The influence of heavy quark potential on heavy quark hadronization in QGP

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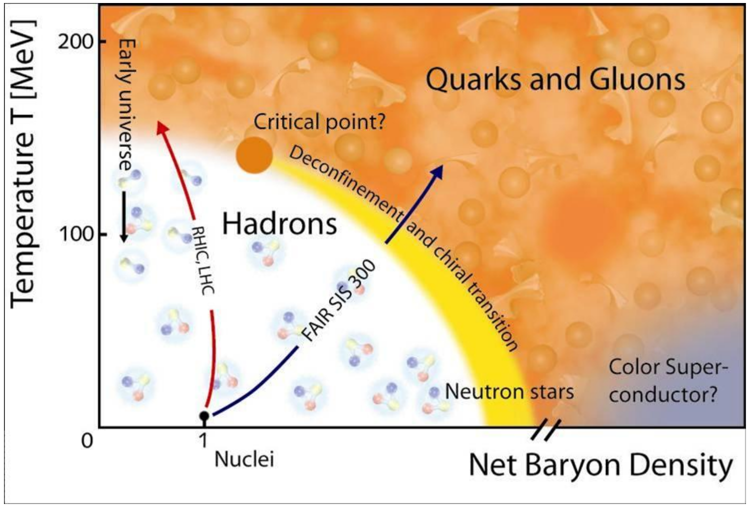
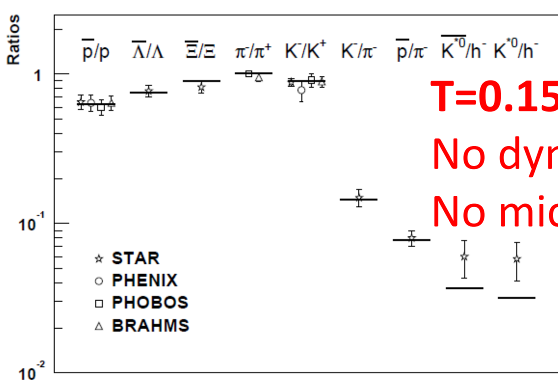
1. Motivation



Relativistic heavy-ion collisions produce a hot and dense matter which changes its phase from QGP to hadron gas (hadronization)

1. Statistical hadronization model



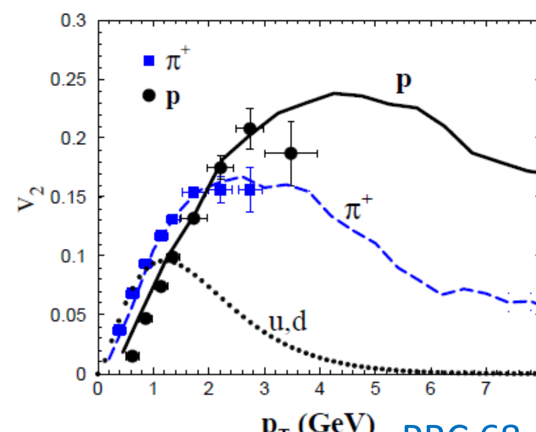
T=0.15-0.16 GeV

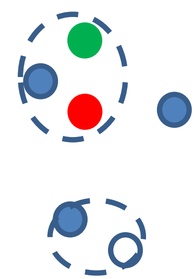
No dynamics

No microscopic picture

arXiv:nucl-th/0304013

2. Parton coalescence model





PRC 68, 034904

- Wigner projection (or coalescence model) coincides with the statistical model in its prediction on particle yield, assuming the binding energy is small

$$N = \int \frac{d^3p_1 d^3r_1}{(2\pi)^3} \int \frac{d^3p_2 d^3r_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} W(r,p)$$
$$\approx V \int \frac{d^3P}{(2\pi)^3} e^{-(M+P^2/(2M))/T},$$

where $M = m_1 + m_2$, $P = p_1 + p_2$ and

$$\int \frac{d^3p d^3r}{(2\pi)^3} W(r,p) = 1. \qquad W = |\langle Q\bar{Q}|\Phi\rangle|^2$$

- If binding energy is large, it underestimates particle yield (compared to the statistical model)

$$(2\pi)^3 \frac{dN}{d^3R d^3P} \approx e^{-(M_\Phi + P^2/2M)/T} e^{-\varepsilon_0/T},$$

