Phase Transition to Hyperon Matter in Neutron Stars using an Effective Model with Chiral Invariant Masses

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(KMI, Nagoya University, JAEA)

Outline

- Introduction & Motivation
- Model Construction
- Onset Density of Hyperons in Neutron Star Matter
- Summary and Outlook

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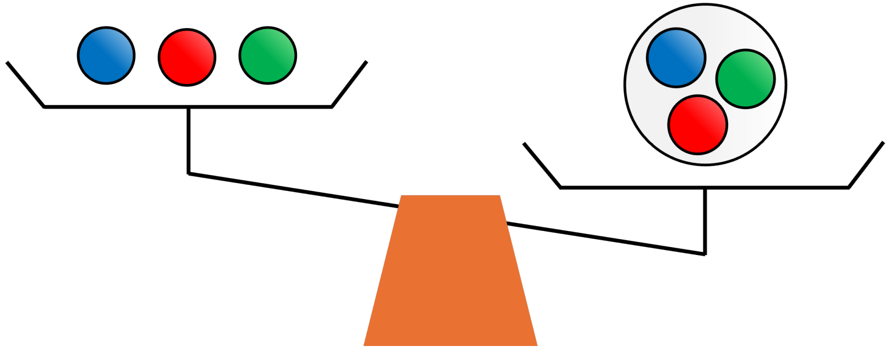
The Origin of Baryon Mass

Quark mass $\ll$ Baryon mass



In parity doublet model: $m = m_0 + g\langle q\bar{q}\rangle$

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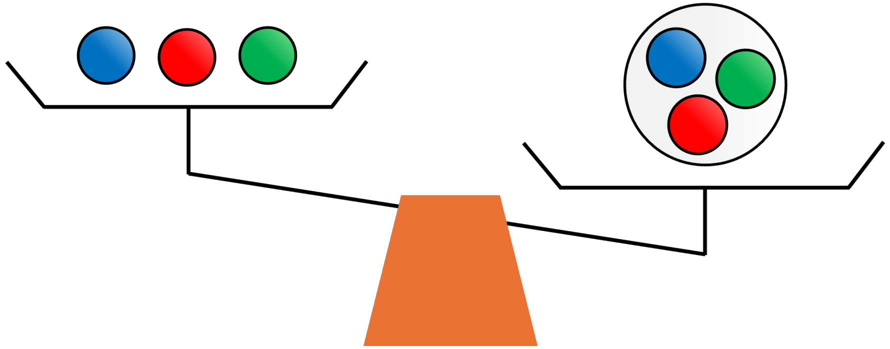

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Chiral invariant mass of nucleons by the study of neutron stars

- $600 \leq m_{N0} \leq 900\,\mathrm{MeV}$ [1]

  [1] T. Minamikawa, et al., Phys. Rev. C 103, 045205 (2021).

- $580 \leq m_{N0} \leq 860\,\mathrm{MeV}$ [2]

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More than half of nucleon mass

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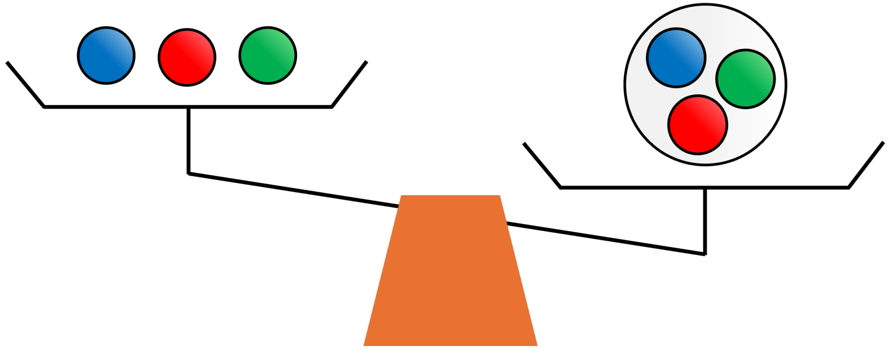

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How large the chiral invariant masses of hyperons?

Motivation of Model Construction

Previous model based on $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ chiral symmetry

- Same chiral invariant mass for octet baryons [3]

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- Different chiral invariant masses, but many parameters [4]

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Construct an $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ effective model reduced from $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$, with fewer parameters, while capturing the effects near hyperon onset density.

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$$
\begin{aligned}
m_{\Lambda 0} &= m_{\Lambda 0}^{\mathrm{SU}(3)} + m_{\langle s\bar{s}\rangle} + m_{\langle u\bar{u}\rangle} + m_{\langle d\bar{d}\rangle} \\
&= m_{\Lambda 0}^{\mathrm{SU}(2)} + m_{\langle u\bar{u}\rangle} + m_{\langle d\bar{d}\rangle}
\end{aligned}
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Nucleon Model

Parity doublet model [5]

Representation of Nucleon and meson

$$N \sim [(2,1) \oplus (1,2)], \quad M \sim [(2,2^*)]$$

Transformation under Chiral Symmetry

$$N_{1L} \to g_L N_{1L},\ N_{1R} \to g_R N_{1R}$$
$$N_{2L} \to g_R N_{2L},\ N_{2R} \to g_L N_{2R}$$
$$M \to g_L M g_R^\dagger$$

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Chiral Invariant Mass Term

$$\mathscr{L}_m = m_0(\bar{N}_{1L} N_{2R} + \cdots)$$

Interaction Term

$$\mathscr{L}_{\mathrm{int}} = -\, g_1(\bar{\psi}_{1L} M \psi_{1R} + \cdots) + g_2(\cdots)$$

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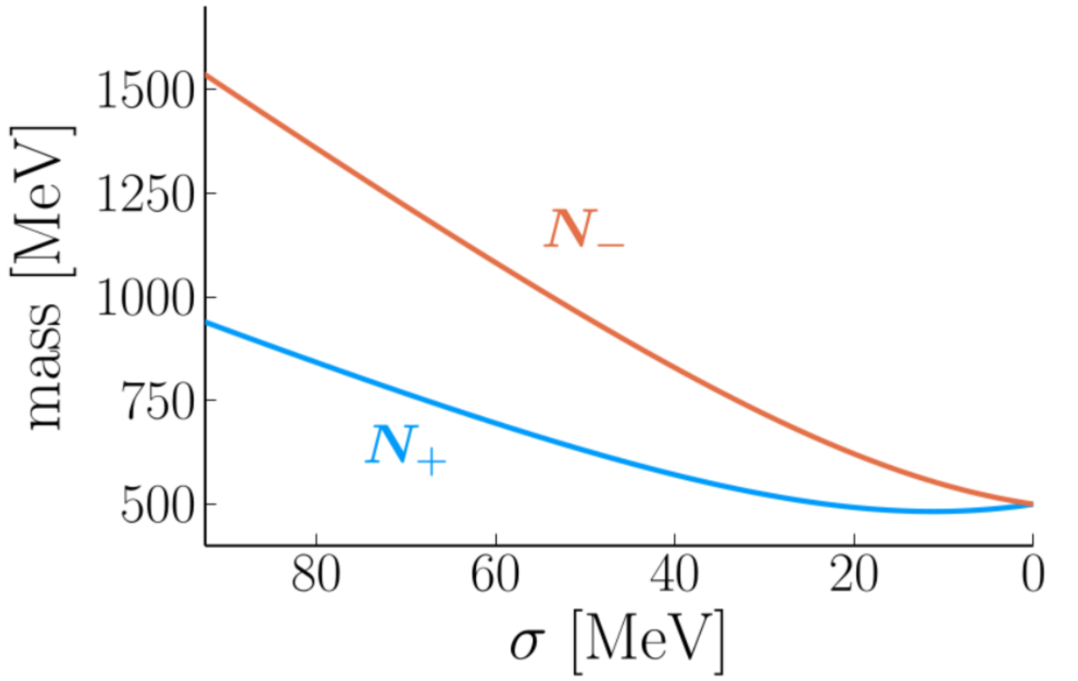
Effective mass of Nucleon

$$m^*_{N_\pm} = \frac{1}{2}\sqrt{(g_{N1}+g_{N2})^2\sigma^2 + 4m_{N0}^2} \pm (g_{N1}-g_{N2})\sigma$$



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Hyperon Model

Representation of Hyperon (s quark is heavy)

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\begin{aligned}
qqs &\sim [(2,1) \oplus (1,2)]^2 \\
&= \underset{\Lambda}{2[(1,1)]} \oplus 2[(2,2)] \oplus \underset{\Sigma}{[(3,1) \oplus (1,3)]}
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$$\Lambda_{L,R} \to \Lambda_{L,R}$$

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Chiral Invariant Mass Term

$$\mathscr{L}_m = -m_{\Lambda 0}\bar{\Lambda}\Lambda - m_{\Sigma 0}\mathrm{tr}\left[\bar{\Sigma}_{1L}\Sigma_{2R} + \cdots\right]$$

Interaction Term

$$\begin{aligned} \mathscr{L}_{\mathrm{int}} = &-G_\Lambda\left[\bar{\Lambda}_L (M)^{l_1}{}_{r_1} (M^\dagger)^{r_1}{}_{l_1} \Lambda_R + \cdots\right] \\ &-G_{\Sigma 1}\left[(\bar{\Sigma}_{1L})_{l_1 l_2} (M)^{l_1}{}_{r_1} (M)^{l_2}{}_{r_2} (\Sigma_{1R})^{r_1 r_2} + \cdots\right] + G_{\Sigma 2}[\cdots] \end{aligned}$$

