Probing Strong Magnetic Fields with Photons and Dileptons

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Magnetic measurement in HIC

- Nonzero μ_5 and $\vec{B}$ drive the chiral magnetic effect (CME)

$$\vec{j} = \frac{e^2\vec{B}}{2\pi^2}\mu_5$$

- Experiment difficulties: Large background!

  Isobar experiments?

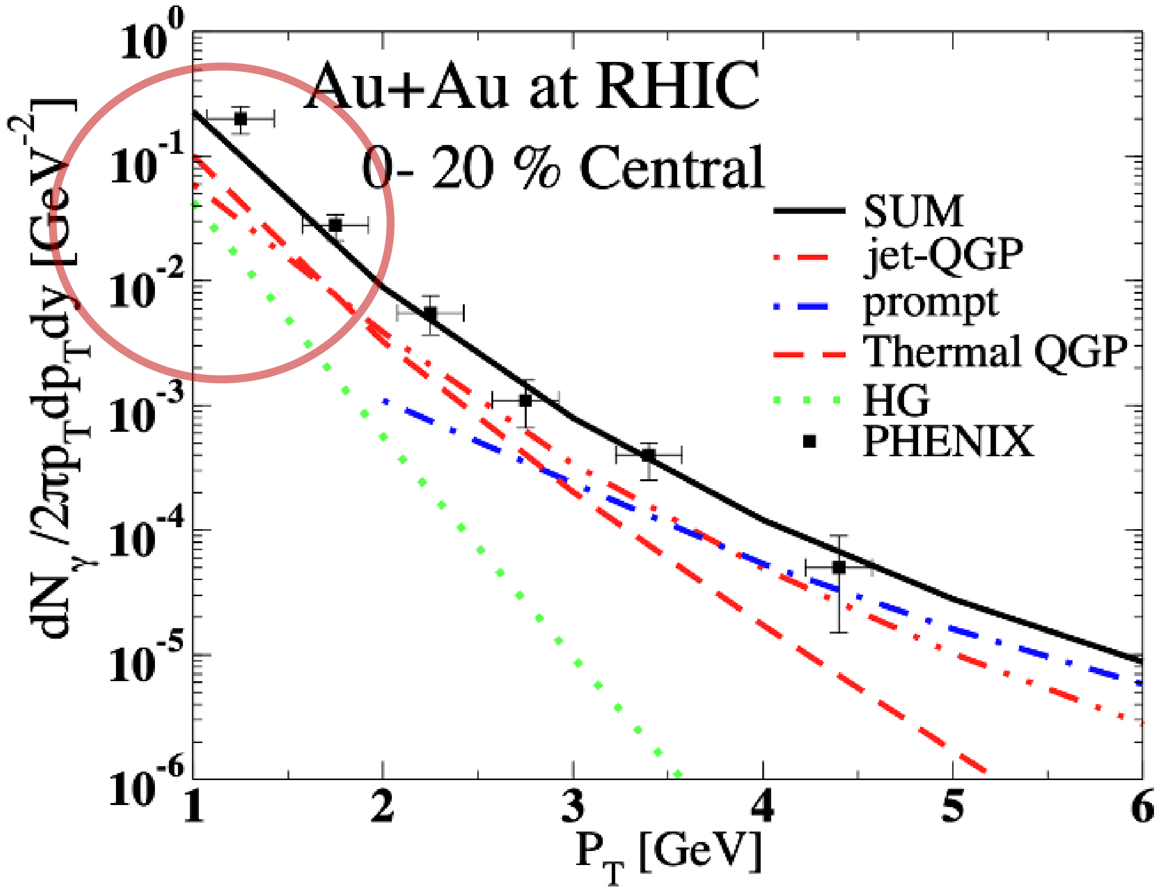
[STAR Collaboration, 2014]

$${}^{96}_{40}\mathrm{Zr} + {}^{96}_{40}\mathrm{Zr} \text{ vs. } {}^{96}_{44}\mathrm{Ru} + {}^{96}_{44}\mathrm{Ru}$$

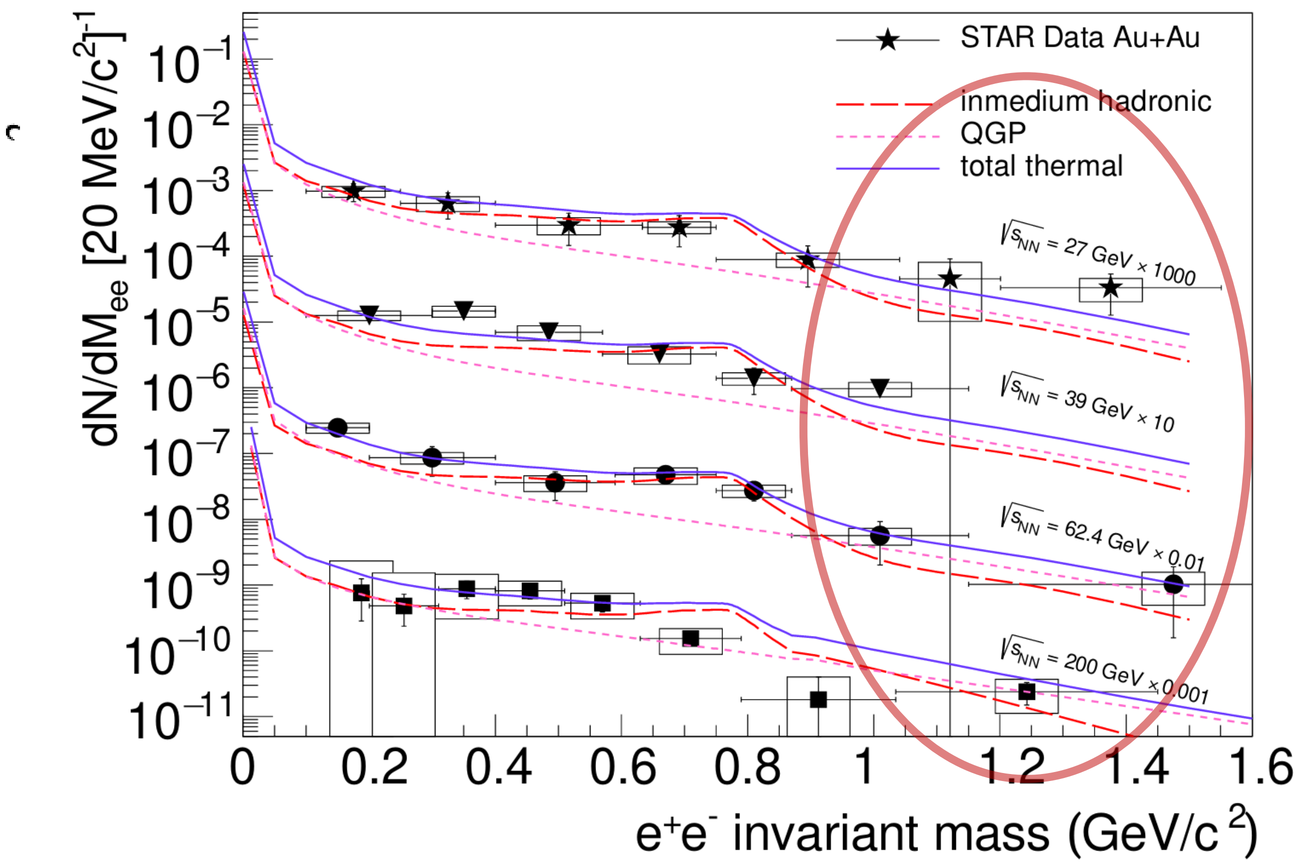
$${}^{130}_{52}\mathrm{Te} + {}^{130}_{52}\mathrm{Te} \text{ vs. } {}^{130}_{56}\mathrm{Ba} + {}^{130}_{56}\mathrm{Ba}$$

No positive official results now!

E&M sources in HIC



[Turbide et al,, Phys. Rev. D, 2008]

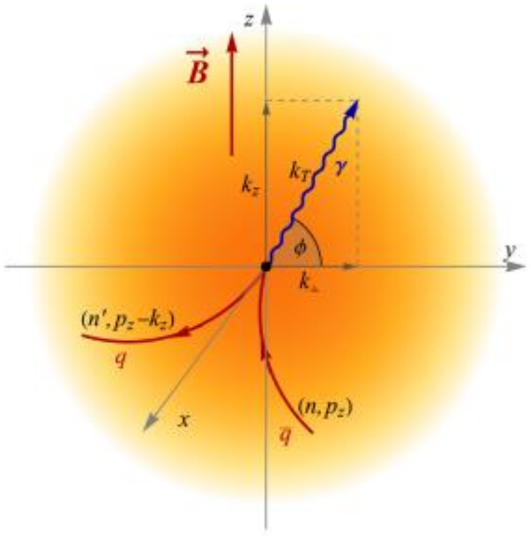


[Jinhui Chen et al., Nucl.Sci.Tech., 2024]

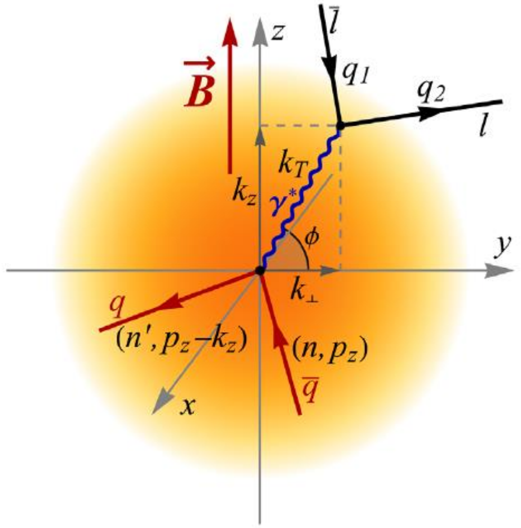
E&M probes thermal rate

- The expression for the photon rate is

$$k^0 \frac{d^3 R}{dk_x dk_y dk_z} = -\frac{1}{(2\pi)^3} \frac{\text{Im}\left[\Pi^{\mu}_{\mu}(k)\right]}{\exp\left(\frac{k_0}{T}\right) - 1}$$



[X.W., Igor Shovkovy, Lang Yu, Mei Huang, PRD, 2020]
[X.W., Igor Shovkovy, PRD, 2021]
$\mu \neq 0$: [X.W., Igor Shovkovy, EPJC, 2021]



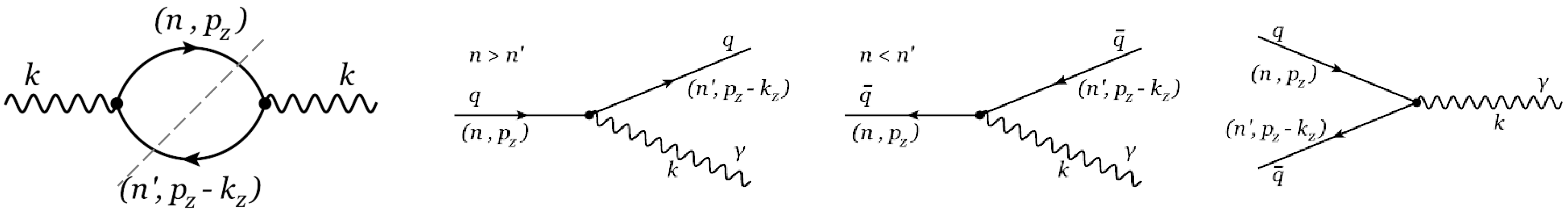
- The expression for the dilepton rate is

$$\frac{dR_{l\bar{l}}}{d^4K} = \frac{\alpha}{12\pi^4} \frac{n_B(\Omega)}{M^2} \text{Im}[\Pi^{\mu}_{\mu}(\Omega, \mathbf{k})]$$

$$M^2 = K^2 \equiv \Omega^2 - k_\perp^2 - k_z^2$$

[X.W., Igor Shovkovy, PRD, 2022]