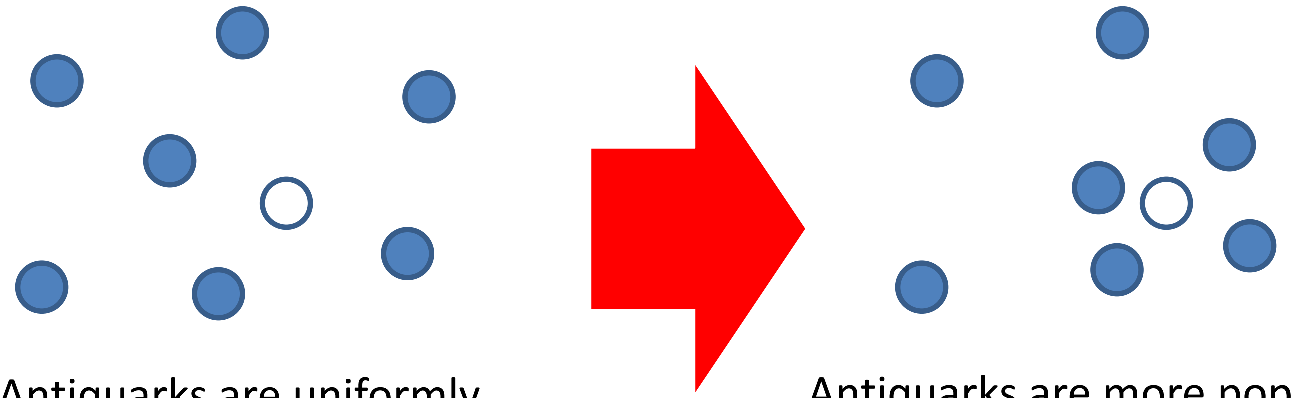
ε_0 is the positive binding energy of Φ ($M = M_\Phi + \varepsilon_0$).

PRC 107, 054906; PRC 113, 024916

How can one solve the discrepancy?

- Parton distribution in QGP might not be uniform in coordinate space. For example,

$$N = \int \frac{d^3p_1 d^3r_1}{(2\pi)^3} \int \frac{d^3p_2 d^3r_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} e^{-V(r)/T} W(r,p)$$



Antiquarks are uniformly distributed in space

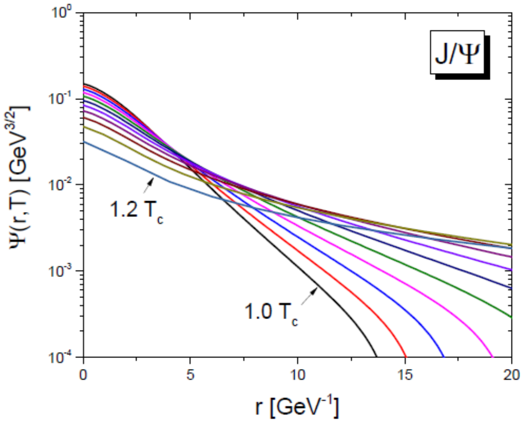
Antiquarks are more populated near a quark (if the pair is a color singlet)

It will increase coalescence probability (or Wigner projection) and thus coalescence model ≈ statistical model

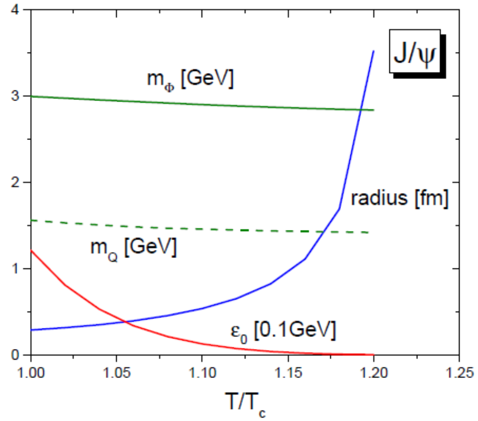
2. Quarkonium

- Solving Schrödinger Eq.,

$$\left[2m_0 - \frac{\nabla^2}{m_0} + V(r,T)\right]\psi(r,T) = M_\Phi(T)\psi(r,T),$$



