Hypercharge Flux in Heterotic Compactifications

With L.Anderson, A.Constantin, A.Lukas

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Introduction

String Phenomenology

- The Standard Model

  a Particular 4D Quantum Gauge Field Theory

  - $\mathbf{G_{std} = SU(3) \times SU(2) \times U(1)}$
  - $\mathbf{3} \times \left[\mathbf{(3,2)_{\frac{1}{6}} + (\bar{3},1)_{\frac{1}{3}} + (\bar{3},1)_{-\frac{2}{3}} + (1,2)_{-\frac{1}{2}} + (1,1)_1 + (1,1)_0} \right]$

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- Superstring Phenomenology
  - Find a String Vacuum with the structure of the (SUSY) SM
  - Strings in 10D seen as particles in 4D

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- Superstring Phenomenology
  - Find a String Vacuum with the structure of the (SUSY) SM
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- Compactify Het. E_8 Strings [Candelas-Horowitz-Strominger-Witten '85]

Outline

1. **Introduction**
   Heterotic Calabi-Yau models
2. **Basic Structure**
   Group theoretical analysis
3. **Geometry Hunt**
   A partial no-go argument for simplest cases
4. **Discussion**
   Generalisation and interpretation

Introduction

Compactification Ingredients

- Low-Energy Theory

  4D Models, upon compactifying the internal geometry

- SUSY Geometry
  - A CY threefold $\mathbf{X}$
  - A holomorphic, slope-poly-stable vector bundle $\mathbf{V}$ over $\mathbf{X}$

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    s.t. the internal gauge field satisfies the HYM eqns

    $\mathbf{F_{ab} = 0 = F_{\bar{a}\bar{b}}}$ and $\mathbf{g^{a\bar{b}}F_{a\bar{b}} = 0}$

    [Donaldson; Uhlenbeck-Yau]

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    $E_8 \longrightarrow \mathcal{H} = \mathbf{SU(5)}_{\perp}$ GUT
  - Further break by Wilson-line if $\pi_1(X) \neq \phi$
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- Massless matter

$\mathcal{G}$	$\mathcal{H}$	Branching of $\mathbf{248}$ under $\mathcal{G} \times \mathcal{H} \subset E_8$
$SU(5)$	$SU(5)_\perp$	$(\mathbf{1},\mathbf{24}) \oplus (\mathbf{5},\mathbf{10}) \oplus (\overline{\mathbf{5}},\overline{\mathbf{10}}) \oplus (\mathbf{10},\overline{\mathbf{5}}) \oplus (\overline{\mathbf{10}},\mathbf{5}) \oplus (\mathbf{24},\mathbf{1})$

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$\mathcal{G}$	$\mathcal{H}$	Particle Spectrum
$SU(5)$	$SU(5)_{\perp}$	$n_{10} = h^1(X, V)$ $n_{\overline{10}} = h^1(X, V^\star) = h^2(V)$ $n_5 = h^1(X, \wedge^2 V^\star)$ $n_{\overline{5}} = h^1(X, \wedge^2 V)$ $n_1 = h^1(X, V \otimes V^\star)$

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In particular,

$\mathbf{Ind(V)} = -\mathbf{3}$

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- Massless matter
  - $\mathbf{Ind(V) = -3}$
- Heterotic SM Study

  [Braun-He-Ovrut-Pantev '05, Bouchard-Donagi '05, Anderson-Gray-He-Lukas '09, Braun-Candelas-Davies '10, Anderson-Constantin-Gray-Lukas-Palti '11-13, $\ldots\ldots$]

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Majority of known CY three-folds are simply-connected

Highly restrictive

- Massless matter
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CY3 Zoo and π_1

- Complete intersection CYs in a multi-proj.
  - Common zero locus of homogeneous polynomials in $\otimes_{r=1}^{m}\mathbb{P}^{n_r}$
  - Classification ~ 8000 [Candelas, Dale, Lutken, Schimmrigk '88]

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  - Classification ~ 500 million [Kreuzer-Skarke '90]

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  - Exactly 16 are non-simply-connected
- A priori no reasons to give them all up...

Basic Structure

Direct Path to the SM Group

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(cf.) [Blumenhagen, Honecker, Moster, Reinbacher, Weigand '05-06]

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- Massless matter

$$\mathbf{248_{E_8} \to (3,1,1) \oplus (1,8,1) \oplus (1,1,35) \oplus (1,3,\overline{15}) \oplus}$$
$$\mathbf{(1,\overline{3},15) \oplus (2,3,6) \oplus (2,\overline{3},\overline{6}) \oplus (2,1,20)}$$

