Bubble Nucleation and Hadronic Phase Conversion in an Expanding Quark–Gluon Plasma

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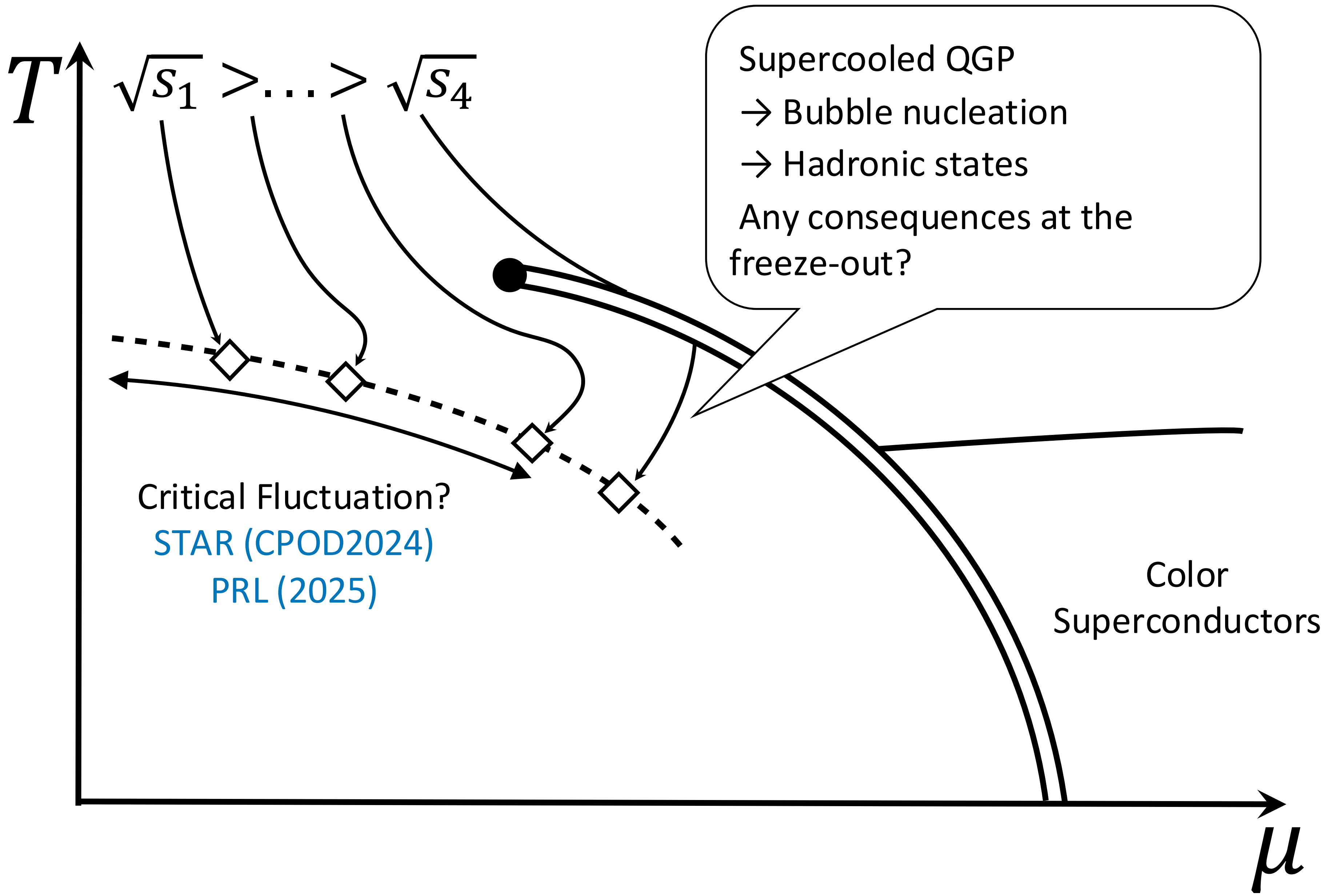
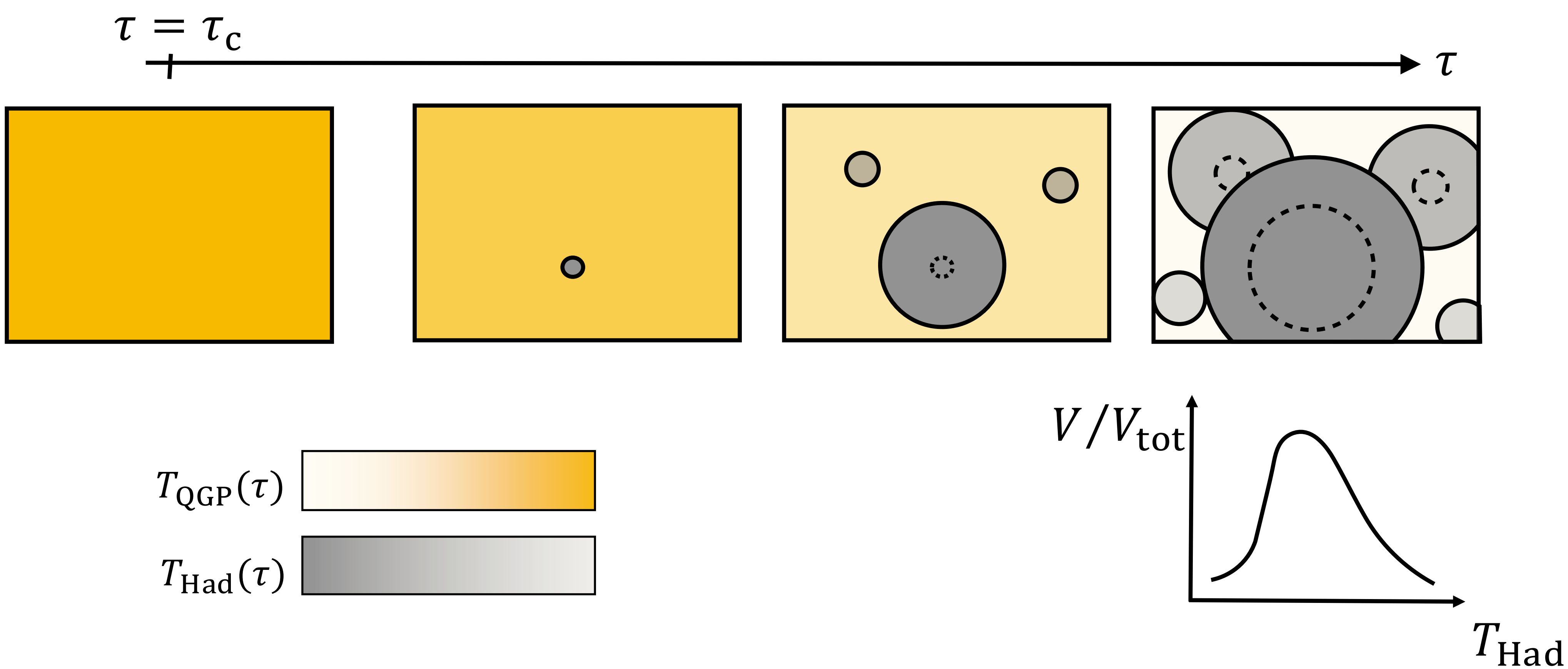
From fluctuations to first-order dynamics

