Theoretical Modeling of Cosmological Momentum Field through the Kinetic SZ Effect

@IBS CTPU (11/4, 2015)

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Cosmology

- How is the beginning of the universe?
- What are components in the universe?
- How have the universe evolved?

Cosmological Measurements

LSS

Galaxy Number density

CMB

Temperature fluctuations

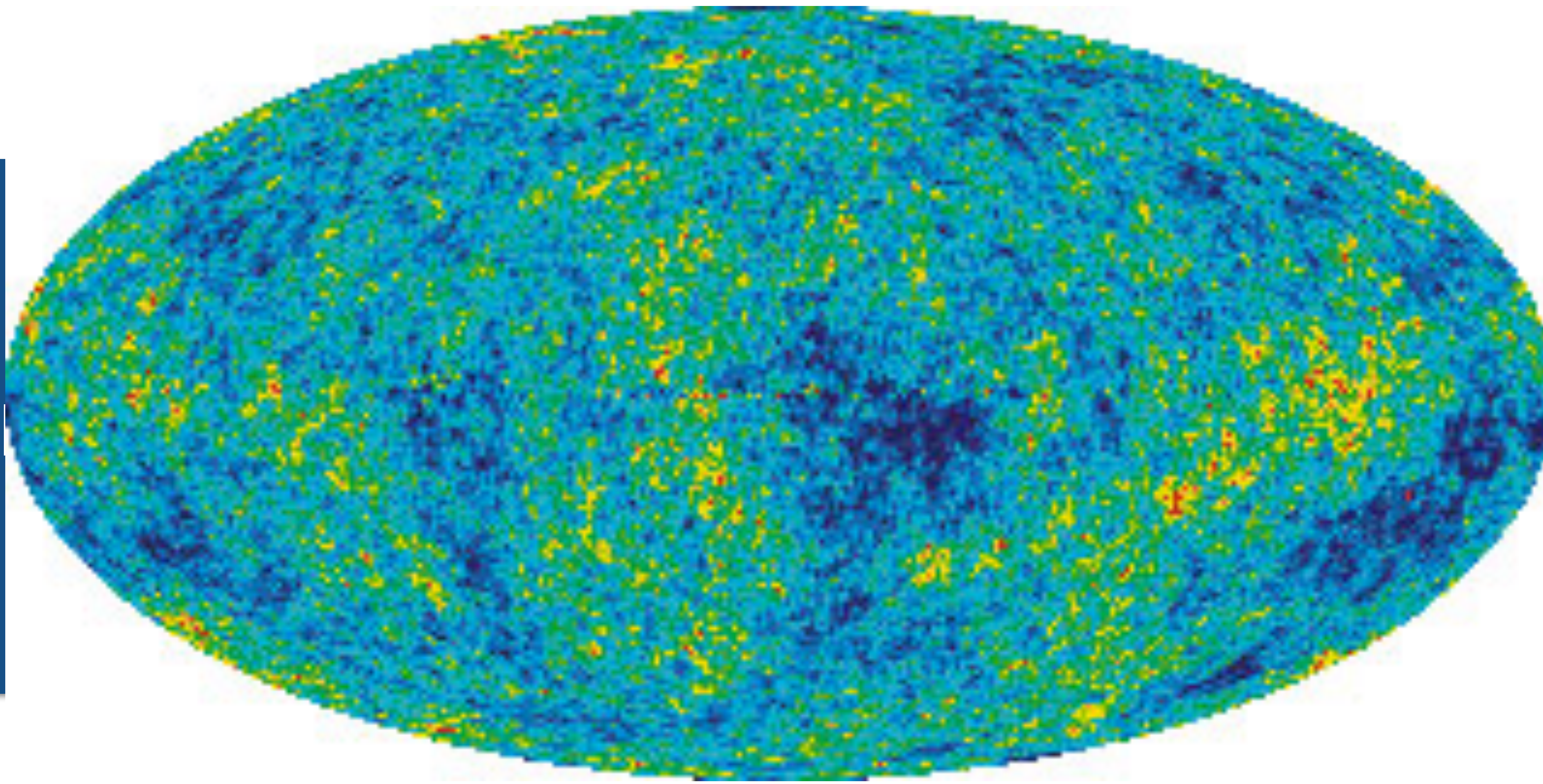
Lensing

Gravitational potential

Beginning
Components
Evolution

Cosmological Measurements

Ga



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WMAP homepage

Cosmological Measurements

LSS

Galaxy Number density

CMB

Temperature fluctuations

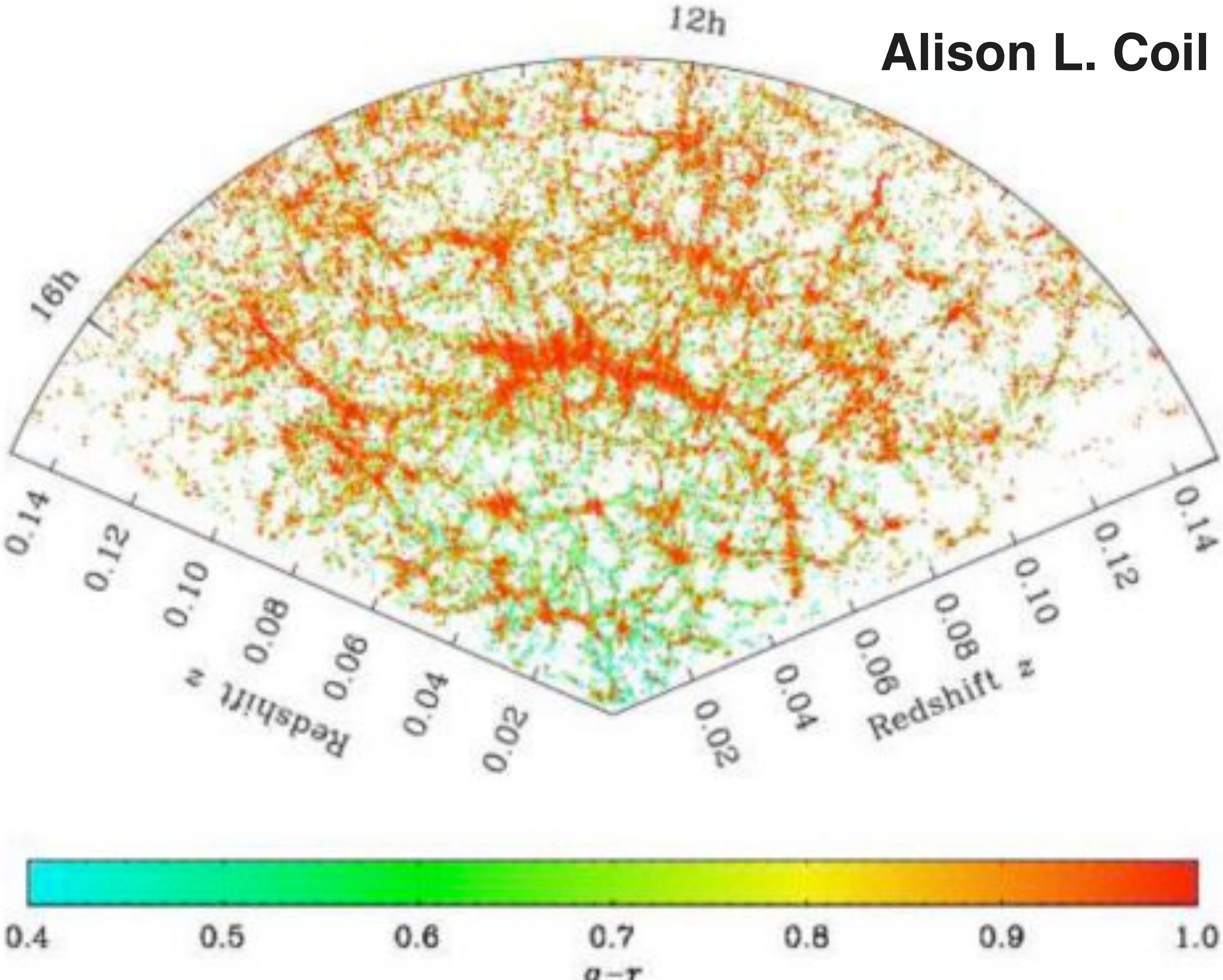
Lensing

Gravitational potential

Beginning
Components
Evolution

Cosmological Measurements

Alison L. Coil (2012)



Gala

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Cosmological Measurements

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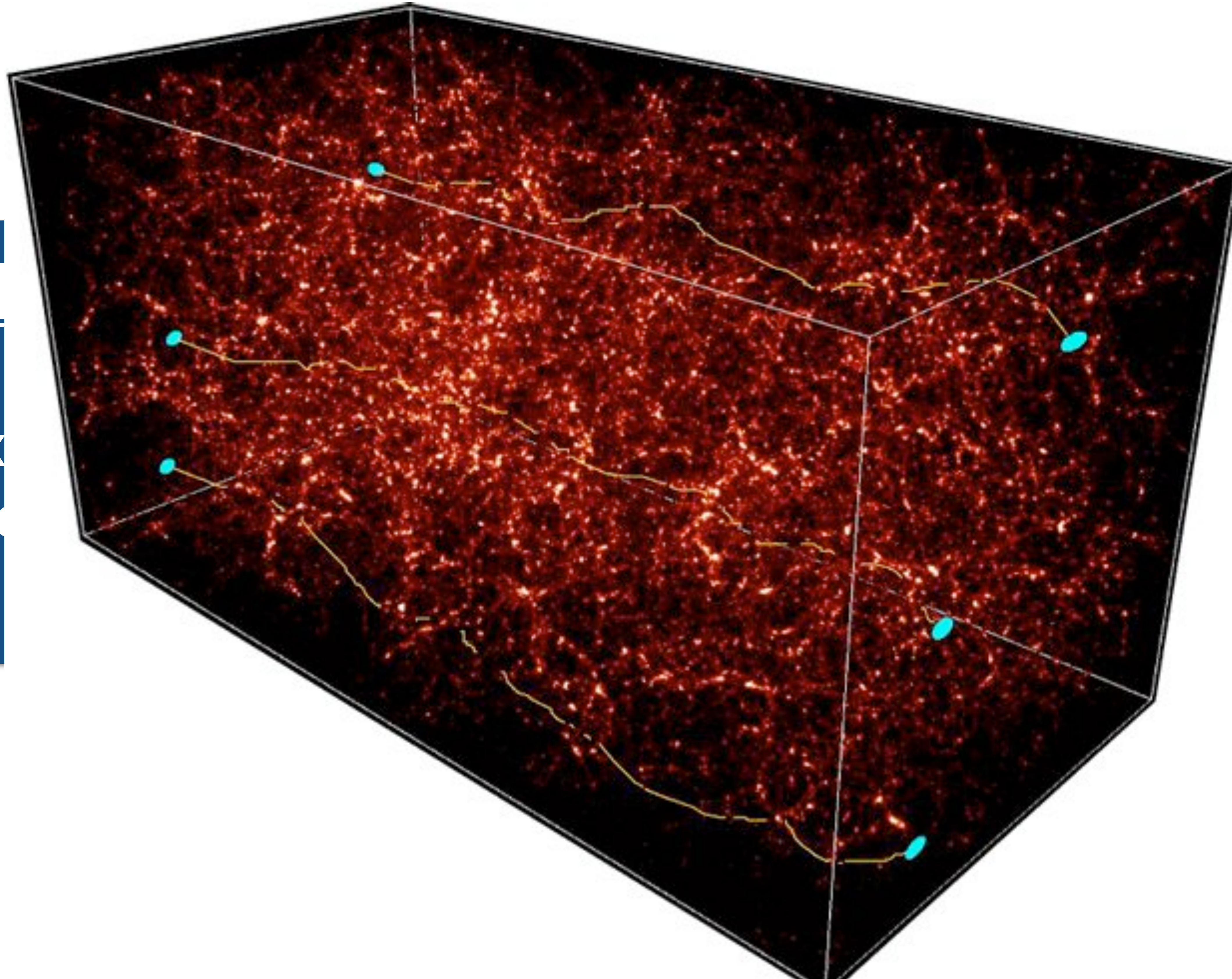
Beginning
Components
Evolution

Cosmological Measurements

Galax
d



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Cosmological Measurements

LSS

Galaxy Number density

CMB

Temperature fluctuations

Lensing

Gravitational potential

Kinetic
Sunyaev
Zel’dovich
(KSZ) effect

Cosmological Measurements

LSS

Gal ρ ber

CMB

Temperature fluctuations

$$\frac{\delta T_{\mathrm{kSZ}}}{T} \propto \tau_{\mathrm{g}} \frac{\hat{n} \cdot \boxed{\vec{v}}}{c}$$

Lensing

Gravitational potential

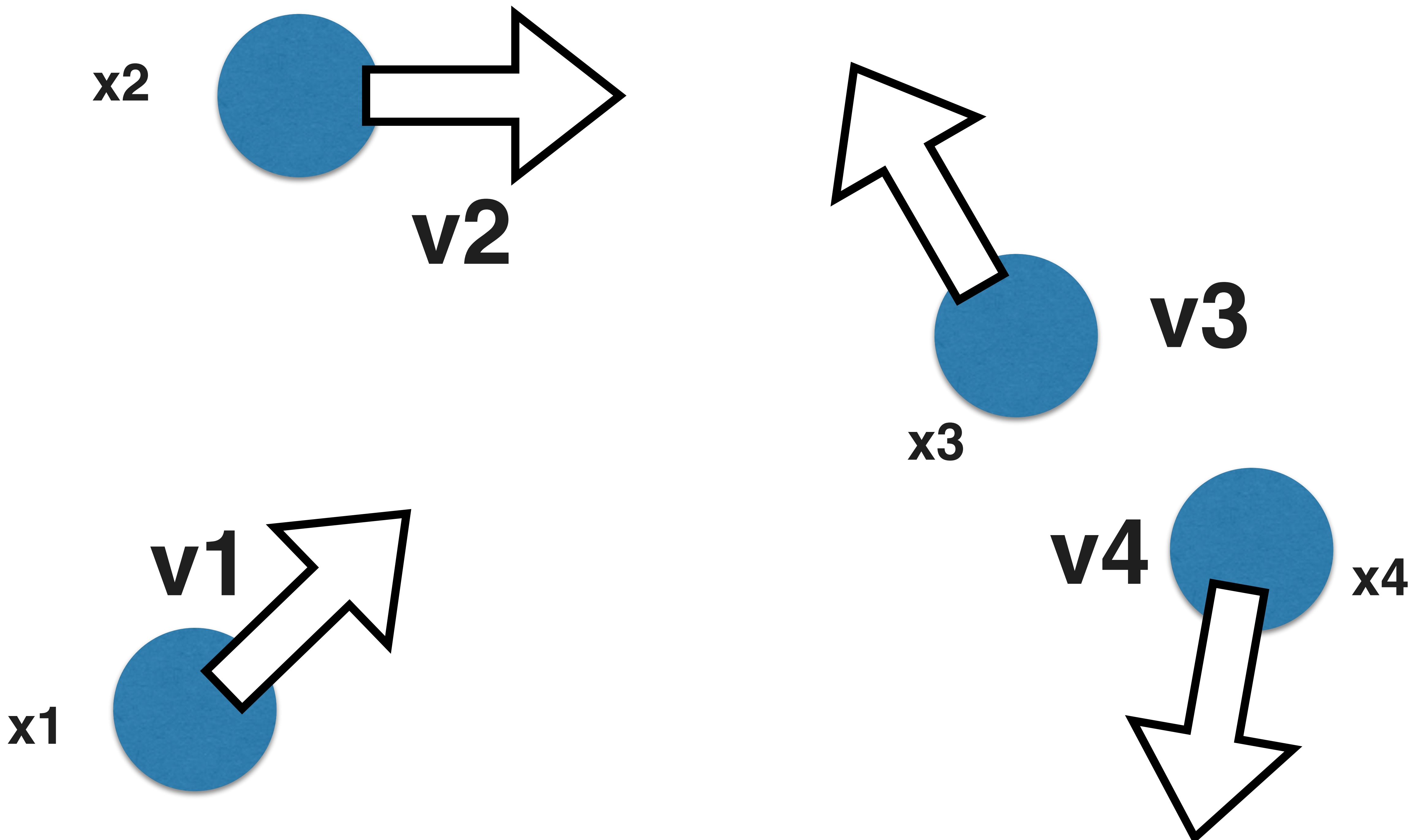
“Momentum Field”

$$\boxed{\vec{p} = \vec{v}\rho}$$

kSZ effect in CMB

Galaxy clustering

Galaxy Distribution



Velocity field is unmeasurable,
while momentum field is measurable.

“Momentum Field”

$$\boxed{\vec{p} = \vec{v}\rho}$$

kSZ effect in CMB

Galaxy clustering

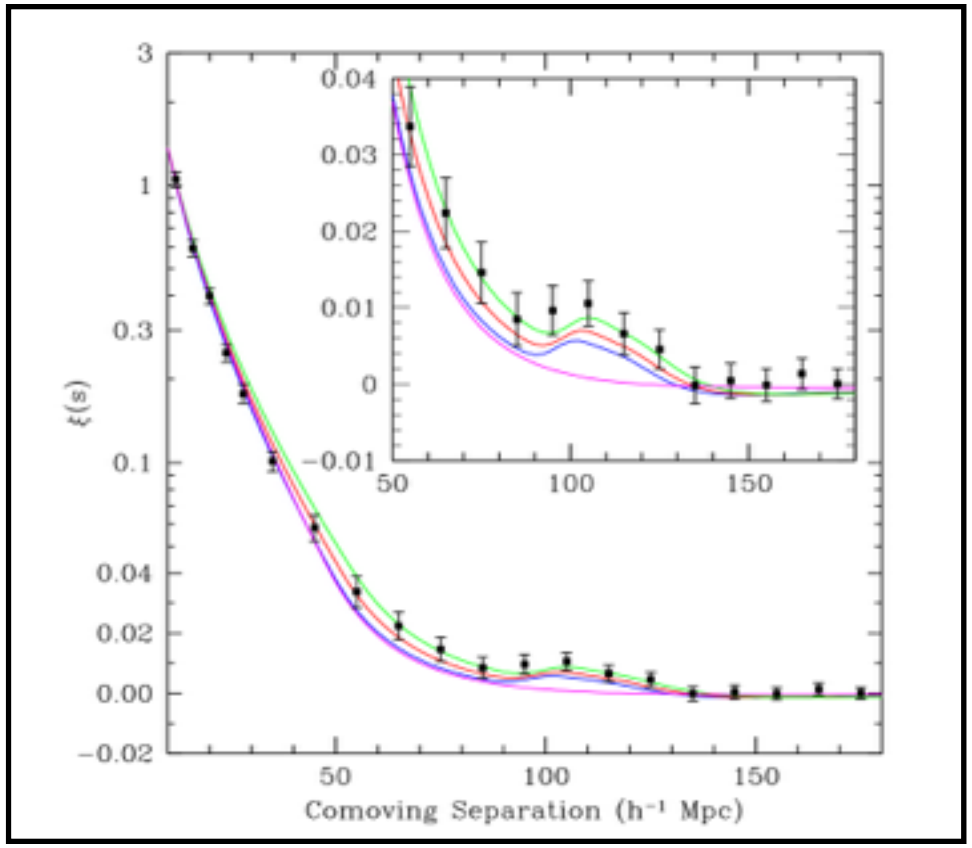
Importance of Velocity Information

- **Sensitive to gravity theories.**

$$\boxed{\frac{d\vec{v}}{dt} \propto -\nabla\delta\phi}$$

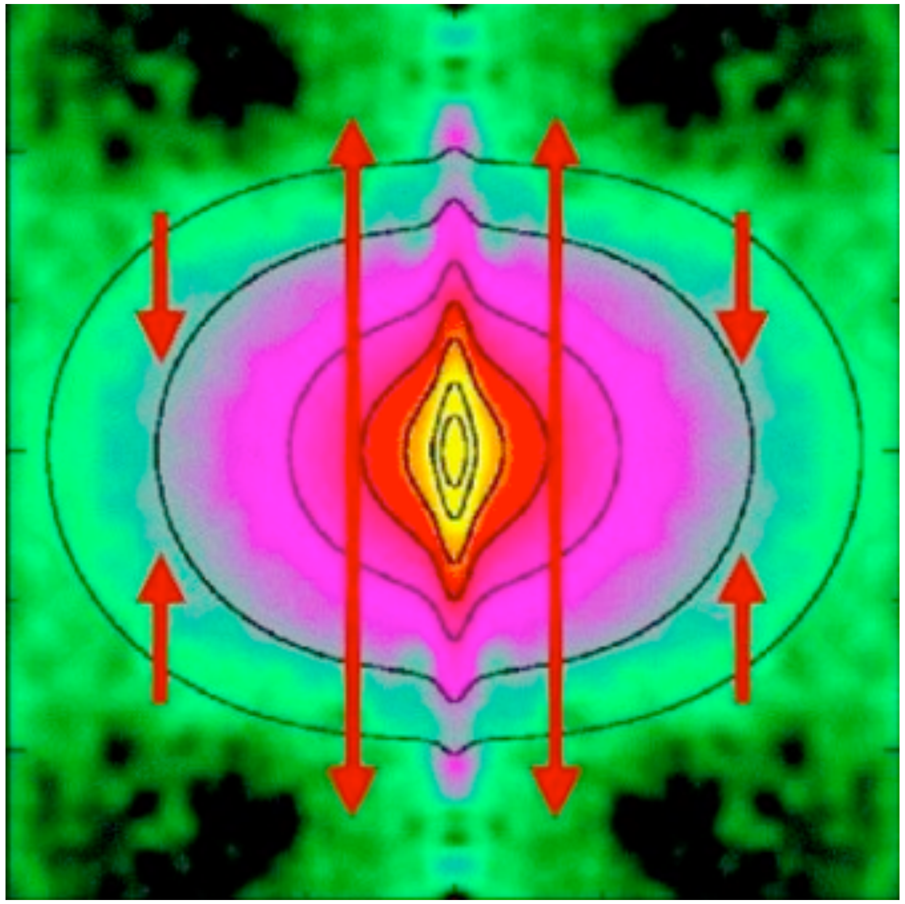
3D Galaxy Distributions

Baryon Acoustic Oscillation (BAO)



Eisenstein et al. (2005)

Redshift Space Distortions (RSD)



Alison L. Coil (2012)

Through RSD, the peculiar velocity correlation function can be measured.

