Natural inflation with and without modulations in string theory

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based on PTEP 2015 6, 063E02
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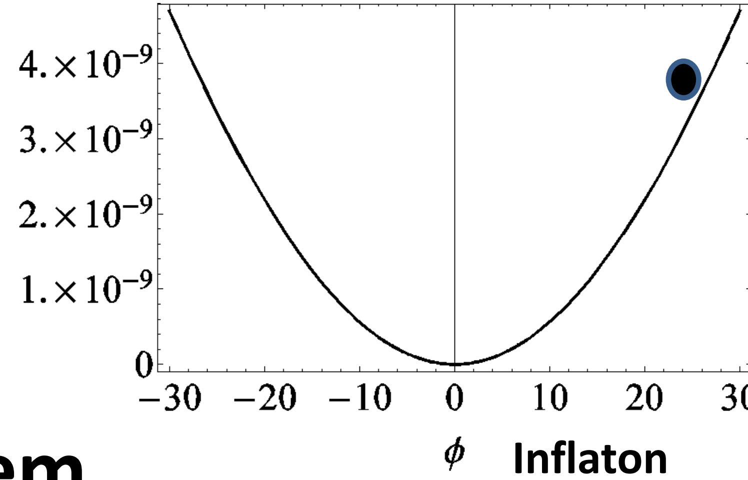
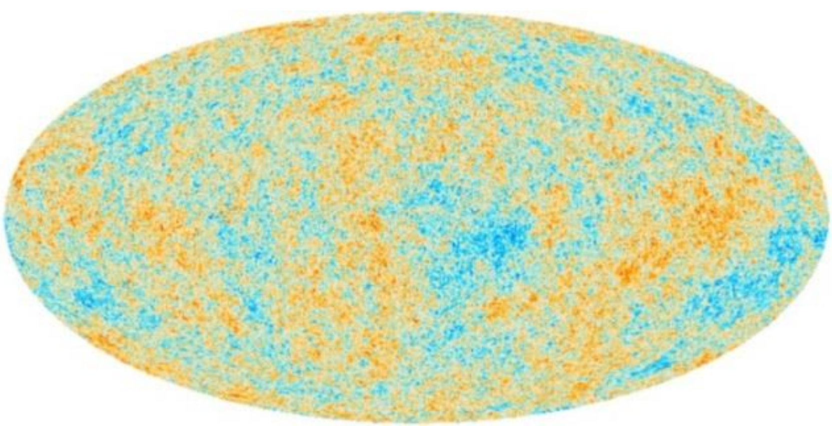
Inflation