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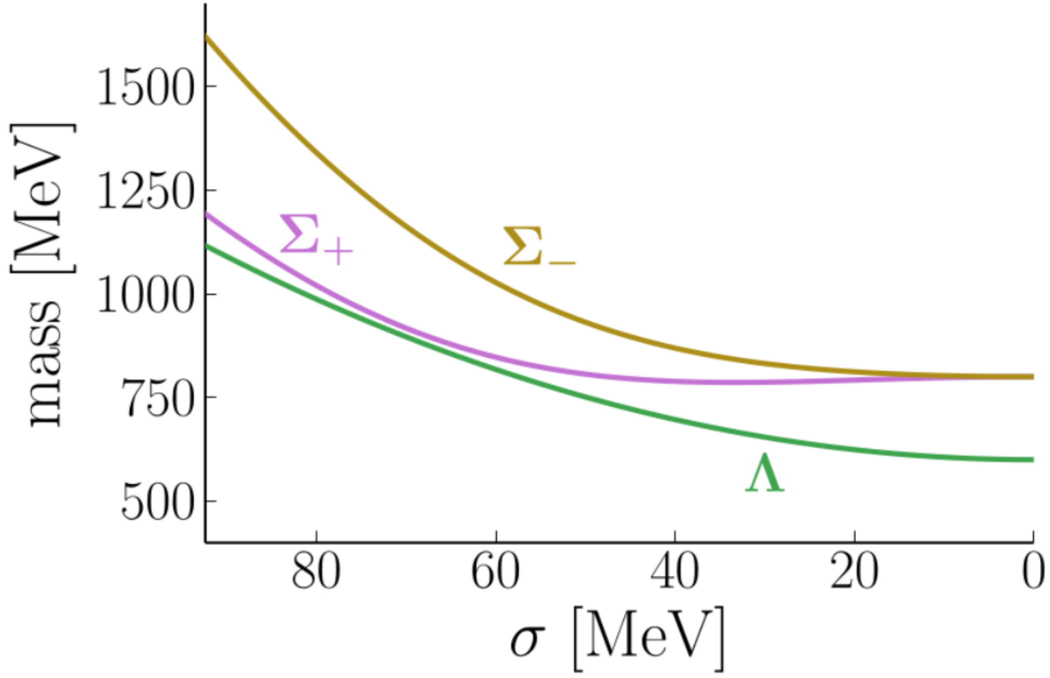
Effective mass of Hyperon

$$m_\Lambda^* = m_{\Lambda 0} + G_\Lambda \sigma^2$$
$$m_{\Sigma\pm}^* = \frac{1}{2}\sqrt{(G_{\Sigma 1} + G_{\Sigma 2})^2\sigma^4 + 4m_{\Sigma 0}^2} \pm \frac{1}{2}(G_{\Sigma 1} - G_{\Sigma 2})\sigma^2$$



Pressure in the mean field approximation

Pressure

$$P = \sum_{i=N,\Lambda,\Sigma} P^F(m^*, \mu^*) + \sum_{l=e,\mu} P^*(m_l, \mu_l) - V(\sigma, \omega, \rho)$$

P^F: Pressure of Fermi Gas

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Effective Chemical Potential

$$\mu_i^* = \mu_B - g_{\omega i}\omega + I_3(\mu_I - g_{\rho i}\rho) + S\mu_s \quad (i = N, \Lambda, \Sigma)$$

I_3: isospin, S: strangeness ($S = 0$ for N, -1 for Λ, Σ)

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Meson Potential [6]

$$V(\sigma, \omega, \rho) = \frac{1}{2}\bar{\mu}^2\sigma^2 - \frac{1}{4}\lambda_4\sigma^4 + \frac{1}{6}\lambda_6\sigma^6 + m_\pi^2 f_\pi \sigma + \frac{1}{2}m_{\omega^2}\omega^2 + \frac{1}{2}m_\rho^2\rho^2$$

[6] Y. Motohiro, et al., Phys. Rev. C 92, 025201 (2015)

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Pion Decay constant: $f_\pi = 92.4\,\mathrm{MeV}$

Saturation Property

- Saturation density: $n_0 = 0.16\,\mathrm{fm}^{-3}$
- Binding energy: $B_0 = -16\,\mathrm{MeV}$
- Symmetry energy: $S_0 = 31\,\mathrm{MeV}$
- Incompressibility: $K_0 = 240\,\mathrm{MeV}$
- Hyperon potential:

$$U_\Lambda = -30\,\mathrm{MeV}, U_\Sigma = +30\,\mathrm{MeV}$$

$$U_i = m_i^*(\sigma) - m_i + g_{\omega i}\omega$$

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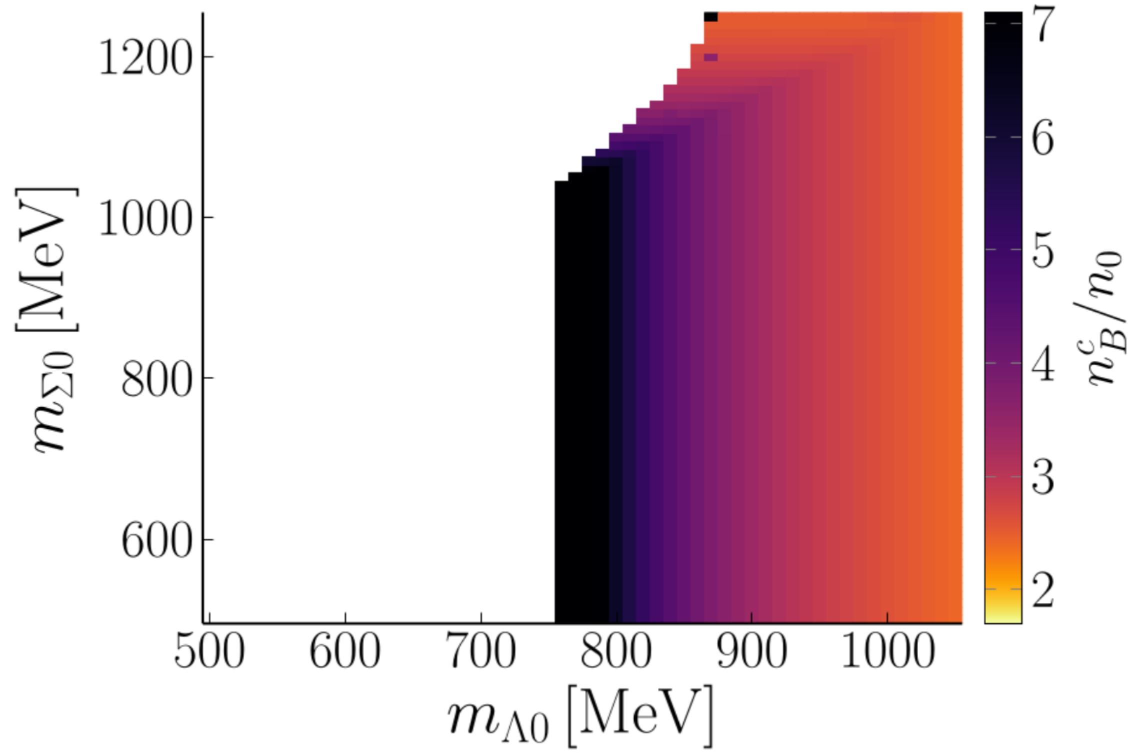
Onset Density of Λ

Results for neutron star matter up to $10n_0$

$m_{N0} = 500\,\mathrm{MeV}$



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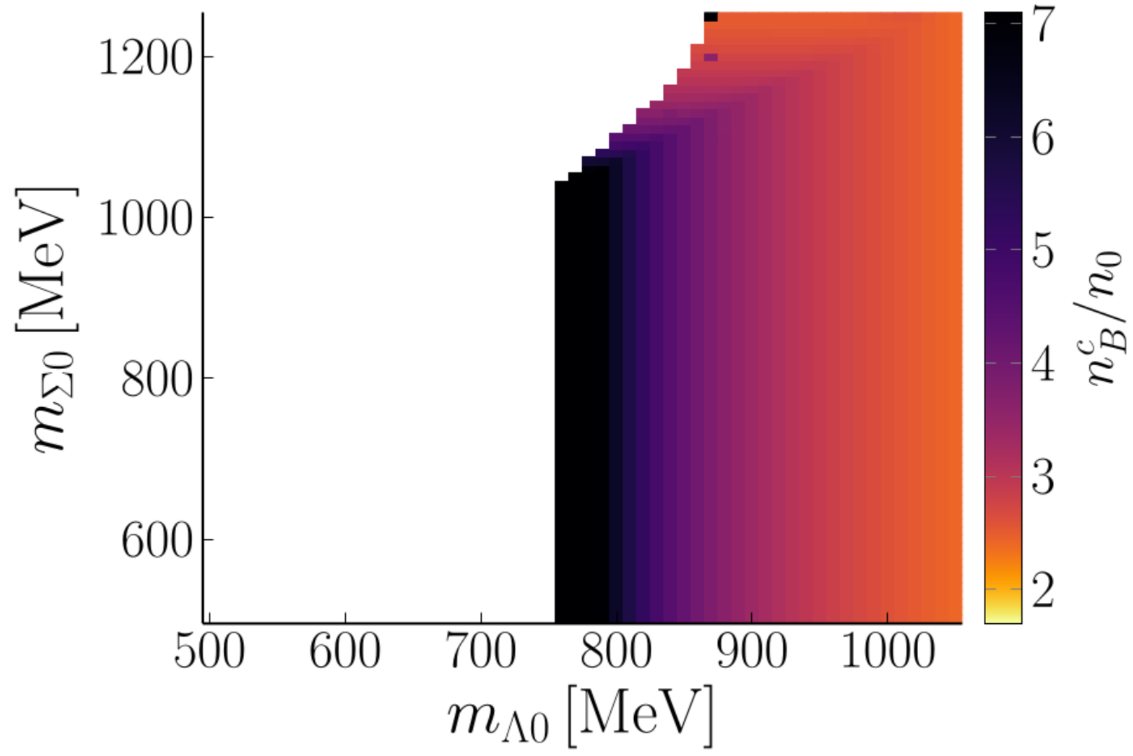