Redshift Space Distortion

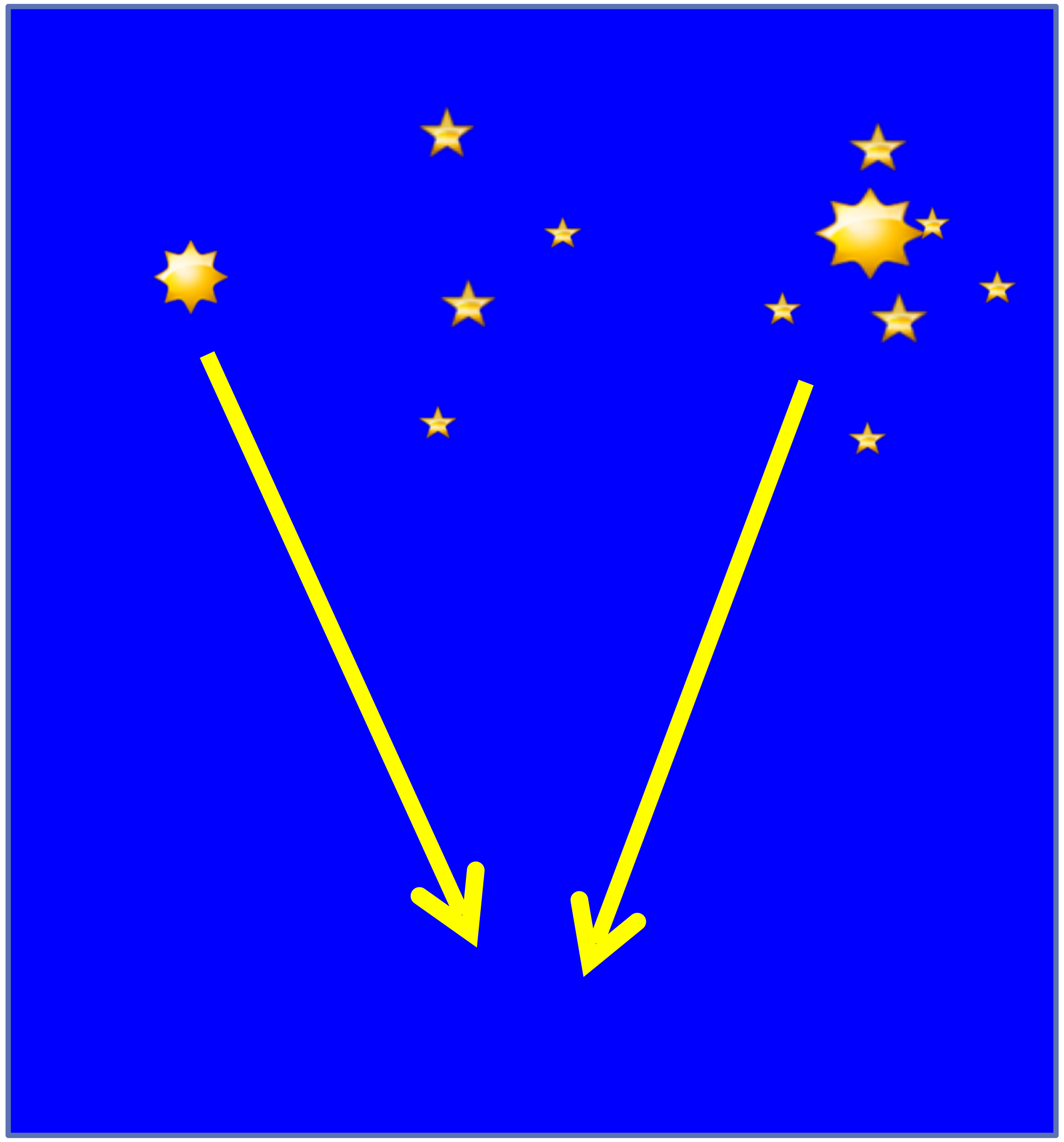
PV SW ISW

$$\mathbf{x} \quad \rightarrow \quad \mathbf{s} = \mathbf{x} + \left[\frac{\mathbf{v}(t,\mathbf{x}) \cdot \hat{n}}{H} - \frac{\delta\phi(t,\mathbf{x})}{aHc} + \frac{2}{aHc} \int dt \frac{\partial \delta\phi(t,\mathbf{x})}{\partial t} \right] \hat{n} + \mathcal{O}\left((\mathbf{v}/c)^2 \right)$$

v : peculiar velociry
δφ : gravitational potential

$$1 + z_{\mathrm{obs}} \equiv \frac{E_{\mathrm{e}}}{E_{\mathrm{o}}} = \frac{-g_{\mu\nu} u^{\mu} P^{\nu}|_{\mathrm{e}}}{-g_{\mu\nu} u^{\mu} P^{\nu}|_{\mathrm{o}}}$$

E_e : energy of emitted photon
E_o : energy of observed photon



Redshift Space Distortion

PV SW ISW

$$\mathbf{x} \quad \rightarrow \quad \mathbf{s} = \mathbf{x} + \left[\frac{\mathbf{v}(t,\mathbf{x}) \cdot \hat{n}}{H} - \cancel{\frac{\delta\phi(t,\mathbf{x})}{aHc} + \frac{2}{aHc} \int dt \frac{\partial \delta\phi(t,\mathbf{x})}{\partial t}} \right] \cancel{\hat{n} + \mathcal{O}\left((\mathbf{v}/c)^2 \right)}$$

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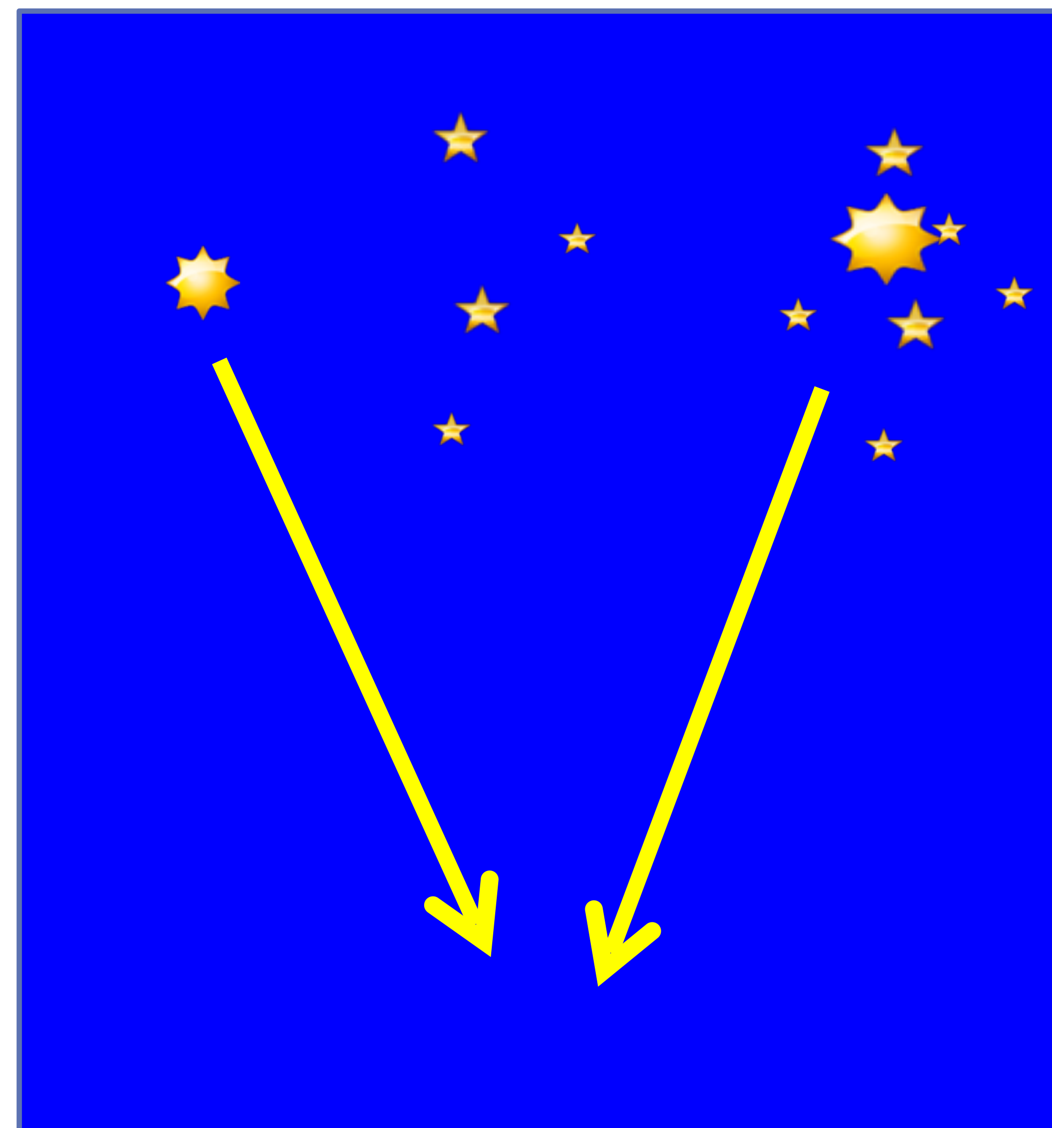
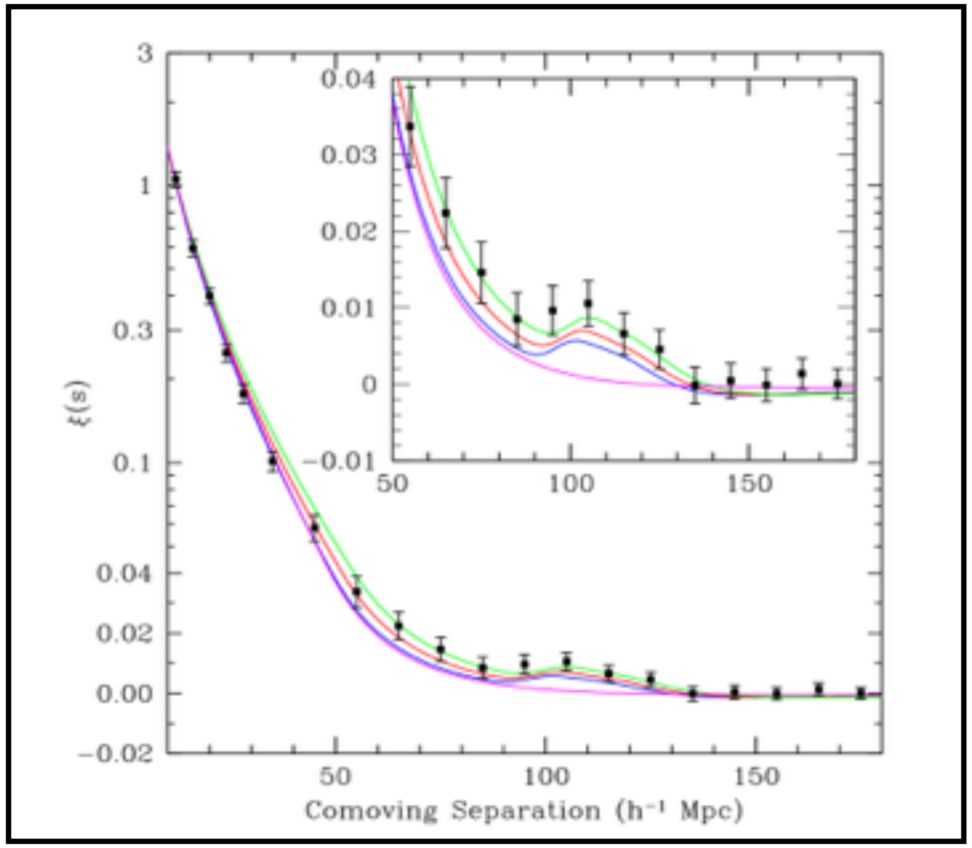


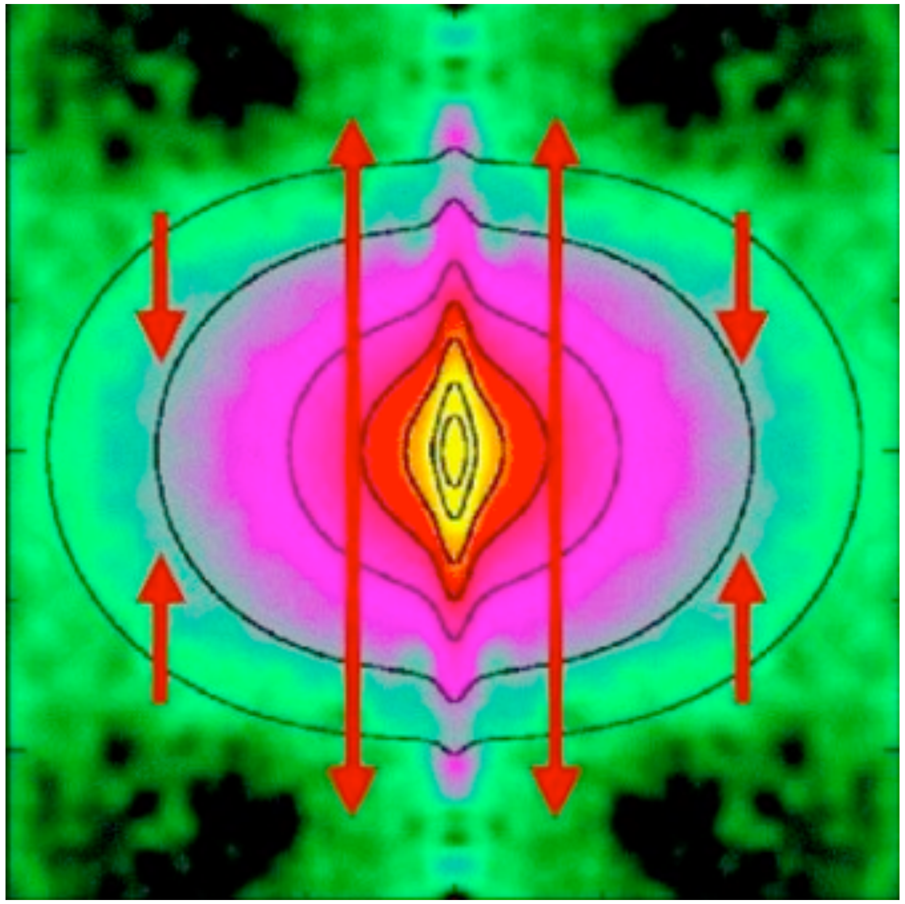
3D Galaxy Distributions

Baryon Acoustic Oscillation (BAO)



Eisenstein et al. (2005)

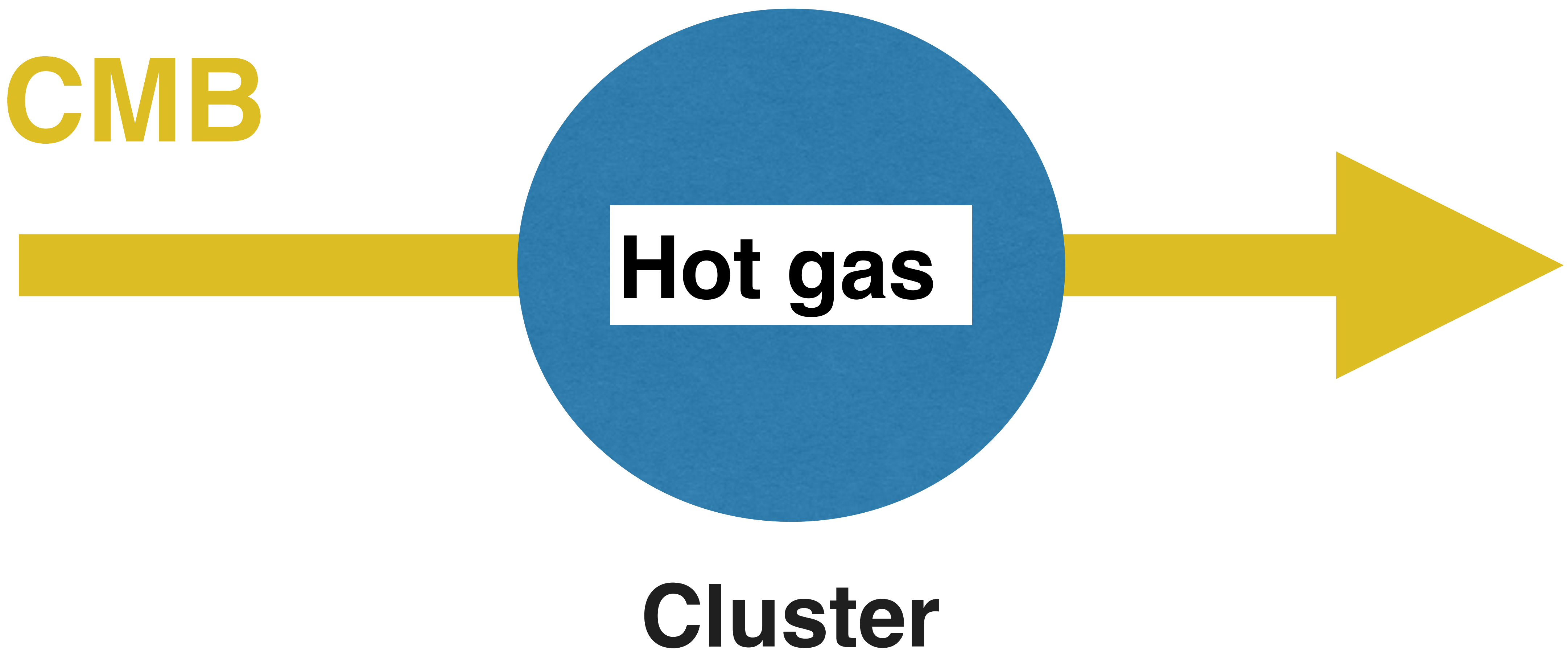
Redshift Space Distortions (RSD)



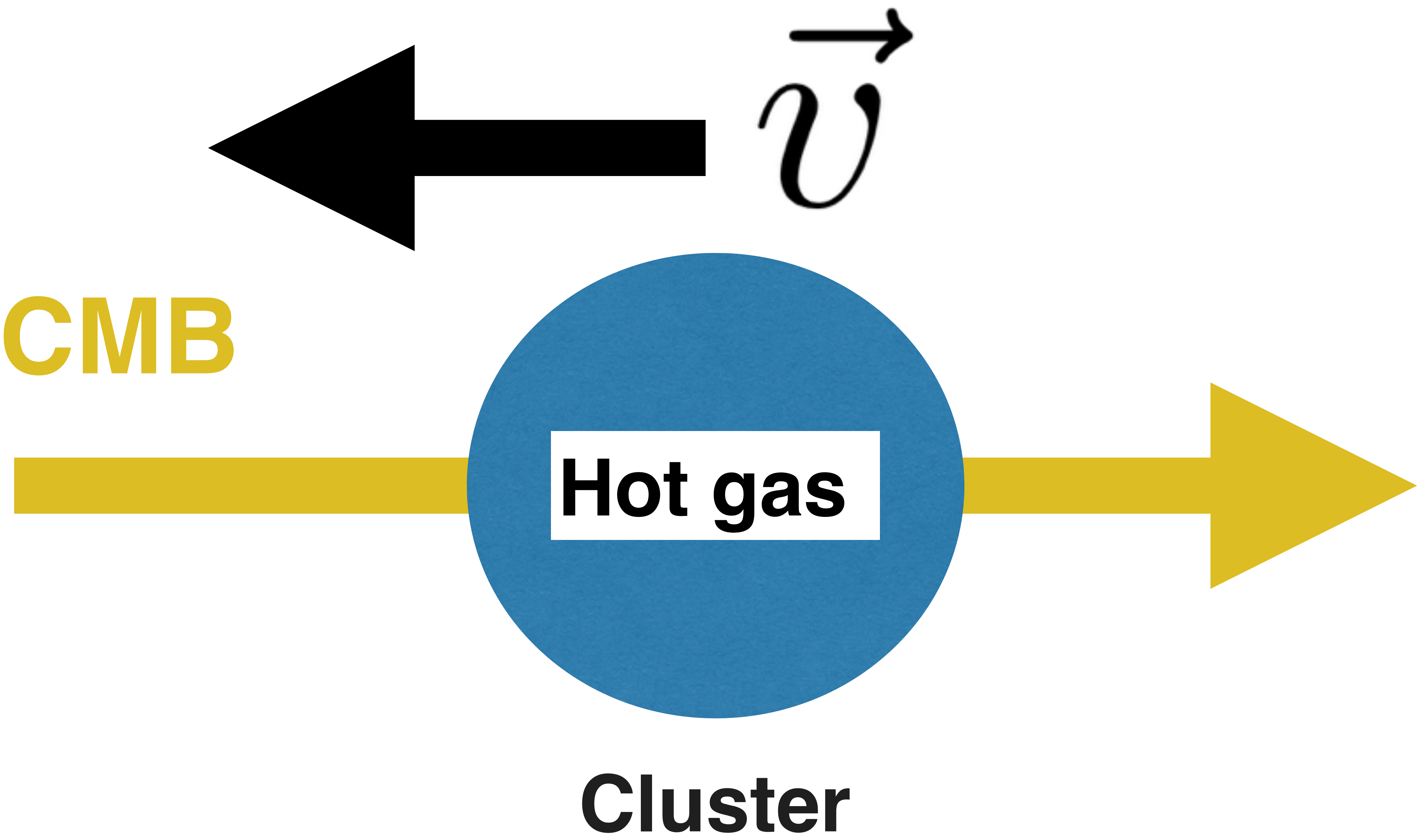
Alison L. Coil (2012)

Through RSD, the peculiar velocity correlation function can be measured.

