# Non-perturbative perspectives on QCD thermodynamics

## -- From Scalar Nonets to Topological Glueballs --


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# In Memoriam
# Mannque Rho (1936-2026)

*Dedicated to a distinguished physicist, a cherished collaborator for 25 years, and a deeply missed friend.*

*His profound insights into chiral symmetry and hadron structure continue to light our path forward.*

# Outline

## Part I: The lowest scalar nonet

1. A puzzle: non-exotic or exotic?
2. Production of scalar mesons and glueballs in HICs
3. A new classification [Yasui et al., arXiv:2603.13764]

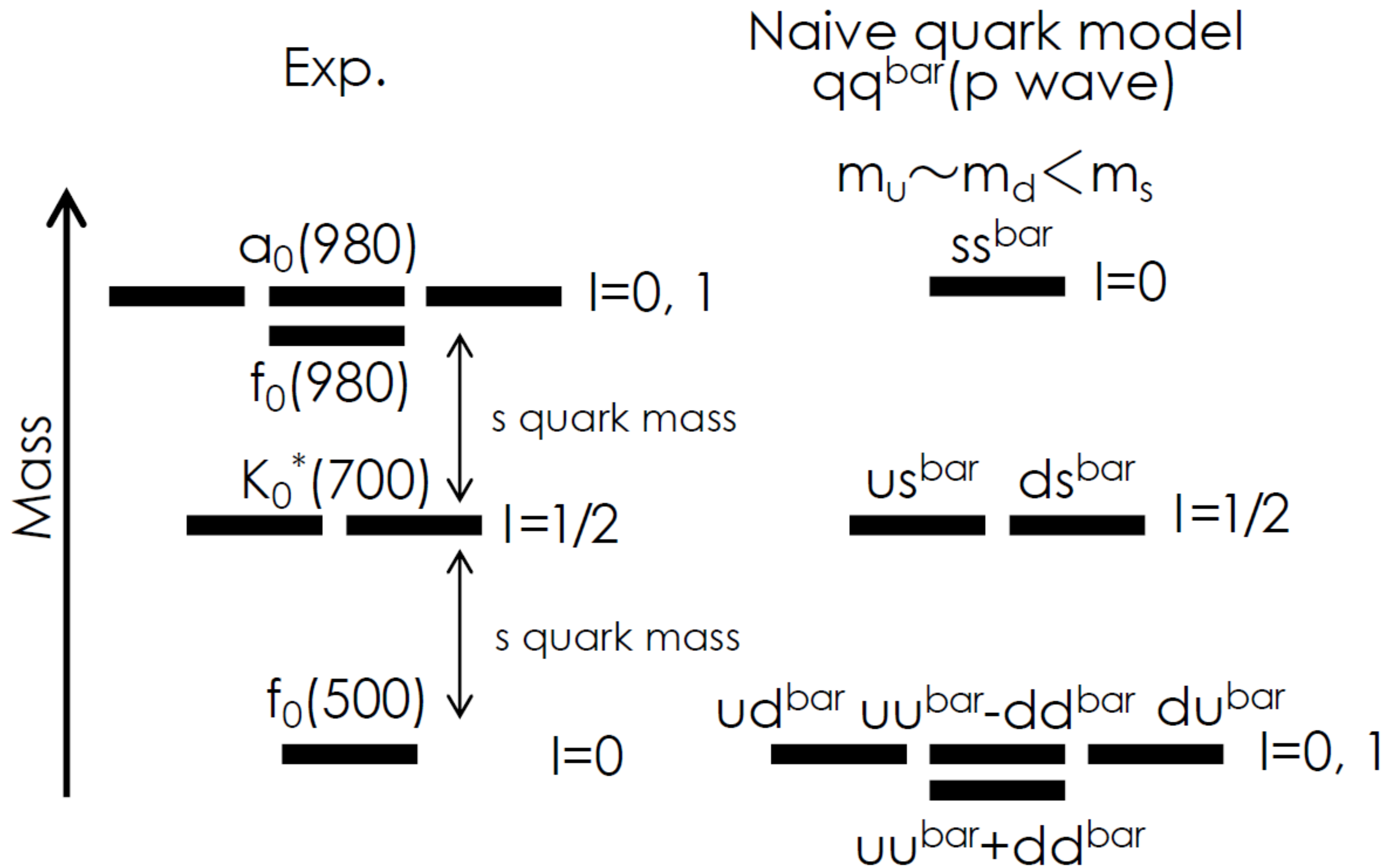
## Part II: Glueballs as topological solitons

1. The Skyrme-Faddeev model
2. Glueballonia [Amari et al., Phys. Lett. B (2025)]
3. Gravitational FFs [Hutauruk et al., to appear on arXiv]

# Part I:
# The lowest scalar nonet

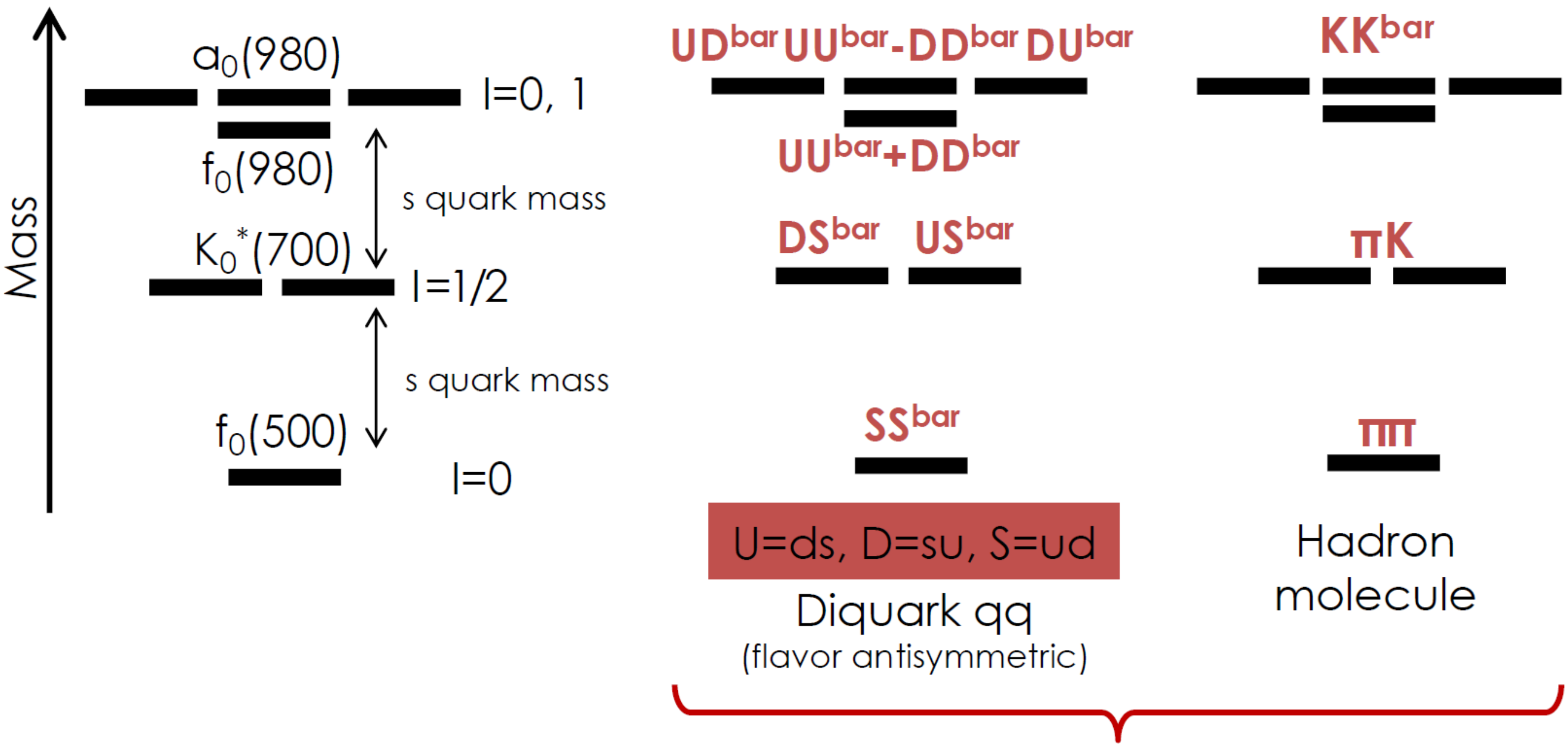
# Revisiting *light* exotic mesons

Long-standing puzzle: why is that mass hierarchy?



