

The fate of the $U(1)_A$ symmetry and the static quark potential at high temperature

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Alexander Rothkopf

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[arXiv:2604.11916, 2608.xxxxx]

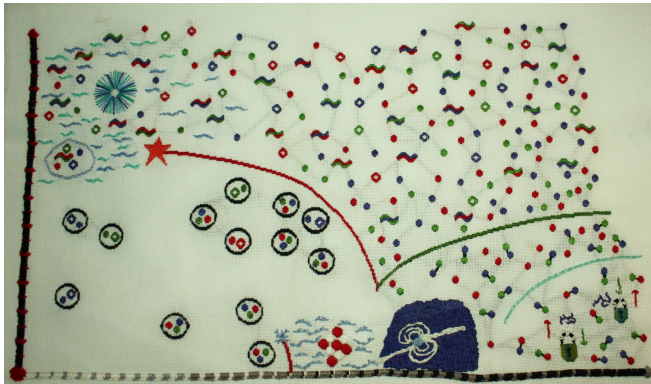
Outline

Introduction, lattice setup

$U(1)_A$ restoration

Static quark potential

Background



Background

- ▶ Hints of nontrivial phase structure of QCD at high T
- ▶ Chiral-spin symmetry?
- ▶ Chirally symmetric but confining?
- ▶ Vortex percolation transition at $\sim 2T_\chi$?
- ▶ $U(1)_A$ symmetry restoration still in dispute

Background

- ▶ Hints of nontrivial phase structure of QCD at high T
- ▶ Chiral-spin symmetry?
- ▶ Chirally symmetric but confining?
- ▶ Vortex percolation transition at $\sim 2T_\chi$?
- ▶ $U(1)_A$ symmetry restoration still in dispute
- ▶ Dispute about static quark potential
 - some results show unscreened $\text{Re } V(r)$ well above T_χ

Simulation parameters

[PRD **76** 194513 (2007), HadSpec Collab, PRD **79** 034502 (2009), JSPC **4** 100258 (2025)/2510.02954]

Gen	N_f	ξ	a_s (fm)	a_τ^{-1} (GeV)	m_π (MeV)	N_s	L_s (fm)
1	2	6.0	0.162	7.35	490	12	1.94
2	2+1	3.45	0.121	5.63	390	24	2.95
						32	3.94
2L	2+1	3.45	0.112	6.08	240	32	3.58
3	2+1	7.0	0.108	12.82	380	32	3.46

Simulation parameters: temperatures

Generation 2			Generation 2L			Generation 3		
N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c
128	44	0.24	128	47	0.42	256	50	0.28
			64	95	0.59	128	100	0.55
			56	109	0.67	112	114	0.64
48	117	0.63	48	127	0.78	96	133	0.73
40	141	0.76	40	152	0.94	80	160	0.88
						76	169	0.93
36	156	0.84	36	169	1.04	72	178	0.98
						68	188	1.04
32	176	0.95	32	190	1.17	64	200	1.10
28	201	1.09	28	217	1.34	56	229	1.26
24	235	1.27	24	253	1.56	48	267	1.47
20	281	1.52	20	304	1.87	40	320	1.76
						36	356	1.96
16	352	1.90	16	380	2.34	32	400	2.20
			12	507	3.12	24	534	2.93
			8	760	4.55			

Chiral and $U(1)_A$ symmetries

Classical symmetry of QCD:

$$U(N_f)_L \otimes U(N_f)_R \simeq SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_A \otimes U(1)_B$$

- ▶ $D\chi$ SB: $SU(N_f)_L \otimes SU(N_f)_R \rightarrow SU(N_f)_V$
Restored above $T_{pc} \approx 150 \text{ MeV}$
- ▶ $U(1)_A$: broken by the axial anomaly
effectively restored at high T (?)
- ▶ Can affect the order and universality class of chiral transition in chiral limit