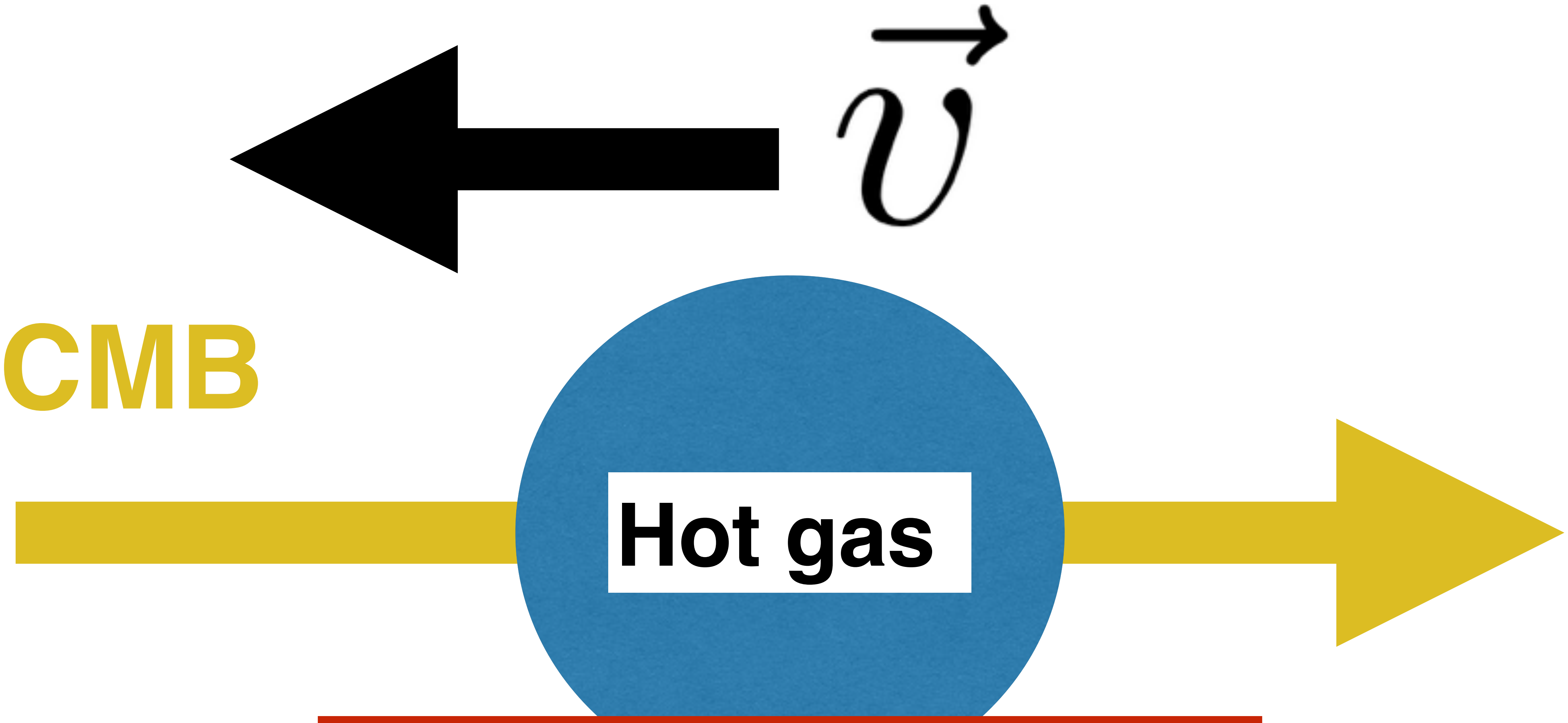
Thermal SZ effect



Kinetic SZ effect



Kinetic SZ effect



$$\boxed{\frac{\delta T_{\mathrm{kSZ}}}{T} \propto \tau_{\mathrm{g}} \frac{\hat{n} \cdot \vec{v}}{c}}$$

“Momentum Field”

$$\boxed{\vec{p} = \vec{v}\rho}$$

kSZ effect in CMB

Galaxy clustering

Observable

Density Correlation Function

$$\xi(r) \propto \sum_{i,j} \delta_{\mathrm{D}}\left(\vec{r} - \left(\vec{x}_i - \vec{x}_j \right) \right)$$

Particle positions

$$\langle \rho \,,\, \rho \rangle$$

Observable

kSZ Correlation Function

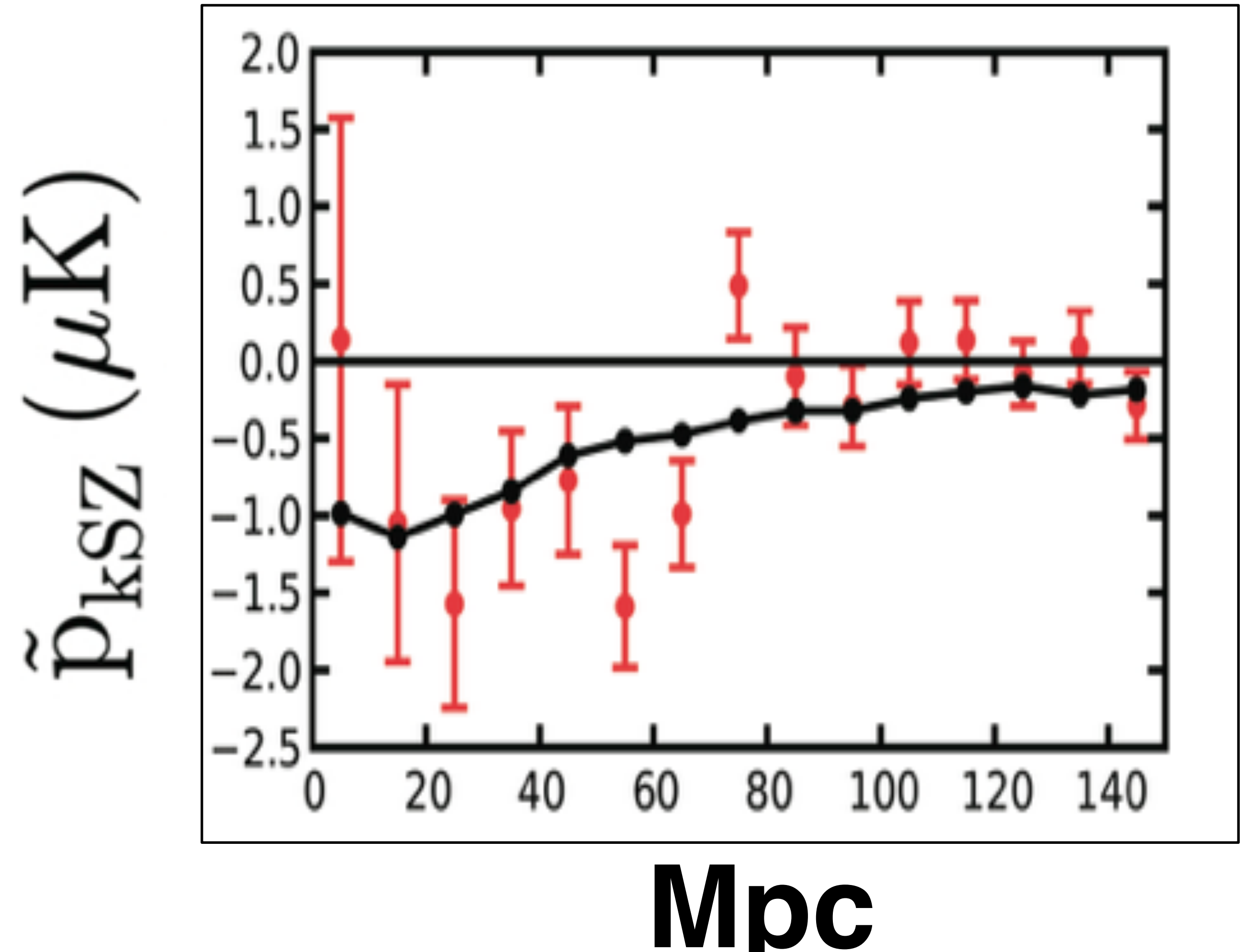
$$\xi_{\mathrm{kSZ}}(r) \propto \sum_{i,j} \underline{[\ \delta T_{\mathrm{kSZ,i}} - \delta T_{\mathrm{kSZ,j}}\ ]}\ \delta_{\mathrm{D}}(\ \vec{r} - (\ \vec{x}_i - \vec{x}_j\ )\ )$$

$$\propto \sum_{i,j} \underbrace{[\ \hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j\ ]}_{\text{weight}}\ \delta_{\mathrm{D}}(\ \vec{r} - (\ \vec{x}_i - \vec{x}_j\ )\ )$$

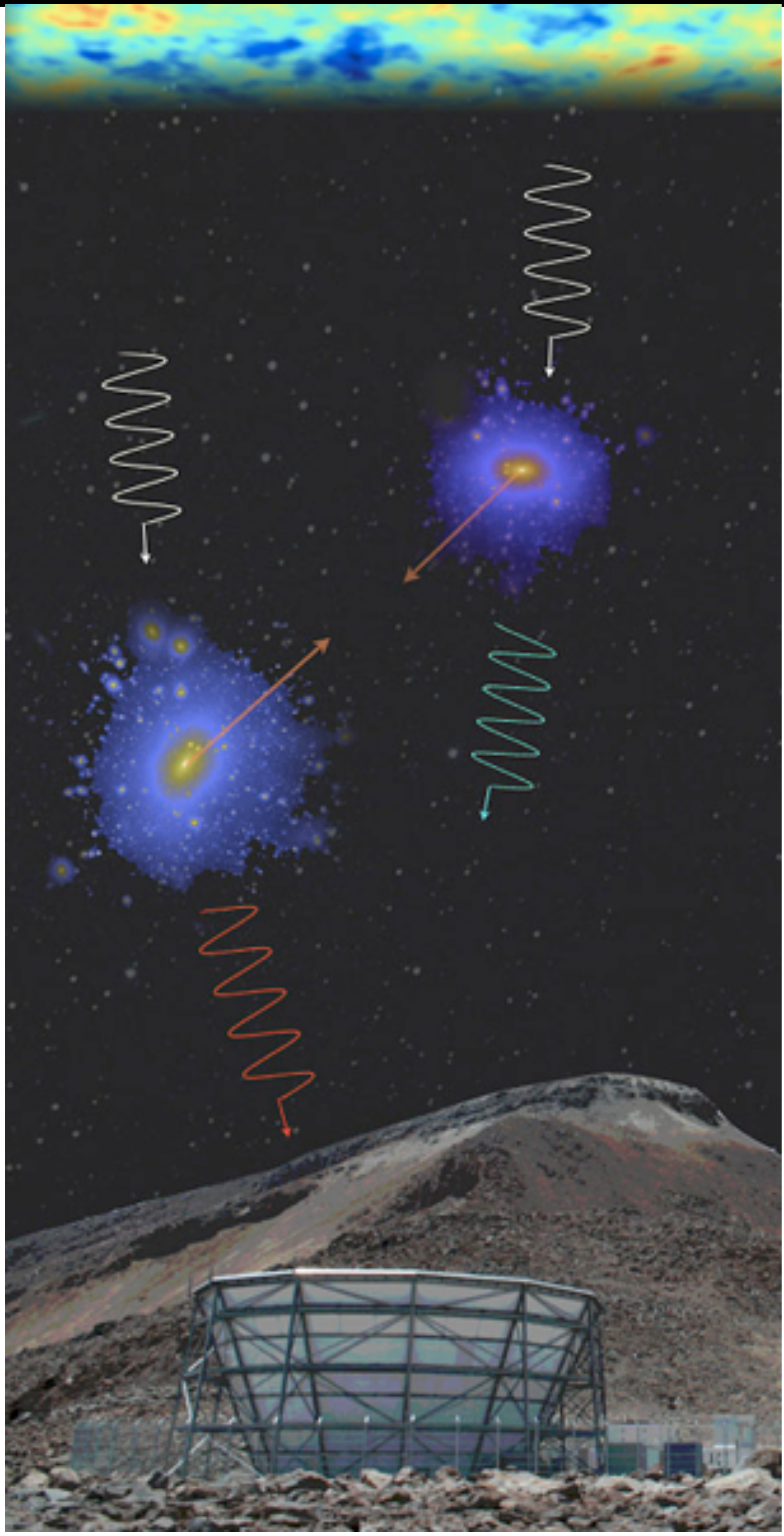
$$\langle\ \hat{n}\cdot\vec{p}\ ,\ \rho\ \rangle - \langle\ \rho\ ,\ \hat{n}\cdot\vec{p}\ \rangle$$

First Detection of kSZ

$$\hat{p}_{\mathrm{kSZ}}(r) = -\frac{\sum_{i<j}(\delta T_i - \delta T_j)\, c_{i,j}}{\sum_{i<j} c_{i,j}^2}$$



Hand et al. (2012) [See also Plank2015]



[Princeton University's Homepage]

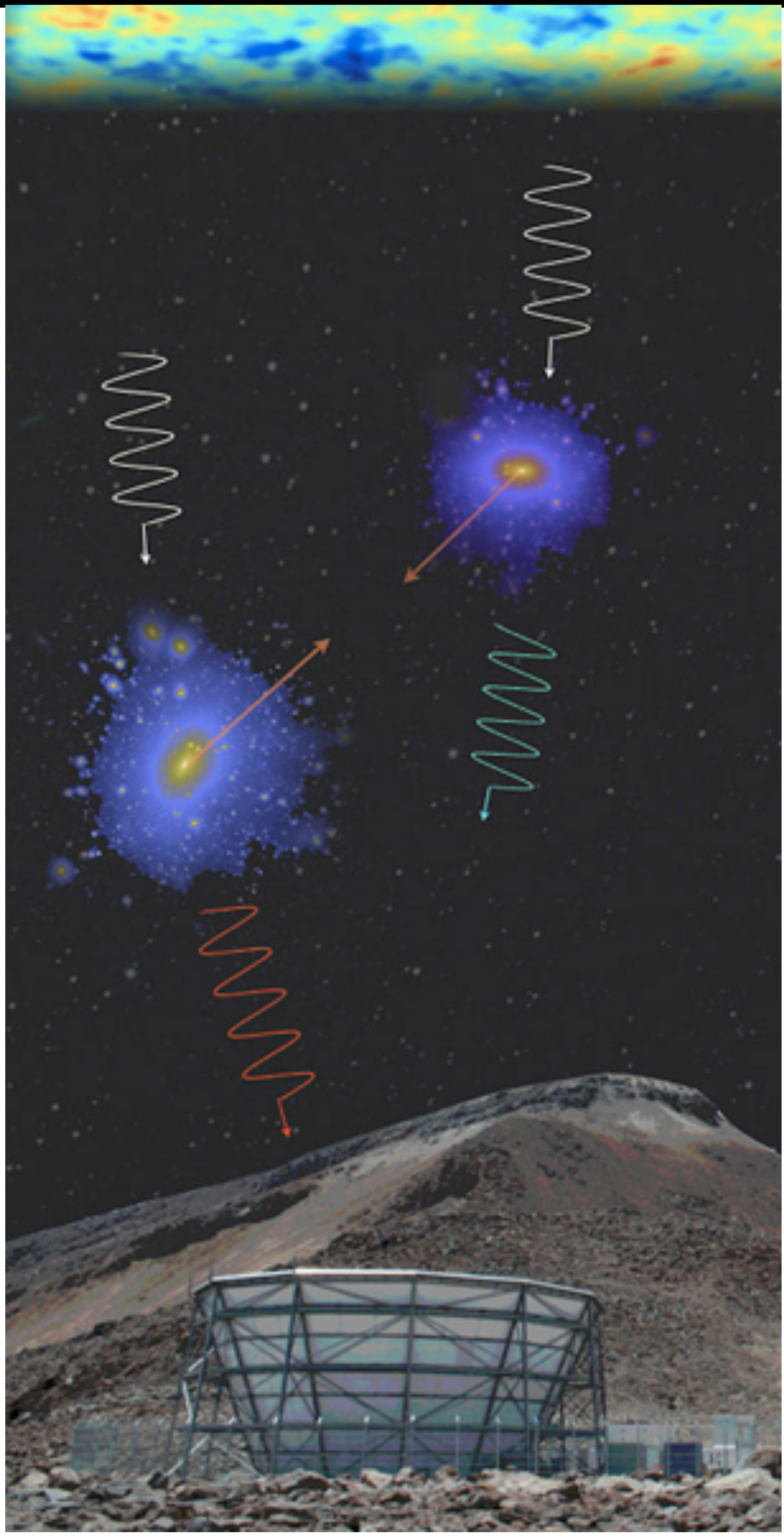
First Detection of kSZ

- **Only two measurements**

(Hand et al. (2012) and Plank 2015)

- **Future survey project**

(Advanced ACTPol)

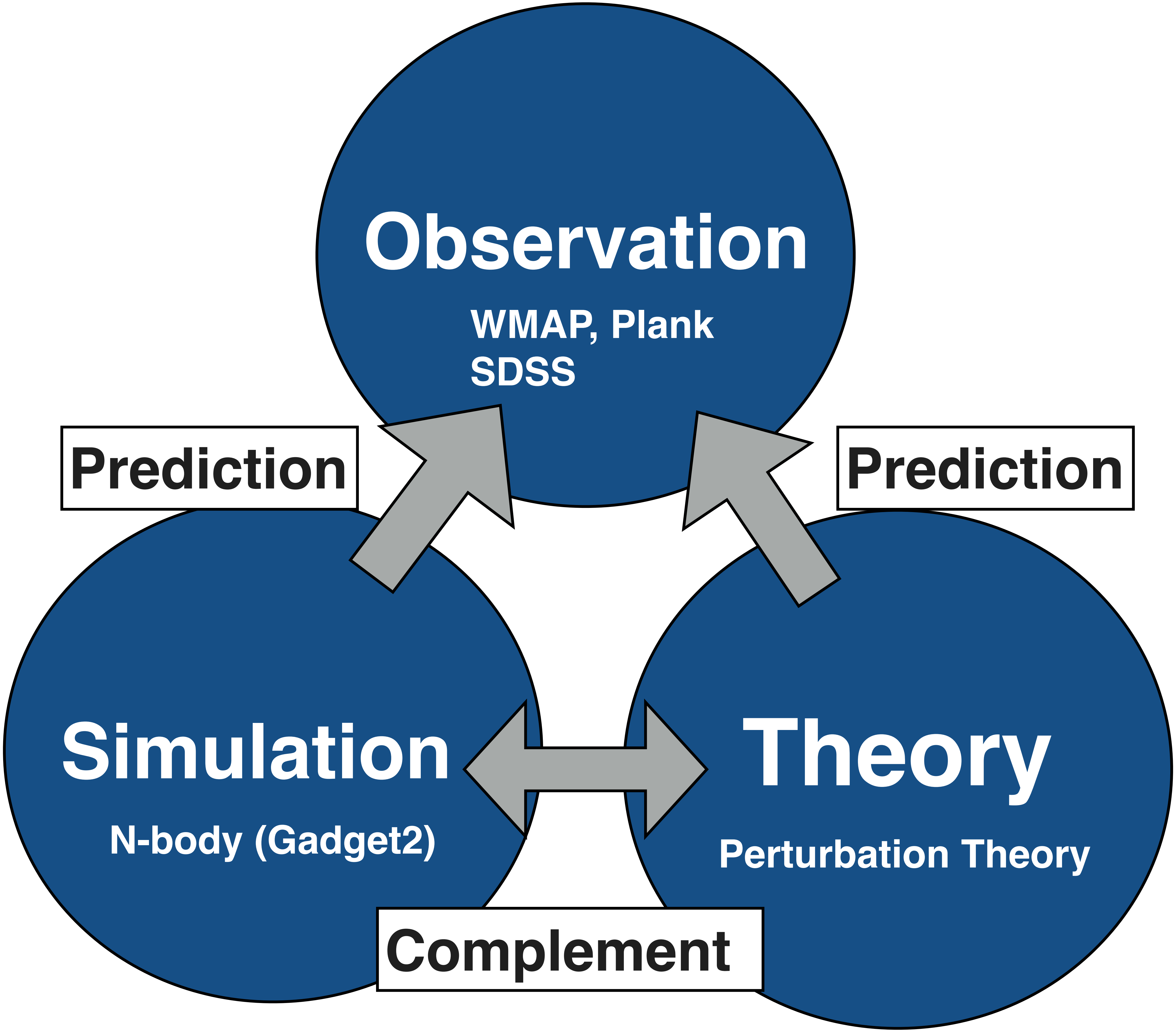


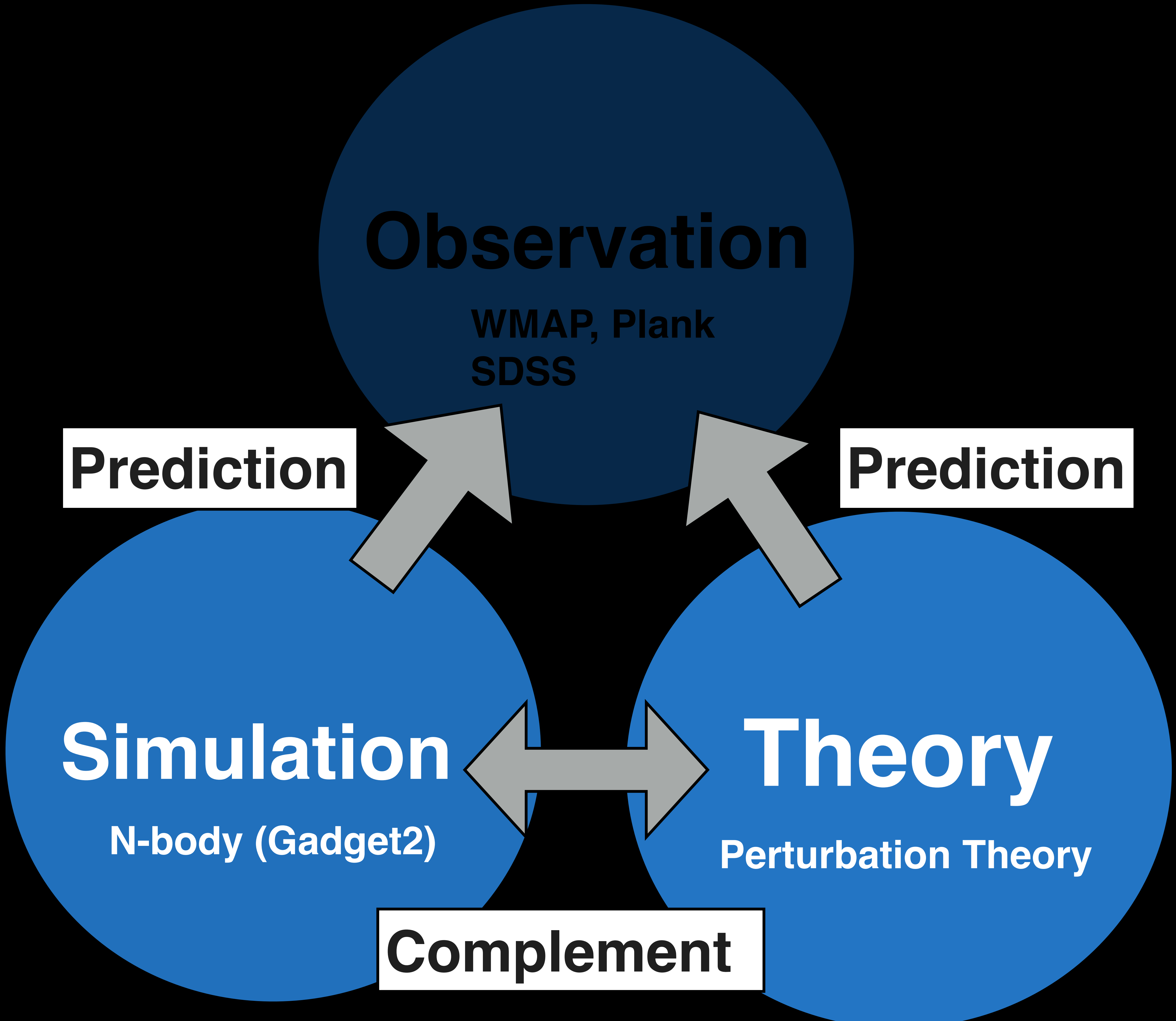
[Princeton University's Homepage]

Theoretical Modering of Momentum Field

Motivation

- **Explaining the observable (Correlation Function)**
- **Predicting kSZ power spectrum**


Observation
WMAP, Plank
SDSS
Prediction
Prediction
Simulation
N-body (Gadget2)
Theory
Perturbation Theory
Complement