○ **Solving the fine tuning problem (Horizon problem and flatness problem)**

○ **Producing the origin of the density perturbations**



$$\frac{\Delta T}{T} \sim 10^{-5}$$

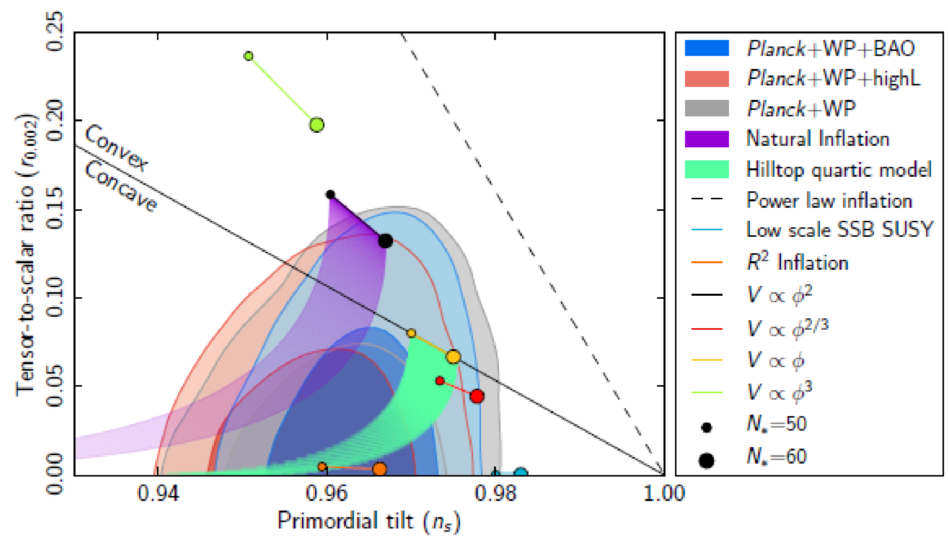


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Planck

Large-field Inflation

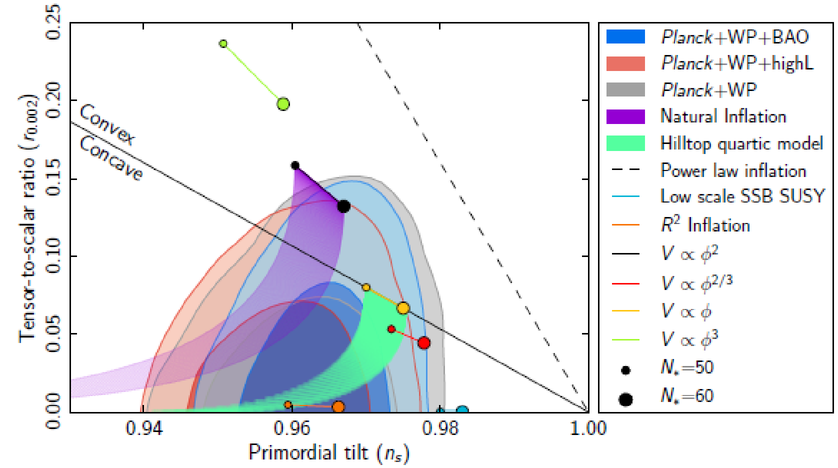


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Planck

〇No detection of B-modes

〇The near future experiments, e.g., BICEP/ Keck Array, Planck,…, have the sensitivity to detect the tensor-to-scalar ratio r～O(10^{-2}).

➔A large-field inflation would be tested by them.

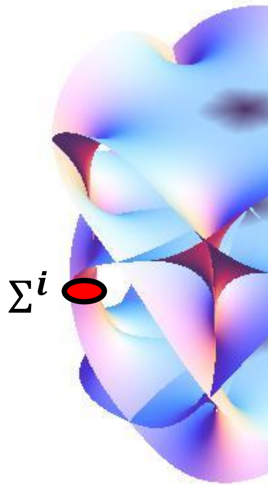
We consider it based on the low-energy effective theory of string theory.

Axions in string theory

A lot of axions originating from high-dimensional form fields:

e.g.

○ Integrating the 2-form A over the internal 2-cycle of internal manifold,



$$a^i(x) = \int_{\Sigma^i} A$$

Axions 2-form

○Axion is the imaginary part of a modulus.($T^i = t^i$+ia^i)

Axions in string theory

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Axions　　　2-form

○Axion is the imaginary part of a modulus.($T^i = t^i$+ia^i)

Perturbative flat potential (shift symmetry)

○Gauge symmetry: $A \rightarrow A + d\Lambda$ becomes a shift symmetry for $a^i(x)$.

○ Some non-perturbative effects generate their potential.

➔A good candidate for the inflaton

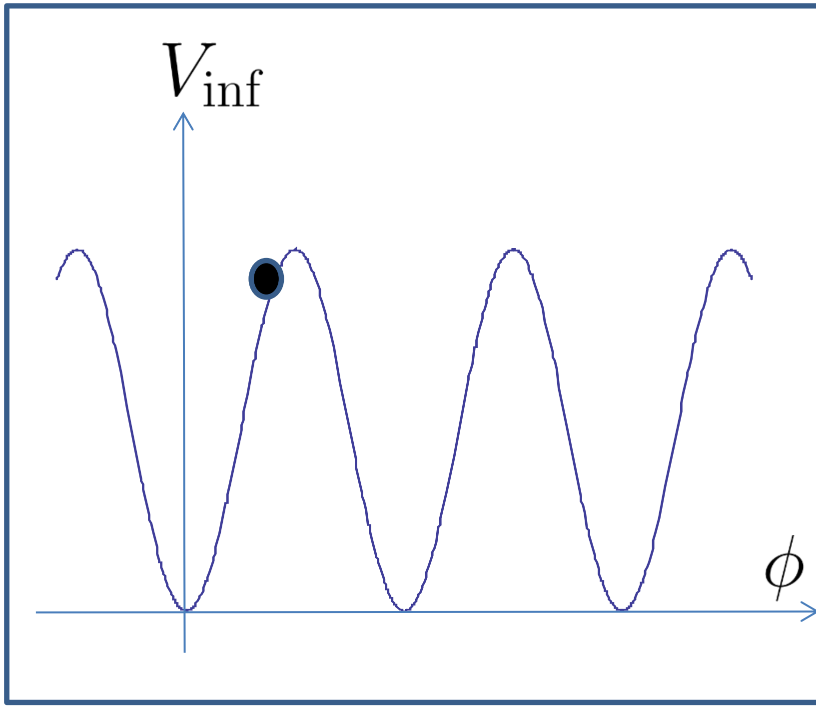
Natural inflation (Axion inflation)

Freese, Frieman and Olinto (1990)

$$V_{\rm inf} \simeq \Lambda^4 \left(1 - \cos\left(\frac{\phi}{f}\right)\right)$$