Photons polarization tensor



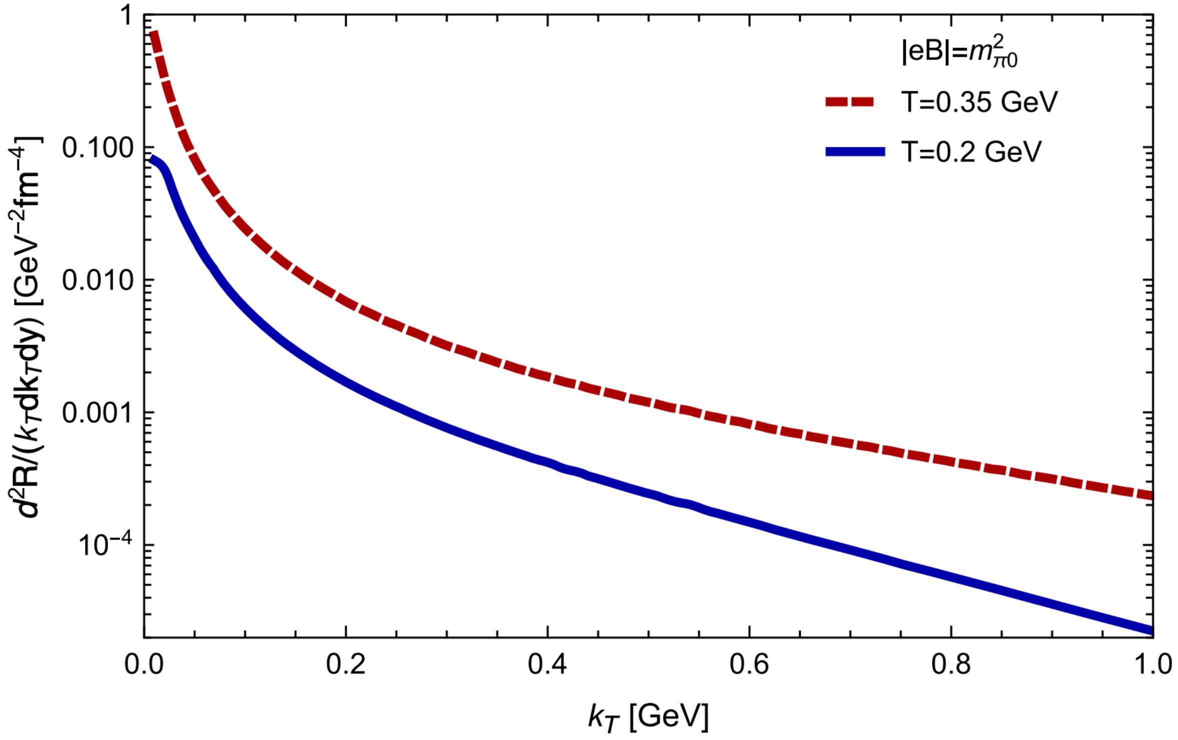
$$\Pi^{\mu\nu}(i\Omega_m;\mathbf{k}) = 4\pi N_c \sum_{f=u,d} \alpha_f T \sum_{k=-\infty}^{\infty} \int \frac{dp_z}{2\pi} \int d^2\mathbf{r}_\perp e^{-i\mathbf{r}_\perp\cdot\mathbf{k}_\perp} \mathrm{tr}[\gamma^\mu \bar{G}_f(i\omega_k, p_z;\mathbf{r}_\perp)\gamma^\nu \bar{G}_f(i\omega_k - i\Omega_m, p_z - k_z; -\mathbf{r}_\perp)],$$

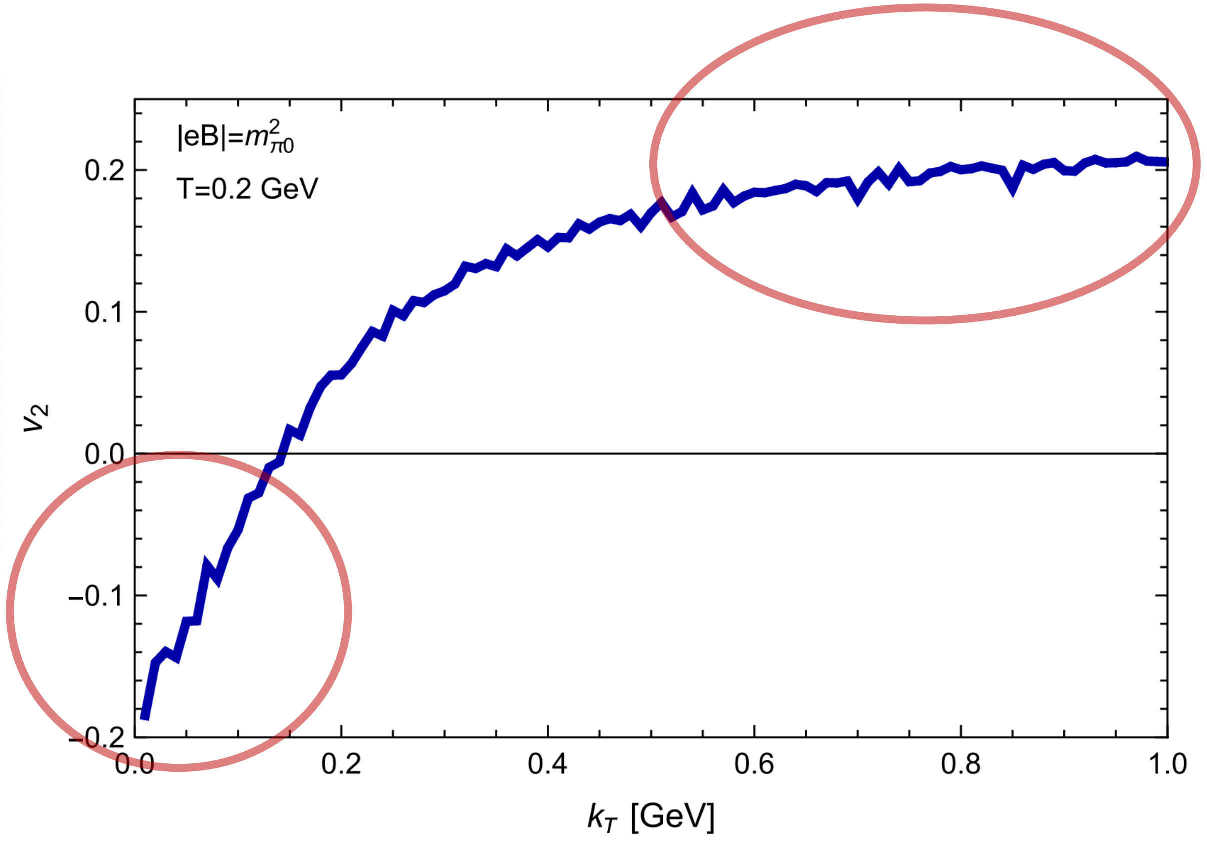
$$\mathrm{Im}[\Pi^{\mu}_{R,\mu}(\Omega;\mathbf{k})] = \sum_{f=u,d} \frac{N_c\alpha_f}{2l_f^4} \sum_{n,n'=0}^{\infty} \int \frac{dp_z}{2\pi} \sum_{\lambda,\eta=\pm 1} \frac{n_F(E_{n,p_z,f}) - n_F(\lambda E_{n',p_z-k_z,f})}{2\eta\lambda E_{n,p_z,f} E_{n',p_z-k_z,f}} \sum_{i=1}^{4} \mathcal{F}_i^f \boxed{\delta(E_{n,p_z,f} - \lambda E_{n',p_z-k_z,f} + \eta\Omega)}$$

$$E_{n,p_z,f} = \sqrt{m^2 + p_z^2 + 2n|e_f B|}$$

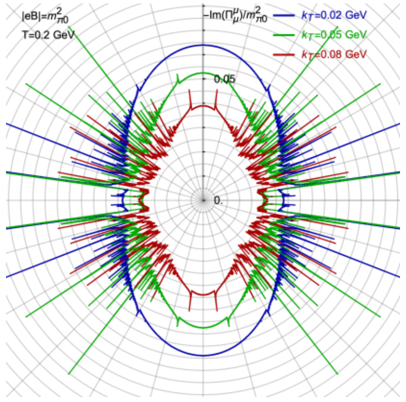
[X.W., Igor Shovkovy, Lang Yu, Mei Huang, PRD, 2020]
[X.W., Igor Shovkovy, PRD, 2021]

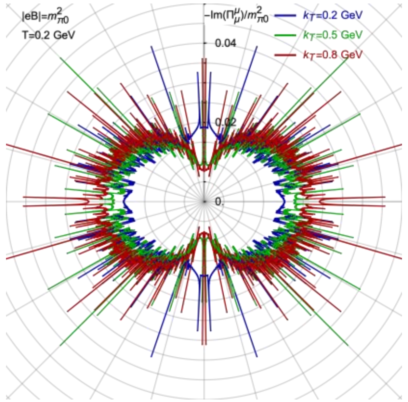
The thermal photon production rate and v_2