Observation
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Prediction
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N-body (Gadget2)
Theory
Perturbation Theory
Complement

Theory vs. N-body

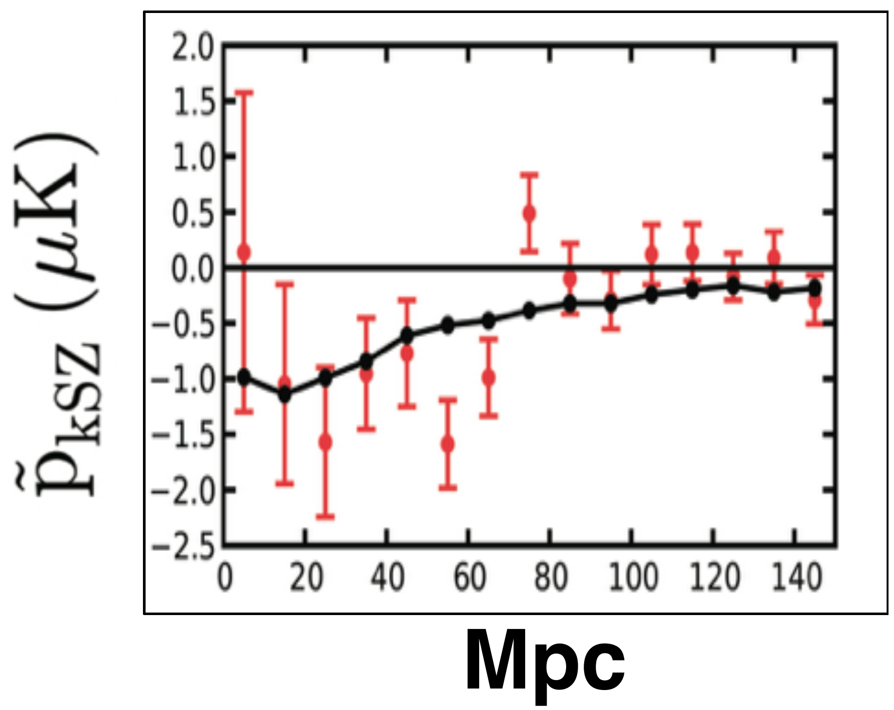
	Gravity	Resolusion	Box	Realization	Speed
Theory	Perturbation	Infinity	Infinity (Ideally)	Infinity	Fast
N-body	Full	Finite	Finite	Finite	Slow

Theoretical Prediction: Linear Theory

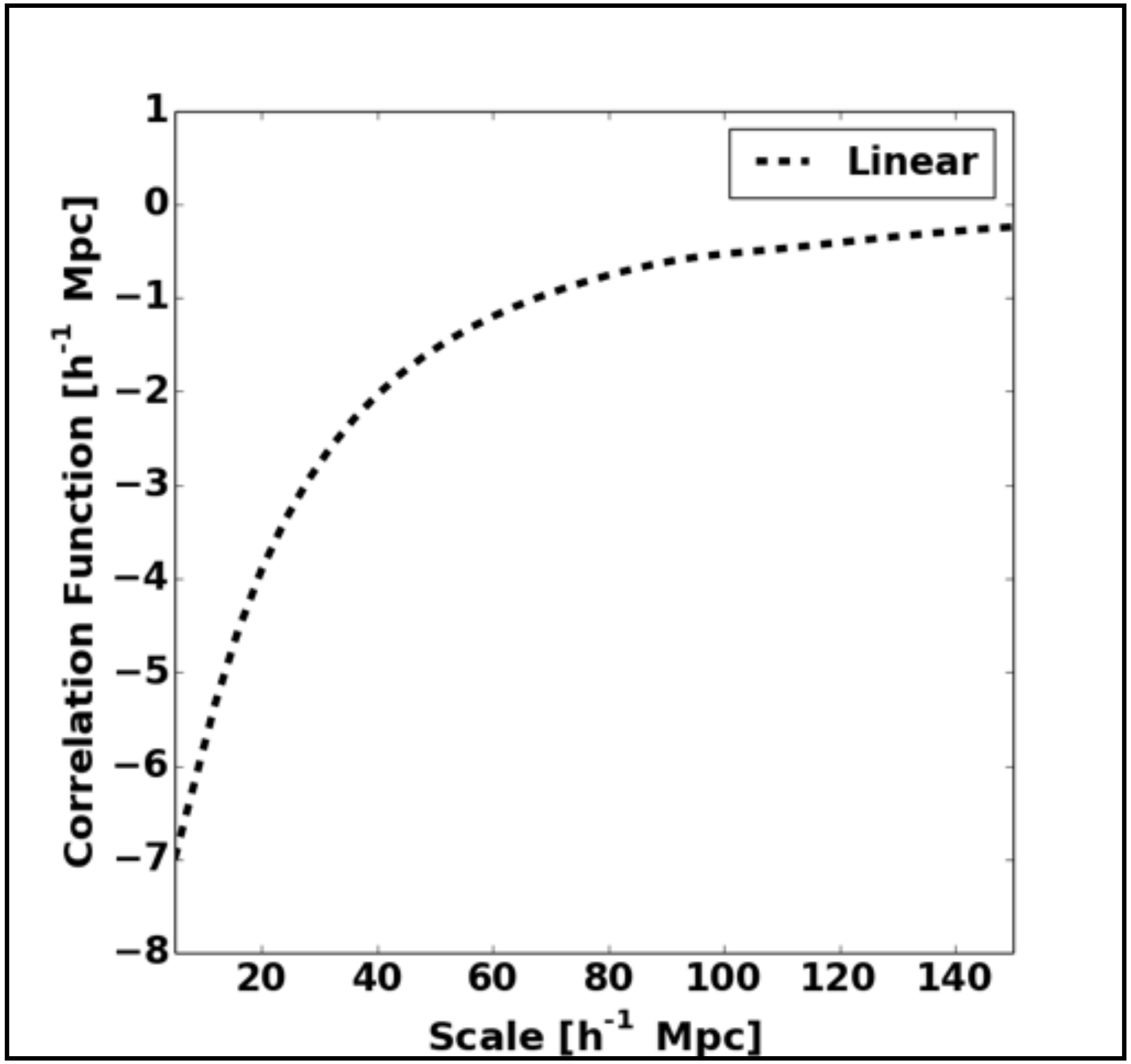
$$\begin{aligned}\langle\, \hat{n} \cdot \vec{p}\,,\ \rho\,\rangle \;&=\; \bar{\rho}^2 \langle\, \hat{n} \cdot \vec{v}\,,\ \delta\,\rangle \\ &+\; \bar{\rho}^2 \langle\, \hat{n} \cdot \vec{v}\,\delta\,,\ \delta\,\rangle \\ &\sim\; \bar{\rho}^2 \langle\, \hat{n} \cdot \vec{v}\,,\ \delta\,\rangle\end{aligned}$$

In linear theory,
the correlation between density field and velocity field only appears.

Theoretical Prediction: Linear Theory



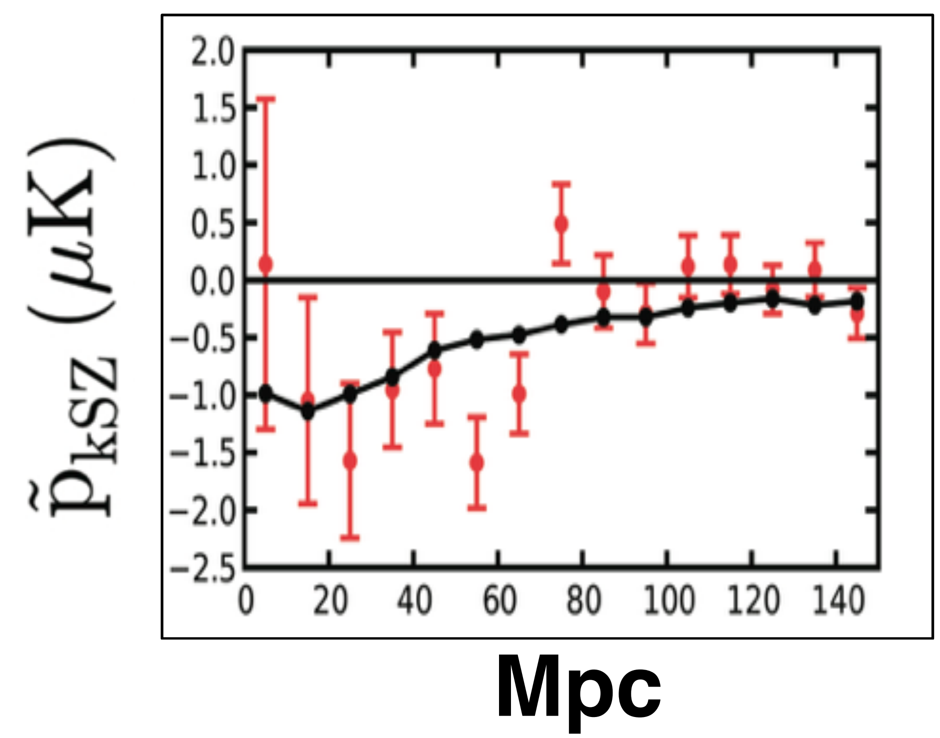
$\propto$

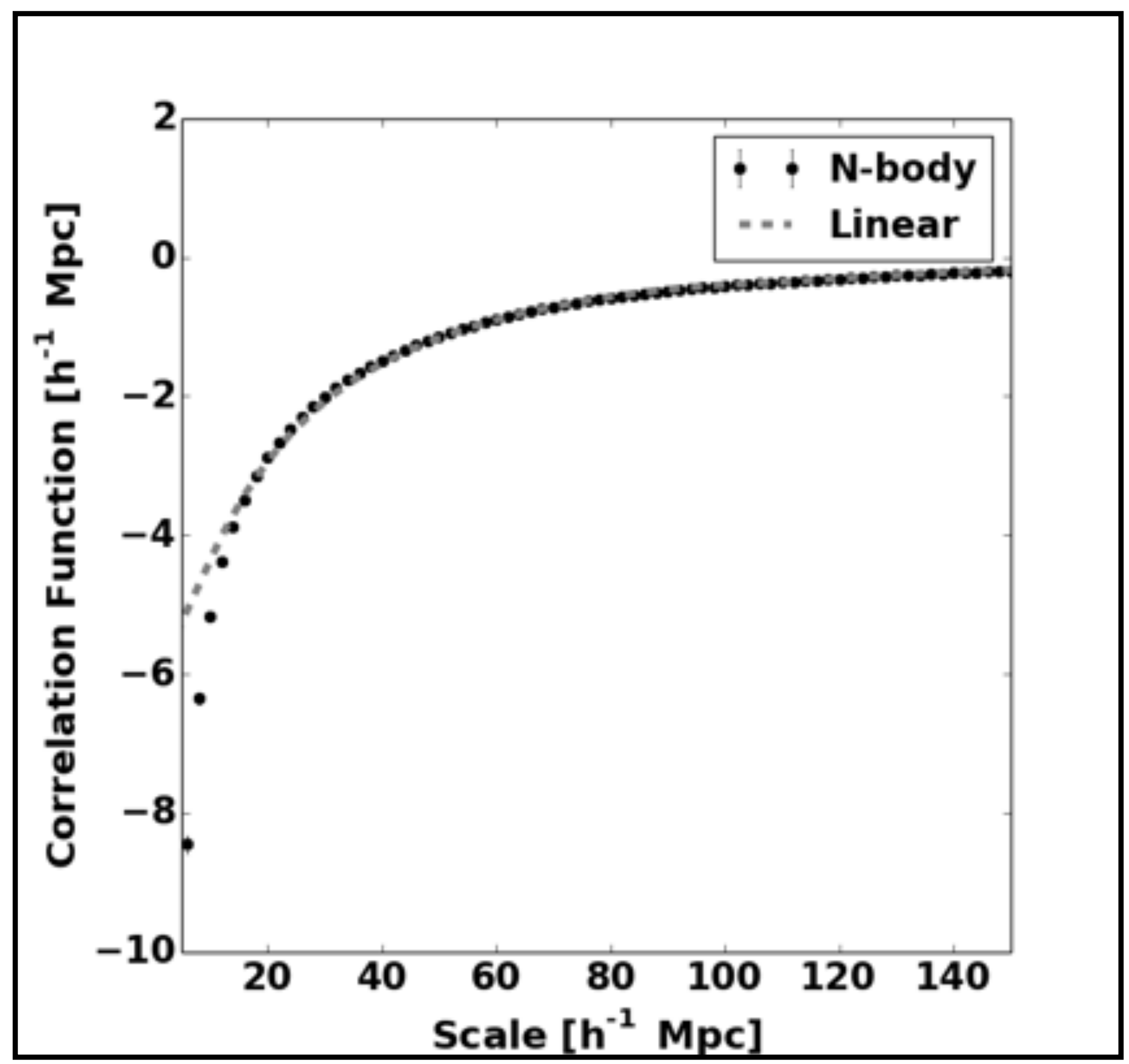


Different normalisation

Theoretical Prediction:
N-body simulation

$$\sum_{i,j} \left[\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j\right] \delta_{\mathrm{D}}\left(\vec{r} - (\vec{x}_i - \vec{x}_j)\right)$$





Non-linear gravitational effect make the amplitude of velocities larger than linear theory.

Redshift Space Distortion

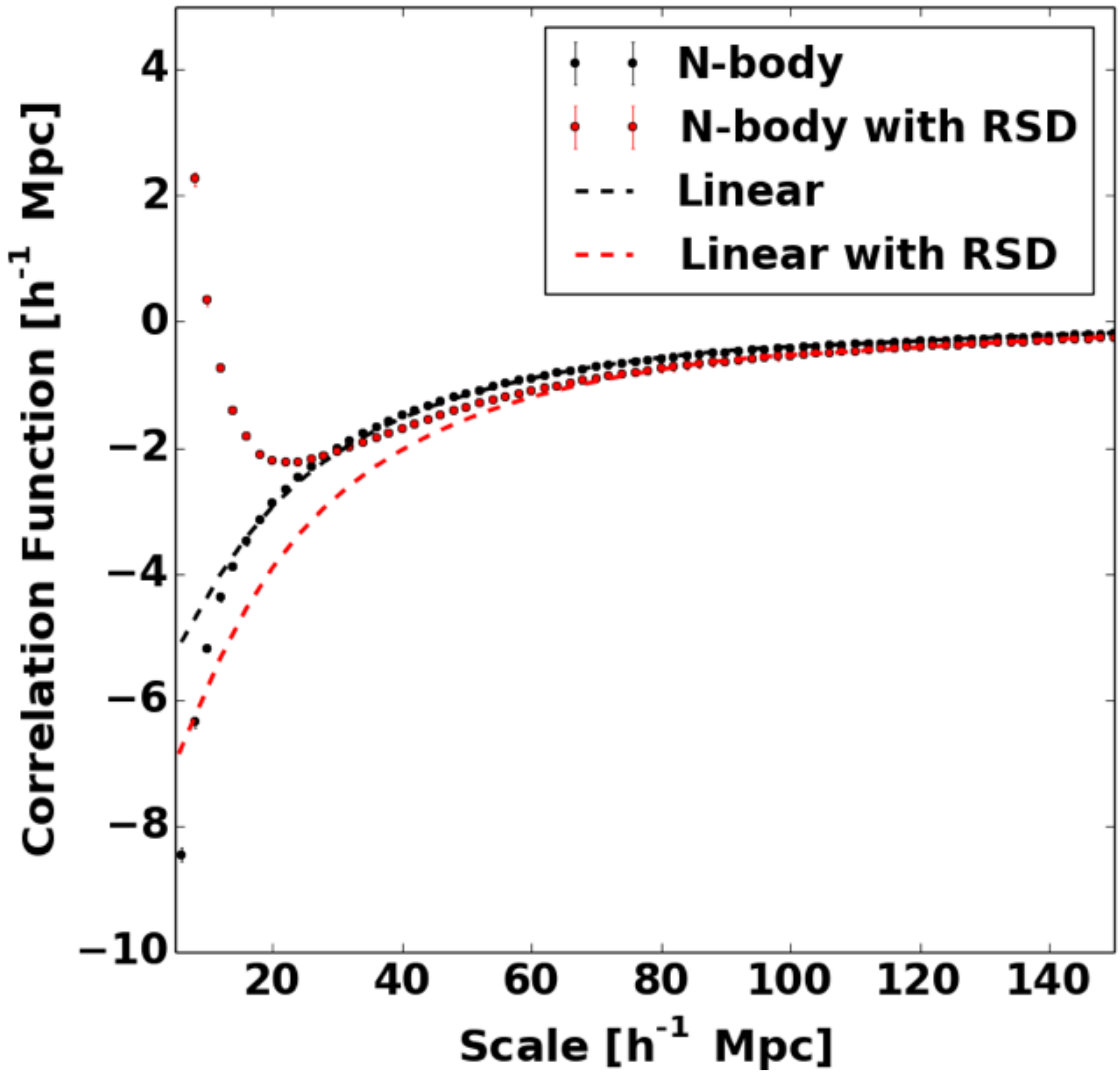
Coordinate transformation of particle positions

$$\boxed{\vec{s} = \vec{x} + \underline{\frac{\hat{n}\cdot\vec{v}(\vec{x})}{aH}\hat{n}},}$$

Coordinate transformation of correlation function

$$\boxed{\sum_{i,j}\left[\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j\right]\delta_{\rm D}\left(\vec{r} - (\vec{x}_i - \vec{x}_j) - \underline{\left(\frac{\hat{n}\cdot\vec{v}_i}{aH} - \frac{\hat{n}\cdot\vec{v}_j}{aH}\right)\hat{n}\,\Big)}\right.}$$