# Bob: “They are compact tetra-quark states.”

Diquarks [Jaffe (1977)] or Molecules [Close & Tornqvist (2002)]

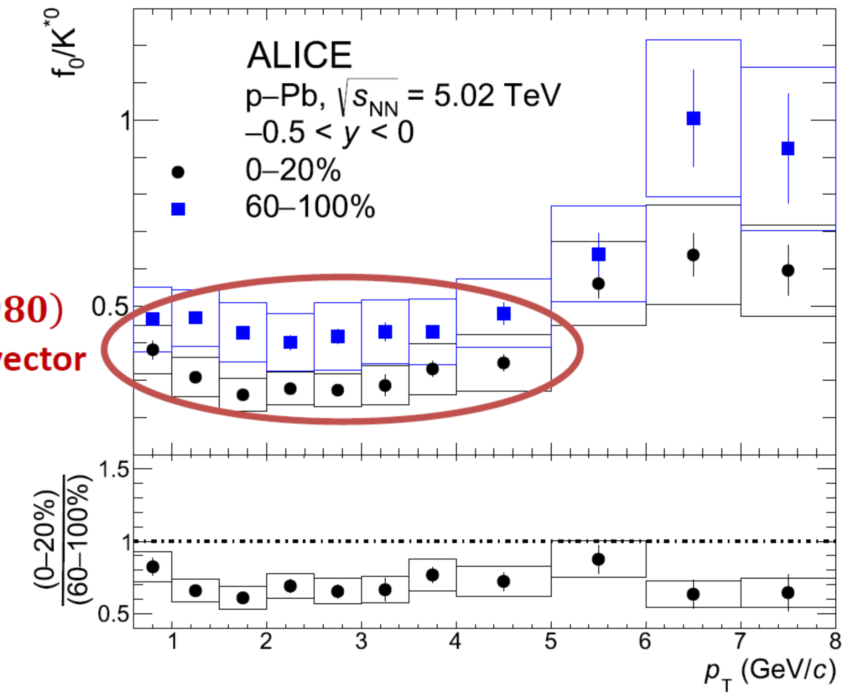



**“$f_0(980)$ should include $s\bar{s}$.”**

[ALICE Collaboration, PLB (2024)]

# ALICE: “No, $f_0(980)$ is not a tetra-quark state.”

Suppression of $f_0(980)$ against $K^{*0}(892)$ (vector meson) at low $p_T$.



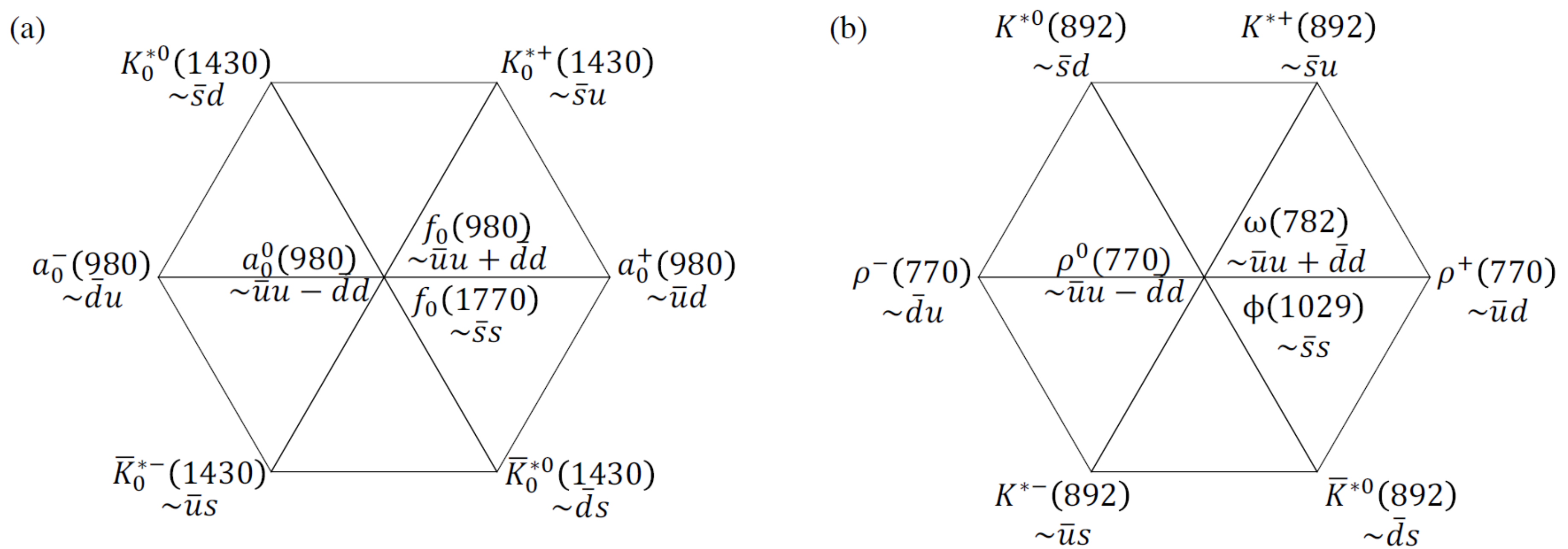

The abnormal suppression in terms of multiplicity and transverse momentum relative to other particles sheds light on the internal structure of $f_0(980)$ suggesting that it is a conventional meson with no hidden strange quarks and provides insight into the properties of the late hadronic phase in p–Pb collisions.

# A new classification

✓Abandon $f_0(500)$ and $K_0^*(700)$ .

✓$f_0(980)$ and $a_0(980)$ as the lowest states.

➔ Nonet: $f_0(980), a_0(980), K_0^*(1430), f_0(1770)$



# Summary of the new nonet

| | particle | $m$ [MeV] | $\Gamma$ [MeV] | $I(J^P)$ | confs.($L$) |
|---|---|---|---|---|---|
| nonet | $f_0(980)$ | $990 \pm 20$ | 10-100 | $0(0^+)$ | $\bar{n}n(P)$ |
| | $a_0(980)$ | $980 \pm 20$ | 50-100 | $1(0^+)$ | $\bar{n}n(P)$ |
| | $K_0^*(1430)$ | $1425 \pm 50$ | $270 \pm 80$ | $1/2(0^+)$ | $\bar{s}n(P)$ |
| | $f_0(1770)$ | $1733^{+8}_{-7}$ | $150^{+12}_{-10}$ | $0(0^+)$ | $\bar{s}s(P)$ |
| glueball | $f_0(1370)$ | 1200-1500 | 200-500 | $0(0^+)$ | $\bar{n}n(P)$ |
| | | | | | $\bar{n}\bar{n}nn(S)$ |
| | | | | | $gg(S)$ |
| | $f_0(1500)$ | $1522 \pm 25$ | $108 \pm 33$ | $0(0^+)$ | $\bar{n}n(P)$ |
| | | | | | $\bar{n}\bar{n}nn(S)$ |
| | | | | | $gg\ (S)$ |
| normal | $\rho(770)$ | 775 | 147 | $1(1^-)$ | $\bar{n}n(S)$ |
| | $\phi(1020)$ | 1019 | 4.3 | $0(1^-)$ | $\bar{s}s(S)$ |
| | $\Omega(1672)$ | 1672 | — | $0(3/2^+)$ | $sss(S)$ |

?

Production in HICs

?

reference particle

# Phenomenological approaches

## ❑Statistical model [Andronic, Braun-Munzinger, Redlich, Stachel (2003)]

$$N_h^{\mathrm{stat}} = V_H \frac{g_h}{2\pi^2} \int_0^\infty \frac{p^2 \mathrm{d}p}{e^{(E_h - \mu_B B - \mu_S S)/T_H} \pm 1}$$

## ❑Coalescence model [Greco, C. M. Ko, P. Levai (2003)]

$$N_h^{\mathrm{coal}} = g_h \int \left[ \prod_{i=1}^{n} \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{\mathrm{d}^3 \mathbf{p}_i}{E_i} f(x_i, p_i) \right] \times f^W(x_1, \ldots, x_n : p_1, \ldots, p_n).$$

① distribution function $f(x_i, p_i)$ for particle i

$$\int p_i \cdot d\sigma_i \frac{d^3\mathbf{p}_i}{(2\pi)^3 E_i} f(x_i, p_i) = N_i$$

