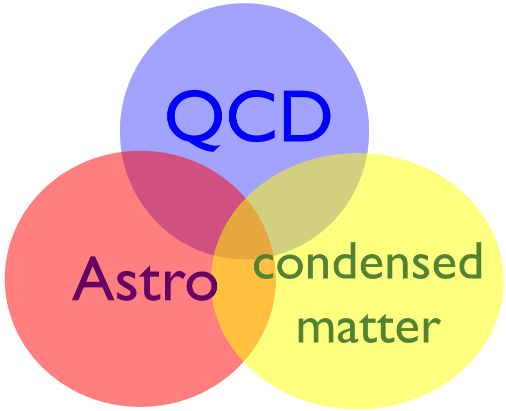
Cold, dense QCD for Neutron Stars



Toru Kojo

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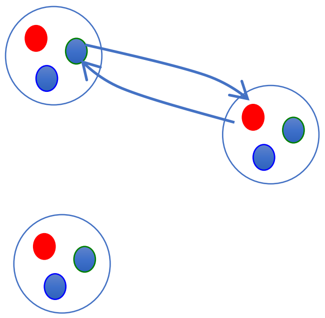
1, Introduction & steady progress in NS observations

2, Microphysics: nuclear & quark matter

3, Quarkyonic matter: rapid stiffening & hyperon puzzle

Three-window model (n_0 = 0.16 fm^{-3}) [Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

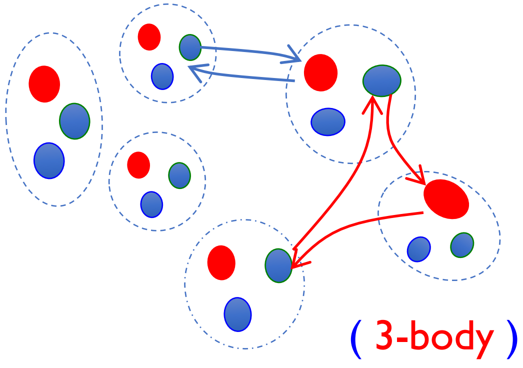


ab-initio nuclear cal.

laboratory experiments

steady progress

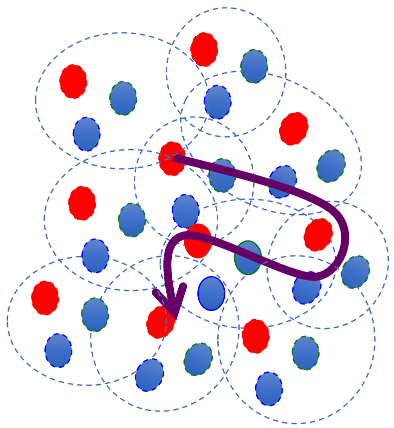
- many-quark exchange
- structural change,...
- hyperons, Δ, ...



most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Gorda+,...]