Simulation Result



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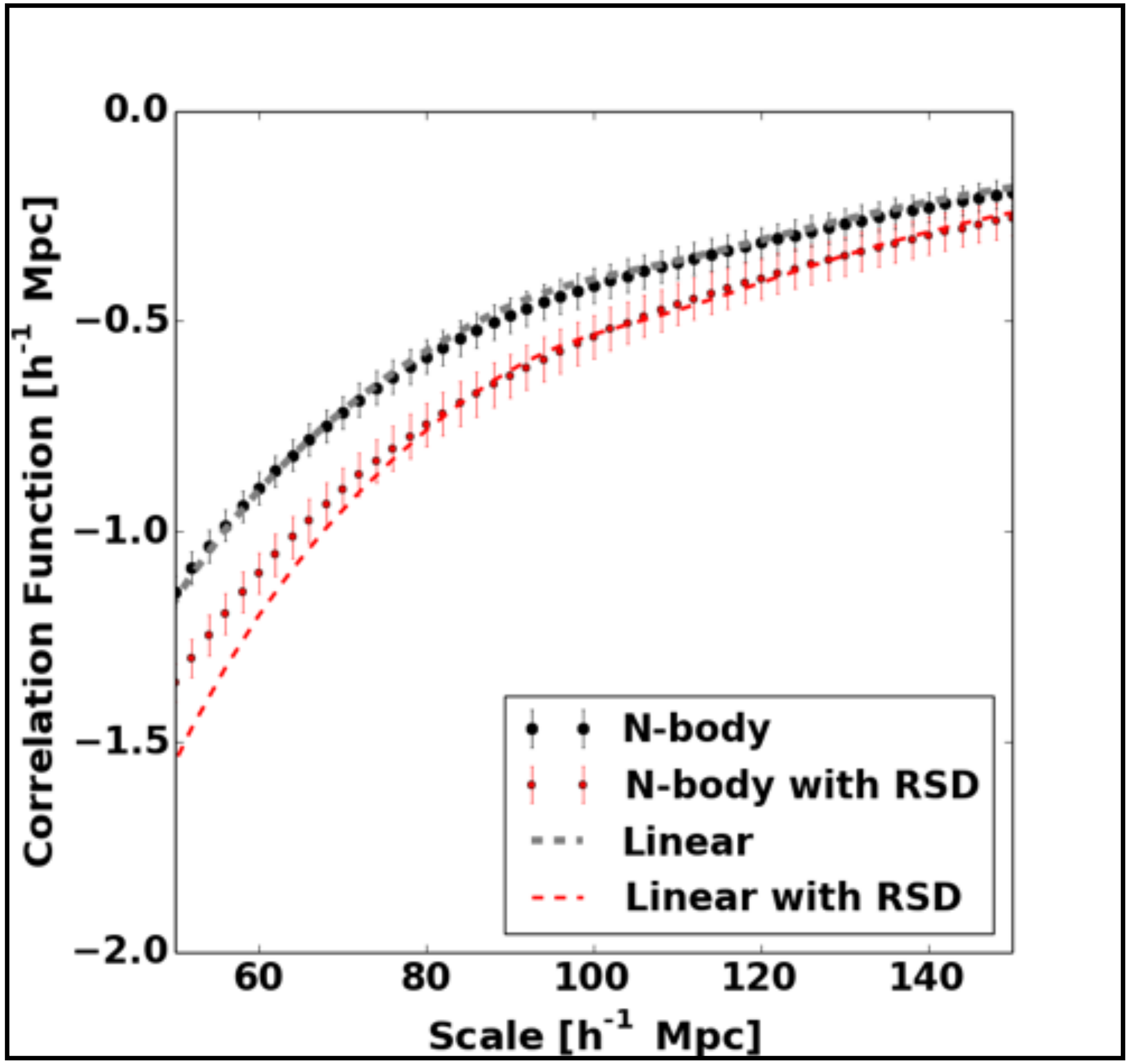
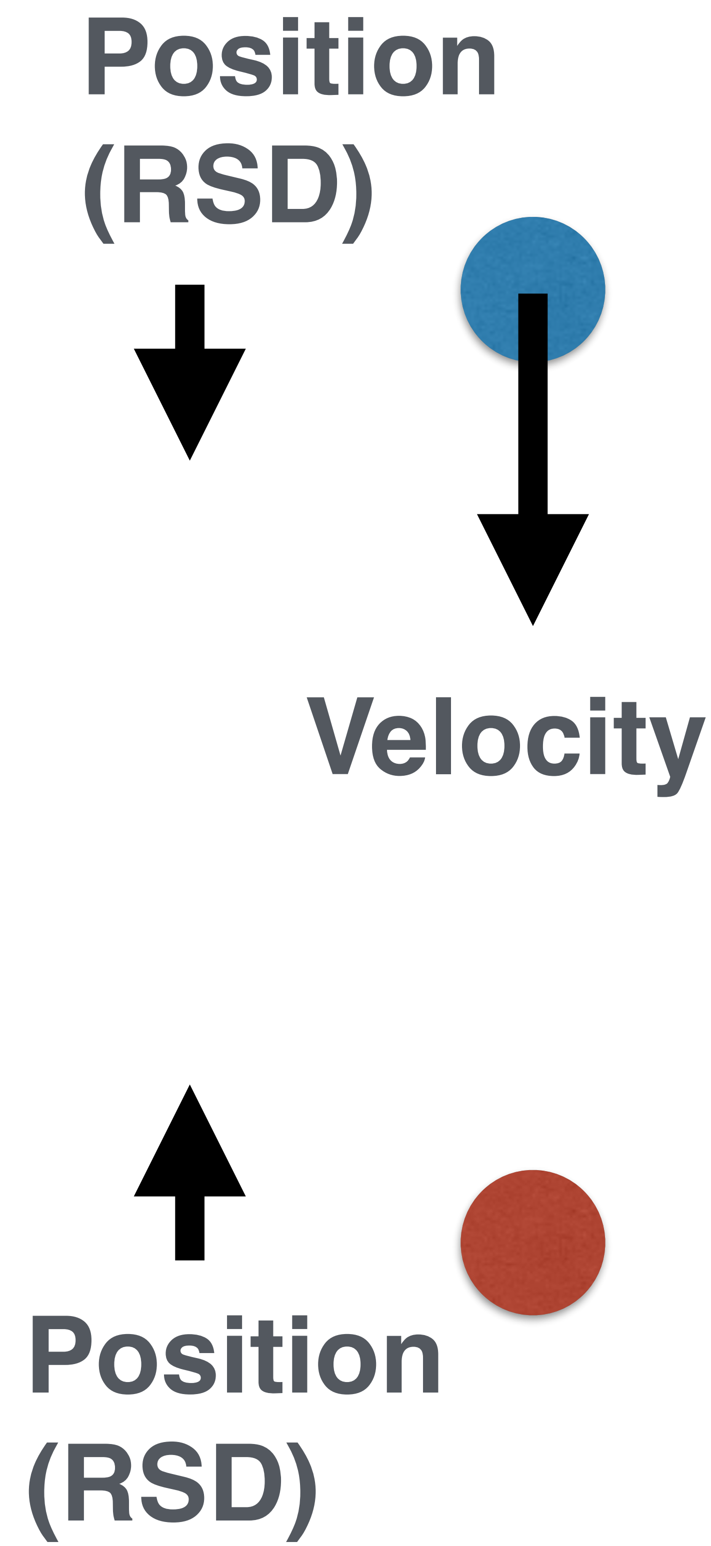
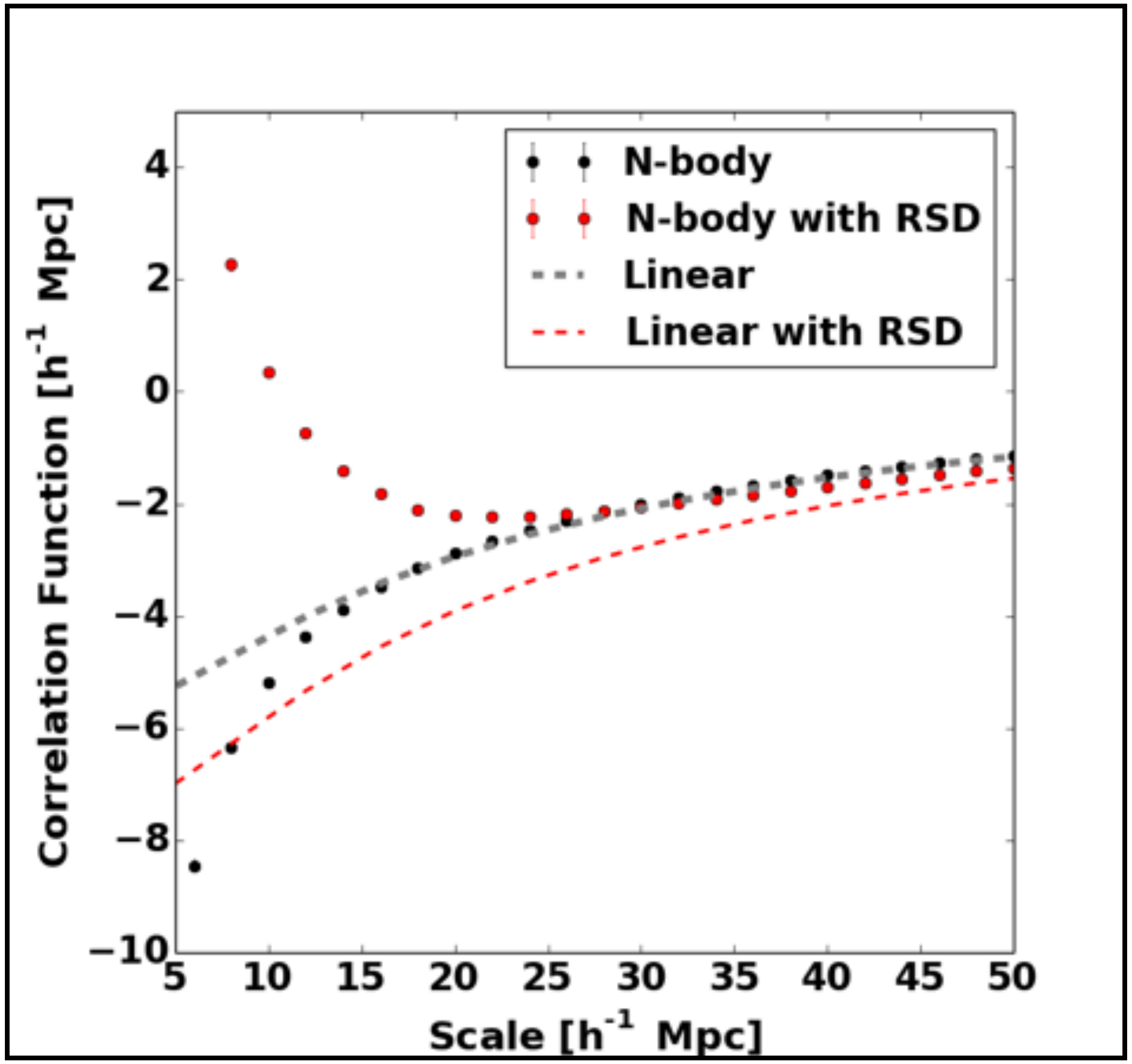


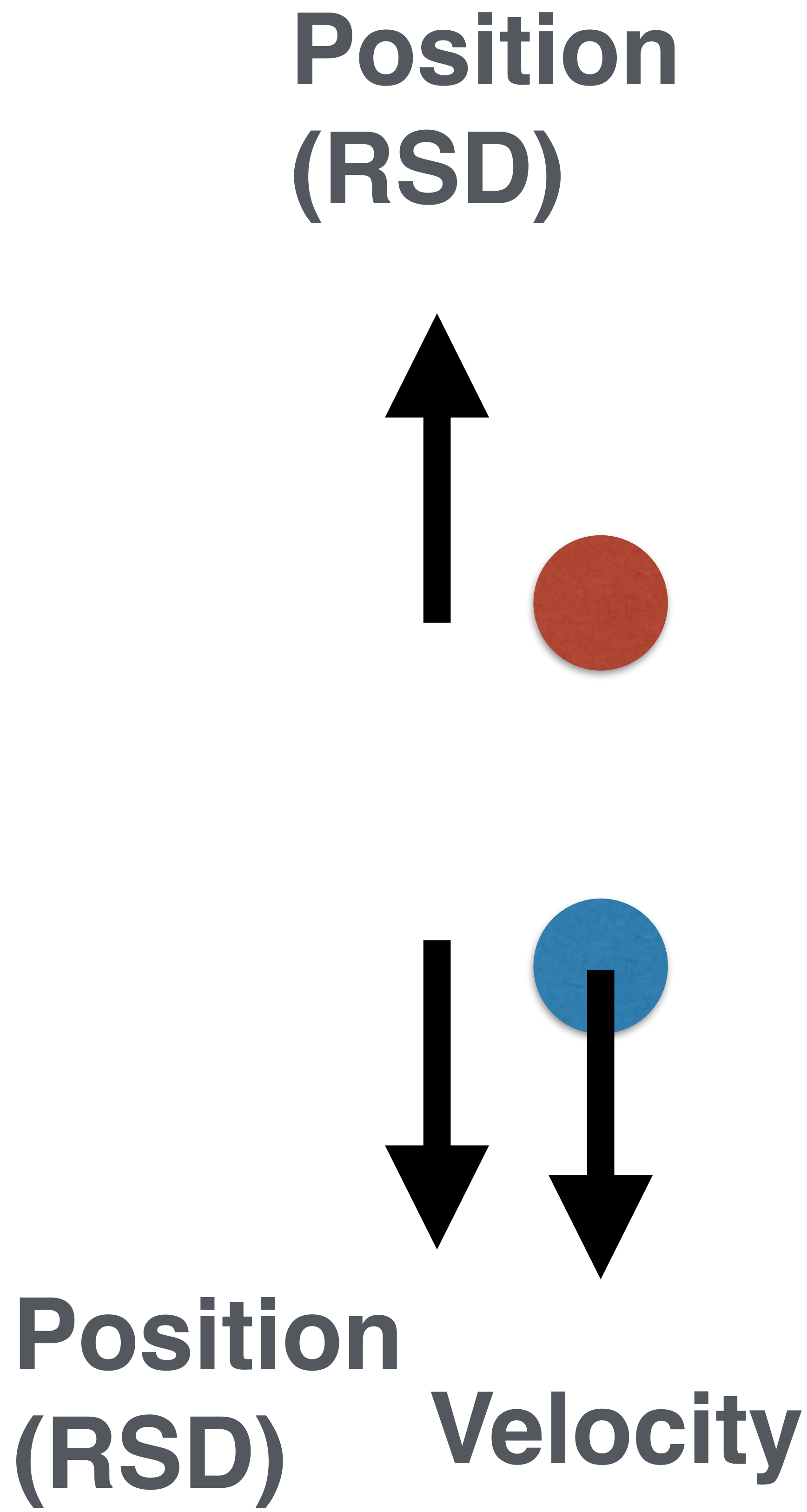
At large scales: Kaiser effect



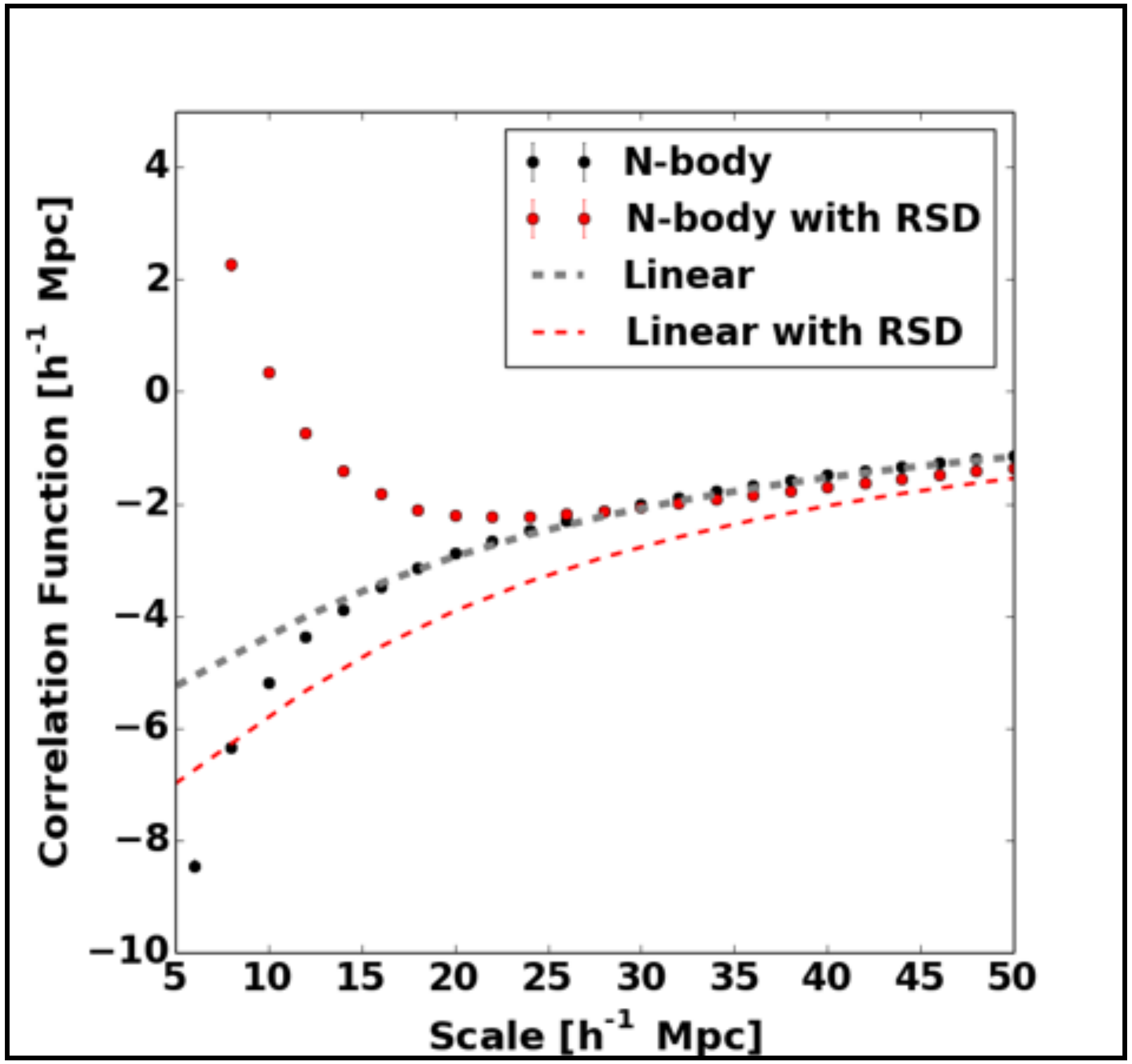
Simulation Result



At small scales:
change of sign of the velocity correlation function.

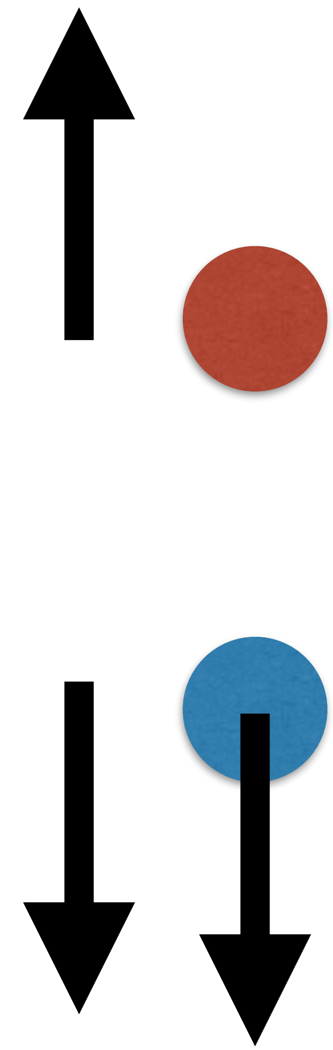


Simulation Result



Importance of RSD

Position (RSD)



Position (RSD)

Velocity

Theoretical modering:
Non-linear Theory

$$\boxed{\begin{aligned} \langle\, \hat{n}\cdot\vec{p}\;,\;\rho\,\rangle \;&=\; \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\;,\;\delta\,\rangle \\ &+\; \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\,\delta\;,\;\delta\,\rangle \end{aligned}}$$

3pt correlation function with RSD

Theoretical modering: Non-linear Theory

$$\boxed{\begin{aligned} \langle\, \hat{n}\cdot\vec{p}\;,\; \rho\,\rangle &= \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\;,\; \delta\,\rangle \\ &+ \bar{\rho}^2\langle\, \hat{n}\cdot\vec{v}\,\delta\;,\; \delta\,\rangle \end{aligned}}$$

We propose a simple method to predict the momentum correlation function.

Theoretical modering: Non-Linear Theory

$$\hat{\xi}(r) = \frac{V}{N_{\mathrm{p}}^2} \sum_{i,j} \underline{[\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j]}\, \delta_{\mathrm{D}}\left(\vec{r} - (\vec{x}_i - \vec{x}_j) - \underline{\left(\frac{\hat{n}\cdot\vec{v}_i}{aH} - \frac{\hat{n}\cdot\vec{v}_j}{aH}\right)\hat{n}}\right)$$

Fourier transformation

$$\hat{P}(k) = \frac{V}{N_{\mathrm{p}}^2} \sum_{i,j} \underline{[\hat{n}\cdot\vec{v}_i - \hat{n}\cdot\vec{v}_j]}\, e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j) \underline{- i\vec{k}\cdot\left(\frac{\hat{n}\cdot\vec{v}_i}{aH} - \frac{\hat{n}\cdot\vec{v}_j}{aH}\right)\hat{n}}}$$

Theoretical modering:
Non-Linear Theory

Momentum Power Spectrum

$$\boxed{\hat{P}^{(n)}_{\mathrm{p}}(\vec{k}) = \left(i\frac{aH}{\vec{k}\cdot\hat{n}}\right)^{n}\left[\frac{d^n}{d^n\gamma}\hat{P}^{(0)}_{\mathrm{p}}(\vec{k};\gamma)\right]\Bigg|_{\gamma=1}}$$

Density Power Spectrum (Generating Function)

$$\boxed{\hat{P}^{(0)}_{\mathrm{p}}\left(\vec{k};\gamma\right) \equiv \frac{V}{N_{\mathrm{p}}^2}\sum_{i,j}\left[e^{-i\vec{k}\cdot\vec{x}_{ij}-\underline{i\gamma}\frac{\vec{k}\cdot\hat{n}}{aH}(\hat{n}\cdot\vec{v}_i-\hat{n}\cdot\vec{v}_j)}\right]}$$

Theoretical modering:
Non-linear Theory

Main result

$$\hat{P}^{(n)}_{\mathrm{p}}(\vec{k},\hat{n}) = \left(i\frac{aHf}{\vec{k}\cdot\hat{n}}\right)^{n}\frac{\partial^{n}}{\partial^{n}f}\hat{P}_{\mathrm{m}}(D,f,\vec{k},\hat{n}).$$

From

$$\vec{v}\propto f=\frac{d\ln D}{d\ln a}\quad \text{for}\quad f=\Omega_{\mathrm{m}}^{0.5}$$

Momentum power spectrum can be computed from the density power spectrum.

Theoretical modering:
Non-linear Theory

Main result

$$\boxed{\hat{P}^{(n)}_{\mathrm{p}}(\vec{k},\hat{n}) = \left(i\frac{aHf}{\vec{k}\cdot\hat{n}}\right)^{n}\frac{\partial^{n}}{\partial^{n}f}\hat{P}_{\mathrm{m}}(D,f,\vec{k},\hat{n}).}$$

- **General**
- **True even for halos.**

Perturbation Theory

Density Field

Particle description

$$\rho = \sum_i \delta_{\mathrm{D}} \left(\vec{x} - \vec{x}_i \right)$$

Continuous limit

Lagrangian description

$$\rho = \bar{\rho} \int d^3 q \delta_{\mathrm{D}} \left(\vec{x} - \vec{q} - \vec{\Psi}(\vec{q}) \right)$$

Infinite Mode-Coupling

Power Spectrum

$$P(\vec{k}) = \left\langle \frac{V}{N_{\mathrm{p}}^2} \sum_{i,j} e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j)} \right\rangle$$

$$= \boxed{\int d^3q}\, e^{-i\vec{k}\cdot(\vec{q}_1 - \vec{q}_2)} \left\langle e^{-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2)\right)} \right\rangle \quad \text{, where} \quad \vec{q} = \vec{q}_1 - \vec{q}_2$$

Continuous limit

In computing the non-linear power spectrum, volume integral necessarily appears.

Infinite Mode-Coupling

$$
\begin{aligned}
P(\vec{k}) &= \left\langle \frac{V}{N_{\rm p}^2} \sum_{i,j} e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j)} \right\rangle \\
&= \int d^3 q e^{-i\vec{k}\cdot(\vec{q}_1 - \vec{q}_2)} \left\langle e^{-i\vec{k}\cdot(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2))} \right\rangle \quad \text{, where} \quad \vec{q} = \vec{q}_1 - \vec{q}_2 \\
&= \int d^3 q e^{-i\vec{k}\cdot(\vec{q}_1 - \vec{q}_2)} \left\langle 1 + \left(-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2)\right)\right) + \frac{1}{2}\left(-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2)\right)^2 + \ldots\right) \right\rangle \\
&= \Gamma^{(1)}(k) P_{\rm lin}(k) + \frac{1}{2} \boxed{\int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3}} (2\pi)^3 \delta_{\rm D}\left(\vec{k} - \vec{k}_1 - \vec{k}_2\right) \left[\Gamma^{(2)}(\vec{k}_1, \vec{k}_2)\right]^2 P_{\rm lin}(k_1) P_{\rm lin}(k_2) + \ldots
\end{aligned}
$$

Without relation to gravity,
infinite mode-coupling integrals
appear in order to compute
the non-linear power spectrum.

