

Spectral Function Analysis of the Vector and Axial-vector Meson Correlators at Non-zero Temperature/Small Baryon Density in Lattice QCD



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Abstract

In the presence of the chiral symmetry, the vector and axial-vector meson channels are expected to be degenerate. With Generation 2 and Generation 2L FASTSUM lattice gauge field ensembles at non-zero temperature, the FASTSUM collaboration studied the mesonic/baryonic correlator ratios at finite temperature. They also studied the effect of small baryon chemical potential on the mesonic correlators using the Taylor expansion method. Here, using the Maximum Entropy Method (MEM), we reconstruct spectral functions of corresponding mesonic channels at non-zero temperature. Starting from zero baryon chemical potential, we investigate the dependence of spectral functions on small baryon chemical potential using the leading-order Taylor expansion, to understand a possible origin of the degeneracy in the chiral symmetry restored phase. In this preliminary report, we discuss the systematic effects and motivate future studies on Generation 3 ensembles which have a finer temporal lattice spacing.

Introduction

• Chiral symmetry and V-A degeneracy

- ✓ Low-T phase (below T_{PC}): chiral symmetry broken, light mesons $\rho(V)$ and $a_1(A)$
- ✓ High-T phase (above T_{PC}): chiral symmetry restored, degenerate V-A meson

• Baryon chemical potential μ_B : Determination of $T_{PC}(\mu_B)$ ^[1]

- ✓ Taylor expansion around $\mu_B = 0$

$$G(\tau; \mu_q) = G(\tau) \Big|_{\mu_q=0} + \frac{1}{2} \frac{\mu_q^2}{T^2} T^2 \frac{\partial^2 G}{\partial \mu_q^2} \Big|_{\mu_q=0} + O\left(\frac{\mu_q^4}{T^4}\right), (\mu_B = 3\mu_q)$$

$$\frac{\partial^2 G}{\partial \mu_q^2} \Big|_{\mu_q=0} = \left\langle g''(\tau) + g \frac{\Delta''}{\Delta} \right\rangle_B - \langle g \rangle_B \left\langle \frac{\Delta''}{\Delta} \right\rangle_B,$$

where $g(\tau) = \langle J_H(\tau) J_H^\dagger(0) \rangle_F$,

$$J_H = \bar{\psi} \Gamma_H \psi \rightarrow \gamma_i(V), \gamma_5 \gamma_i(A)$$

- ✓ To understand the origin of the degeneracy

• Spectral functions $\rho(\omega)$

- ✓ contribution of the states
- ✓ real time physics
- ✓ qualitative / quantitative analysis

$$G(\tau) = \int d\omega K(\omega, \tau) \rho(\omega),$$

$$K(\omega, \tau) = \frac{\cosh\left(\omega\tau - \frac{\omega}{2T}\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

- ✓ How does the degeneracy appear in the spectral functions?

Maximum Entropy Method(MEM)

• Reconstruction of spectral functions: An ill-posed problem

- ✓ $G(\tau) \sim O(10) \rightarrow \rho(\omega) \sim O(10^3)$ not uniquely determined
- ✓ Strategies : Maximum Entropy method, BR method, Backus-Gilbert method, Neural Networks, ...

• MEM: Entropy as a regulator^[2]

- ✓ Minimize χ^2 with maximizing entropy
- ✓ Positive semi-definite and unique solution if it exists

• Benchmark: $T \sim 0$ ($N_\tau = 128$) results

- ✓ Gen2 / Gen2L ensembles^[1] $\rightarrow$ Gen2L: lighter pion mass

	Generation 2	Generation 2L
M_π [MeV]	391(3)	239(1)
T_{pc} [MeV]	182(2)	167(3)
a_τ [fm]	0.03482(26)	0.03246(7)
a_s [fm]	0.11992(92)	0.11208(31)
N_s	24	32