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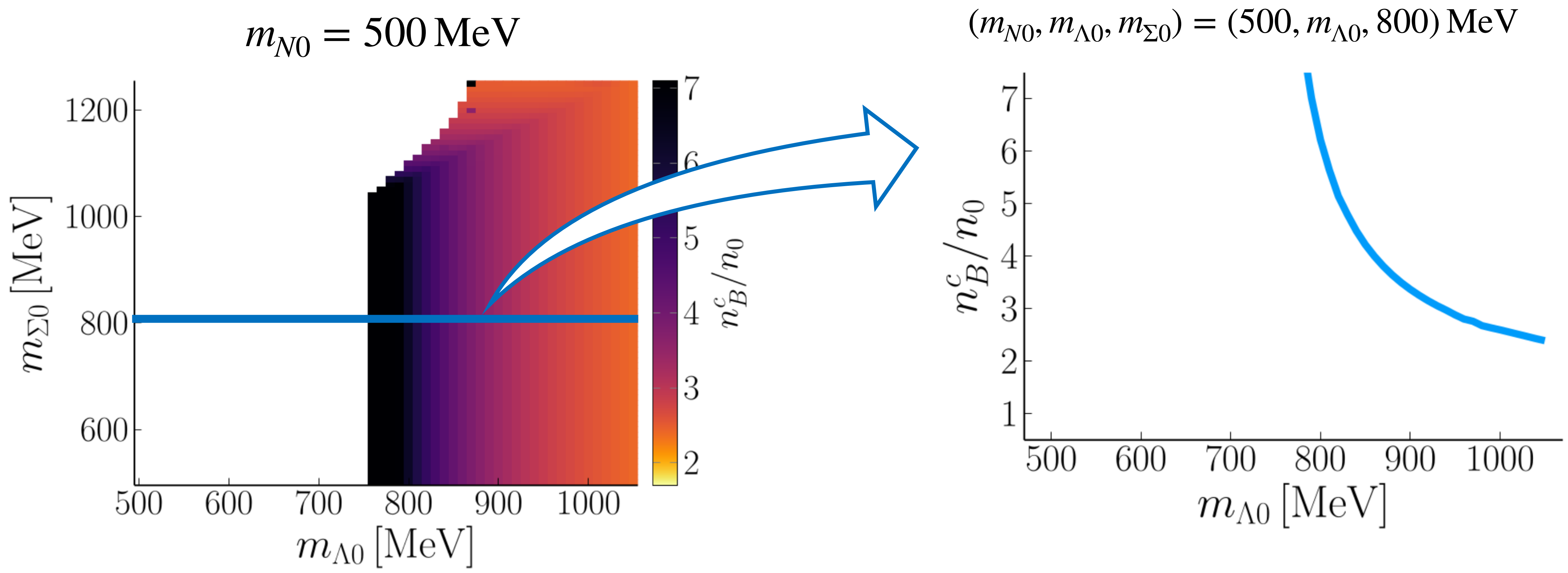

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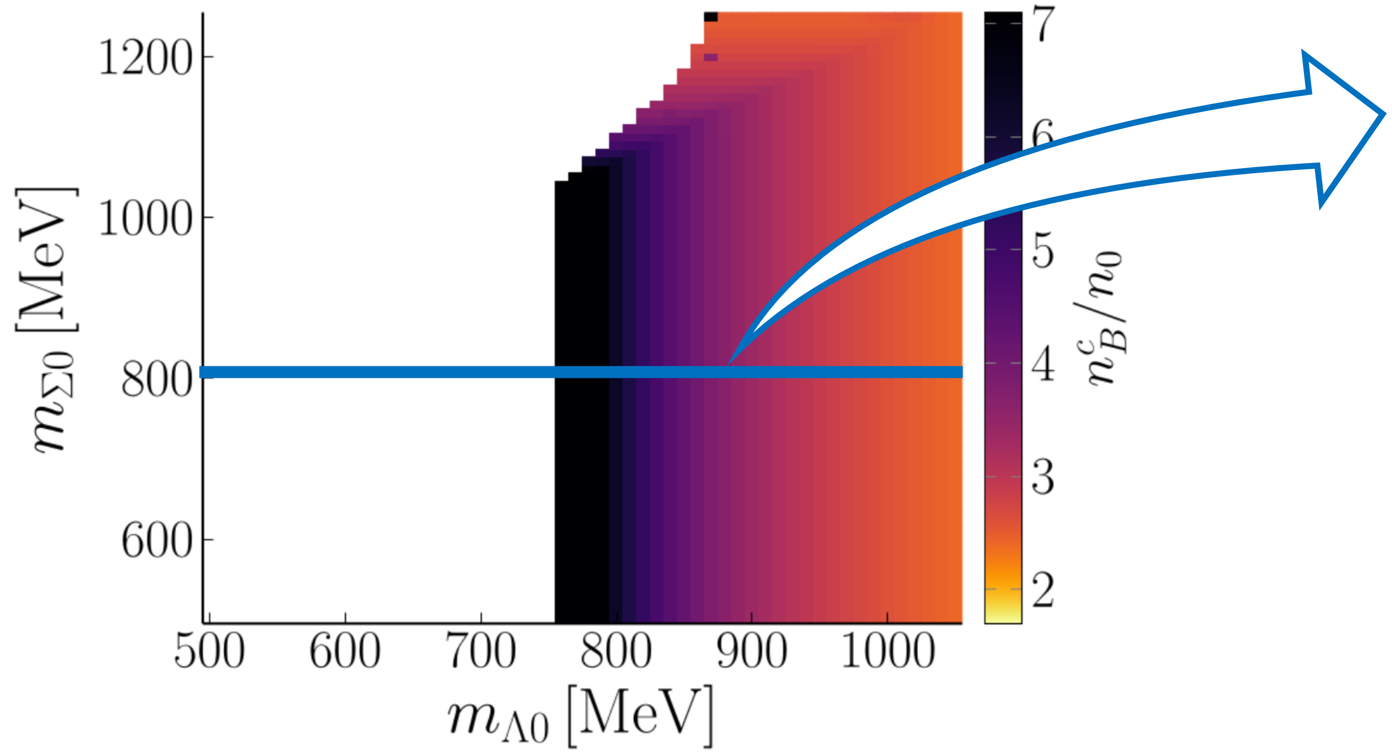
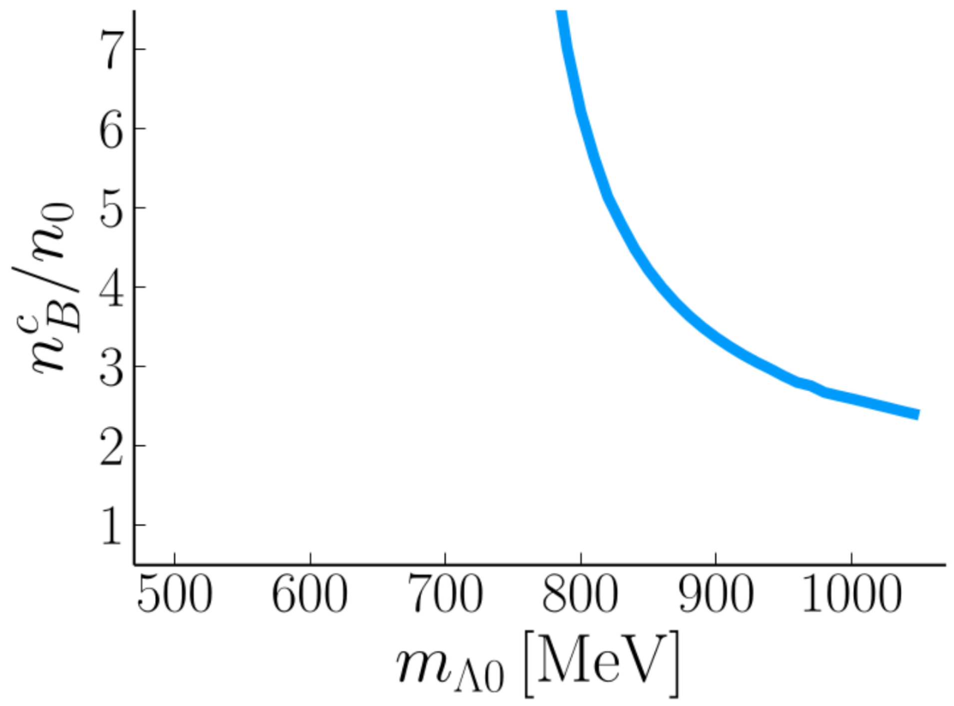
Onset Density of Λ