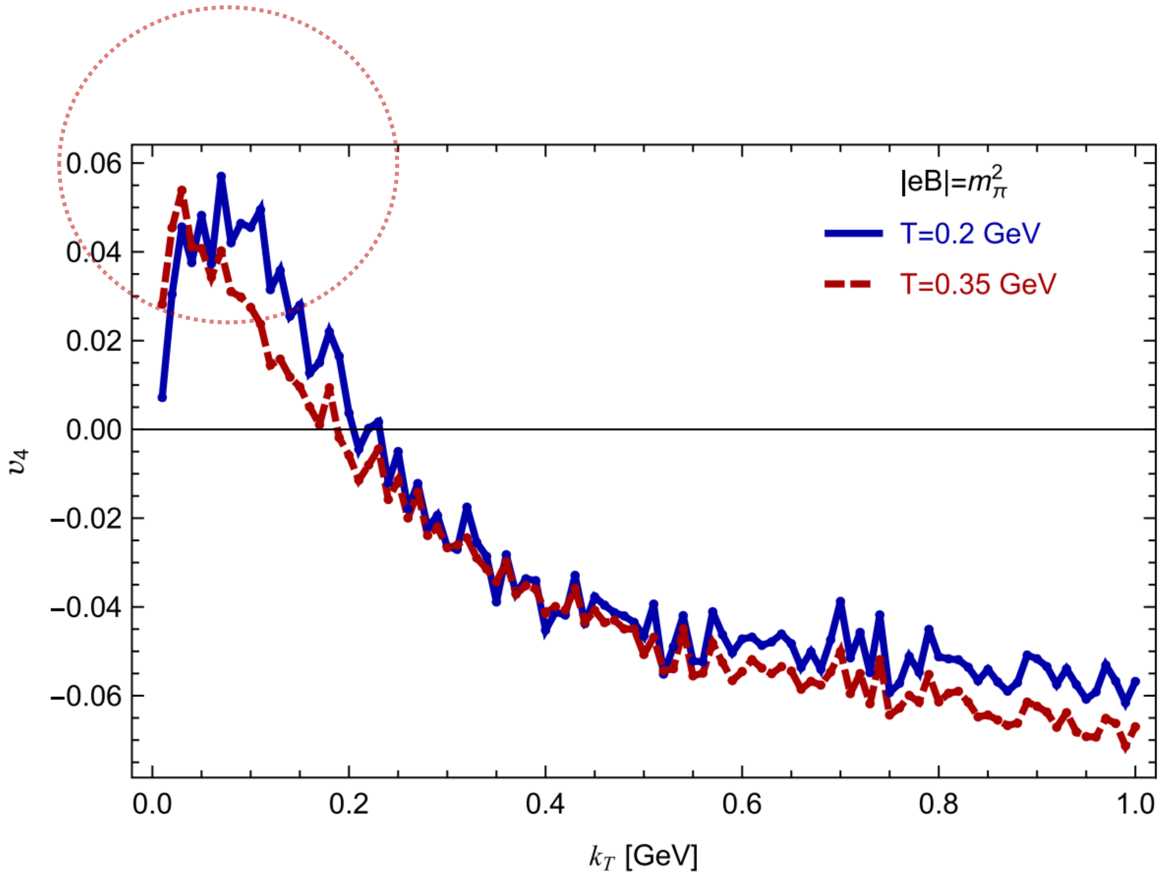


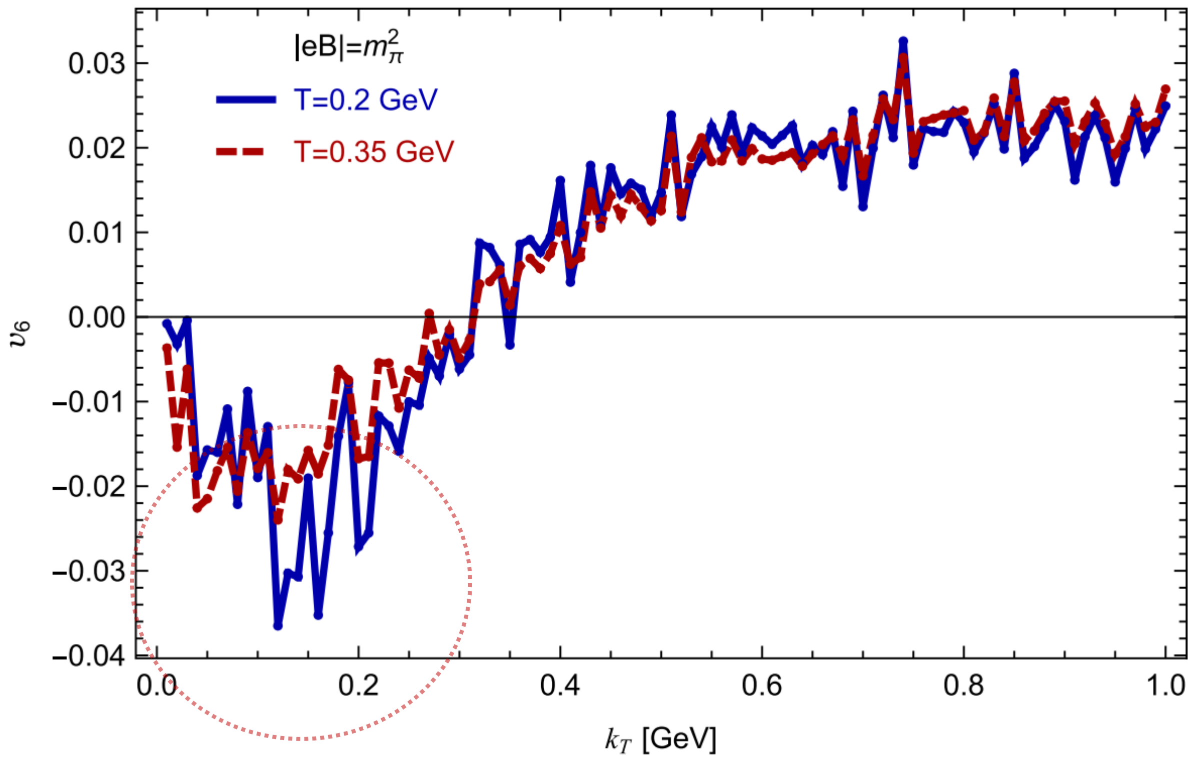
$$E\frac{\mathrm{d}^3N}{\mathrm{d}^3\mathbf{p}} = \frac{1}{2\pi}\frac{\mathrm{d}^2N}{p_T\mathrm{d}p_T\mathrm{d}y}\left(1+2\sum_{n=1}^{\infty} v_n\cos[n(\phi-\Psi_{\mathrm{RP}})]\right)$$





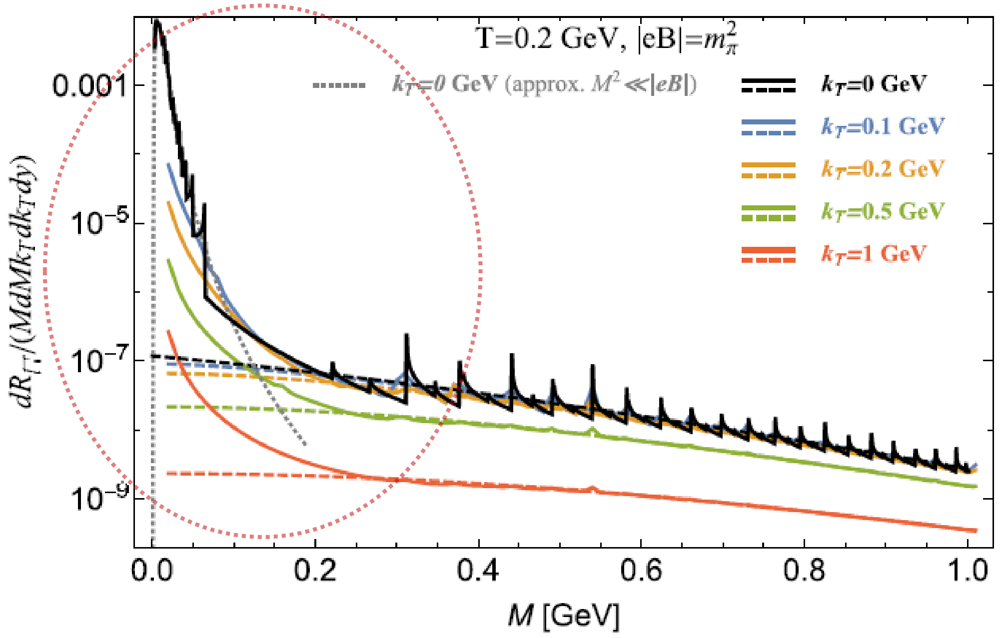
Photon v_4 and v_6

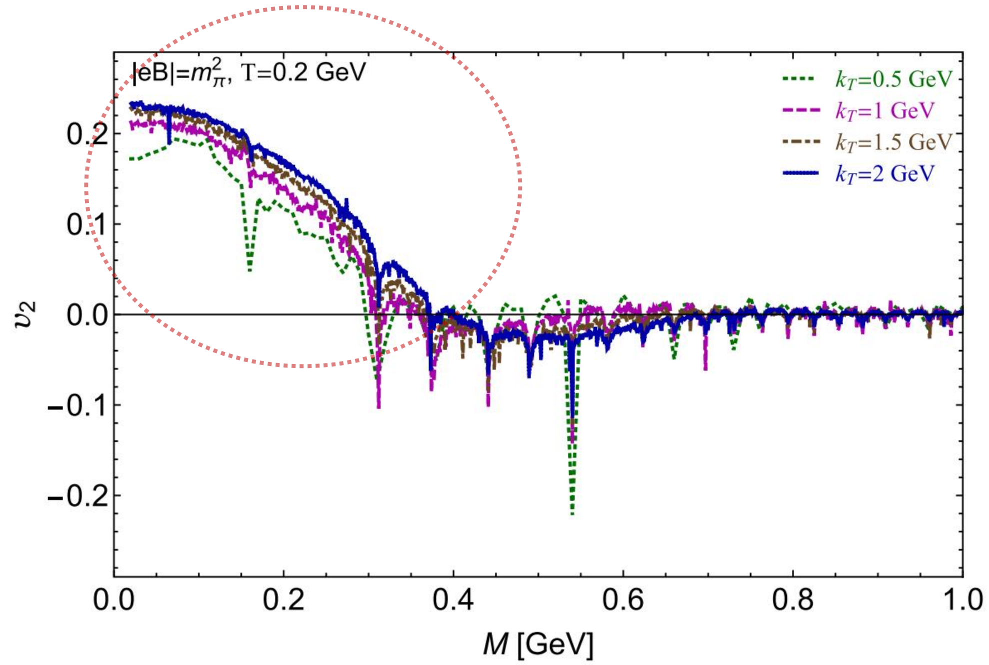


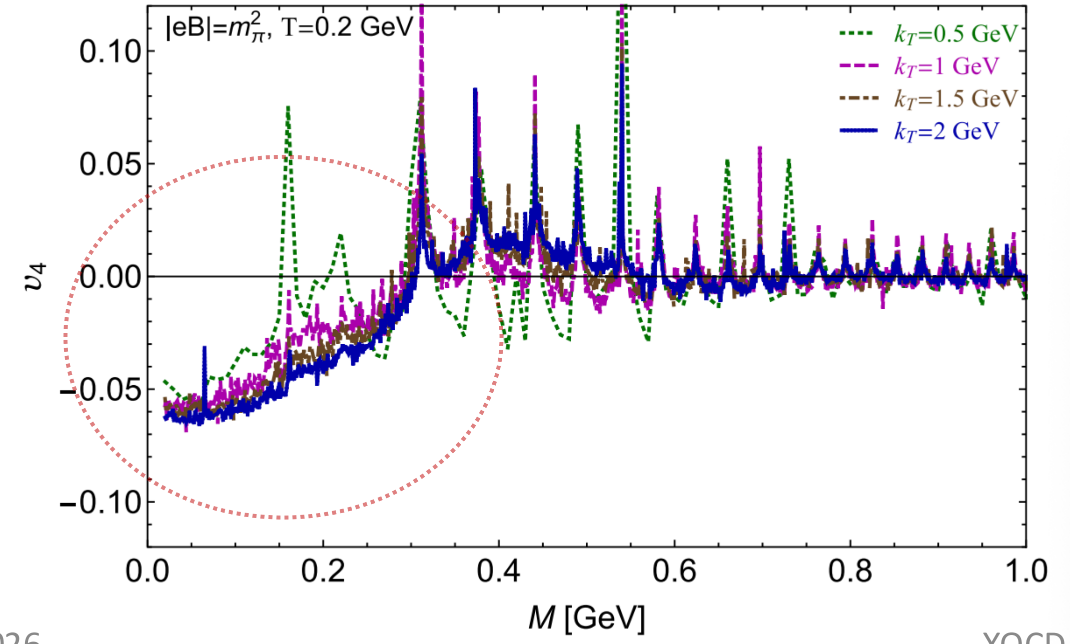


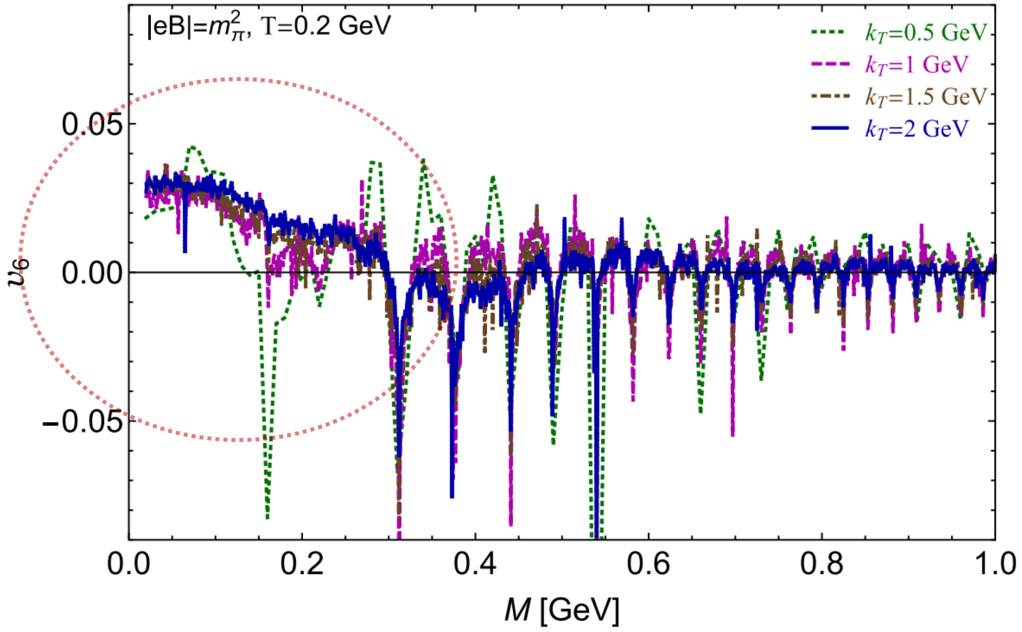
[X.W., Igor Shovkovy, PRD, 2024]

Dilepton rate and v_n









Polarization decomposition

$$g^{\mu\nu} = \epsilon_0^\mu \epsilon_0^{\nu *} - \epsilon_+^\mu \epsilon_+^{\nu *} - \epsilon_-^\mu \epsilon_-^{\nu *} - \epsilon_\parallel^\mu \epsilon_\parallel^{\nu *}$$

$$\epsilon_0^\mu = (1,0,0,0) \qquad \epsilon_\parallel^\mu = (0,0,0,1) \qquad \epsilon_\pm^\mu = (0,1,\pm i,0)/\sqrt{2}$$

$$g^{\mu\nu}\Pi_{\mu\nu}(k) \to \Pi^{(0)}(k) + \Pi^{(+)}(k) + \Pi^{(-)}(k) + \Pi^{(\parallel)}(k)$$

