xQCD2026

14-17 July 2026 PKNU Busan Korea

Extracting the QCD speed of sound in the presence of quantum fluctuations

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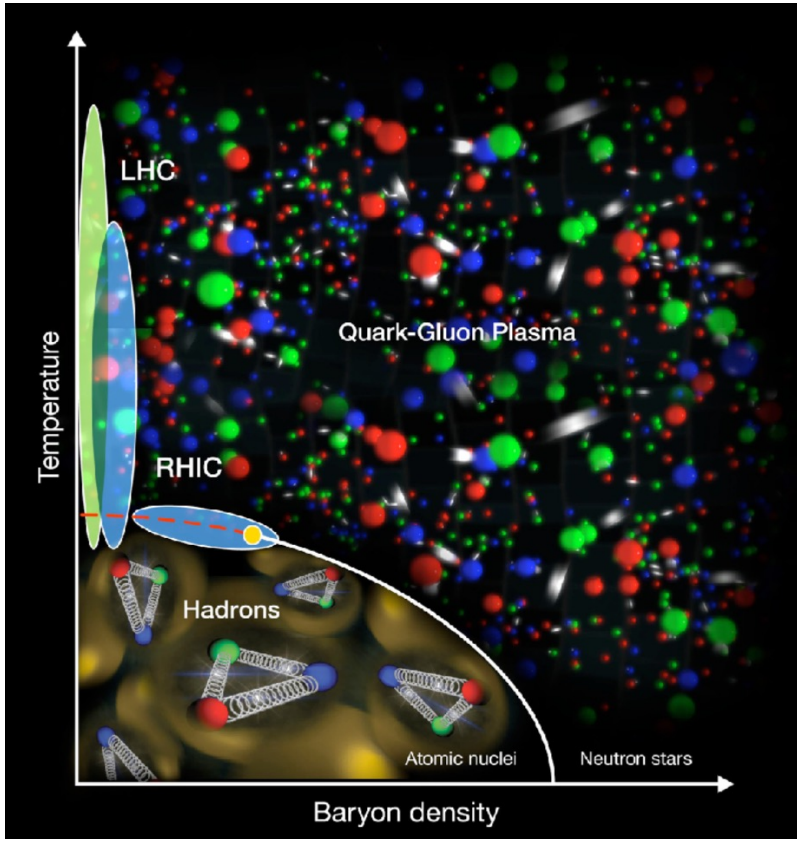
Based on *Phys.Rev.Lett.* 135 (2025) 16, 162301 (arXiv:2501.02777),

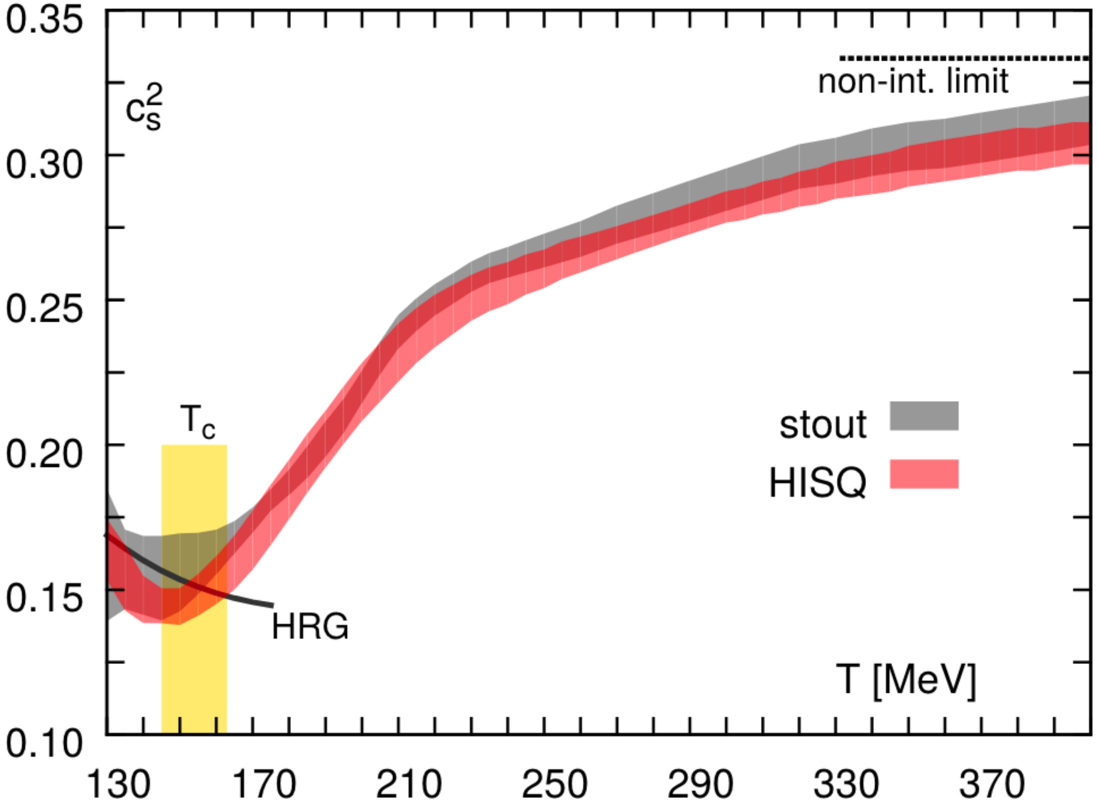
In collaboration with Li Yan and Xu-Guang Huang

July 14 2026, Busan Korea

Relativistic heavy-ion collisions



μ_B=0



A direct measurement of QCD equation of state is challenging in heavy-ion study.

The QCD speed of sound in heavy-ion collisions

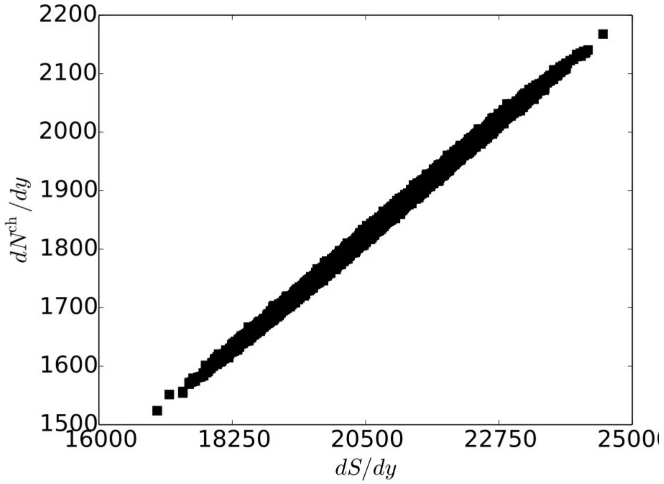
$$c_s^2 \equiv \frac{dP}{de} \xrightarrow{dP = sdT,\ de = Tds} = \frac{d\ln T}{d\ln s} \xrightarrow[s \propto S]{\text{constant volume}} = \frac{d\ln T}{d\ln S} \xrightarrow[d\ln S = d\ln N_{ch},\ d\ln T = d\ln\langle p_T\rangle]{S \propto N_{ch},\ T \propto \langle p_T\rangle} = \frac{d\ln\langle p_T\rangle}{d\ln N_{ch}}$$

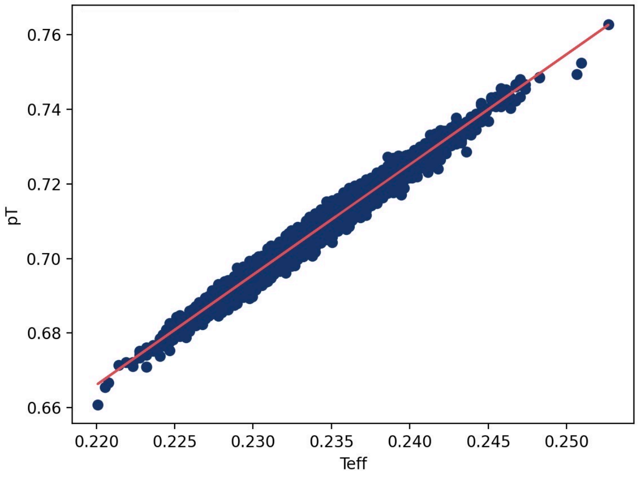

nature physics
LETTERS
https://doi.org/10.1038/s41567-020-0846-4
Check for updates

Thermodynamics of hot strong-interaction matter from ultrarelativistic nuclear collisions

Fernando G. Gardim[1,2], Giuliano Giacalone[2], Matthew Luzum[3] and Jean-Yves Ollitrault[2]