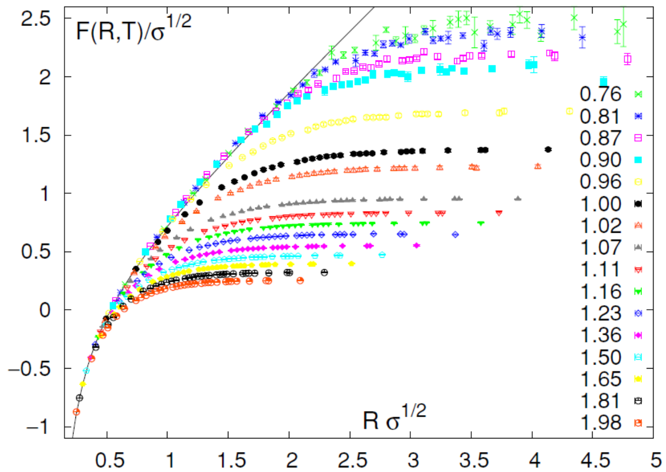
One can obtain eigenfunctions



eigenvalues

PRC113,024916

- We use free energy from lattice QCD as the heavy quark potential



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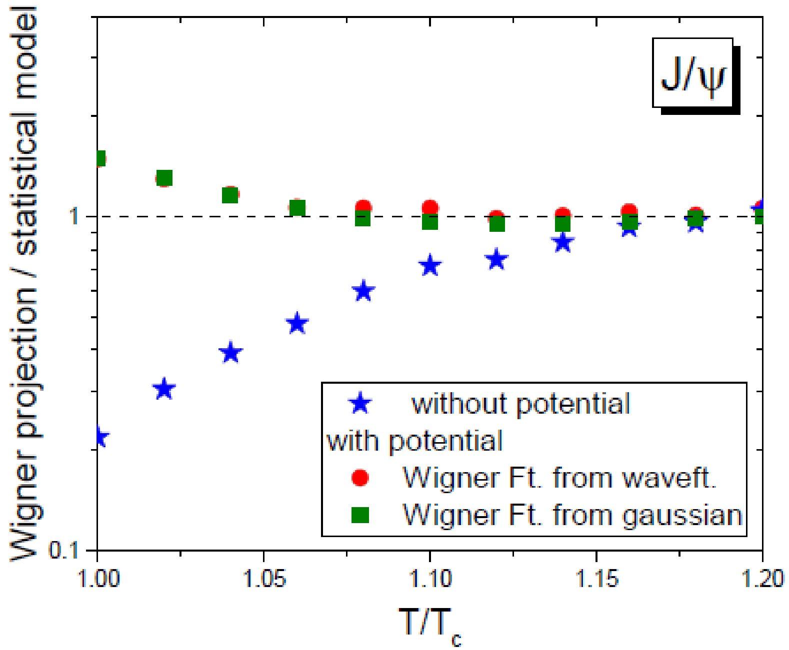
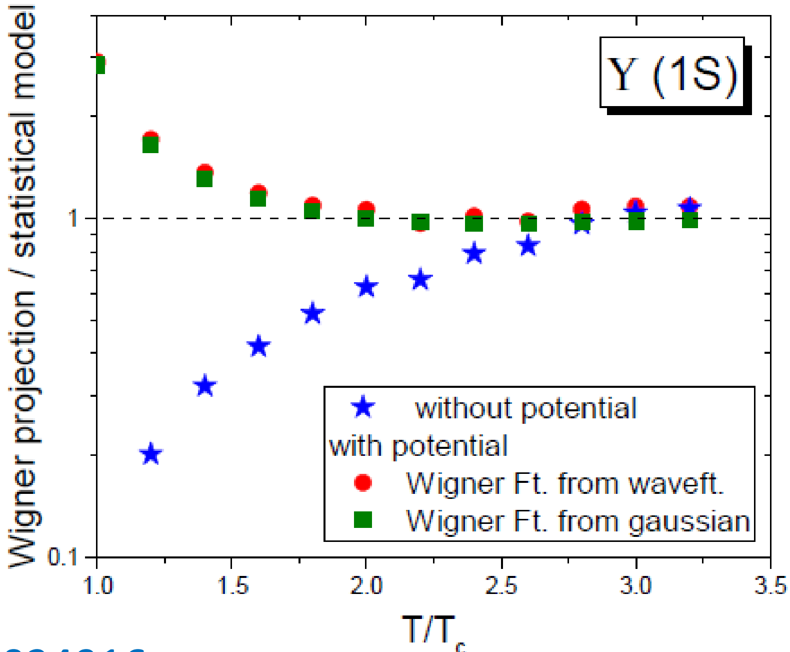
PRD101,114029

