Phase structure of QCD at intermediate temperature and baryon density

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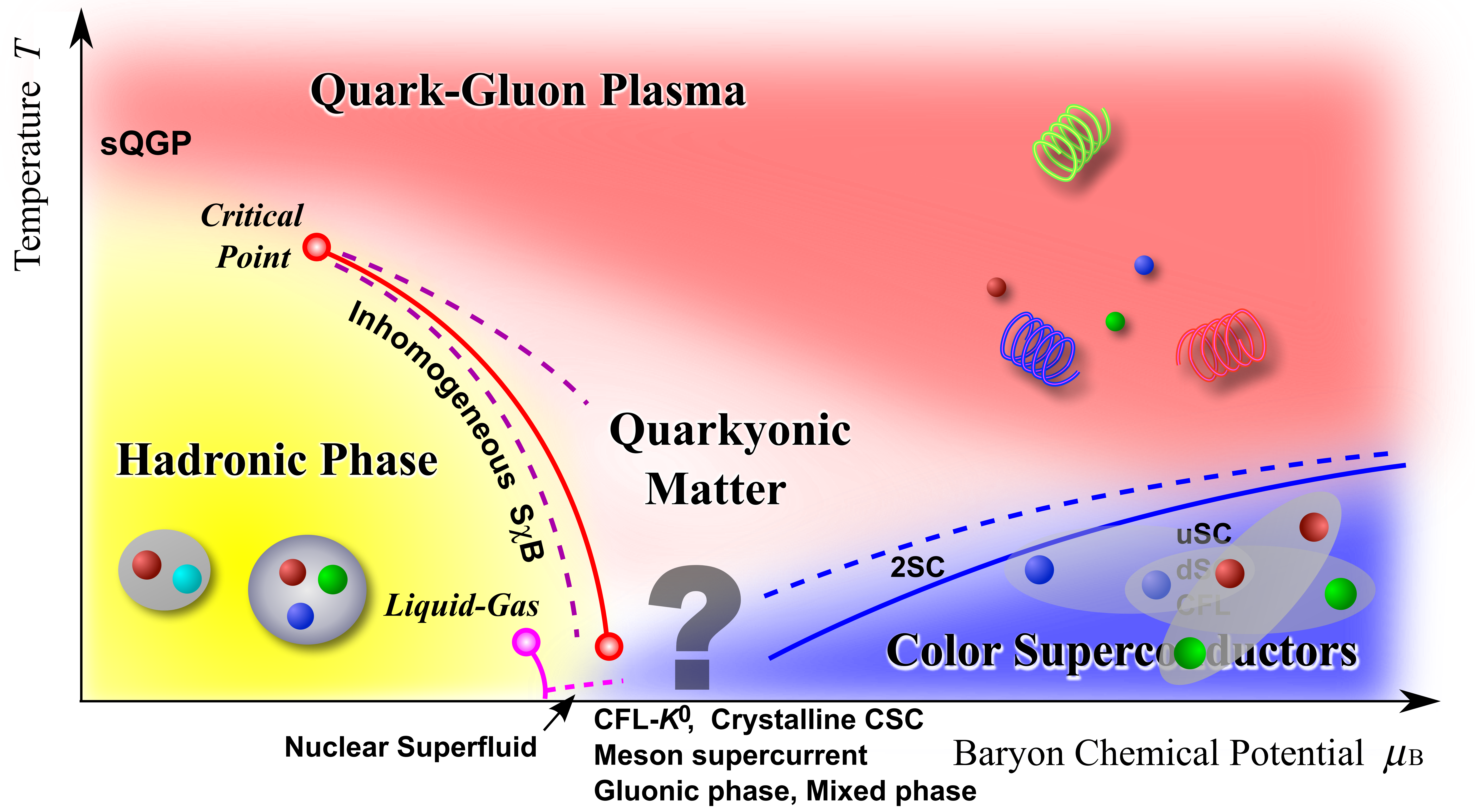
based on: Fujimoto, Fukushima, YH, McLerran, Phys. Rev. D 112, 074006 (2025)
Hayata, YH, Nishimura, JHEP 07, 106 (2024)
Fujikura YH, 2605.17183

QCD phase diagram

Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001

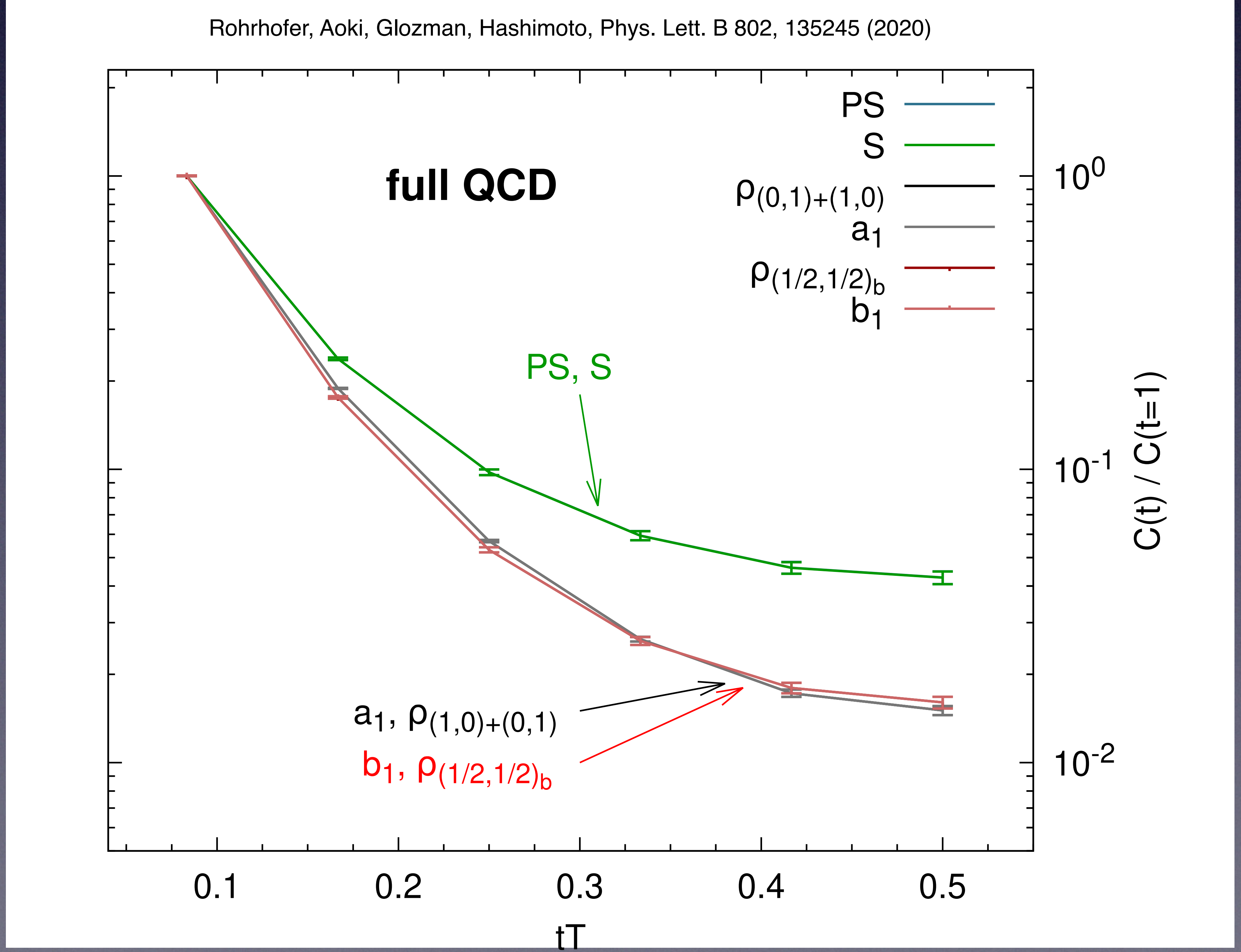


Finite-Temperature

Chiral pseudo-critical temperature $T_C \sim 160\ \mathrm{MeV}$

Pure-glue deconfinement $T_d \simeq 285\,\mathrm{MeV}$

What is the intermediate state of QCD matter?



Chiral spin symmetry? Glozman ('15)

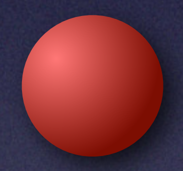
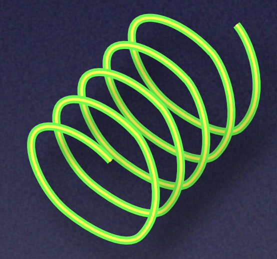
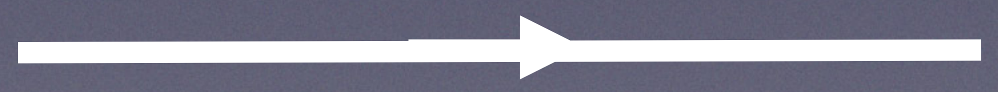
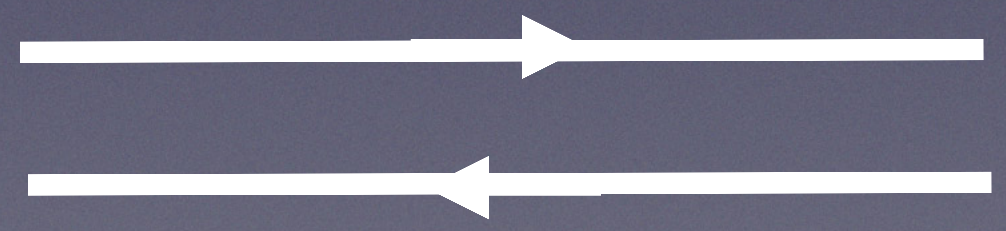
Stringy fluid?

Large-N_c QCD

't Hooft ('74)

QCD: SU(3) gauge theory with coupling constant g

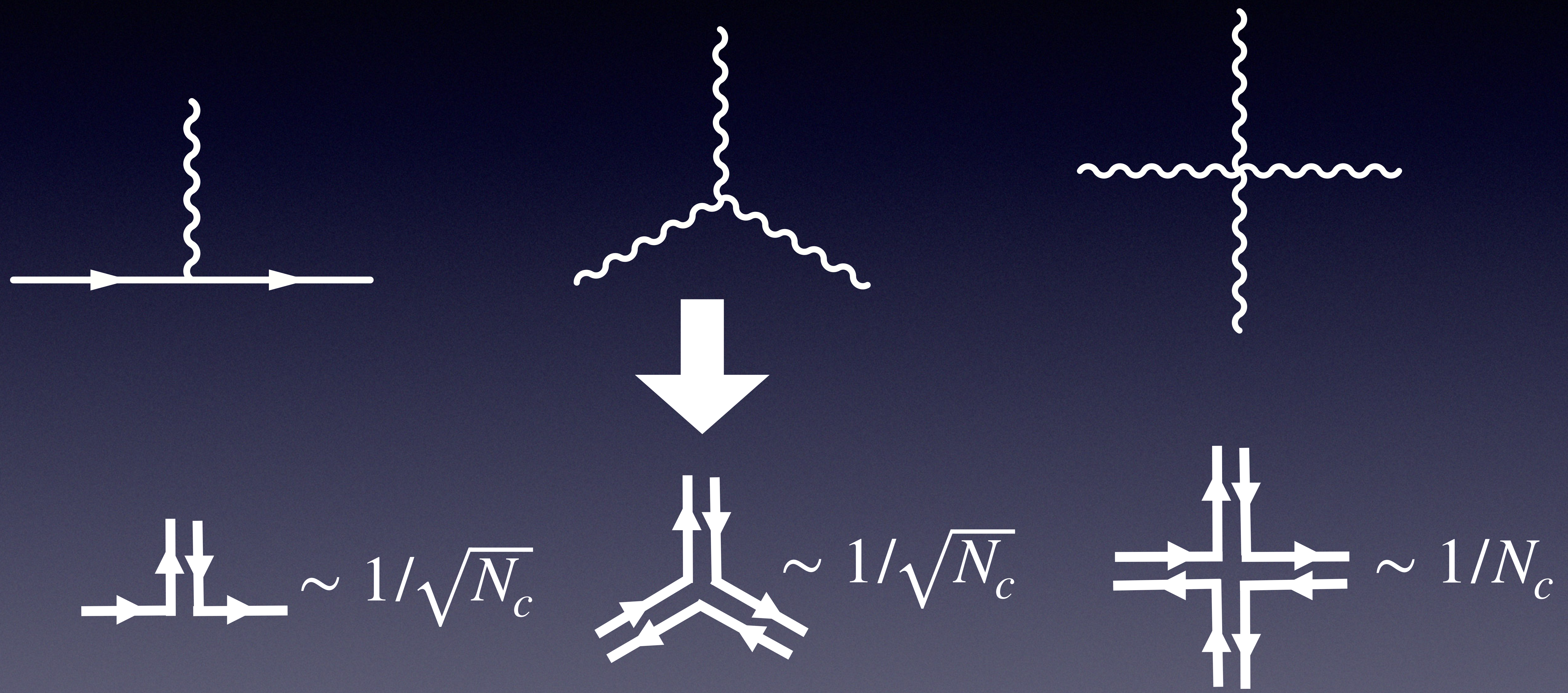
Generalization: SU(3) → SU(N_c)

	Quark	Gluon
Field		
Color	Fundamental N_c	Adjoint $N_c^2 - 1$
Double line notation		 $-\frac{1}{N_c}$ 

Large-N_c QCD

't Hooft ('74)

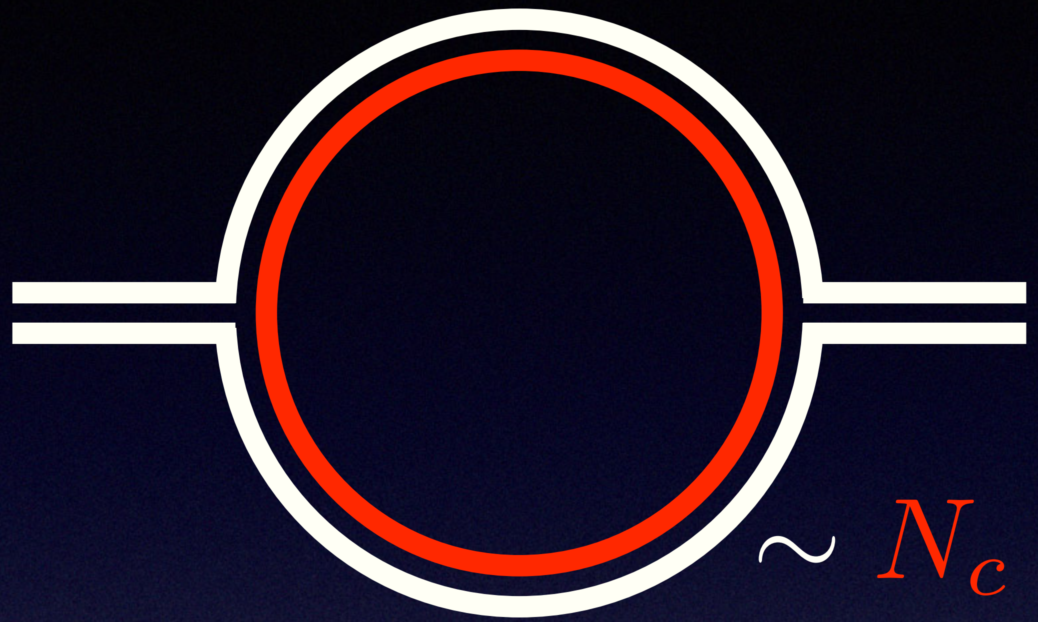
Quark-gluon **Gluon-gluon**



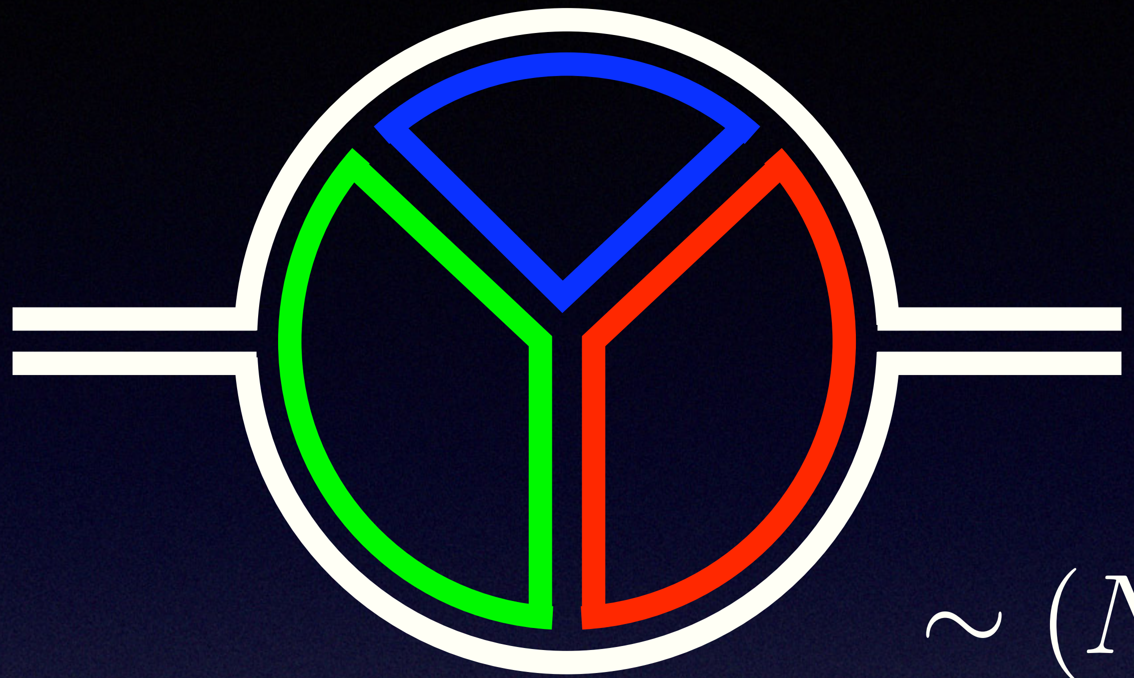
't Hooft limit: $\lambda = N_c g^2$ **is fixed** N_c **large** $\Rightarrow g \sim 1/\sqrt{N_c}$

Loop corrections

For example, gluon self-energy,



$\sim N_c \times \left(\frac{1}{\sqrt{N_c}}\right)^2 \sim N_c^0$

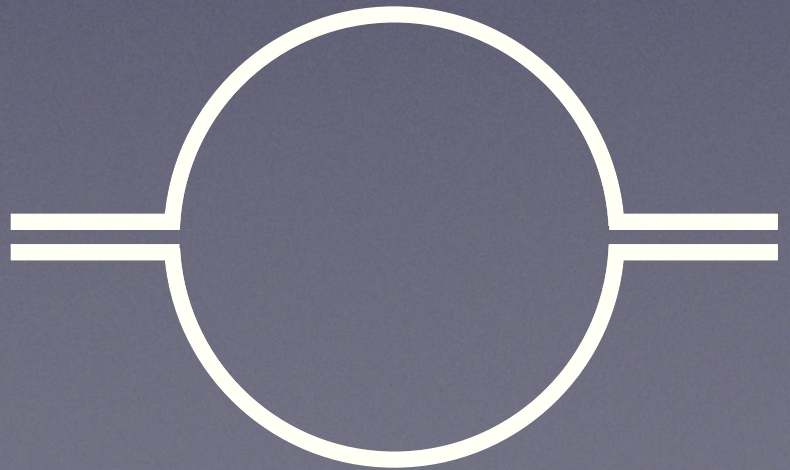


$\sim (N_c)^3 \times \left(\frac{1}{\sqrt{N_c}}\right)^6 \sim N_c^0$

In general, planar diagrams $\sim N_c^0$

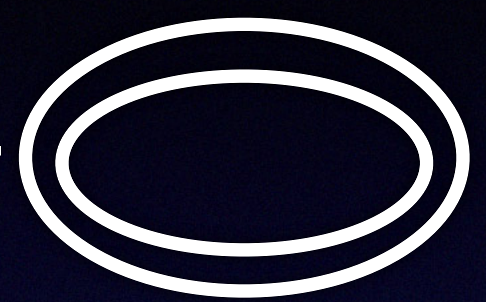
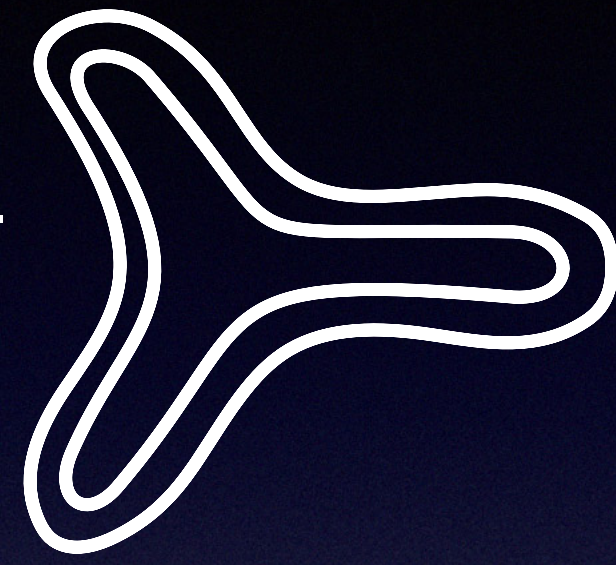
Quark loop and Non-planar diagrams are suppressed