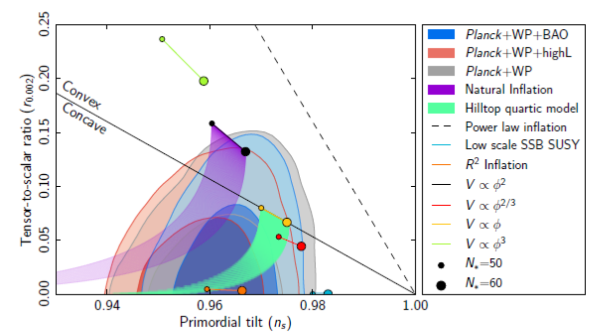
ϕ : inflaton (axion)

f : axion decay constant



Planck results require the trans-Planckian axion decay constant:

$$f > 5M_{\rm Pl}$$

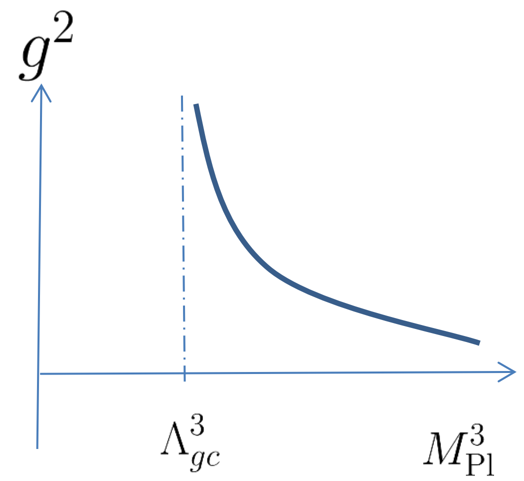


Axion potential in string theory

Non-perturbative effects generate the axion potential.

e.g., gaugino condensation

$$\Lambda_{gc}^3 = \langle \lambda\lambda \rangle \simeq e^{-24\pi^2/bg^2}$$



b **: one-loop beta-function coefficient**
(b=3N for SU(N) SYM.)

Superpotential

$$W = \Lambda_{gc}^3 = Ae^{-\frac{2\pi}{N}T}$$

$$T = t + ia$$

➔ Axion potential

$$V = \Lambda^4 \left(1 - \cos\left(\frac{2\,\pi}{N\,d}\phi\right)\right)$$

$d \simeq 1/\langle t \rangle$ **: normalization factor**

ϕ **: canonically normalized axion (inflaton)**

The trans-Planckian axion decay constant is achieved by N=O(100) and d=O(1).

In order to enhance the axion decay constant,

Multiple axions

○Alignment mechanism, N-flation, Kinetic mixing,…

J. E. Kim, H. P. Nilles and M. Peloso (‘04)

S. Dimopoulos, S. Kachru, J. McGreevy, J. G. Wacker (‘05)

Bachlechner et. Al (‘14)
G. Shiu (’15)

e.g., two axions (ϕ_1, ϕ_2)

$$V = \Lambda_1^4 \left(1 - \cos\left(\frac{\phi_1}{f_1} + \frac{\phi_2}{g_1}\right)\right) + \Lambda_2^4 \left(1 - \cos\left(\frac{\phi_1}{f_2} + \frac{\phi_2}{g_2}\right)\right)$$

When their axion decay constants are tuned as

$$\frac{g_1}{f_1} \simeq \frac{g_2}{f_2}$$

effective axion decay constant becomes trans-Planckian for a light mode, even if $f_{1,2}, g_{1,2} < M_{\rm Pl}$

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Single axion

○Axion monodromy

○Threshold correction

H. Abe, T. Kobayashi, and H. O. ('14)

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We propose the single-field natural inflation by (closed) string axion.

Outline

i) Introduction

ii) Trans-Planckian axion decay constant

iii) Natural inflation w and w/o modulations
(type IIB string)

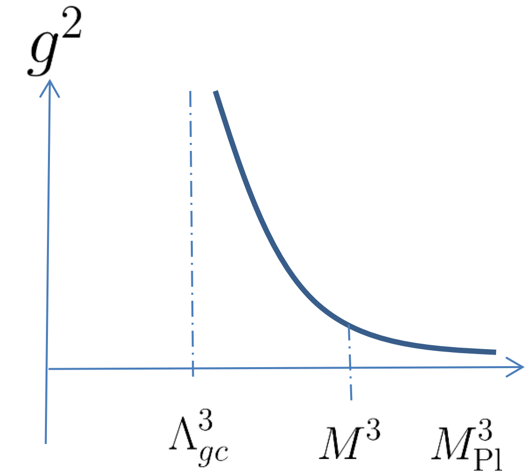
iv) Moduli stabilization

v) Natural inflation (heterotic string)

vi) Conclusion

Threshold corrections

The gauge coupling receives the threshold corrections due to the massive modes,



$$\frac{1}{g^2} = \frac{T}{4\pi} + \frac{\Delta b}{16\pi^2} \ln\left(\frac{M}{M_s}\right)^2$$

Δb : beta-function coefficient due to massive modes

E.g., such corrections come from the KK modes.

$$M_k = \frac{k}{R}$$

R : volume of internal manifold (moduli)

k : integer

$$\sum_k \frac{\Delta b}{8\pi^2} \ln\left(\frac{M_k}{M_s}\right) \simeq \frac{\Delta b}{8\pi^2} \ln RM_s + \cdots$$

