The fate of the $U(1)_A$ symmetry and the static quark potential at high temperature

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With special thanks to Ryan Bignell, Antonio Smecca, Rachel Horohan D'Arcy, Alexander Rothkopf

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[arXiv:2604.11916, 2608.xxxxx]

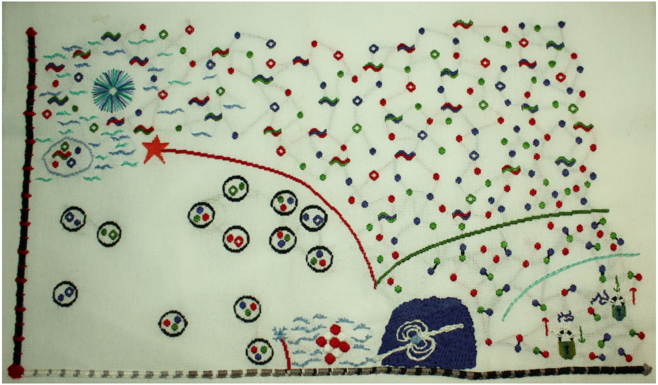
Outline

Introduction, lattice setup

$U(1)_A$ restoration

Static quark potential

Background



Background

- Hints of nontrivial phase structure of QCD at high T
- Chiral-spin symmetry?
- Chirally symmetric but confining?
- Vortex percolation transition at $\sim 2T_\chi$?
- $U(1)_A$ symmetry restoration still in dispute

Background

- Hints of nontrivial phase structure of QCD at high T
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- Vortex percolation transition at $\sim 2T_\chi$?
- $U(1)_A$ symmetry restoration still in dispute
- Dispute about static quark potential
  — some results show unscreened $\mathrm{Re}\, V(r)$ well above T_χ

Simulation parameters

[PRD **76** 194513 (2007), HadSpec Collab, PRD **79** 034502 (2009), JSPC **4** 100258 (2025)/2510.02954]

Gen	N_f	ξ	a_s (fm)	a_τ^{-1} (GeV)	m_π (MeV)	N_s	L_s (fm)
1	2	6.0	0.162	7.35	490	12	1.94
2	2+1	3.45	0.121	5.63	390	24	2.95
						32	3.94
2L	2+1	3.45	0.112	6.08	240	32	3.58
3	2+1	7.0	0.108	12.82	380	32	3.46

Simulation parameters: temperatures

Generation 2			Generation 2L			Generation 3		
N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c
128	44	0.24	128	47	0.42	256	50	0.28
			64	95	0.59	128	100	0.55
			56	109	0.67	112	114	0.64
48	117	0.63	48	127	0.78	96	133	0.73
40	141	0.76	40	152	0.94	80	160	0.88
						76	169	0.93
36	156	0.84	36	169	1.04	72	178	0.98
						68	188	1.04
32	176	0.95	32	190	1.17	64	200	1.10
28	201	1.09	28	217	1.34	56	229	1.26
24	235	1.27	24	253	1.56	48	267	1.47
20	281	1.52	20	304	1.87	40	320	1.76
						36	356	1.96
16	352	1.90	16	380	2.34	32	400	2.20
			12	507	3.12	24	534	2.93
			8	760	4.55			

Chiral and $U(1)_A$ symmetries

Classical symmetry of QCD:

$$U(N_f)_L \otimes U(N_f)_R \simeq SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_A \otimes U(1)_B$$

- DχSB: $SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$
  Restored above $T_{pc} \approx 150\,\mathrm{MeV}$
- $U(1)_A$: broken by the axial anomaly
  effectively restored at high T (?)
- Can affect the order and universality class of chiral transition in chiral limit

Correlator degeneracies

$$\text{Chiral:} \qquad \rho \longleftrightarrow a_1$$

$$U(1)_A: \qquad \pi \longleftrightarrow \delta(a_0)$$

Degeneracy ratios:

$$\mathcal{R}_{XY}(\tau) = \frac{\widetilde{G}_X(\tau) - \widetilde{G}_Y(\tau)}{\widetilde{G}_X(\tau) + \widetilde{G}_Y(\tau)} \qquad \widetilde{G}_X(\tau) = \frac{G_X(\tau)}{G_x(N_\tau/2)}$$