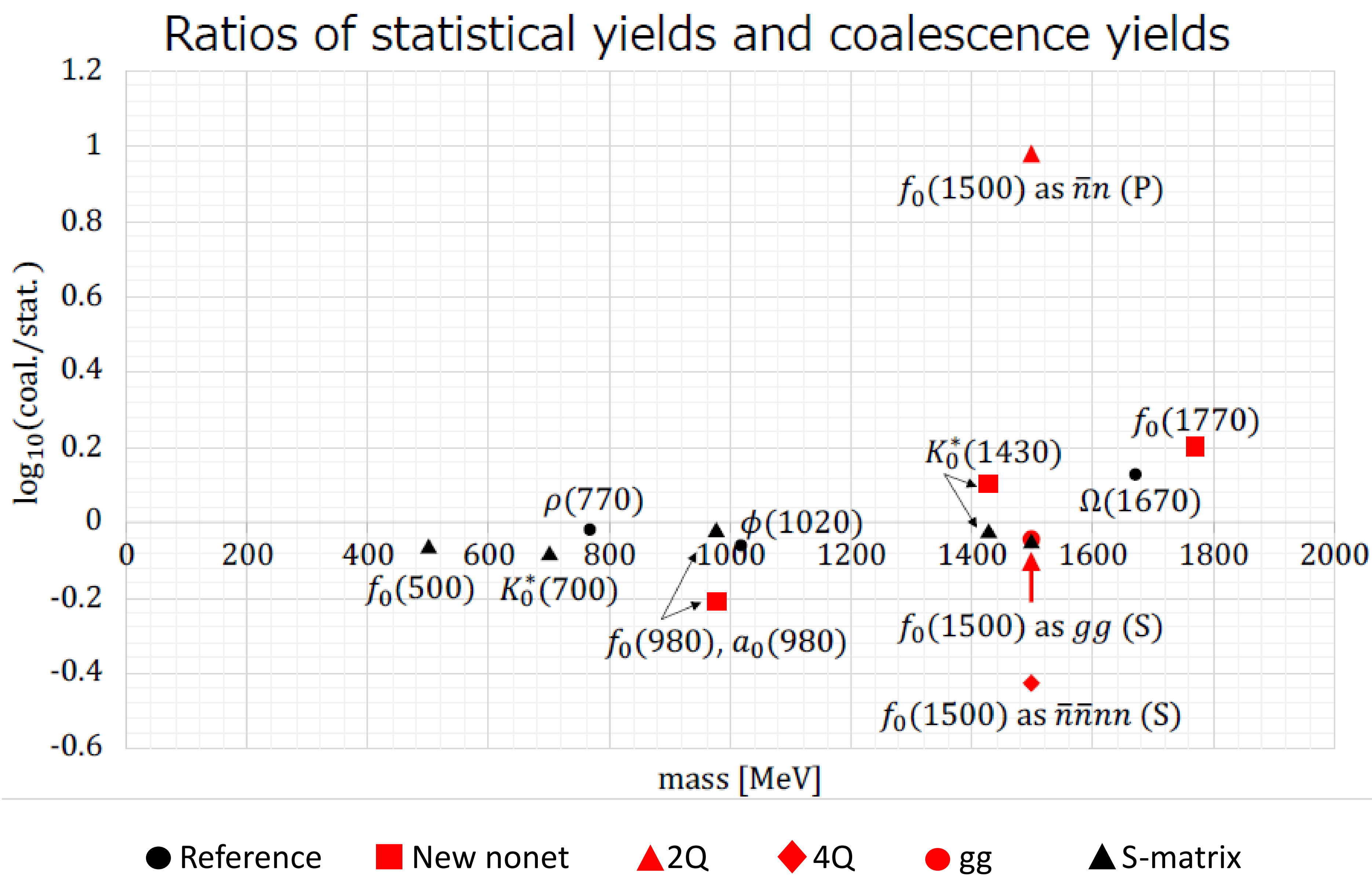
② Wigner function ($n$ # of constituent particles)

$$\begin{aligned} f^W&(x_1, \ldots, x_n : p_1, \ldots, p_n) \\ &= \int \prod_{i=1}^{n} dy_i e^{i p_i y_i} \psi^*(x_1 + y_1/2, \ldots, x_n + y_n/2) \\ &\quad \times \psi(x_1 - y_1/2, \ldots, x_n - y_n/2), \end{aligned}$$

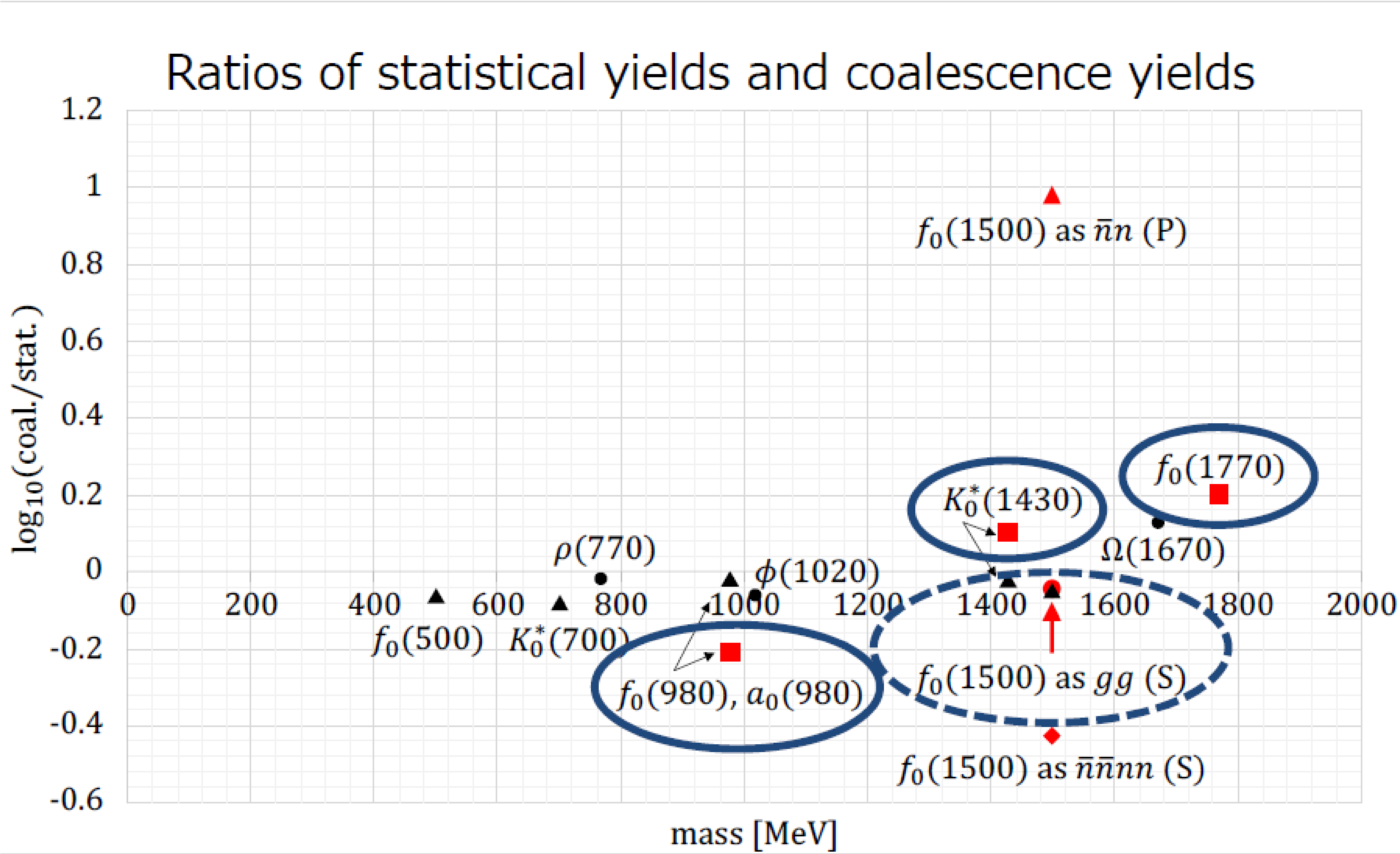
## ❑S-matrix approach

$$N_h^{\mathrm{S}} = \frac{g_h V_H}{2\pi^2} \int \frac{\mathrm{d}\sqrt{s}}{2\pi} B(s) \int_0^\infty \frac{p^2 \, \mathrm{d}p}{e^{(\varepsilon(p,s) - \mu_B B - \mu_S S)/T_H} \pm 1}$$