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- Massless matter

$(SU(3) \times SU(2))_{\mathbf{q}}$	$(\mathbf{1},\mathbf{1})_{\mathbf{e}_a-\mathbf{e}_b}$	$(\bar{\mathbf{3}},\mathbf{1})_{\mathbf{e}_a+\mathbf{e}_b}$	$(\mathbf{3},\mathbf{2})_{\mathbf{e}_a}$	$(\mathbf{1},\mathbf{2})_{\mathbf{e}_a+\mathbf{e}_b+\mathbf{e}_c}$
Range	$a,b = 1,\ldots,6$	$a < b$	$a = 1,\ldots 6$	$a < b < c$
Particle	$e_{a,b},\ S_{a,b}$	$d_{a,b},\ u_{a,b}$	Q_a	$L_{a,b,c},\ H_{a,b,c},\ \bar{H}_{a,b,c}$
Line bundle	$L_a \otimes L_b^*$	$L_a \otimes L_b$	L_a	$L_a \otimes L_b \otimes L_c$
Contained in	$V \otimes V^*$	$\wedge^2 V$	V	$\wedge^3 V$

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Range	$a, b = 1, \ldots, 6$	$a < b$	$a = 1, \ldots 6$	$a < b < c$
Particle	$e_{a,b},\ S_{a,b}$	$d_{a,b},\ u_{a,b}$	Q_a	$L_{a,b,c},\ H_{a,b,c},\ \bar{H}_{a,b,c}$
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where $\mathbf{q} = (q_1, \cdots, q_f)$ denotes the $\mathbf{S(U(1)^f)}$ charges

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Hypercharge	$2,\ 0$	$2/3,\ -4/3$	$1/3$	$-1,\ -1,\ 1$

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Embedding of hypercharge

- $\mathbf{U(1)_Y \subset S(U(1)^f)}$
  - Embedding vector, $\mathbf{y} = (y_1, \cdots, y_f)$
  - Hypercharge of a multiplet F, $Y(F) = \mathbf{y} \cdot \mathbf{q}(F)$

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  - $\mathbf{U(1)_Y}$ normalisation, $\frac{g}{g_Y^2} = \frac{1}{120}\mathrm{Tr}(Y^2) = \frac{5}{3}$

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Embedding of hypercharge

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  - Embedding vector, $\mathbf{y} = (y_1, \cdots, y_f)$
  - Hypercharge of a multiplet F, $Y(F) = \mathbf{y} \cdot \mathbf{q}(F)$
- Constraints on the embedding
  - Fix the U(1)-charge freedom by $\mathbf{n} \cdot \mathbf{y} = 0 \quad (\mathbf{n} = (n_1, \cdots, n_f))$
  - $\mathbf{U(1)_Y}$ normalisation, $\sum_{a=1}^{f} n_a y_a^2 = \frac{5}{3}$

Basic Structure

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  - Masslessness, $\sum_{a=1}^{f} y_a \mathbf{c}_1(W_a) = 0$

Basic Structure

<u>Classification of the embedding vectors</u>

Classification

splitting type $\mathbf{n}$	allowed $\mathbf{y}$ vectors
$(4,1,1)$	~~$(1/3,1/3,-5/3)$~~
$(3,2,1)$	~~$(1/3,1/3,-5/3)$~~, $(-2/3,1/3,4/3)$
$(2,2,2)$	no solution
$(3,1,1,1)$	~~$(1/3,1/3,1/3,-5/3)$~~, $(-2/3,1/3,1/3,4/3)$
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$(1,1,1,1,1,1)$	~~$(1/3,1/3,-5/3,1/3,1/3,1/3)$~~,$(1/3,4/3,-2/3,-2/3,-2/3,1/3)$ $(1/3,5/6,-7/6,-1/6,-2/3,5/6)$,$(1/3,7/12,-17/12,1/12,-5/12,5/6),\ldots$

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Remark

- Embedding vectors with a reduced structure group are crossed out

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Remark

- Embedding vectors with a reduced structure group are crossed out
- Infinite family of embeddings found for the Abelian split case

Basic Structure

At the level of group theory, hypercharge flux provides with a valid alternative, by which the conventional gauge unification persists and all the SM matters are made to have the correct hypercharges.

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What about at the level of string geometries?

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What about at the level of string geometries?

Start by the simplest split type, $\mathbf{n} = (1, 1, 1, 1, 1, 1)$

Geometry Hunt

Index Constraints

- Index Constraints for the Abelian Split, $\bigoplus_{a=1}^{6} L_a$

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$$\sum_{a:y_a=1/3} \mathrm{Ind}(L_a) = -3$$

$$\sum_{a<b:y_a+y_b=2/3} \mathrm{Ind}(L_a \otimes L_b) = -3$$

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- No exotic-Y quarks

$\mathrm{Ind}(L_a) = 0$ if $y_a \neq 1/3$
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- No mirror quarks

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Parametrisation of the geometry

- Geometry Label

$$\mathbf{c}_1(L_a) \equiv x_a \in H^2(X,\mathbb{Z}) \text{ for } a = 1, \cdots, 6 \; ; \; \mathbf{c}_2(X) \in H^4(X,\mathbb{Z})$$

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Two of the x_a's can be eliminated due

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- Diophantine Variables

$$X_{abc} \equiv x_a \cdot x_b \cdot x_c \; ; \quad Z_a \equiv x_a \cdot \mathbf{c}_2(X) \text{ for } a, b, c = 1, \cdots, 4$$

$$\mathrm{Ind}(L_a) = \frac{1}{6}x_a^3 + \frac{1}{12}x_a \cdot \mathbf{c}_2(X)$$

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Each of the embedding vectors, $\mathbf{y}$, gives rise to
a Linear Diophantine system on the 24 variables,

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    - no exotic particles arise
- Robustness of the No-Go
  - Does not rely on any particular (class of) CYs

Geometry Hunt

Relaxing the requirement

- Evading the no-go?
  - What if the gauge unification condition, $\sum_{a=1}^{6} y_a^2 = \frac{10}{3}$, holds approximately, within 5%?