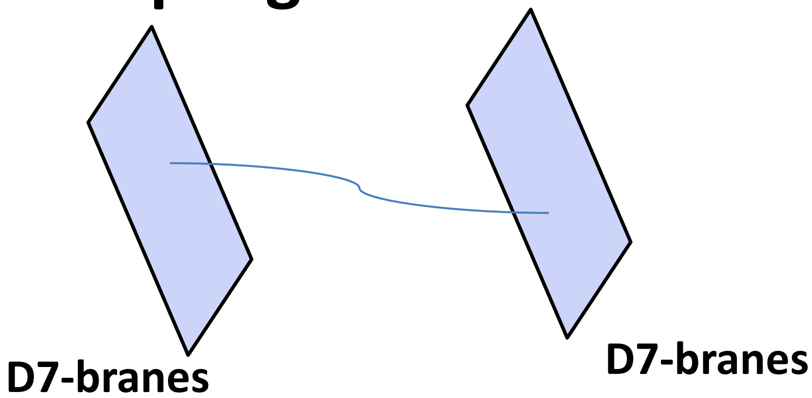
They are moduli-dependent.

Threshold corrections in string theory

○Type IIB string on toroidal orientifold and orbifold with D3/D7-branes

$$T^6/(Z_2 \times Z_2)$$

The gauge coupling on D7-branes:



D. Lüst and S. Stieberger, ('03)

$$\frac{1}{g^2} = \frac{T}{4\pi} + \frac{\Delta(U)}{16\pi^2}$$

U :Complex structure moduli (Shape of torus)

T :Kähler moduli (Volume of torus)

Moduli-dependent threshold corrections

$$\Delta = -4b_{N=2} \ln \eta(i\,U)$$

Dedekind function

$$\eta(iU) = e^{-\pi U/12} \prod_{n=1} (1 - e^{-2\pi n U})$$

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D. Lüst and S. Stieberger, (ʻ03)

The gaugino condensation on D7-branes (SU(L) SYM)

$$W = Ae^{-8\pi^2/Lg^2} = Ae^{-\frac{2\pi T}{L} - \frac{2b_{N=2}}{L} \ln \eta(iU)}$$

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$$\eta(i\,U) \to e^{-\frac{\pi}{12}U}\left[1 - \mathcal{O}(e^{-2\pi U})\right], \mathrm{Re}\,U \gg 1$$

$$W = Ae^{-\frac{2\pi T}{L}} e^{-\frac{b_{N=2}\pi}{6L}U}$$

Natural inflation in string theory

○Type IIB string on toroidal orientifold and orbifold with D3/D7-branes

$T^6/(Z_2 \times Z_2)$

Superpotential

$$W = Ae^{-\frac{2\pi T}{L}} e^{-\frac{b_{N=2}\pi}{6L}U}$$

Integrate out the moduli except for $\mathrm{Im}\, U$

Axion potential

$$V_{\mathrm{eff}} = \Lambda^4 \left(1 - \cos\left(\frac{b_{N=2}\,\pi\sqrt{2}\langle \mathrm{Re}\, U\rangle}{6L}\phi\right)\right)$$

$$f \simeq \frac{1.4\, L}{b_{N=2}\langle \mathrm{Re}\, U\rangle} \qquad \phi \simeq \frac{\mathrm{Im}\, U}{\sqrt{2}\langle \mathrm{Re}\, U\rangle}$$

In the case of $L/b_{N=2} > 1$ and $\langle \mathrm{Re}\, U\rangle = \mathcal{O}(1)$, the trans-Planckian axion decay can be obtained, even if $L \ll O(100)$.

We can realize the successful natural inflation.

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i) Introduction
ii) Trans-Planckian axion decay constant
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(type IIB string)
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Natural inflation with modulations

Superpotential

$$W = A\,e^{-\frac{2\pi}{L}T}\eta(iU)^{2b_{N=2}/L} + \cdots$$

$$\eta(i\,U) \to e^{-\frac{\pi}{12}U}\left[1 - e^{-2\pi U} - \mathcal{O}(e^{-4\pi U})\right], \mathrm{Re}\,U > 1$$

$$\Downarrow$$

$$Ae^{-\frac{2\pi\,T}{L}}e^{-\frac{b_{N=2}\,\pi}{6L}U}\left(1 - e^{-2\pi U}\right)$$

Inflaton potential

T. Kobayashi and F. Takahashi ('10)

$$V_{\mathrm{eff}} \simeq \Lambda - \Lambda_1 \cos\left(\lambda_1\phi\right) + \boxed{\Lambda_2 \cos\left(\lambda_2\phi\right)}$$

Modulation term

$$\Lambda_2 = \Lambda_1\,\frac{2b}{L}e^{-\left(2\pi + \frac{b_{N=2}\,\pi}{6L}\right)\langle\mathrm{Re}\,U^2\rangle} < \Lambda_1$$