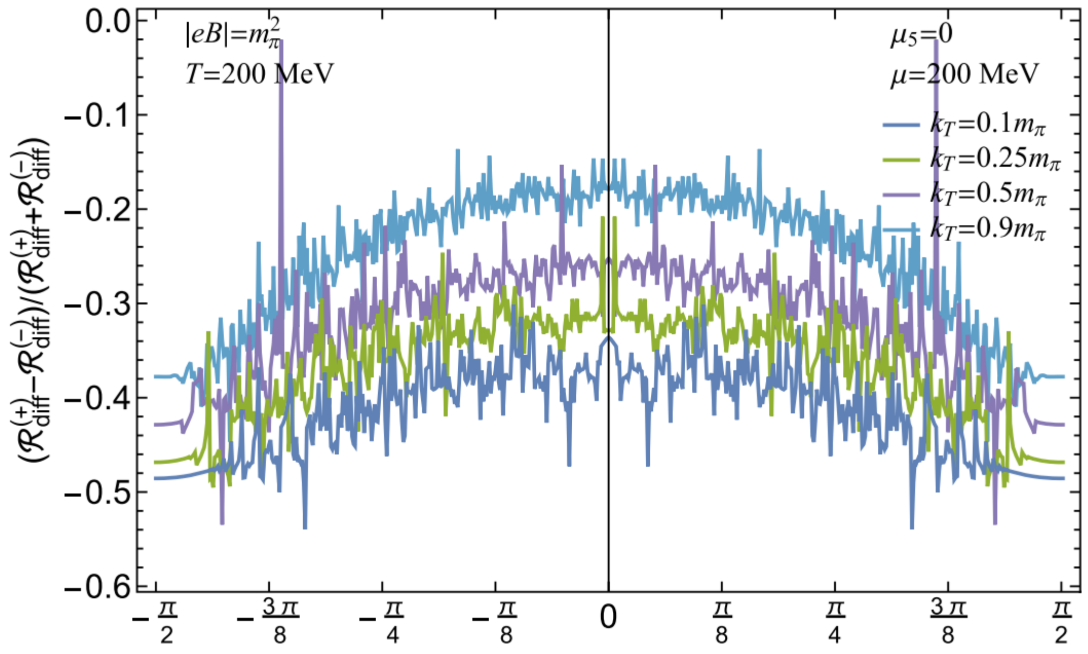
$$\mathcal{R}_{\text{diff}}^{(+)} \equiv k^0 \frac{d^3R}{dk_x dk_y dk_z} = -\frac{1}{(2\pi)^3} \frac{\text{Im}\left[\Pi^{(+)}(k)\right]}{\exp\left(\frac{k_0}{T}\right) - 1}$$

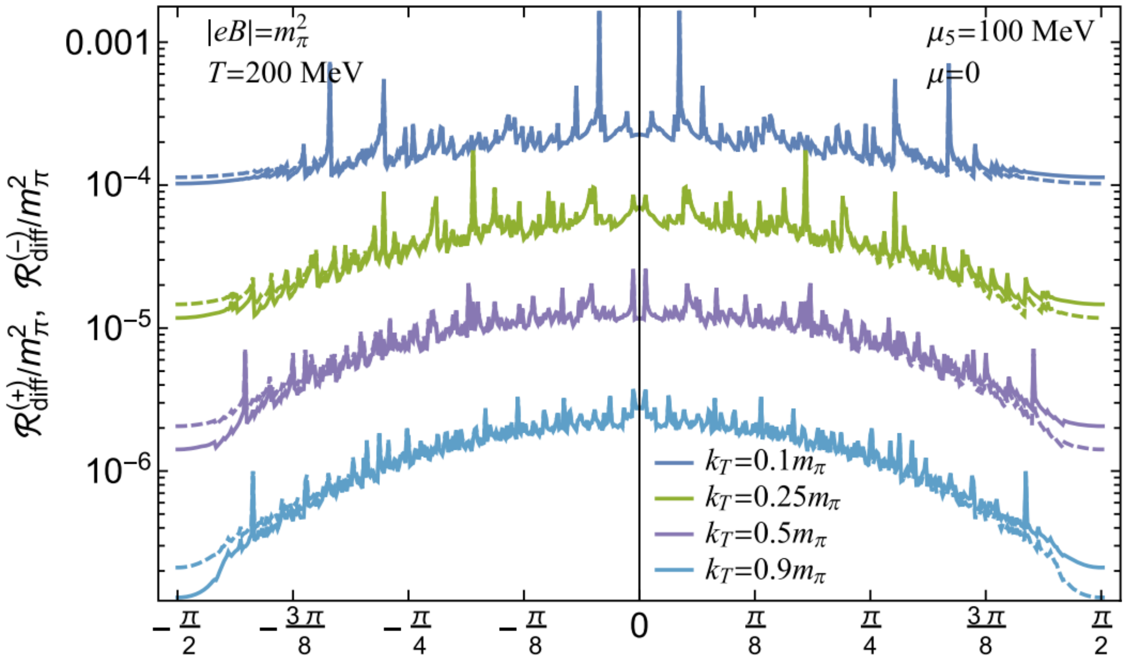
$$\mathcal{R}_{\text{diff}}^{(-)} \equiv k^0 \frac{d^3R}{dk_x dk_y dk_z} = -\frac{1}{(2\pi)^3} \frac{\text{Im}\left[\Pi^{(-)}(k)\right]}{\exp\left(\frac{k_0}{T}\right) - 1}$$

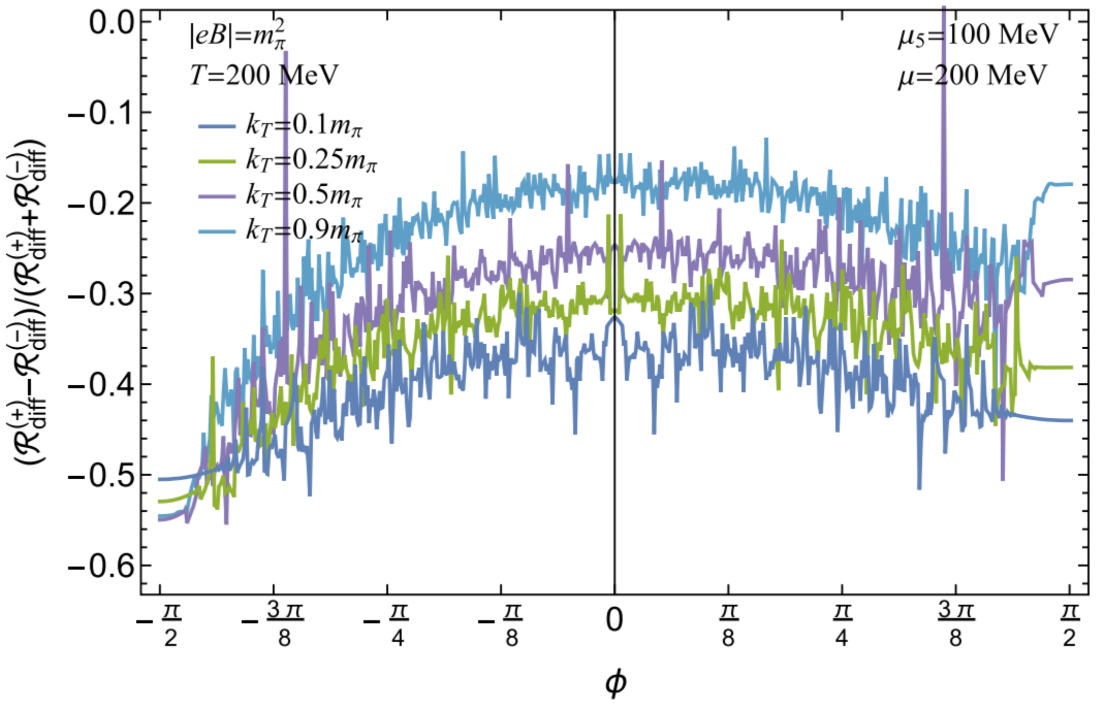
$$\mathcal{R}_{\text{diff}}^{(\pm)}(k_T, \phi, y) = \frac{d^3R^{(\pm)}}{k_T dk_T d\phi dy}$$

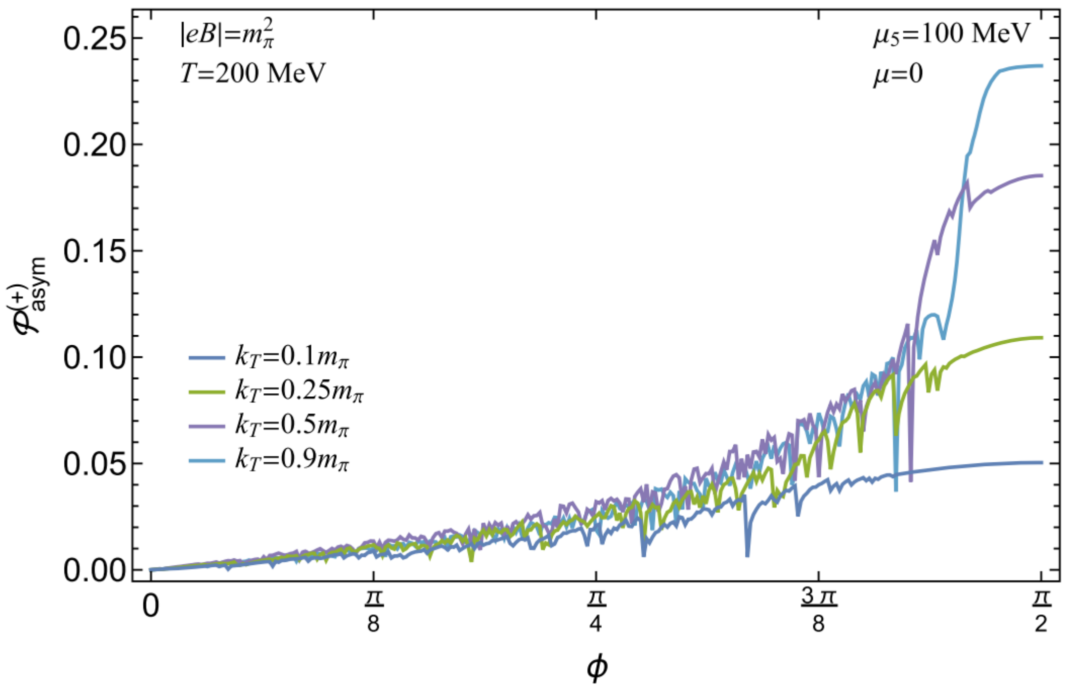
[X.W., Igor Shovkovy, PRD, 2024]