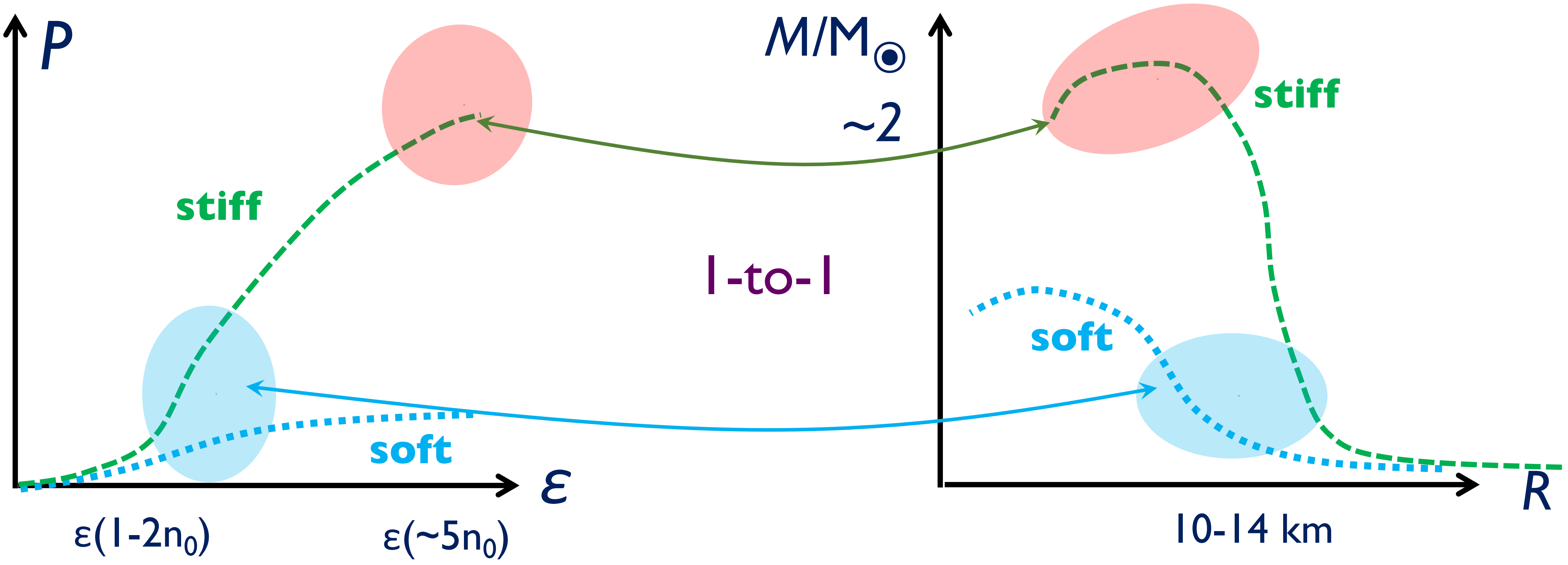
~ 1.4 $M_\odot$ ~ 2 $M_\odot$ n_B

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

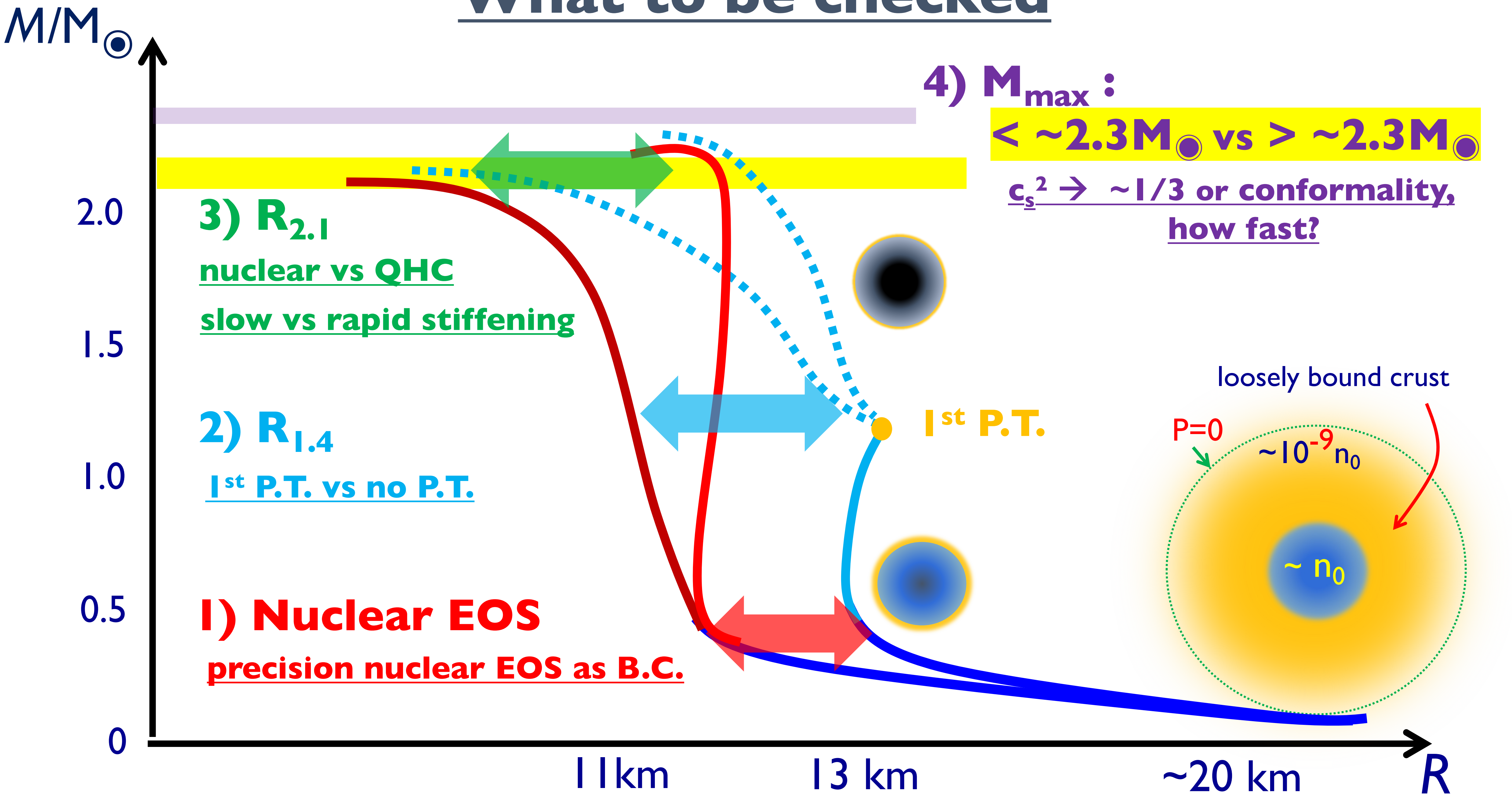
EoS stiffness & M-R

Ref) Lattimer & Prakash (2001)

measure: P vs ε



What to be checked



Nuclear EOS: complementary constraints

Why so difficult to estimate?

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

small (!) crucial for pressure

$$\varepsilon(n_B) = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^{\alpha} \quad \Rightarrow \quad P = \frac{2}{3} a \frac{n_B^{5/3}}{m_N} + b(\alpha - 1) n_B^{\alpha}$$

Symmetric nuclear matter (SNM) at ~n_0

experimentally accessible near n_0 (nuclear exp., PREX-II & CREX, HIC,....)