Correlator degeneracies

Chiral:

$$\rho \longleftrightarrow a_1$$

$U(1)_A$:

$$\pi \longleftrightarrow \delta(a_0)$$

Degeneracy ratios:

$$\mathcal{R}_{XY}(\tau) = \frac{\tilde{G}_X(\tau) - \tilde{G}_Y(\tau)}{\tilde{G}_X(\tau) + \tilde{G}_Y(\tau)} \qquad \tilde{G}_X(\tau) = \frac{G_X(\tau)}{G_X(N_\tau/2)}$$

Susceptibilities:

$$\chi_X = \frac{1}{2} \sum_{\tau} G_X(\tau), \quad \tilde{\chi}_X^{\text{norm}}(T, \tau_{\min}) = \frac{1}{N_\tau} \sum_{\tau=\tau_{\min}}^{N_\tau/2} \tilde{G}_X(\tau; T)$$

Degeneracy if $\chi_X - \chi_Y = 0$

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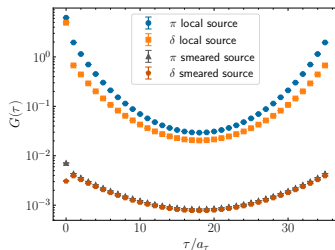
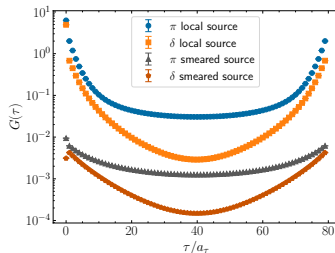
Degeneracy if $\chi_X - \chi_Y = 0$

Normalised correlators $\tilde{G}(\tau)$ eliminate renormalisation factors.

All quantities may be evaluated with **local** or **smeared** operators

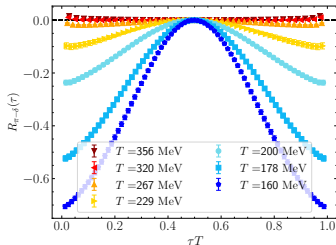
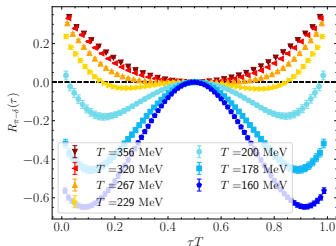
PS-S degeneracy: correlators

- Top: $T = 160$ MeV
 Bottom: $T = 356$ MeV
- PS-S degeneracy at high T



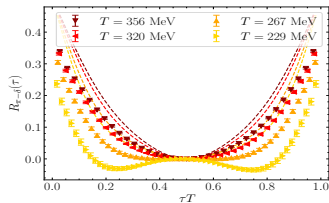
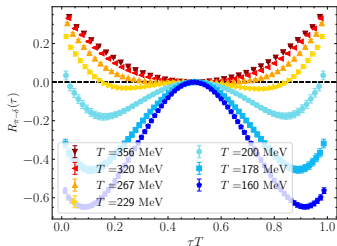
PS-S degeneracy: correlators

- ▶ Top: local
Bottom: smeared
- ▶ PS-S degeneracy at high T
- ▶ **Smearing** gives improved short-distance behaviour



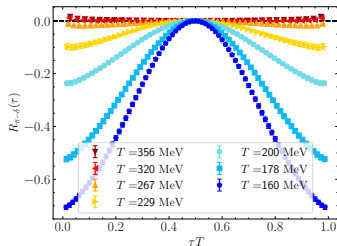
PS-S degeneracy: correlators

- ▶ PS-S degeneracy at high T
- ▶ **Smearing** gives improved short-distance behaviour
- ▶ — removes short-distance lattice artefacts



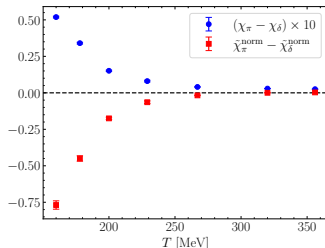
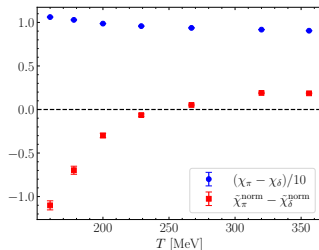
PS-S degeneracy: correlators