Decomposition rate

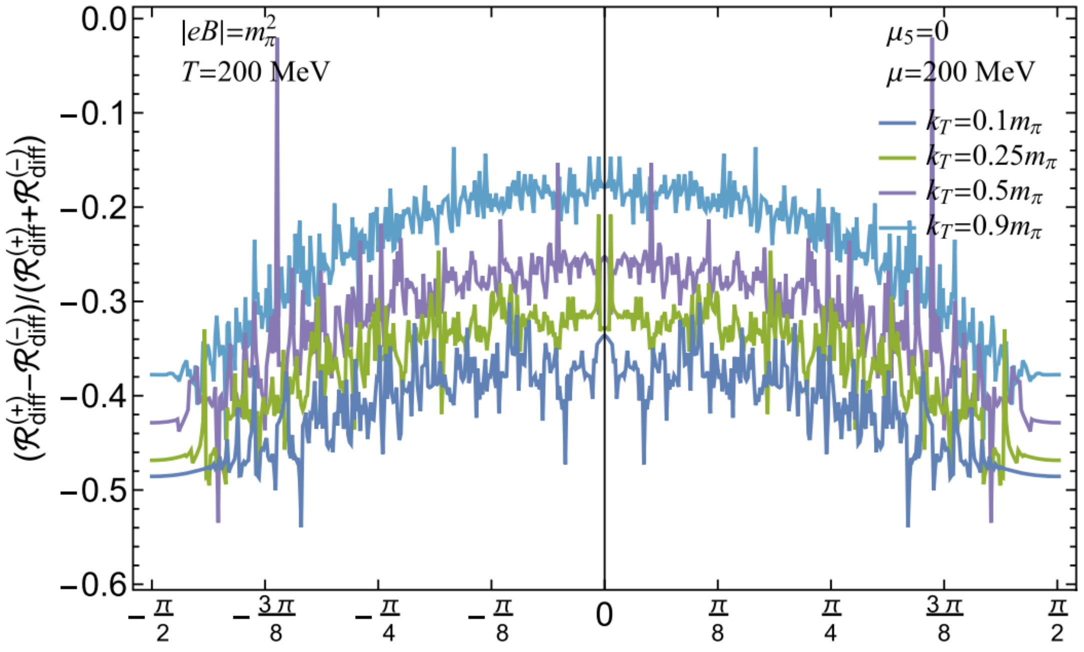






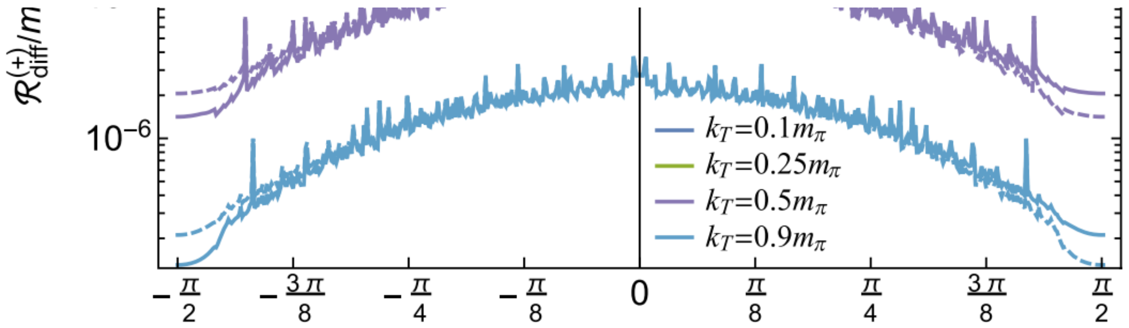


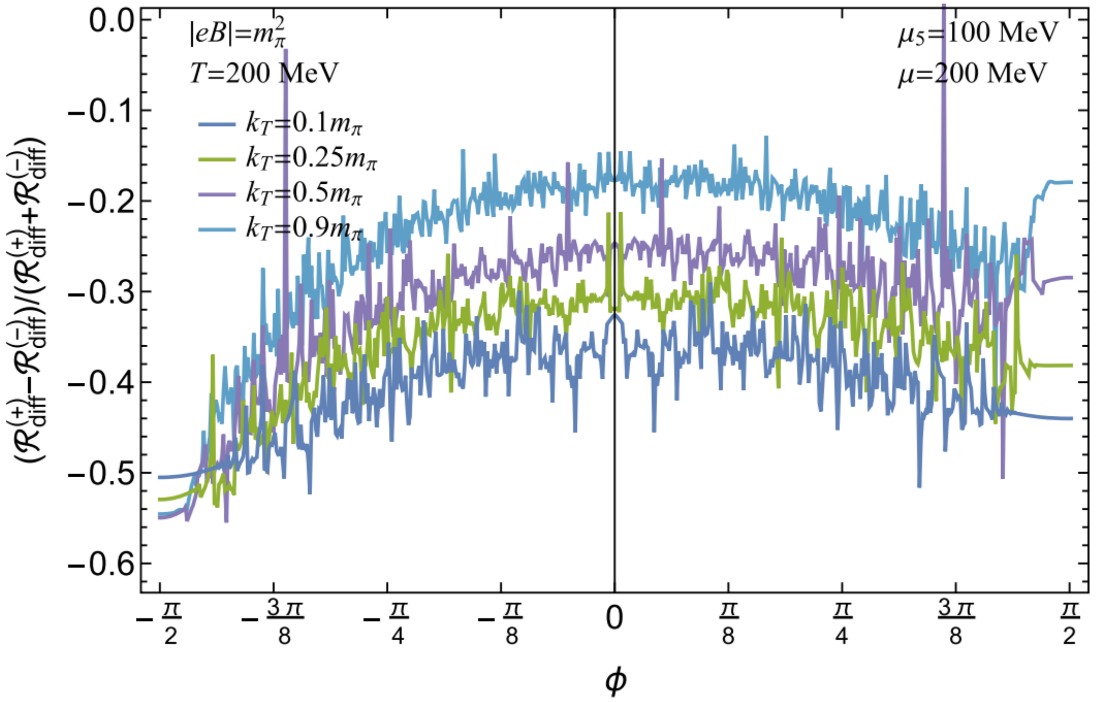
Decomposition rate

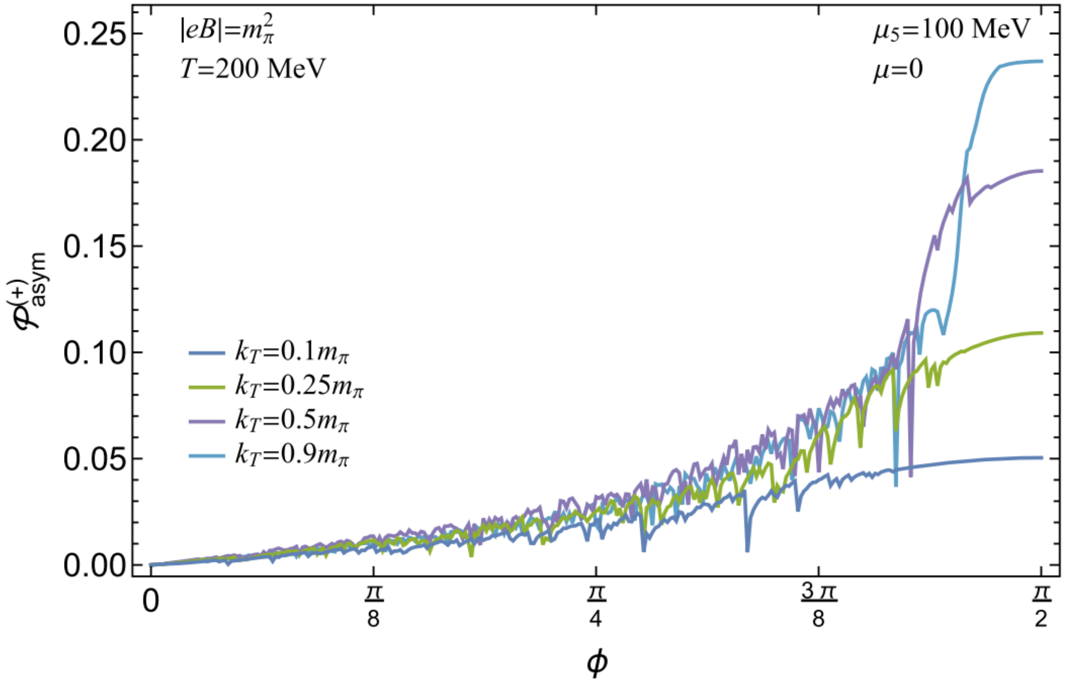


$$\mathcal{P}_{\rm asym}^{(+)}(k_T,\phi,y) = \frac{\mathcal{R}_{\rm diff}^{(+)}(k_T,\phi,y) - \mathcal{R}_{\rm diff}^{(+)}(k_T,-\phi,y)}{\mathcal{R}_{\rm diff}^{(+)}(k_T,\phi,y) + \mathcal{R}_{\rm diff}^{(+)}(k_T,-\phi,y)}$$

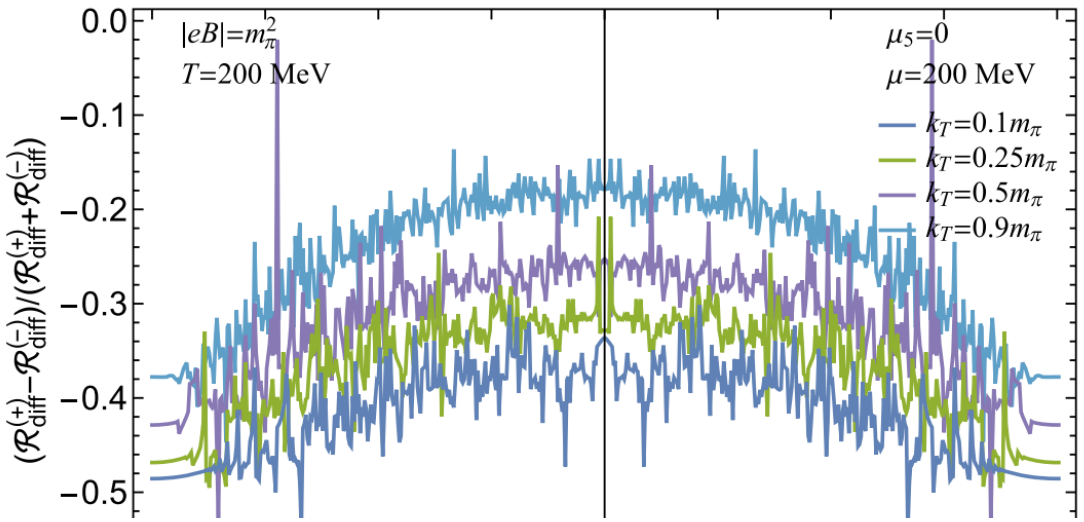
$$\mathcal{P}_{\rm asym}^{(-)}(k_T,\phi,y) = \frac{\mathcal{R}_{\rm diff}^{(-)}(k_T,\phi,y) - \mathcal{R}_{\rm diff}^{(-)}(k_T,-\phi,y)}{\mathcal{R}_{\rm diff}^{(-)}(k_T,\phi,y) + \mathcal{R}_{\rm diff}^{(-)}(k_T,-\phi,y)}$$