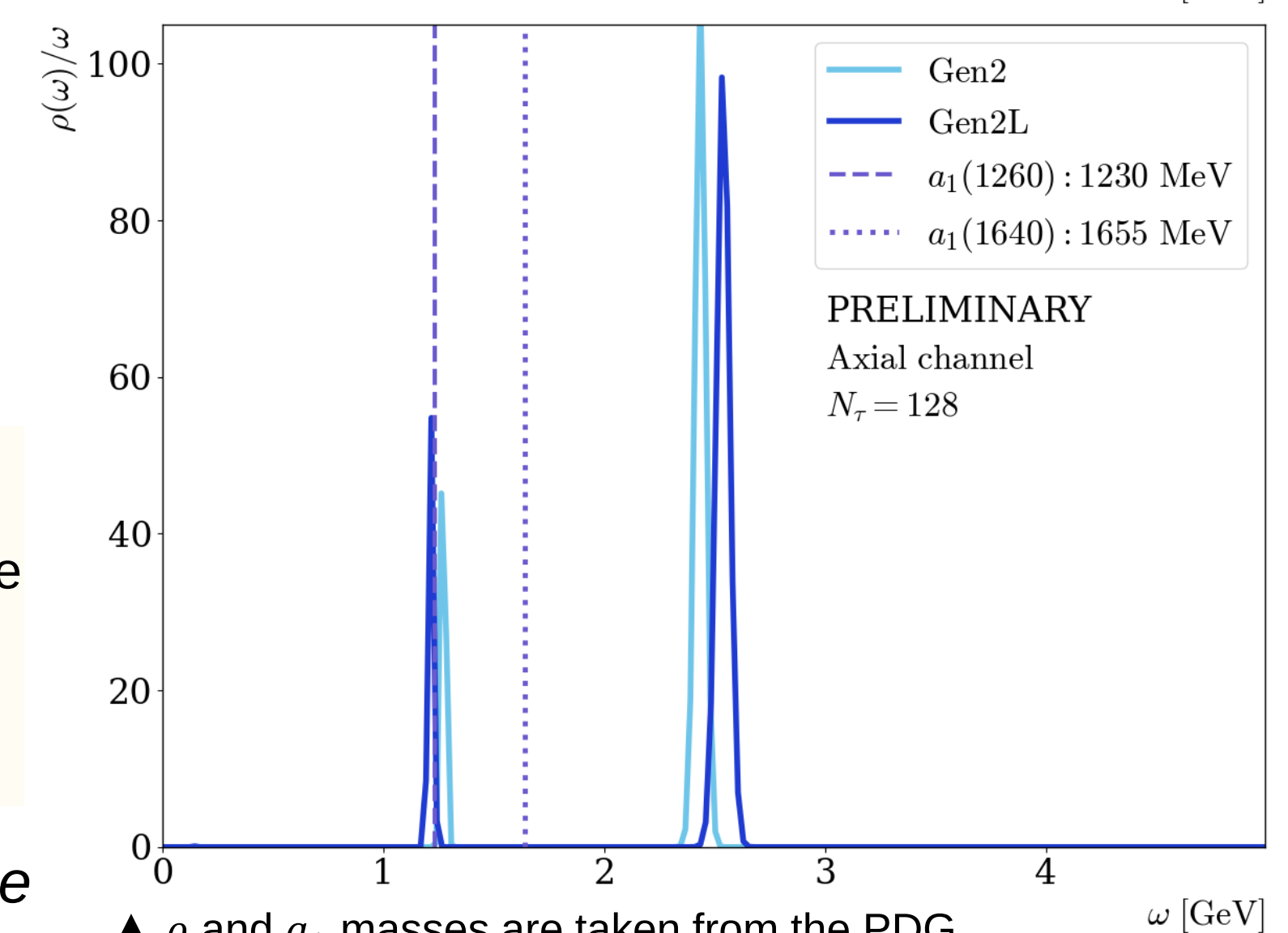