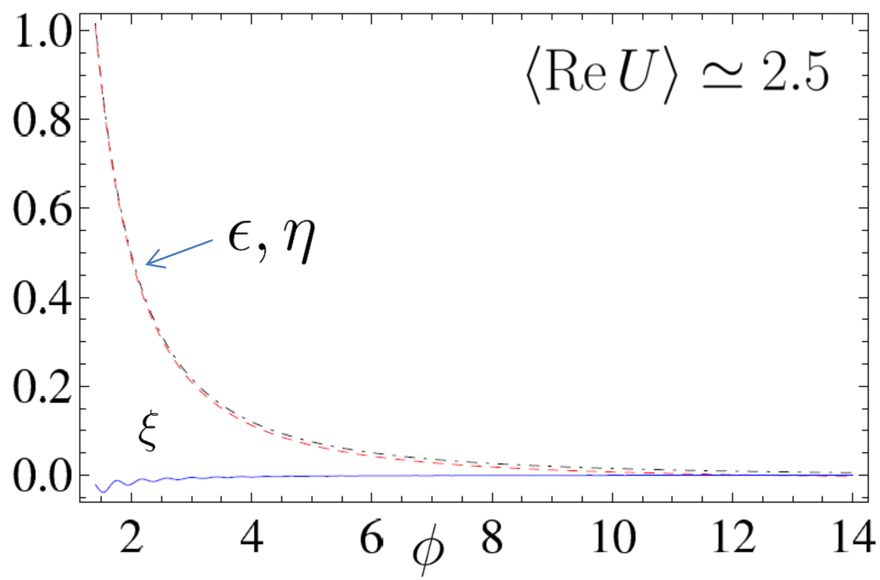
$$\lambda_1 = \frac{b_{N=2}\,\pi\sqrt{2}\langle\mathrm{Re}\,U\rangle}{6L}$$

$$\lambda_2 = \left(2\pi + \frac{b_{N=2}\,\pi}{6L}\right)\sqrt{2}\langle\mathrm{Re}\,U\rangle \simeq \mathcal{O}(2\pi)$$

The behavior of the slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{\partial_\phi V_{\rm eff}}{V_{\rm eff}}\right)^2 \quad \eta = \frac{\partial_\phi \partial_\phi V_{\rm eff}}{V_{\rm eff}} \quad \xi = \frac{\partial_\phi V_{\rm eff} \partial_\phi \partial_\phi \partial_\phi V_{\rm eff}}{V_{\rm eff}^2}$$

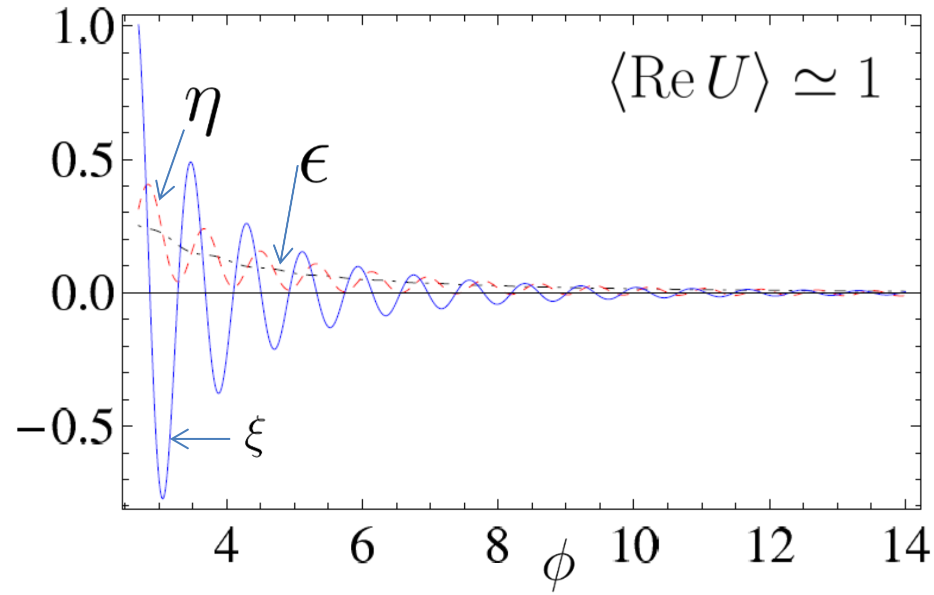
Without modulations



$$V_{\rm eff} \simeq \Lambda_1 \left(1 - \cos\left(\lambda_1 \phi\right)\right)$$