Results for neutron star matter up to $10n_0$

$m_{N0} = 500\,\mathrm{MeV}$

$(m_{N0}, m_{\Lambda 0}, m_{\Sigma 0}) = (500, m_{\Lambda 0}, 800)\,\mathrm{MeV}$

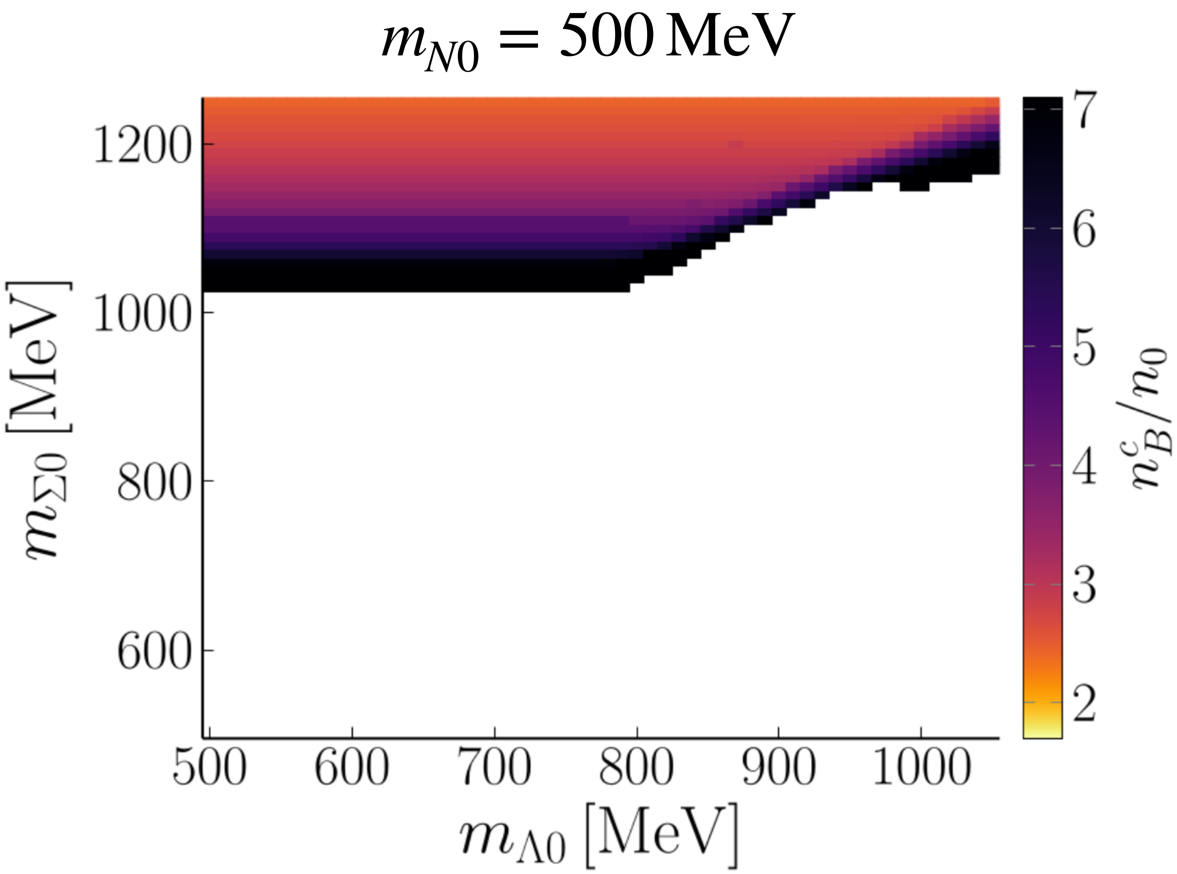




- Λ appears at a lower density when its chiral invariant mass is greater
- Λ can appear in neutron stars
  when its chiral invariant mass is sufficiently large

$n^c \sim 2.57 n_0$ when $m_{\Lambda 0} = 1000\,\mathrm{MeV}$

Onset Density of Σ



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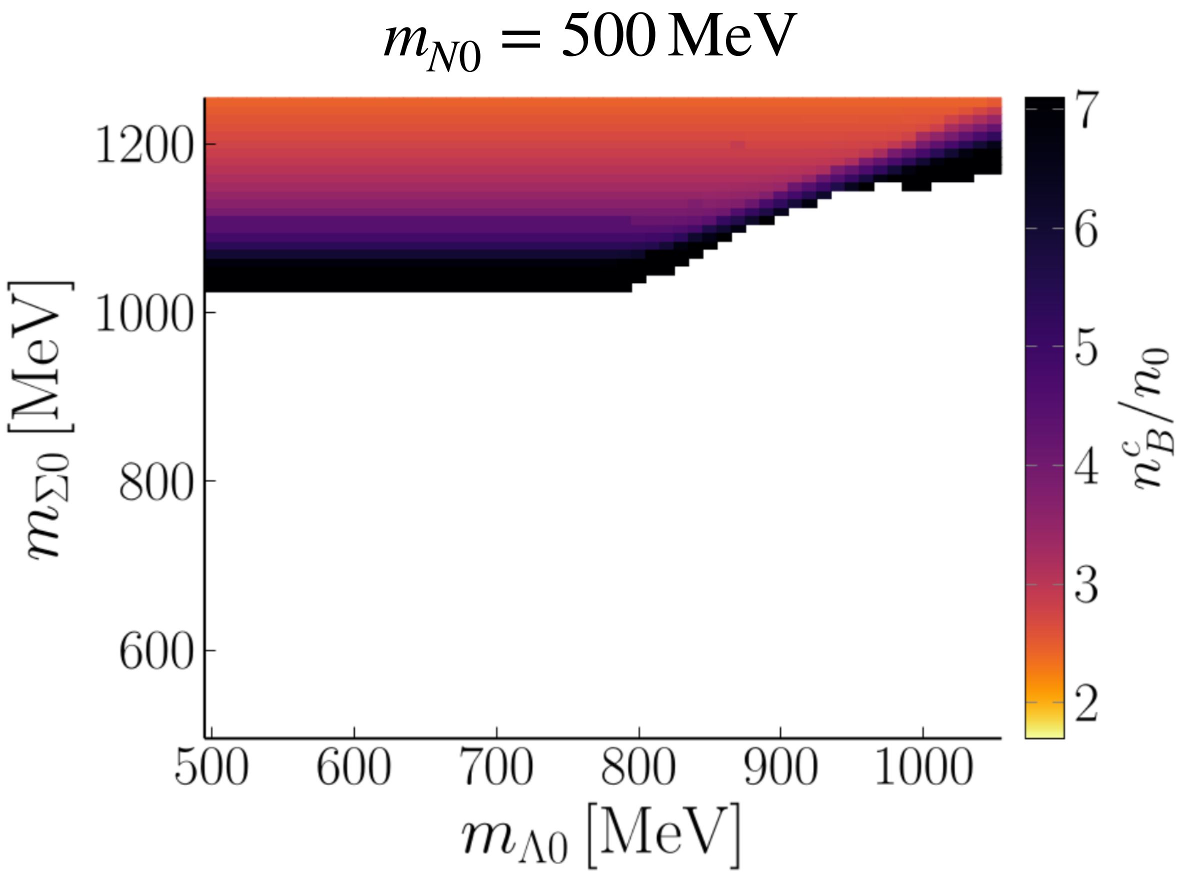