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  - With $\frac{9.5}{3} \leq \sum_{a=1}^{6} y_a^2 \leq \frac{10.5}{3}$, (re)classify the viable embeddings and solve for the Diophantine system!
  - No embedding survives except for

$$\mathbf{y} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} - \alpha, \frac{1}{3} + \alpha, -\frac{5}{3})$$

where $|\alpha| \leq 0.288\cdots$

Geometry Hunt

Relaxing the requirement - what remains

- The surviving case
  - Embedding: $y = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} - \alpha, \frac{1}{3} + \alpha, -\frac{5}{3})$ with $|\alpha| \leq 0.288\cdots$
  - The SM spectrum is highly restricted:

$$
\begin{array}{rcl}
\mathrm{ind}(L_1) = \mathrm{ind}(L_2) = \mathrm{ind}(L_3) = -1 & \rightarrow & Q_1,\ Q_2,\ Q_3 \\
\mathrm{ind}(L_1 \otimes L_4) = \mathrm{ind}(L_2 \otimes L_4) = \mathrm{ind}(L_3 \otimes L_4) = -1 & \rightarrow & u_{1,4},\ u_{2,4},\ u_{3,4} \\
\mathrm{ind}(L_5 \otimes L_6) = -3 & \rightarrow & 3\, d_{5,6} \\
\mathrm{ind}(L_4 \otimes L_5 \otimes L_6) = -\mathrm{ind}(L_1 \otimes L_2 \otimes L_3) = -3 & \rightarrow & 3\, L_{4,5,6} \\
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\mathrm{ind}(L_1 \otimes L_2^*) = -\mathrm{ind}(L_2 \otimes L_1^*) = 2 - X_{112} & \rightarrow & S_{1,2} \text{ or } S_{2,1} \\
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  - Any geometries realising the above?

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    - NO for CICYs

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\mathrm{ind}(L_1) = \mathrm{ind}(L_2) = \mathrm{ind}(L_3) = -1 \quad &\to \quad Q_1,\ Q_2,\ Q_3 \\
\mathrm{ind}(L_1 \otimes L_4) = \mathrm{ind}(L_2 \otimes L_4) = \mathrm{ind}(L_3 \otimes L_4) = -1 \quad &\to \quad u_{1,4},\ u_{2,4},\ u_{3,4} \\
\mathrm{ind}(L_5 \otimes L_6) = -3 \quad &\to \quad 3\, d_{5,6} \\
\mathrm{ind}(L_4 \otimes L_5 \otimes L_6) = -\mathrm{ind}(L_1 \otimes L_2 \otimes L_3) = -3 \quad &\to \quad 3\, L_{4,5,6} \\
\mathrm{ind}(L_1 \otimes L_4^*) = \mathrm{ind}(L_2 \otimes L_4^*) = \mathrm{ind}(L_3 \otimes L_4^*) = -1 \quad &\to \quad e_{1,4},\ e_{2,4},\ e_{3,4} \\
\mathrm{ind}(L_1 \otimes L_2^*) = -\mathrm{ind}(L_2 \otimes L_1^*) = 2 - X_{112} \quad &\to \quad S_{1,2} \text{ or } S_{2,1} \\
\mathrm{ind}(L_1 \otimes L_3^*) = -\mathrm{ind}(L_3 \otimes L_1^*) = 2 + X_{111} + X_{112} \quad &\to \quad S_{1,3} \text{ or } S_{3,1} \\
\mathrm{ind}(L_2 \otimes L_3^*) = -\mathrm{ind}(L_3 \otimes L_2^*) = 6 - X_{112} + X_{222} \quad &\to \quad S_{2,3} \text{ or } S_{3,2}
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  - Any geometries realising the above?
    - NO for CICYs; unclear in general.

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THANK YOU

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- Polystability test is difficult in general, but . . .
  - Line bundle, $L = \mathcal{O}_X(\mathbf{k})$ is stable
  - Line-bundle sum, $V = \bigoplus_{a=1}^{n} L_a = \bigoplus_{a=1}^{n} \mathcal{O}_X(\mathbf{k}_a)$ is polystable iff

$$\mu(O_X(\mathbf{k}_a)) = 0,\ \forall a$$

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  - By having the non-Abelian entries in y split equally into the Abelian counterparts, the No-Go extends in an obvious manner.