$$\eta(i\,U) \to e^{-\frac{\pi}{12}U}\left[1 - \mathcal{O}(e^{-2\pi U})\right]$$

With modulations

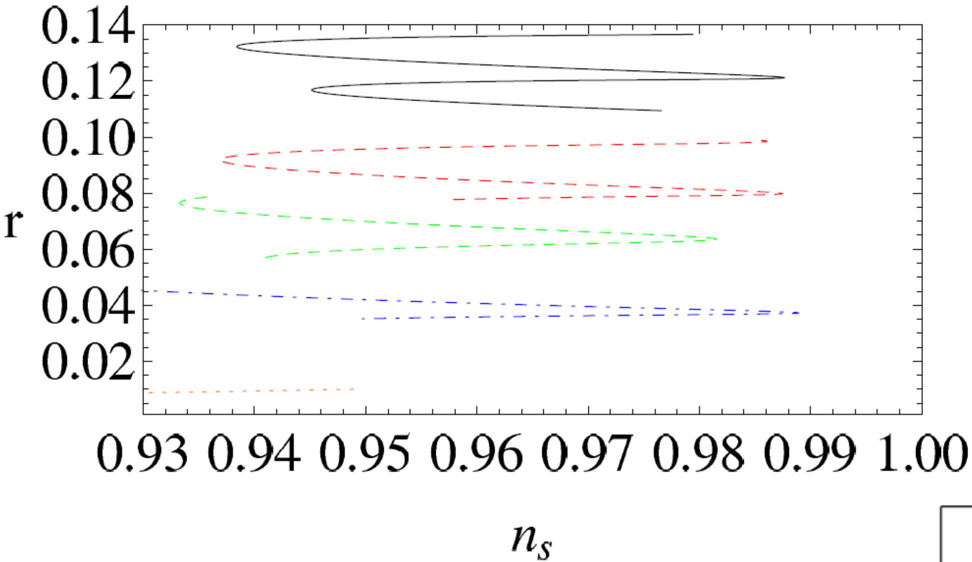


$$V_{\rm eff} \simeq \Lambda - \Lambda_1 \cos\left(\lambda_1 \phi\right) + \Lambda_2 \cos\left(\lambda_2 \phi\right)$$

$$\eta(i\,U) \to e^{-\frac{\pi}{12}U}\left[1 - e^{-2\pi U} - \mathcal{O}(e^{-4\pi U})\right]$$

$$V_{\rm eff} \simeq \Lambda - \Lambda_1 \cos(\lambda_1 \phi) + \Lambda_2 \cos(\lambda_2 \phi)$$

Modulation term



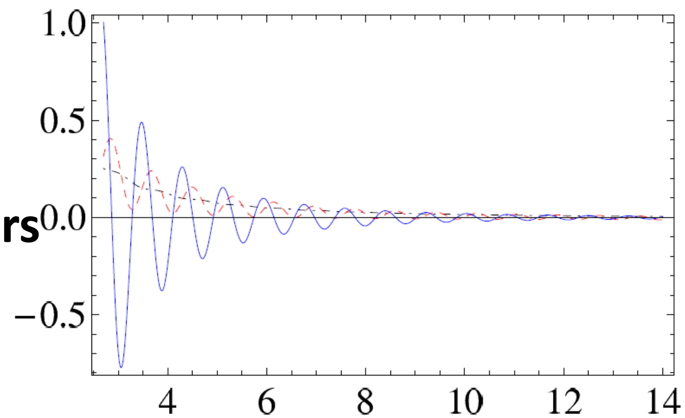
$$n_s = 1 + 2\eta - 6\epsilon + \cdots$$

$$r = 16\epsilon$$

$$\lambda_1 = \frac{b_{N=2}\,\pi\sqrt{2}\langle {\rm Re}\,U\rangle}{6L}$$

$$\lambda_2 = \left(2\pi + \frac{b_{N=2}\,\pi}{6L}\right)\sqrt{2}\langle {\rm Re}\,U\rangle$$

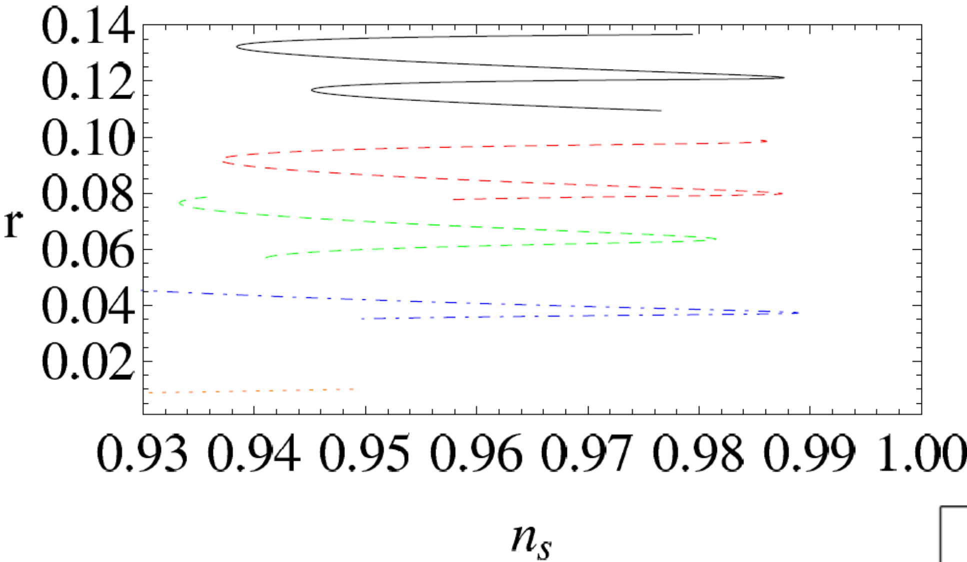
Slow-roll parameters



b/L	$\langle {\rm Re}\,U^2\rangle$	N_e	n_s	r	$dn_s/d\ln k$
1/10	1.3	50	0.96	0.14	−0.0008
1/10	1.3	57	0.96	0.12	−0.012
1/5	1.2	55	0.96	0.08	−0.002
1/5	1.2	60	0.96	0.08	−0.001
1/4	1.2	53	0.96	0.07	−0.002
1/4	1.2	58	0.96	0.06	−0.001
1/3	1.1	54	0.96	0.04	−0.002
1/3	1.1	60	0.96	0.04	−0.001
1/2	1.1	50	0.95	0.01	−0.0003