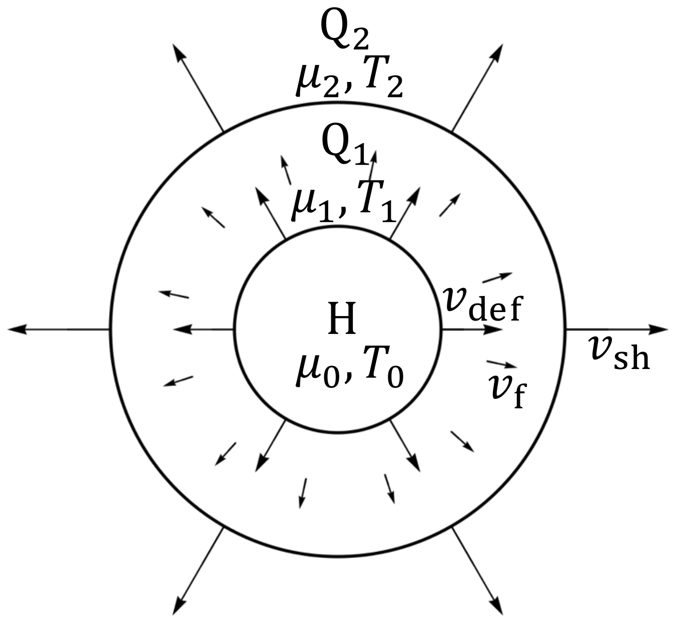


Bubble nucleation and growth in expanding media



- $T_{\mathrm{QGP}}(\tau)$ as expanding media → τ dependent hadronic states $T_{\mathrm{Had}}(\tau)$
- Most probable hadronic state distribution $(\mu_{\mathrm{Had}}, T_{\mathrm{Had}})$ (volume weighted)

Setup



- Three-layer problem:
  Gyulassy, Kajantie, Kurki-Sunio, McLerran (1984)
  - Q_2 : Supercooled QGP
  - Q_1 : Reheated QGP (required by kinematics)
  - H : Hadronic state
- BEST EOS (3D Ising + lattice data)
  + metastable extension beyond mean-field approximation (MFA).
  c.f., see also Karthein, Koch, and Ratti (2025) for the metastable BEST MFA
- Bjorken expansion of Q_2

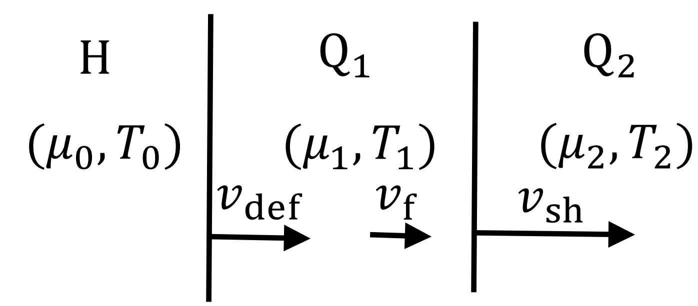
$$s_2 = s_{2c}\frac{\tau_{2c}}{\tau},\ \hat{s}_2 \equiv \frac{s_2}{n_2} = \hat{s}_{\mathrm{ini}} \Rightarrow T_2 = T_2(\tau),\ \mu_2 = \mu_{\hat{s}_2}(T_2)$$

Hydrodynamic matching

- Energy, momentum, and baryon number conservations

  Expanding QGP states → Reheated QGP and hadronic states:

$$(\mu_2(\tau), T_2(\tau)) \xrightarrow[\hat{s}_1(\geq \hat{s}_2)]{\text{3 eqs.}} \underset{v_{\mathrm{sh}}, v_{\mathrm{f}}}{(\mu_1, T_1)} \xrightarrow{\text{3 eqs.}} \underset{v_{\mathrm{def}}}{(\mu_0, T_0)}$$



$\mathcal{S}_{\hat{s}_1}(\tau) \equiv (\mu_0, T_0, \mu_1, T_1, v_{\mathrm{def}}, v_{\mathrm{f}}, v_{\mathrm{sh}})$: one-parameter family of solutions

- Assuming all bubbles $\mathcal{S}_{\hat{s}_1}(\tau)$ created stochastically in space

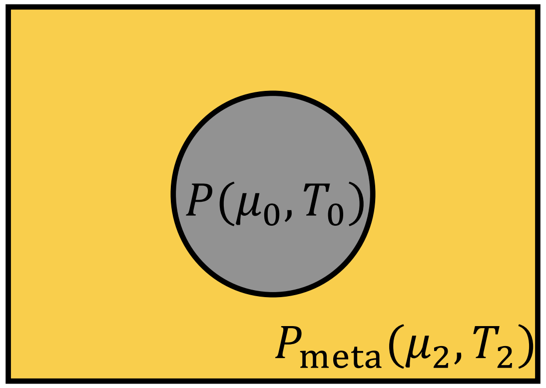
Nucleation rate

- Nucleation rate: $\Gamma(\mathcal{S}) \propto \exp\left(-\frac{\Delta F(R_{\mathrm{crit}})}{T_2}\right))$ per unit time and volume)