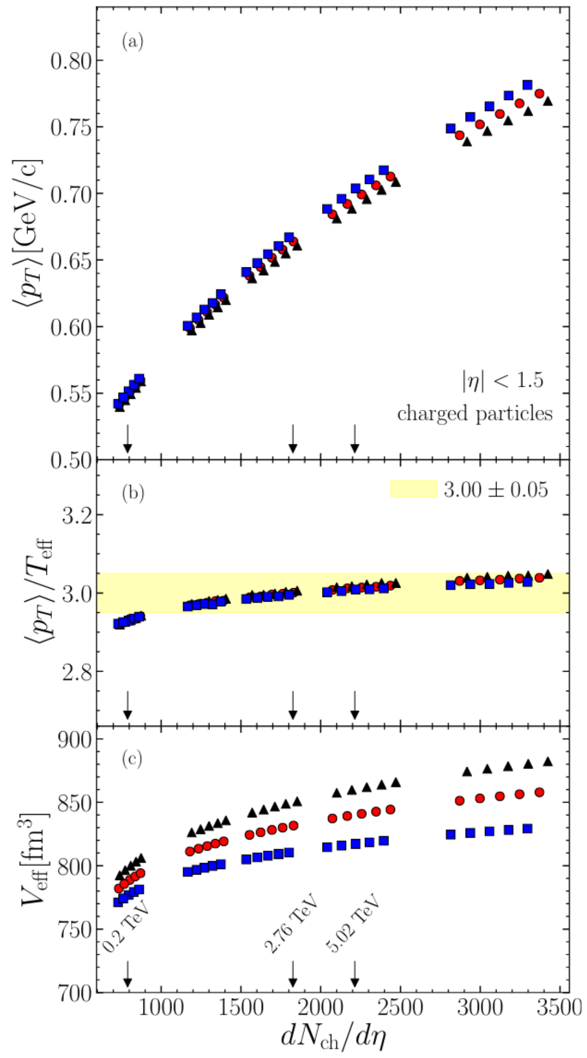
[F. Gardim et al , 1908.09728, Nature Physics]





[C. Shen, Comput.Phys.Commun. 199 (2016) 61-85]

Effective temperature and effective volume



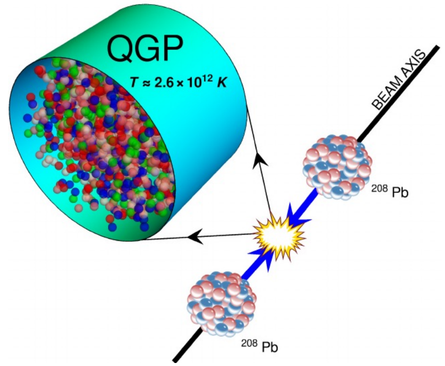
[F. Gardim et al , 2403.06052]

- A uniform fluid at rest with $T_{\rm eff}$ and $V_{\rm eff}$:

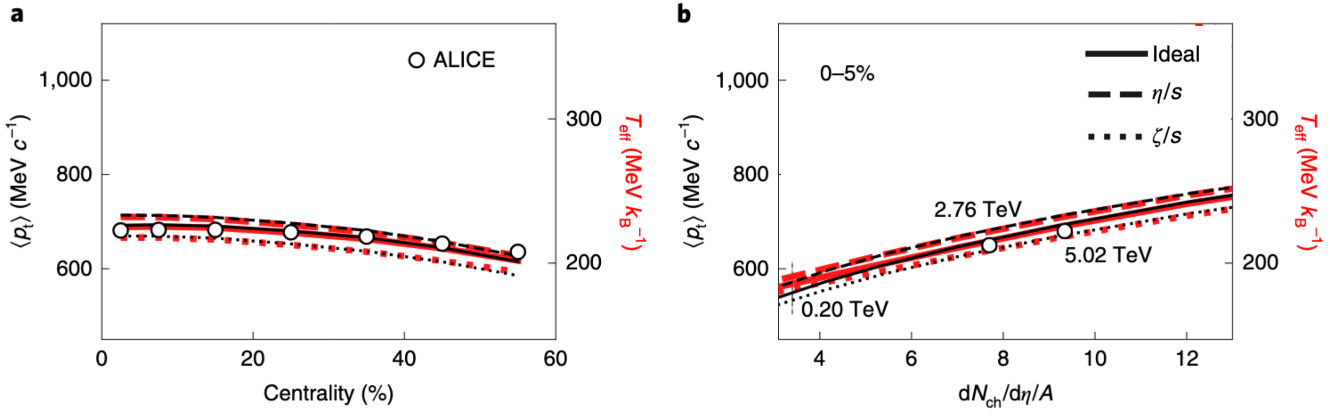
$$E = \boxed{\int_{\rm f.o.} T^{0\mu} d\sigma_\mu} = \boxed{\epsilon(T_{\rm eff}) V_{\rm eff}}$$

$$S = \boxed{\int_{\rm f.o.} s u^\mu d\sigma_\mu} = \boxed{s(T_{\rm eff}) V_{\rm eff}}$$

QGP uniform fluid

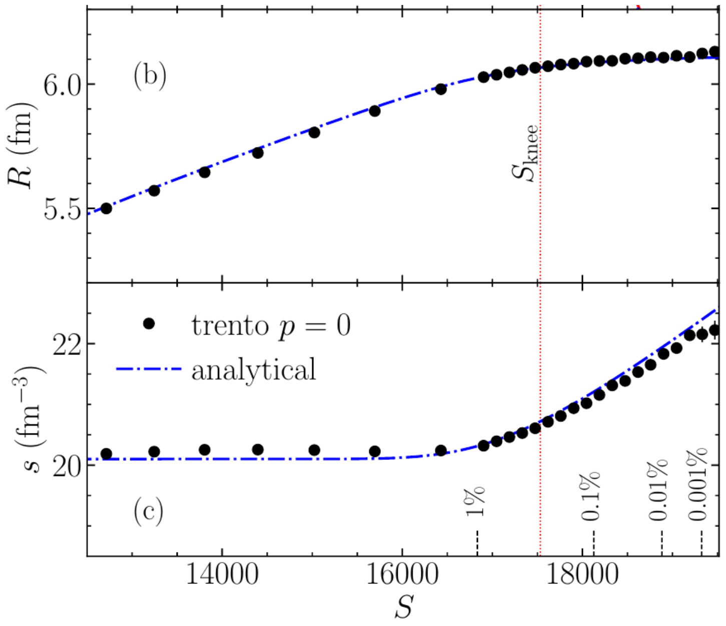


- In boost-invariant fluid, $\langle p_T \rangle \approx 3\, T_{\rm eff}$

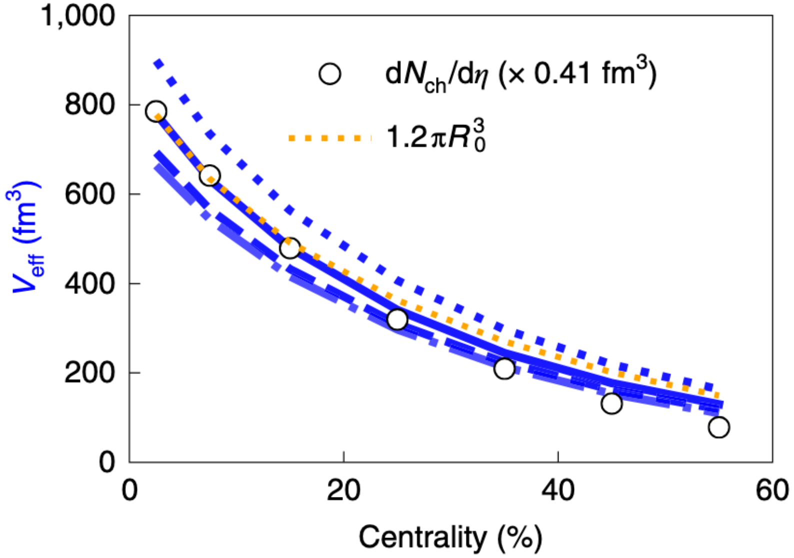


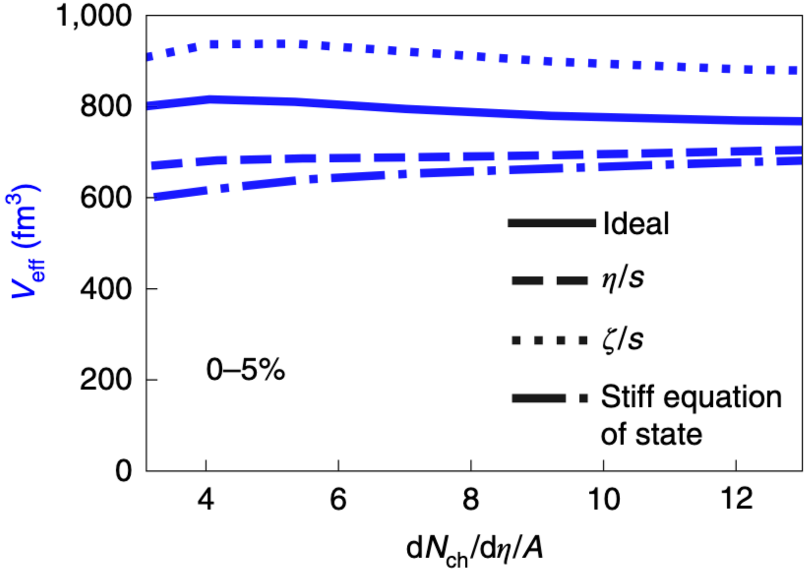
[F. Gardim et al , 1908.09728, Nature Physics]