Wigner projection/statistical model



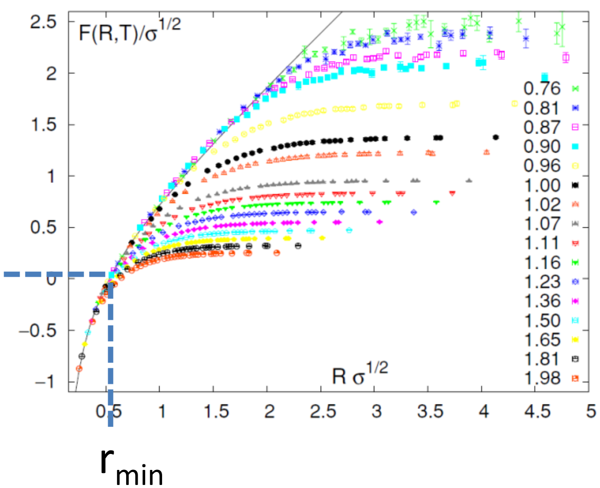


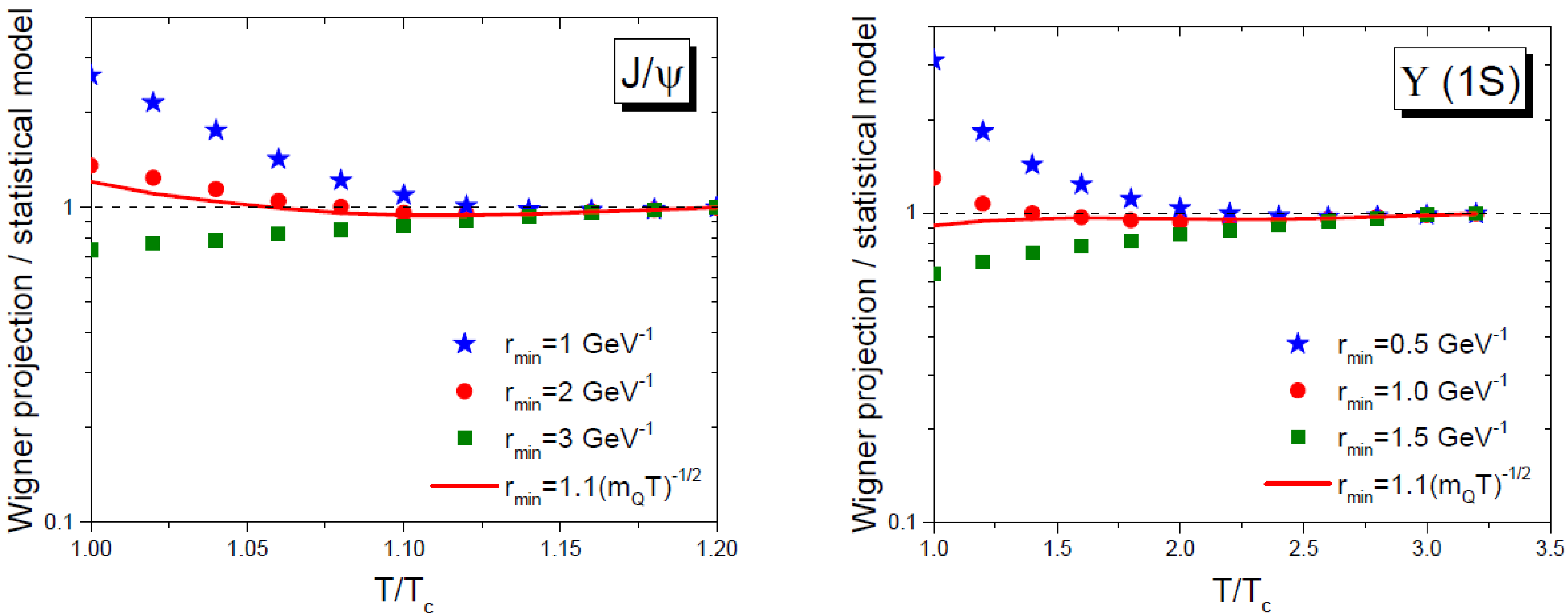
PRC113,024916

1. Near the dissociation temperature (small binding energy), the Wigner projection and statistical model agree with each other
2. Without potential, Wigner projection underestimates, as T decreases (binding energy increases)
3. Including potential helps, but it overshoots at low T

In quantum mechanics, position and momentum cannot be specified simultaneously.

We simply introduce a cutoff r_{min} to the potential V(r) to smear position





Charm needs a larger r_{min} , while bottom needs a smaller r_{min}

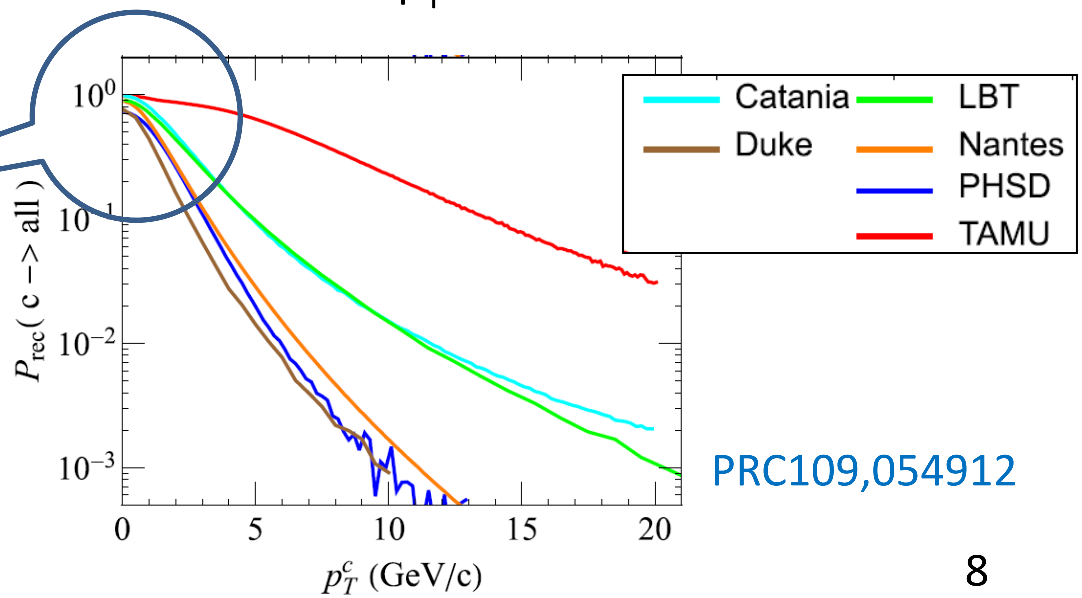
r_{min} proportional to thermal wave length ($\sqrt{2\pi/mT}$) seems to be universal

3. Heavy quark hadronization

- **Heavy quark fragmentation:**
- Heavy quark emits soft gluons for hadronization
- Heavy quark loses energy-momentum
- Dominant hadronization process at high p_T

- **Heavy quark coalescence:**
- Heavy quark is combined with antiquark
- Heavy quark gains energy-momentum
- Dominant hadronization process at low p_T

Coalescence probability increases with decreasing p_T and is supposed to be unity at p_T=0
So **I will focus on static heavy quark hadronization**