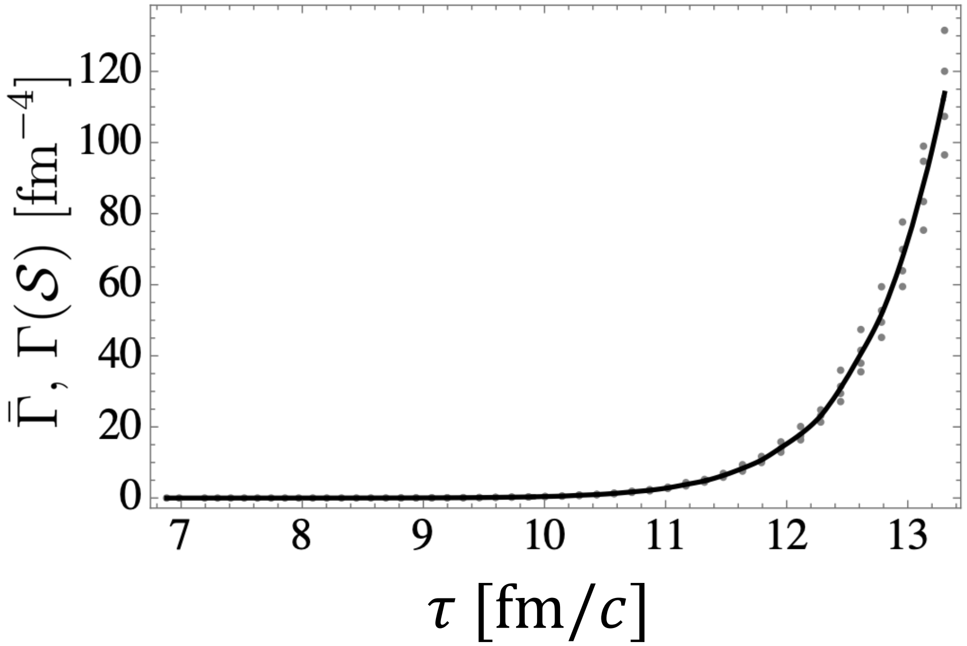
Csernai and Kapusta (1992)

$$\Delta F(R) = 4\pi R^2 \sigma - \frac{4\pi}{3} R^3 [P(\mu_0, T_0) - P_{\mathrm{meta}}(\mu_2 - T_2)]$$

$\sigma \sim \frac{(\phi_{\mathrm{meta}} - \phi)^2}{\xi}$) surface tension)



- Mean nucleation rate: $\overline{\Gamma}(\tau) = \langle \Gamma_{\hat{s}_1}(\tau) \rangle_{\mathrm{family}}$



First nucleation

- Total nucleation: $\overline{N}(\tau) \equiv \int_{\tau_{2c}}^{\tau} \mathrm{d}\tau' \overline{\Gamma}(\tau') V_{\mathrm{tot}}(\tau')$, $\qquad V_{\mathrm{tot}}(\tau) = \pi R_{\mathrm{sys}}^2 \tau \Delta\eta$

  $\left(R_{\mathrm{sys}} = 4\ \mathrm{fm},\ \tau = 5\ \mathrm{fm}/c,\ \Delta\eta = 0.5\ \mathrm{fm}\right)$

- The first nucleation probability density:

$$\mathcal{P}_{\mathrm{first}}(\tau) = \underbrace{\dot{\overline{N}}(\tau)}_{\text{Nucleation at } \tau} \underbrace{\exp[-\overline{N}(\tau)]}_{\text{No nucleation until } \tau}$$

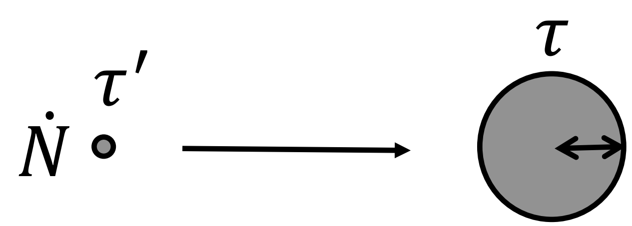
- The first nucleation time: $\dot{\mathcal{P}}_{\mathrm{first}}(\tau_{\mathrm{first}}) = 0$