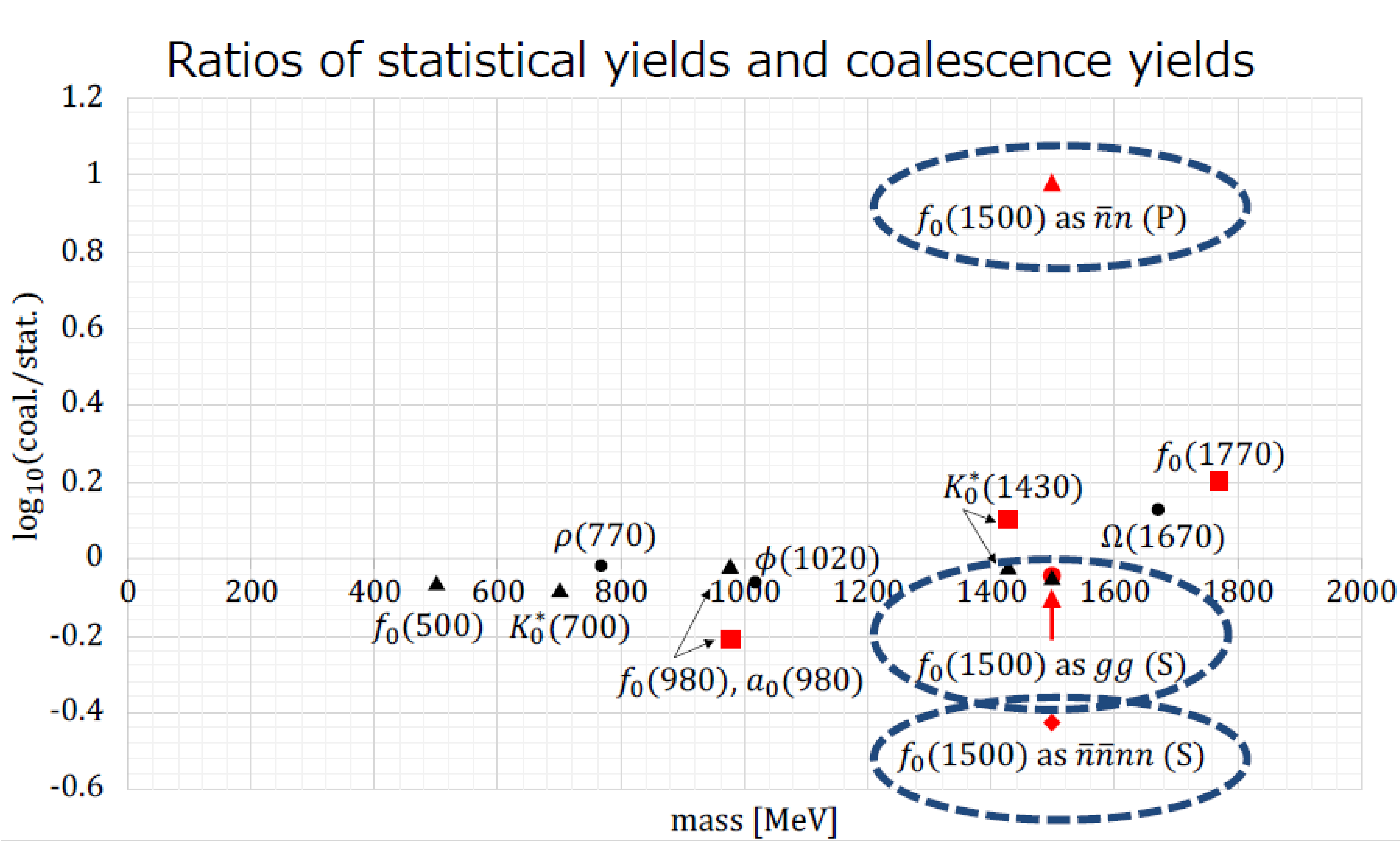
[Dashen, Ma, Bernstein (‘69); Venugopalan, Prakash (‘92); Lo (‘20-)]



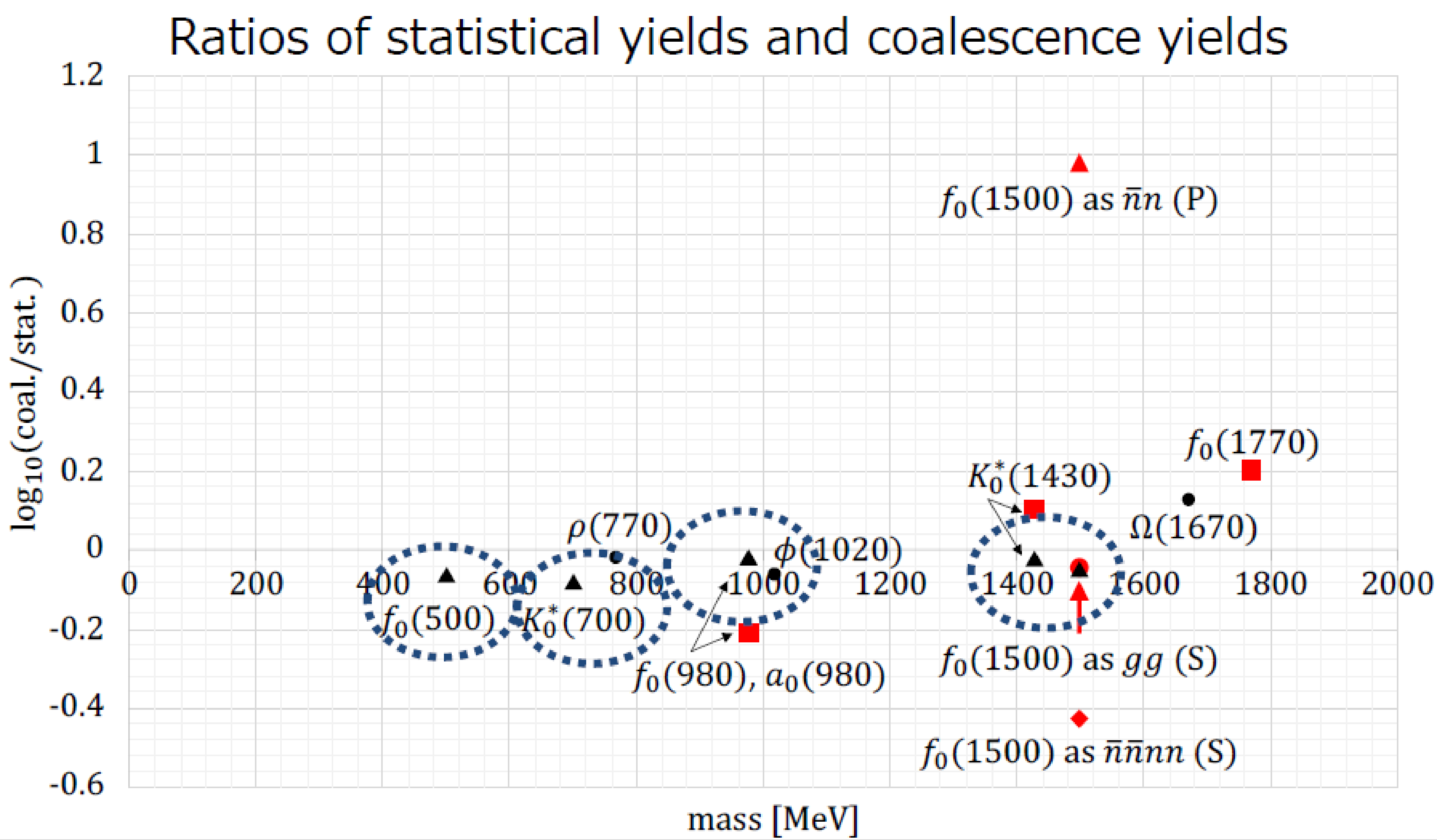
Ratios of statistical yields and coalescence yields
log10(coal./stat.)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
200
400
600
800
1000
1200
1400
1600
1800
2000
mass [MeV]
$f_0(1500)$ as $\bar{n}n$ (P)
$f_0(1770)$
$K_0^*(1430)$
$\Omega(1670)$
$\rho(770)$
$\phi(1020)$
$f_0(500)$
$K_0^*(700)$
$f_0(980)$, $a_0(980)$
$f_0(1500)$ as $gg$ (S)
$f_0(1500)$ as $\bar{n}\bar{n}nn$ (S)
Reference
New nonet
2Q
4Q
gg
S-matrix




**Point 1: Ratios are almost close to *one* like normal hadrons.**




nt 2: Glueball ($gg$) is favored for $f_0(1500)$ for the ratio consistenc




Dashen, Ma, Bernstein (1969), Weinhold, Friman, Norenberg (1998), Lo (2017, 2020)

## Point 3: S-matrix formulation gives consistent values.

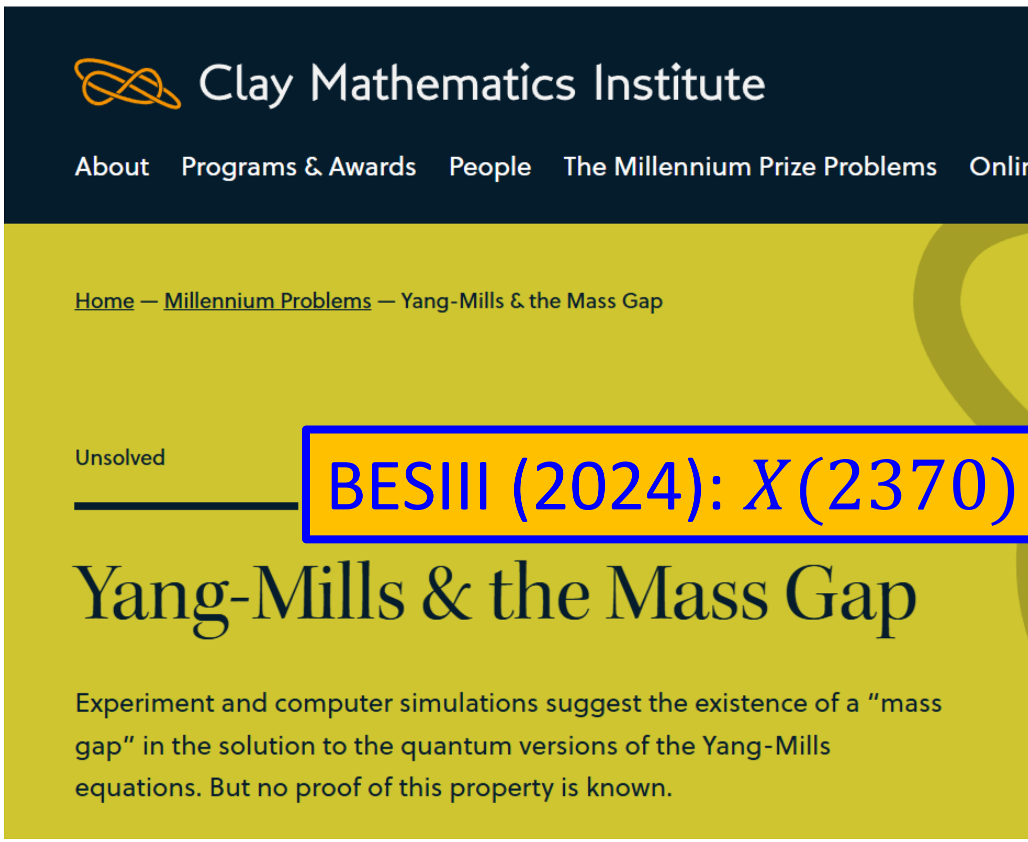
# Summary of Part I

- New nonet scheme
  - $f_0(980), a_0(980), K_0^*(1430), f_0(1770)$
- Their production in HICs
  - Statistical model
  - Coalescence model
  - S-matrix model
- $f_0(1500)$ as a scalar glueball