Non-planar diagrams  $\sim N_c \times \left(\frac{1}{\sqrt{N_c}}\right)^6 \sim N_c^{-2}$

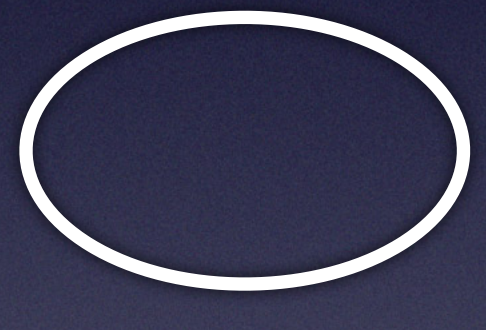
Quark loop  $\sim \times \left(\frac{1}{\sqrt{N_c}}\right)^2 \sim N_c^{-1}$

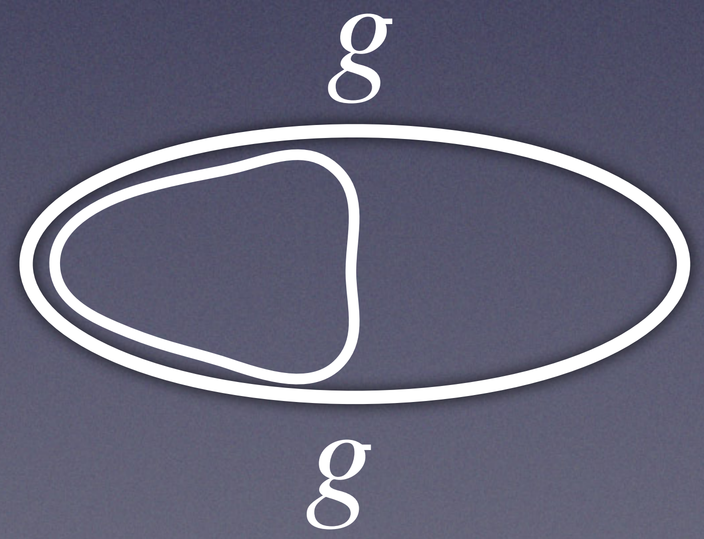
Interactions

Glueball operator

$\mathcal{G} = \frac{1}{N_c} F^a_{\mu\nu} F^a_{\rho\sigma}$ $\langle \mathcal{G}\mathcal{G} \rangle \sim \frac{1}{N_c^2}$  $\sim N_c^0$ $\langle \mathcal{G}\mathcal{G}\mathcal{G} \rangle \sim \frac{1}{N_c^3}$  $\sim \frac{1}{N_c}$

Meson operator

$\mathcal{M} = \frac{1}{\sqrt{N_c}} \bar{\psi}\Gamma\psi$ $\langle \mathcal{M}\mathcal{M} \rangle \sim \frac{1}{N_c}$  $\sim N_c^0$ $\langle \mathcal{M}\mathcal{M}\mathcal{M} \rangle \sim \frac{1}{N_c^{3/2}}$  $\sim \frac{1}{\sqrt{N_c}}$

Glueball-meson mixing $\langle \mathcal{G}\mathcal{M} \rangle \sim \frac{1}{N_c^{3/2}}$ 

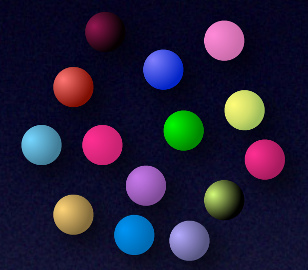
$\sim \frac{1}{N_c^{1/2}}$

Summary:Large-N_c counting

't Hooft ('74) Witten ('79)

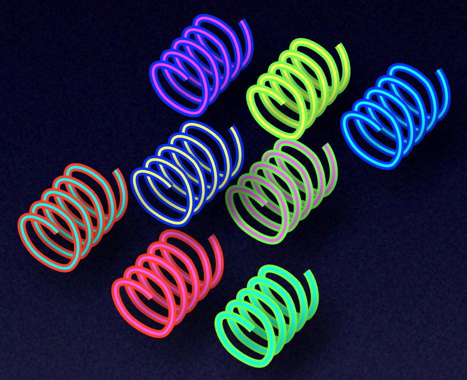
Degrees of freedom

quarks



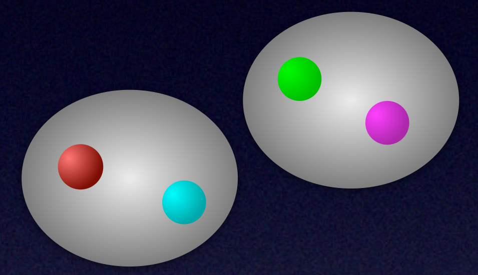
$\sim N_c$

gluons



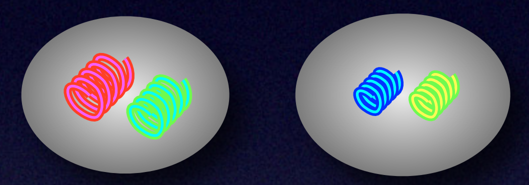
$\sim N_c^2$

mesons



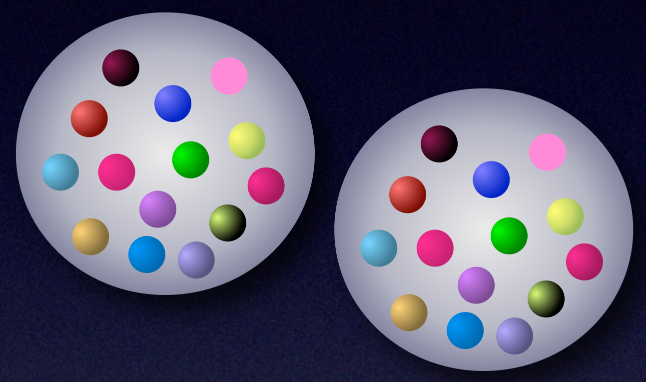
$\sim N_c^0$

glueball



$\sim N_c^0$

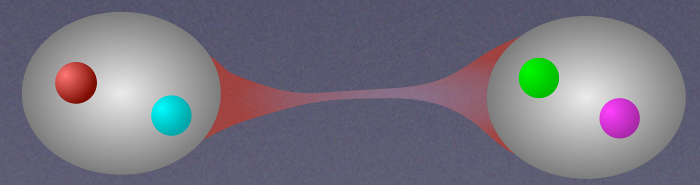
Baryons



$\sim N_c^0$

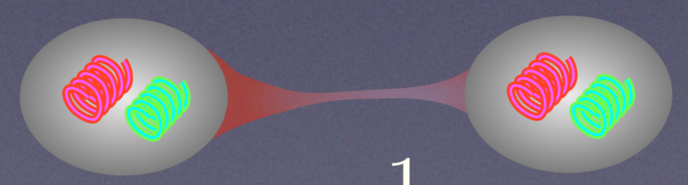
Interactions

meson -meson



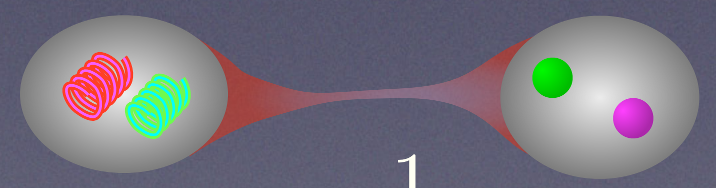
$\sim \frac{1}{N_c}$

glueball -glueball



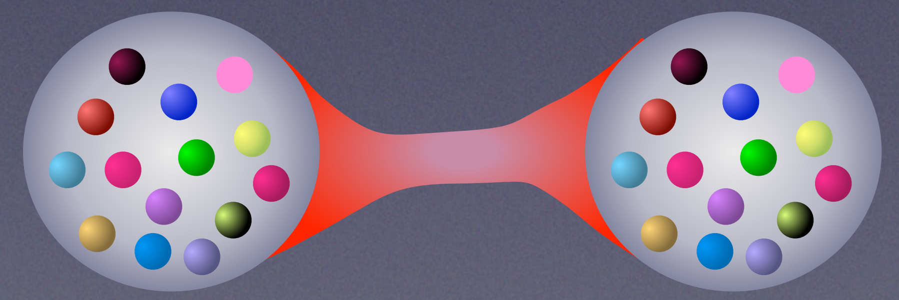
$\sim \frac{1}{N_c^2}$

glueball -meson



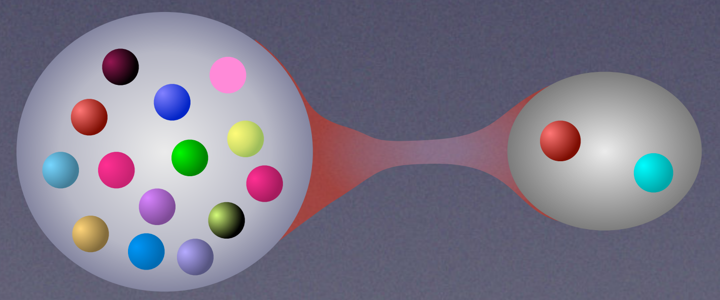
$\sim \frac{1}{N_c^2}$

baryon -baryon



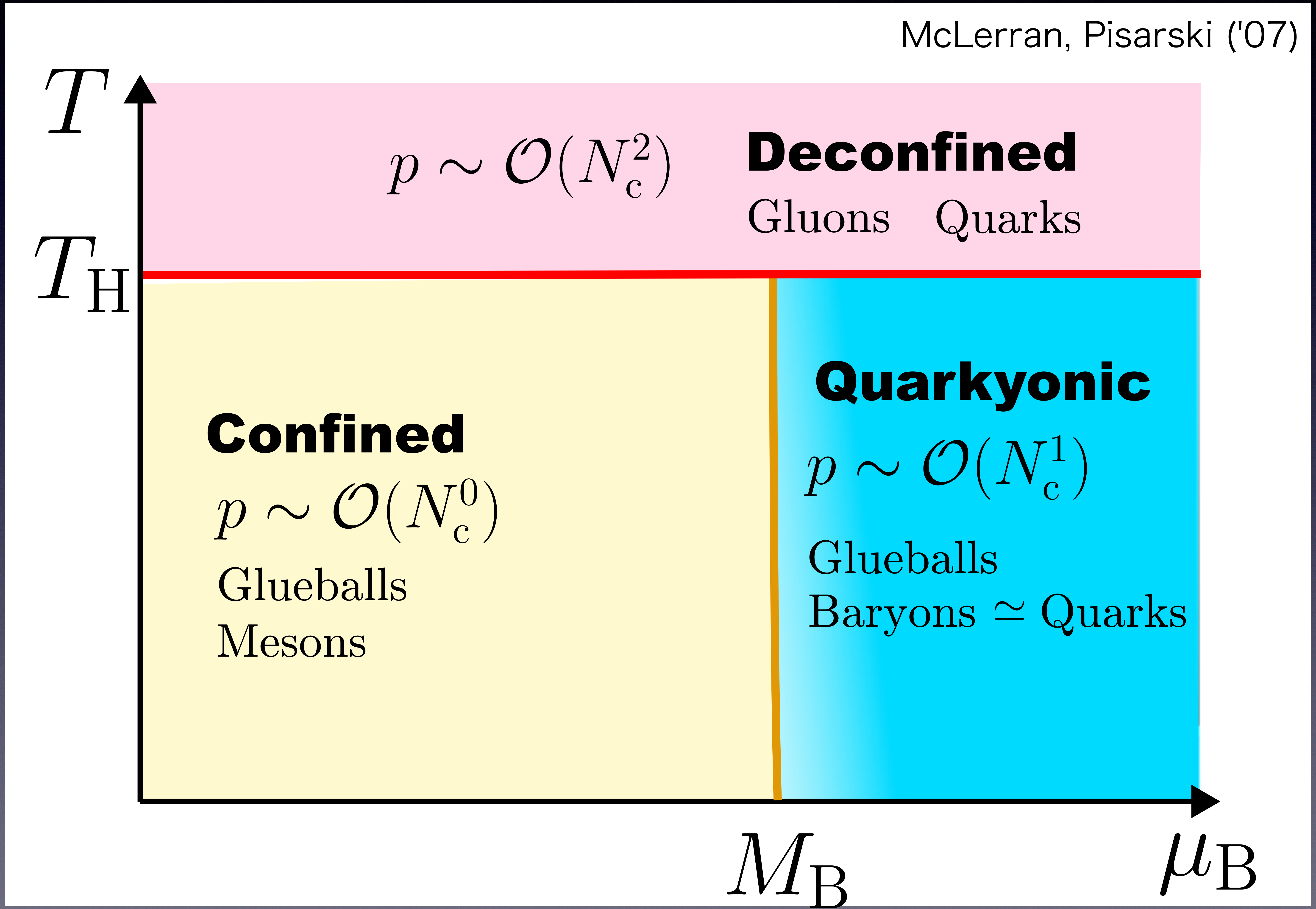
$\sim N_c$

baryon -meson



$\sim N_c^0$

Large-N_c phase diagram



Debye screening

$$m_D^2 \sim \frac{1}{N_c}(N_c g^2)\mu^2$$

Intermediate region

When entropy $s \sim n_m \sim N_c$

Contribution of interaction of mesons

to the pressure $\sim N_c$

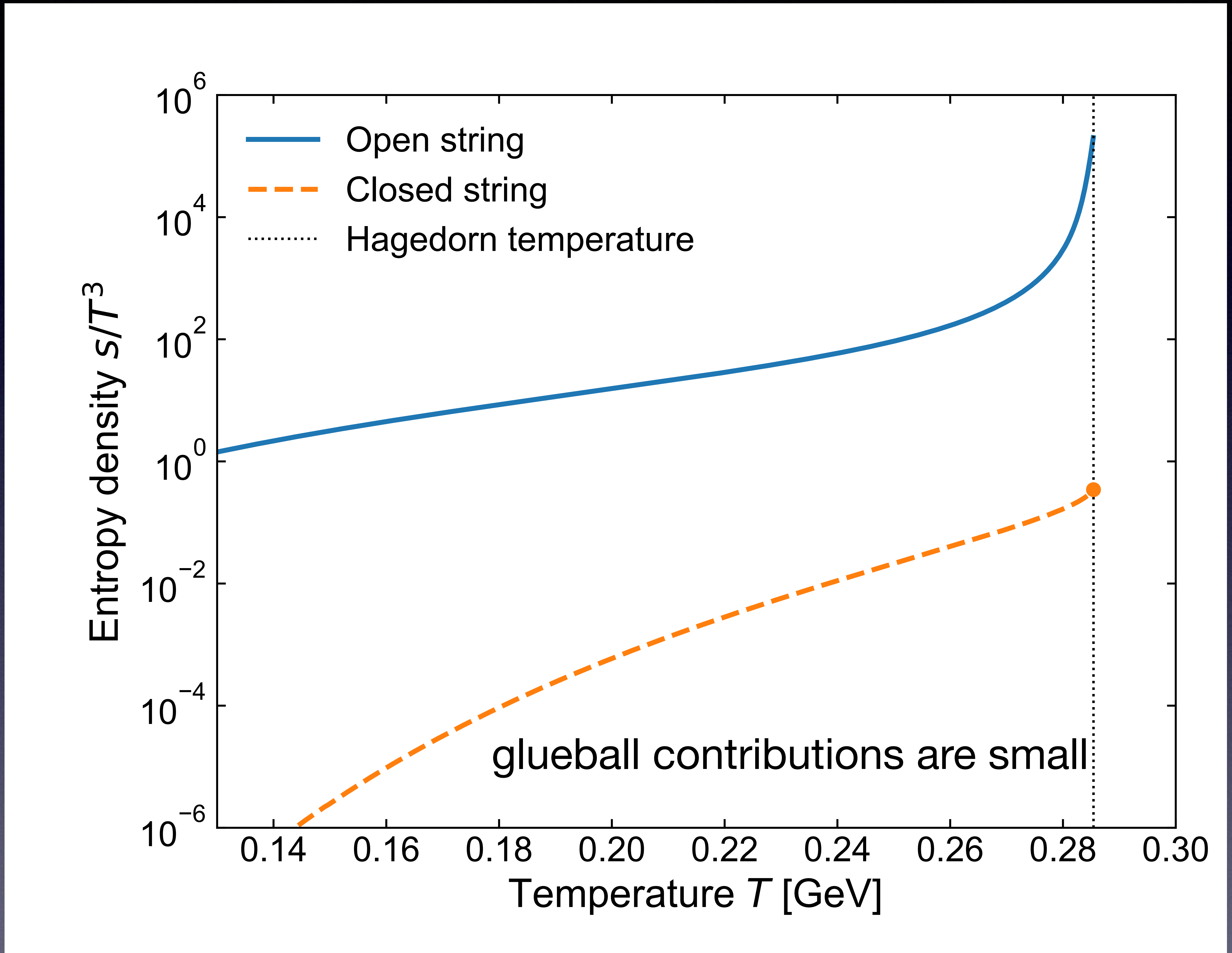
$$g^2 n_m^2 \sim \frac{1}{N_c} N_c^2 \sim N_c$$

The degrees freedom will look like quarks.