Bubble growth and conversion dynamics

See also Kolmogorov (1937), Johnson and Mehl (1939), Avrami (1939)

- Hadronic volume fraction for each matching solution:

$$K(\tau,\tau';\mathcal{S}) \equiv \dot{N}(\tau';\mathcal{S})\,\frac{V_{\rm bub}(\tau,\tau';\mathcal{S})}{V_{\rm tot}(\tau)}$$



$$R_{\rm bub}(\tau,\tau';\mathcal{S}) = v_{\rm def}(\mathcal{S})(\tau-\tau')$$

Accumulated mean volume fraction: $I(\tau) = \int_{\tau_{\rm 2c}}^{\tau} \mathrm{d}\tau' \overline{K}(\tau,\tau')$

- Conversion rate: $\mathcal{P}_{\rm conv} \equiv \dot{I}(\tau)\,\underbrace{\exp[-I(\tau)]}_{\text{Survival rate (no bubbles)}} \rightarrow \dot{\mathcal{P}}_{\rm conv}(\tau_{\rm conv}) = 0$

- Bubble creation history: $\mathcal{P}_{\rm bub} \sim \overline{K}(\tau_{\rm comp},\tau) \rightarrow \dot{\mathcal{P}}_{\rm bub}(\tau_{\rm bub}) = 0$