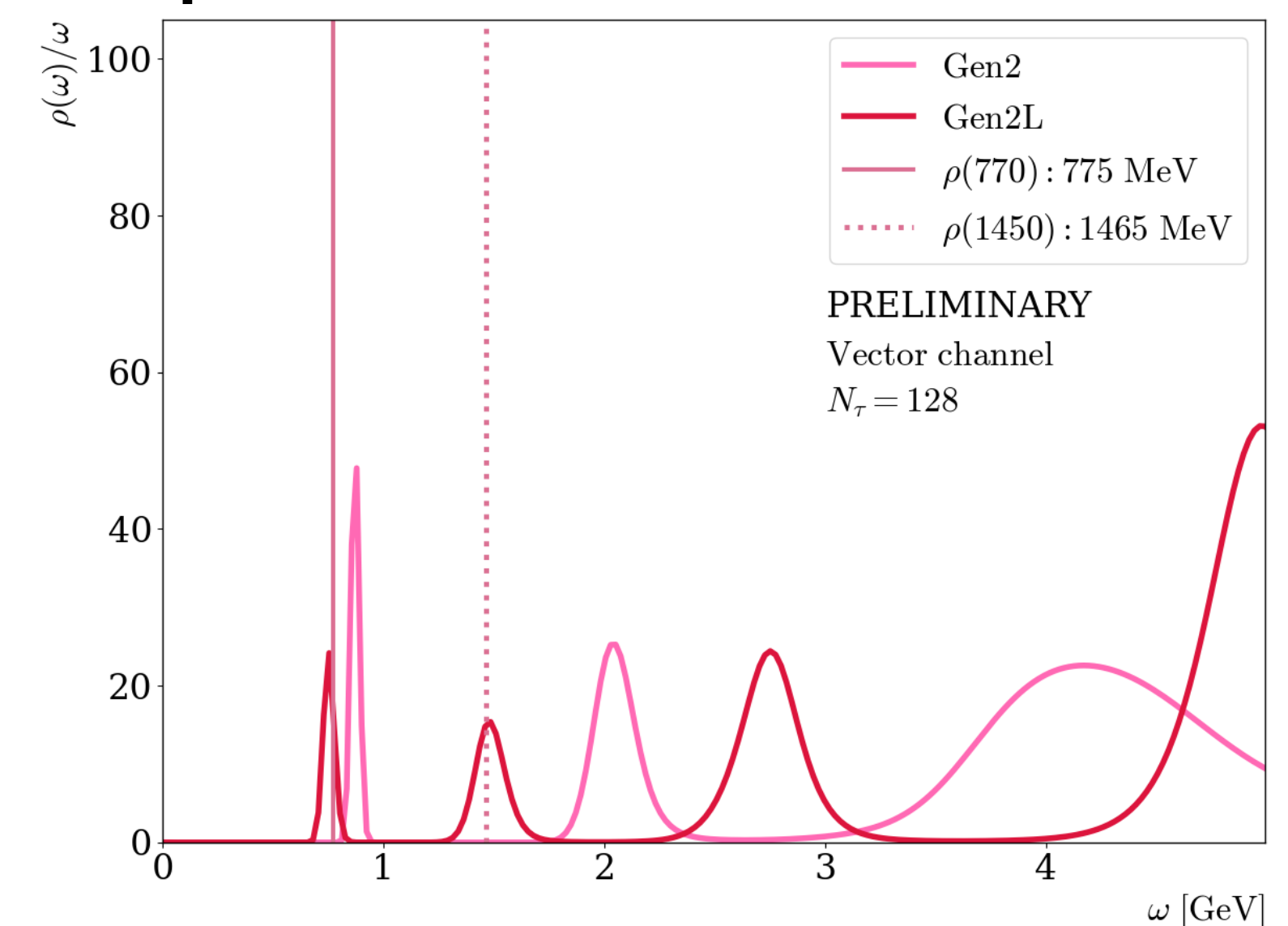
✓ Vector channel