Ultra-central nucleus-nucleus collisions (UCC)



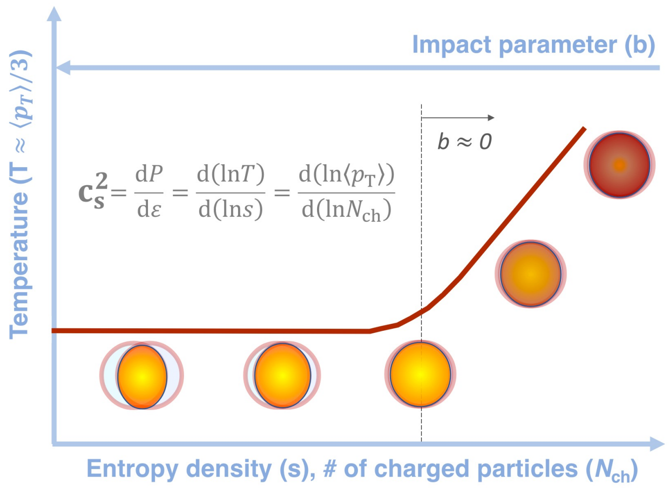
[F. Gardim et al , 1909.11609]





[F. Gardim et al , 1908.09728, Nature Physics]

- $S_{\rm knee}$ is the mean value of the entropy at b = 0.
- The volume is fixed in UCC events.

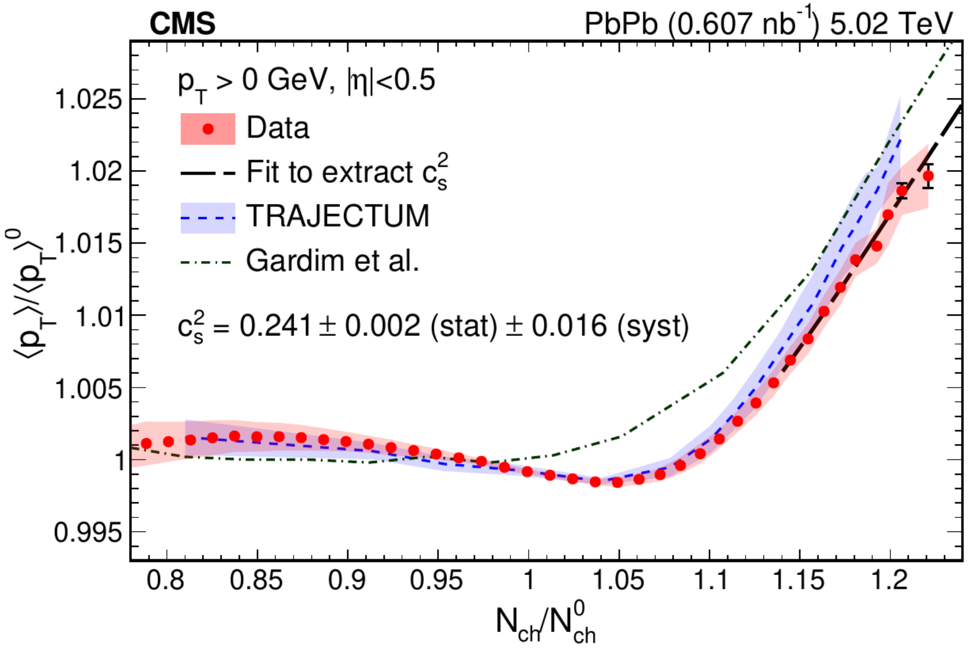


Extract the speed of sound in experiments

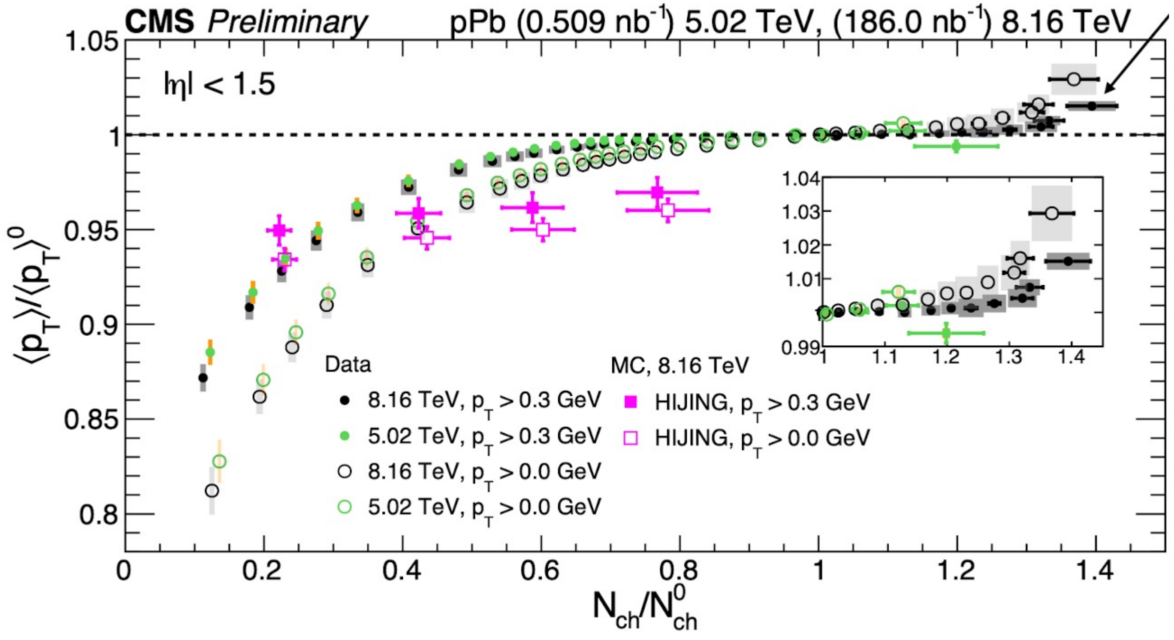
$$c_s^2 = \frac{d\ln\langle p_t\rangle}{d\ln N_{\rm ch}} \longrightarrow \frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2 \frac{\Delta_N}{N_0} \quad \text{with} \quad \begin{cases} \Delta_p \equiv \langle p_T\rangle - \langle p_T\rangle_0 \\ \Delta_N \equiv N_{ch} - N_0 \end{cases}$$

where $\langle p_T\rangle_0$ and N_0 are the averaged values over the ultra-central events

[CMS collaboration, 2401.06896]



[CMS collaboration, QM2025]