theoretically **delicate**; sensitive to small 3N forces

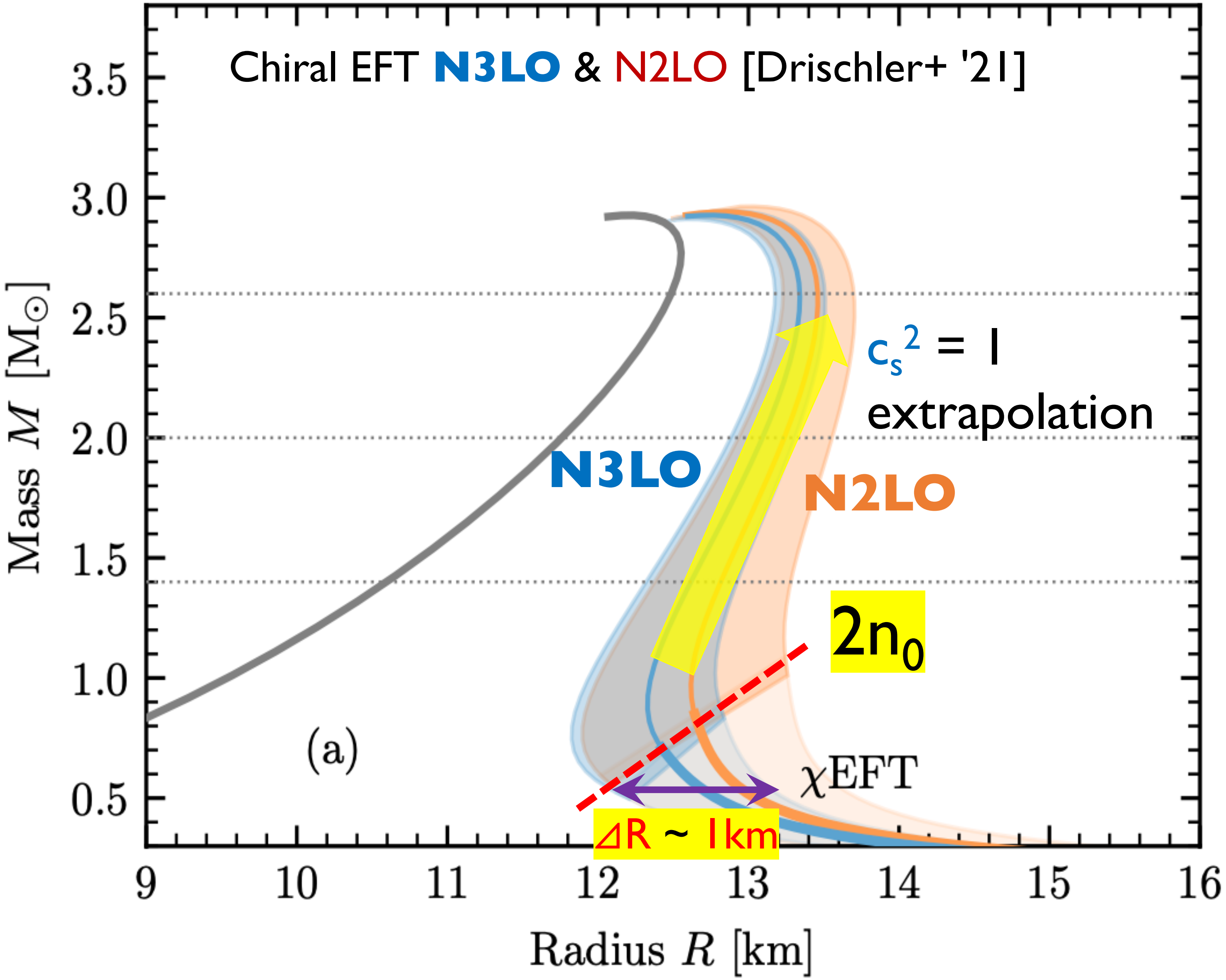
Pure neutron matter (PNM) at ~n_0

NOT directly accessible in experiments

theoretically **cleaner**: Pauli + isospin constraints on 3n forces

e.g.) N2LO ChEFT $V_{3N} = V_{2\pi}(c_1, c_3, \cancel{c_4}) + V_{1\pi}(\cancel{c_D}) + V_{contact}(\cancel{c_D})$

Neutron star EOS as a low-density B.C.