- ▶ PS-S degeneracy at high T
- ▶ **Smearing** gives improved short-distance behaviour
- ▶ Degeneracy for $T \gtrsim 300$ MeV

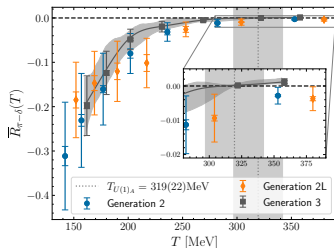


PS-S degeneracy: susceptibilities

- Top: local
 Bottom: smeared
- Normalised $\Delta\chi \rightarrow 0$ at
 ~ 320 MeV



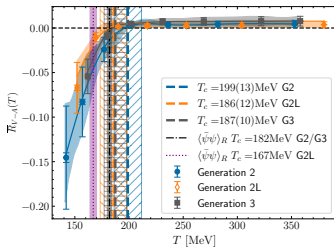
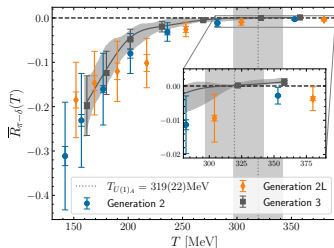
PS-S degeneracy: susceptibilities



- Normalised $\Delta\chi \rightarrow 0$ at $\sim 320 \text{ MeV}$
- Similar results for Gen2, Gen2L

PS-S degeneracy: susceptibilities

- Normalised $\Delta\chi \rightarrow 0$ at ~ 320 MeV
- Similar results for Gen2, Gen2L
- Well above T_χ



Summary [$U(1)_A$]

- ▶ Correlator ratios can be used to probe symmetry restoration
- ▶ T_χ (V–A degeneracy) agrees with traditional methods
- ▶ $U(1)_A$ effectively restored at $T \gtrsim 300 \text{ MeV} \sim 2T_\chi$

Summary [$U(1)_A$]

- ▶ Correlator ratios can be used to probe symmetry restoration
- ▶ T_χ (V–A degeneracy) agrees with traditional methods
- ▶ $U(1)_A$ effectively restored at $T \gtrsim 300 \text{ MeV} \sim 2 T_\chi$
- ▶ Repeat with lighter/physical quarks: Gen2P, Gen3L, Gen3P
- ▶ Other degeneracies: chiral-spin symmetry?

Static quark potential

- ▶ Long (pre-)history of potential models
- ▶ Definition of real-time potential, relation to Wilson loop/lines
- ▶ Real and imaginary parts (peak position and width)
- ▶ Screening or no screening: different methods give different results

Static quark potential: Methods (I)

Bayesian spectral function reconstruction

Bayes theorem

$$P(\rho|DI) = \frac{P(D|\rho I)P(\rho|I)}{P(D|I)}$$

Parametrise prior probability by

$$P(\rho|I) \propto e^{\alpha S[\rho]} \implies P(\rho|DI) \propto e^{-L[D,\rho] + \alpha S[\rho]}$$

where L is the standard likelihood (χ^2)

Spectral function $\rho(\omega)$ is expressed in terms of default model $m(\omega)$

$$\rho(\omega) = m(\omega) \exp\left[\sum_{k=1}^{N_b} b_k u_k(\omega)\right]$$

Different prescriptions for S, u_k : MEM and BR

Static quark potential: Methods (II)

UV subtraction method

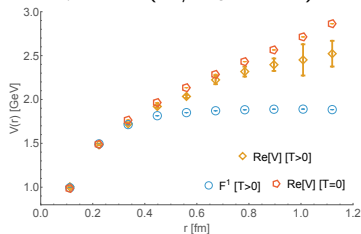
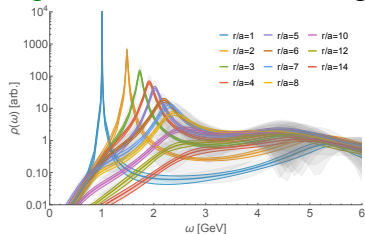
- ▶ Subtract T -independent “UV” correlator
- ▶ UV corr taken to be $T = 0$ corr minus lowest state(s)
- ▶ Fit UV subtracted m_{eff} to straight line (assume gaussian $\rho(\omega)$)
- ▶ **or** fit to lorentzian $\rho(\omega)$
- ▶ Consider **ground** and **first excited** state