Still gluons are decoupled so it's "confined"

Quark Spaghetti with Glue Balls (SQGB)

Our Model



Meson:
Goldstones+open string

Glueballs:
Glueballs ($E < 2M_{\mathrm{GB}}$)
+ closed string

We choose

$$T_H = T_d = 285\ \mathrm{MeV}$$

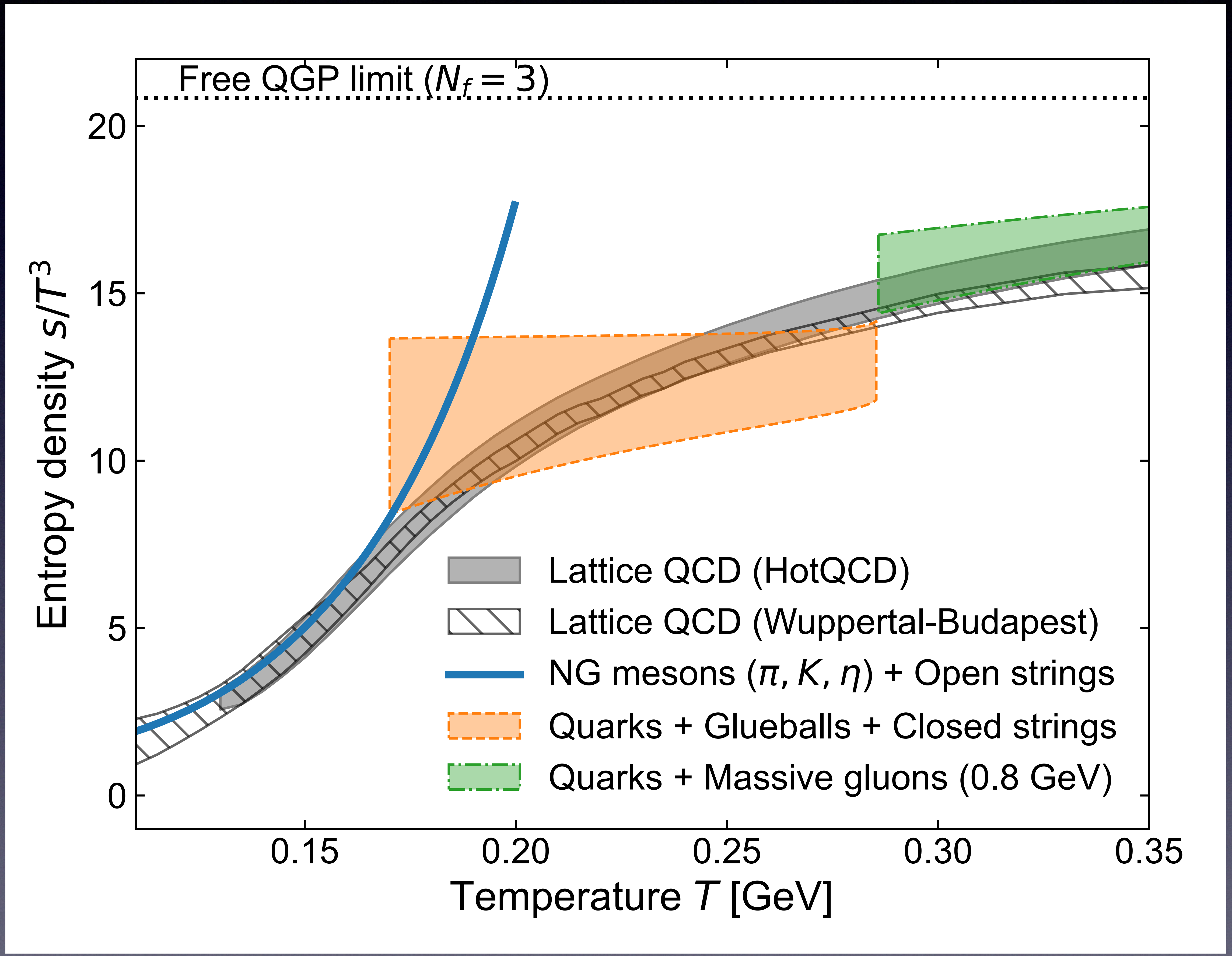
Hagedorn deconfined

$$T_H^2 = \frac{3}{2\pi}\sigma \qquad \sqrt{\sigma} = 440\ \mathrm{MeV}$$

String tention

Entropy vs Temperature

cf. Vovchenko's talk



There are three regimes

Low-T: Goldstone + open string

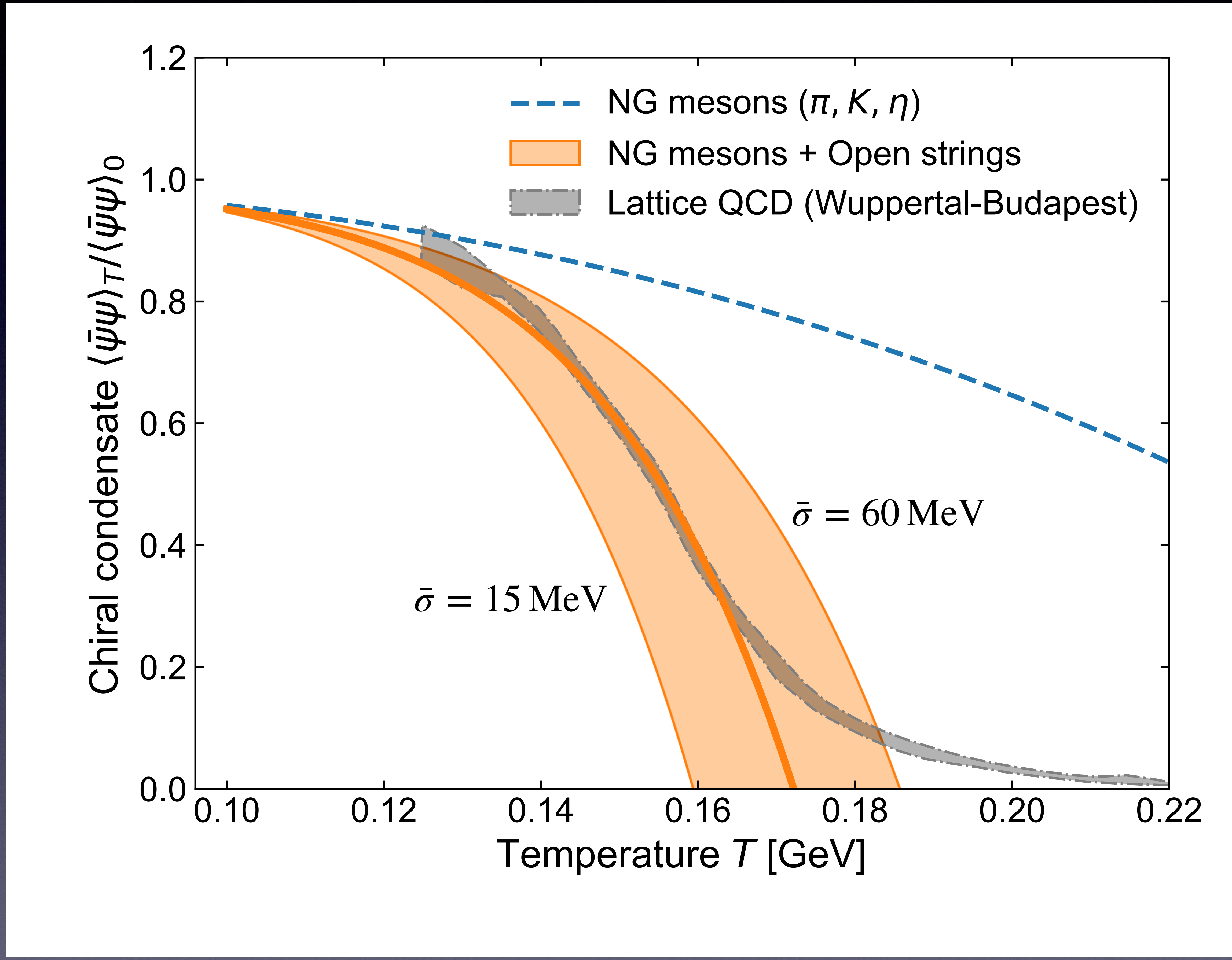
Intermediate region $T_c < T < T_d$

: Quark+ Glueball + closed string

Quark Spaghetti with Glue Balls (SQGB)

High-T: Quark-gluon plasma

Chiral symmetry



Chiral condensate

$$\langle\bar{\psi}\psi\rangle_T = \langle\bar{\psi}\psi\rangle_0 - \frac{\partial p}{\partial m_q}$$

For the contribution of mass M

$$\frac{\partial p(M)}{\partial m_q} = \frac{\partial p(M)}{\partial M}\frac{\sigma_M}{m_q}$$

Sigma term: $\sigma_M = m_q \dfrac{\partial M}{\partial m_q} \overset{\text{assumption}}{\approx} \underbrace{n_q}_{\text{\# of light quarks}}\bar{\sigma}$

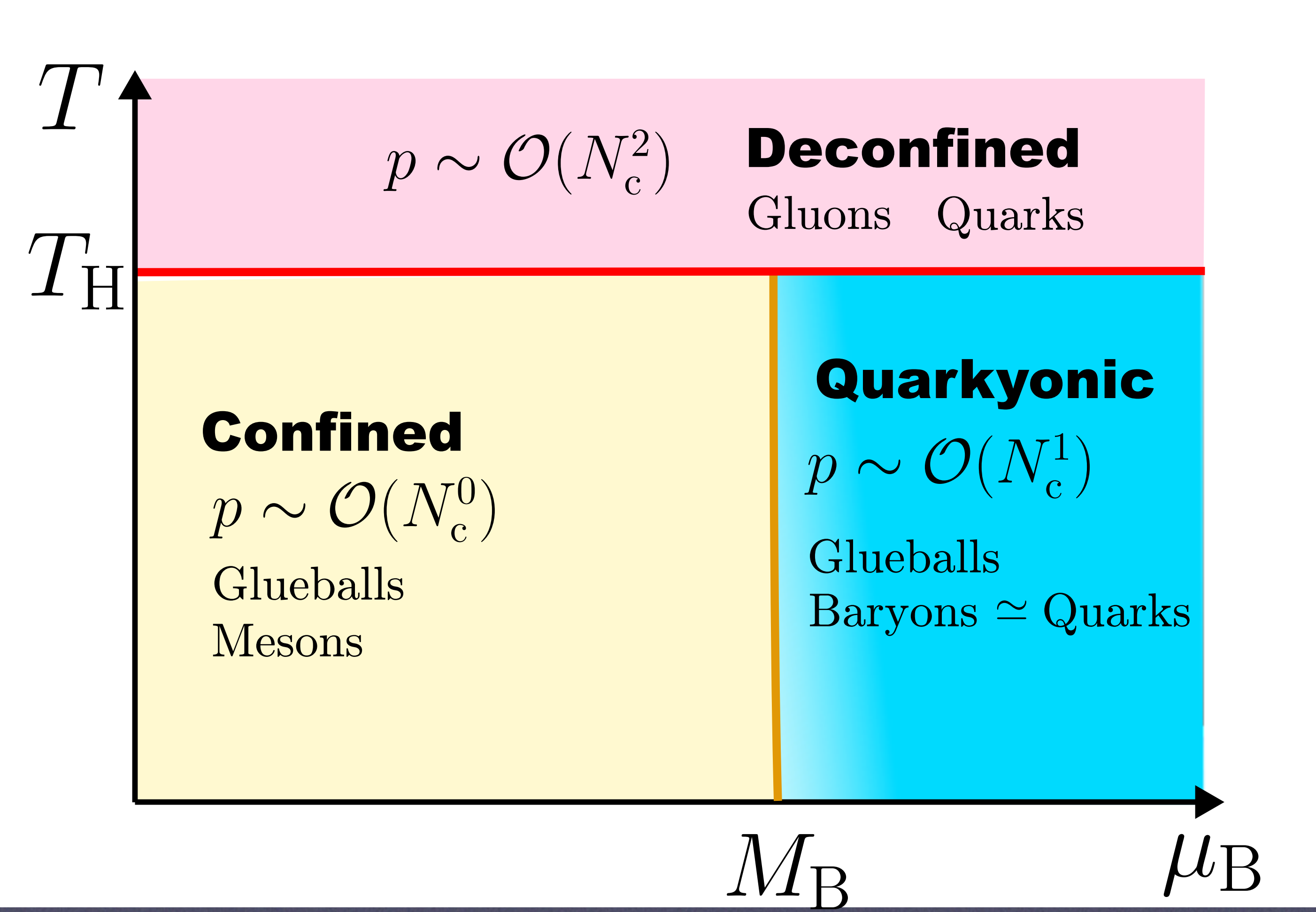
We choose $\bar{\sigma} = 30\ \mathrm{MeV}$

Chiral symmetry is restored.

Large-N_c to $N_c = 3$

Large-N_c limit

McLerran, Pisarski ('07)