ChEFT up to $2n_0$ + **causality**

$R_{1.4}$ < 12.9 km

ChEFT up to $1.5n_0$ + **causality**

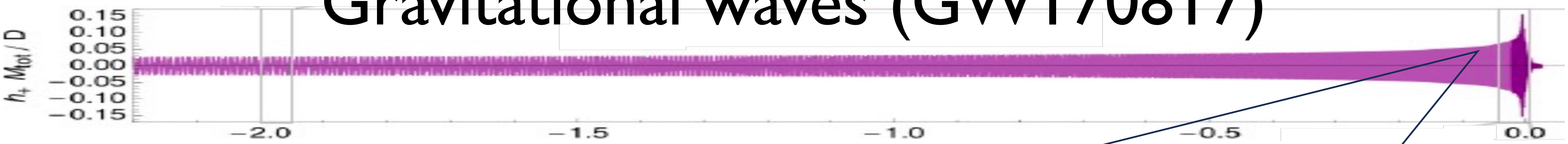
$R_{1.4}$ < 13.6 km

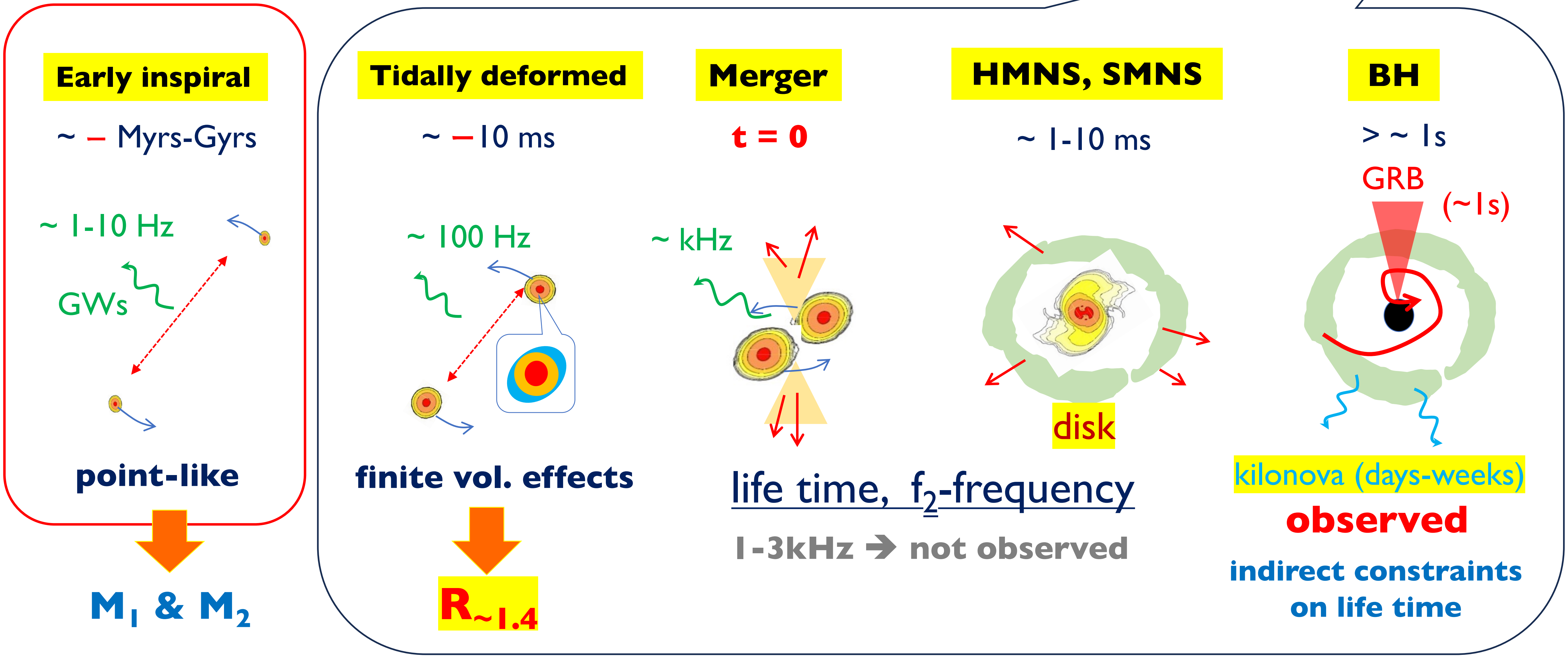
These ChEFT **N3LO** results