Bala–Datta method

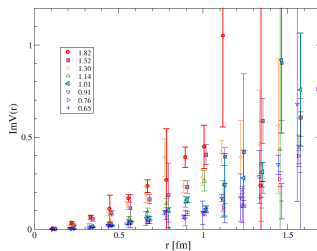
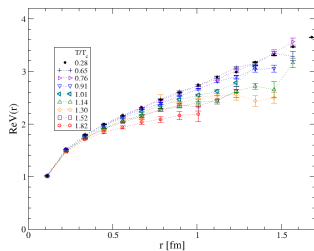
- ▶ Based on requirement for existence of real-time potential
- ▶ Decompose into linear (ReV) + periodic (ImV) terms
- ▶ Requires assumptions about shape of spectral peak

BR results in detail

High statistics: ~ 3000 configs at $N_\tau = 24$ ($T/T_c = 1.5$)

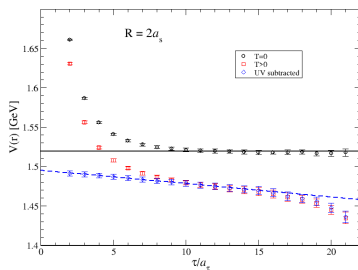


BR results: summary



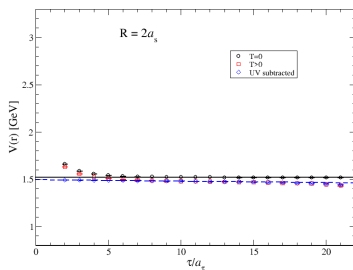
UV subtraction method: details

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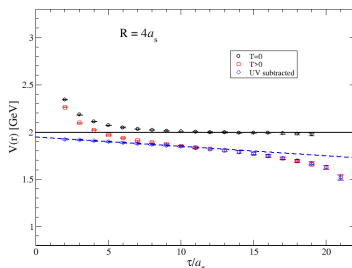
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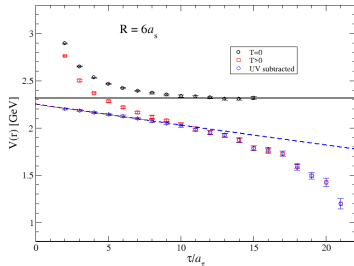
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- UV-sub V_{eff} has shape typical of **lorentzian** not gaussian