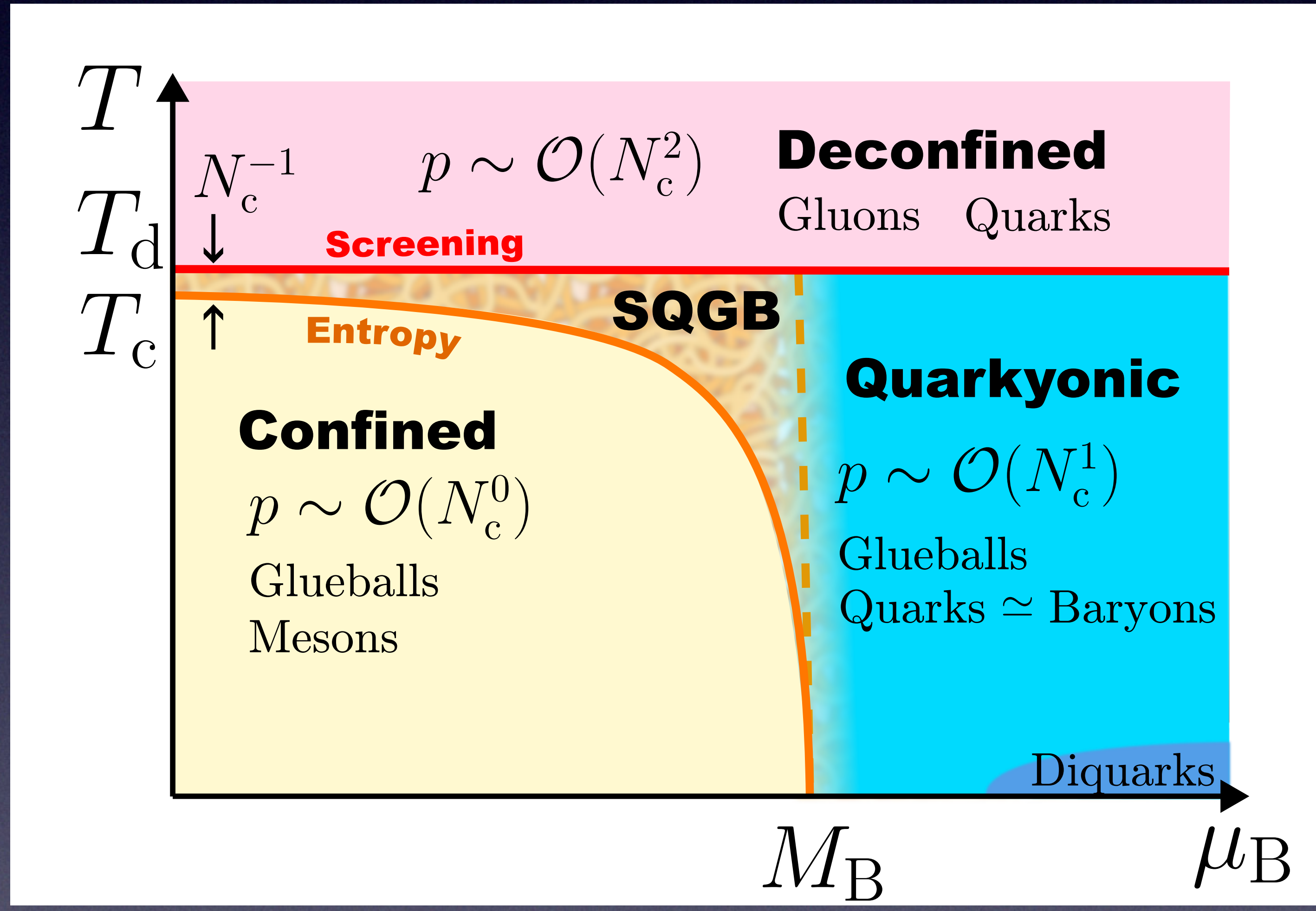


Three phases based on degrees of freedom

Baryon contribution is suppressed

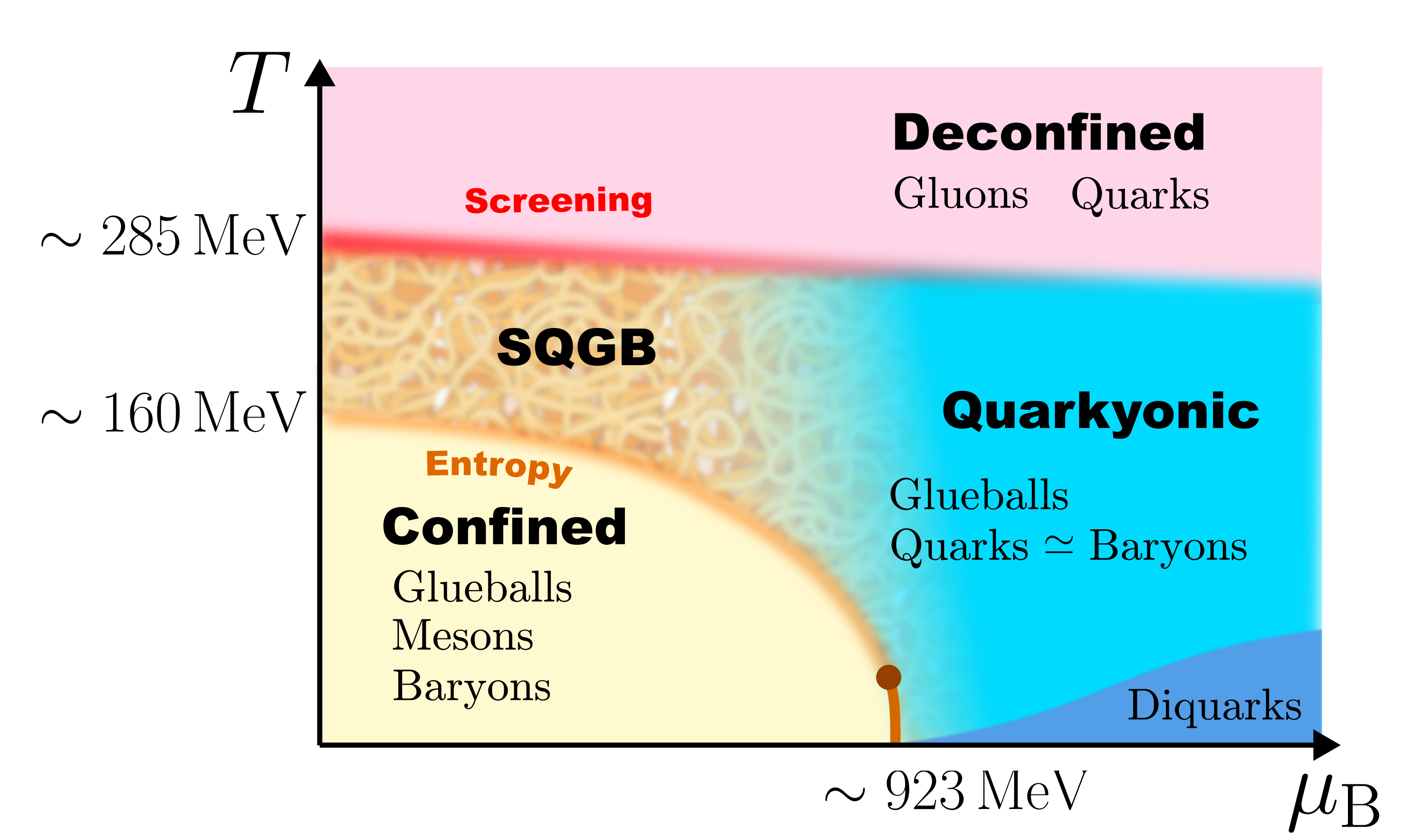
$$\exp(-M_B/T) \sim \exp(-N_c)$$

Large-N_c



SQGB is $1/N_c$ at $\mu_B = 0$

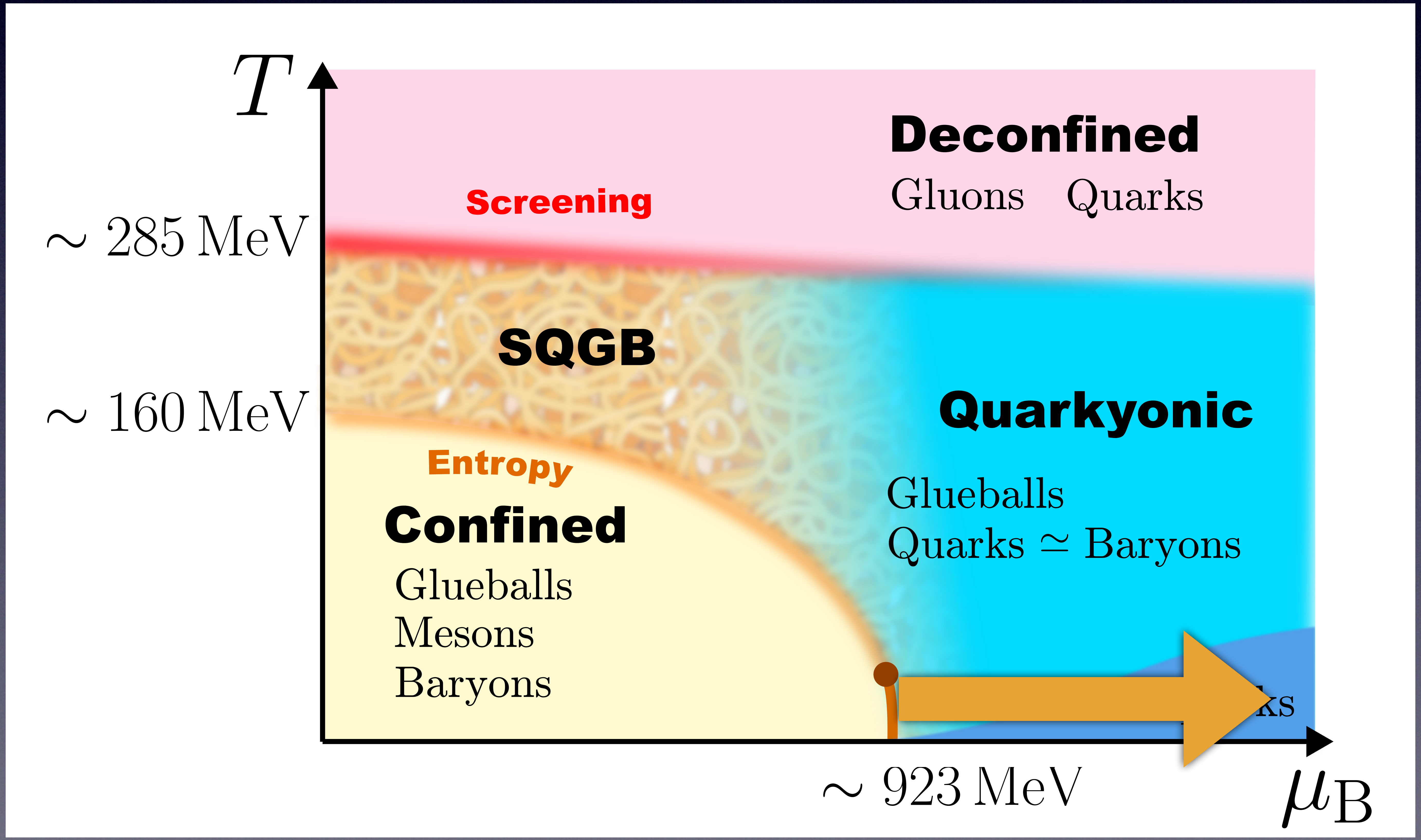
$N_c = 3$



$N_c = 3$ SQGB

$T_c \sim 160\,\text{MeV} < T < T_d \sim 285\,\text{MeV}$

Dense QCD matter

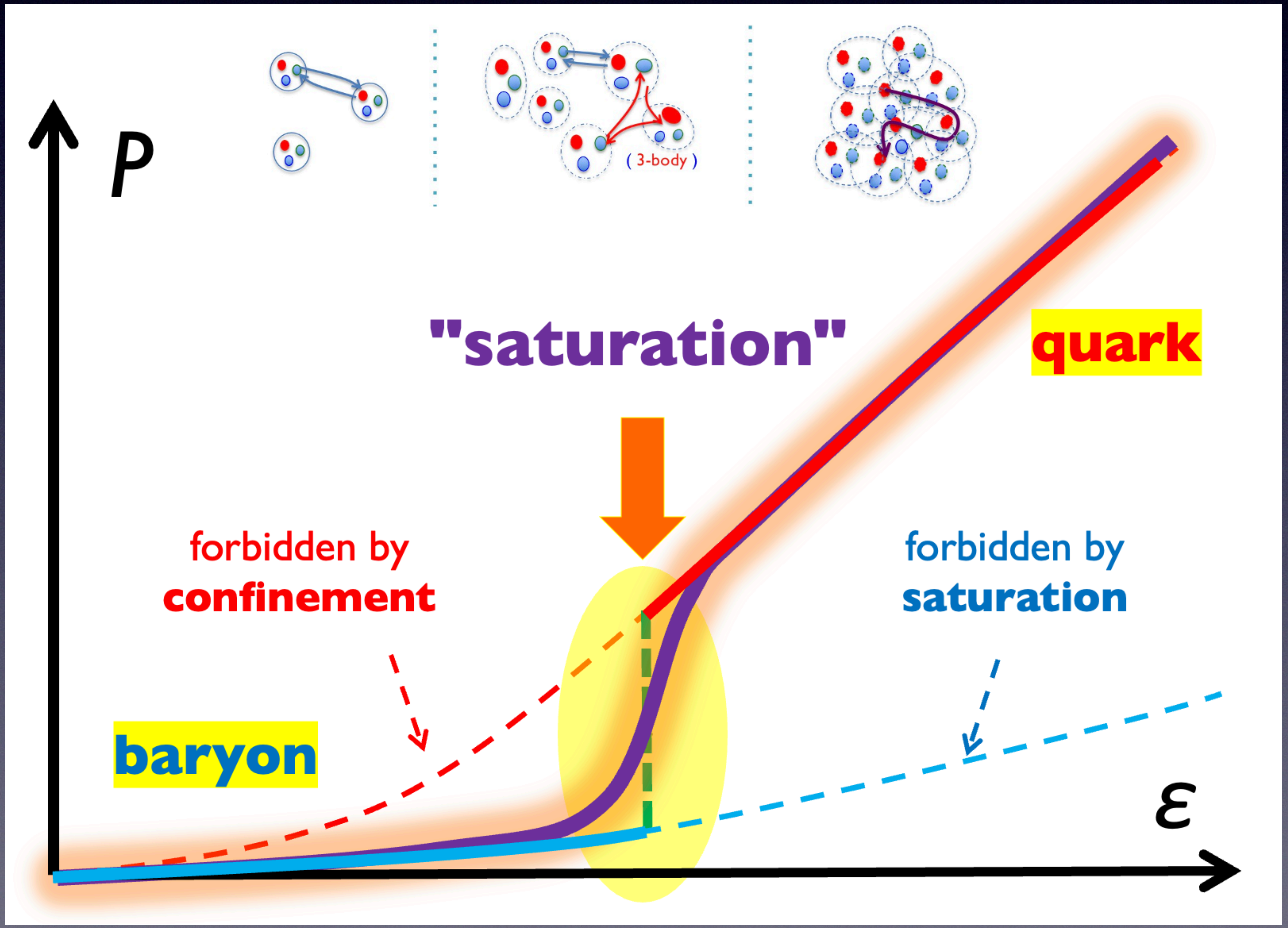


Ideal dual Quarkyonic (IdylliQ) model

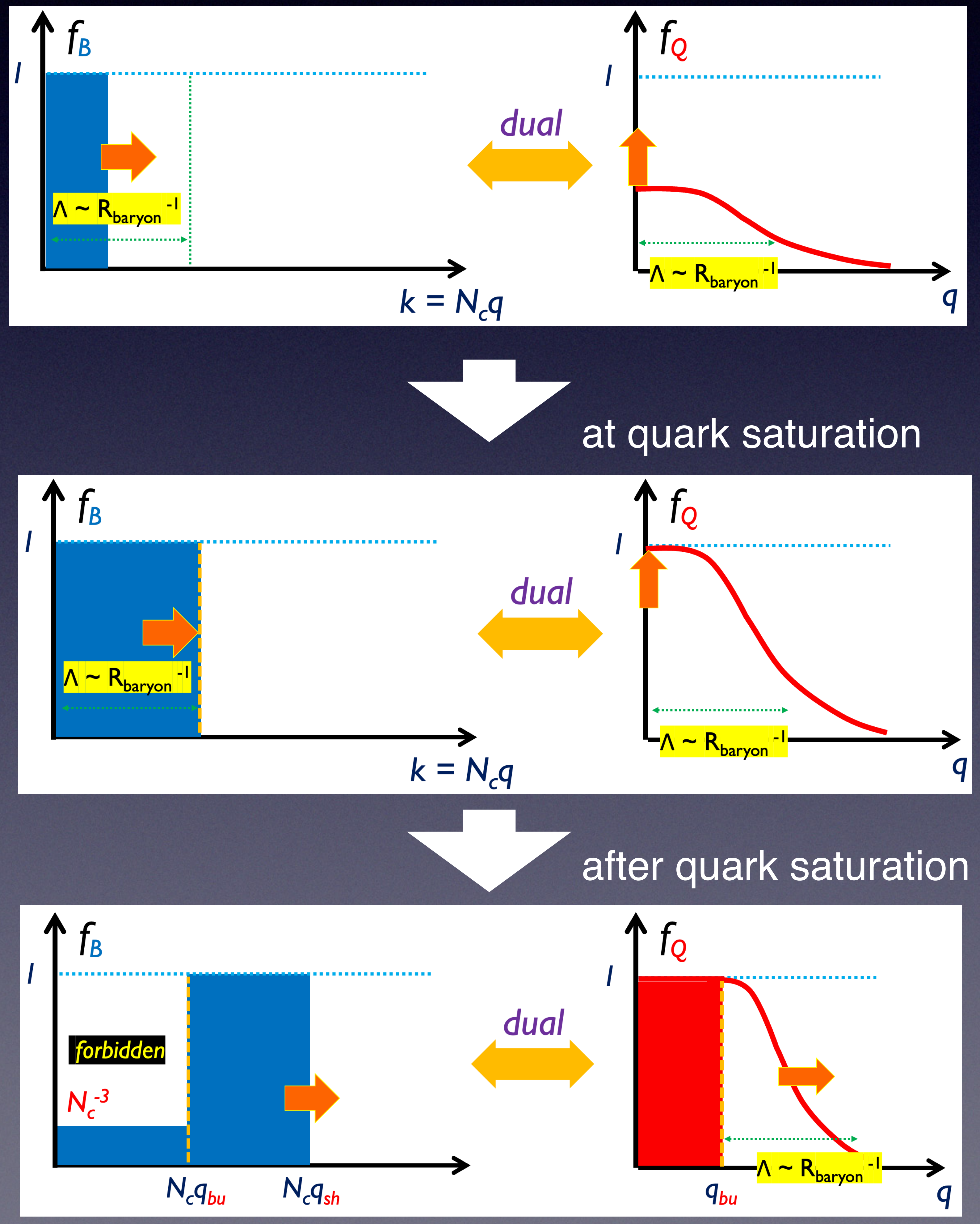
cf. Kojo's talk   Kojo ('21), McLerran, Reddy ('19), Fujimoto, Kojo, McLerran ('24)

$$f_Q(\boldsymbol{q}, n_B) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B; n_B)\varphi_Q^B(\boldsymbol{q}_B; \boldsymbol{P}_B)$$

Figure is taken from Kojo, 2412.20442



before quark saturation



Study of QCD in (1+1)d

cf. see Itou's talk for two-color QCD in 3+1d

A simple model describing quarkyonic matter

Numerical calculations
can be done by using tensor network technique

(Bosonization tequnique also useful.)

Baluni ('80), P. J. Steinhardt ('80) Kojo ('11), Lajer, Konik, Pisarski, Tsvelik ('21)

- **Finite box simulation**

Banuls, Cichy, Cirac, Jansen, Khun ('17) Hayata, YH, Nishimura, JHEP 07 (2024) 106

- **Infinite Volume simulation**

Fujikura YH, 2605.17183

(dimensionless) QCD_2 Hamiltonian

$$J = \frac{ag_0}{2}, w = \frac{1}{2g_0 a}, m = m_0/g_0$$

We use $g_0 = 1$ unit

$$H/g_0 = J \sum_{n=1}^{N-1} E_i^2(n)$$ **Electric field term**

$$+w \sum_{n=1}^{N-1} \left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1) \right)$$

Hopping term

$$+m \sum_{n=1}^{N} (-1)^n \chi^\dagger(n)\chi(n)$$ **Mass term**

As a variational ansatz of wave function

- **We employ a matrix product state**

$$|\psi\rangle = \sum_{\{n_i\}} |n_1\rangle \cdots |n_N\rangle \mathrm{tr} M_1^{n_1} \cdots M_N^{n_N}$$

$[M_i^{n_i}]_{ij}$ **:** $D \times D$ **matrix**

- **Optimize the wave function by density matrix renormalization group technique**

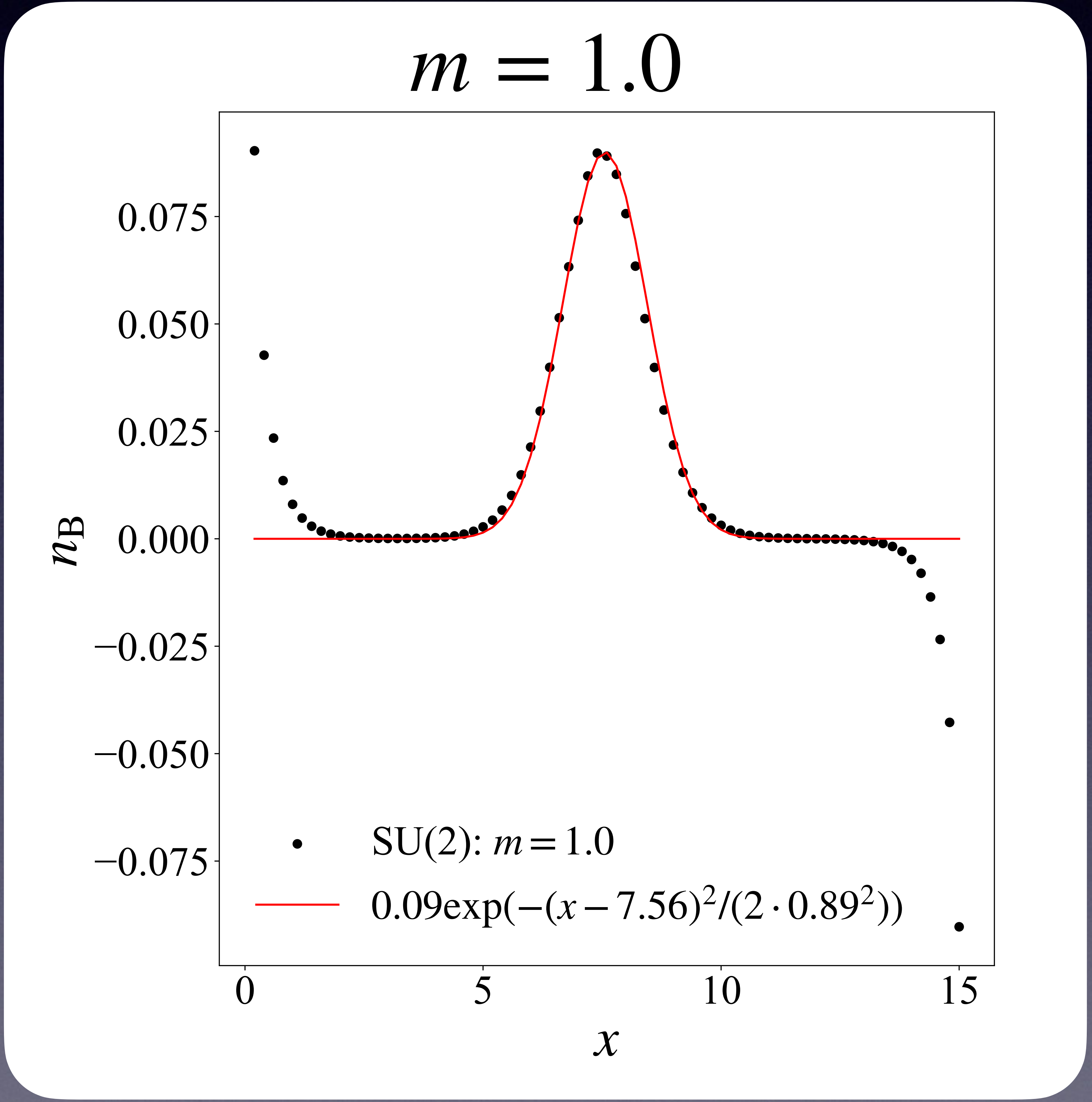
$$E = \min_{\psi} \langle \psi | H | \psi \rangle$$

Numerical results

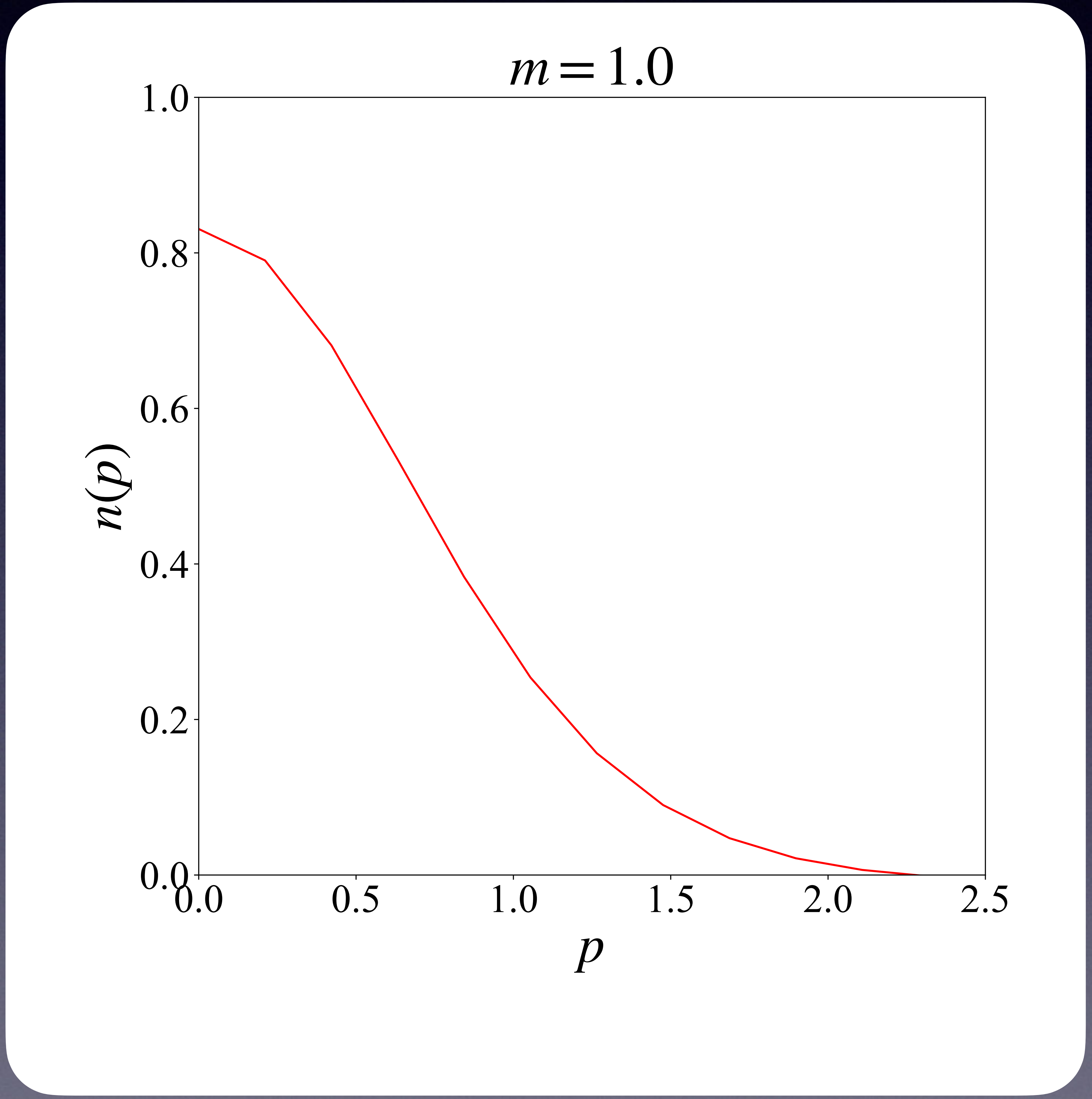
Color SU(2), 1 flavor, vacuum

single baryon state $\dim\mathscr{H} = 2^{300}$ $J = 1/20$ $w = 5$ volume $V = 15$

Baryon number density



Quark distribution function



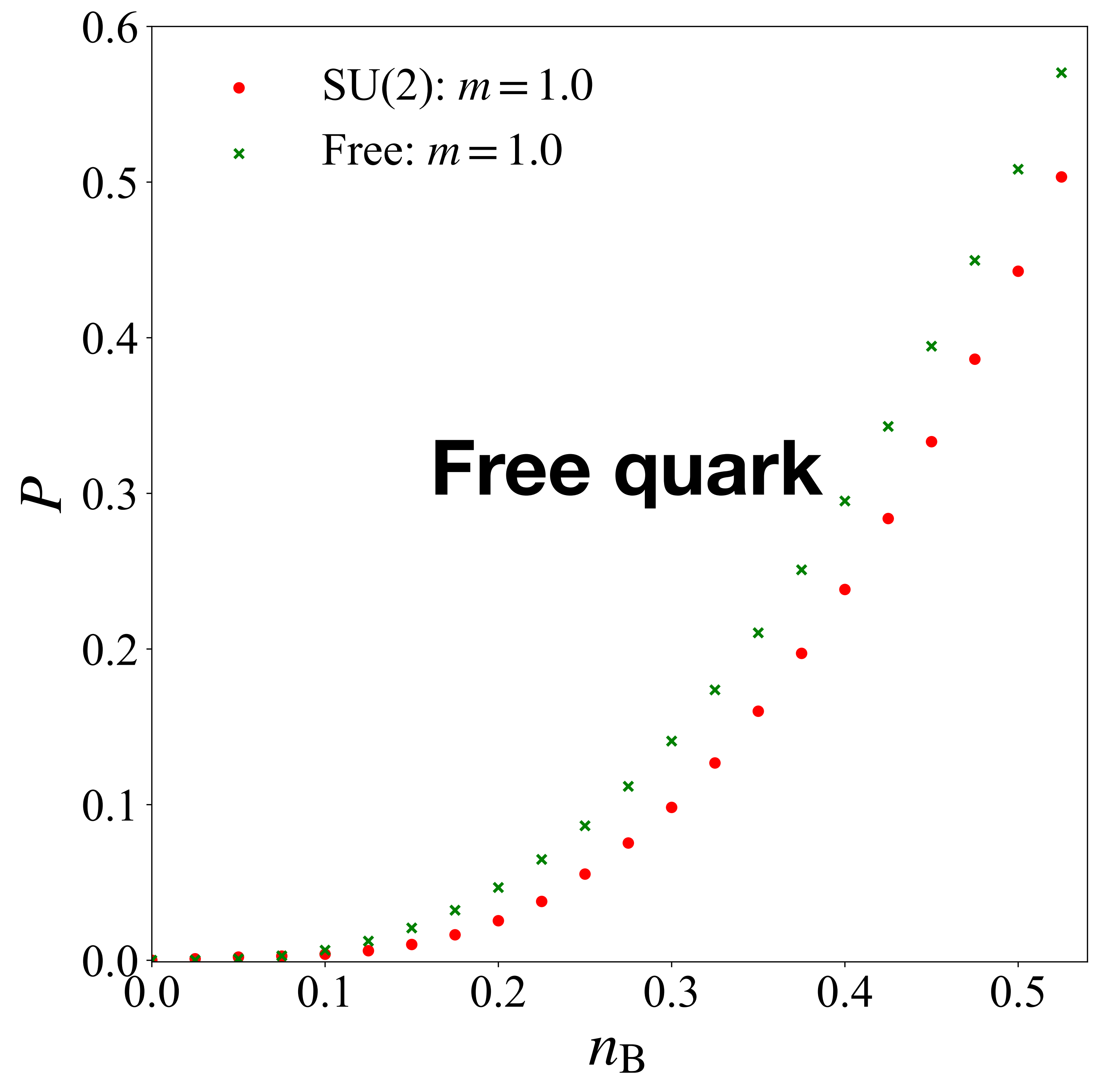
Baryon size ~ 1

QCD_2 Hamiltonian Lattice simulation with density matrix renormalization technique