$$V_{\rm eff} \simeq \Lambda - \Lambda_1 \cos(\lambda_1 \phi) + \Lambda_2 \cos(\lambda_2 \phi)$$

Modulation term



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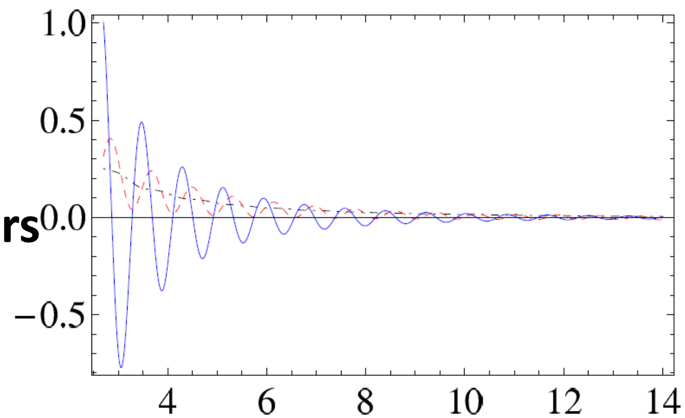
Slow-roll parameters



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(type IIB string)
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v) Natural inflation (heterotic string)
vi) Conclusion

Moduli stabilization

So far, we have assumed that
the other moduli except for the inflaton are already decoupled.

In the framework of toroidal orientifold and orbifold,

$$T^6/(Z_2 \times Z_2)$$

Moduli :

S ・・・Dilaton,
T・・・Overall Kähler moduli (for simplicity)
U_i・・・Three complex structure moduli
(One of Im(U_i) is the inflaton.)

Moduli stabilization

We consider the scenario similar to the KKLT scenario.

The procedures:

i) Flux compactification to stabilize the linear combination of S and $\mathbf{U_i}$.

$$W_{\rm flux}(U,S) = \int G_3 \wedge \Omega$$

ii) Non-perturbative effects for S and T (racetrack superpotential)

iii) Uplifting the scalar potential (E.g., F-term uplifting)

i) Flux compactification

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) - \sum_{i=1}^{3} \ln(U_i+\bar{U}_i)$$

$$\begin{aligned} W_{\rm flux} = & \, w_1 + i w_2 (U_1 - U_2) + i w_3 U_3 + i w_4 S \\ & + w_5 S U_3 + (U_1 - U_2)(w_6 U_3 + w_7 S + i w_8 S U_3) \end{aligned}$$

$\mathbf{U_4 = U_1 - U_2}$

$$\begin{aligned} W_{\rm flux} = & \, w_1 + i w_2 U_4 + i w_3 U_3 + i w_4 S \\ & + w_5 S U_3 + U_4 (w_6 U_3 + w_7 S + i w_8 S U_3) \end{aligned}$$

Only S, $\mathbf{U_3}$ and $\mathbf{U_4 = U_1 - U_2}$ appear in the superpotential.

$$\langle D_I W \rangle = 0, \ \ \langle W_{\rm flux} \rangle = 0$$

$\mathbf{U_2}$ and the linear combination of $\mathbf{S, U_3, U_4}$ remain massless.

ii) Non-perturbative superpotential (racetrack)

$\mathbf{T}, \mathbf{U_2}$ and the linear combination of $\mathbf{S}, \mathbf{U_3}, \mathbf{U_4}$ remain massless.

$$W_{\rm non} = A\exp[-8\pi^2 f_1/N_1] + B\exp[-8\pi^2 f_2/N_2] + C\exp[-8\pi^2 f_3/N_3] + D\exp[-8\pi^2 f_4/N_4]$$

$$f_1 = f_2 = T'/(4\pi)$$

$$f_3 = f_4 = S/(4\pi)$$

$$T' = T + c\,U_2$$

$$c = b_{N=2}/12$$

$$K = -\ln(U_2+\bar{U}_2) - \ln(U_4+\bar{U}_4+U_2+\bar{U}_2) - 3\ln(T'+\bar{T}'-c(U_2+\bar{U}_2))$$

S and T' are stabilized and Re $\mathrm{U_2}$ is also stabilized by nonvanishing superpotential ($\langle W_{\rm non}\rangle \neq 0$).

$$K_{U_2}W = 0 \;\Longrightarrow\; {\rm Re}\,U_2 = 2{\rm Re}\,T'/(5c)$$

iii) Uplifting the AdS vacuum

$$V_F = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2)$$

$$\langle V_F \rangle < 0$$

F-term uplifting scenario:
We have assumed the F-term of SUSY breaking field lifts up the potential.