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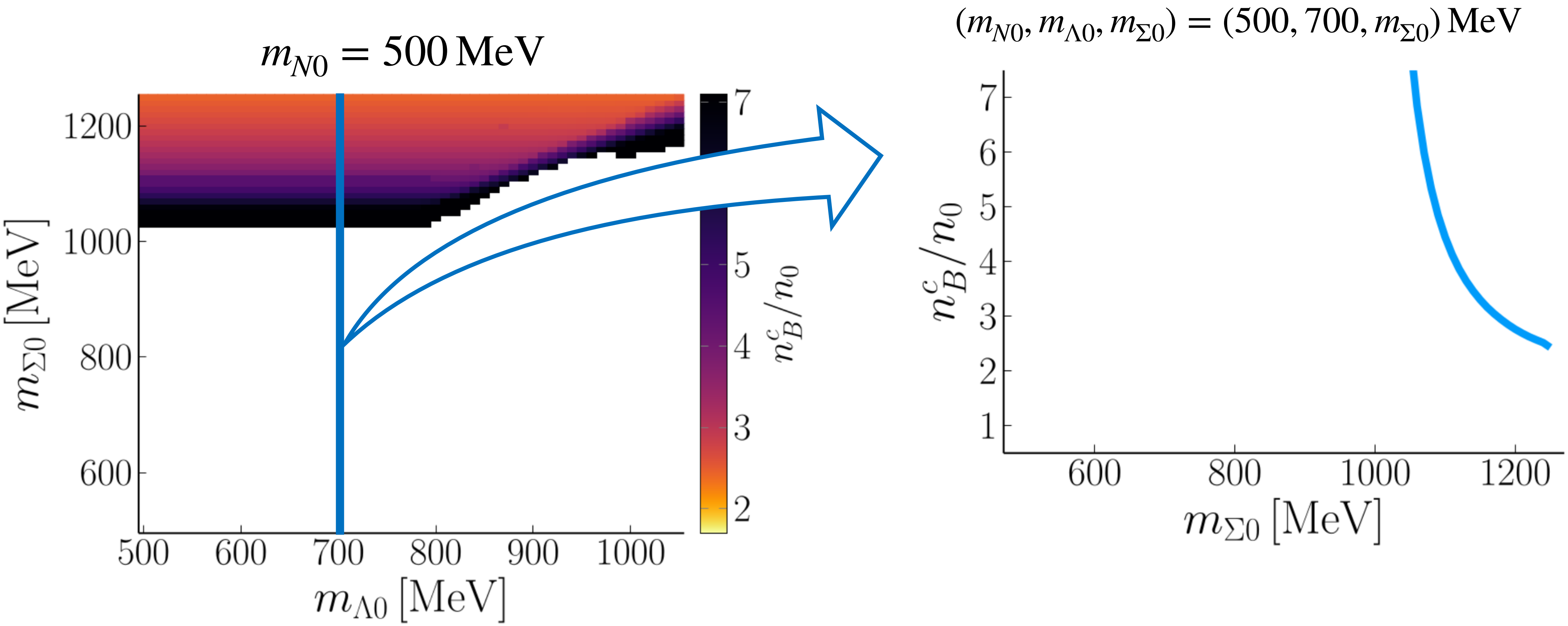

- Σ also appears at a lower density when its chiral invariant mass is greater
- Σ appears only when its chiral invariant mass is as large as its vacuum mass

Why hyperons appear at lower density when $m_{\Lambda 0}$, $m_{\Sigma 0}$ are greater?

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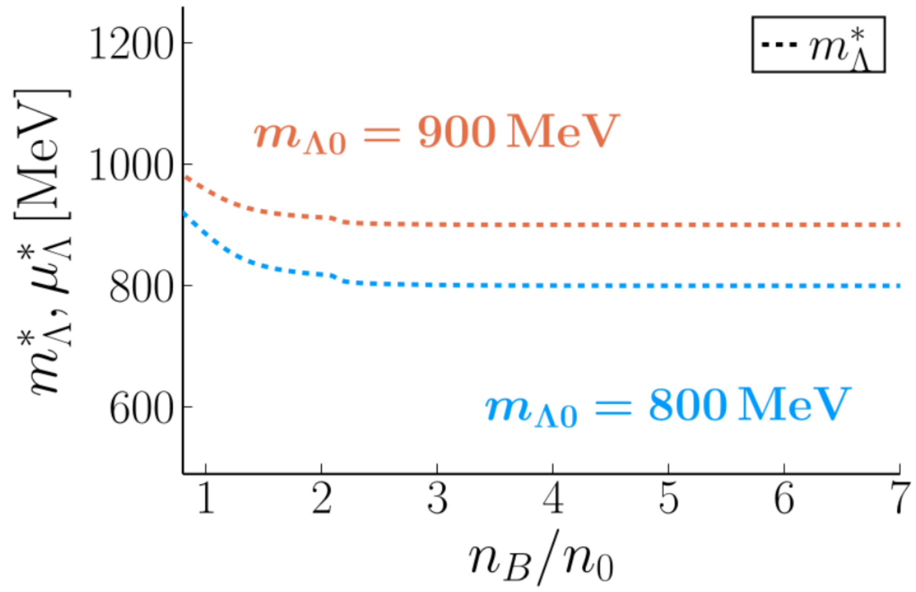
Hyperon mass is close to chiral invariant mass in the matter



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Because the repulsive force is weaker for larger chiral invariant masses.

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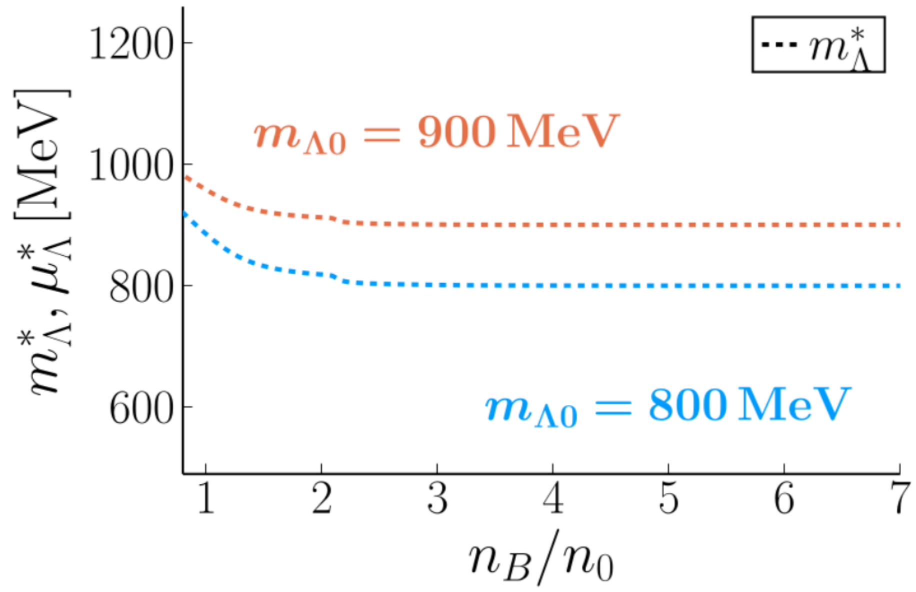

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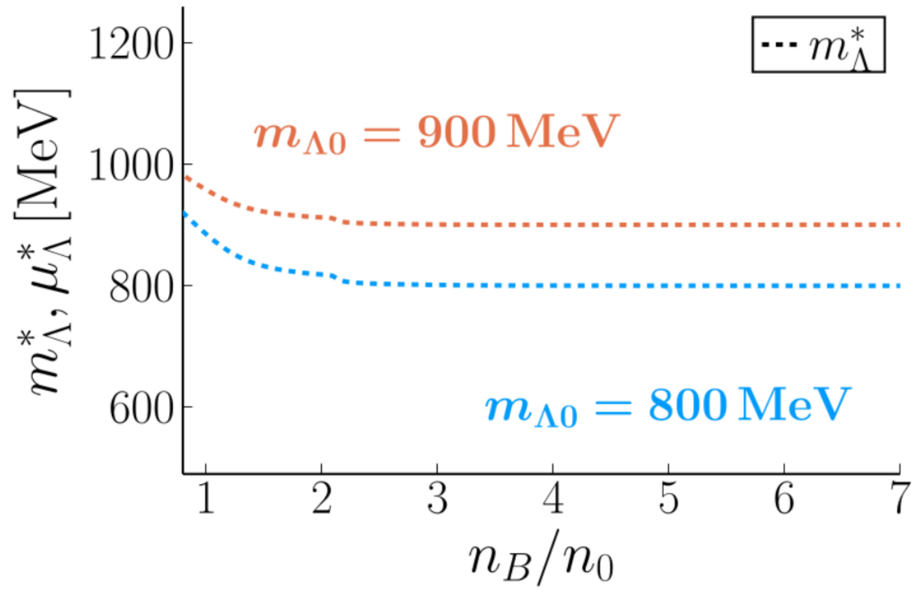