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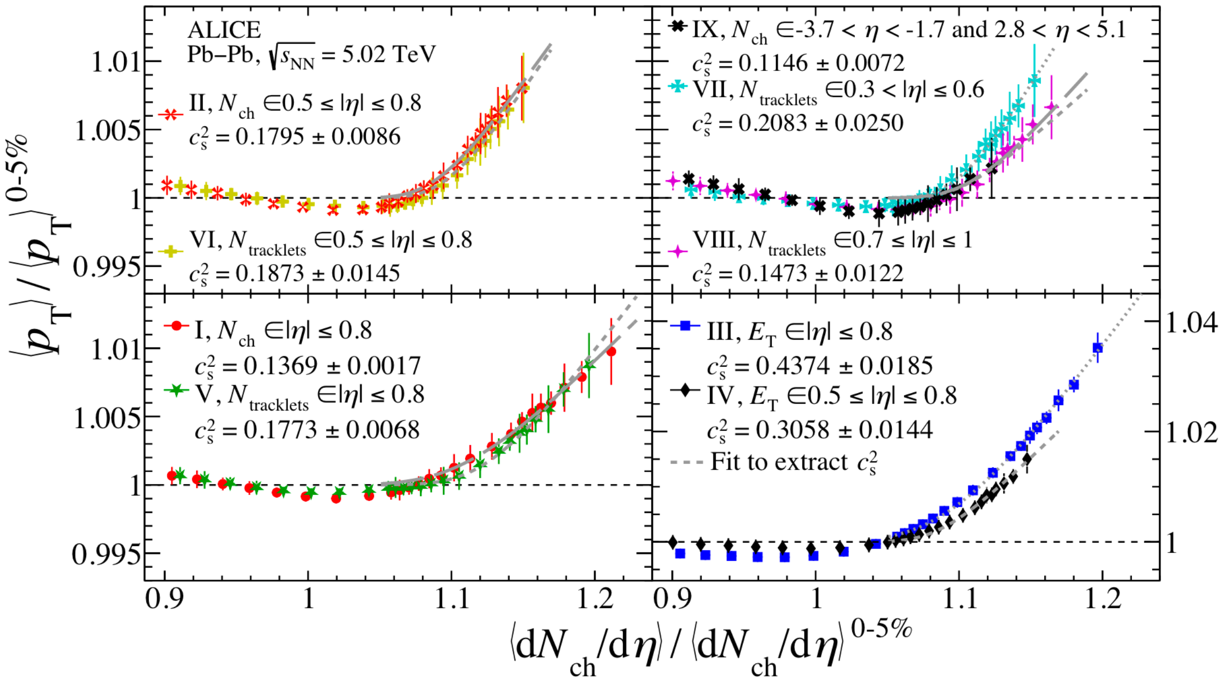
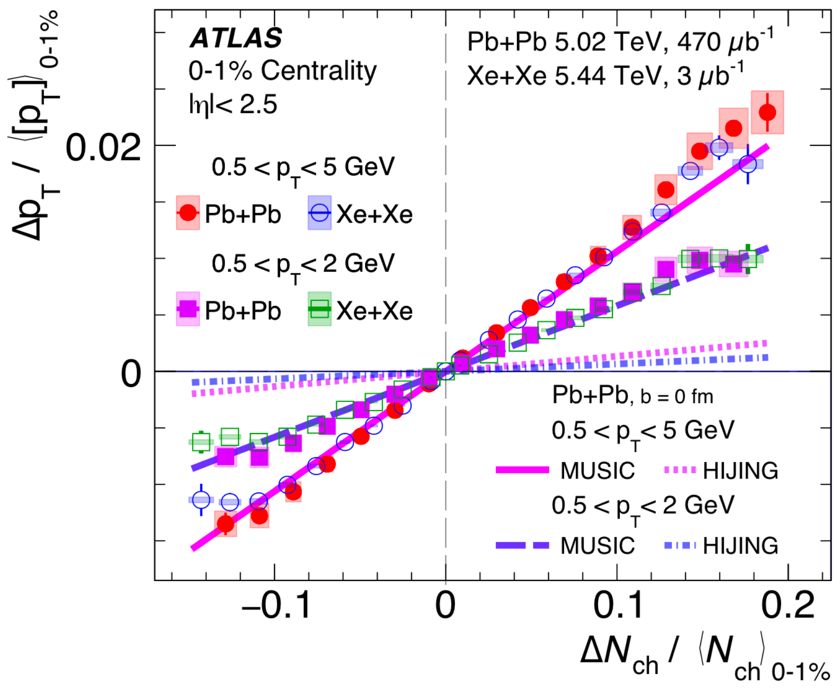
[ALICE collaboration, 2506.10394]



[ATLAS collaboration, 2407.068413]



Extract the speed of sound from [pt] fluctuations

$$\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N}{N_0}$$

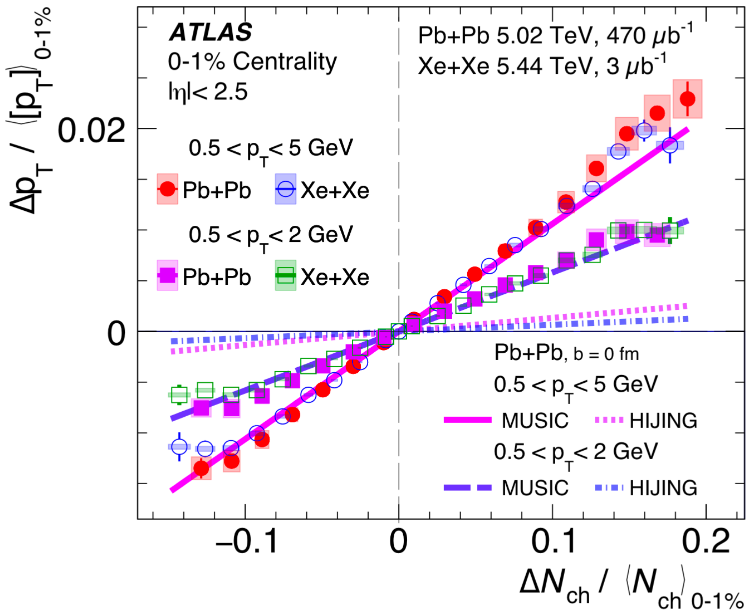
$$\frac{\delta N_A}{\langle N_A \rangle} = \frac{\delta N}{\langle N \rangle} + D_A \frac{\delta [p_T]}{\langle [p_T] \rangle}$$

$$\frac{\delta [p_{TA}]}{\langle [p_{TA}] \rangle} = C_A \frac{\delta [p_T]}{\langle [p_T] \rangle}$$

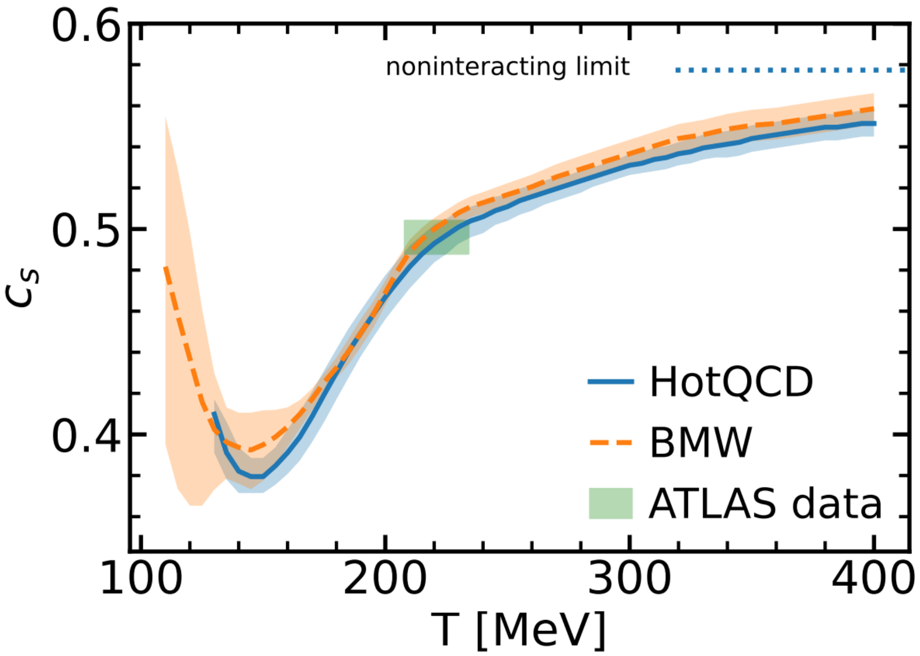
$$D_A \equiv \frac{\int_A \frac{v_0(p_T)}{v_0} \langle N(p_T) \rangle}{\int_A \langle N(p_T) \rangle}$$

$$C_A \equiv \frac{\int_A \frac{v_0(p_T)}{v_0} (p_T - \langle p_{TA} \rangle) \langle N(p_T) \rangle}{\int_A p_T \langle N(p_T) \rangle}$$

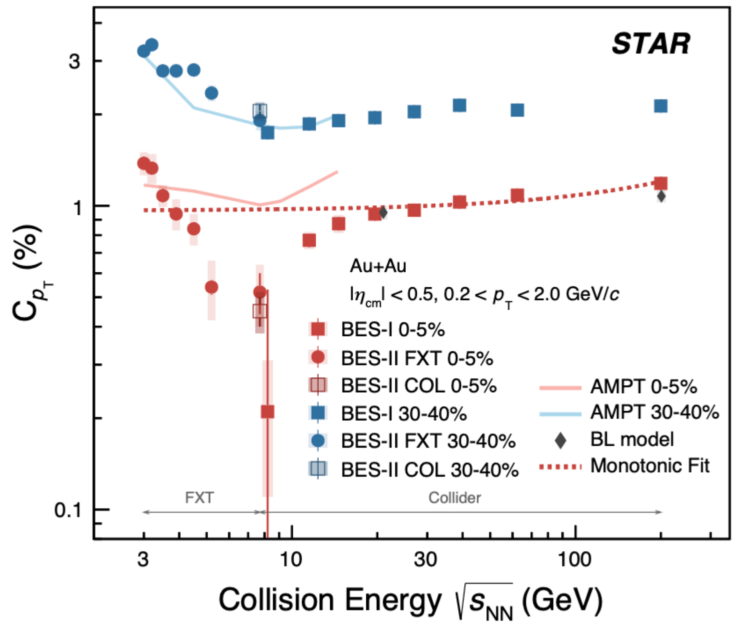
[ATLAS collaboration, 2407.068413]



