{x}_o)\mathscr{Q}(\eta, \boldsymbol{x}_o)$ must be zero

Summary & Future Directions

- Cosmology entered high-precision era (Euclid, DESI, ACT, LSST, SKA, Simons Observatory…)
- Theoretical descriptions of what we measure need to match level of observations
- We proposed two sanity checks beyond linear theory
  - I) Coordinate independence of observables (geometrical) - extend to weak lensing
  - II) Infrared insensitivity or compatibility with EP (dynamical) - to be completed
- Apply most robust second-order descriptions to three-point statistics, e.g. galaxy bispectrum