Infinite Mode-Coupling

Power Spectrum

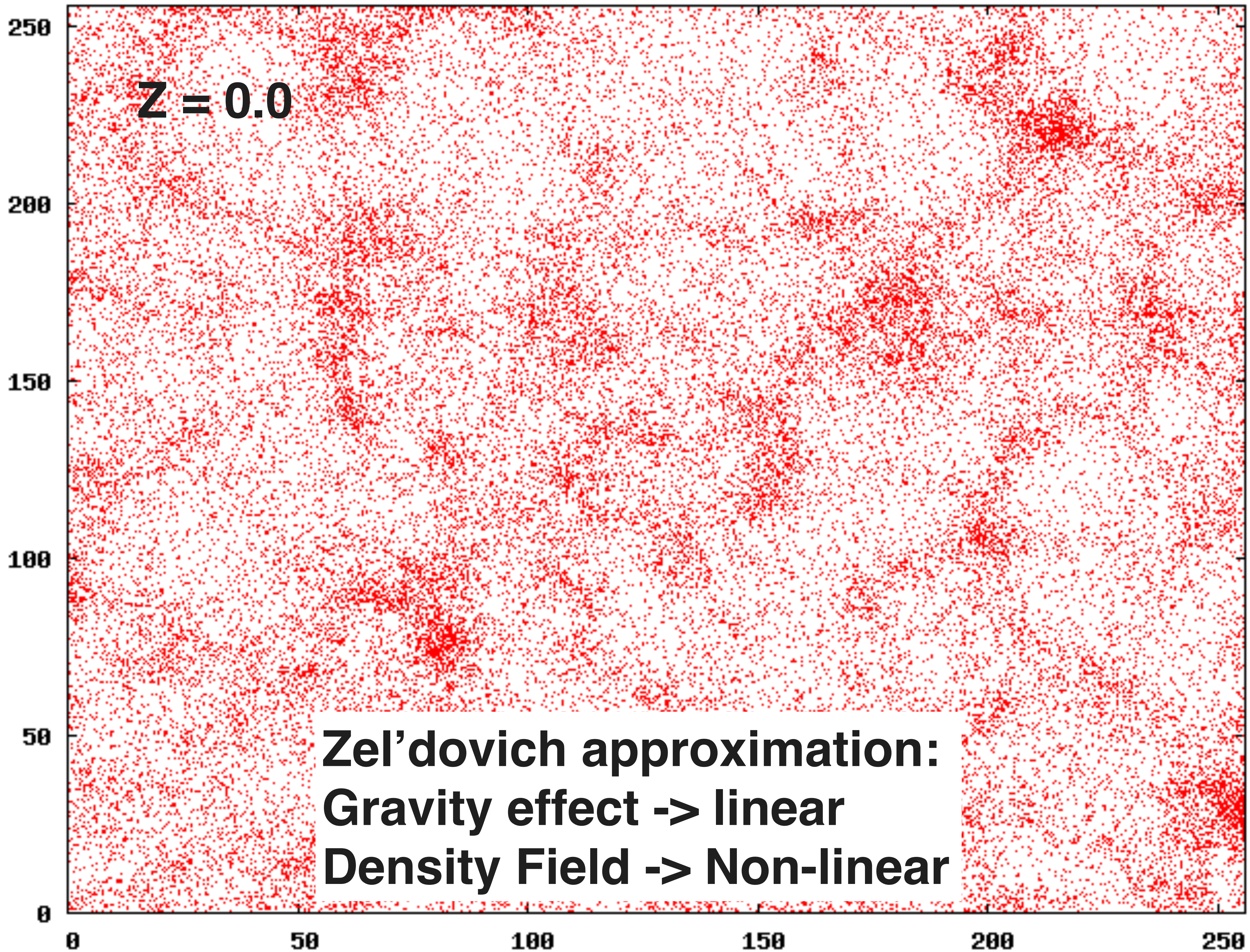
$$P(\vec{k}) = \left\langle \frac{V}{N_{\rm p}^2} \sum_{i,j} e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j)} \right\rangle$$

$$= \int d^3 q e^{-i\vec{k}\cdot(\vec{q}_1 - \vec{q}_2)} \left\langle e^{-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1) - \vec{\Psi}(\vec{q}_2)\right)} \right\rangle \quad \text{, where} \quad \vec{q} = \vec{q}_1 - \vec{q}_2$$

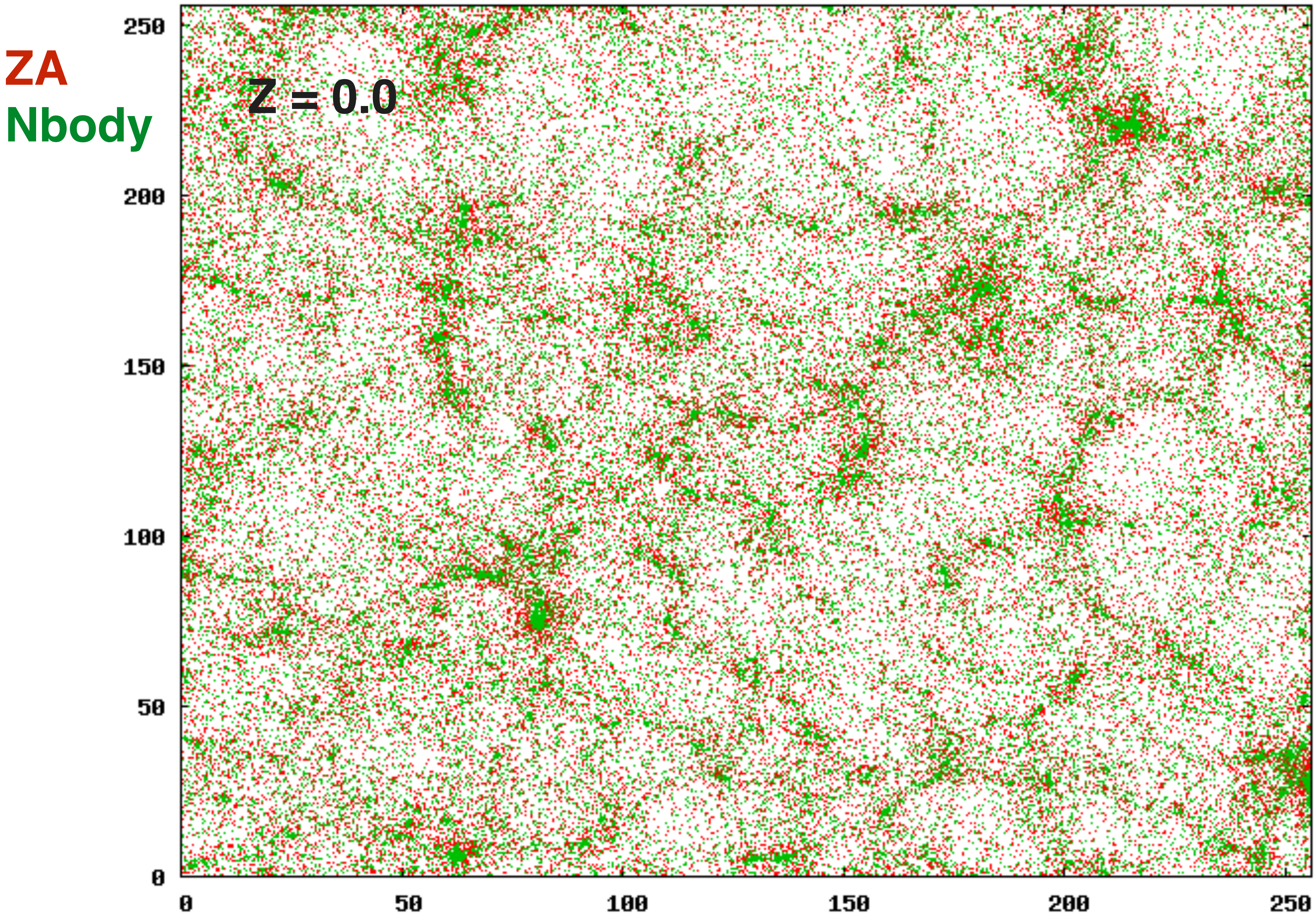
Method:
computing the volume integral without using Fourier space integrals.

Zel’dovich Approximation

ZA

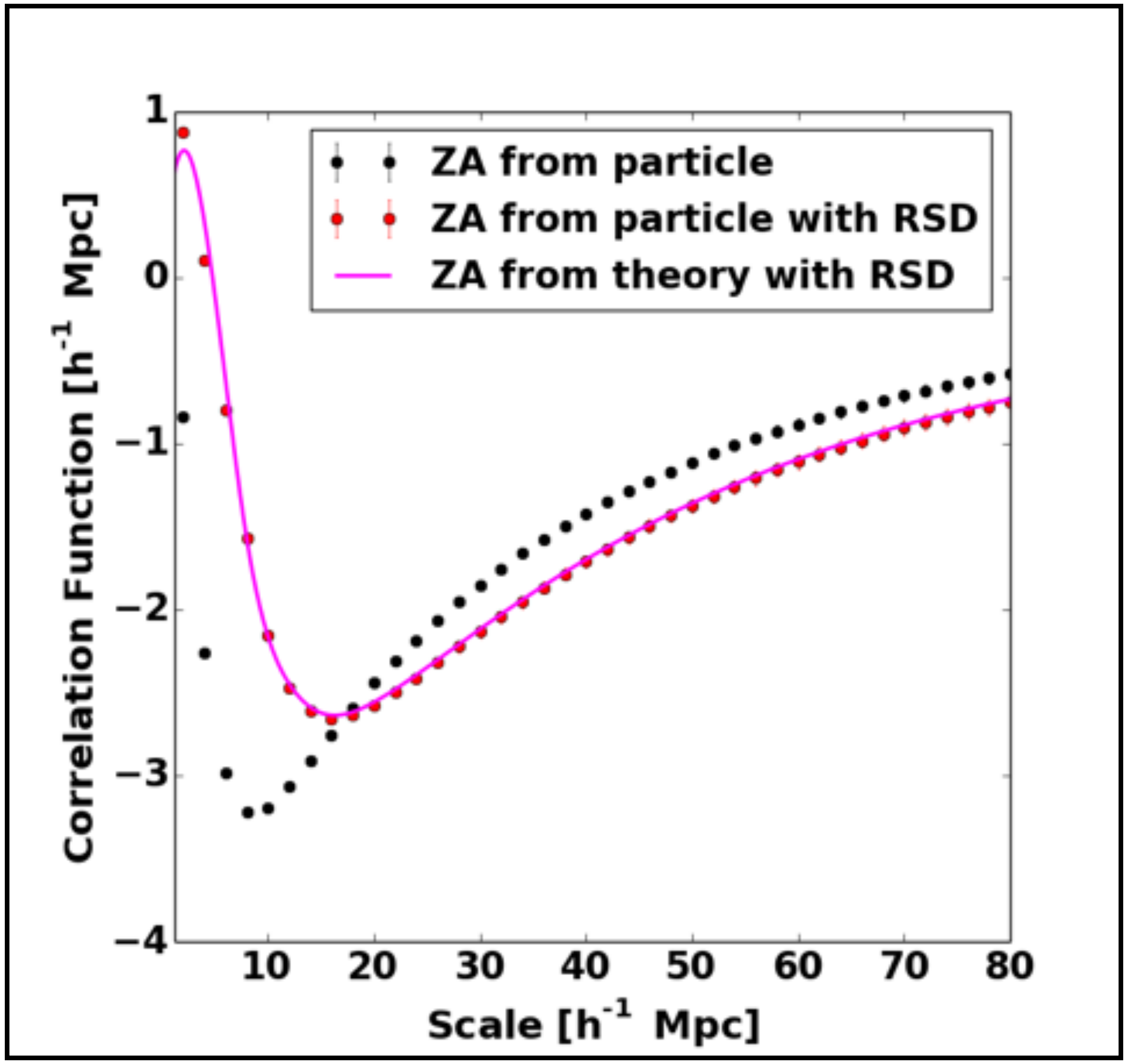


Zel'dovich Approximation

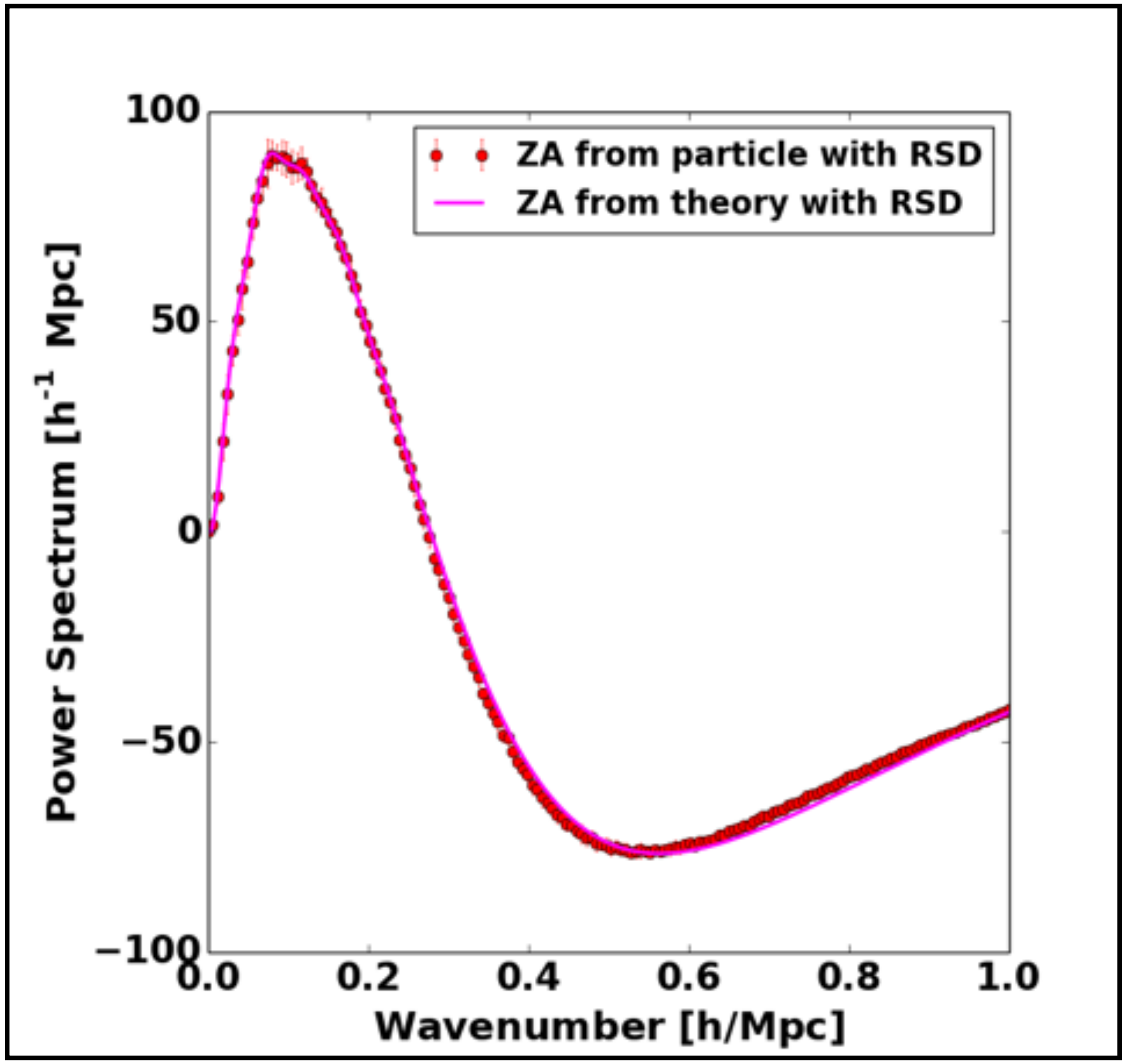


Zel'dovich Approximation

Momentum Correlation Function



Momentum Power Spectrum

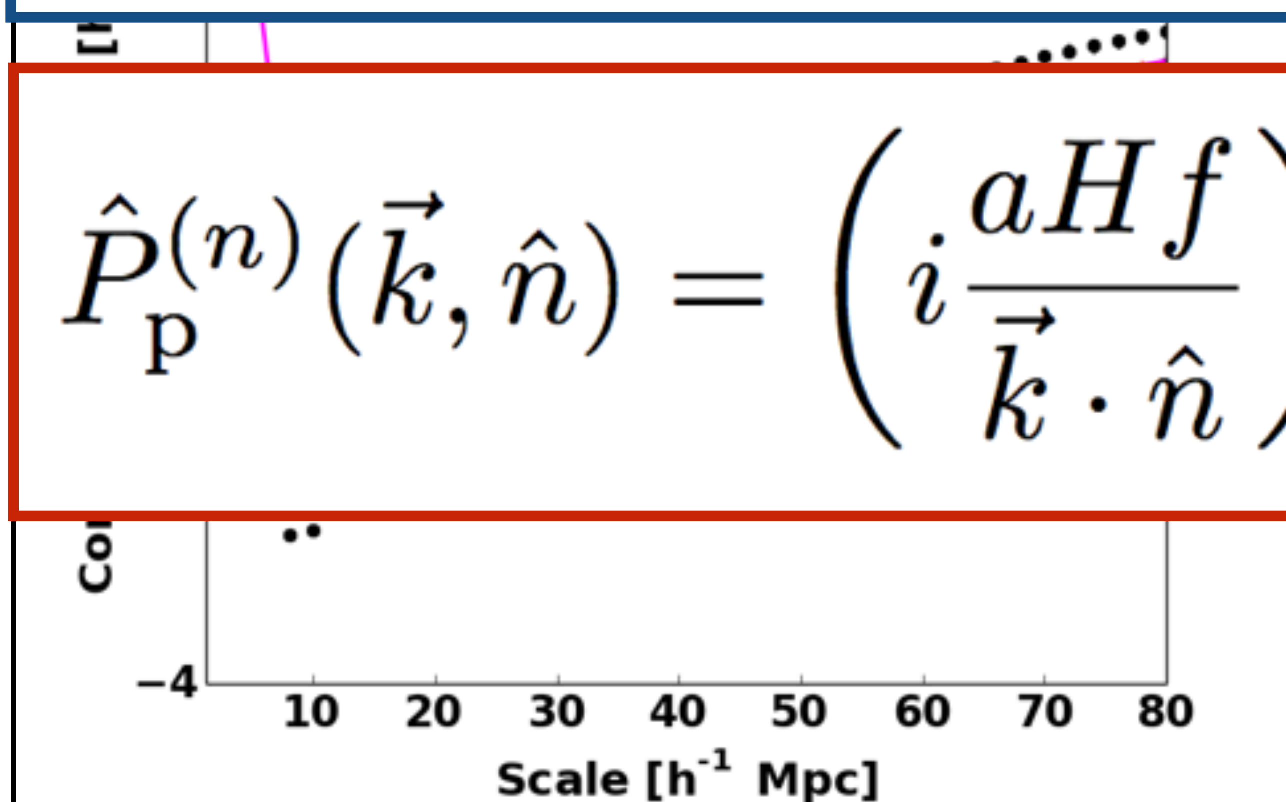


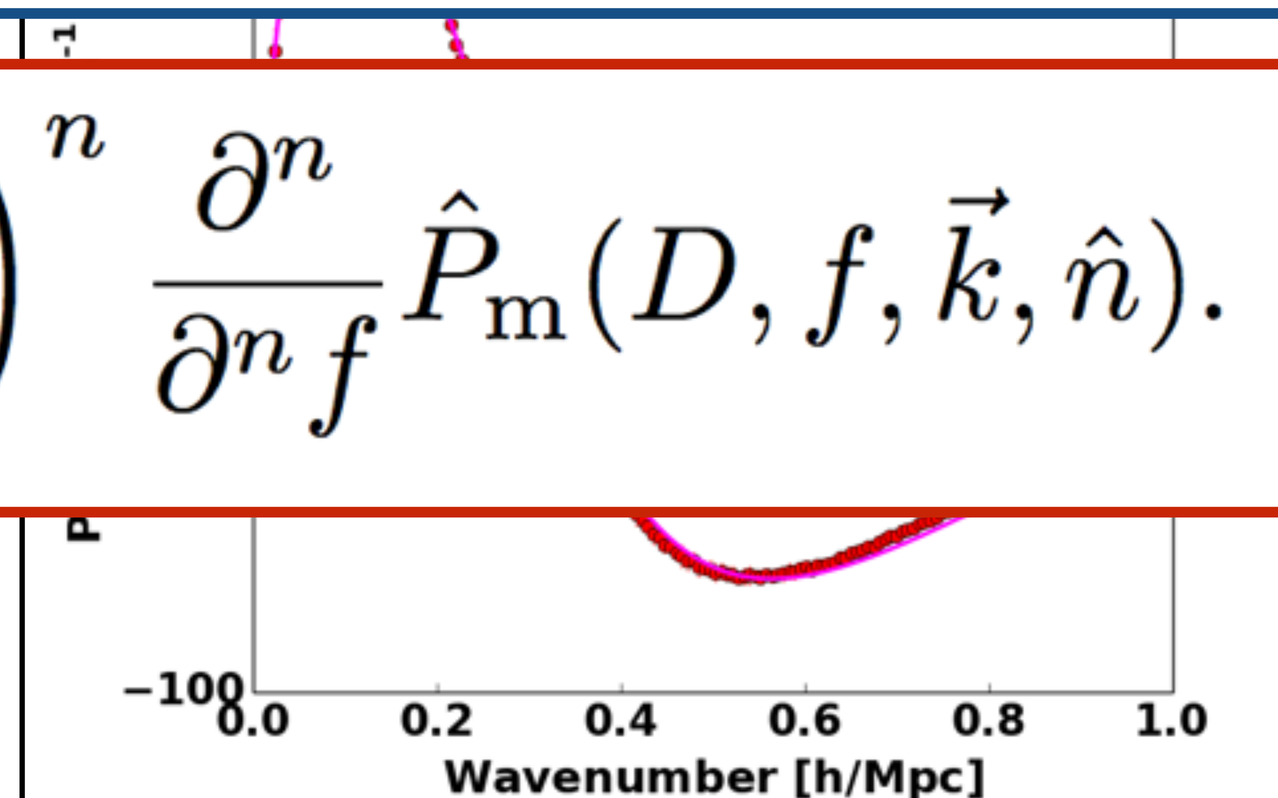
Zel’dovich Approximation

$$P(\vec{k}) = \left\langle \frac{V}{N_{\mathrm{p}}^2} \sum_{i,j} e^{-i\vec{k}\cdot(\vec{x}_i - \vec{x}_j)} \right\rangle$$

$$= \int d^3q e^{-i\vec{k}\cdot(\vec{q}_1-\vec{q}_2)} \left\langle e^{-i\vec{k}\cdot\left(\vec{\Psi}(\vec{q}_1)-\vec{\Psi}(\vec{q}_2)\right)} \right\rangle \quad \text{, where} \quad \vec{q} = \vec{q}_1 - \vec{q}_2$$