[M.Alqahtani, 2603.09647]

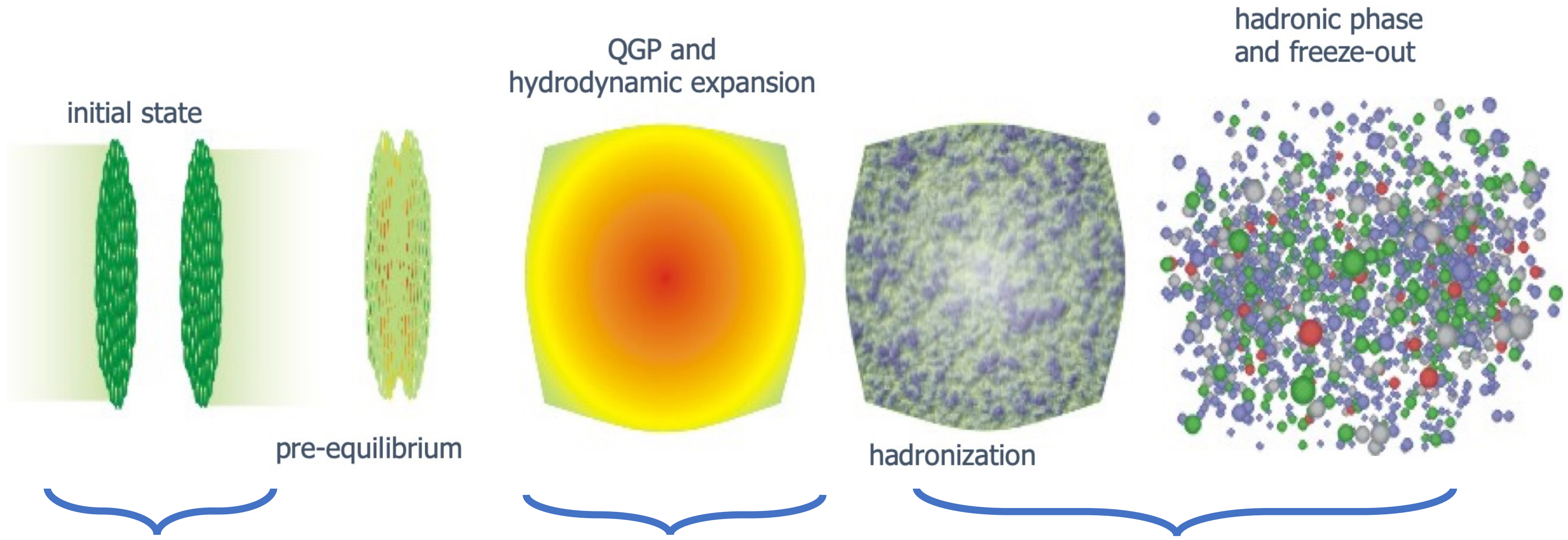


[STAR collaboration, 2604.06434]



Two sets of model studies

- Thermal model

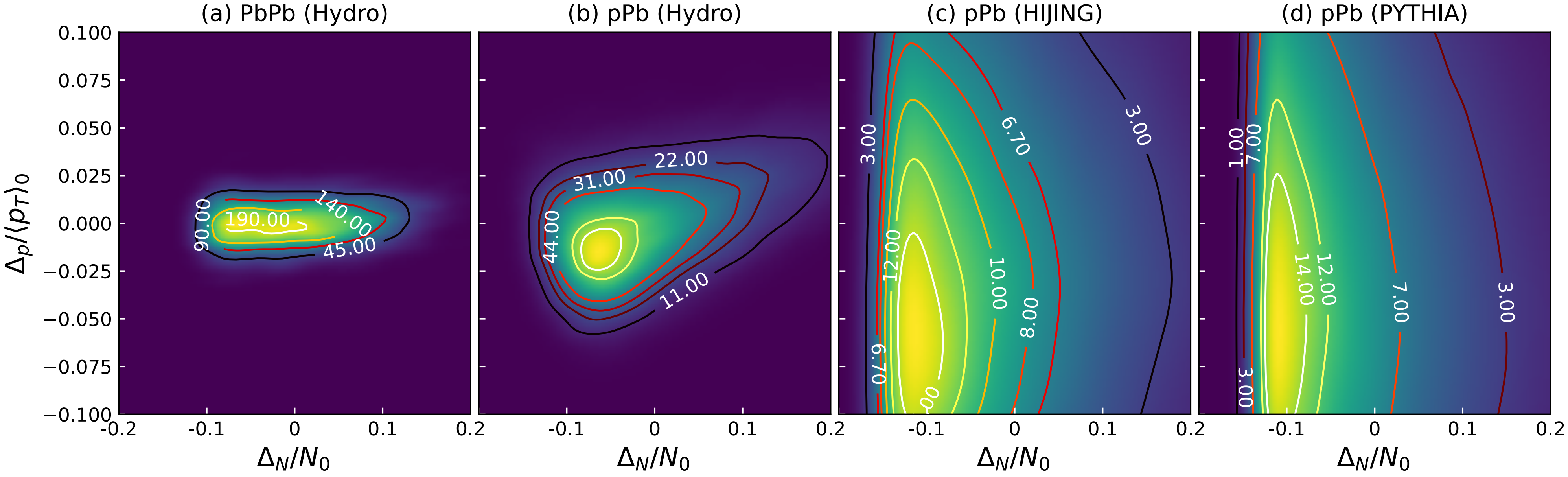


TRENTo + MUSIC + UrQMD

main source of
quantum fluctuations

- Non-thermal model   HIJING, Pythia

Quantum fluctuations in realistic collisions



hydro model

transport model

$$\frac{\{\Delta_p\}_I}{\langle p_T\rangle_0} = c_s^2 \frac{\{\Delta_N\}_I}{N_0} \xrightarrow{\text{quantum fluctuation term}} \frac{\{\Delta_p\}_I}{\langle p_T\rangle_0} = c_s^2 \frac{\{\Delta_N\}_I + \{\delta\}_I}{N_0}$$

The presence of quantum fluctuations leads to 2D distribution of Δ_N and Δ_p.

Extracting the speed of sound in the presence of quantum fluc.

- In a thermalized system,

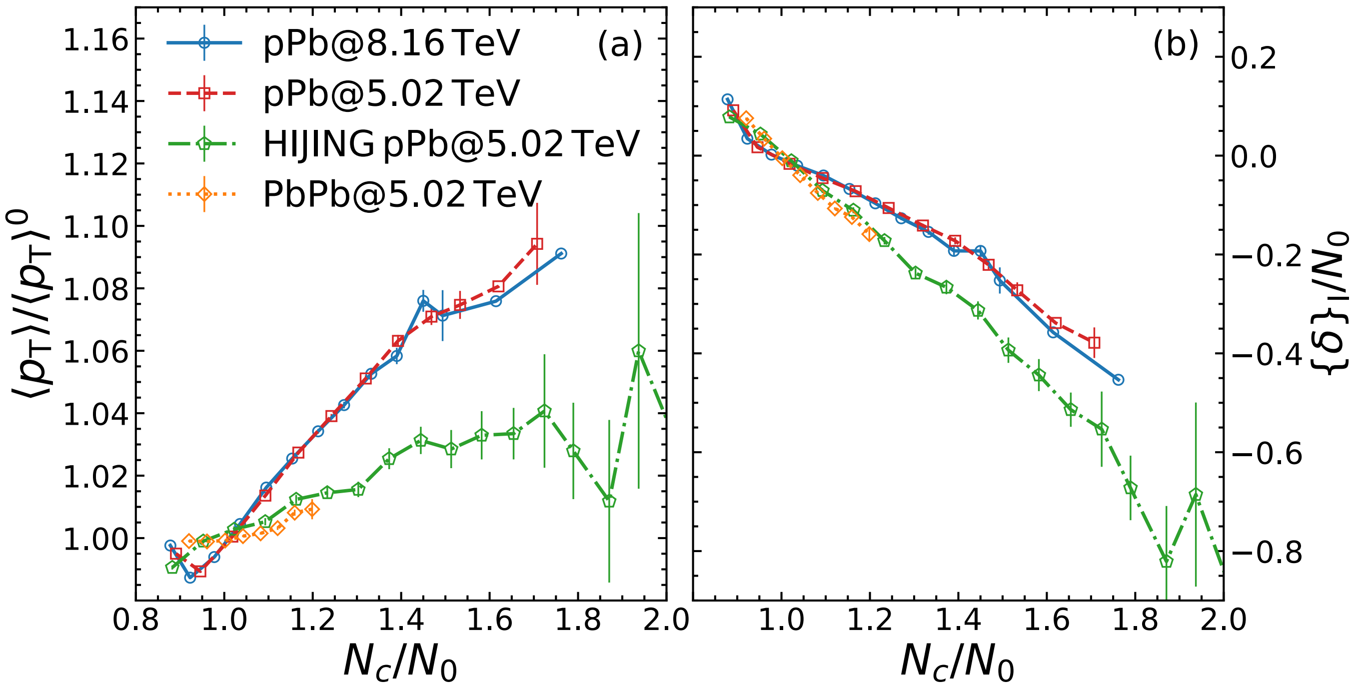
$$\frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \longrightarrow \delta = \frac{N_0}{c_s^2\langle p_T\rangle_0}\Delta_p - \Delta_N$$