Finally, we have added the gaugino condensation term which includes the threshold correction for Im U_2

$$W = \langle W_{\rm non} \rangle + E(\langle T \rangle) \exp[-2b_{N=2} \ln \eta(iU_2)/L]$$

We obtain the inflaton potential as explained before.

Comment on the moduli stabilization

We have shown the specific example compatible with the inflation mechanism.

Key idea:
If the other moduli expect for one of Im U_i are already stabilized at their minima,
threshold corrections can enhance the axion decay constant by the inverse of loop factor.

Comment on the gauge couplings

In our scenario, the gauge couplings are determined at the tree-level.

The axion (inflaton) appears in the non-perturbative effects through the threshold corrections.

Regardless of the size of gauge coupling,
the axion decay constant can be enhanced by the threshold corrections.

Comment on the other compactification

In the case of CY, we do not know the explicit form of threshold corrections.

However, we expect that certain corrections enhance the axion decay constant,
if the axions appear by non-perturbative effects through such corrections.

$E_8{}^{(vis)} \times E_8{}^{(hid)}$ Heterotic string theory

The threshold corrections can be also appeared through the one-loop Green-Schwarz term.

Gaugino condensation on $E_8{}^{(hid)}$

$$W = A\, e^{-\frac{8\pi^2}{a}(S-\beta_i T^i)}$$

$$a = 30, \quad \beta_i = \mathcal{O}(1/(8\pi))$$

When we identify one of Im T^i with the inflaton, the trans-Planckian axion decay constant can be realized.

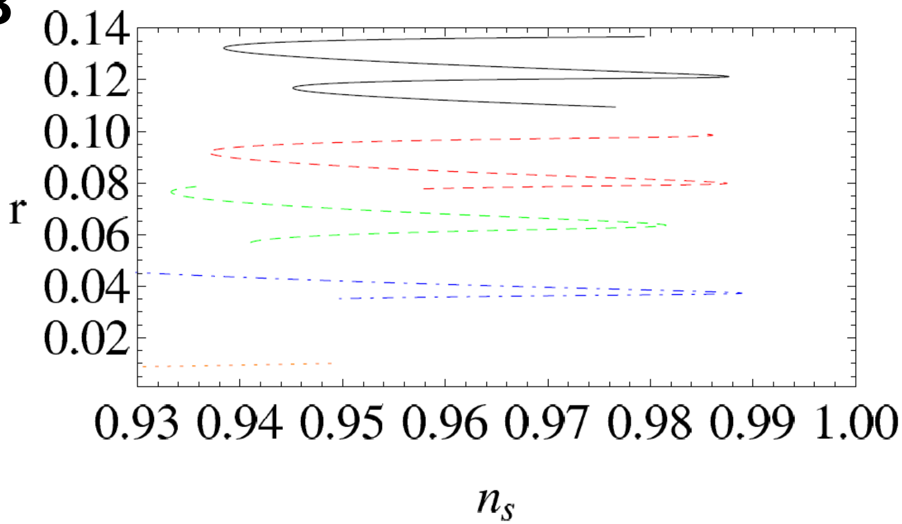
$$V_{\rm eff} \simeq \Lambda^4 (1 - \cos(\beta\, \phi)) \quad \beta = \mathcal{O}\left(\frac{8\pi^2}{a\, d}\beta_i\right) = \mathcal{O}\left(\frac{1}{8}\right) \quad E_8^{(\rm hid)}: a = 30$$

$d \simeq \mathcal{O}(1) \cdots$ normalization factor

Conclusion

i) We propose the single-field natural inflation w/ and w/o modulations in string theory.

Type IIB



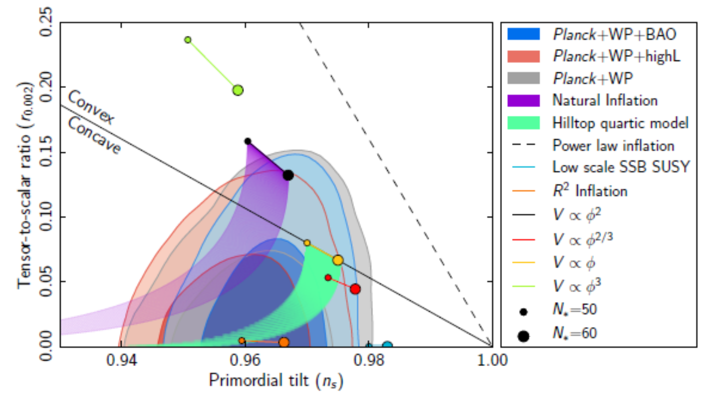


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

ii) String threshold corrections include the modulation terms.

iii) The axion decay constant is enhanced by the inverse of loop factor.