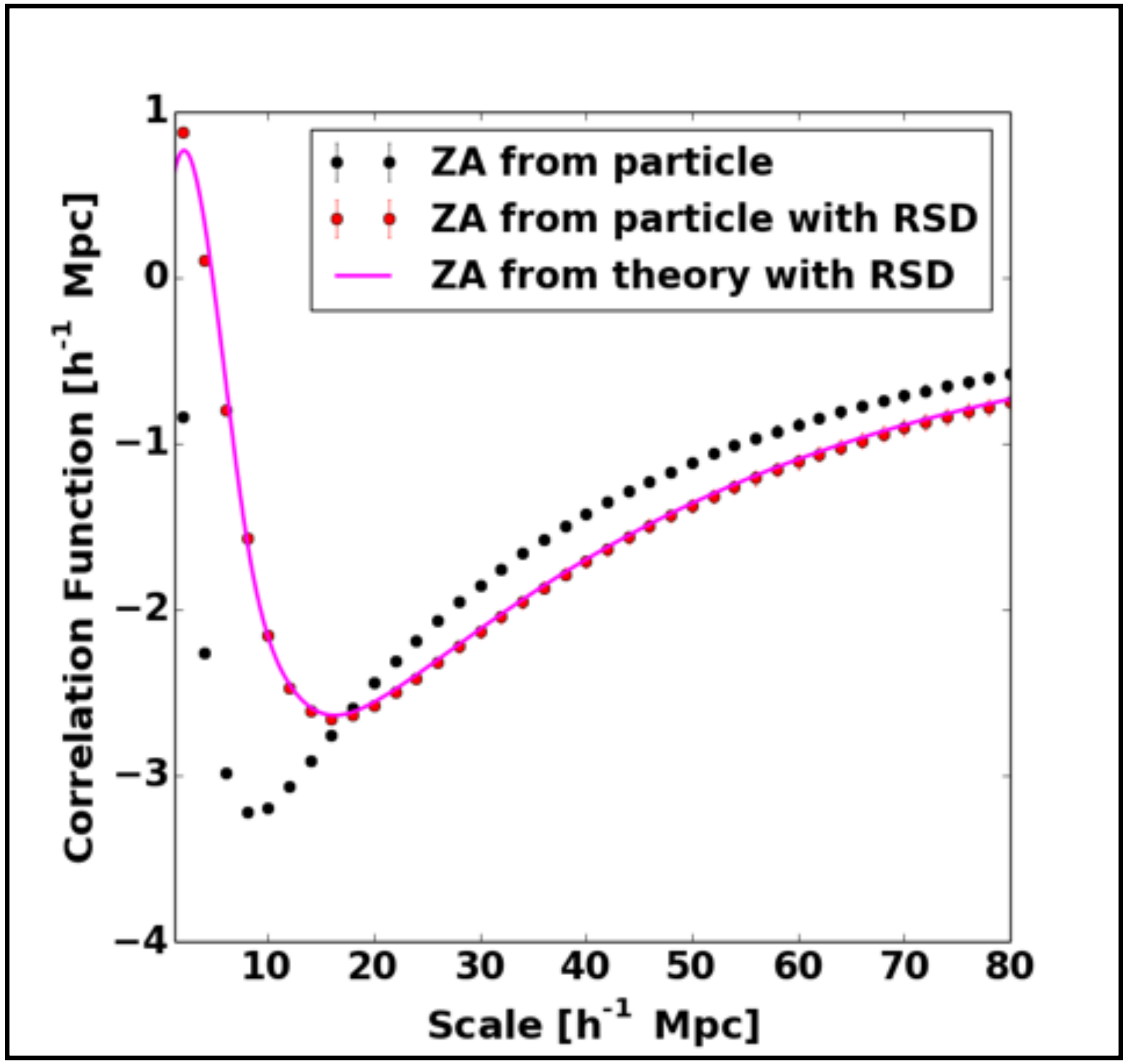
$$\hat{P}_{\mathrm{p}}^{(n)}(\vec{k}, \hat{n}) = \left(i \frac{aHf}{\vec{k}\cdot\hat{n}} \right)^n \frac{\partial^n}{\partial^n f} \hat{P}_{\mathrm{m}}(D, f, \vec{k}, \hat{n}).$$



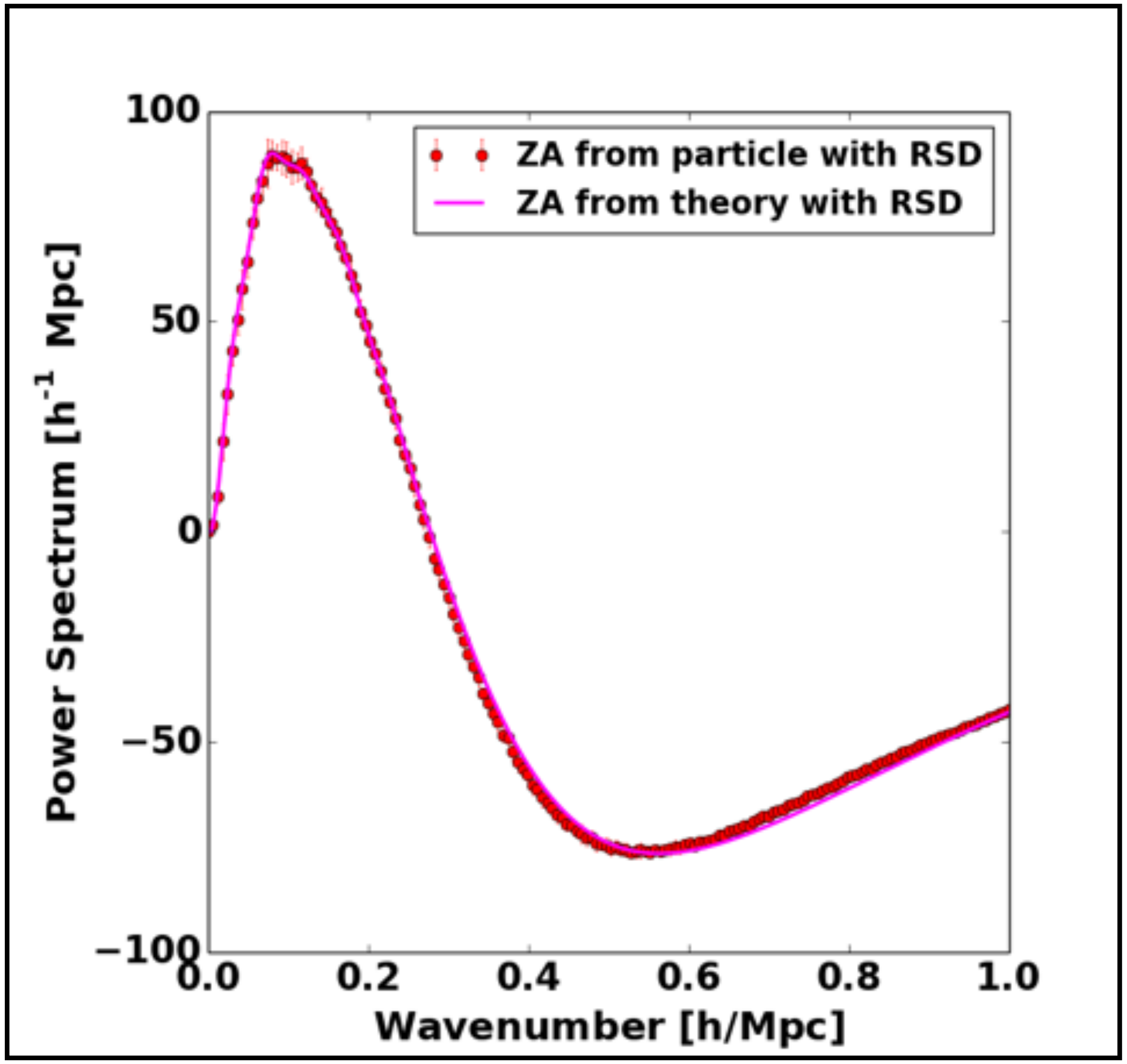


Zel'dovich Approximation

Momentum Correlation Function

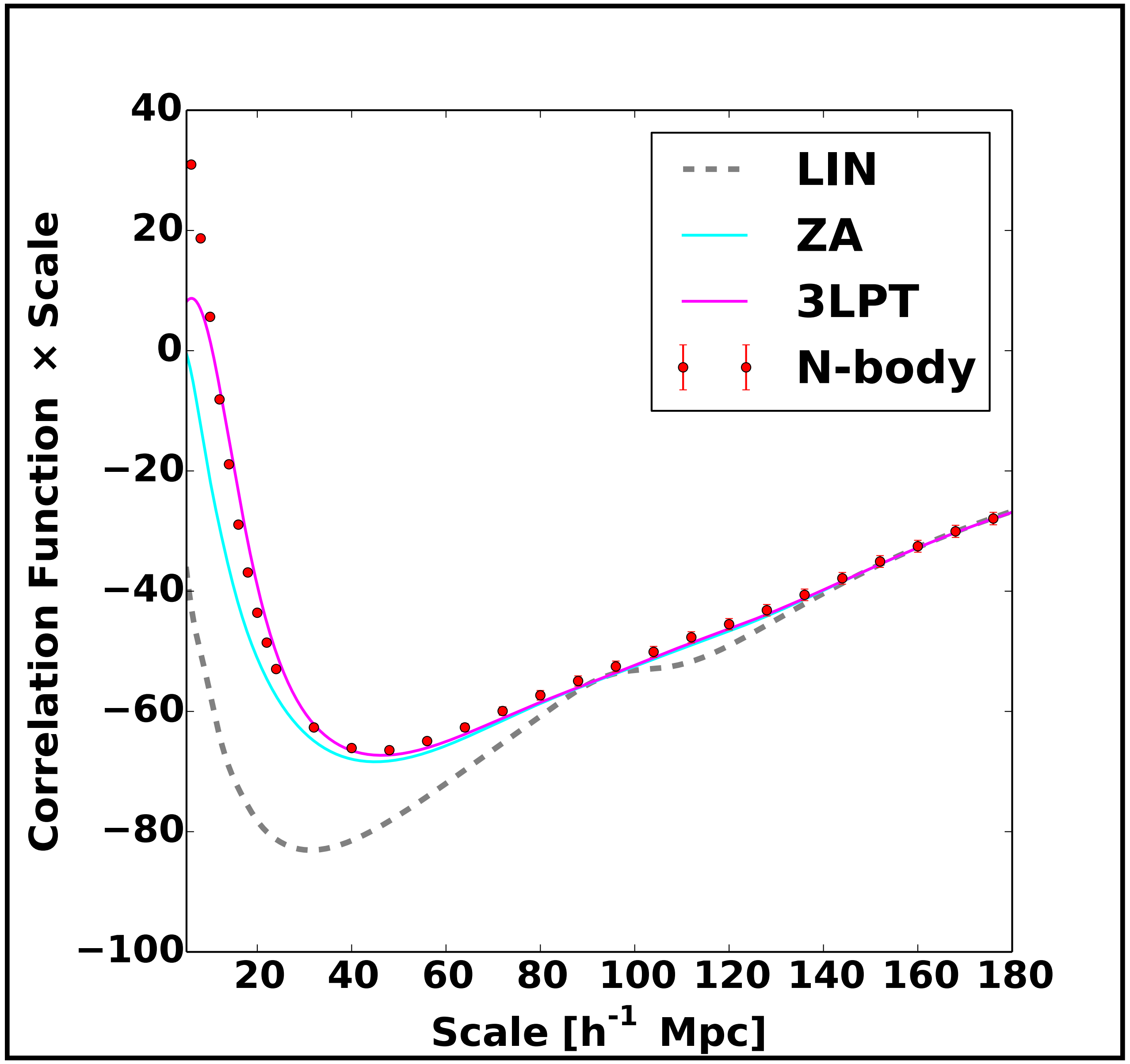


Momentum Power Spectrum

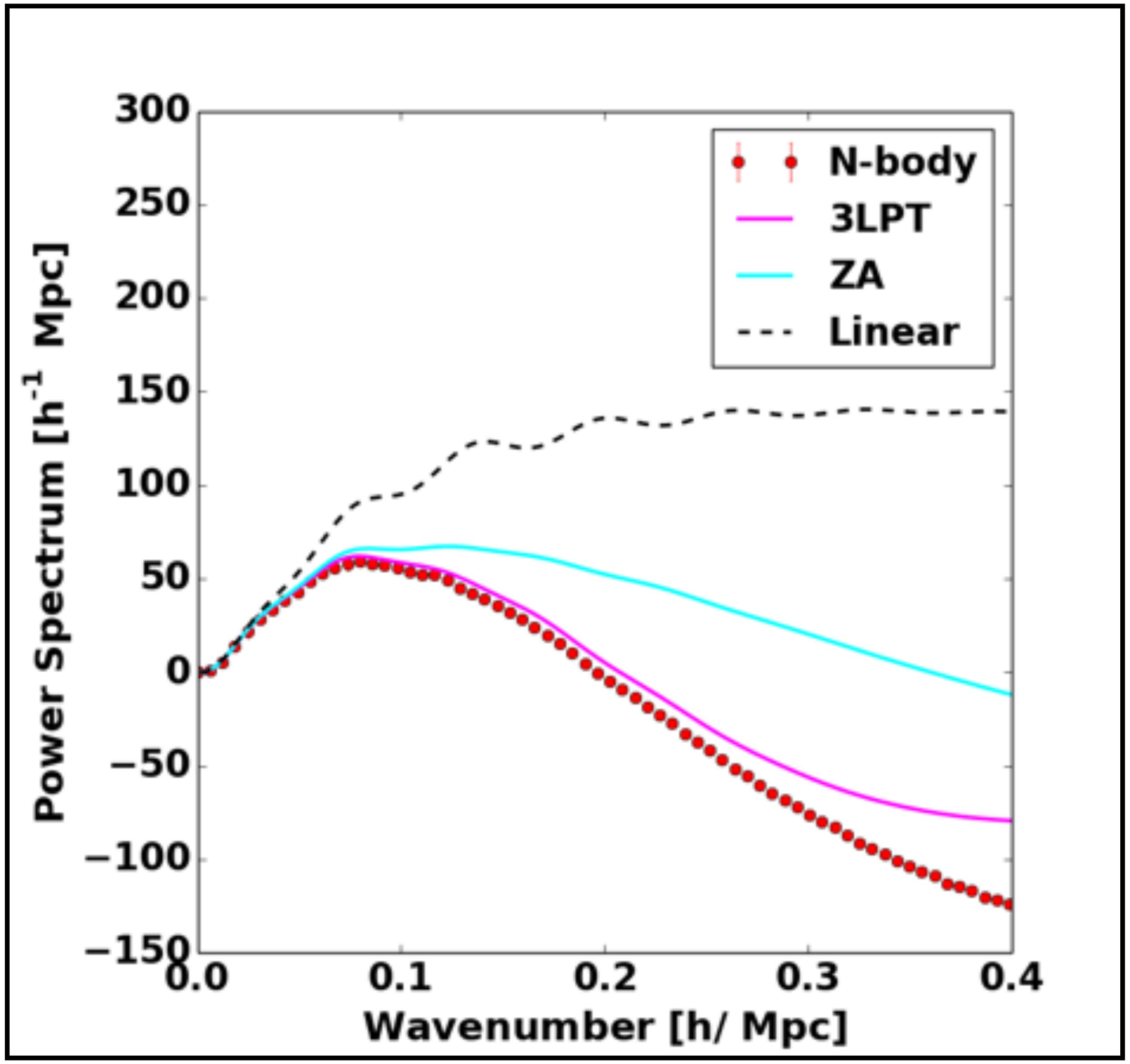


Third Order PT

Momentum Correlation Function



Momentum Power Spectrum

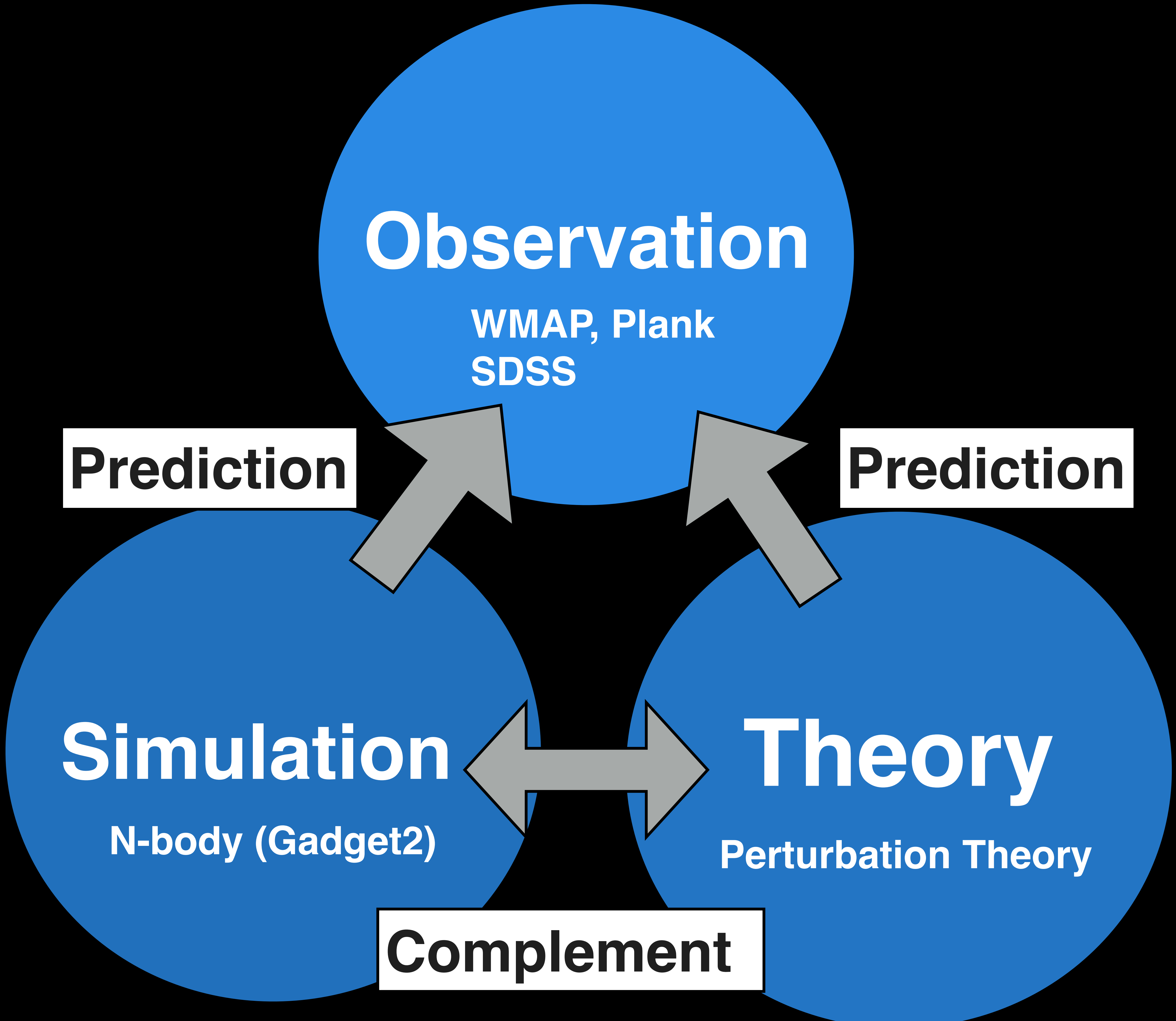


Summary

$$\boxed{\hat{P}_{\mathrm{p}}^{(n)}(\vec{k},\hat{n}) = \left(i\frac{aHf}{\vec{k}\cdot\hat{n}}\right)^{n}\frac{\partial^{n}}{\partial^{n} f}\hat{P}_{\mathrm{m}}(D,f,\vec{k},\hat{n}).}$$

Future Work

- **Covariance matrix (computing)**
- **Halo model**
- **Measurement of kSZ power spectrum**


Observation
WMAP, Plank
SDSS
Prediction
Prediction
Simulation
N-body (Gadget2)
Theory
Perturbation Theory
Complement

Extra Slides

Fisher Analysis

$$\boxed{\frac{\partial P_{\mathrm{s}}^{(n)}(\vec{k})}{\partial \ln f} = nP_{\mathrm{s}}^{(n)}(\vec{k}) + \left(i\frac{aH}{\vec{k}\cdot\hat{n}}\right)^{-1} P_{\mathrm{s}}^{(n+1)}(\vec{k})}$$

Measurable in simulations

Covariance Matrix

$$\mathrm{Cov}\left(\hat{P}^{(n_1)}_{\ell_1}(k_1), \hat{P}^{(n_2)}_{\ell_2}(k_2)\right) = \frac{\delta^{(\mathrm{K})}_{k_1,k_2}}{N_{\mathrm{mode}}(k_1)} \underline{C^{(n_1)(n_2)}_{\ell_1\ell_2}(k_1)} + \underline{T^{(n_1)(n_2)}_{\ell_1\ell_2}(k_1, k_2)}$$

Gaussian term　non-Gaussian term

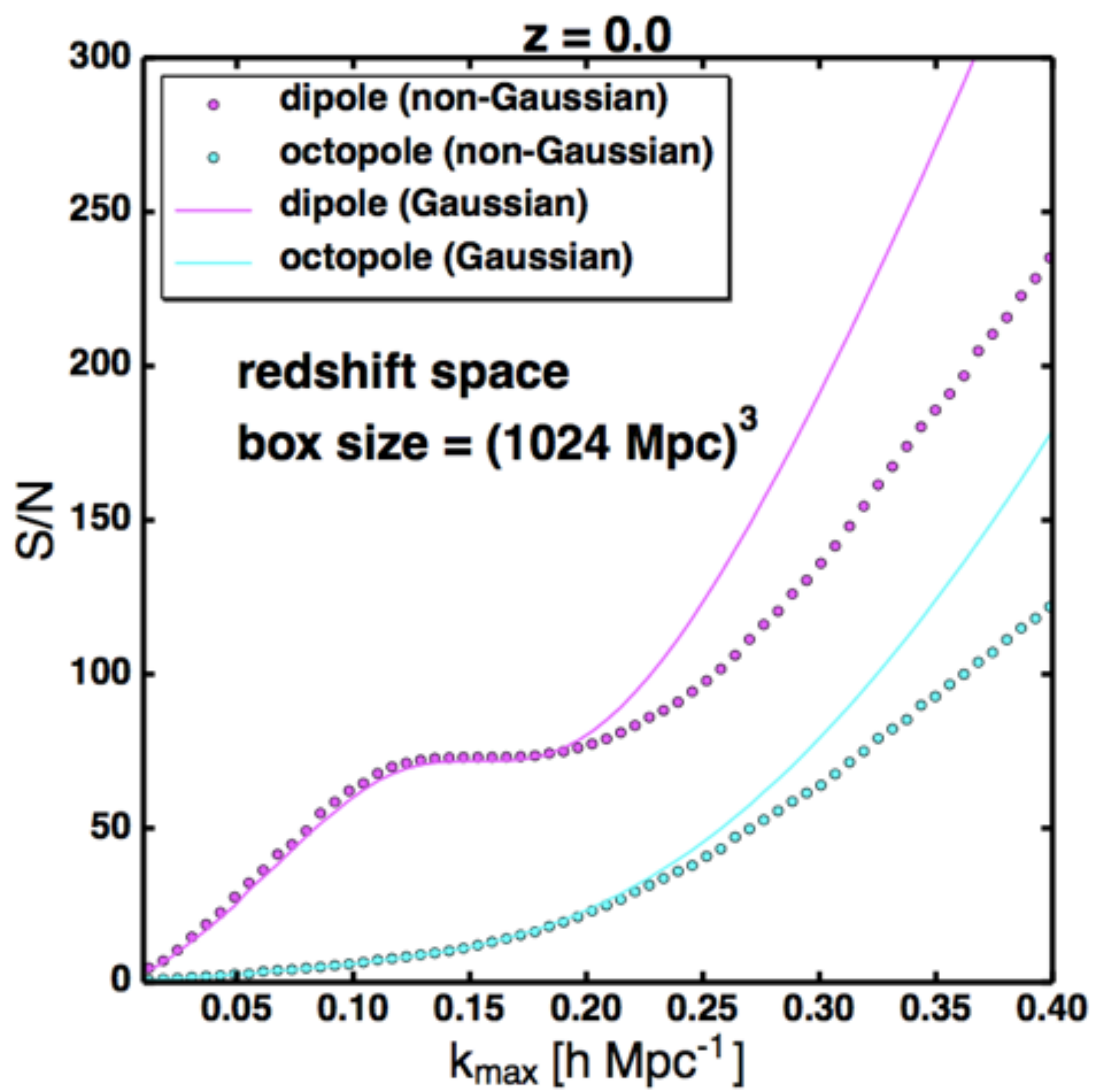
Gaussian term

$$C^{(1)(1)}_{\ell_1\ell_2}(k) = \left[1 + (-1)^{\ell_2+1}\right] \frac{(2\ell_1+1)(2\ell_2+1)}{2} \int d\mu \mathcal{L}_{\ell_1}(\mu)\mathcal{L}_{\ell_2}(\mu) \times \left[-2P^{(0)(0)}(\vec{k})P^{(1)(1)}(\vec{k}) + 2P^{(1)(0)}(\vec{k})P^{(1)(0)}(\vec{k})\right].$$