$$\frac{\text{(hadronic volume)}}{\text{(total volume)}} \sim 1$$

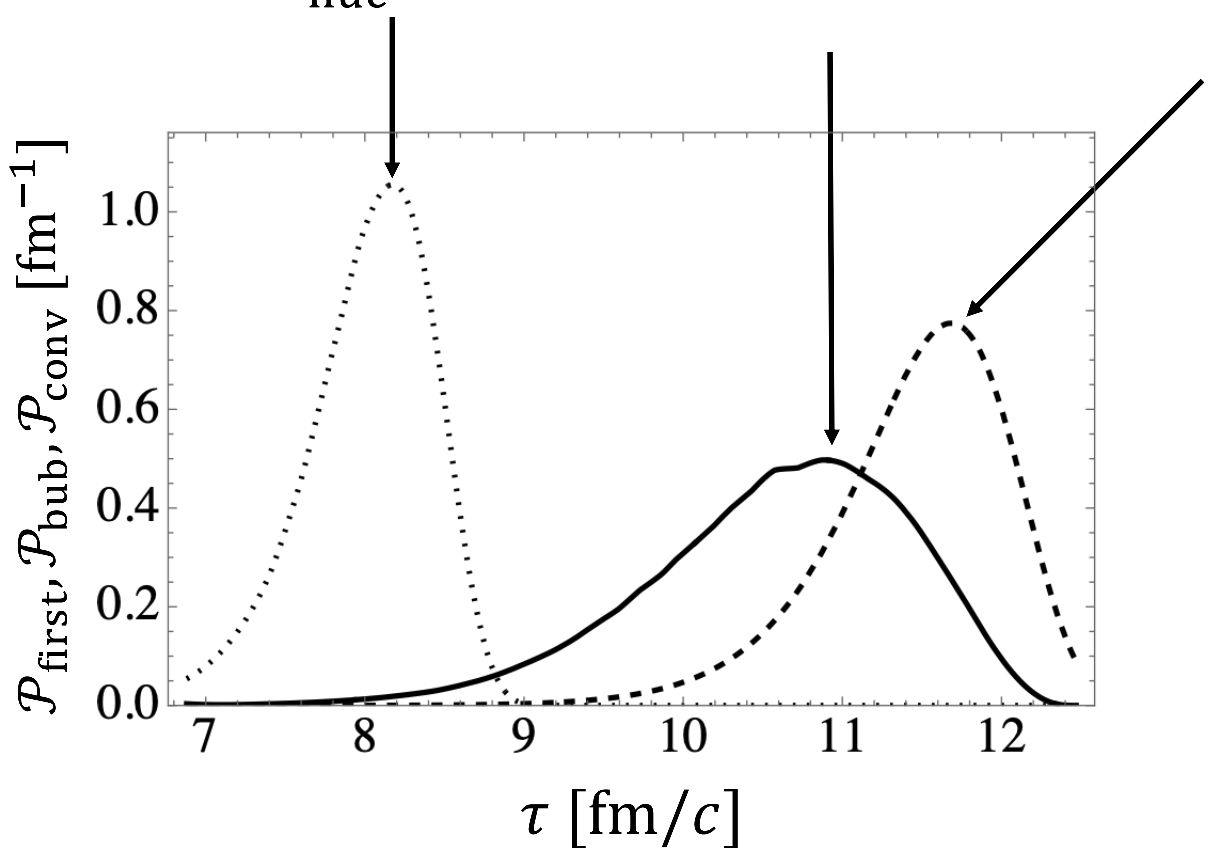
Three characteristic stages

NS, Stephanov, Yee (in preparation)

Volume-dominant bubble creation τ_{bub}