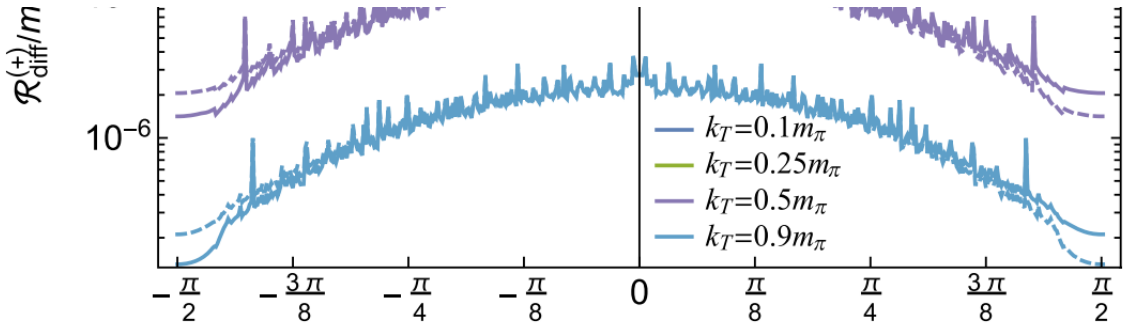


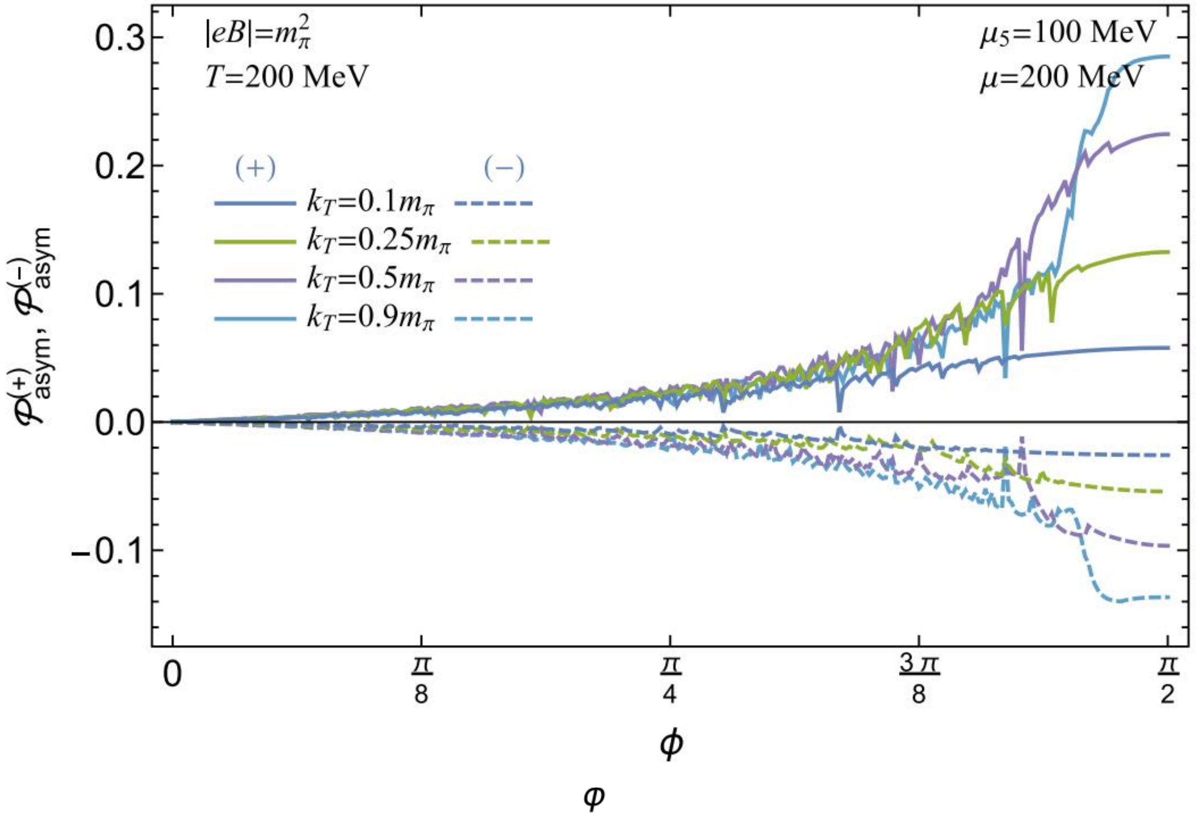
Decomposition rate

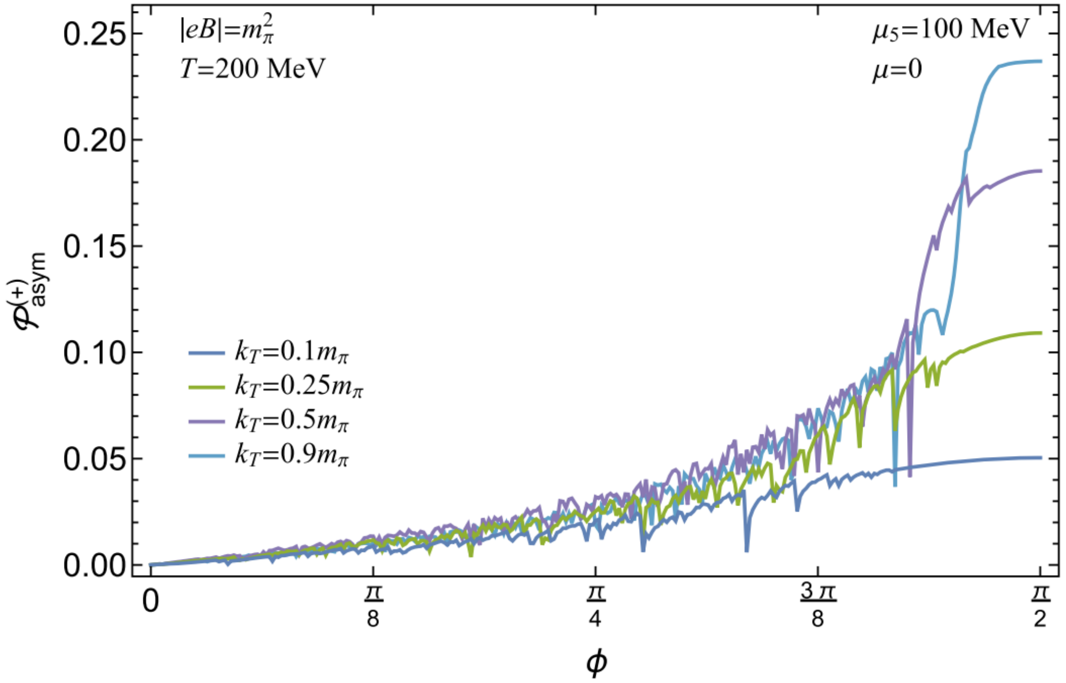


$$\mathcal{P}^{(+)}_{\mathrm{asym}}(k_T,\phi,y)=\frac{\mathcal{R}^{(+)}_{\mathrm{diff}}(k_T,\phi,y)-\mathcal{R}^{(+)}_{\mathrm{diff}}(k_T,-\phi,y)}{\mathcal{R}^{(+)}_{\mathrm{diff}}(k_T,\phi,y)+\mathcal{R}^{(+)}_{\mathrm{diff}}(k_T,-\phi,y)}$$

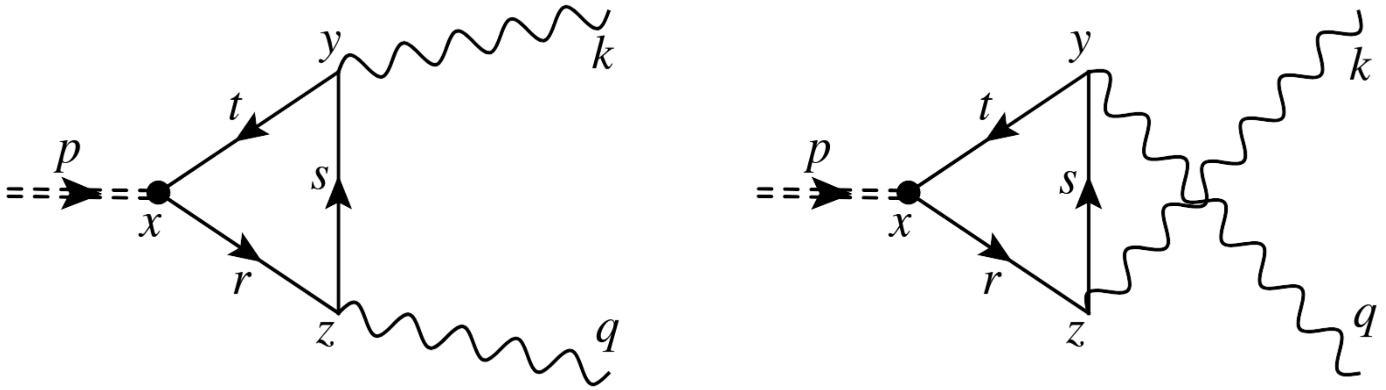
$$\mathcal{P}^{(-)}_{\mathrm{asym}}(k_T,\phi,y)=\frac{\mathcal{R}^{(-)}_{\mathrm{diff}}(k_T,\phi,y)-\mathcal{R}^{(-)}_{\mathrm{diff}}(k_T,-\phi,y)}{\mathcal{R}^{(-)}_{\mathrm{diff}}(k_T,\phi,y)+\mathcal{R}^{(-)}_{\mathrm{diff}}(k_T,-\phi,y)}$$







Pion decay in a weak magnetic hadron gas



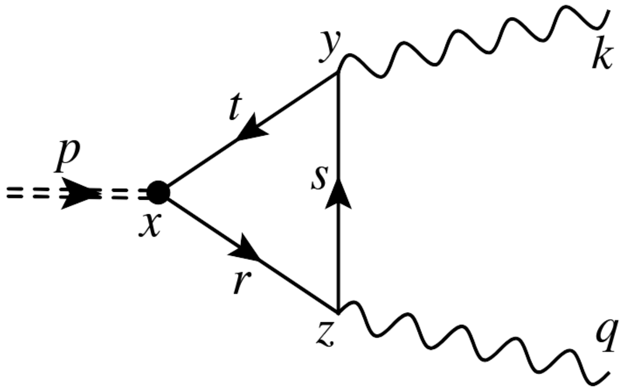
$$\Gamma^{\mathrm{CS}}(B) \approx \Gamma(0)\left(1 - \frac{2|eB|^2}{5m_P^4} - \frac{2|eB|^2 m_\pi^2}{189 m_P^6}\right) \qquad \Gamma(0) = \frac{\alpha_e^2 \lambda^2 m_\pi^3}{64\pi^3 m_P^2} = \frac{\alpha_e^2 m_\pi^3}{64\pi^3 f_\pi^2}$$

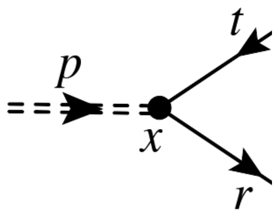
$$\Gamma_{\mathbf{p}}^{\mathrm{SP}}(B) \approx \Gamma_{\mathbf{p}}^{\mathrm{SP}}(0)\left(1 - \frac{2|eB|^2}{5m_P^4} - \frac{64 m_\pi |eB|^2}{63\pi^2 m_P^6}\mathcal{A}_{\mathbf{p}}\right) \qquad \Gamma_{\mathbf{p}}^{\mathrm{SP}}(0) = \frac{\alpha_e^2 \lambda^2 m_\pi^4}{64\pi^3 m_P^2 \sqrt{m_\pi^2 + \mathbf{p}^2}} = \frac{\alpha_e^2 m_\pi^4}{64\pi^3 f_\pi^2 \sqrt{m_\pi^2 + \mathbf{p}^2}}$$

$$\mathcal{A}_{\mathbf{p}} = \frac{1}{2m_\pi}\int \frac{d^3Q}{(2\pi)^3}\frac{1}{2q_0}\int \frac{d^3K}{(2\pi)^3}\frac{1}{2k_0}(\mathbf{q}_\perp^2 + \mathbf{q}_\perp\cdot\mathbf{k}_\perp + \mathbf{k}_\perp^2)(2\pi)^4\delta^{(4)}(p-k-q)$$

[X.W., Fan Lin, Igor Shovkovy, PRD, 2026]

Pion decay in a weak magnetic hadron gas

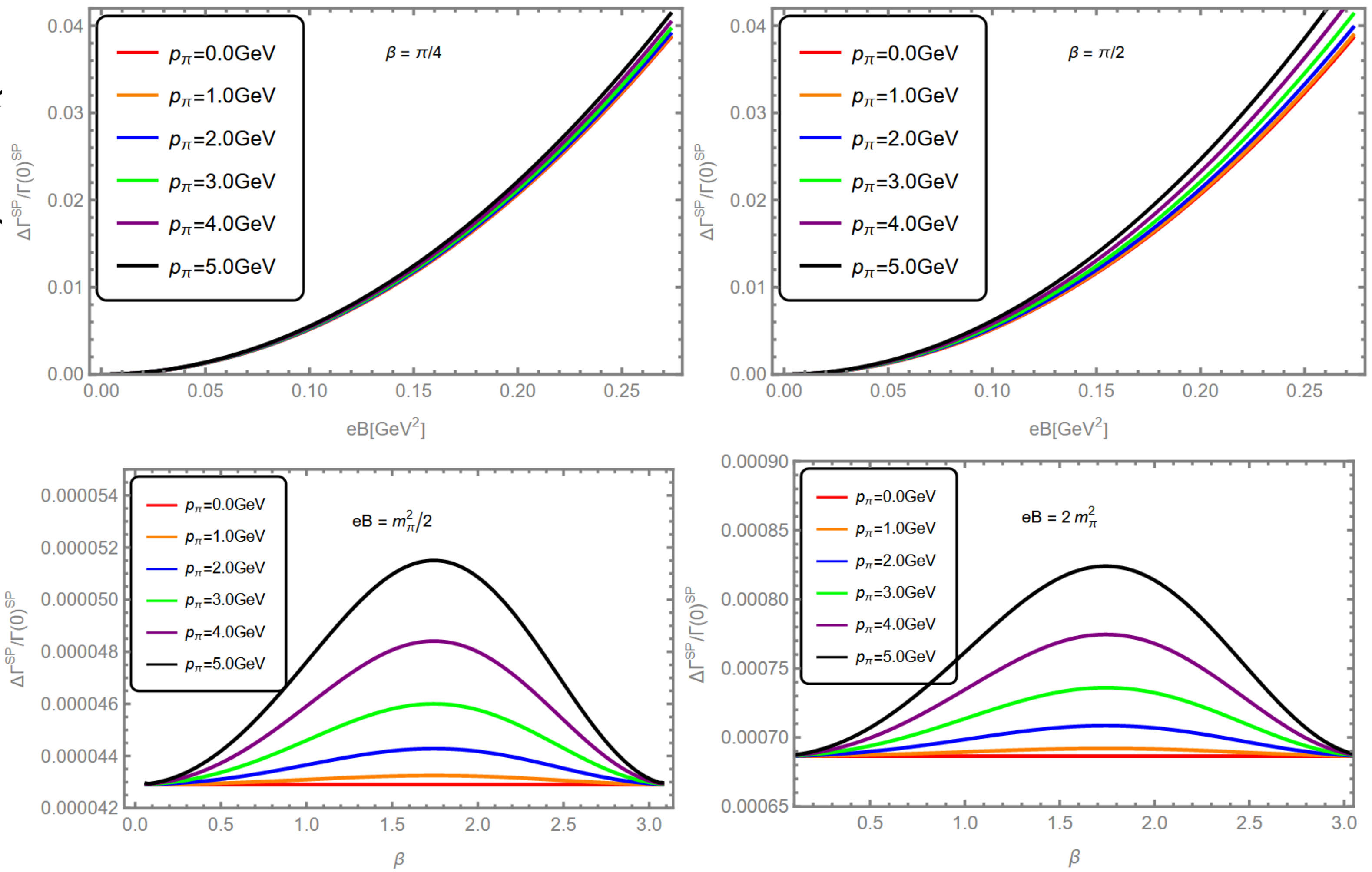




$$\Gamma^{\rm CS}(B) \approx \Gamma(0)\left(1 - \frac{2|eB|^2}{5m_P^4} - \frac{2|eB|^2 m_\pi^2}{189 m_P^6}\right)$$

$$\Gamma_{\mathbf{p}}^{\rm SP}(B) \approx \Gamma_{\mathbf{p}}^{\rm SP}(0)\left(1 - \frac{2|eB|^2}{5m_P^4} - \frac{64 m_\pi |eB|^2}{63\pi^2 m_P^6}\right.$$