Gaussian limit でも複雑なスケール依存性を持つ。

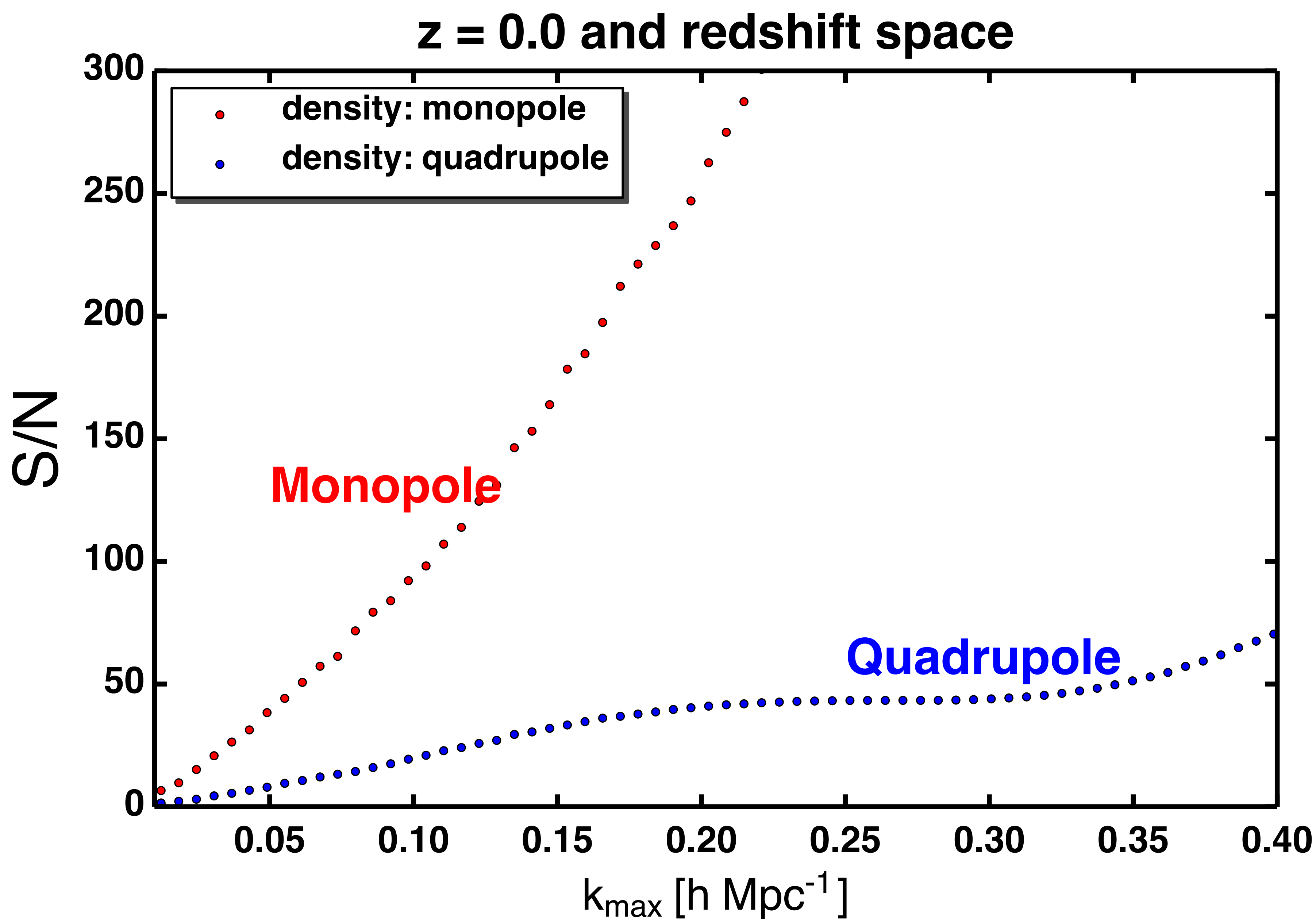
(単純にpower の２乗ではない。)

S/N



Gaussian vs. non-Gaussian

S/N

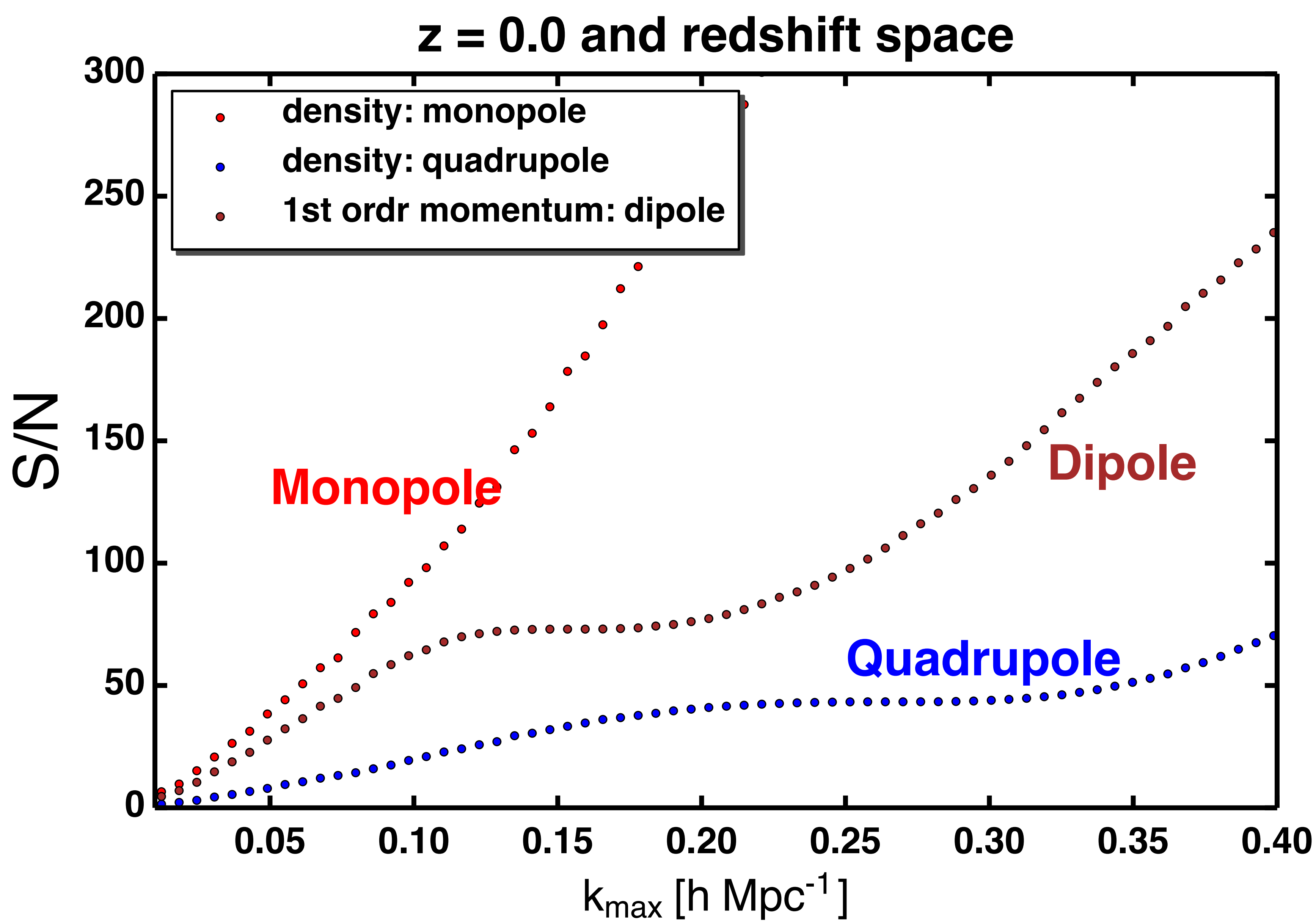


Density Field

<δ δ> : Monopole

<δ v> : Quadrupole

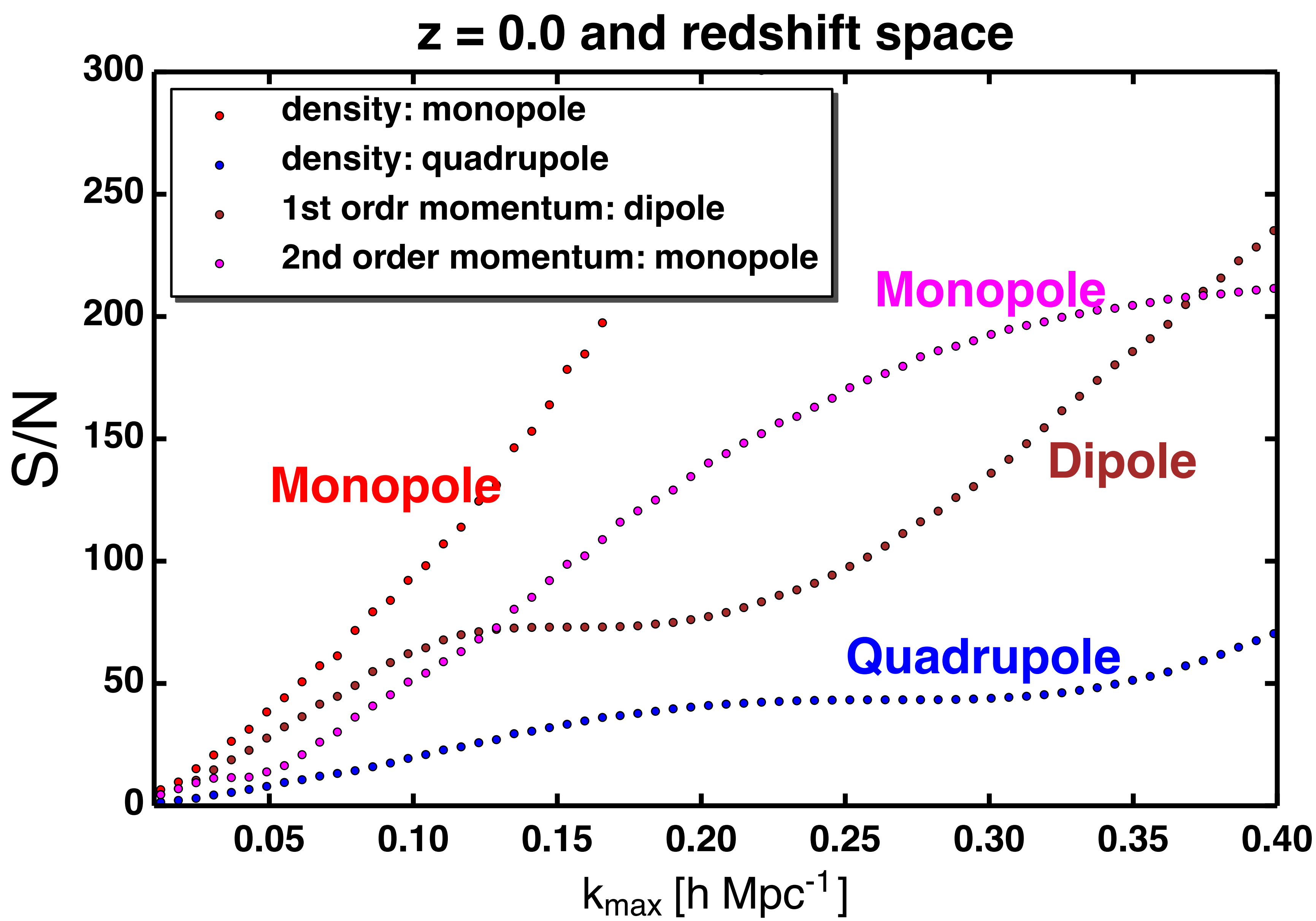
S/N



1st order Momentum Field

<δ v> : Dipole

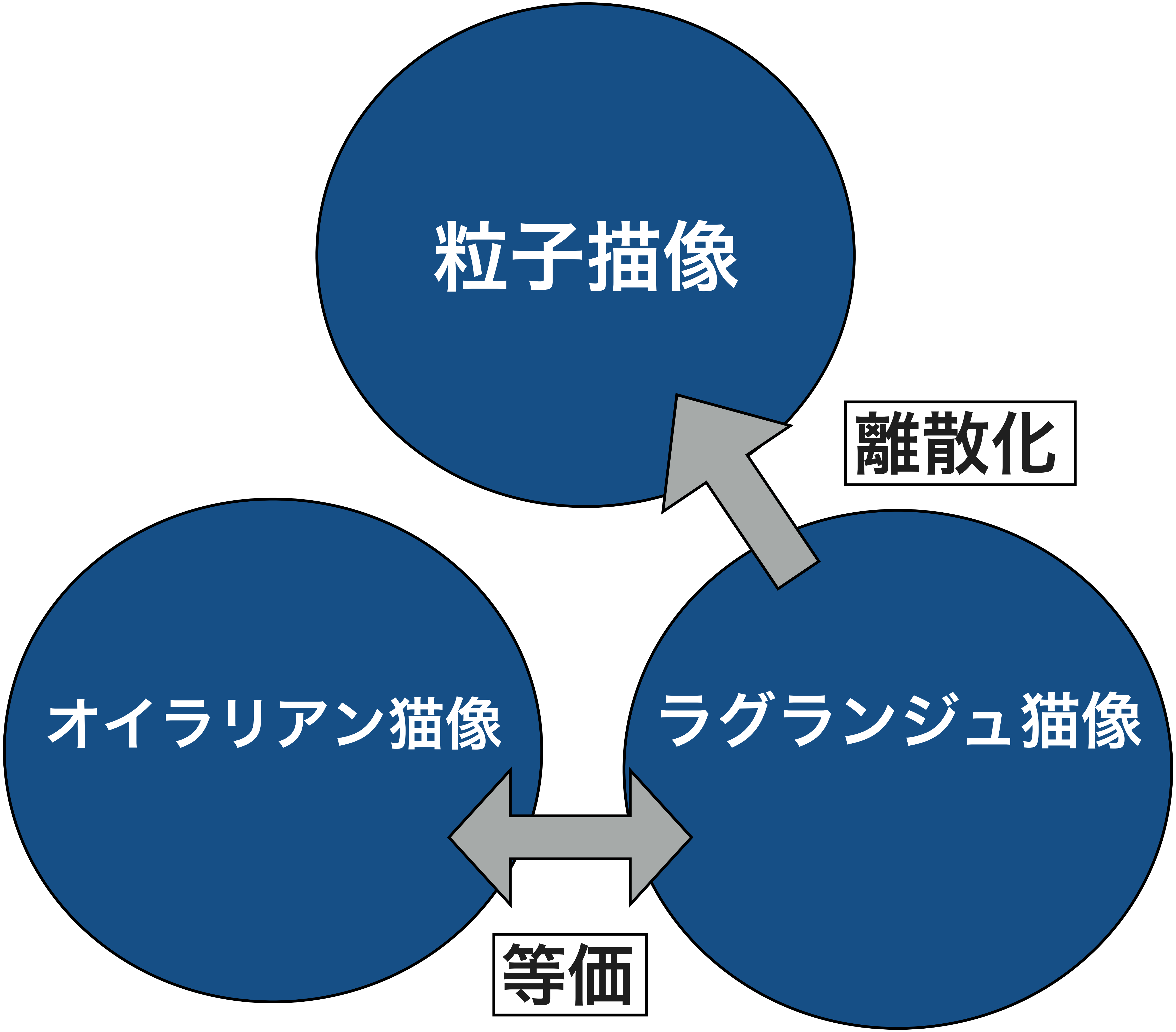
S/N

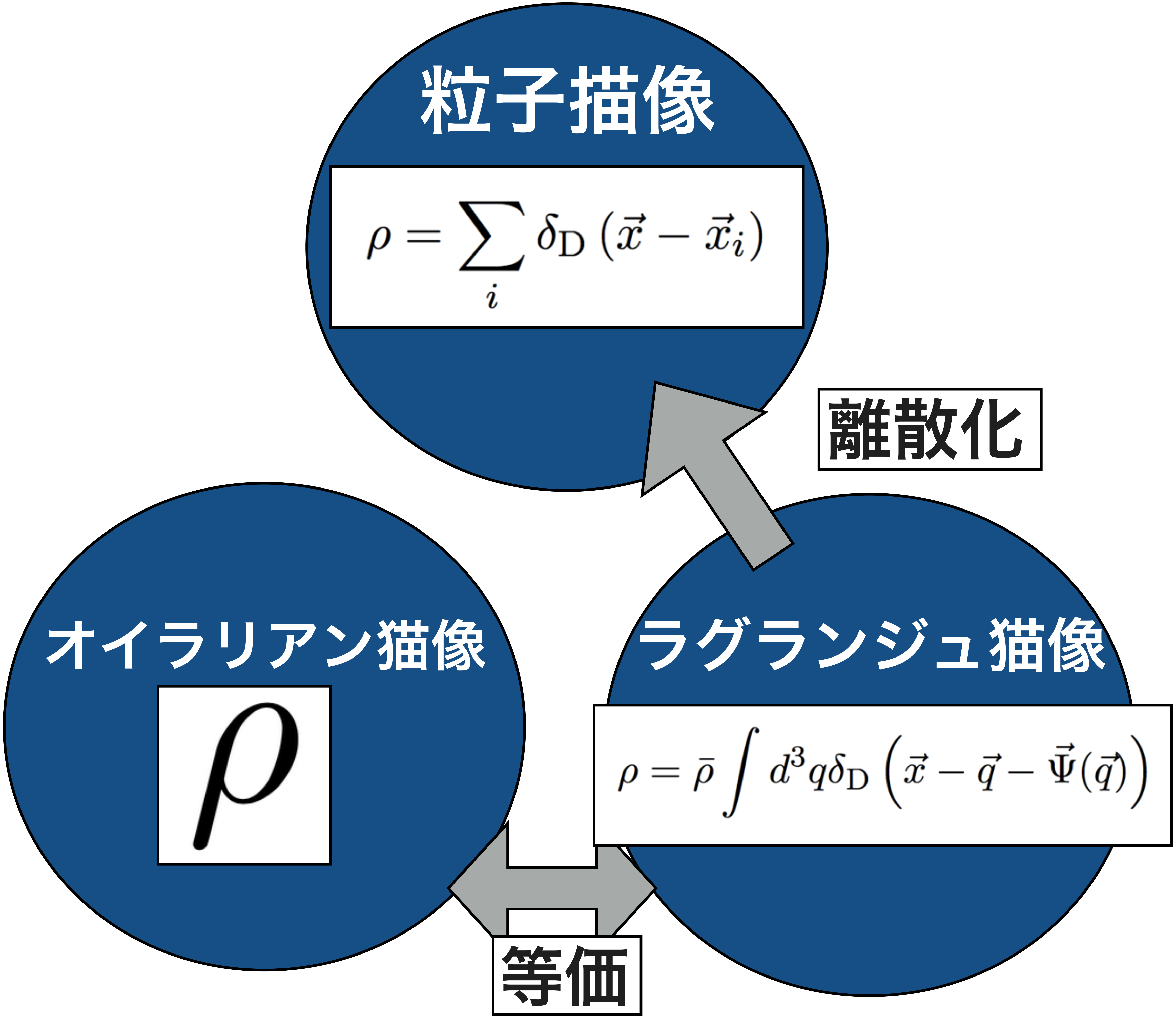


2nd order Momentum Field

<v v> : Monopole

Bias free


粒子描像
離散化
オイラリアン描像
ラグランジュ描像
等価


粒子描像
$\rho = \sum_i \delta_{\mathrm{D}}\left(\vec{x} - \vec{x}_i\right)$
離散化
オイラリアン描像
ρ
ラグランジュ描像
$\rho = \bar{\rho}\int d^3q\,\delta_{\mathrm{D}}\left(\vec{x} - \vec{q} - \vec{\Psi}(\vec{q})\right)$
等価

Euleriam vs. Lagrangian Perturbation Theory

Eulerian PT

$$\begin{aligned}\rho &= \mathcal{O}(1) + \mathcal{O}(\delta_{\rm L}) + \mathcal{O}\left(\delta_{\rm L}^2\right) + \ldots, \\ \vec{v} &= \mathcal{O}(\delta_{\rm L}) + \mathcal{O}\left(\delta_{\rm L}^2\right) + \ldots\end{aligned}$$

Lagrangian PT

$$\begin{aligned}\rho &= \text{Full}, \\ \vec{\Psi} &= \dot{\vec{\Psi}} = \vec{v} = \mathcal{O}(\delta_{\rm L}) + \mathcal{O}\left(\delta_{\rm L}^2\right) + \ldots\end{aligned}$$

展開するものが違う。

Euler vs. Lagrangian Perturbation Theory

Improved Eulerian PT

$$\rho = \mathcal{O}(1) + \mathcal{O}(\delta_{\rm L}) + \cdots$$

Full

$$\vec{v} = \mathcal{O}(\delta_{\rm L}) + \mathcal{O}(\delta_{\rm L}^2) + \ldots$$

Resummation of Lagrangian PT

$$\rho =$$

Expanding

$$\vec{\Psi} = \dot{\vec{\Psi}} = \vec{v} = \mathcal{O}(\delta_{\rm L}) + \mathcal{O}(\delta_{\rm L}^2) + \ldots$$

Perturbation Theory

$$\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}$$

Displacement vector (Velocity field)
(Equation of Motion, Gravity)

Density field
(Continuity Equation)

$$\rho$$

	1	3	5	7	$\sim\sim$	Full
1	Linear					ZA
3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
$\sim\sim$						
Full						N-body

Perturbation Theory

$$\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}$$

Displacement vector (Velocity field)
(Equation of Motion, Gravity)

Density field
(Continuity Equation)

$$\rho$$

	1	3	5	7	~~	Full
1	Linear					ZA
3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
~~						
Full						N-body

Improved PT

Perturbation Theory

$$\boxed{\vec{v} \sim \dot{\vec{\Psi}} \sim \vec{\Psi}}$$

Displacement vector (Velocity field)
(Equation of Motion, Gravity)

Density field
(Continuity Equation) $\boxed{\rho}$

	1	3	5	7	~~	Full
1	Linear					ZA
3		3SPT (1-loop)				3LPT
5			5SPT (2-loop)			5LPT
7				7SPT (3-loop)		
~~						
Full						N-body

Zel'dovich Approximation

Density Power Spectrum in
Gamma-Expansion method
(Wiener Hermite expansion (Sugiyama and Futamase), iPT, or etc.)