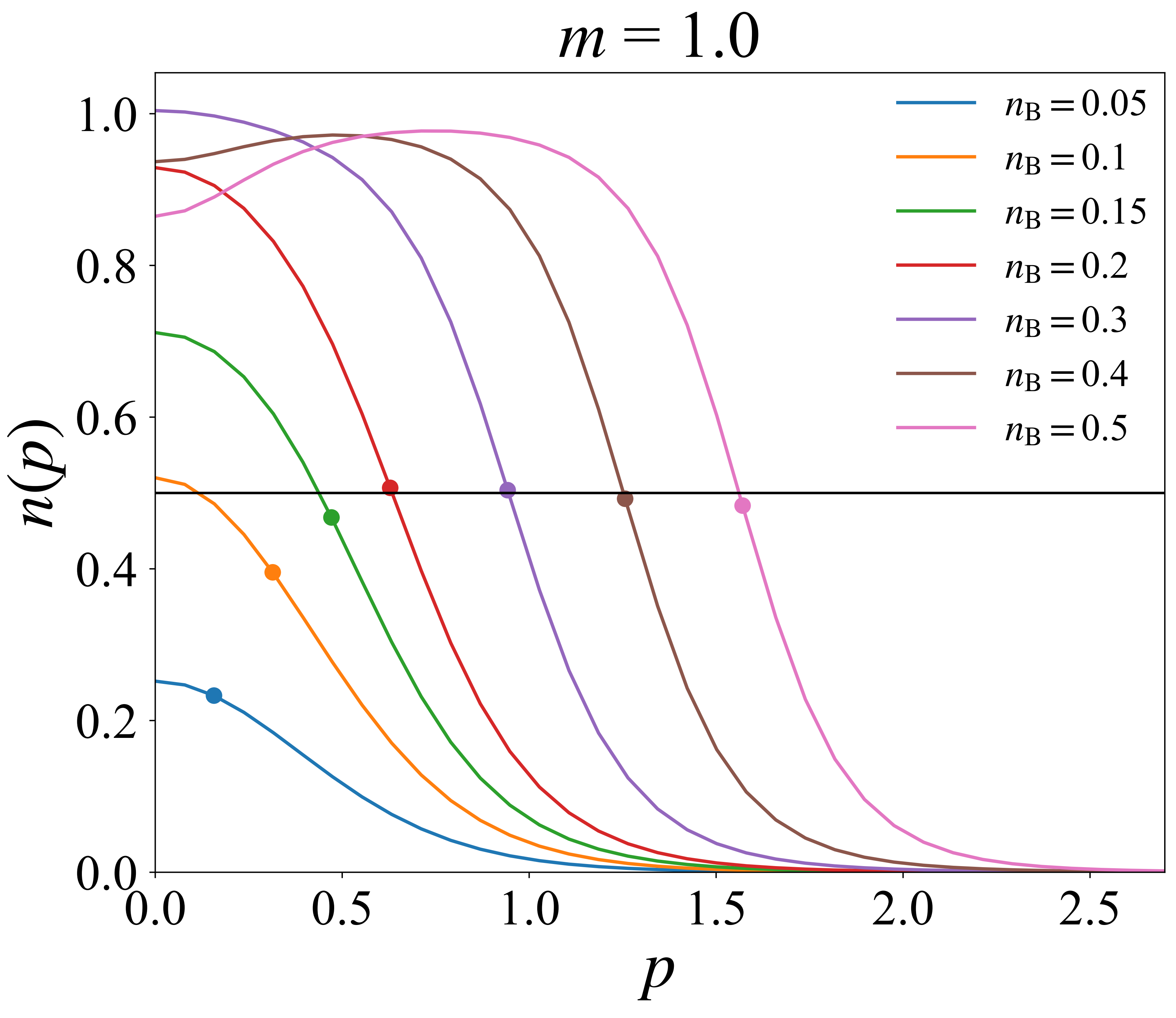
Tomoya Hayata, YH, Nishimura ('23)

Two color QCD $N_f = 1$

$\dim\mathscr{H} = 2^{320}$



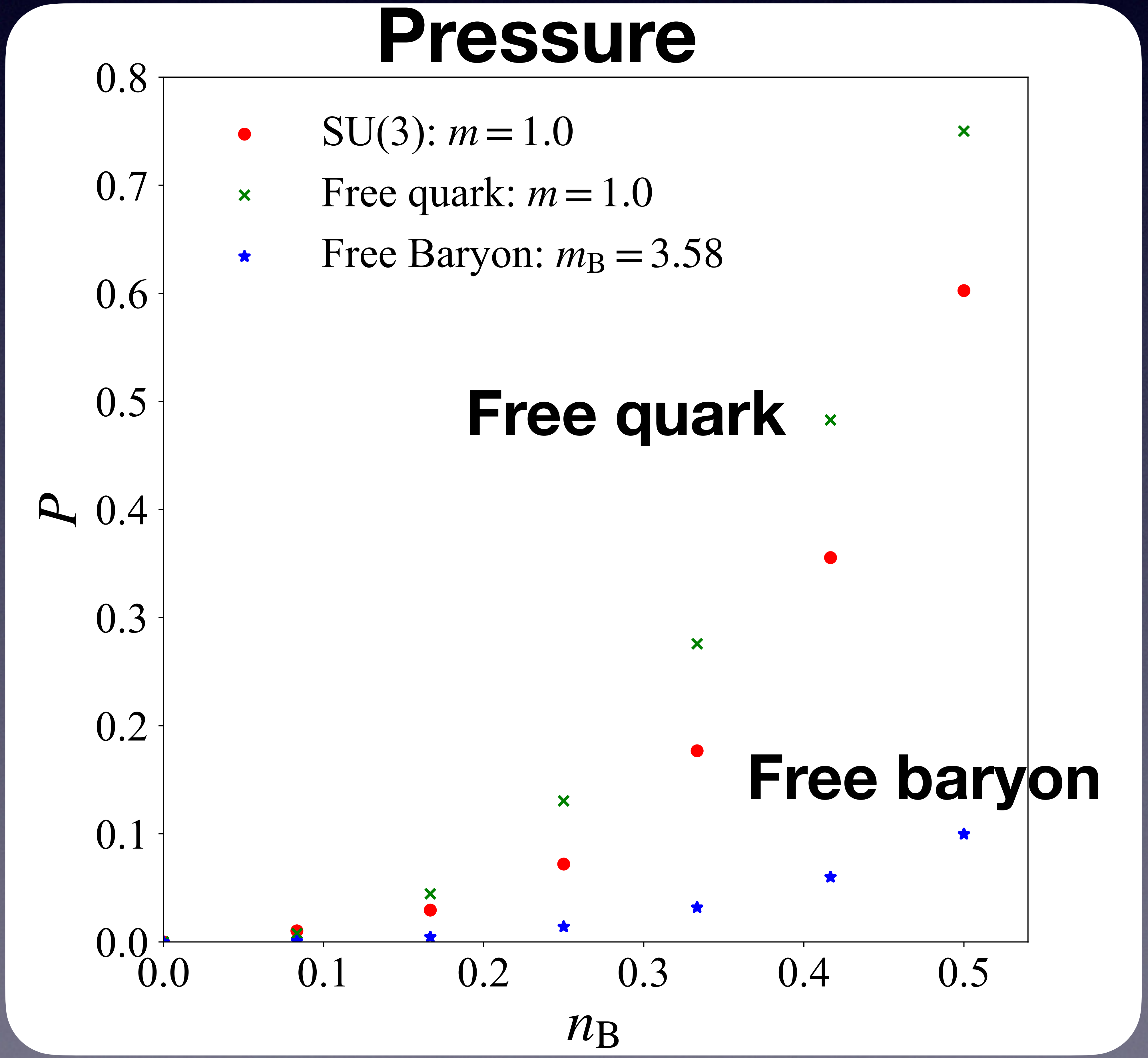
$\dim\mathscr{H} = 2^{480}$

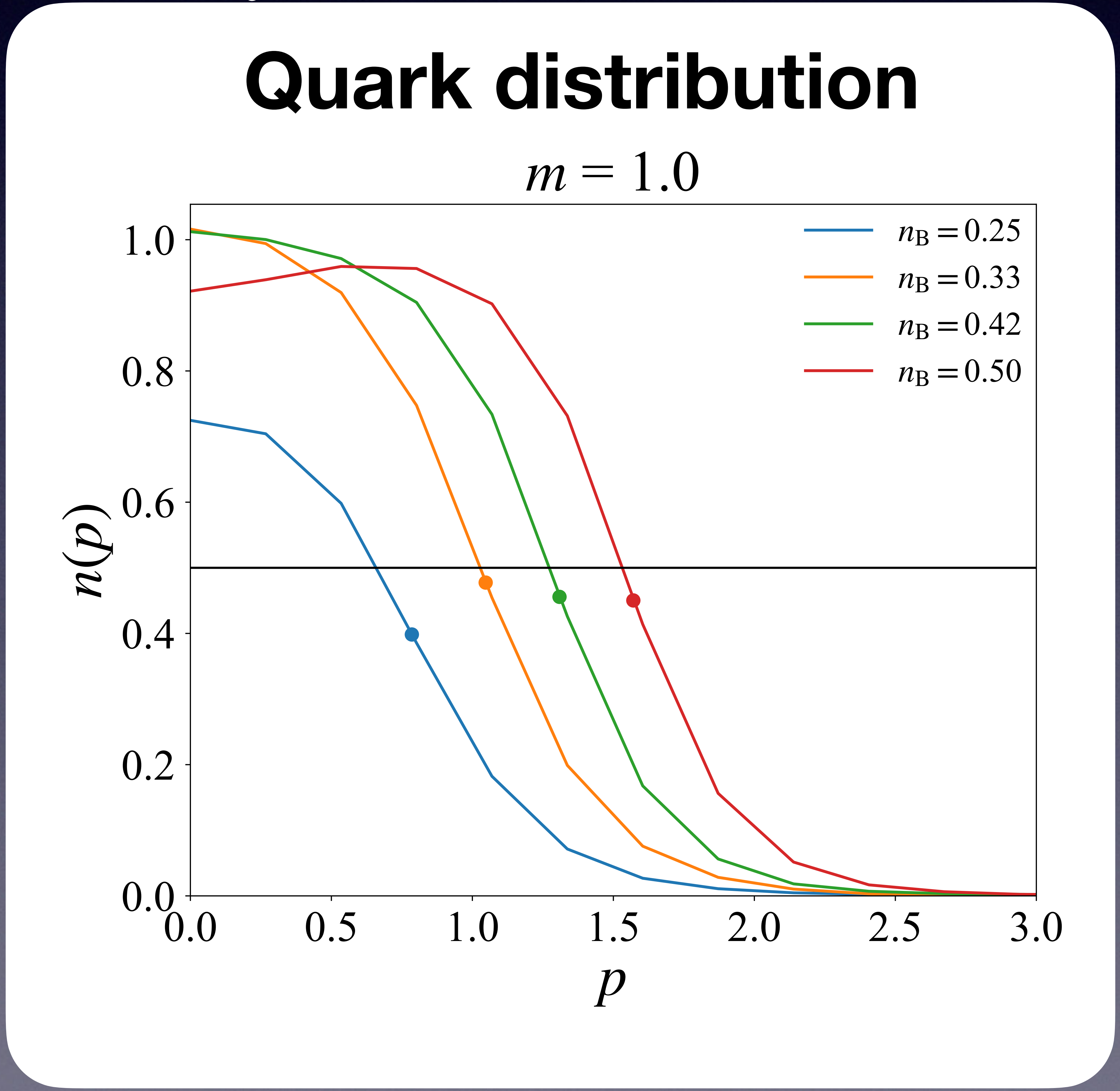


QCD_2 Hamiltonian Lattice simulation
with density matrix renormalization technique

Tomoya Hayata, YH, Nishimura ('23)

Three color QCD, $N_f = 1$ $\dim\mathscr{H} = 2^{144}$



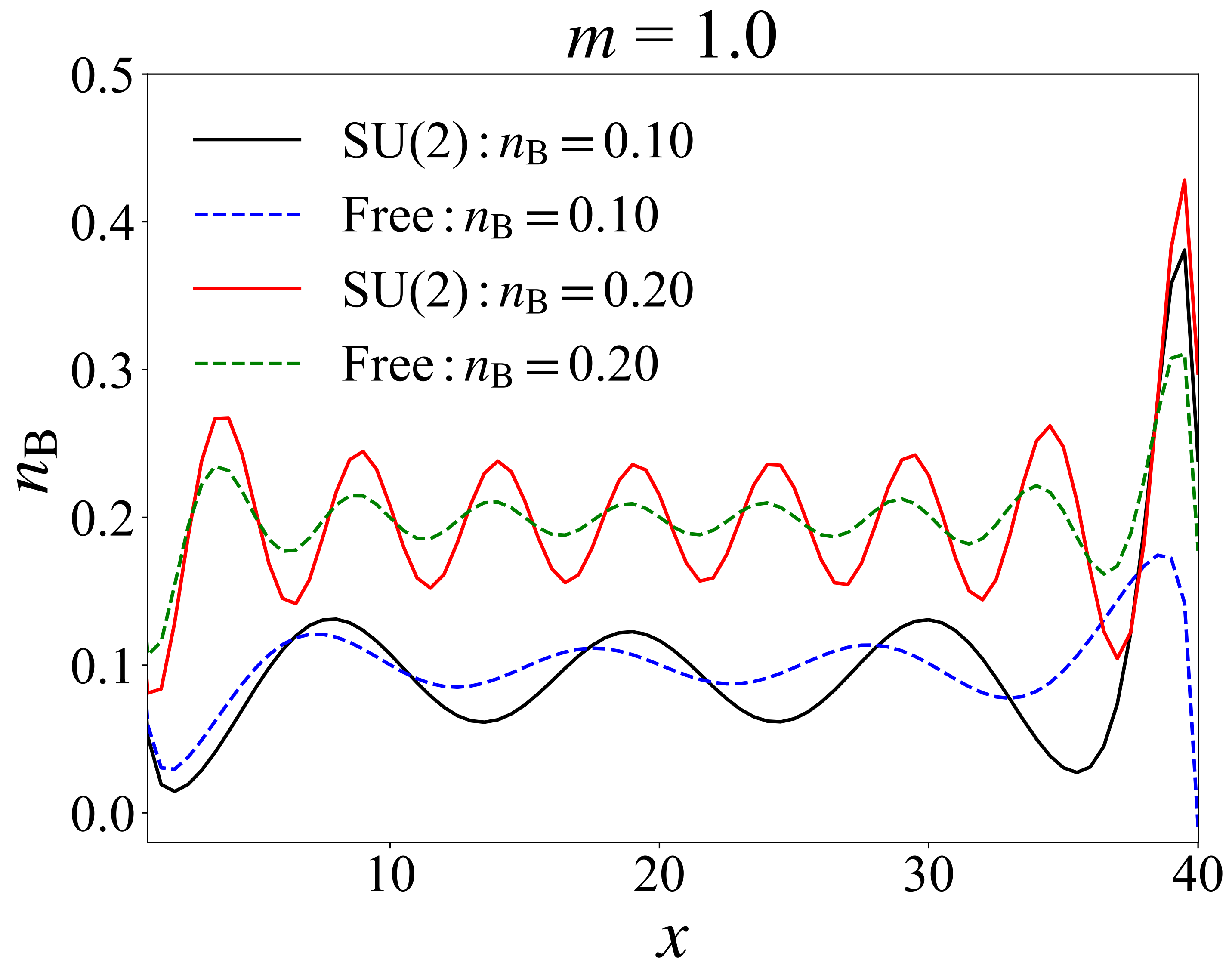


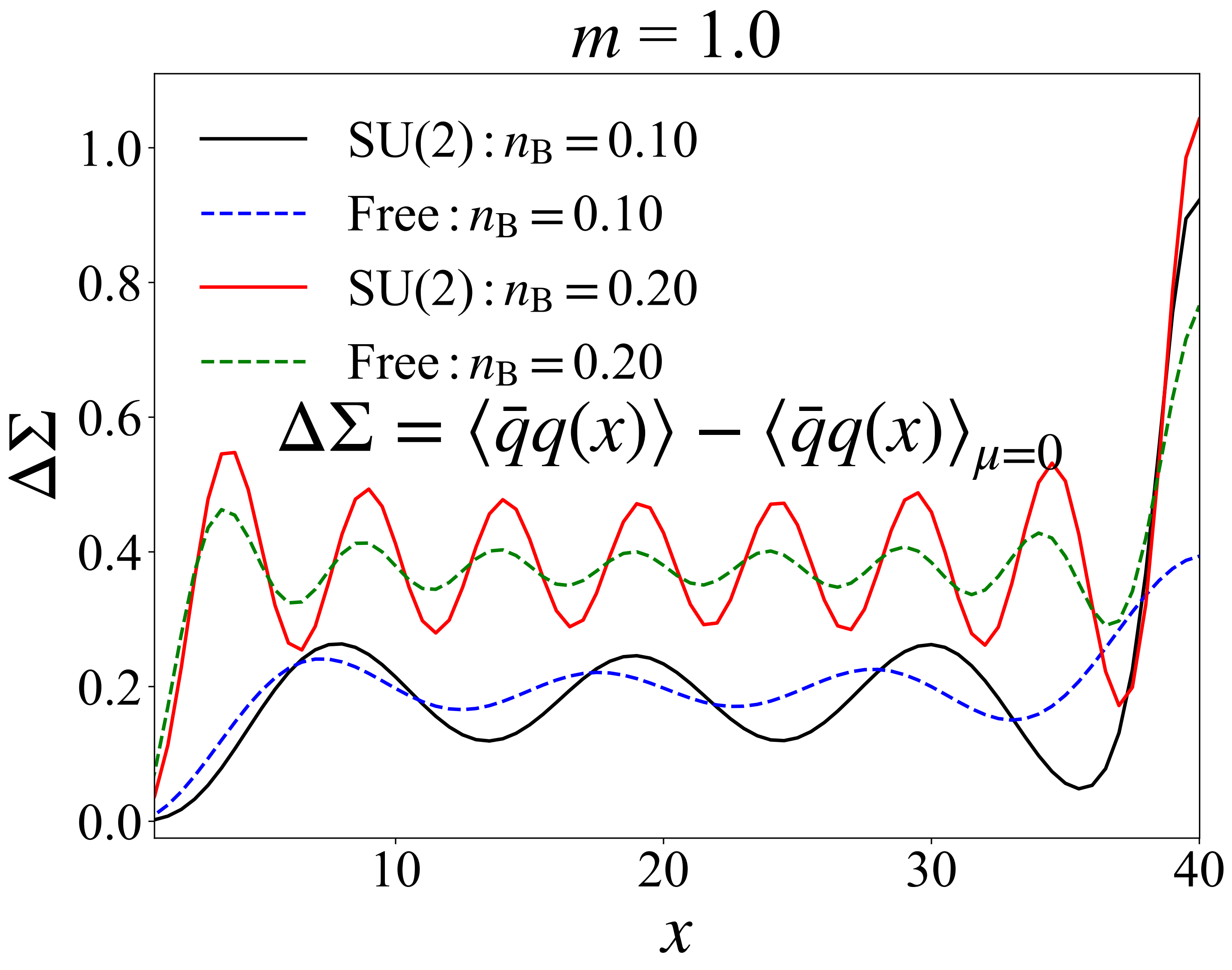
Inhomogeneous phase in QCD_2

corresponding to 'quarkyonic chiral spirals' Kojo, Hidaka, McLerran, Pisarski (2010)