Susceptibilities:

$$\chi_X = \frac{1}{2}\sum_\tau G_X(\tau)\,, \qquad \widetilde{\chi}_X^{\text{norm}}(T, \tau_{\text{min}}) = \frac{1}{N_\tau}\sum_{\tau=\tau_{\text{min}}}^{N_\tau/2} \widetilde{G}_X(\tau; T)$$

Degeneracy if $\chi_X - \chi_Y = 0$

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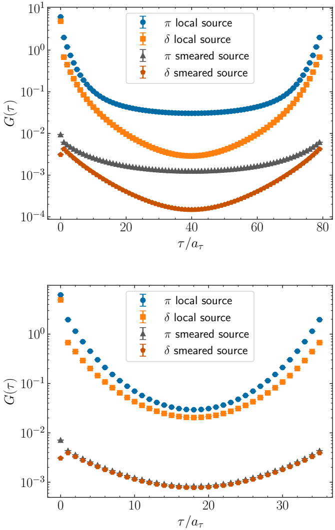
Degeneracy if $\chi_X - \chi_Y = 0$
Normalised correlators $\widetilde{G}(\tau)$ eliminate renormalisation factors.
All quantities may be evaluated with local or smeared operators

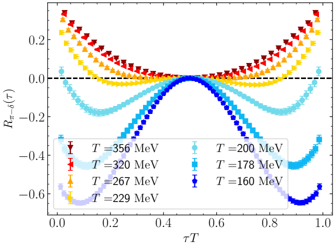
PS–S degeneracy: correlators

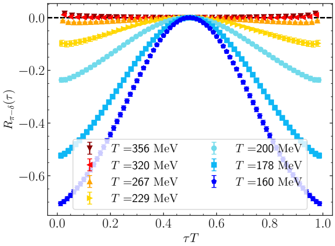
- Top: $T = 160\,\mathrm{MeV}$
  Bottom: $T = 356\,\mathrm{MeV}$
- PS–S degeneracy at high T



PS–S degeneracy: correlators

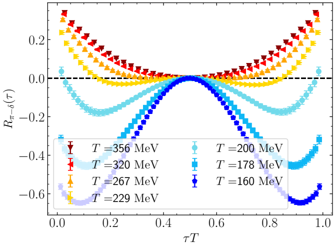
- Top: local
  Bottom: smeared
- PS–S degeneracy at high T
- Smearing gives improved short-distance behaviour

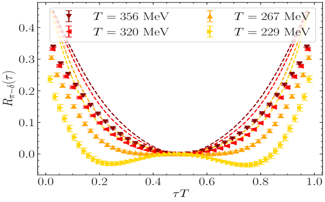




PS–S degeneracy: correlators

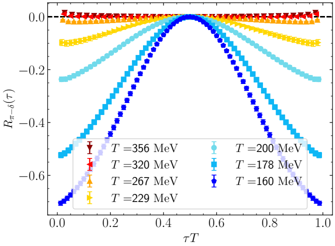
- PS–S degeneracy at high T
- Smearing gives improved short-distance behaviour
- — removes short-distance lattice artefacts





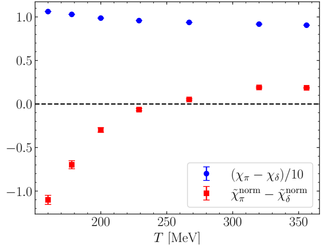
PS–S degeneracy: correlators

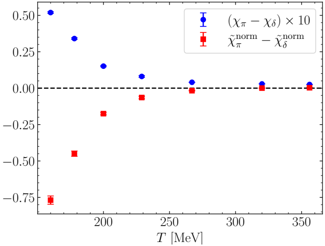
- PS–S degeneracy at high T
- Smearing gives improved short-distance behaviour
- Degeneracy for $T \gtrsim 300\,\mathrm{MeV}$