PRC109,054912

Coalescence probability of charm quark at p_T=0

$$N_3 = \frac{1}{(2\pi)^6}\frac{D_3}{D_1 D_2}\int d^3\mathbf{r_1}d^3\mathbf{r_2}d^3\mathbf{p_1}d^3\mathbf{p_2} \times f_1(\mathbf{r_1},\mathbf{p_1})f_2(\mathbf{r_2},\mathbf{p_2})\Phi(r,k),$$

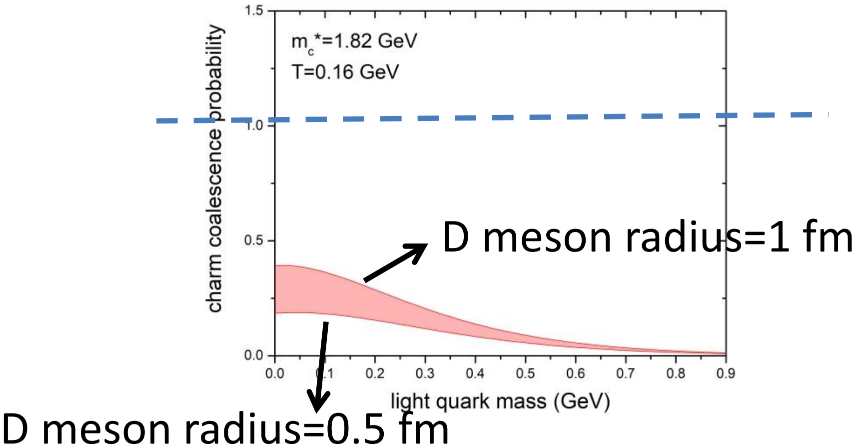
$$\boxed{f(\mathbf{r_1},\mathbf{p_1}) = (2\pi)^3\delta^{(3)}(\mathbf{r_1})\delta^{(3)}(\mathbf{p_1}).}$$

Then N3 changes to coalescence probability.
First, calculate the coalescence probability for D meson, then multiply by

$$\frac{n^{\text{charm hadron}}(T_c)}{n^{D^0}(T_c)} = 7.5,$$

to obtain the total coalescence probability of charm quark at Tc

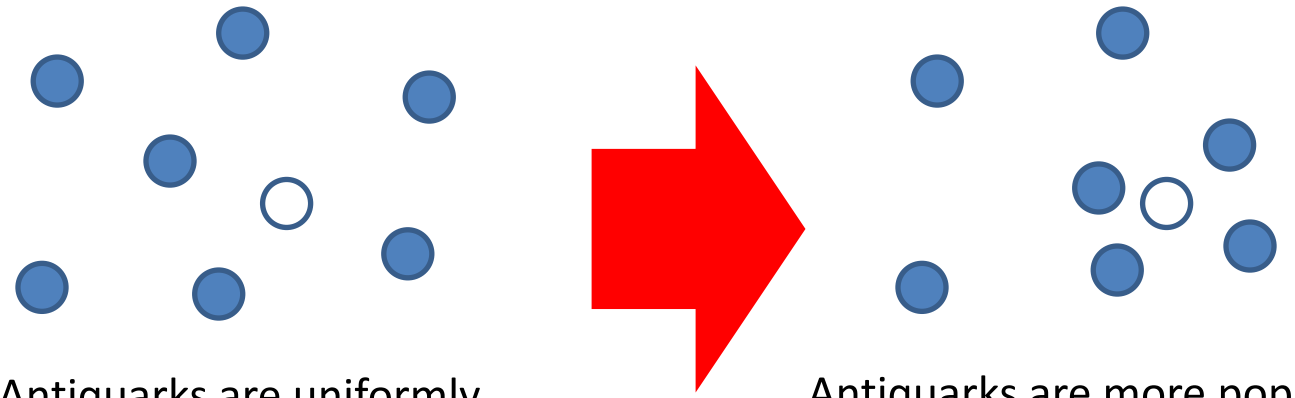
far below unity !



Again we introduce potential

- Parton distribution in QGP might be not completely uniform in coordinate space. For example,

$$N = \int \frac{d^3p_1 d^3r_1}{(2\pi)^3} \int \frac{d^3p_2 d^3r_2}{(2\pi)^3} e^{-E_1/T} e^{-E_2/T} e^{-V(r)/T} W(r,p)$$



Antiquarks are uniformly distributed in space

Antiquarks are more populated near a quark (if it is a color singlet)