UV subtraction method: details

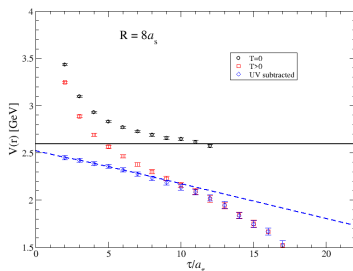
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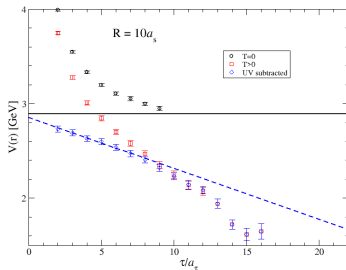
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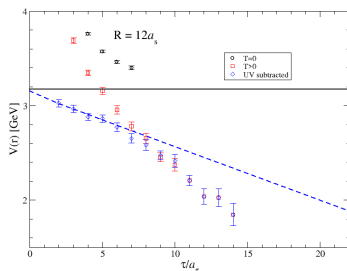
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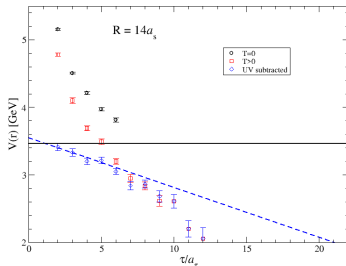
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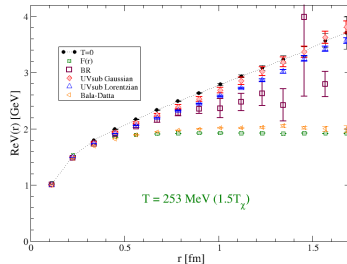
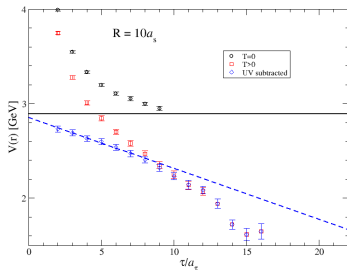
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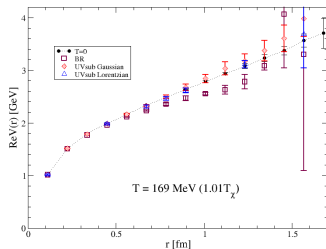
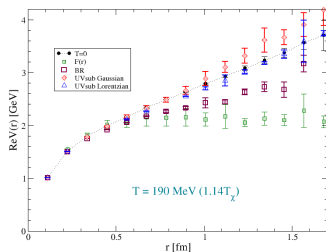
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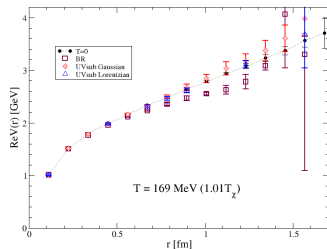
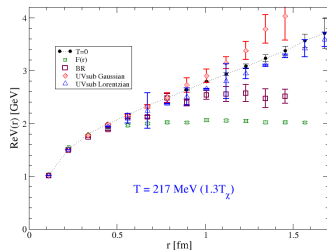


- ▶ UV-sub V_{eff} has shape typical of **lorentzian** not gaussian
- ▶ $V(r; T) < V(R; 0)$ at small + moderate r
- ▶ Agreement between methods at small r
- ▶ All methods break down at large r ?
- ▶ Only Bala–Datta reproduces free energy (**note assumptions!**)

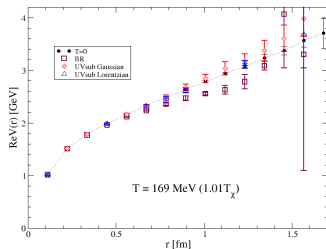
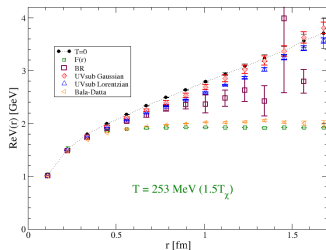
Comparison, temperature dependence