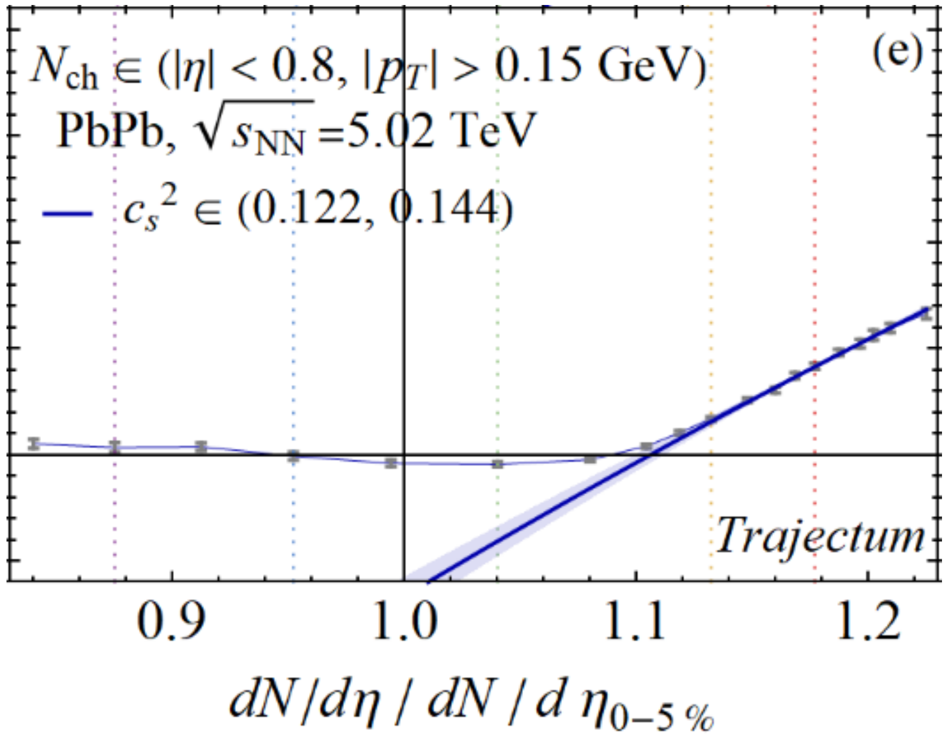
- δ denotes a quantum contribution.
- The distribution of δ is Gaussian according to Central Limit Theorem (CLT).
- Zero skewness condition: $\{\delta^3\}_c = 0$

$$(c_s^2)^3\underline{\frac{\{\Delta_N^3\}}{N_0^3}} - 3(c_s^2)^2\underline{\frac{\{\Delta_N^2\Delta_p\}}{N_0^2\langle p_T\rangle_0}} + 3c_s^2\underline{\frac{\{\Delta_N\Delta_p^2\}}{N_0\langle p_T\rangle_0^2}} - \underline{\frac{\{\Delta_p^3\}}{\langle p_T\rangle_0^3}} = 0$$

mixed skewness of $\langle p_T\rangle$ and N_{ch}
which can be measured in experiments

Quantum fluctuations are anti-correlated with N_{ch}

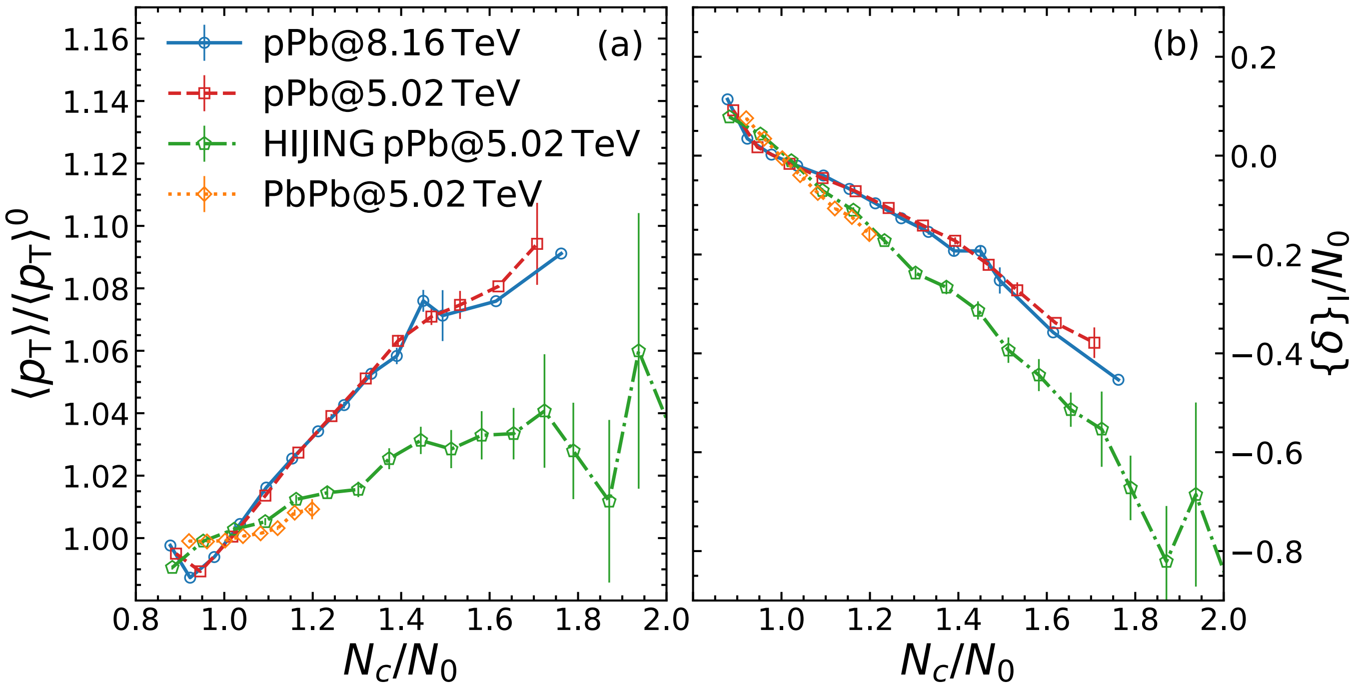


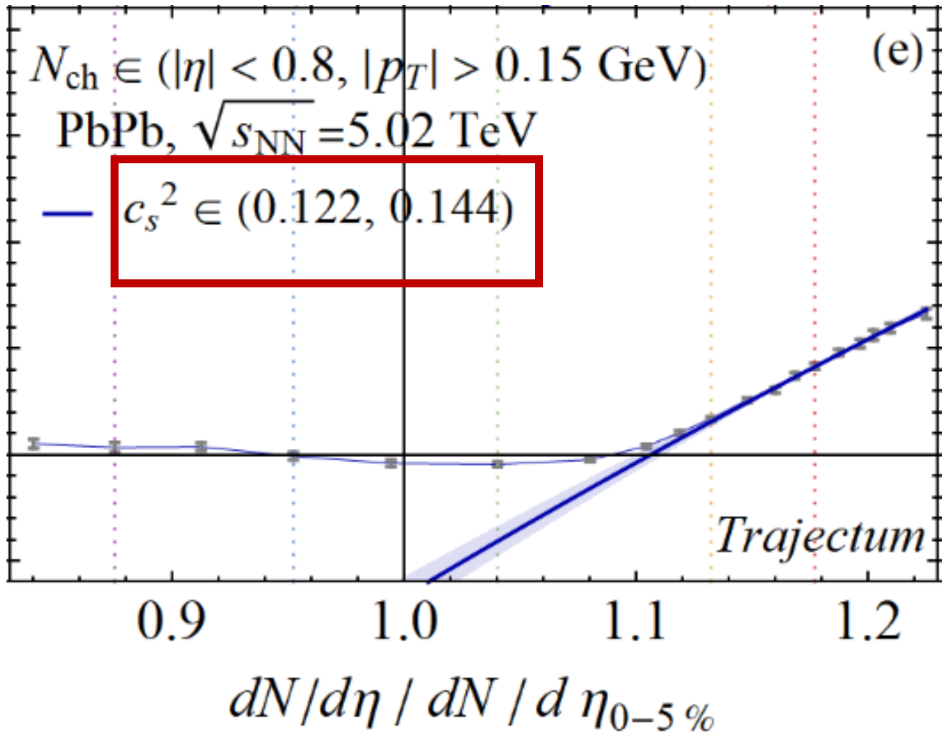


[G. Nijs, 2312.04623]

- $\dfrac{\{\Delta_p\}_I}{\langle p_T\rangle_0} = c_s^2 \, \dfrac{\{\Delta_N\}_I + \{\delta\}_I}{N_0}$
- $\{\delta\}_I = -\alpha\{\Delta_N\}_I$ which leads to $c_s^2 \to c_s^2 - \alpha$.
- The slope from simple linear fit can be suppressed by the quantum fluctuations.