PS–S degeneracy: susceptibilities

- Top: local
  Bottom: smeared
- Normalised $\Delta\chi \to 0$ at $\sim 320\,\text{MeV}$





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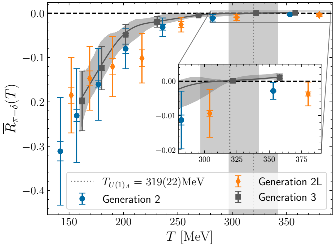
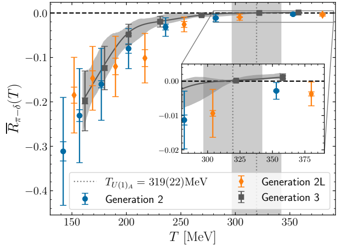
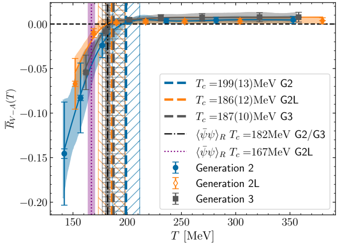


- Normalised $\Delta\chi \to 0$ at $\sim 320\,$MeV
- Similar results for Gen2, Gen2L

PS–S degeneracy: susceptibilities



- Normalised $\Delta\chi \to 0$ at $\sim 320\,\mathrm{MeV}$
- Similar results for Gen2, Gen2L
- Well above T_χ



Summary [$U(1)_A$]

- Correlator ratios can be used to probe symmetry restoration
- T_χ (V–A degeneracy) agrees with traditional methods
- $U(1)_A$ effectively restored at $T \gtrsim 300\,\mathrm{MeV} \sim 2\,T_\chi$

Summary [$U(1)_A$]

- Correlator ratios can be used to probe symmetry restoration
- T_χ (V–A degeneracy) agrees with traditional methods
- $U(1)_A$ effectively restored at $T \gtrsim 300\,\mathrm{MeV} \sim 2T_\chi$
- Repeat with lighter/physical quarks: Gen2P, Gen3L, Gen3P
- Other degeneracies: chiral-spin symmetry?

Static quark potential

- Long (pre-)history of potential models
- Definition of real-time potential, relation to Wilson loop/lines
- Real and imaginary parts (peak position and width)
- Screening or no screening: different methods give different results

Static quark potential: Methods (I)

Bayesian spectral function reconstruction

Bayes theorem

$$P(\rho|DI) = \frac{P(D|\rho I)P(\rho|I)}{P(D|I)}$$

Parametrise prior probability by

$$P(\rho|I) \propto e^{\alpha S[\rho]} \quad \Longrightarrow \quad P(\rho|DI) \propto e^{-L[D,\rho]+\alpha S[\rho]}$$

where L is the standard likelihood (χ^2)
Spectral function $\rho(\omega)$ is expressed in terms of default model $m(\omega)$

$$\rho(\omega) = m(\omega)\exp[\sum_{k=1}^{N_b} b_k u_k(\omega)]$$

Different prescriptions for S, u_k: MEM and BR

Static quark potential: Methods (II)

UV subtraction method

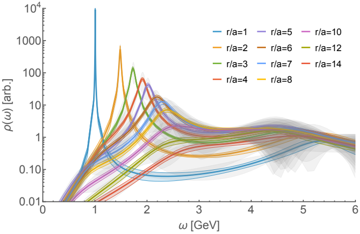
- Subtract T-independent "UV" correlator
- UV corr taken to be $T = 0$ corr minus lowest state(s)
- Fit UV subtracted m_{eff} to straight line (assume gaussian $\rho(\omega)$)
- or fit to lorentzian $\rho(\omega)$
- Consider ground and first excited state