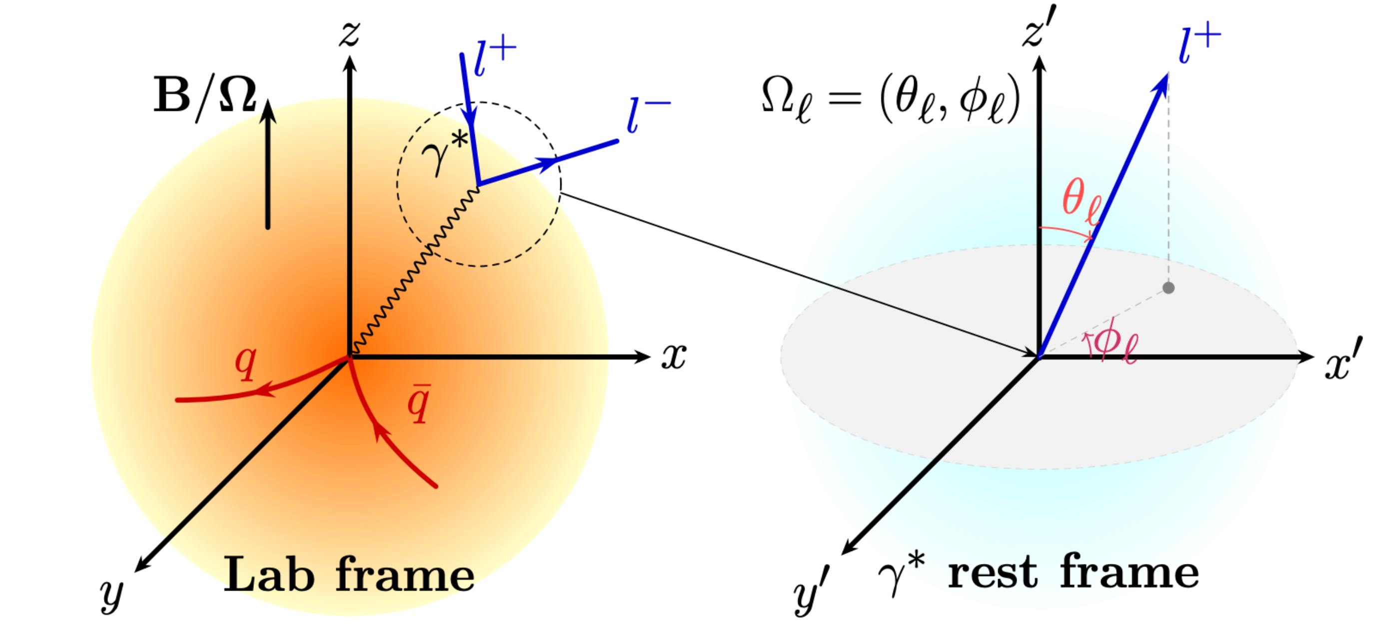
$$\mathcal{A}_{\mathbf{p}} = \frac{1}{2m_\pi}\int \frac{d^3Q}{(2\pi)^3}\frac{1}{2q_0}\int \frac{d^3K}{(2\pi)^3}\frac{1}{2k_0}(\mathbf{q}_\perp^2 + \mathbf{q}_{\text{-}}$$



[X.W., Fan Lin, Igor Shovkovy, PRD, 2026]

Thermal Dilepton Polarization

[Haowen He, Bowen Hou, X.W., Minghua Wei, arXiv:2607.11634]



$$\frac{dR}{d^4q d\Omega_\ell} = \mathcal{N}\,\big(1 + \lambda_\theta \cos^2\theta_\ell + \lambda_\phi \sin^2\theta_\ell \cos 2\phi_\ell + \lambda_{\theta\phi} \sin 2\theta_\ell \cos\phi_\ell + \lambda_\phi^\perp \sin^2\theta_\ell \sin 2\phi_\ell + \lambda_{\theta\phi}^\perp \sin 2\theta_\ell \sin\phi_\ell\big)$$

$$\sigma_\parallel(\omega) = \sum_{f=u,d} \frac{4\alpha N_c q_f^2}{\omega^2 \ell_f^2} \tanh\left(\frac{|\omega|}{4T}\right) \sum_{n=0}^{n_\parallel^{\max}} (2-\delta_{n,0}) \frac{M_{n,f}^2 \theta\left(\omega^2 - 4M_{n,f}^2\right)}{\sqrt{\omega^2 - 4M_{n,f}^2}}$$

$$\lambda_\theta = \frac{\rho^{\gamma^*}_{-1,-1} + \rho^{\gamma^*}_{1,1} - 2\rho^{\gamma^*}_{0,0}}{\rho^{\gamma^*}_{-1,-1} + \rho^{\gamma^*}_{1,1} + 2\rho^{\gamma^*}_{0,0}}$$

$$\lambda_{\theta\phi} = \sqrt{2}\,\frac{\mathrm{Re}\left(\rho^{\gamma^*}_{0,1} - \rho^{\gamma^*}_{-1,0}\right)}{\rho^{\gamma^*}_{-1,-1} + \rho^{\gamma^*}_{1,1} + 2\rho^{\gamma^*}_{0,0}}$$

$$\lambda_\phi = 2\,\frac{\mathrm{Re}\left(\rho^{\gamma^*}_{-1,1}\right)}{\rho^{\gamma^*}_{-1,-1} + \rho^{\gamma^*}_{1,1} + 2\rho^{\gamma^*}_{0,0}}$$

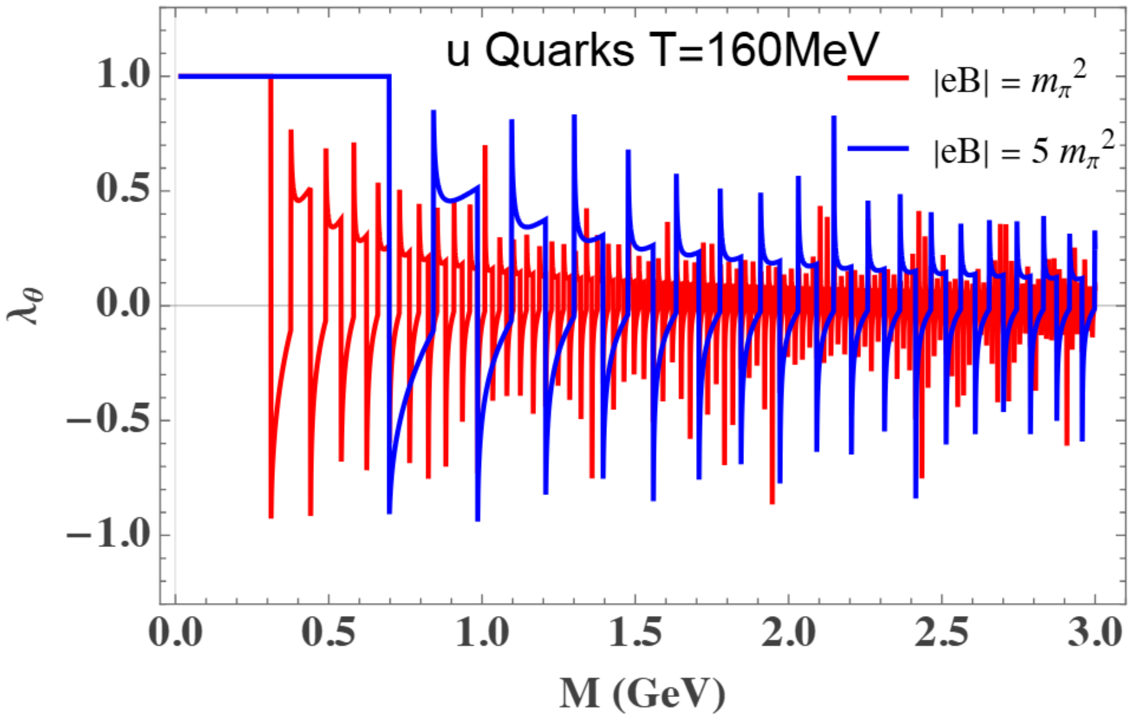
$$\frac{dR_{l^+l^-}}{d\cos\theta_\ell} \propto \sigma_\perp (1 + \cos^2\theta_\ell) + \sigma_\parallel (1 - \cos^2\theta_\ell)$$

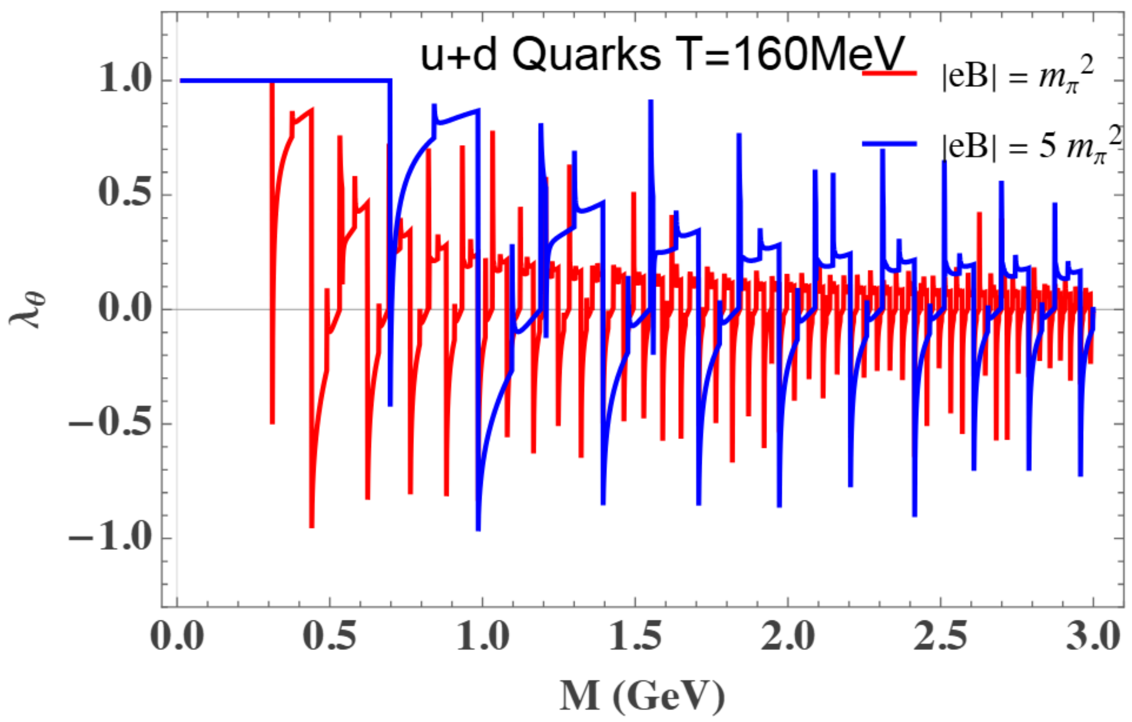
[E. Speranza, Ph.D thesis]