First nucleation τ_{nuc}

Maximal conversion τ_{conv}



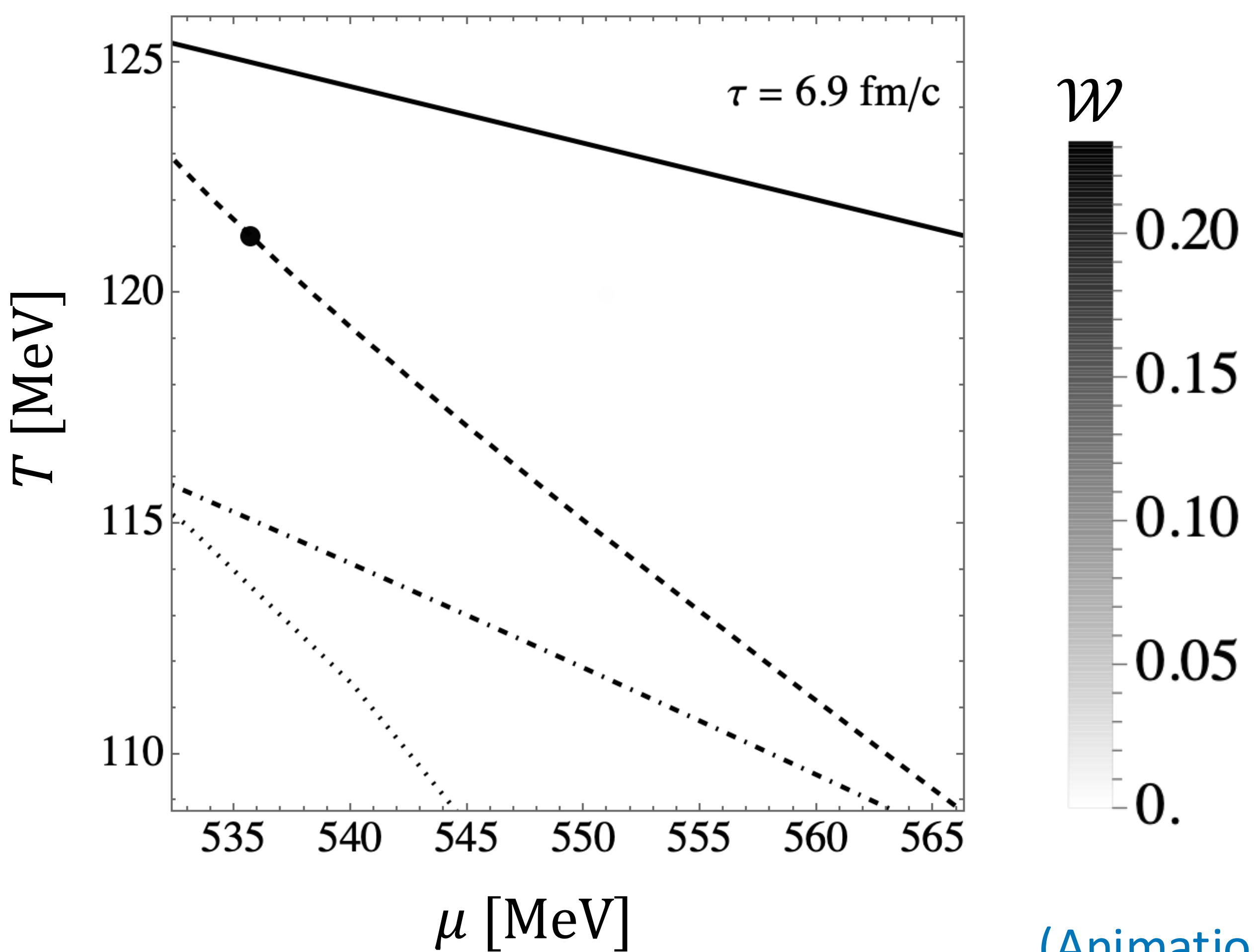
General hierarchy: $\tau_{\mathrm{nuc}} < \tau_{\mathrm{bub}} < \tau_{\mathrm{conv}}$ for