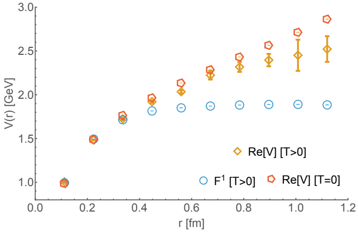
Bala–Datta method

- Based on requirement for existence of real-time potential
- Decompose into linear (ReV) + periodic (ImV) terms
- Requires assumptions about shape of spectral peak

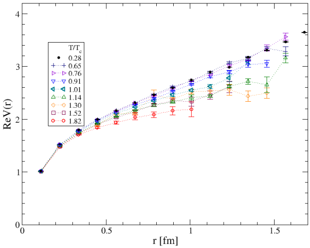
BR results in detail

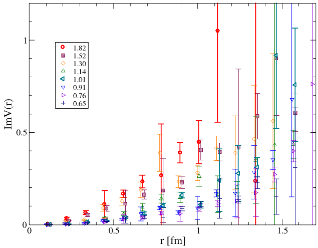
High statistics: ~ 3000 configs at $N_\tau = 24(T/T_c = 1.5)$





BR results: summary





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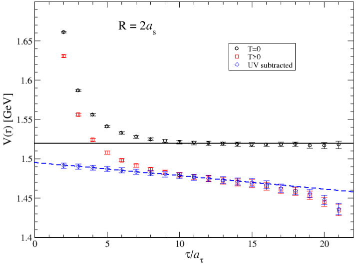

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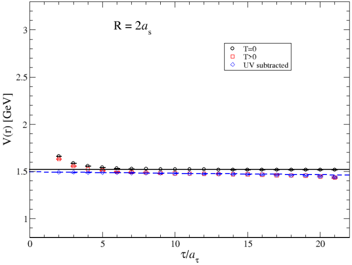

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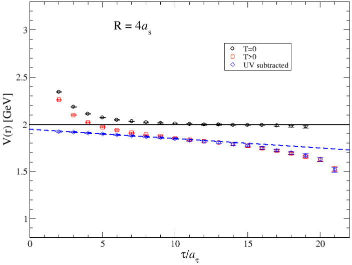

- UV-sub V_{eff} has shape typical of lorentzian not gaussian

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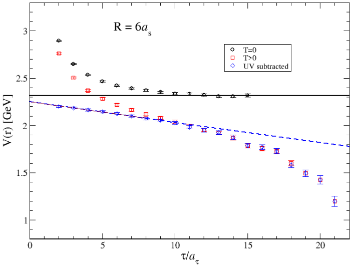

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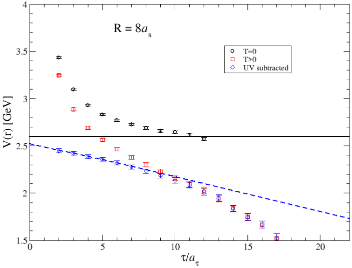

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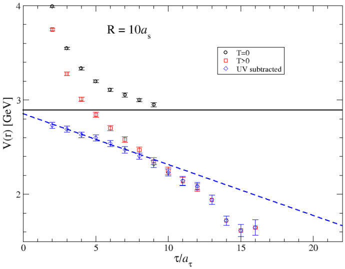

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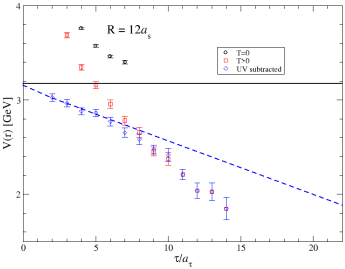

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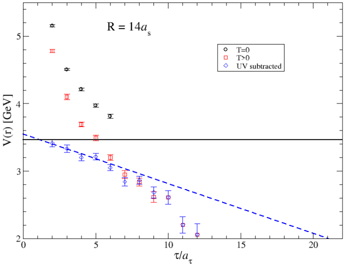