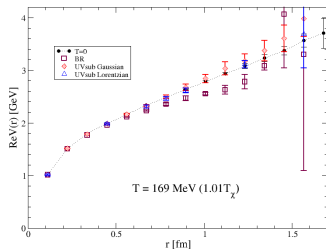
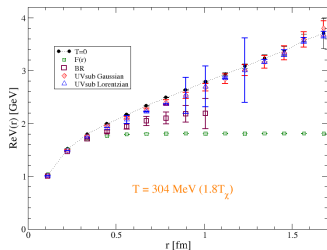
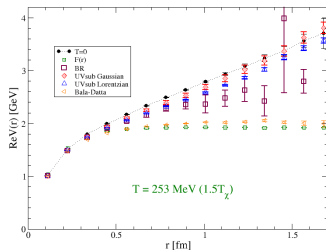
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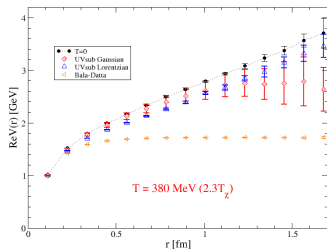
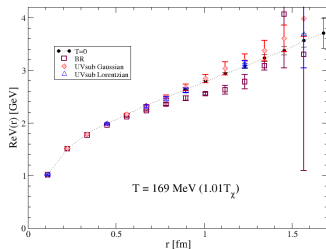
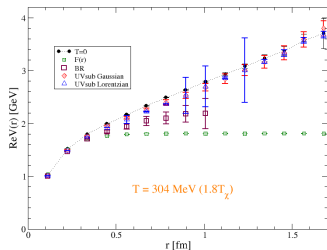
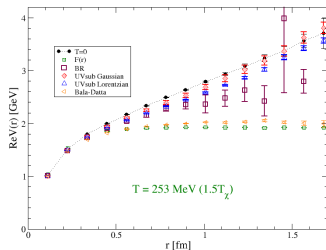
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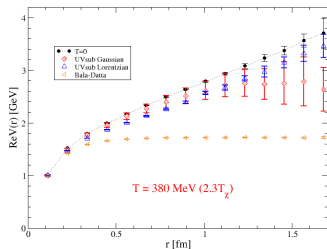
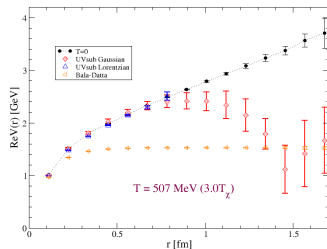
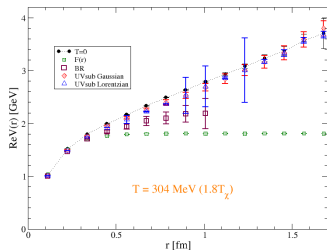
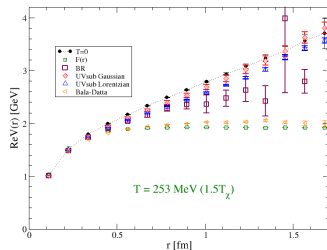
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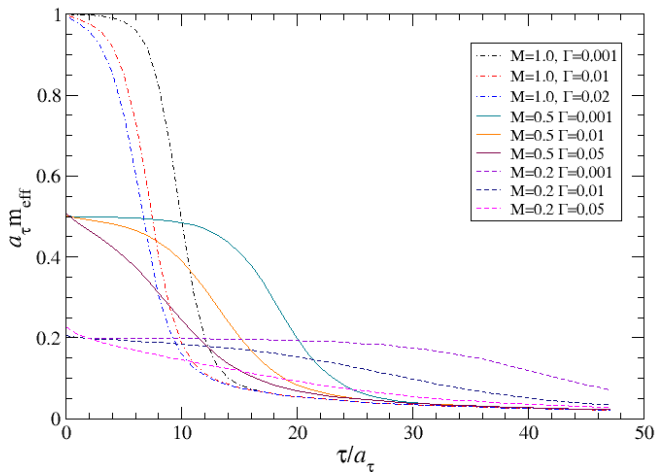
Comparison, temperature dependence



Summary (static quark potential)

- ▶ Fate of static quark potential above T_χ remains unsettled
- ▶ UV subtraction method very sensitive to details of fits
- ▶ Small but robust suppression at small and intermediate r
- ▶ Clear evidence of screening with all methods above $2T_\chi$
- ▶ Repeat for Gen3: better temporal resolution?

BACKUP SLIDE(S)



Bayesian methods

Maximum entropy method (Bryan)

$$S = \int d\omega \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)} \quad (\text{Shannon-Jaynes entropy})$$

Singular value decomposition:

$$K(\omega, \tau) \rightarrow K(\omega_i, \tau_j) = K_{ij} = U \Xi V^T$$

u_k are column vectors of U : $N_b = N_s \leq N_{\text{data}}$

BR method

$$S = \int d\omega \left(1 - \frac{\rho(\omega)}{m(\omega)} + \ln \frac{\rho(\omega)}{m(\omega)} \right)$$

$$N_b = N_\omega, u_k(\omega) = \delta_{k\omega}$$