BEST EOS
Bag model

Hadronic state distribution

NS, Stephanov, Yee (in preparation)

$$\mathcal{W}(\tau,\mu,T) \sim \int_{\tau_{2c}}^{\tau} \mathrm{d}\tau' \langle K(\tau,\tau';\mathcal{S})\delta(\mu-\mu_0^{(\mathcal{S})}(\tau'))\delta(T-T_0^{(\mathcal{S})}(\tau'))\rangle_{\text{family}}$$



Phase transition

Q_2 Bjorken evolution

Freeze-out curve

Metastable boundary

BEST EOS

$\mu_c = 450\text{MeV}$ $\quad T_c = 136\text{MeV}$

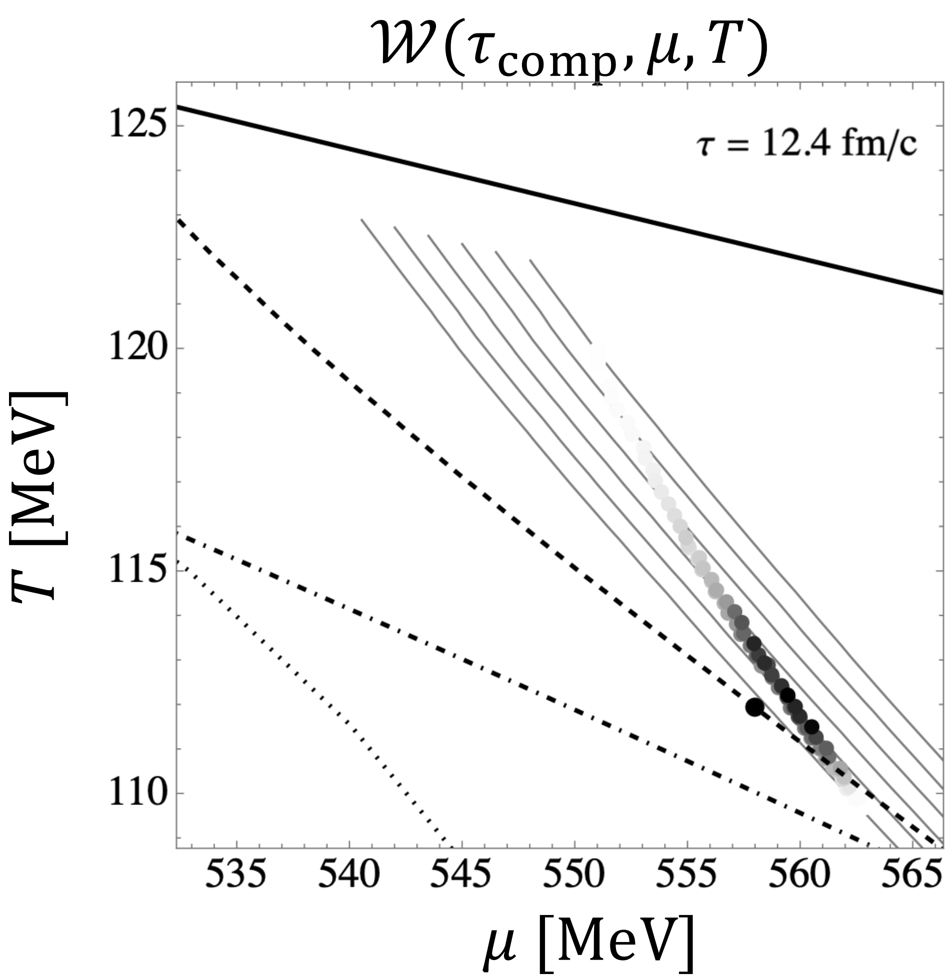
$\alpha_1 = 7^\circ$ $\quad \alpha_2 = -5^\circ$