Gen2L: ground state peak closer to physical $\rho(770)$ mass, $\rho(1450)$ -like excited state

✓ Axial-vector channel

$a_1(1260)$ -like ground state excited states not well constrained

- ✓ Clear V-A splitting expected in symmetry broken phase



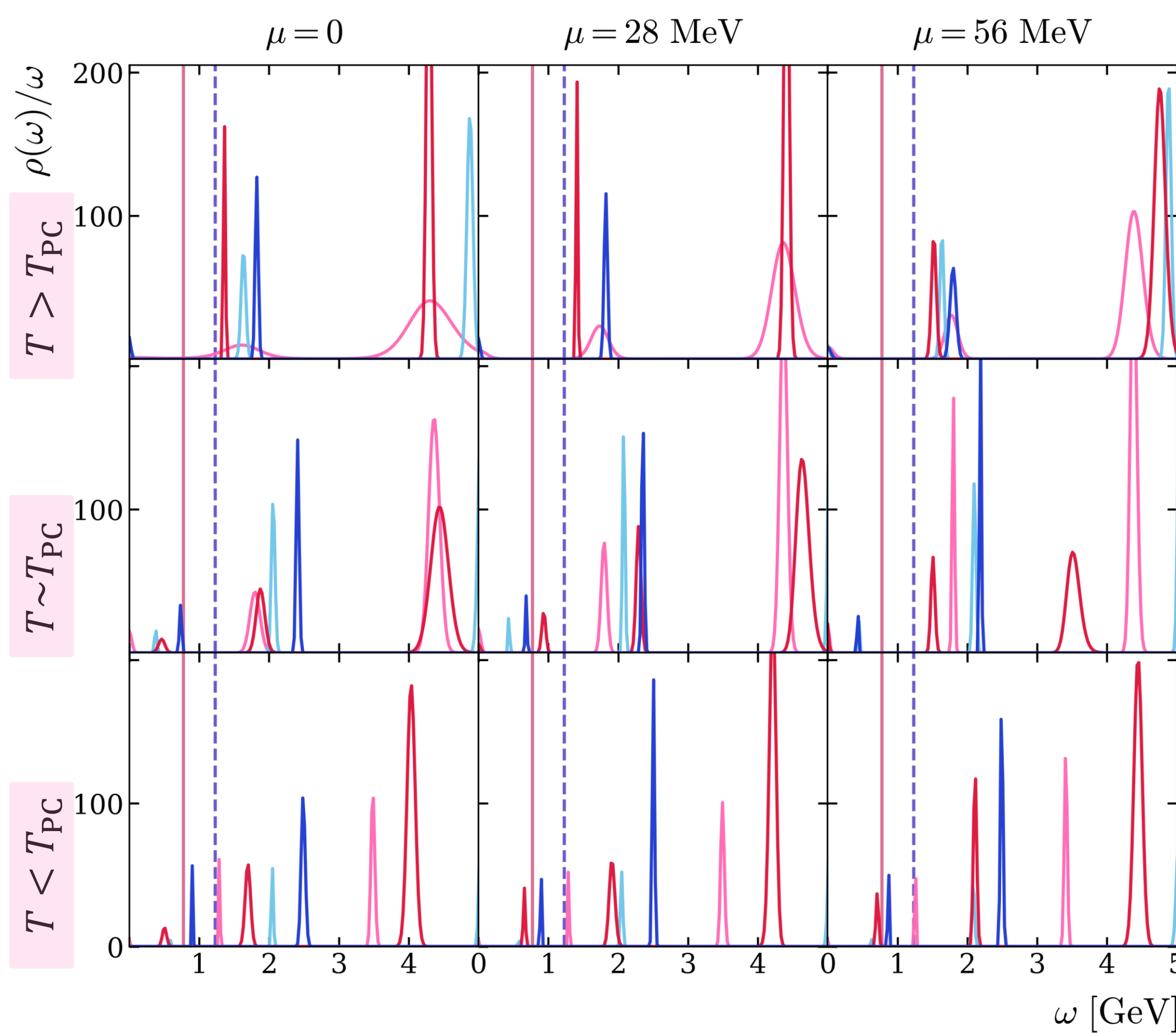
▲ ρ and a_1 masses are taken from the PDG
The same colors and ρ/a_1 lines are used in the following plots

Preliminary Results and Systematic Studies

Non-zero T and μ_B dependence of spectral functions

• PRELIMINARY

- Rows: increasing T from bottom to top
- Columns: increasing μ_B from left to right