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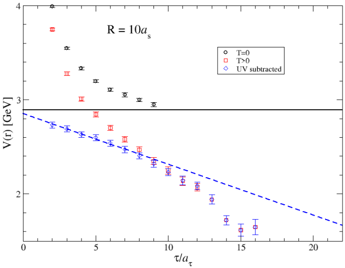
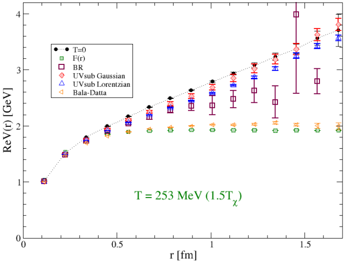
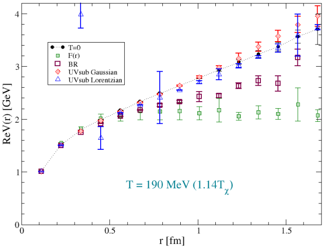
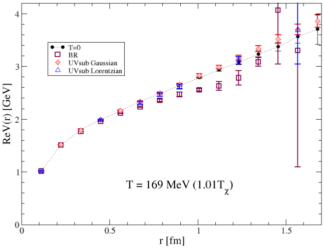




- UV-sub V_{eff} has shape typical of lorentzian not gaussian
- $V(r;T) < V(R;0)$ at small + moderate r
- Agreement between methods at small r
- All methods break down at large r?
- Only Bala–Datta reproduces free energy (note assumptions!)

Comparison, temperature dependence





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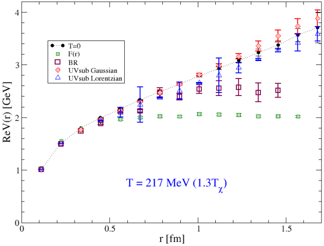

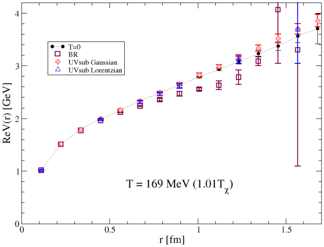

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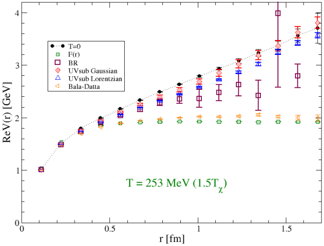

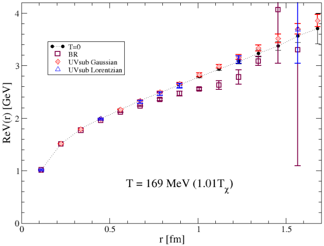

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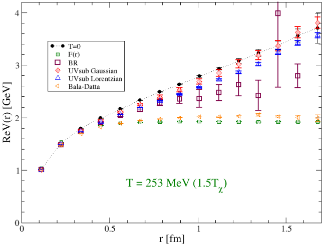

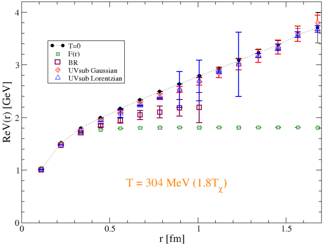

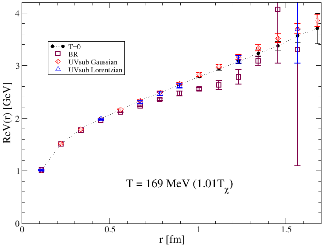

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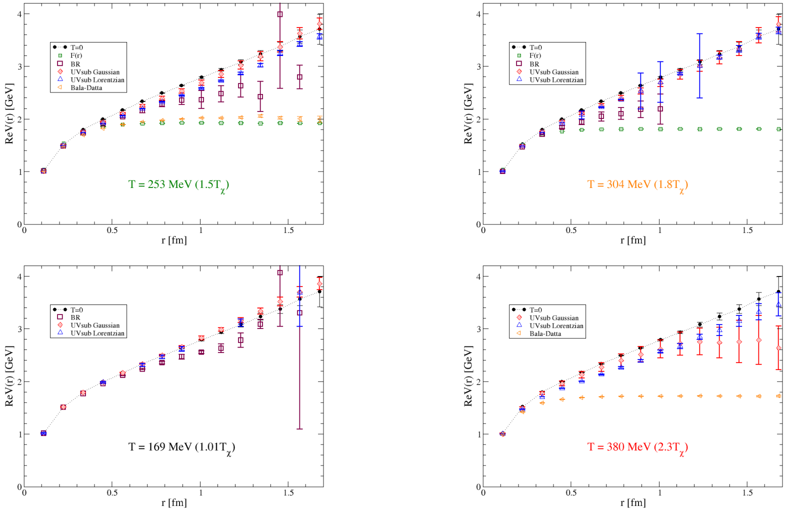