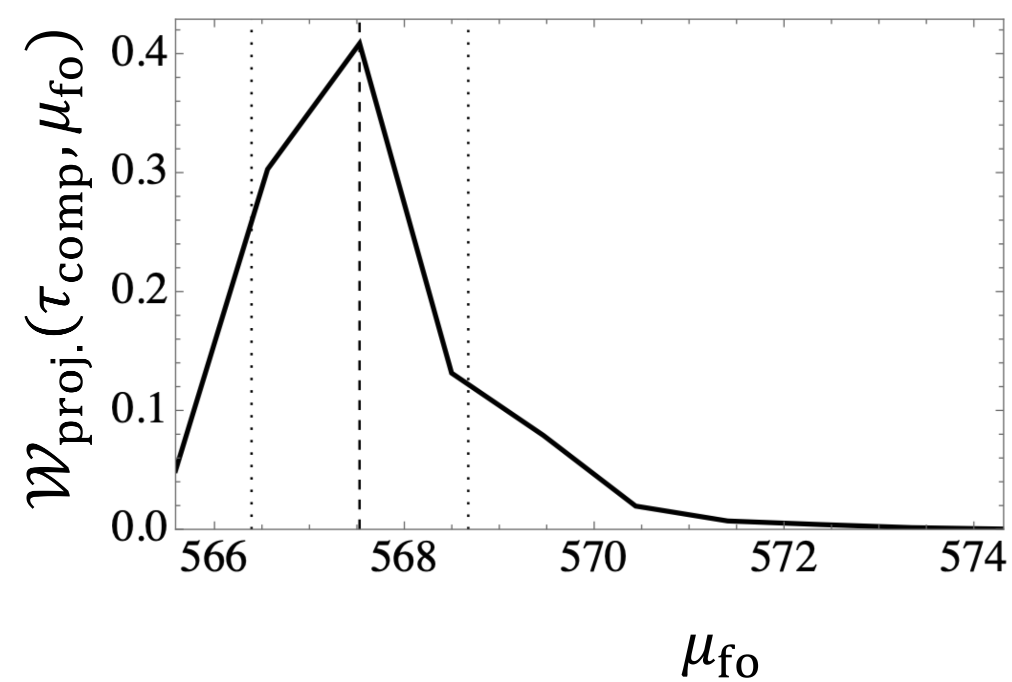
$\rho = 1.0$ $\quad w = 0.5$

(Animation available in the PowerPoint version)

Hadronic state evolution

NS, Stephanov, Yee (in preparation)





- Broad distribution from nonequilibrium bubble nucleation
- Volume-weighted states remain localized
- Different fluid elements freeze out at different μ_{fo}

Summary and Advertisment

NS, Stephanov, Yee (in preparation)

- Searching for the QCD critical point and the first-order chiral phase transition in heavy-ion collisions
- Hadronic conversion dynamics in an expanding medium: Three characteristic times: $\tau_{\mathrm{nuc}} < \tau_{\mathrm{bub}} < \tau_{\mathrm{conv}}$
- Distribution of hadronic states in (μ, T) (volume weighted) dynamically selected by the volume-dominant production time τ_{bub}

[Outlook] More sophisticated BEST EOS Kahangirwe et al. (2024)

[Adv.] Quantum-Metric Dominance in the Meissner Mass of Magnetized Color

Superconductors, Fri 4 pm~, by Kazuya Mameda