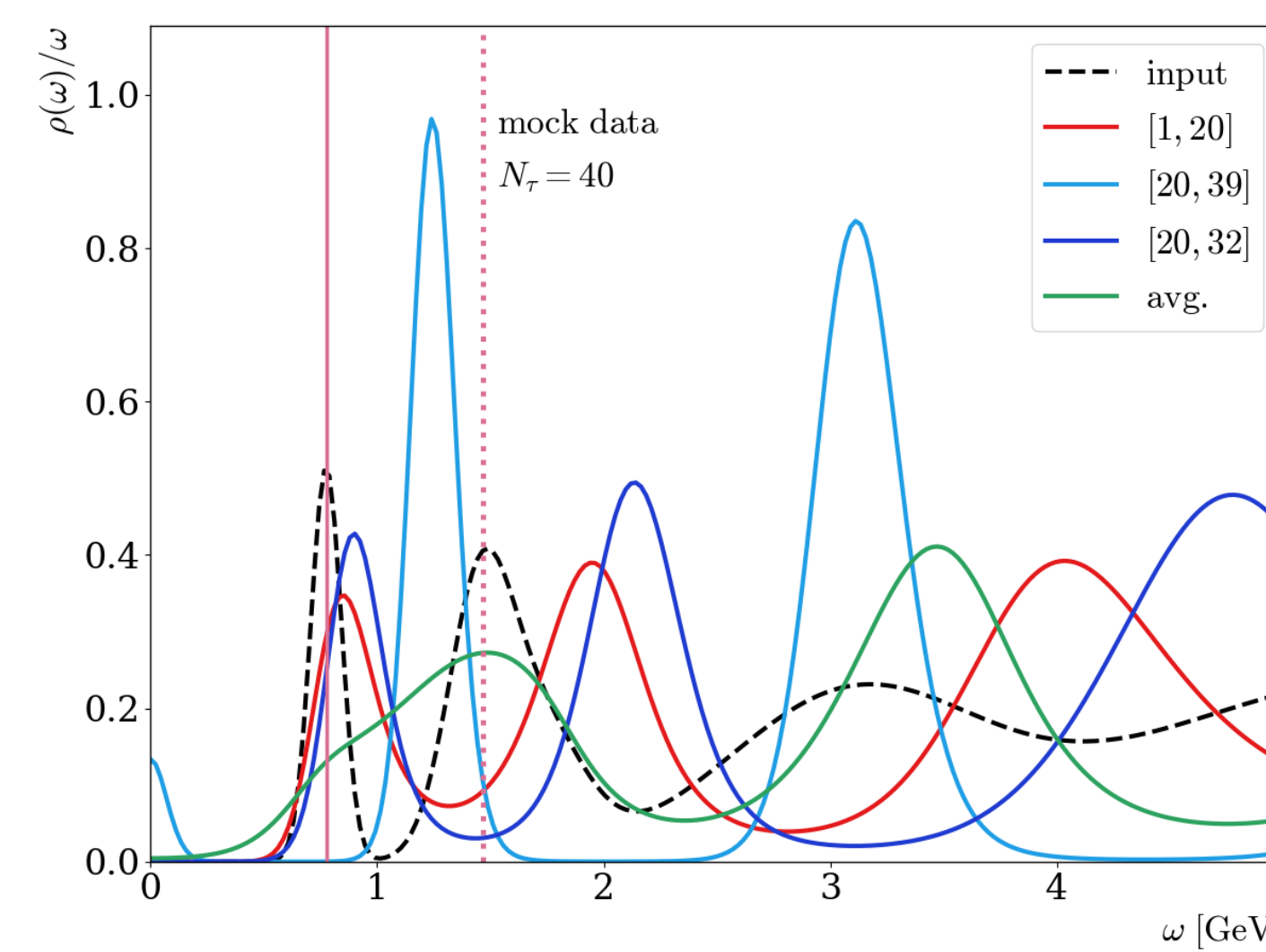
- ✓ Strong T dependence
- ✓ No clear μ_B trends
- ✓ Reduced V-A separation at low- ω above T_{PC}