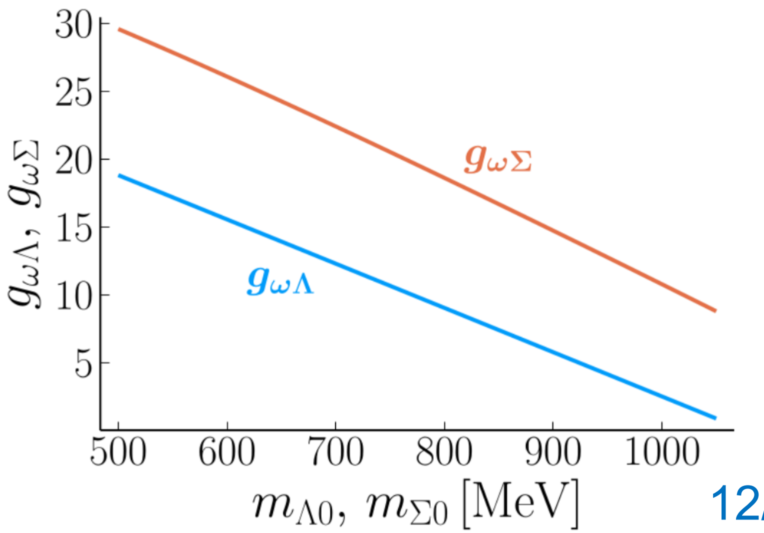

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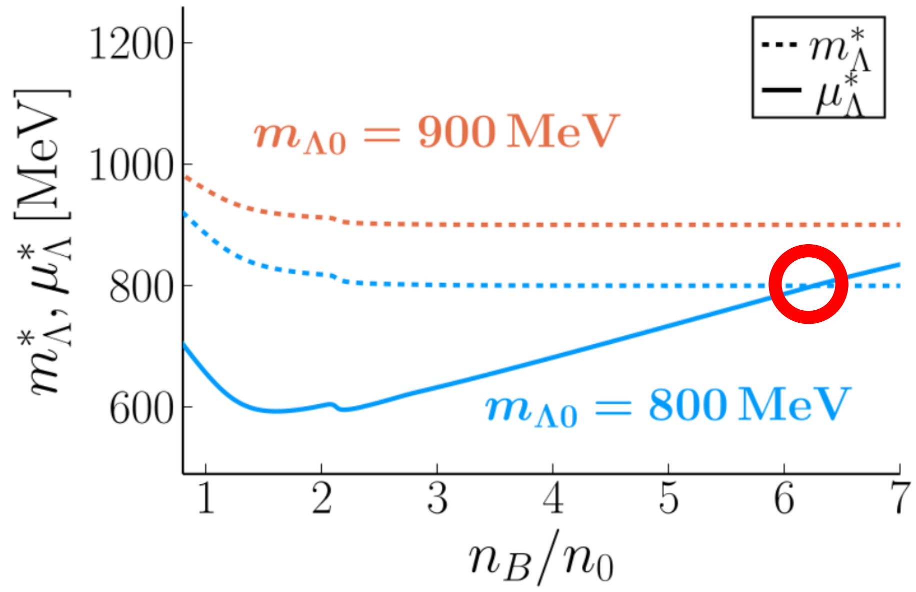
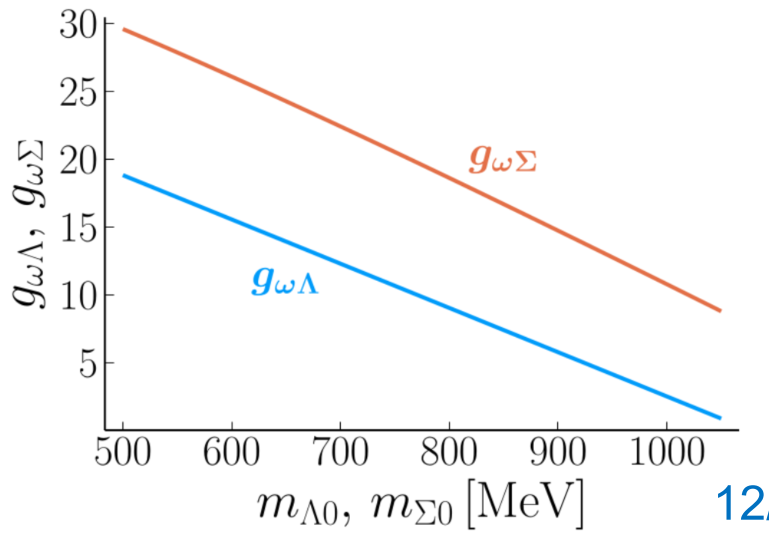


Effective Chemical Potential

$$\mu_\Lambda^* = \mu_B - g_{\omega\Lambda}\omega - \mu_s$$

greater $m_{\Lambda 0}$ ⇨ smaller $g_{\omega\Lambda}$ $\Rightarrow$ larger μ_Λ^* $\Rightarrow$ likely to appear



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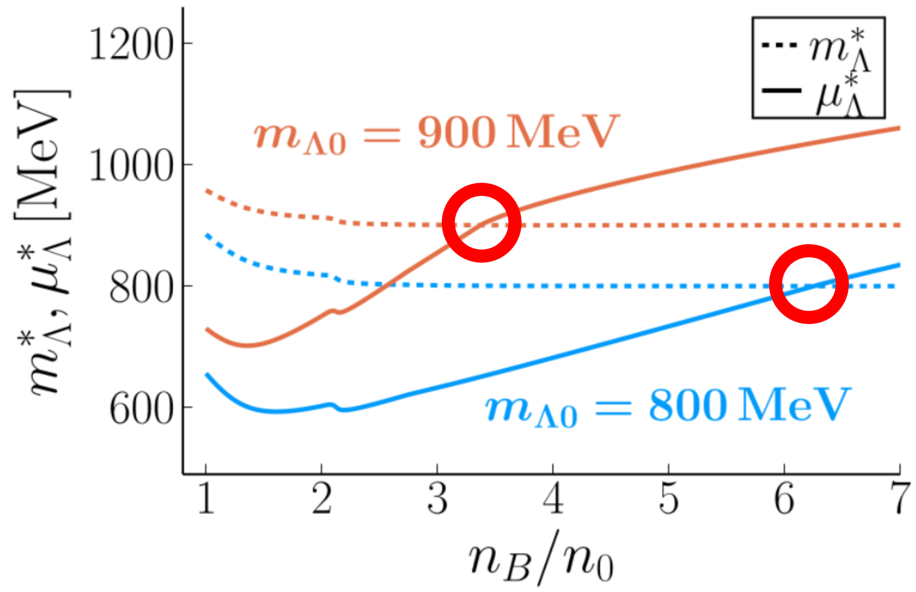

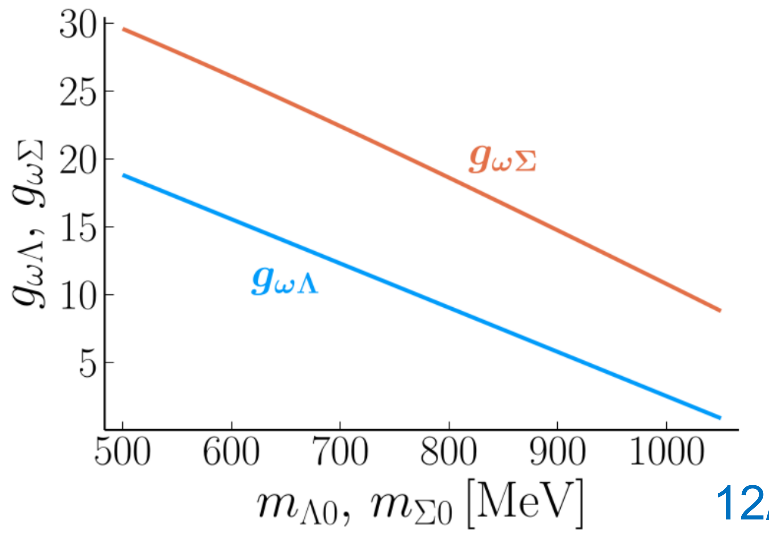

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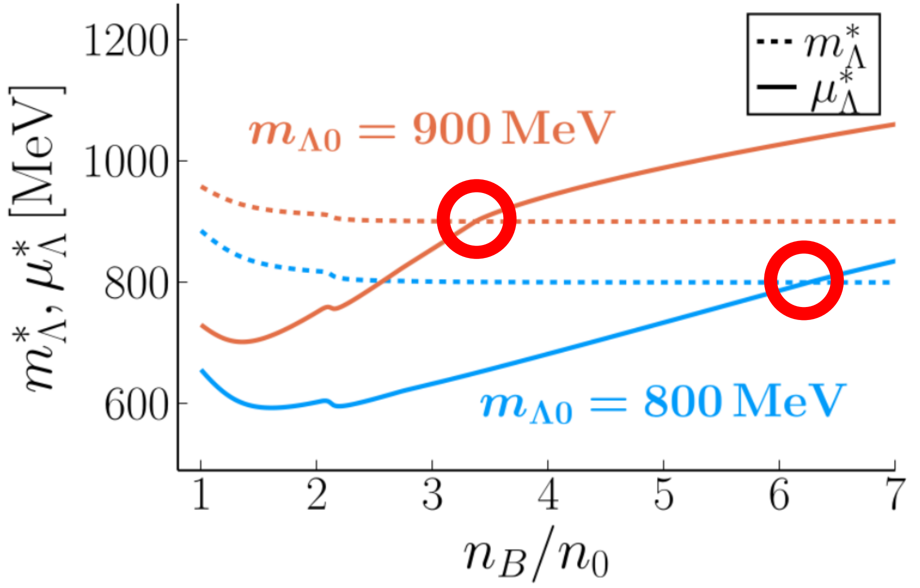
Effective Chemical Potential