Potential for heavy mesons

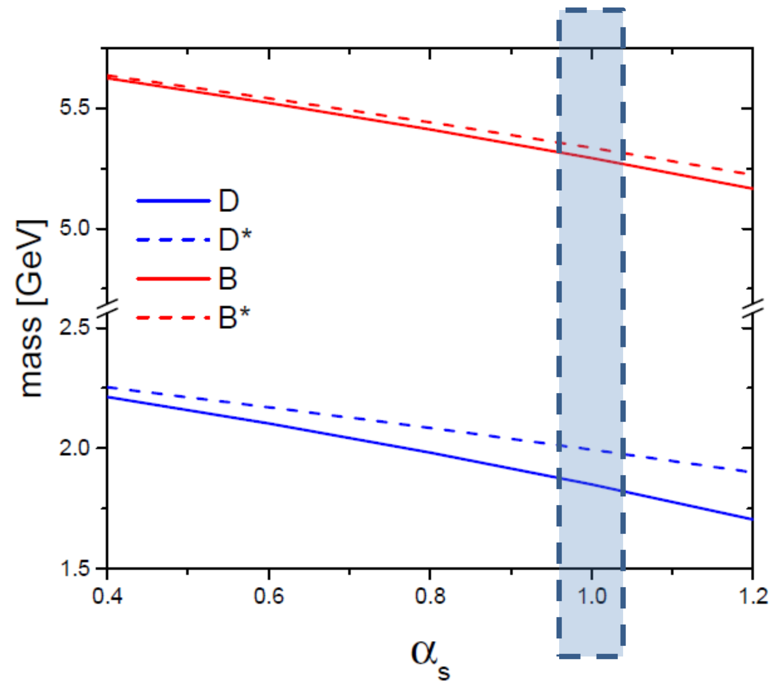
- Solving Schrödinger Eq.,

$$\left[-\frac{\nabla^2}{m_\mu}+\widetilde{V}(r,T)\right]\psi(r,T)=-\varepsilon\psi(r,T),$$

$$V(r,T)=F(r,T)+\alpha_s\frac{\sigma_Q\cdot\sigma_q}{m_Q m_q}\frac{e^{-r/r_0}}{r_0^2 r},$$

Inspired by the lattice free energy
but with different α_s
+ spin-spin interaction

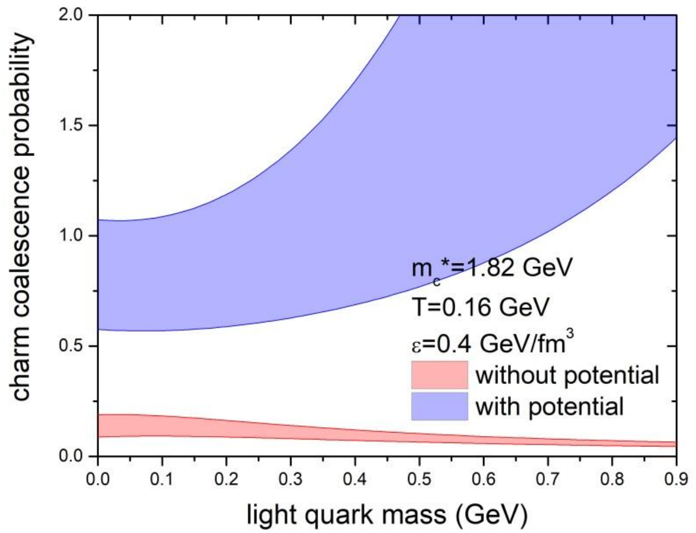
Best fit is obtained $\alpha_s \approx 1$



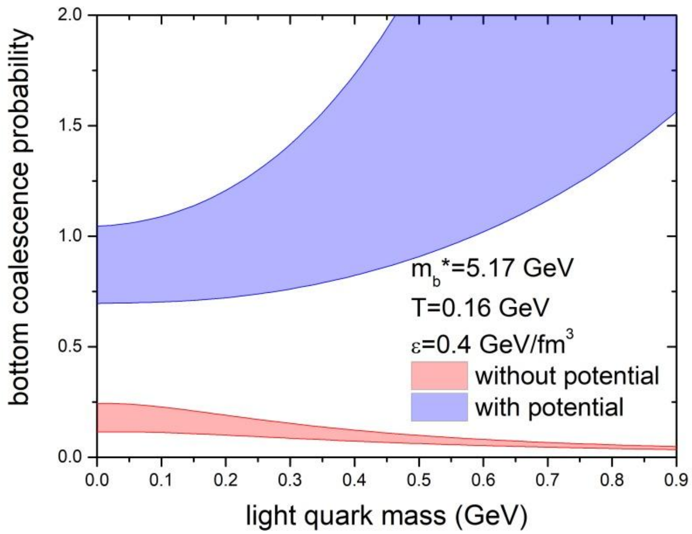
arXiv:2607.08707

After introducing the potential

Charm coalescence probability



Bottom coalescence probability

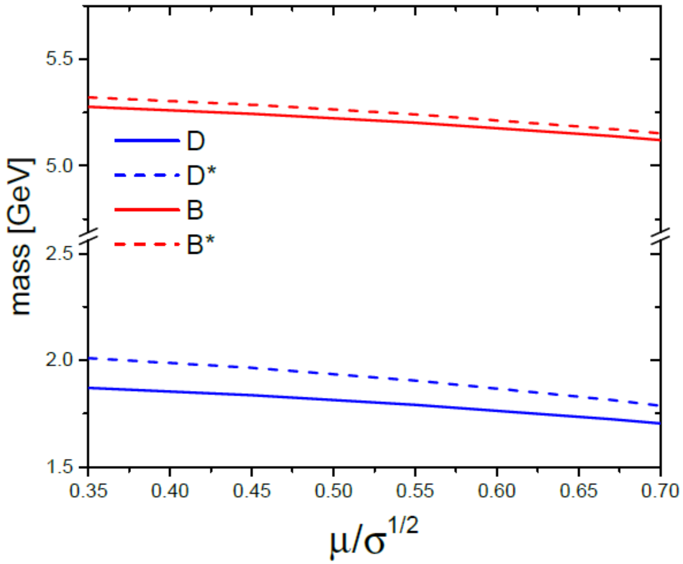


Now coalescence probabilities are near unity!