Reconstruction sensitivity: mock data analysis

How to generate mock data

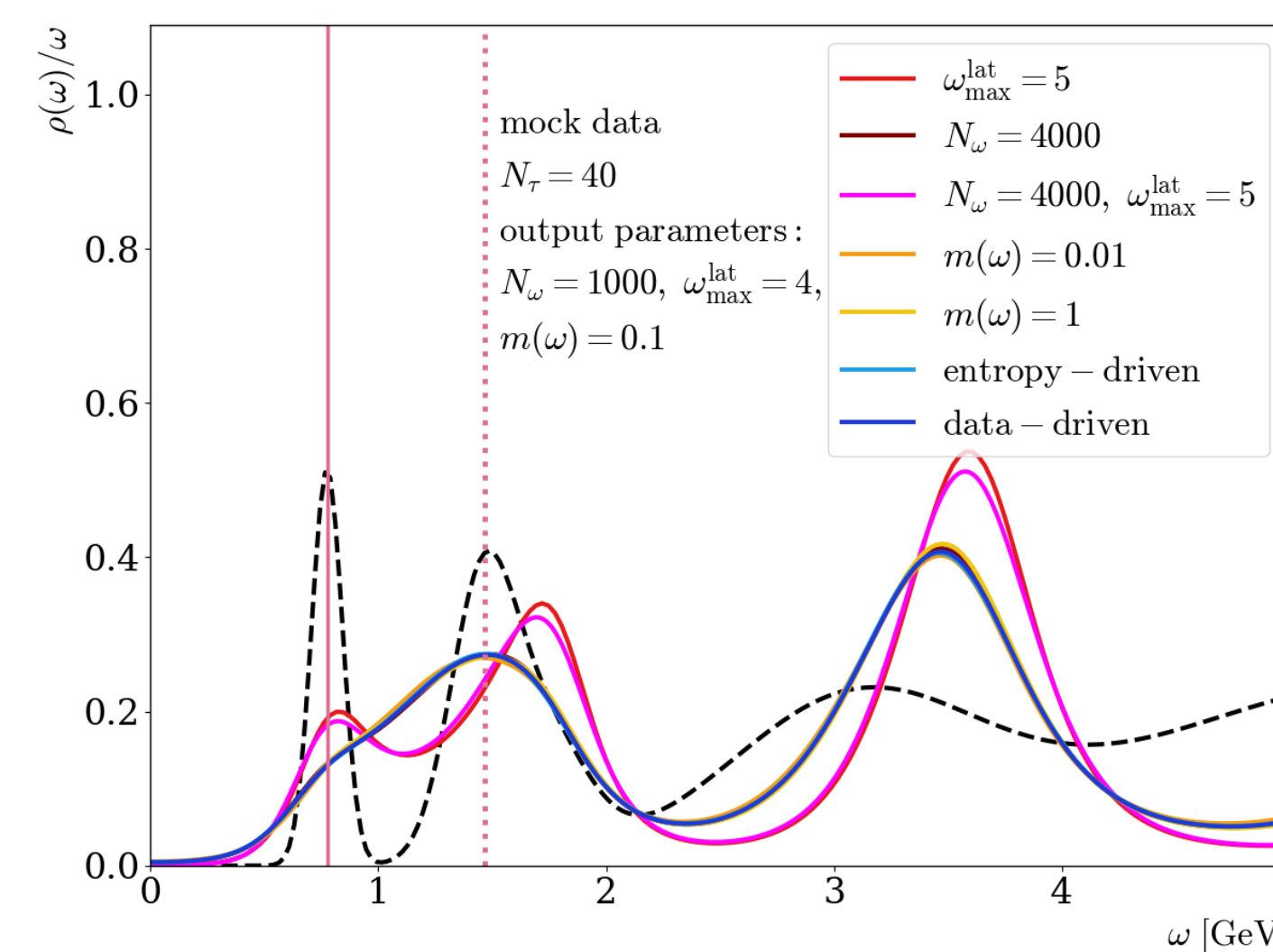
1. Input spectrum: finite-width peaks and continuum structures
2. Generate $N_\tau = 40$ mock correlators with Gen2-like magnitude and noise correlations
3. Reconstruct spectra with the same MEM setup for different temporal windows
4. Compare input and output to assess reconstruction systematics

- ✓ To distinguish physical spectral features from reconstruction artifacts



◀ Temporal window dependence of mock data reconstruction

- ✓ Peak structures sensitive to the temporal window
- ✓ Similar τ_{min} dependence to the lattice data
- ✓ Short distance effect and MEM artifacts entangled