SU(2)

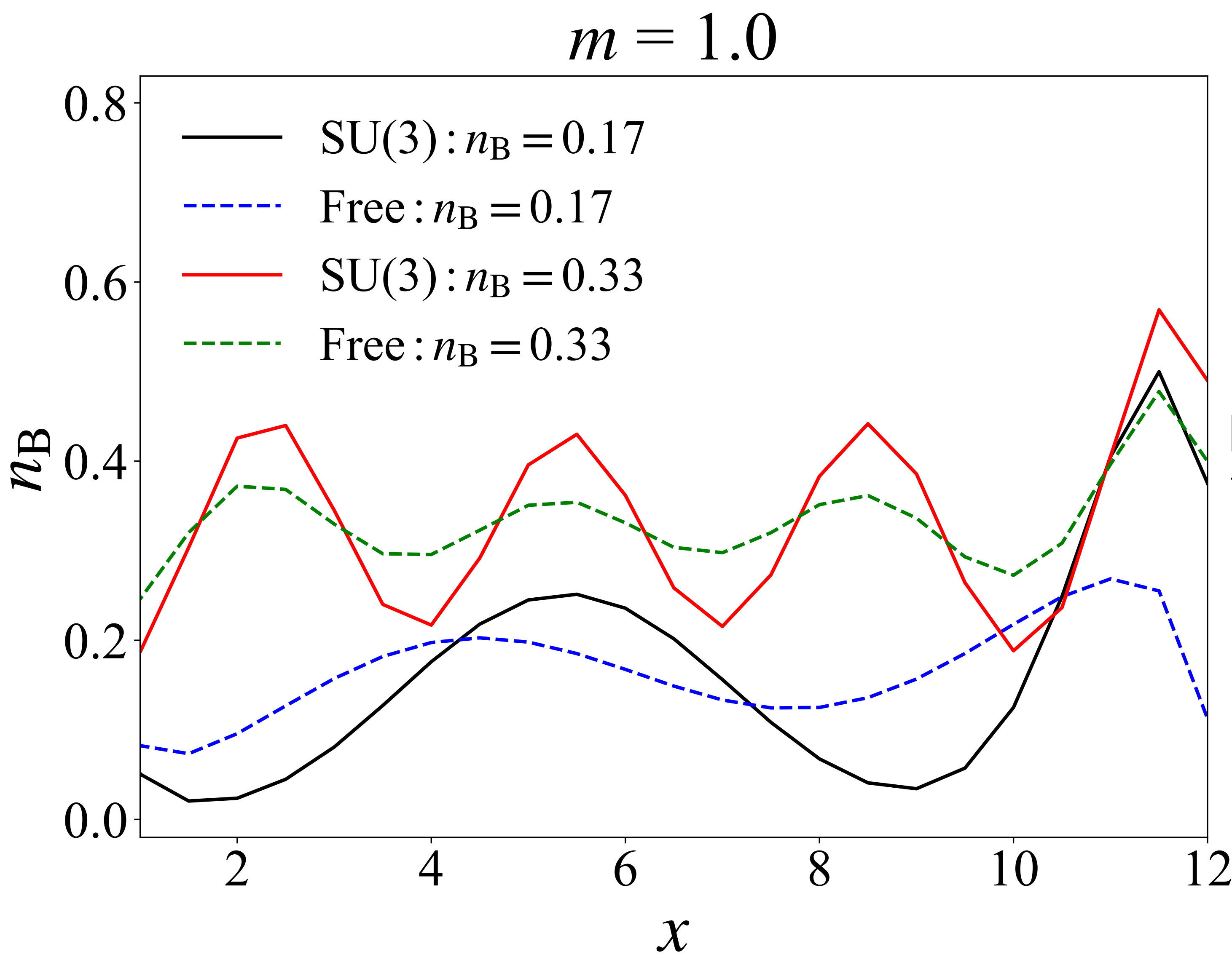
$\dim\mathscr{H} = 2^{320}$

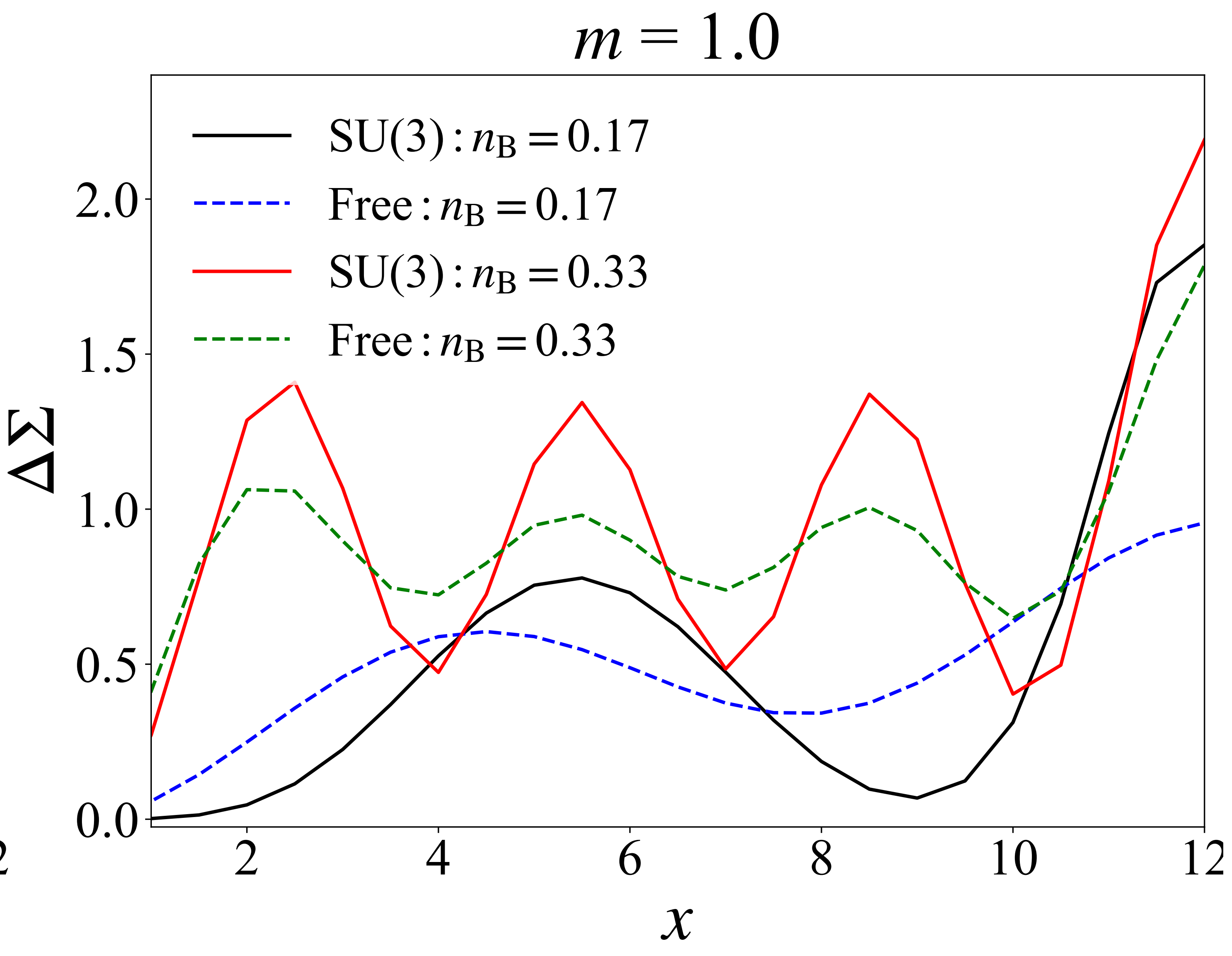




SU(3)

$\dim\mathscr{H} = 2^{320}$

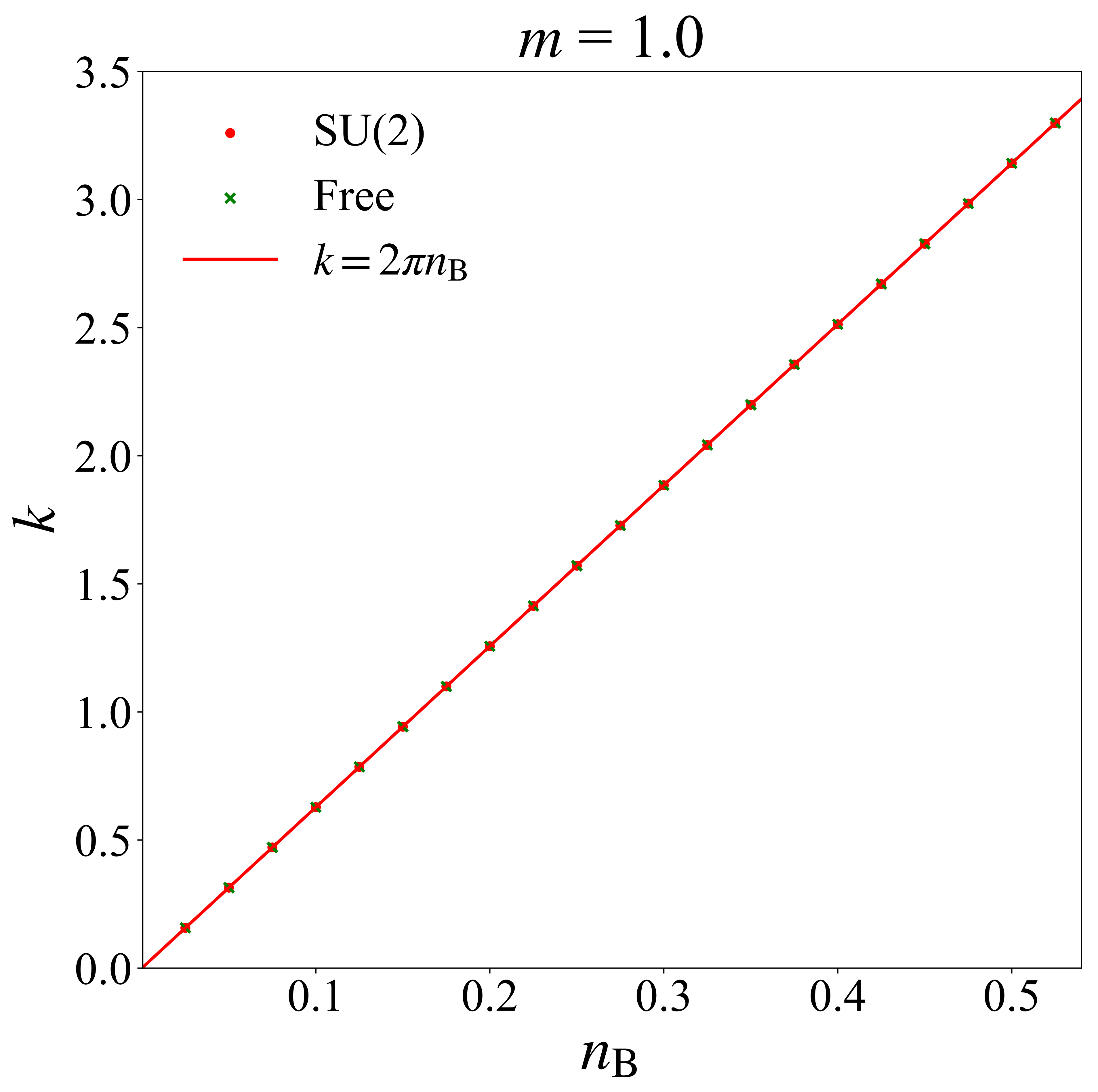




Wave number dependence

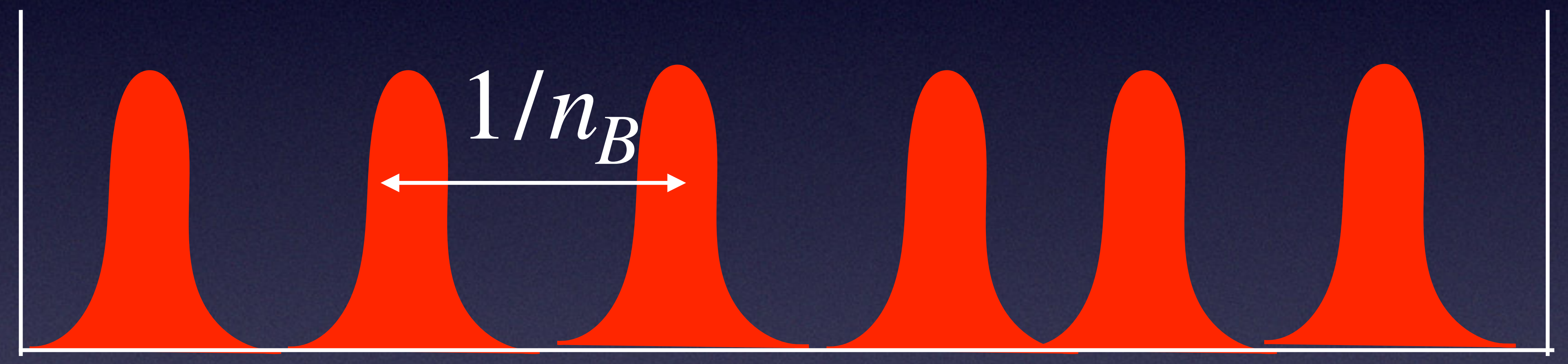
$$J = 1/8 \quad w = 2 \quad V = 40 \quad \dim\mathscr{H} = 2^{320}$$

Wave number dependence



Hadronic picture

If hadron interactions are repulsive



distance $1/n_B \Rightarrow k = 2\pi n_B$

Quark picture

If interactions between quarks
Fermi surface is unstable

$\Rightarrow$**density wave** $k = 2p_{\mathrm{F}} = 2\pi n_{\mathrm{B}}$

Infinite volume

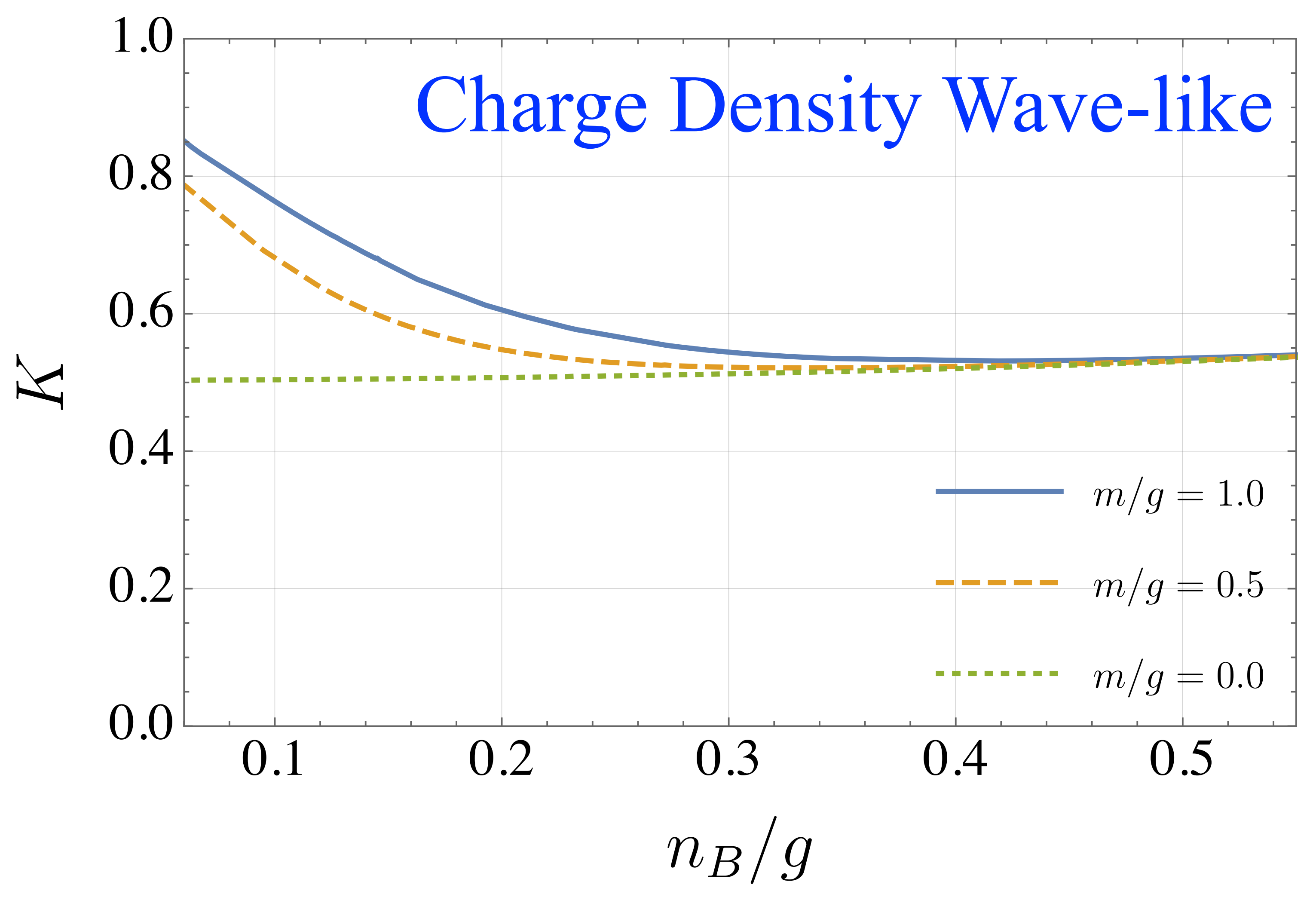
Using Variational Uniform Matrix Product State,   Fujikura YH, 2605.17183

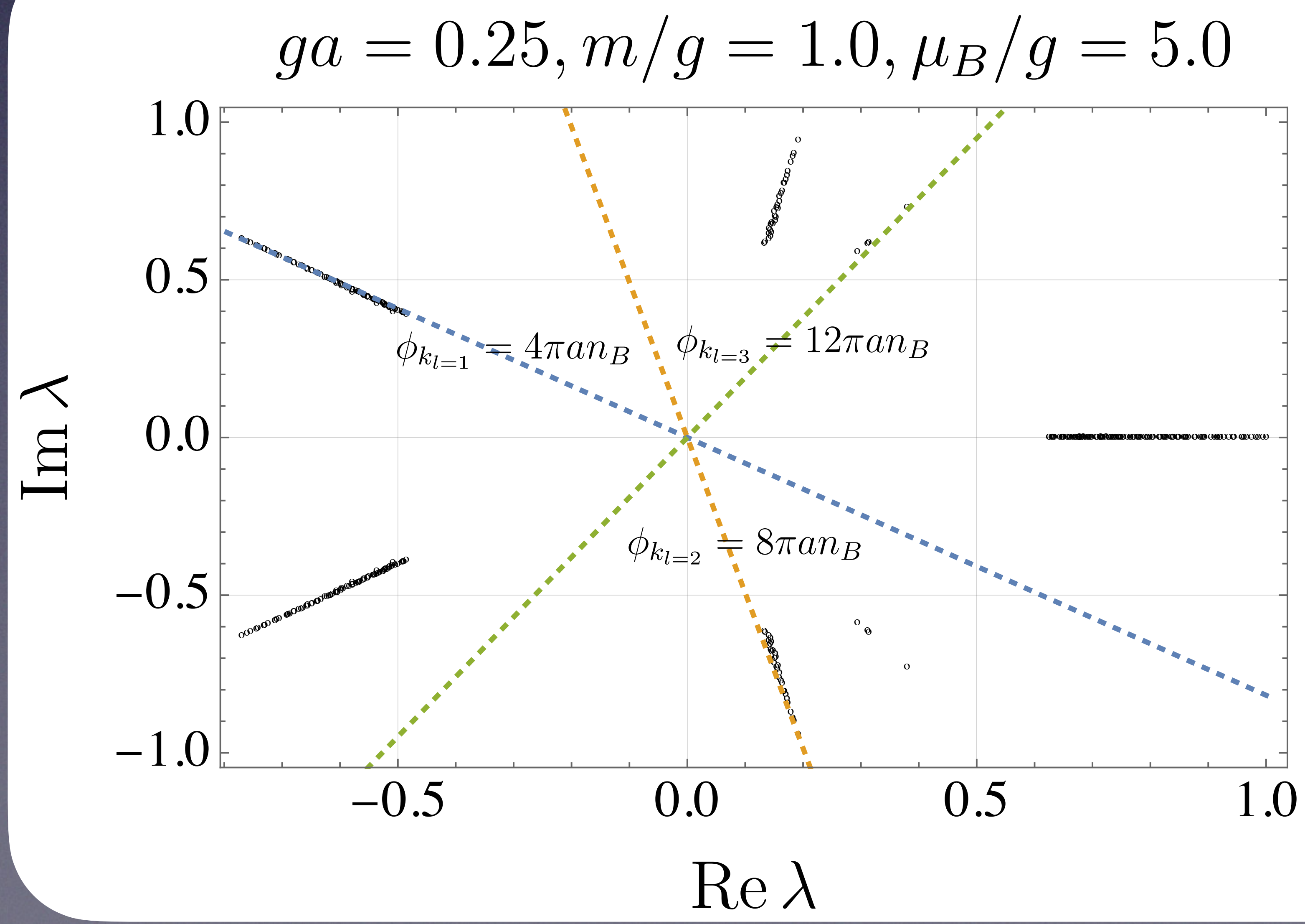
Zauner-Stauber, Vanderstraeten, Fishman, Verstraete, Haegeman ('18)