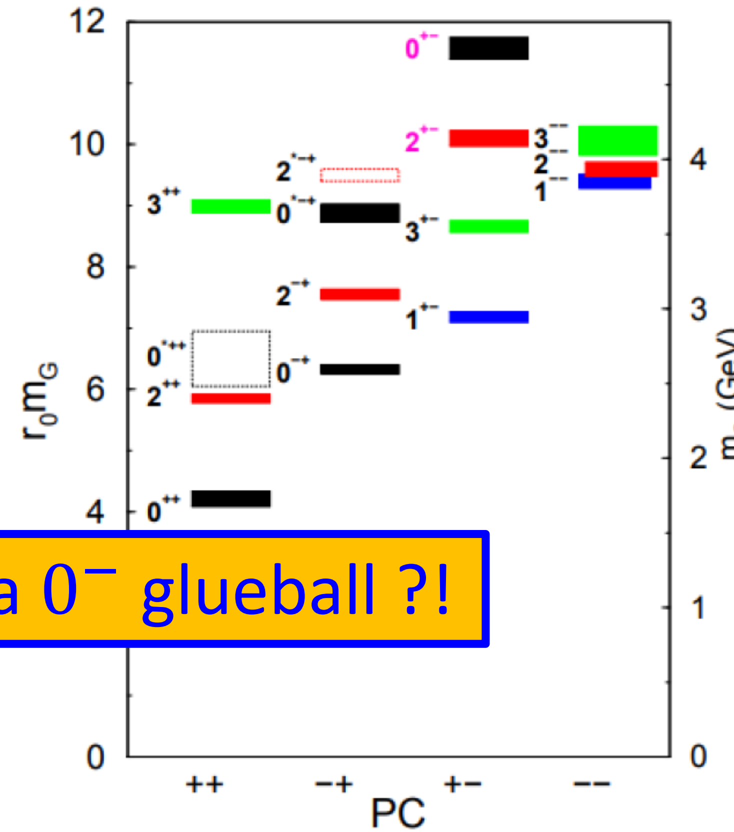
# Part II:
# Glueballs as topological solitons

# Why glueballs?

The key objects to understand the mass gap in pure Yang-Mills theory




**BESIII (2024): $X(2370)$ as a $0^{-}$ glueball ?!**




Morningstar & Peardon PRD **60**, 034509 (1999)

# The goal

Topological approach to non-perturbative physics

- ❑IR phenomenology of SU(2) Yang-Mills theory
- ❑Glueballs as topological solitons

  cf. Faddeev and Niemi ('99), the lowest glueball as a Hopfion

➔ Higher-lying states and their structure, comparison to available experimental & lattice data

Based on the work in collaboration with Amari, Nitta, Sasaki, Shigaki, Yano and Yasui, Phys. Lett. B 869, 139805 (2025)

# The Skyrme-Faddeev model (SFM)

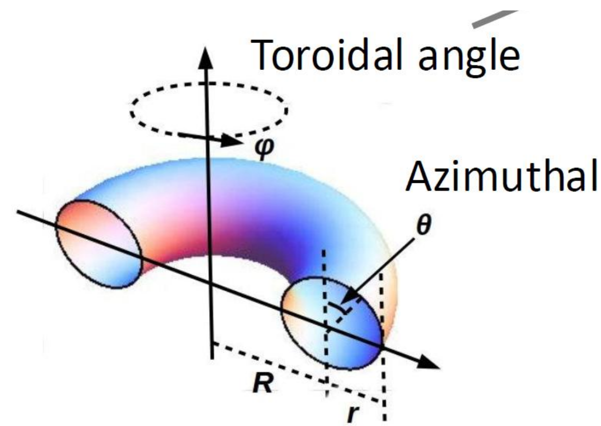
A low-energy effective model of SU(2) YM

[Faddeev & Niemi (99)]

$$\mathcal{L} = \frac{\kappa^2}{4} \mathrm{Tr}\left(\partial_\mu \mathfrak{n} \partial^\mu \mathfrak{n}\right) + \frac{1}{32e^2} \mathrm{Tr}\left[\partial_\mu \mathfrak{n}, \partial_\nu \mathfrak{n}\right]^2$$

$$\mathfrak{n} = \boldsymbol{\tau} \cdot \boldsymbol{n} \qquad \boldsymbol{n} = (n_1, n_2, n_3) \text{ with } |\boldsymbol{n}| = 1$$

- Topological solitons: Hopfions $\leftarrow \pi_3(S^2) = \mathbb{Z}$
- Hopf charges: $Q = lm$
  [$l, m \in \mathbb{Z}$ winding numbers]



# Energy eigenvalues

$$E = \frac{\kappa}{e} M_{\mathrm{cl}}^{*} + \frac{\kappa e^{3}}{2} \left\{ \frac{J(J+1)}{V_{11}^{*}} + \left( \frac{1}{U_{33}^{*}} - \frac{\ell^{2}}{V_{11}^{*}} \right) K_{3}^{2} \right\}$$

Ansatz for a torus-shape Hopfion
$n(l, m; \theta, \varphi)$

| $Q_{\mathrm{top}} = \ell m$ | $(\ell, m)$ | $M_{\mathrm{cl}}^{*}$ | $V_{11}^{*}$ | $U_{33}^{*}$ |
|---|---|---|---|---|
| 1 | (1,1) | 274.79 | 372.17 | 227.78 |
| 2 | (2,1) | 447.20 | 968.33 | 267.72 |
| 2 | (1,2) | 508.16 | 1203.11 | 398.09 |

# Phenomenological parameters

Energy specified by

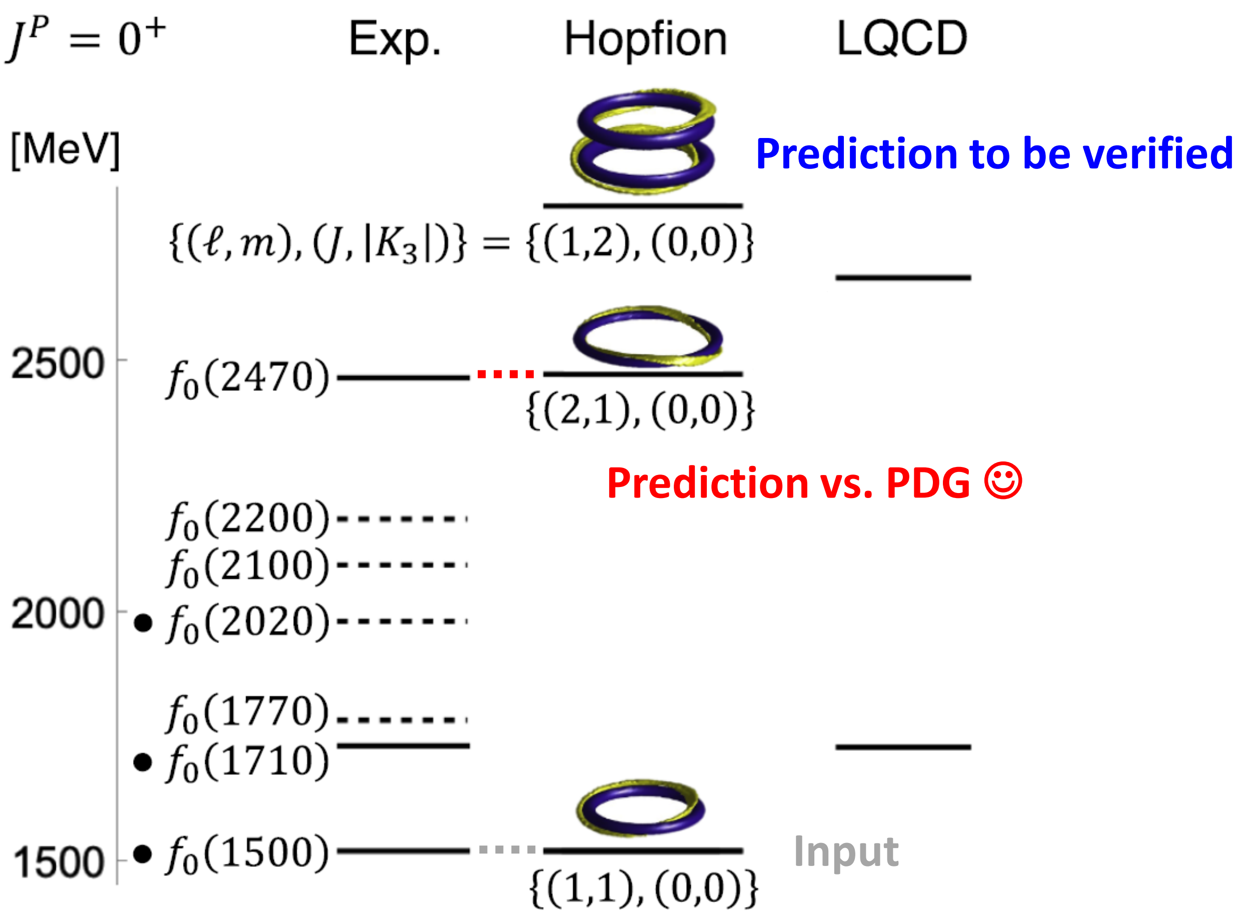
- Winding numbers $(l, m)$
- Quantum numbers $(J, K_3)$
- Model parameters $(\kappa, e)$