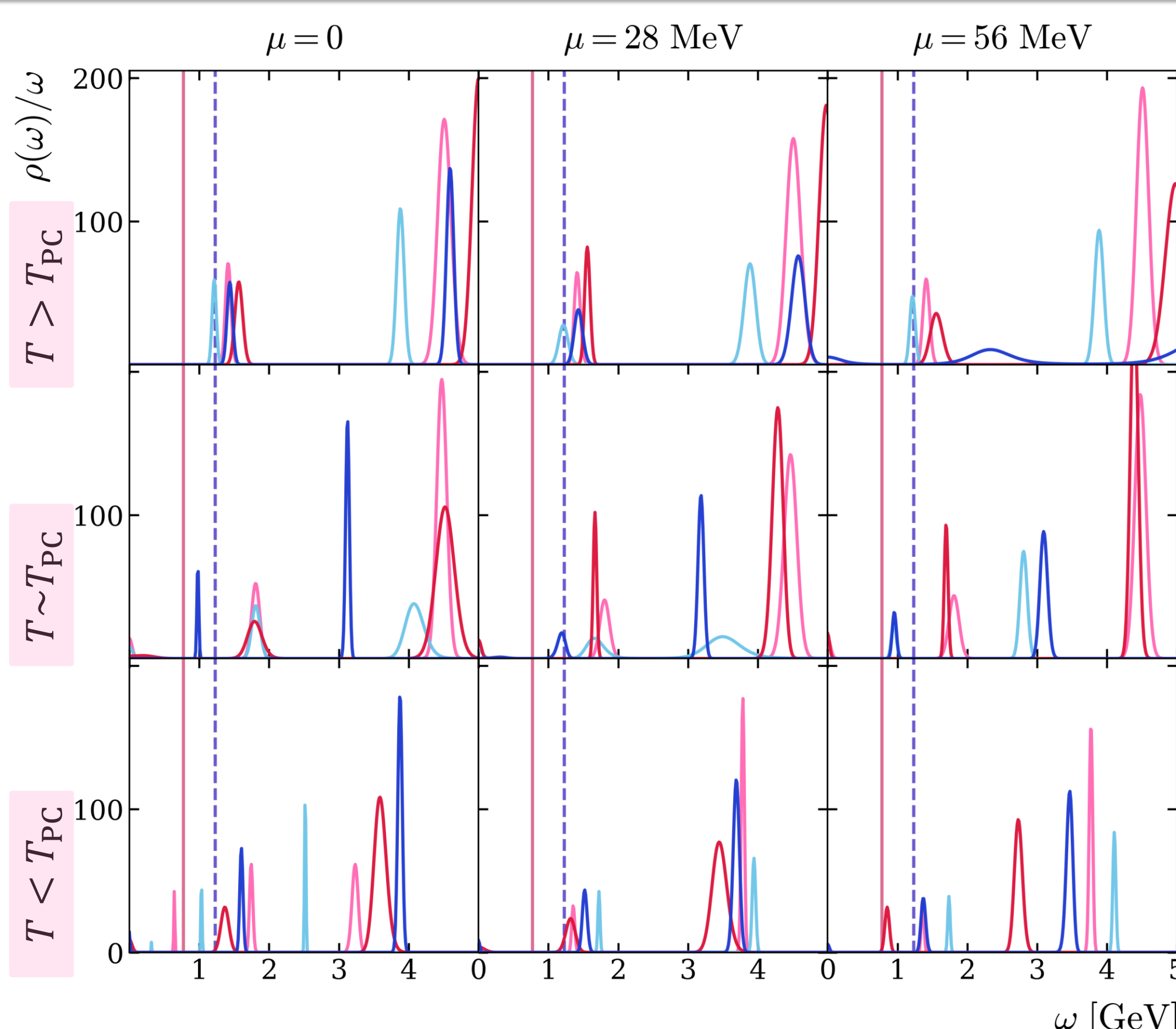
◀ Sensitivity to MEM parameters, input and avg. lines included

- ✓ Energy window modifying the overall spectral structures
- ✓ Peak details sensitive to energy window and $\Delta\omega$ convergence
- ✓ Robust against auxiliary parameters and optimization path

τ_{min} dependence of spectral functions

• PRELIMINARY

- Same $T - \mu_B$ layout
- Larger $\tau_{min} = N_\tau/4$ reducing short distance effect and # data points



- ✓ Strong τ_{min} sensitivity
- ✓ Enhanced V-A similarity at larger τ_{min}
- ✓ Physical interpretation at peak-level inconclusive

Conclusions and Outlook

- V-A channel spectral functions are studied using MEM on FASTSUM lattice ensembles
- V-A splitting expected in chiral symmetry broken phase reproduced at $T \sim 0$
- Spectral functions become similar above T_{PC}
- Degeneracy and μ_B dependence is not yet resolved
- Current short distance effect and MEM systematics requires better temporal resolution
- Gen3 ensembles with finer temporal lattice spacing will provide improved constraints

References

- [1] A. Smecca et al., PHYSICAL REVIEW D 112, 114509 (2025)
- [2] M. Asakawa et al. Prog. Part. Nucl. Phys. 46 (2001) 459-508