No continuous symmetry breaking occurs in (1+1)d

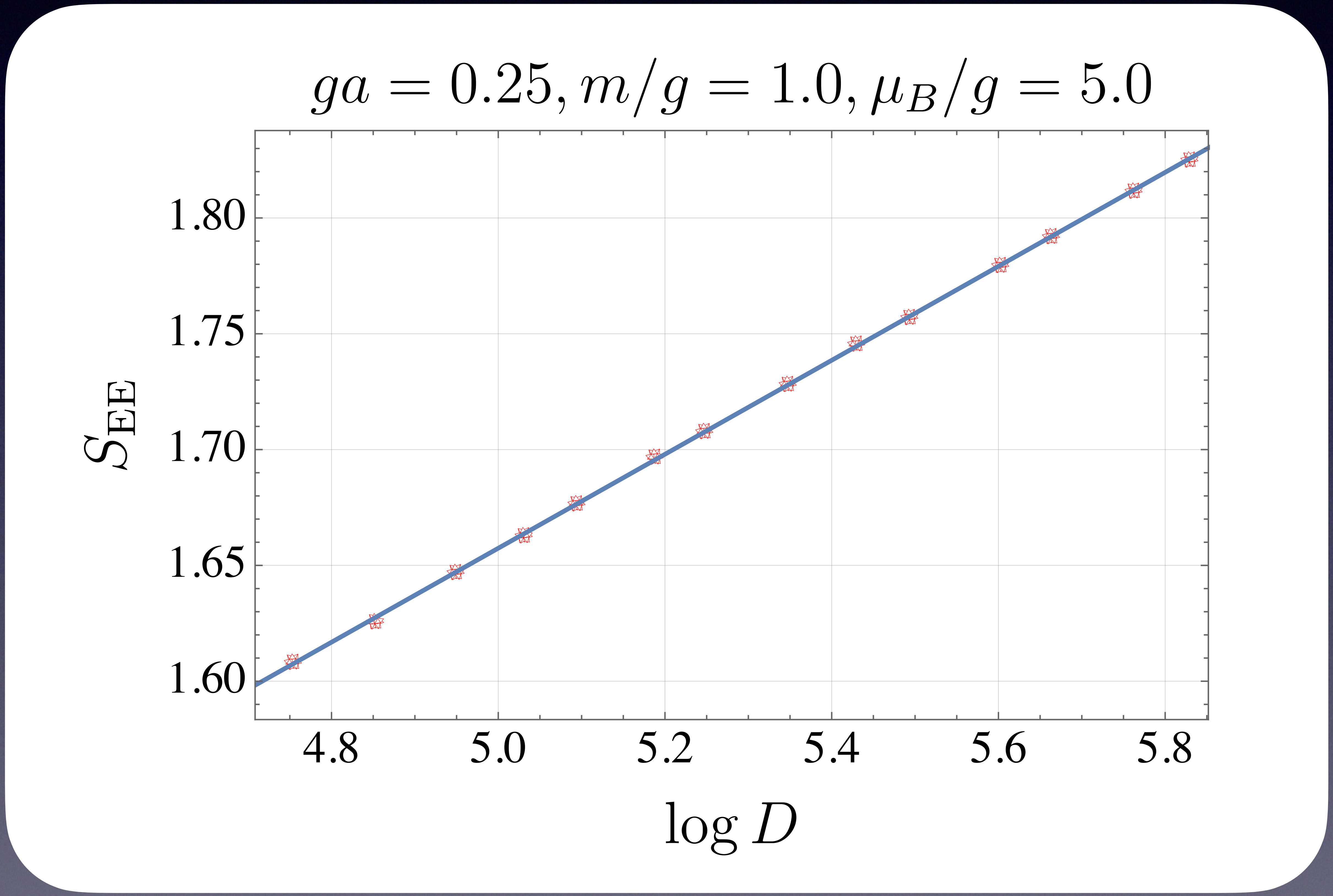
Translation symmetry is restored ⇒ Tomonaga-Luttinger liquid

$$\langle n_B(x)\rangle = \mathrm{const} \qquad \langle \delta n_B(x)\delta n_B(0)\rangle \sim \sum_\ell B_\ell \frac{\cos(2\pi\ell x)}{|x|^{2\ell^2 K}}$$





Entanglement entropy



$$S_{\mathrm{EE}} \sim \frac{c}{6} \log \xi/a$$

$c \simeq 1.0$

implies single low energy excitation

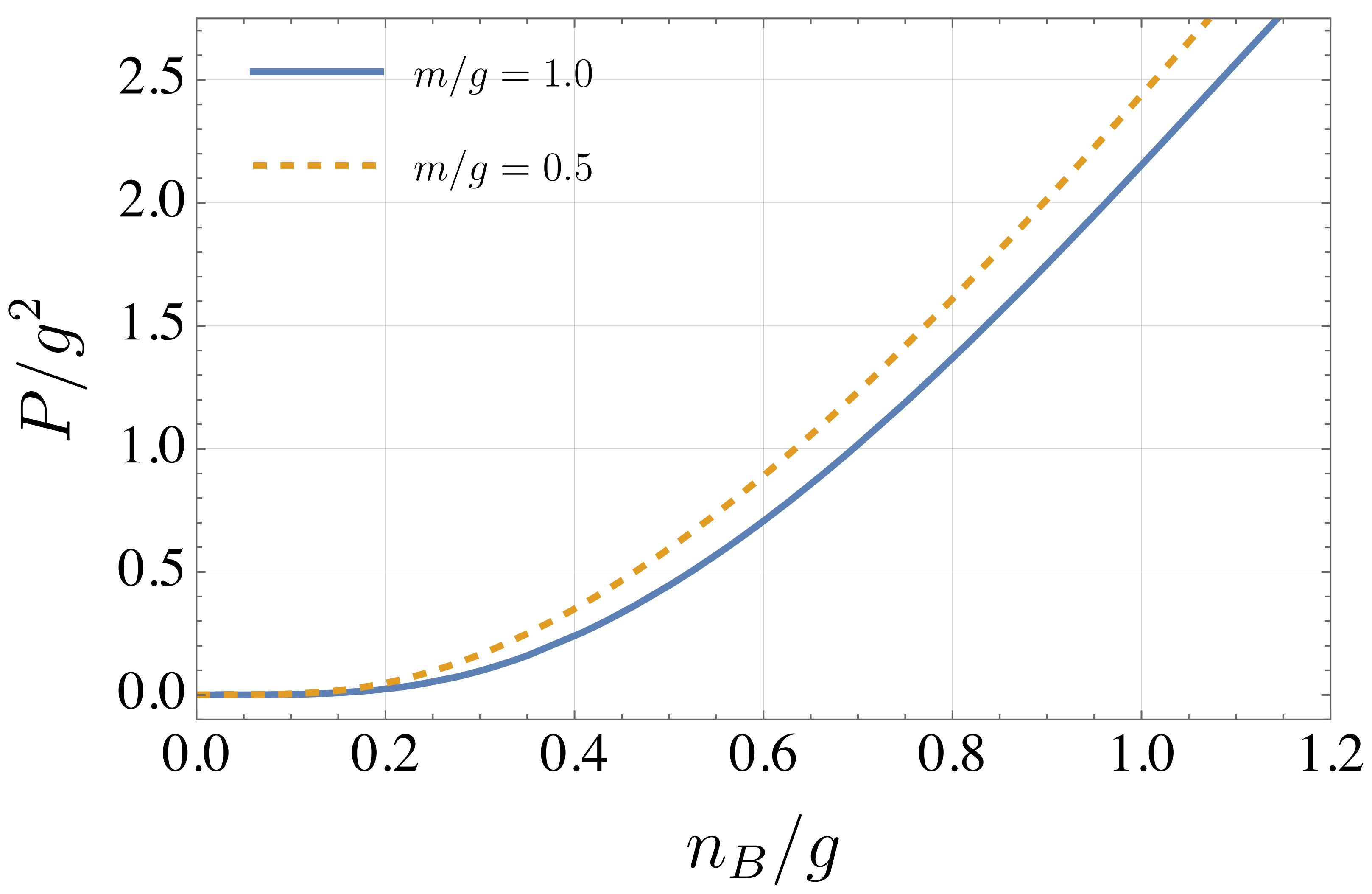
($c \simeq 2$ for free quarks)

Thermodynamics

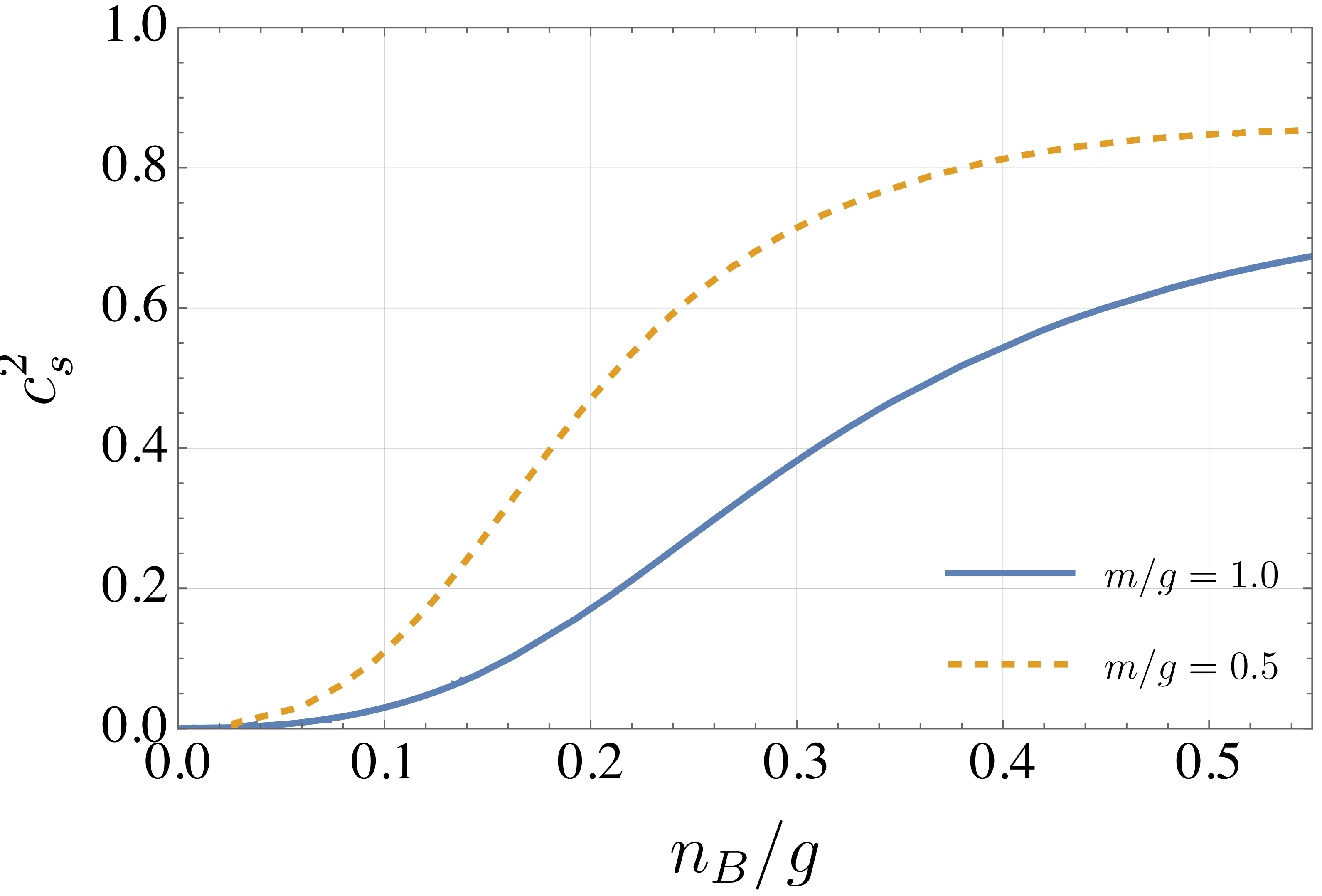
Using Variational Uniform Matrix Product State, Fujikura YH, 2605.17183

Zauner-Stauber, Vanderstraeten, Fishman, Verstraete, Haegeman ('18)

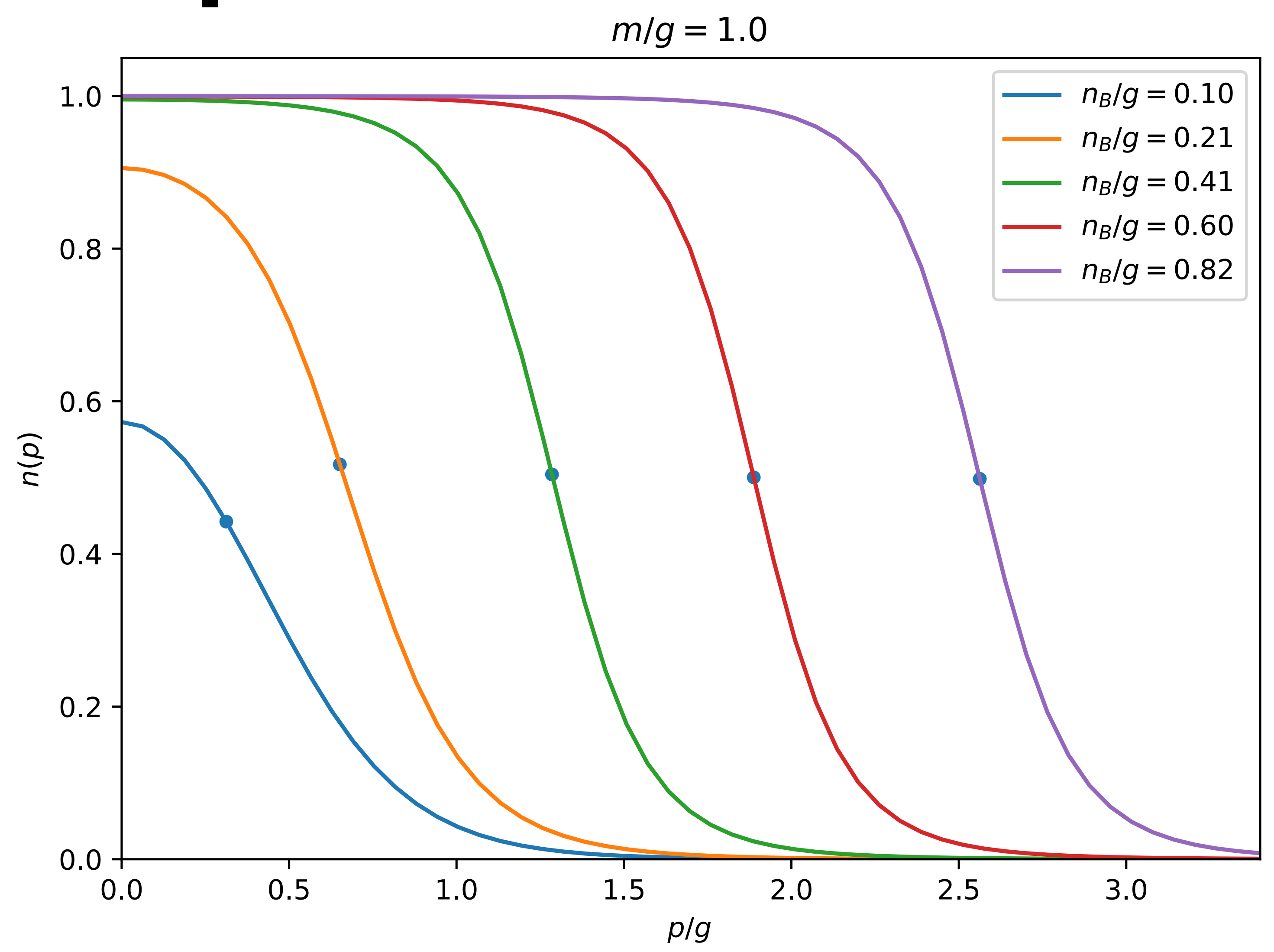
Equation of state



sound velocity



quark distribution



Baryonic to quark Luttinger liquid crossover

Special case, $m = 0$

- Baryons are gapless $m_B = 0$
- color sector and flavor sector are decoupled
- Thermodynamic properties can be exactly obtained

$$P = e = \frac{\mu_B^2}{4\pi} \qquad n_B = \frac{\mu_B}{2\pi} \qquad c_s^2 = \frac{dP}{de} = 1$$

Quark distribution function

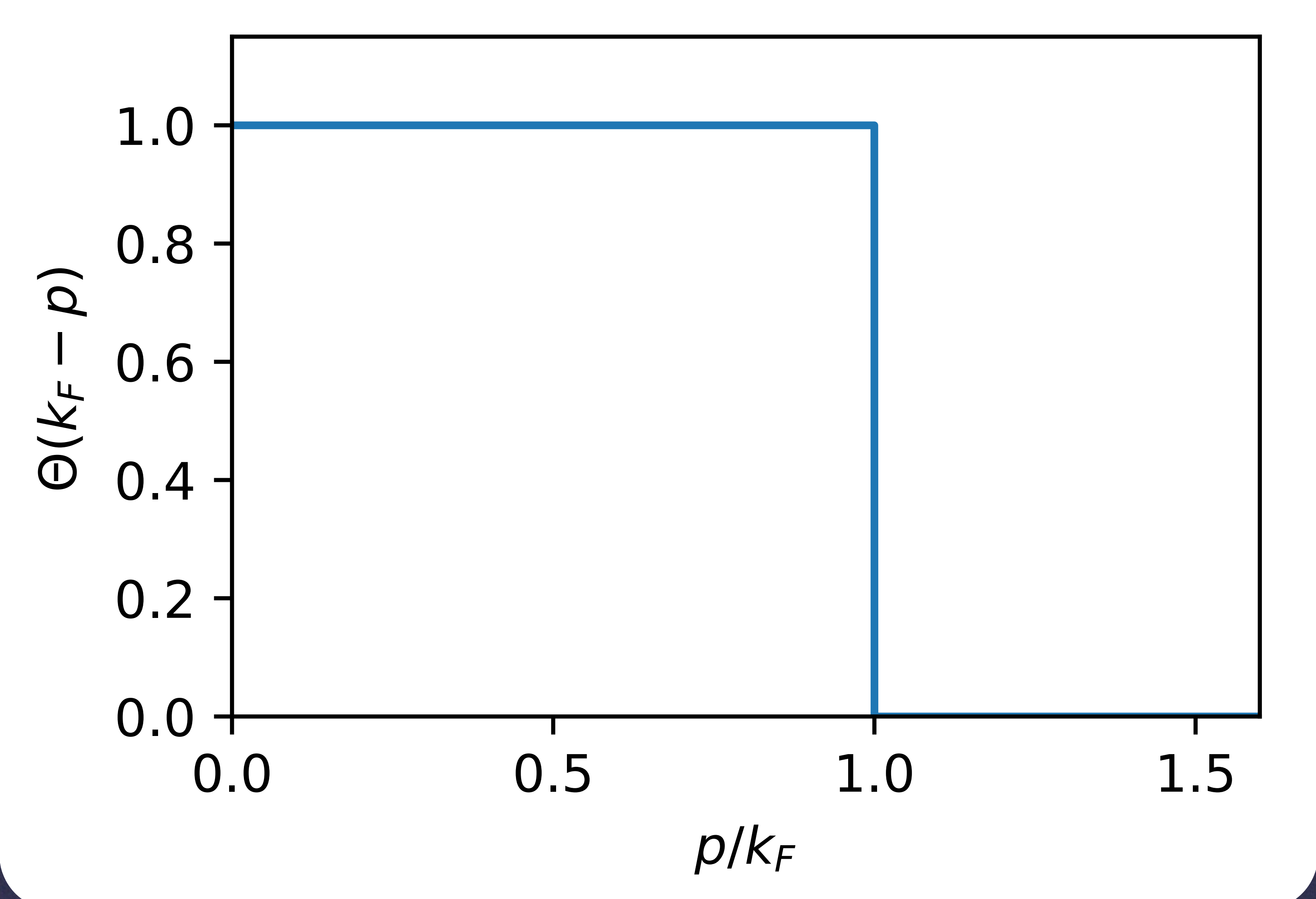
$$n_q(p) = \frac{1}{N_c}\int dx e^{-ipx} W(x) - 1 \quad W(x) = \langle \psi^\dagger(x) \mathscr{P} e^{i\int_0^x A dz} \psi(0) \rangle$$