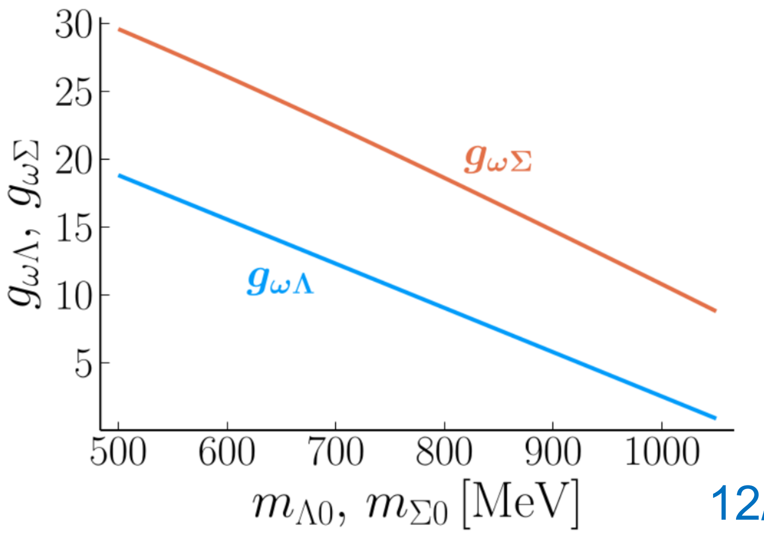
$$\mu^*_\Lambda = \mu_B - g_{\omega\Lambda}\omega - \mu_s$$

greater $m_{\Lambda 0} \Rightarrow$ smaller $g_{\omega\Lambda} \Rightarrow$ larger $\mu^*_\Lambda \Rightarrow$ likely to appear

Σ has same mechanism, but $U_\Lambda = -30\,\mathrm{MeV}$, $U_\Sigma = +30\,\mathrm{MeV}$

$U_\Lambda < U_\Sigma \Rightarrow g_{\omega\Lambda} < g_{\omega\Sigma} \Rightarrow$ less likely to appear than Λ





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- We constructed a hyperon model based on the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry
- We introduced Λ and Σ as $[(1,1)]$ and $[(3,1) \oplus (1,3)]$ representations
- Λ is more likely to appear when its chiral invariant mass is greater because the repulsive force becomes weaker
- The repulsive force for Σ is stronger than that of Λ, so Σ is less likely to appear

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- Λ is more likely to appear when its chiral invariant mass is greater because the repulsive force becomes weaker
- The repulsive force for Σ is stronger than that of Λ, so Σ is less likely to appear

Outlook

- Mass – Radius relation of neutron star
- Constraint on the chiral invariant masses
- Introduction of Ξ

BACKUP SLIDES

$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ vs. $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

- The $\bar{s}s$ condensate shows very weak density dependence up to the hyperon onset regime
- The model is expected to be valid up to about the hyperon onset density

  $\Longrightarrow$ It is possible to investigate the hyperon onset density using the framework of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry

Full Lagrangian (Nucleon)

$$
\begin{aligned}
\mathscr{L}_N = &\bar{N}_1 i\gamma^\mu D_\mu N_1 - g_{N1}\left(\bar{N}_{1L} M N_{1R} + \bar{N}_{1L} M^\dagger N_{1L}\right) \\
&+\bar{N}_2 i\gamma^\mu D_\mu N_2 + g_{N2}\left(\bar{N}_{2L} M^\dagger N_{2R} + \bar{N}_{2R} M N_{2L}\right) \\
&-m_{N0}\left(\bar{N}_1\gamma_5 N_2 - \bar{N}_2\gamma_5 N_1\right) \\
\xrightarrow[\text{SSB}]{} &\bar{N} i\gamma^\mu D_\mu N - \bar{N}\begin{pmatrix} m_{N_+} & \\ & m_{N_-}\end{pmatrix} N,
\end{aligned}
$$

$$
\text{where} \quad N = \begin{pmatrix} \cos\theta & \gamma_5 \sin\theta \\ -\gamma_5\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \end{pmatrix}, \quad \tan 2\theta = \frac{2m_{N0}}{(g_{N1}-g_{N2})\sigma}
$$

$$
m^*_{N_\pm} = \frac{1}{2}\sqrt{(g_{N1}+g_{N2})^2\sigma^2 + 4m_{N0}^2} \pm (g_{N1}-g_{N2})\sigma
$$

Full Lagrangian (Hyperon)

$$
\begin{aligned}
\mathscr{L}_{\Lambda,\Sigma} = \bar{\Lambda} i\gamma^\mu D_\mu \Lambda - G'_\Lambda \left[\bar{\Lambda}_L (M)^{l_1}{}_{r_1} (M^\dagger)^{r_1}{}_{l_1} \Lambda_R + \bar{\Lambda}_R (M)^{l_1}{}_{r_1} (M^\dagger)^{r_1}{}_{l_1} \Lambda_L\right] \\
+\mathrm{tr}\left[\bar{\Sigma}_1 i\gamma^\mu D_\mu \Sigma_1\right] - G_{\Sigma 1}\left[(\bar{\Sigma}_{1L})_{l_1 l_2} (M)^{l_1}{}_{r_1} (M)^{l_2}{}_{r_2} (\Sigma_{1R})^{r_1 r_2} + \mathrm{h.c.}\right] \\
+\mathrm{tr}\left[\bar{\Sigma}_2 i\gamma^\mu D_\mu \Sigma_2\right] + G_{\Sigma 2}\left[(\bar{\Sigma}_{2L})_{r_1 r_2} (M^\dagger)^{r_1}{}_{l_1} (M^\dagger)^{r_2}{}_{l_2} (\Sigma_{2R})^{l_1 l_2} + \mathrm{h.c.}\right] \\
-m_{\Lambda 0} \bar{\Lambda}\Lambda - m_{\Sigma 0} \mathrm{tr}\left[\bar{\Sigma}_1 \gamma_5 \Sigma_2 - \bar{\Sigma}_2 \gamma_5 \Sigma_1\right] \\
\xrightarrow{\text{SSB}} \bar{\Lambda} i\gamma^\mu D_\mu \Lambda - m_\Lambda \bar{\Lambda}\Lambda + \mathrm{tr}\left[\bar{\Sigma} i\gamma^\mu D_\mu \Sigma\right] - \mathrm{tr}\left[\bar{\Sigma}\begin{pmatrix} m_{\Sigma_+} & \\ & m_{\Sigma_-}\end{pmatrix}\Sigma\right]
\end{aligned}
$$

where $\Sigma = \begin{pmatrix} \cos\theta & \gamma_5 \sin\theta \\ -\gamma_5 \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix}\Sigma_1 \\ \Sigma_2\end{pmatrix}, \ \tan 2\theta = \dfrac{2m_{\Sigma 0}}{(G_{\Sigma 1} - G_{\Sigma 2})\sigma^2}$

$$
m^*_\Lambda = m_{\Lambda 0} + G_\Lambda \sigma^2
$$

$$
m^*_{\Sigma_\pm} = \frac{1}{2}\sqrt{(G_{\Sigma 1} - G_{\Sigma 2})^2 \sigma^4 + 4m^2_{\Sigma 0}} \pm \frac{1}{2}(G_{\Sigma 1} - G_{\Sigma 2})\sigma^2
$$

Full Lagrangian (Vector Meson)

- Decomposition of meson field: $M = \xi_L^\dagger \sigma \xi_R, \quad \xi_{L,R}(x) \xrightarrow{G\times H} h(x)\xi_{L,R}(x)g_{L,R}^\dagger$
- Maurer-Cartan one form: $\alpha_\mu^{\parallel,\perp} = \frac{1}{2i}\left[(D_\mu\xi_R)\xi_R^\dagger \pm (D_\mu\xi_L)\xi_L^\dagger\right], \quad \alpha_\mu^{\parallel,\perp} \xrightarrow{G\times H} h(x)\alpha_\mu^{\parallel,\perp}h^\dagger(x)$
- Nucleon - Vector meson coupling

$$\mathscr{L}_N^{\text{vector}} = \left\{ a_{\rho N}\left[\bar{N}_{1L}\xi_L^\dagger\gamma^\mu\alpha_\mu^{\parallel}\xi_L N_{1L} + (L \leftrightarrow R)\right] + a_{0N}\left(\bar{N}_{1L}\gamma^\mu \text{tr}\left[\alpha_\mu^{\parallel}\right]N_{1L}\right)\right\} + (1 \leftrightarrow 2)$$

$$\xrightarrow[\text{MFA}]{} - g_{\omega N}\omega \sum_{i=1,2} \bar{N}_i\gamma^0 N_i - \frac{1}{2}g_{\rho N}\rho \sum_{i=1,2} \bar{N}_i\gamma^0\tau^3 N_i$$

- Hyperon - Vector meson coupling

$$\mathscr{L}_{\Lambda,\Sigma}^{\text{vector}} = b_{0\Lambda}\left(\bar{\Lambda}_L\gamma^\mu \text{tr}\left[\alpha_\mu^{\parallel}\right]\Lambda_L + \bar{\Lambda}_R\gamma^\mu \text{tr}\left[\alpha_\mu^{\parallel}\right]\Lambda_R\right)$$

$$+c_{\rho\Sigma}\text{tr}\left[\bar{\Sigma}_{1L}\xi_L^\dagger\gamma^\mu\alpha_\mu^{\parallel}\xi_L\Sigma_{1L} + (L \leftrightarrow R)\right] + c_{0\Sigma}\text{tr}\left[\bar{\Sigma}_{1L}\gamma^\mu \text{tr}\left[\alpha_\mu^{\parallel}\right]\Sigma_{1L}\right] + (1 \leftrightarrow 2)$$

$$\xrightarrow[\text{MFA}]{} - g_{\omega\Lambda}\omega\bar{\Lambda}\gamma^0\Lambda - g_{\omega\Sigma}\omega \sum_{i=1,2} \text{tr}\left[\bar{\Sigma}_i\gamma^0\Sigma_i\right] - g_{\rho\Sigma}\rho \sum_{i=1,2} \text{tr}\left[\bar{\Sigma}_i\gamma^0\tau^3\Sigma_i\right]$$