Quantum fluctuations are anti-correlated with N_{ch}





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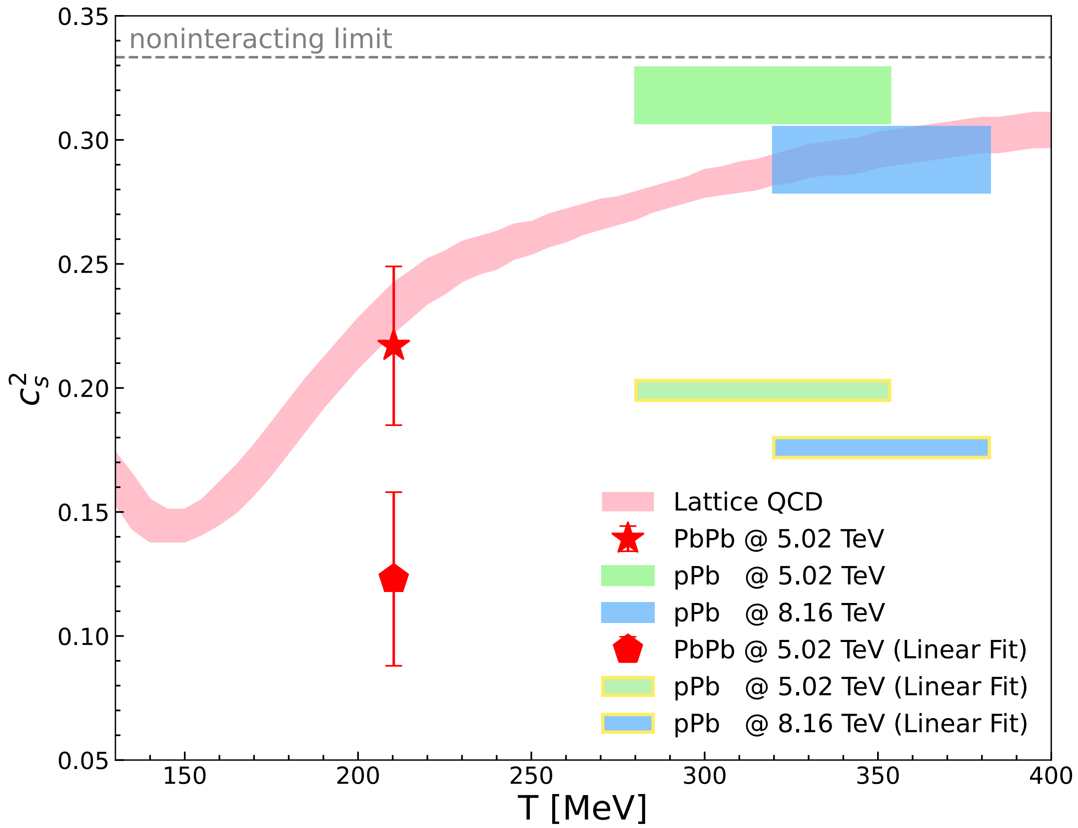
- The slope from simple linear fit can be suppressed by the quantum fluctuations.

Extracted speed of sound from different methods



c_s^2	Sub-bin slope	$\{\delta^3\}_c = 0$	$\{\delta^5\}_c = 0$	LEOS	T_{eff}(MeV)
PbPb (Hydro, 5.02 TeV)	0.123 ± 0.035	0.217 ± 0.032	0.216 ± 0.041	0.222–0.242	210.3
pPb (Hydro, 8.16 TeV)	0.176 ± 0.004	0.292 ± 0.013	0.287 ± 0.012	0.282–0.309	319.9–382.1
pPb (Hydro, 5.02 TeV)	0.197 ± 0.004	0.318 ± 0.011	0.313 ± 0.008	0.269–0.304	280.2–353.4

Probe of thermalization

- δ behaves differently in thermalized and non-thermalized systems.

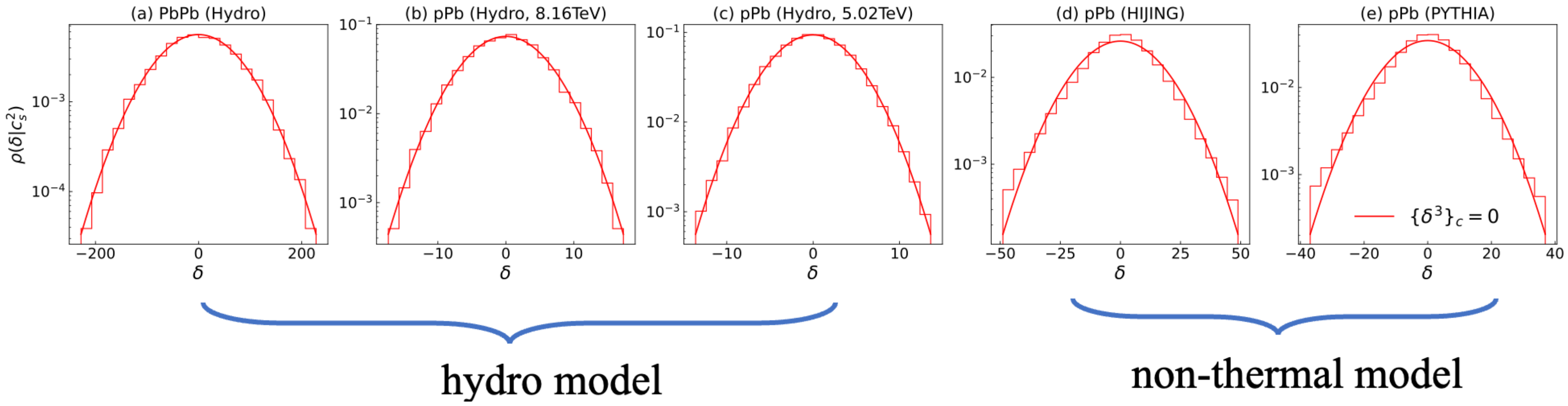
Thermalized system: $$\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0}$$ $\longrightarrow$ thermal response + quantum noise

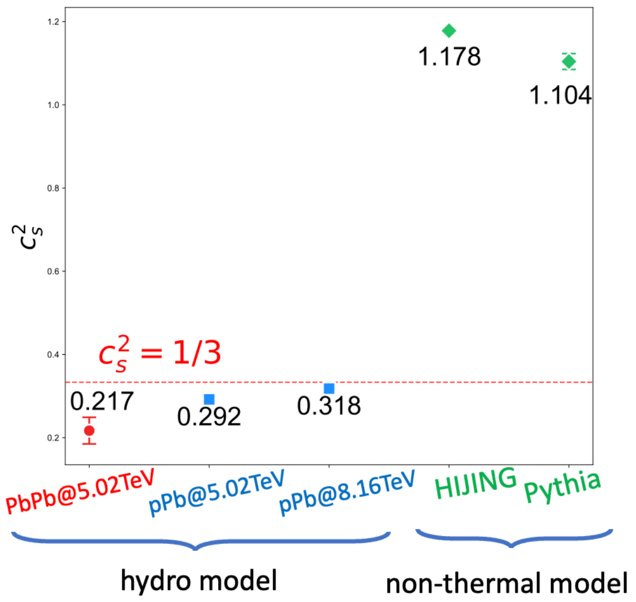
Non-thermalized system: $$\frac{\Delta_p}{\langle p_T \rangle_0} = \kappa \frac{\Delta_N + \delta}{N_0}$$ $\longrightarrow$ quantum response + quantum noise