Input (Q=1)

- J=0: $\{(l, m), (J, K_3)\} = \{(1,1), (0,0)\}$ as $f_0(1500)$
- J=2: $\{(1,1), (2,0)\}$ as e.g. $f_2(2300)$

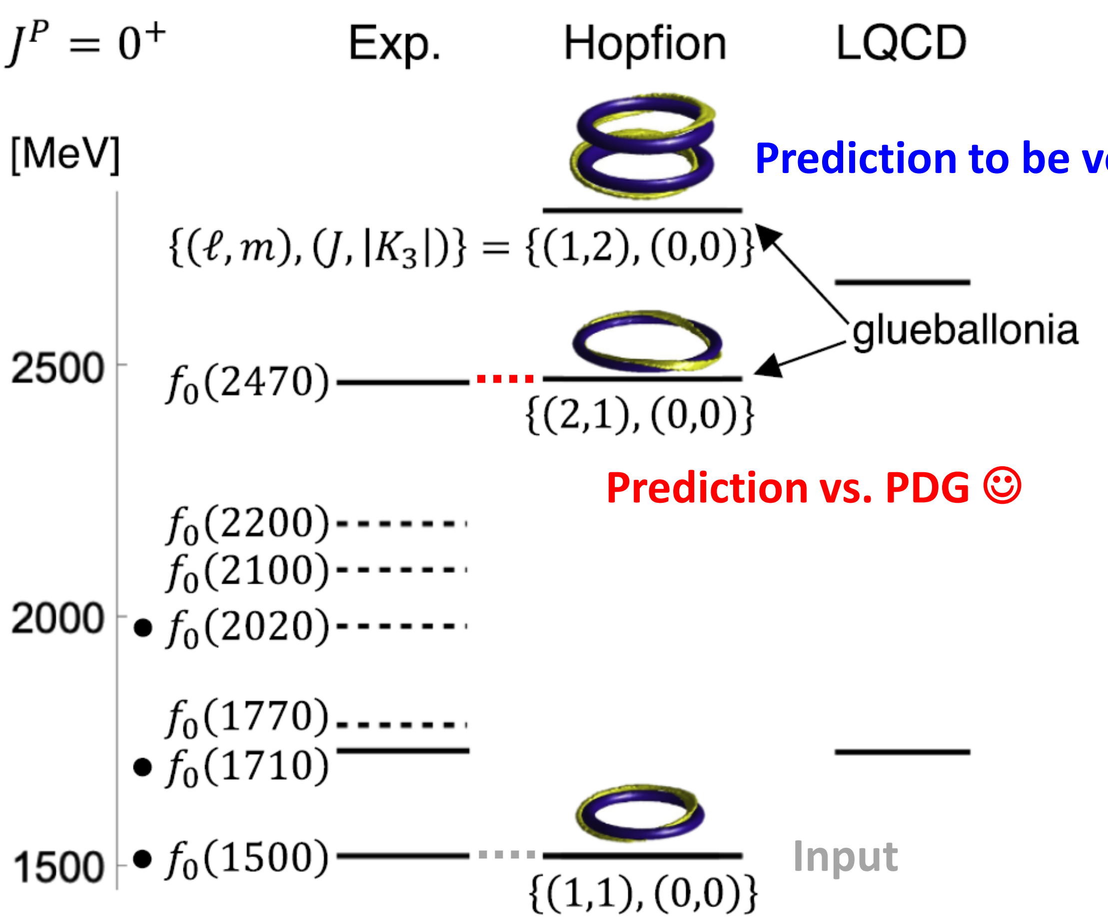
$$\kappa/e = 5.5 \text{ MeV}, \quad \kappa e^3 = 88 \text{ GeV}$$

# Mass spectrum of scalar glueballs




● Listed as established states in PDG
Solid: narrow states ($\Gamma < 150$ MeV)
Broken: broad states ($\Gamma > 150$ MeV)

# Mass spectrum of scalar glueballs




● Listed as established states in PDG
Solid: narrow states ($\Gamma < 150$ MeV)
Broken: broad states ($\Gamma > 150$ MeV)

Q=1 Hopfion
➔ The lowest glueball

Q=2 Hopfion w/ $(l, m) = (2,1)$
➔ Bound state of 2 Q=1 Hopfions
➔ *Tightly* bound glueballonium
B.E. = 577 MeV

Q=2 Hopfion w/ $(l, m) = (1,2)$
➔ Bound state of 2 Q=1 Hopfions
➔ *Loosely* bound glueballonium
B.E. = 230 MeV

# Remarks

- Non-topological approach
  - A model for dilatons [Giacosa, Pilloni & Trotti, ('22)]
  - The lowest glueballonium of the mass 3.4 GeV
  - cf. our Hopfion approach predicts 2.4 GeV

On our agenda:

- Extension to color SU(3)
- Coupling to light quarks
- …
- GFFs toward EIC

# Gravitational form factors

- Encode the gluon momentum fraction and distribution of quantities e.g. the energy inside hadrons. Defined from the matrix elements of $T^{\mu\nu}$.
- In YM for the scalar glueball,

$$\langle G[0^{++}](p')|T^{\mu\nu}|G[0^{++}](p)\rangle$$

$$= 2P^\mu P^\nu A(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{2} D(t)$$

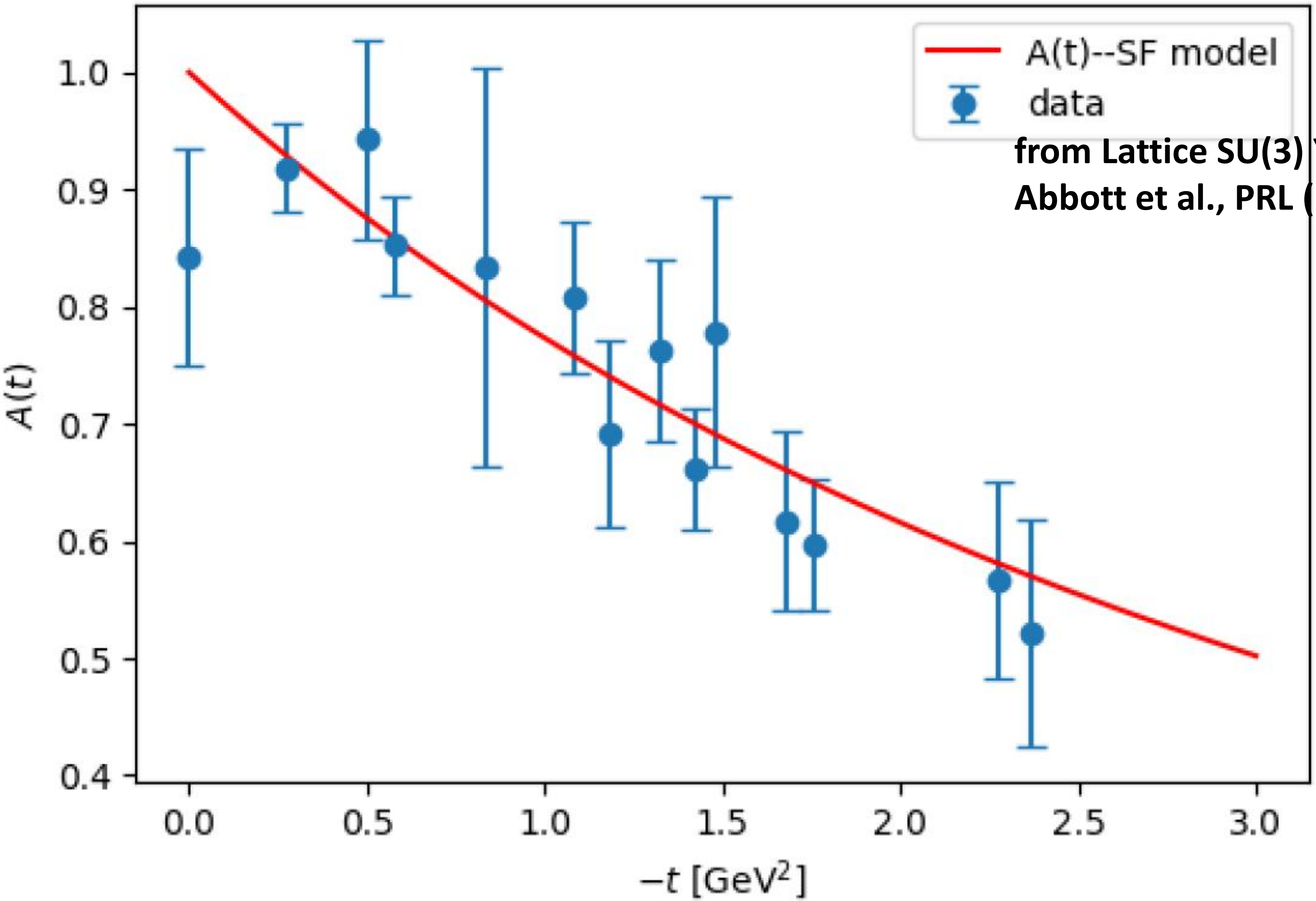
$$P = (p+p')/2,\ \Delta = p' - p,\quad t = \Delta^2$$