Parameter Determination

Parameters

Nucleon:

m_{N0} g_{N1} g_{N2} $g_{\omega N}$ $g_{\rho N}$

Λ baryon:

$m_{\Lambda 0}$ G_{Λ} $g_{\omega\Lambda}$

Σ baryon:

$m_{\Sigma 0}$ $G_{\Sigma 1}$ $G_{\Sigma 2}$ $g_{\omega\Sigma}$ $g_{\rho\Sigma}$

Meson:

$\bar{\mu}^2$ λ_4 λ_6

Inputs

- Vacuum Mass:

  $N(939),\ N(1535),\ \Lambda(1116),\ \Sigma(1193),\ \Sigma(1620)$

- Pion Decay constant: $f_\pi = 92.4\,\mathrm{MeV}$
- Saturation Property
  - Saturation density: $n_0 = 0.16\,\mathrm{fm}^{-3}$
  - Binding energy: $B_0 = -16\,\mathrm{MeV}$
  - Symmetry energy: $S_0 = 31\,\mathrm{MeV}$
  - Incompressibility: $K_0 = 240\,\mathrm{MeV}$
  - Hyperon potential: $U_\Lambda = -30\,\mathrm{MeV}, U_\Sigma = +30\,\mathrm{MeV}$

$$U_i = m_i^*(\sigma) - m_i + g_{\omega i}\omega$$

Assumption

$$g_{\rho\Sigma} = g_{\omega\Sigma}$$

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$\color{red}{m_{N0}}$ $\boxed{g_{N1} \quad g_{N2}}$ $g_{\omega N}$ $g_{\rho N}$

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Hyperon model candidates

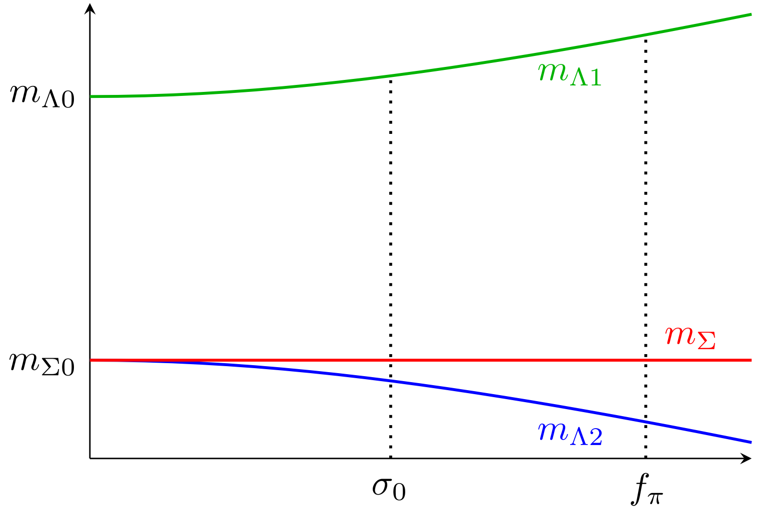
$$U_i = m_i^*(\sigma) - m_i + g_{\omega i}\omega, \quad U_\Lambda = -30\,\mathrm{MeV},\ U_\Sigma = +30\,\mathrm{MeV}$$

- $[(1,1)] \oplus [(2,2)]$

$$m_{\Lambda_{1,2}} = \frac{1}{2}(m_{\Lambda 0} + m_{Y0}) \pm \frac{1}{2}\sqrt{(m_{\Lambda 0} - m_{Y0})^2 + 4g_\Lambda^2\sigma^2}$$

$$m_\Sigma = m_{Y0}$$

$$g_{\omega\Lambda} < 0 : \text{atractive}$$



- $[(2,2)] \oplus [(3,1) \oplus (1,3)]$

$$m_{\Lambda 0} = m_{Y0}$$

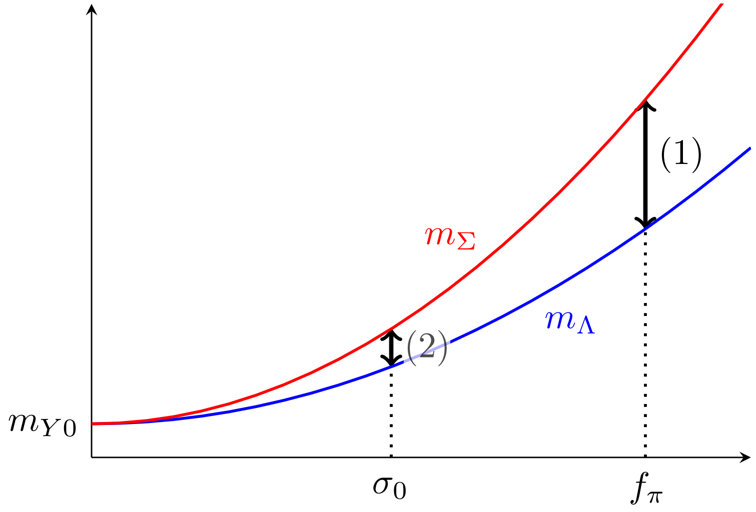
$$g_{\omega\Lambda} < 0 : \text{atractive}$$

- $[(2,2)]$

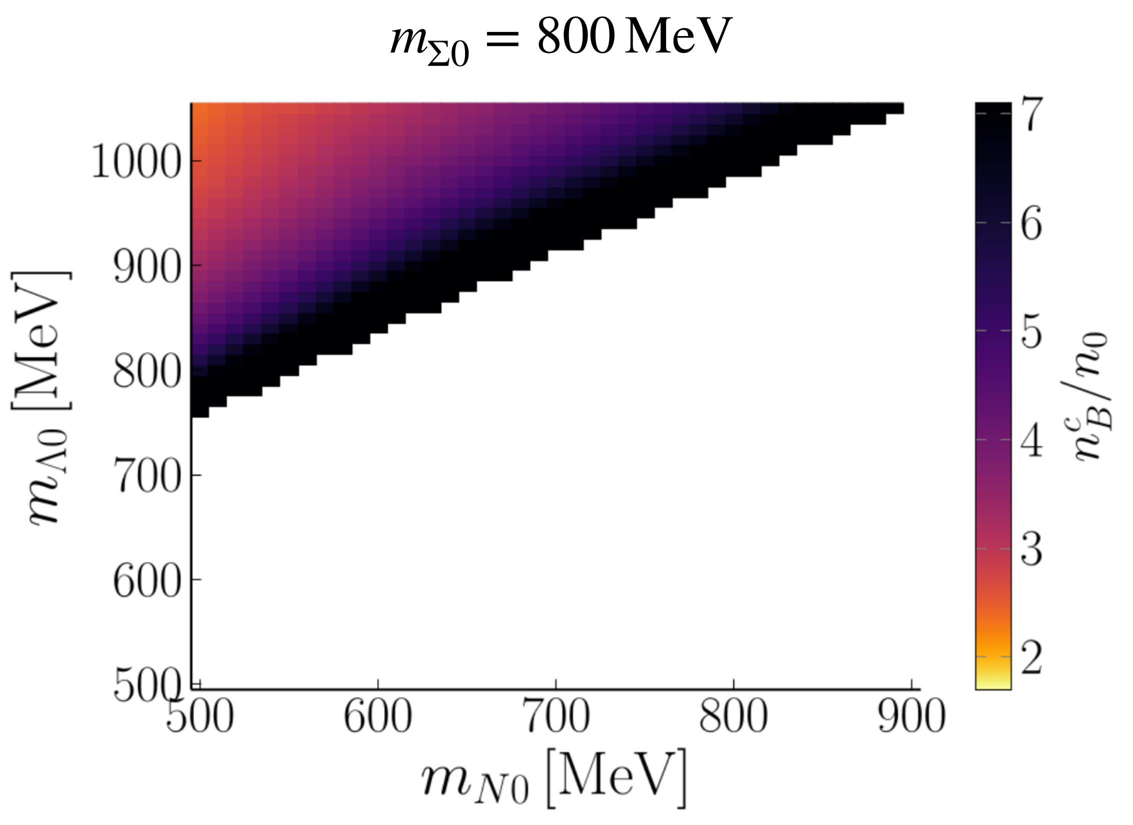
$$m_\Lambda = m_{\Lambda 0} + G_\Lambda \sigma^2$$
$$m_\Sigma = m_{\Sigma 0} + G_\Sigma \sigma^2$$

If $g_{\omega\Lambda} > 0$,

$$U_\Lambda < U_\Sigma \Rightarrow (2) > (1)$$



Λ Onset Density Dependence on m_{N0}



- Λ is less likely to appear when chiral invariant mass of nucleon is greater

The reason of this result

- $\omega - \Lambda$ coupling constant

$$g_{\omega\Lambda} \sim \frac{U_\Lambda - m_{\Lambda 0} - m_\Lambda}{\omega_0}$$

- ω mean field at saturation

$$\omega_0 = \frac{g_{\omega N}}{m_\omega^2} n_0$$

- $g_{\omega N}$ becomes smaller when m_{N0} is greater to balance attractive force