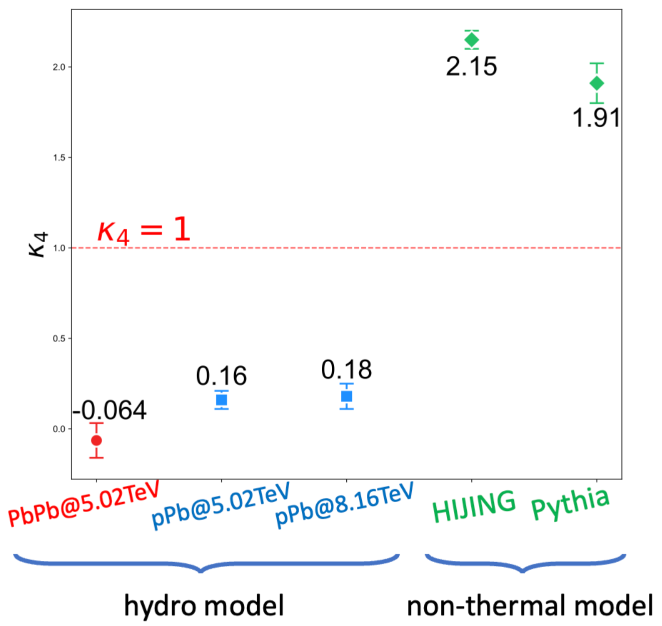
- The extracted value is physical only in thermalized systems.
- δ distribution is Gaussian only in thermalized systems.
- Deviations from thermalization can be quantified by the standard kurtosis.

$$\kappa_4 = \{\delta^4\}/\{\delta^2\}^2 - 3$$

Probe of thermalization







Summary

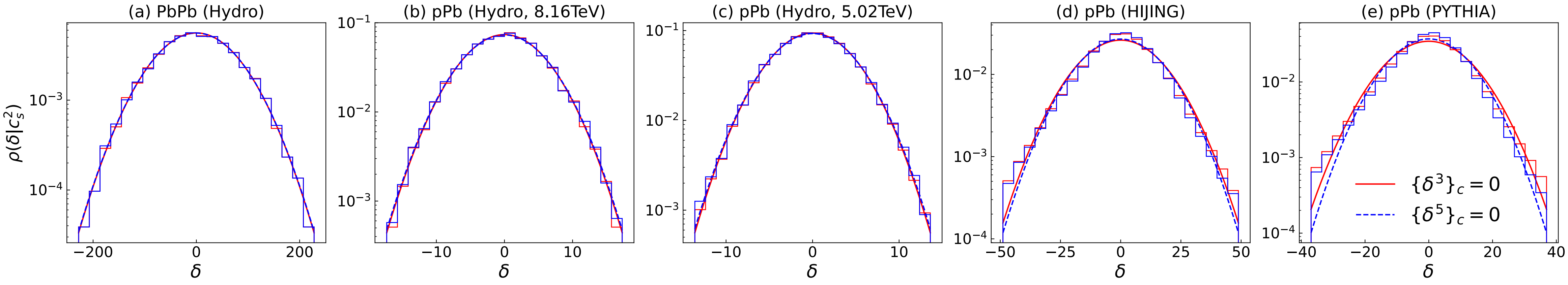
- The QCD speed of sound can be extracted even in the presence of quantum fluctuations;
- Thermalization can be probed by examining the Gaussianity of quantum fluctuation δ and the physical validity of the extracted speed of sound, which can be quantified by the standardized kurtosis of δ.

Backup

Back up

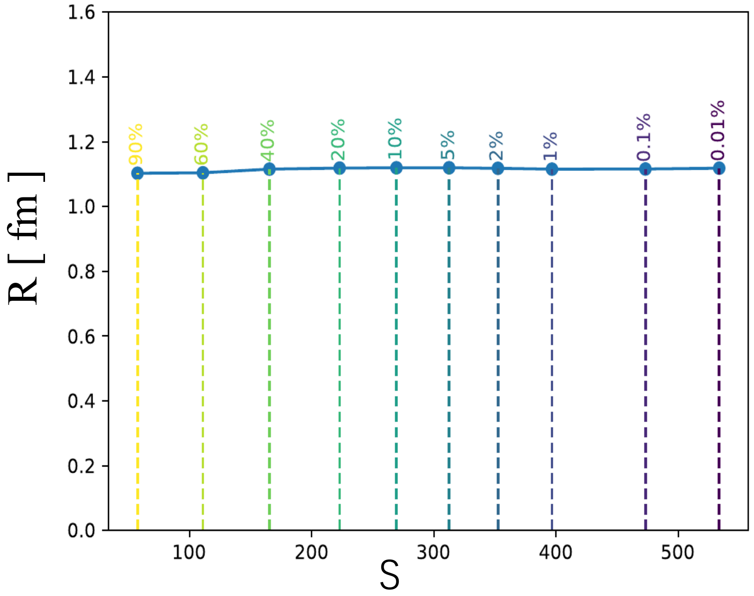
$$\{\delta^5\}_c = 0$$

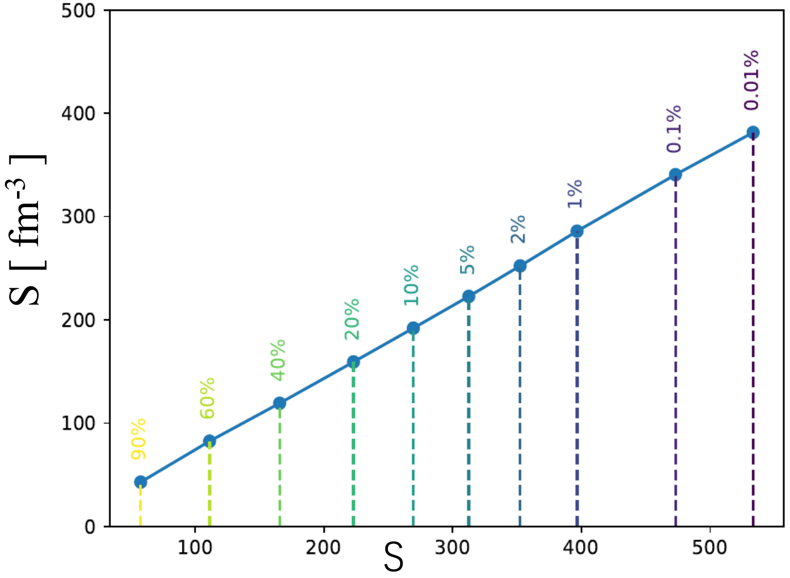
$$(c_s^2)^5 \frac{\{\Delta_N^5\}}{N_0^5} - 5(c_s^2)^4 \frac{\{\Delta_N^4 \Delta_p\}}{N_0^4 \langle p_T \rangle_0} + 10(c_s^2)^3 \frac{\{\Delta_N^3 \Delta_p^2\}}{N_0^3 \langle p_T \rangle_0^2} - 10(c_s^2)^2 \frac{\{\Delta_N^2 \Delta_p^3\}}{N_0^2 \langle p_T \rangle_0^3} + 5c_s^2 \frac{\{\Delta_N \Delta_p^4\}}{N_0 \langle p_T \rangle_0^4} - \frac{\{\Delta_p^5\}}{\langle p_T \rangle_0^5} = 0.$$



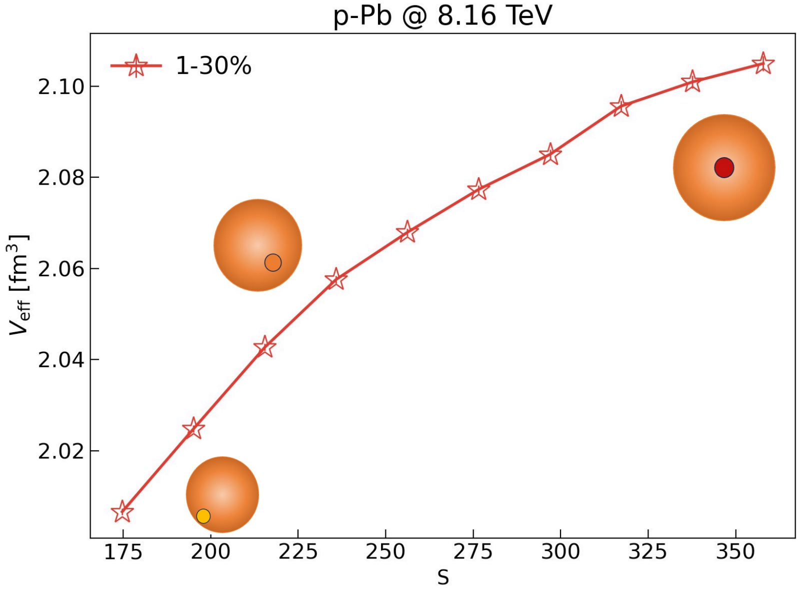
Back up

- Small systems





TRENTo model



hydro model

- Initial geometric size determined by the proton size is almost fixed in all centralities.
- Effective volume only saturates in ultra-centra events.

Back up

$$c_s^2 = \frac{d\ln\langle p_t\rangle}{d\ln N_{\text{ch}}} \rightarrow \frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2\frac{\Delta_N}{N_0} \quad \text{with} \quad \begin{cases} \Delta_p \equiv \langle p_T\rangle - \langle p_T\rangle_0 \\ \Delta_N \equiv N_{ch} - N_0 \end{cases}$$

where $\langle p_T\rangle_0$ and N_0 are the averaged values over the ultra-central events

[STAR collaboration, SQM2026]