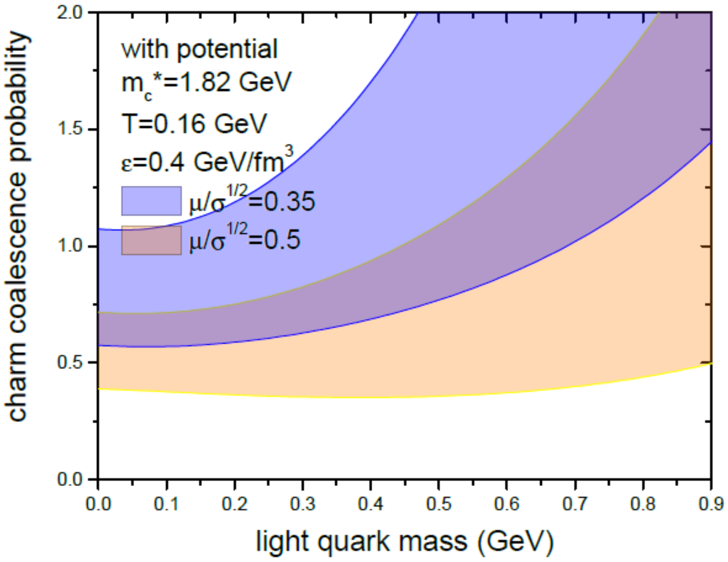
arXiv:2607.08707

In-medium potential

- Screening mass μ increases in medium and D(B) meson mass decreases



- As the screening mass μ increases in medium, potential depth becomes swallow and coalescence probability decreases.

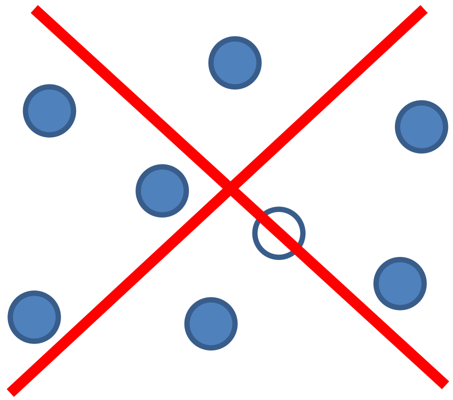


In order for the coalescence probability to be unity, allowed mass shift of D(B) meson is about -60 MeV

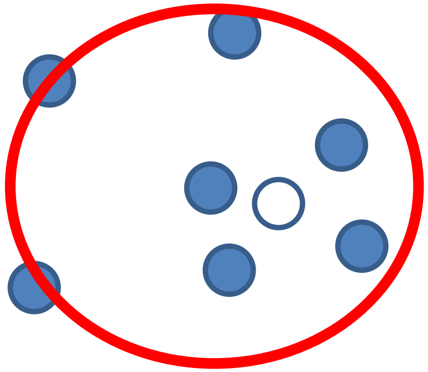
arXiv:2607.08707

4. Summary

- Partons in QGP (especially near Tc) are not uniformly distributed in coordinate space, but more populated near antiparton which forms a color-singlet together
- It explains the discrepancy between the coalescence model (Wigner projection) and statistical model for quarkonium production
- and enables the coalescence probability of heavy quark to be unity at p_T=0



Antiquarks are uniformly distributed in space



Antiquarks are more populated near a quark (if it is a color singlet)

Free energy potential

For the interaction between a heavy quark and a light antiquark forming a heavy meson, we adopt a heavy-light quark potential motivated by lattice-QCD calculations of the heavy-quark free energy [25–27],

$$F(r,T) = F_c(r,T) + F_s(r,T), \tag{1}$$

where F_c and F_s denote the Coulombic and string contributions, respectively, and are given by

$$\begin{aligned} F_c(r,T) &= -\frac{\alpha_s}{r}[e^{-\mu r} + \mu r], \qquad (2) \\ F_s(r,T) &= \frac{\sigma}{\mu}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{\mu r}}{2^{3/4}\Gamma(3/4)} K_{1/4}[(\mu r)^2]\right], \end{aligned}$$

with α_s, σ and μ representing the effective strong coupling, string tension, and screening mass, respectively [25].

EPJC43,71; PRD101,114029

Why does it overshoot?