$$\lambda_\theta = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel}$$

$$\begin{aligned}\sigma_\perp(\omega) = &\sum_{f=u,d} \frac{2\alpha N_c q_f^2 \sinh\left(\frac{\omega}{2T}\right)}{\omega \ell_f^4 \left[\cosh\left(\frac{\omega}{2T}\right) + \cosh\left(\frac{|q_f B|}{T\omega}\right)\right]} \\ &\times \sum_{n=1}^{n_\perp^{\max}} \frac{\left[2(2n-1) - \omega^2 \ell_f^2\right] \theta\left[(M_{n,f} - M_{n-1,f})^2 - \omega^2\right]}{\sqrt{\left[(M_{n,f} - M_{n-1,f})^2 - \omega^2\right]\left[(M_{n,f} + M_{n-1,f})^2 - \omega^2\right]}} \\ &+ \sum_{f=u,d} \frac{2\alpha N_c q_f^2 \sinh\left(\frac{\omega}{2T}\right)}{\omega \ell_f^4 \left[\cosh\left(\frac{\omega}{2T}\right) + \cosh\left(\frac{|q_f B|}{T\omega}\right)\right]} \\ &\times \sum_{n=1}^{n_\perp^{\max}} \frac{\left[\omega^2 \ell_f^2 - 2(2n-1)\right] \theta\left[\omega^2 - (M_{n,f} + M_{n-1,f})^2\right]}{\sqrt{\left[\omega^2 - (M_{n,f} - M_{n-1,f})^2\right]\left[\omega^2 - (M_{n,f} + M_{n-1,f})^2\right]}}\end{aligned}$$

Thermal Dilepton Polarization

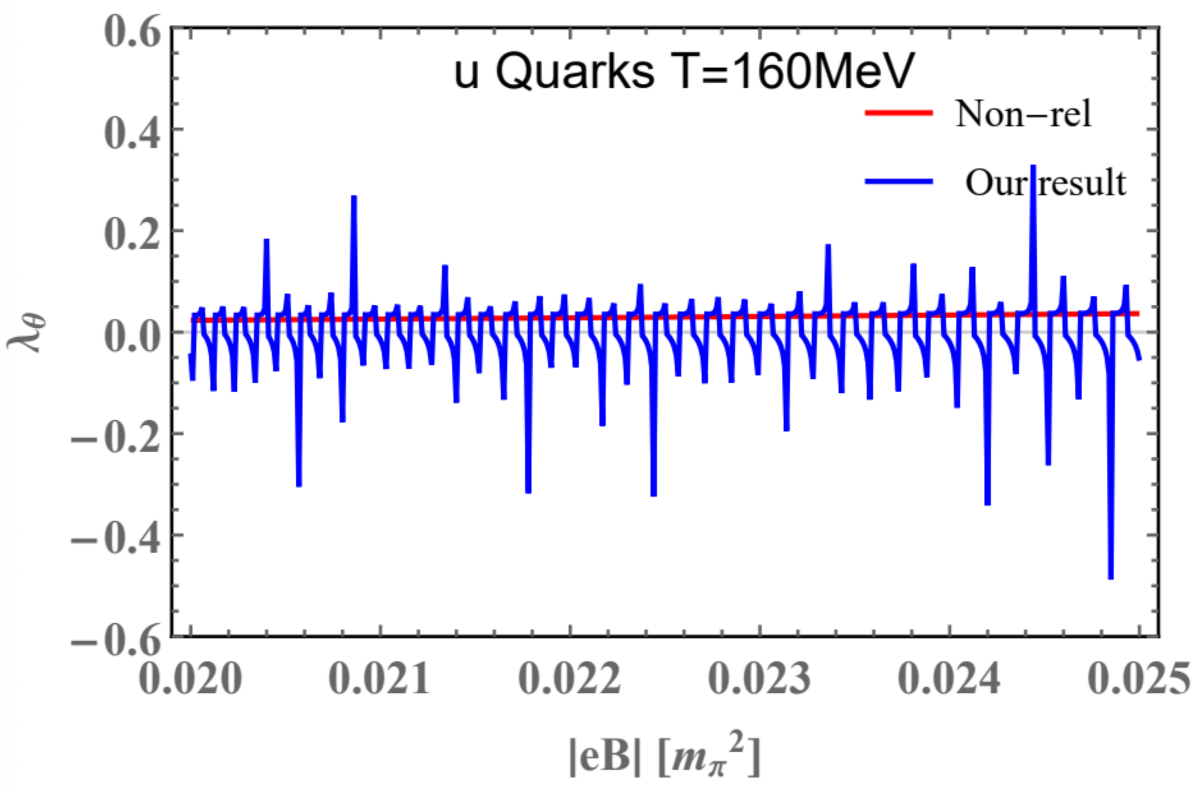


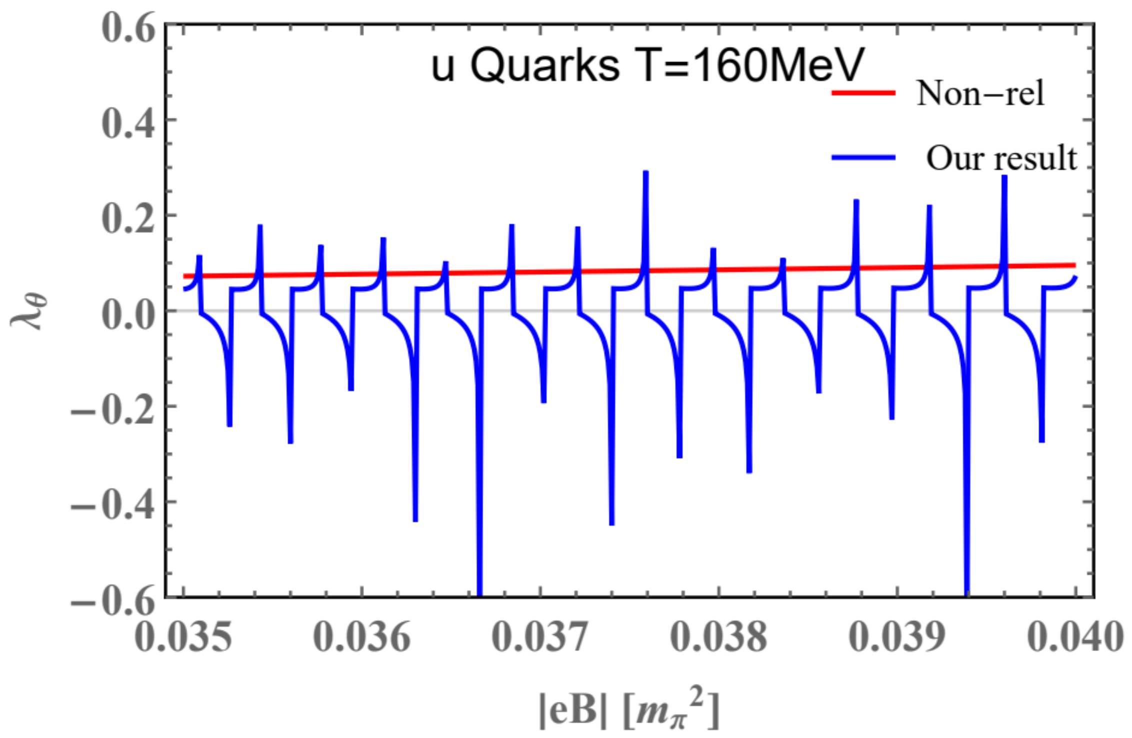


Thermal Dilepton Polarization

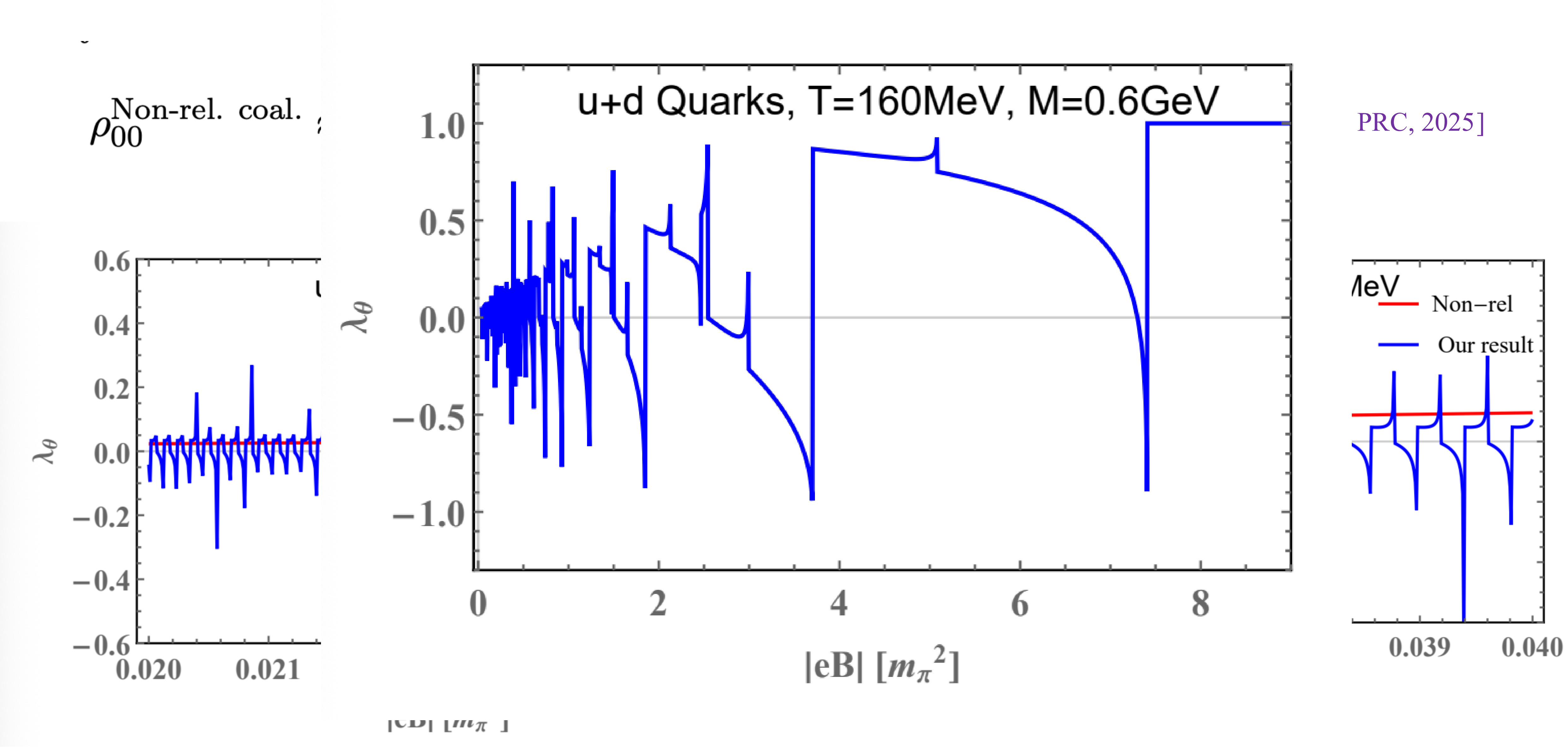
$$\rho_{00}^{\text{Non-rel. coal.}} \approx \frac{1}{3} - \frac{1}{9T^2}\frac{q_f q_{f'}}{M_f M_{f'}} B^2$$

[Y. G. Yang, R.H. Fang, Q. Wang and X.N. Wang, PRC, 2025]





Thermal Dilepton Polarization



Summary

Based on the difficulties of the measurement magnetic field in HIC through CME effect, we think the following indirect measurement may help:

- Thermal Photon & Dilepton rate (big enhancement in small transverse momentum/invariant mass regime)
- Thermal Photon & Dilepton anisotropic behavior
- Polarization decomposition of Photon
- Thermal dilepton(Virtual photon) polarization

Thank you!