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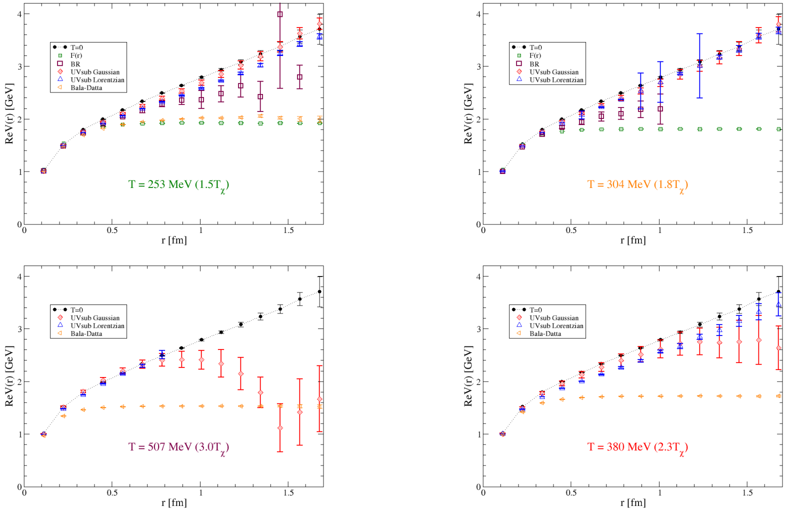
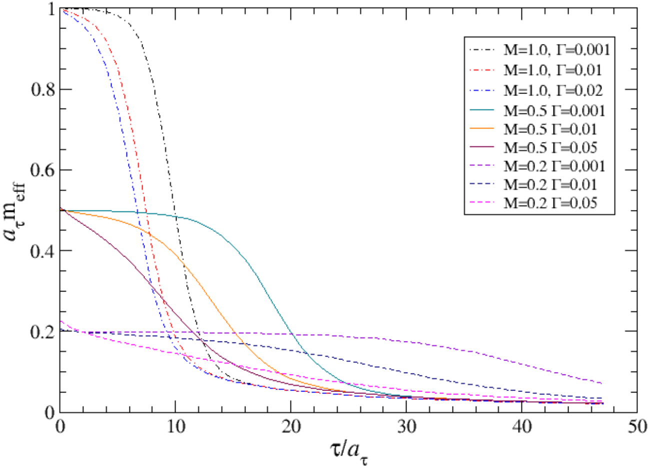


Summary (static quark potential)

- Fate of static quark potential above T_χ remains unsettled
- UV subtraction method very sensitive to details of fits
- Small but robust suppression at small and intermediate r
- Clear evidence of screening with all methods above $2T_\chi$
- Repeat for Gen3: better temporal resolution?
- Compare imaginary part from different methods

BACKUP SLIDE(S)



Bayesian methods

Maximum entropy method (Bryan)

$$S = \int d\omega \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)} \quad \text{(Shannon–Jaynes entropy)}$$

Singular value decomposition:

$$K(\omega, \tau) \to K(\omega_i, \tau_j) = K_{ij} = U \Xi V^T$$

u_k are column vectors of U: $N_b = N_s \leq N_{\text{data}}$

BR method

$$S = \int d\omega \left(1 - \frac{\rho(\omega)}{m(\omega)} + \ln \frac{\rho(\omega)}{m(\omega)}\right)$$

$N_b = N_\omega, u_k(\omega) = \delta_{k\omega}$