- Energy density $\mathcal{E}(n) = T^{00}$
- Pressure & shear forces in $T^{ij}$

A- & D-terms

[Hutauruk, Amari, Sasaki, to appear]

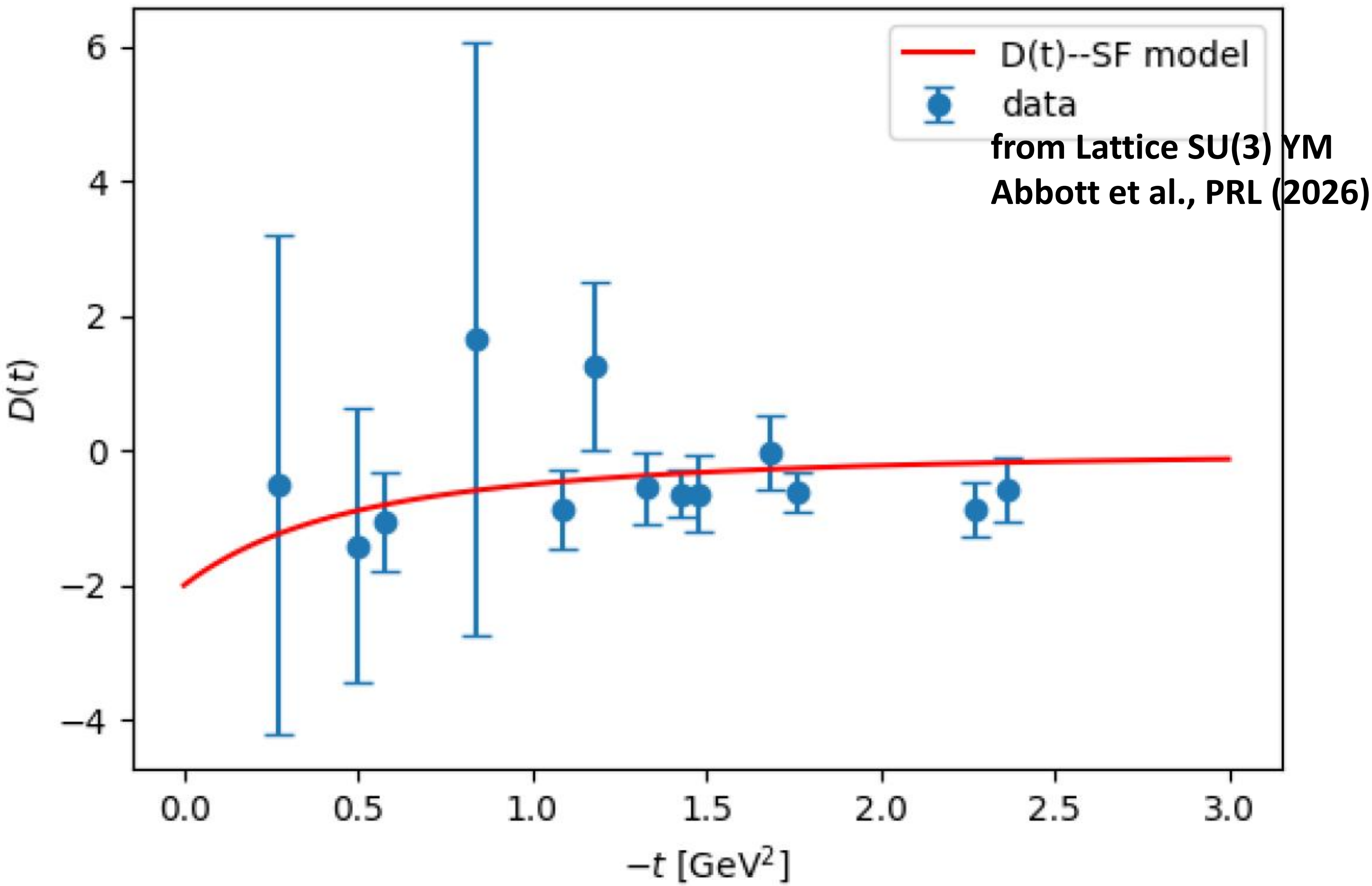
# Glueball GFFs in the SF model




**from Lattice SU(3) YM**
**Abbott et al., PRL (2026)**

[Hutauruk, Amari, Sasaki, to appear]

# Glueball GFFs in the SF model



[Hutauruk, Amari, Sasaki, to appear]

# Mass radius

How gluons are concentrated?

→Mass radius of the scalar glueball ≈ spatial distribution of mass or energy inside the glueball.

$$\langle r^2 \rangle_{\rm mass} = 6 \frac{dA(t)}{dt}\bigg|_{t=0}$$

❑SF model: 0.253 fm

- Input: f0(1500) ↔ k/e = 5.5 MeV

❑SU(3) Lattice YM: 0.263 fm [Abbott et al. (2026)]

❑Earlier studies: SU(2) Latice YM [de Forcrand-Liu (92)], instanton model [Schaefer-Shuryak (95)]

# Summary of Part II

- ❑ Conjecture: $f_0(2470)$ as a molecule of 2 glueballs (glueballonium) with B.E. = 577 MeV
  - Their internal structures and binding energies
  - Characteristic multiplets of glueballonia (scalar & tensor)
- ❑ GFFs of scalar glueball (preliminary)
  - A- & D-terms and the mass radius align with Lattice YM.
  - Toward the nucleon GFFs, in progress
- ❖ BESIII $X(2370)$ as another benchmark test
  - So far, only rotational modes [Amari et al., in progress]
  - Vibration modes of a Hopfion → excited states