Decoupling of color sector and flavor sector

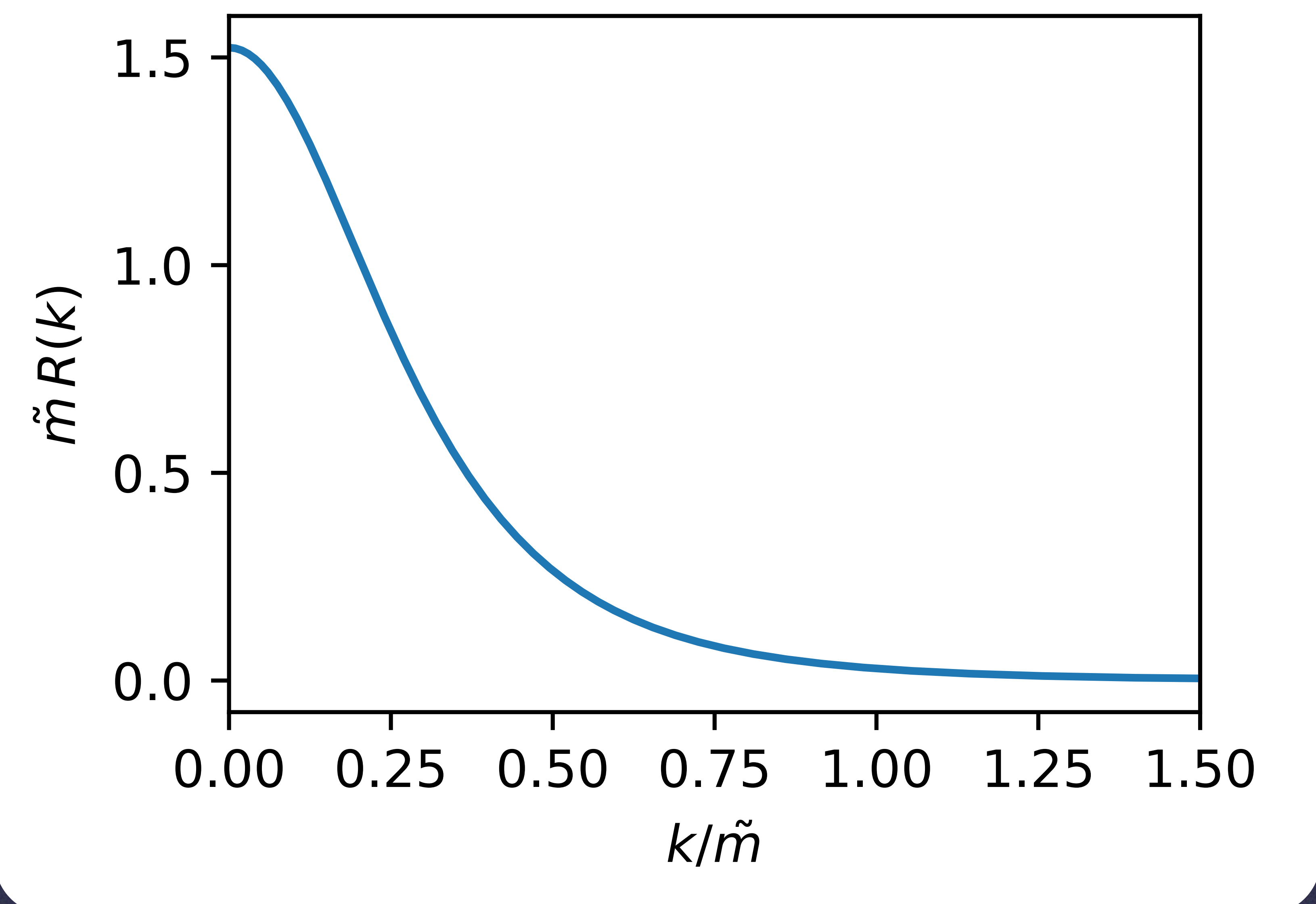
$$\Rightarrow \quad n_q(p) = \int dq \theta(k_F - q) R(p-q) \quad \text{with} \int dp R(p-q) = 1$$

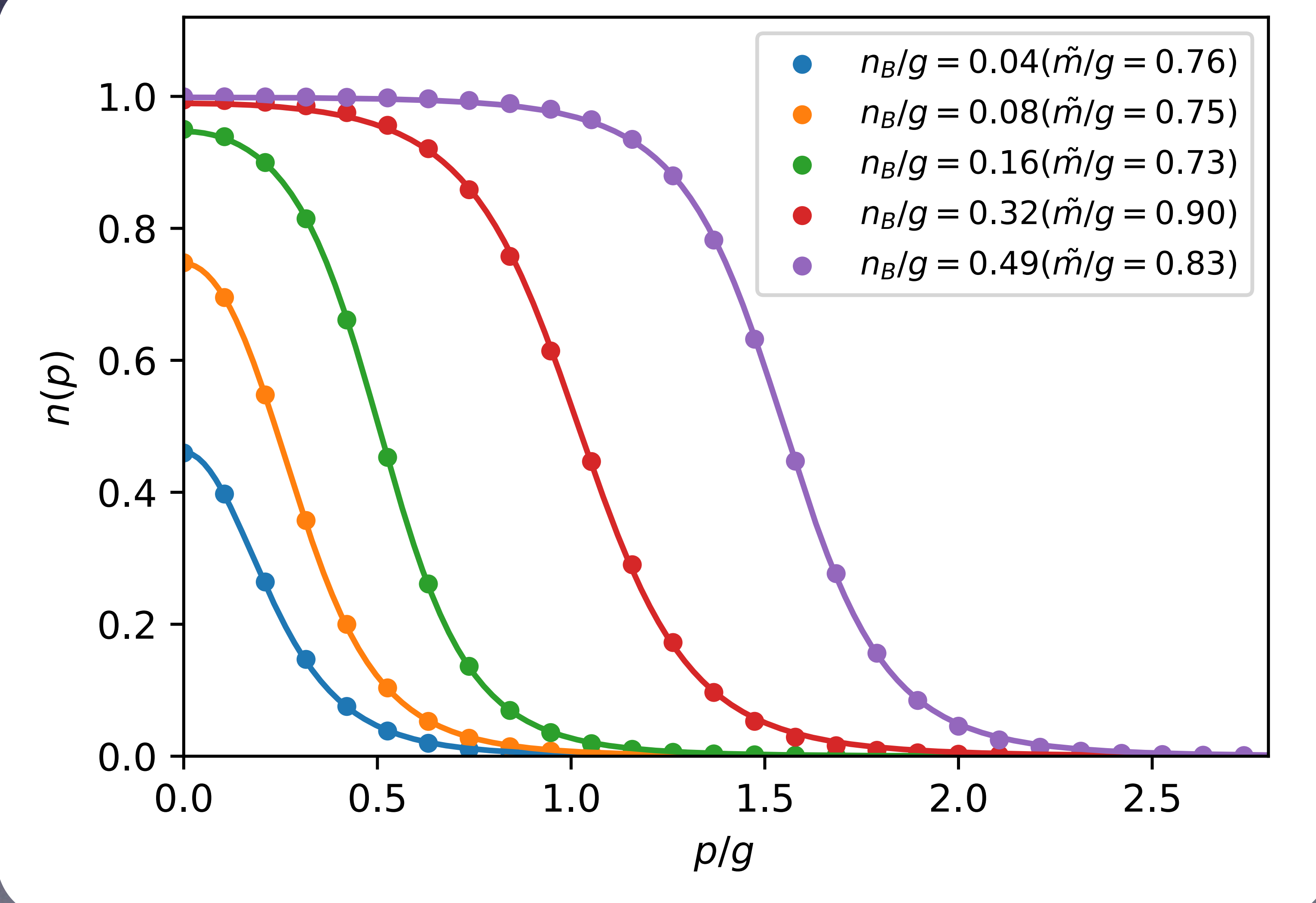
cf. IdylliQ model $f_Q(\boldsymbol{q}, n_B) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B; n_B) \varphi_Q^B(\boldsymbol{q}_B; \boldsymbol{P}_B)$

$$n(q) = \int dp\,\theta(k_F - p) R(q - p)$$



$\otimes$





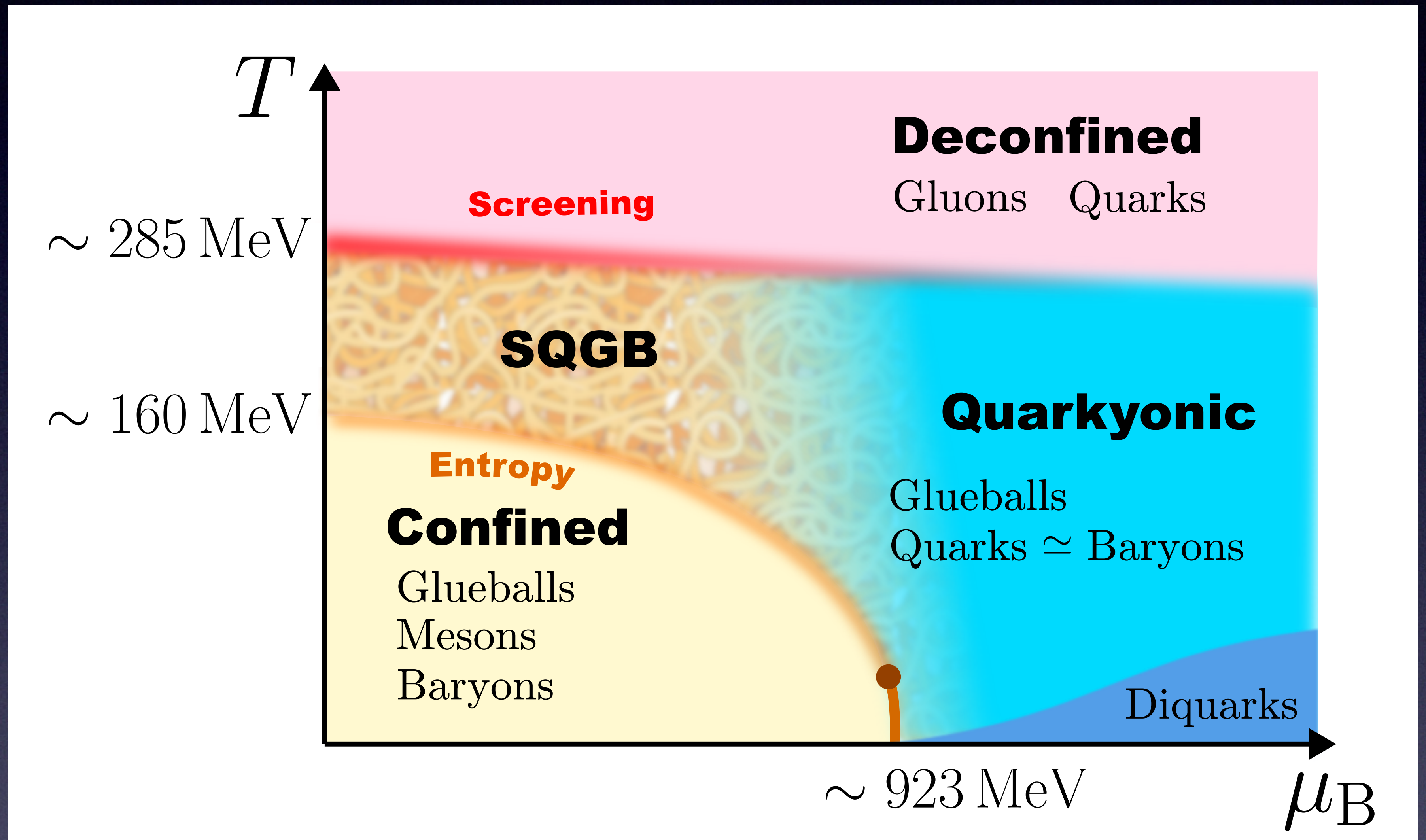
$R(q)$ is not known for SU(N)

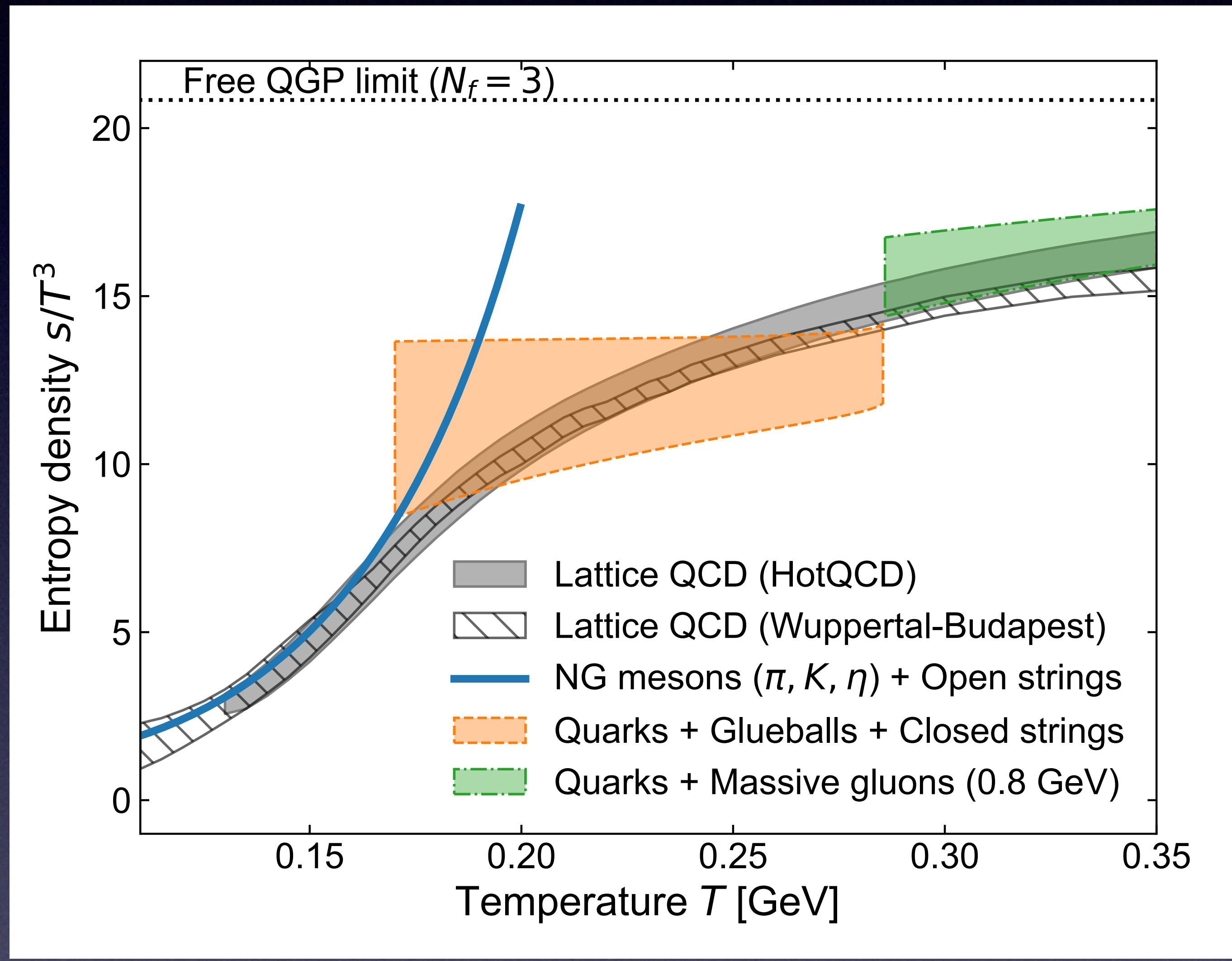
We use the functional form of of two flavor Schwinger model and fit the mass parameter

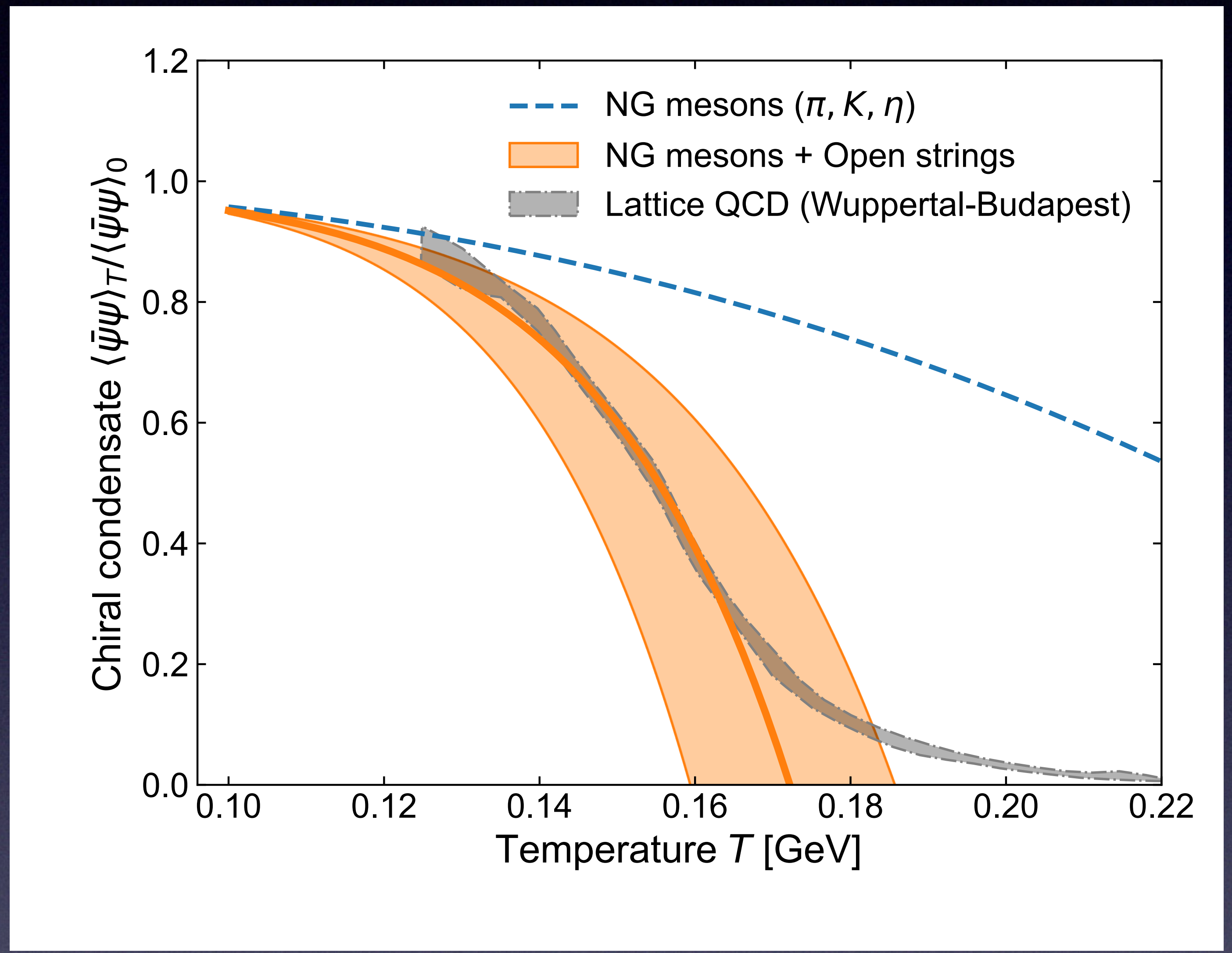
Summary

Finite-temperature QCD:

We have proposed Quark Spaghetti with Glue Balls







Finite-density QCD:

We discussed crossover between Hadronic and quark matter

We examined QCD in (1+1)d using tensor network technique.

⇒Hadronic to quark Luttinger liquid cross over