- 1-dim. simple harmonic oscillator

in quantum statistics the probability for the ground state

$$P_0 = \frac{1}{Z} e^{-\omega/(2T)} = 1 - e^{-\omega/T},$$

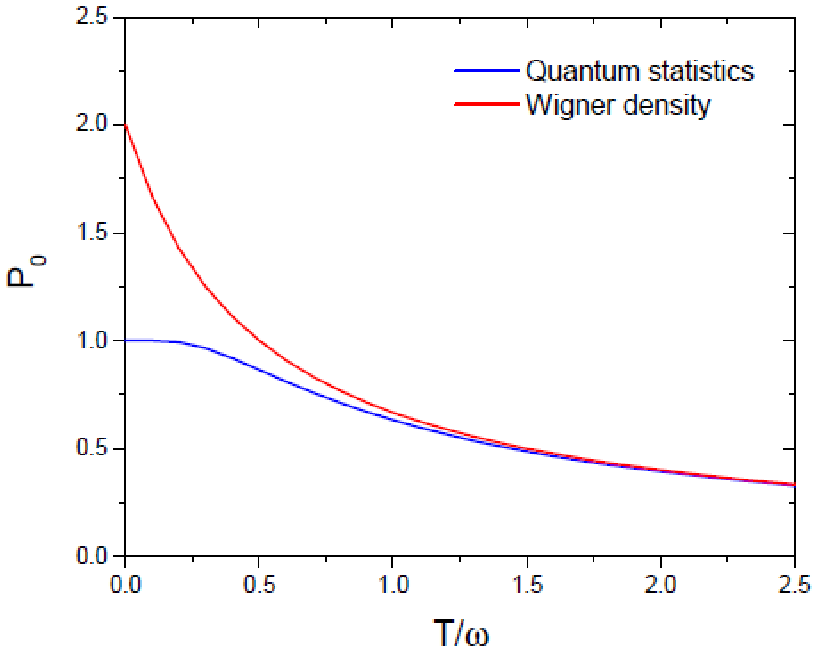
where $Z = [2\sinh(\omega/(2T))]^{-1}$ is the partition function. Applying the Remler formalism,

$$P_0 = \int \frac{dxdp}{2\pi} f(x,p) W_0(x,p) = \frac{2}{1+2T/\omega},$$

where $\omega = \sqrt{K/m}$ and normalized distribution function and Wigner function respectively

$$f(x,p) = \sqrt{\frac{K}{mT^2}} \exp\left[-\frac{p^2 + mKx^2}{2mT}\right] \rightarrow \delta(x)\delta(p),$$

$$W_0(x,p) = 2e^{-x^2/\sigma^2 - \sigma^2 p^2} \rightarrow 2, \quad \text{if } T = 0$$



However, position and momentum cannot simultaneously be specified in QM.

For example, the density operator of free gas $\langle \vec{r'} | \rho | \vec{r} \rangle = \frac{1}{V} \exp\left[-\frac{\pi}{\lambda^2} (\vec{r'} - \vec{r})^2\right],$

where the thermal wavelength $\lambda = \sqrt{2\pi/mT}$

Coalescence probability of charm quark at p_T=0

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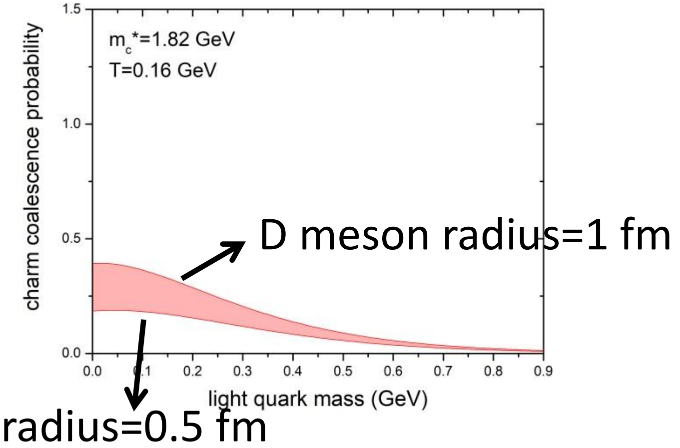
$$f(\mathbf{r_1},\mathbf{p_1}) = (2\pi)^3 \delta^{(3)}(\mathbf{r_1})\delta^{(3)}(\mathbf{p_1}).$$



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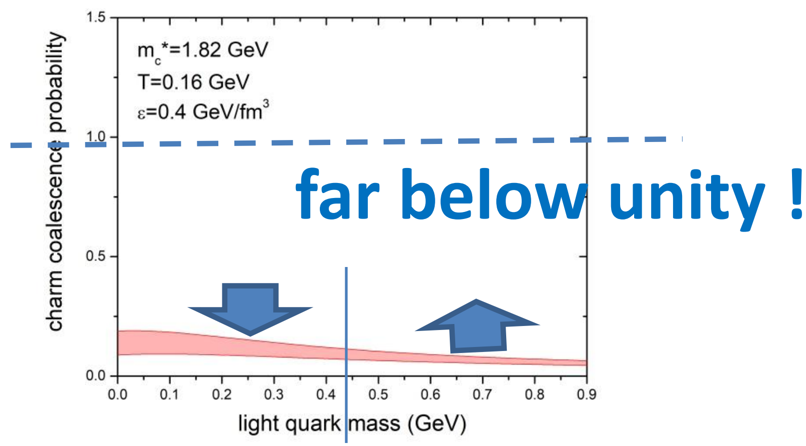
to obtain the total coalescence probability of charm quark at Tc

After fixing the energy density to 0.4 GeV/fm^3



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