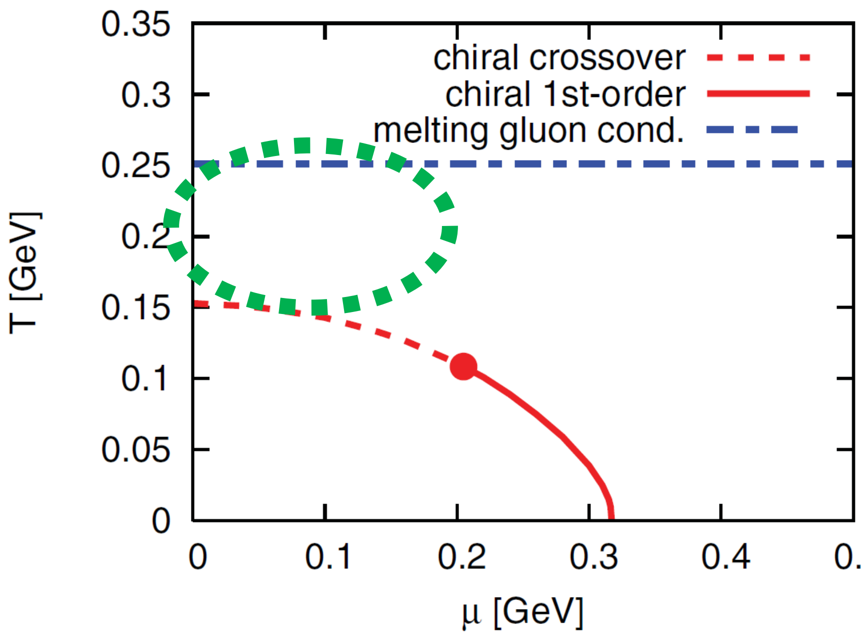
# Summary

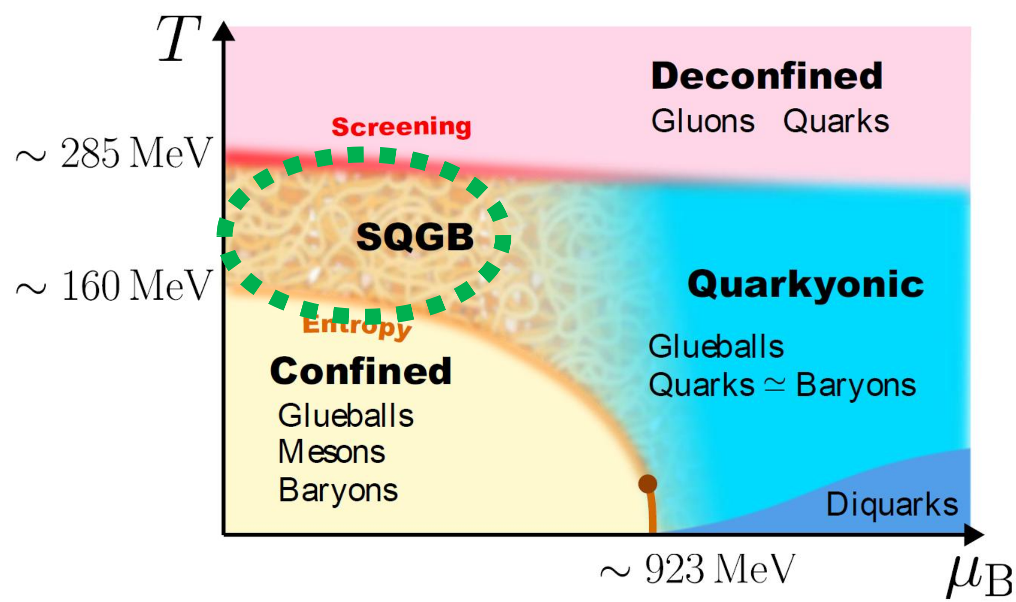
# Implications

QCD phase diagram: Hopfions at $T \gtrsim T_c$?

❑ Chiral-scale interplay [Left: Sasaki-Mishustin (2012)]

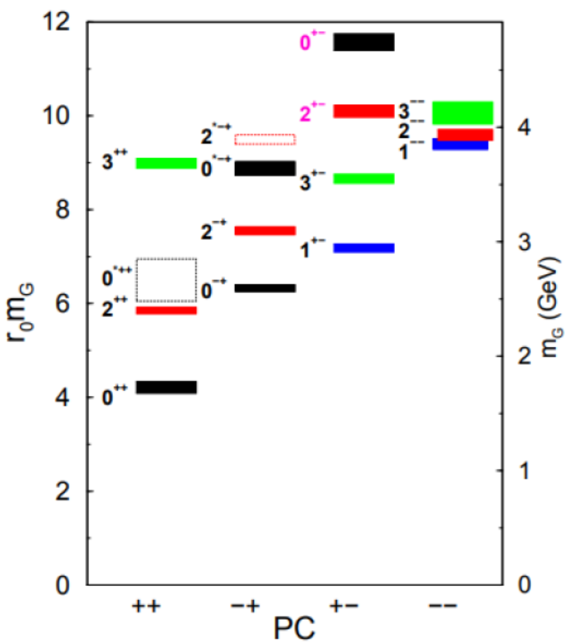
❑ Quark Spaghetti & Glueballs [Right: McLerran et al. (2025)]



# Fascinating Light Scalars: Bridging from Quark to Gluon Dynamics

# Backup




Morningstar & Peardon
PRD **60**, 034509 (1999)

**$0^{++}$ light unflavored mesons from PDG**

| | | Mass[MeV] | Width[MeV] |
|---|---|---|---|
| ● | $f_0(980)$ | 990 | $10-100$ |
| ● | $f_0(1370)$ | $1200-1500$ | $200-500$ |
| ● | $f_0(1500)$ | 1522 | 108 |
| ● | $f_0(1710)$ | 1733 | 150 |
| | $f_0(1770)$ | 1784 | 161 |
| ● | $f_0(2020)$ | 1982 | 440 |
| | $f_0(2100)$ | 2095 | 287 |
| | $f_0(2200)$ | 2187 | 210 |
| | $f_0(2330)$ | | |
| | $f_0(2470)$ | 2470 | 75 |

● indicates established particles

**$2^{++}$ light unflavored mesons from PDG**

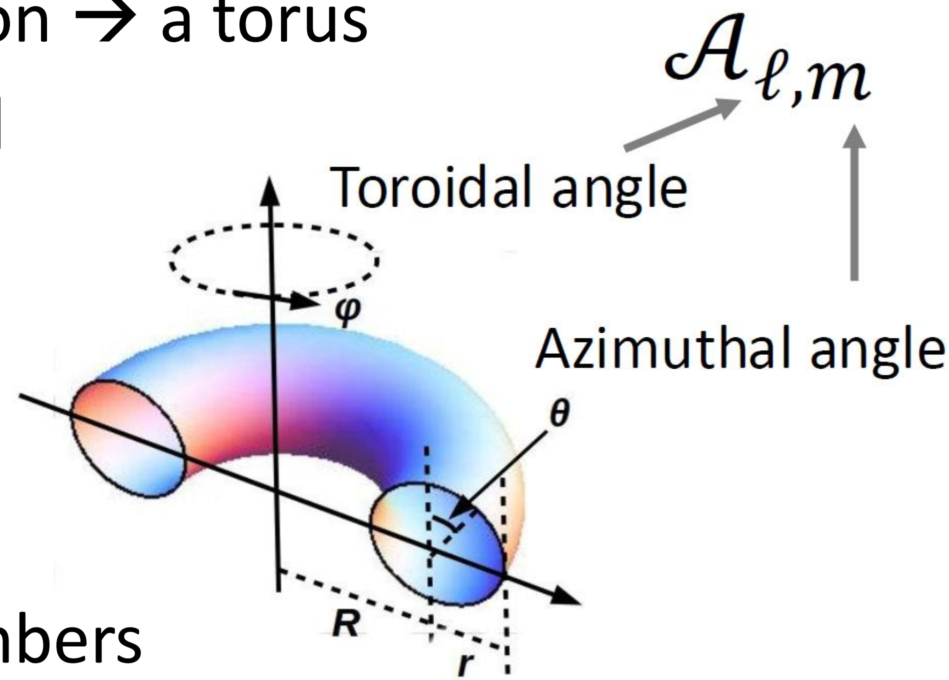
| | | Mass[MeV] | Width[MeV] |
|---|---|---|---|
| ● | $f_2(1270)$ | 1275 | 186 |
| | $f_2(1430)$ | 1430 | |
| ● | $f_2'(1525)$ | 1517 | 72 |
| ● | $f_2(1565)$ | 1571 | 132 |
| | $f_2(1640)$ | 1639 | 100 |
| | $f_2(1810)$ | 1815 | 197 |
| | $f_2(1910)$ | | |
| ● | $f_2(1950)$ | 1936 | 464 |
| ● | $f_2(2010)$ | 2010 | 200 |
| | $f_2(2150)$ | | |
| | $f_J(2220)$ | 2231 | 23 |
| ● | $f_2(2300)$ | 2297 | 150 |
| ● | $f_2(2340)$ | 2346 | 331 |

# The Skyrme-Faddeev model (SFM)

An axial-symmetric Hopfion → a torus

[Gladikowski & Hellmund (97)]

[Battye & Sutcliffe (98)]




- Hopf charges: $Q = lm$

  $l, m \in \mathbb{Z}$ winding numbers
- Static energy configurations via rational map

# Energy eigenvalues

- Classical mass [$M_{\text{cl}} = \frac{\kappa}{e} M_{\text{cl}}^*$]
- Inertia tensors [$X_{jk} = \frac{1}{\kappa e^3} X_{jk}^*$]

$$M_{\text{cl}}^* = \int d^3x \left[\frac{1}{2}(\partial_i \boldsymbol{n})^2 + \frac{1}{4}\left(\partial_i \boldsymbol{n} \times \partial_j \boldsymbol{n}\right)^2\right]$$

$$U_{jk}^* = \int d^3x \left[\delta_{jk} - n_j n_k + \partial_l n_j \partial_l n_k\right]$$

$$V_{jk}^* = \int d^3x \left[\left(i\mathrm{L}_j \boldsymbol{n}\right) \cdot \left(i\mathrm{L}_k \boldsymbol{n}\right) + \left(i\mathrm{L}_j \boldsymbol{n} \times \boldsymbol{n}\right) \cdot \left(i\mathrm{L}_k \boldsymbol{n} \times \boldsymbol{n}\right)\right]$$

with $\mathrm{L}_j = -i\varepsilon_{jkl} x^k \partial_l$