remain broadly consistent with the recent updates, e.g.,

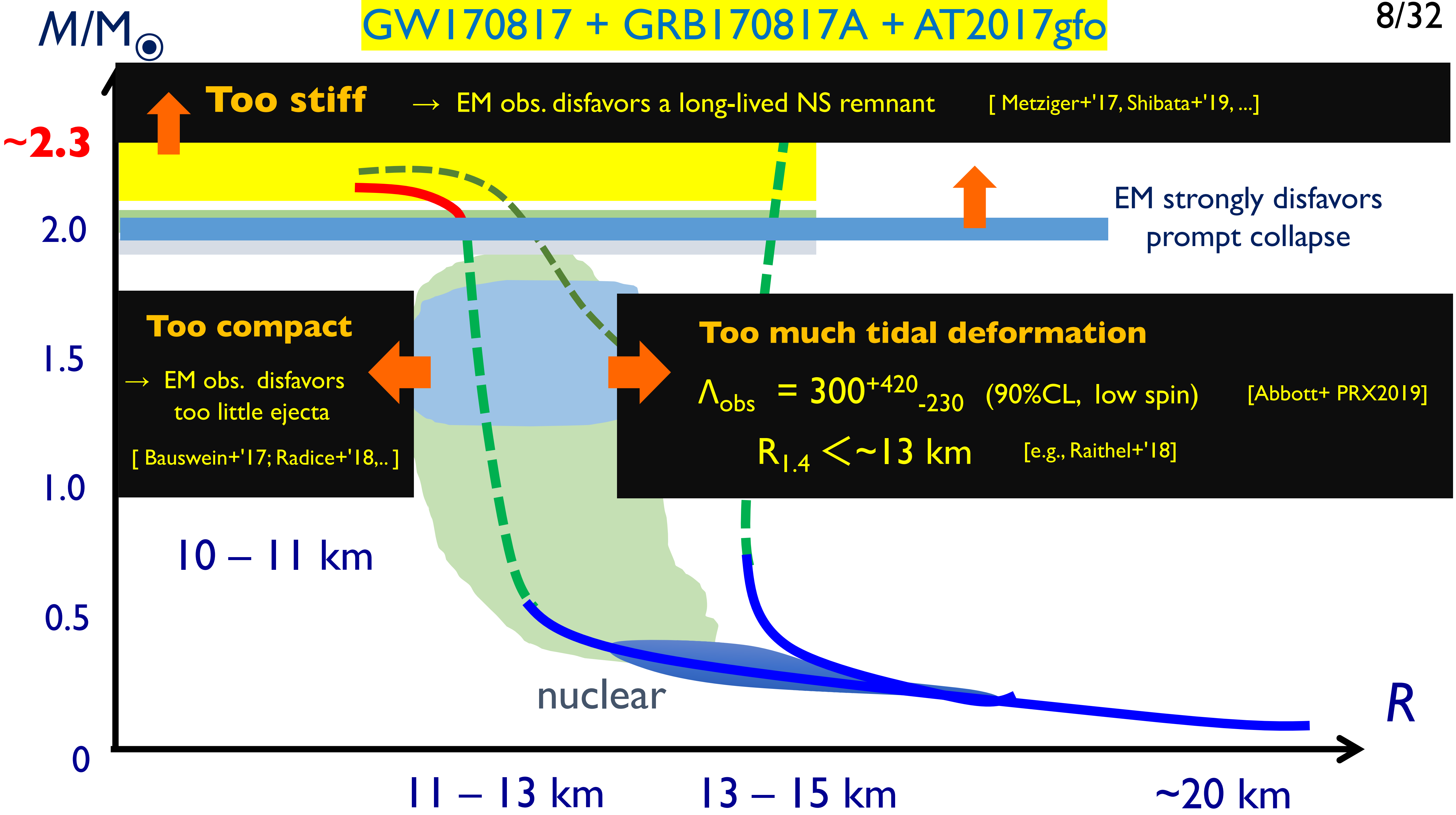
Tews+'24: QMC, less regular artifacts
Gottling+'25: reduced EFT uncertainty
Drischler+'26: check of MBPT convergence

Gravitational waves (GW170817)



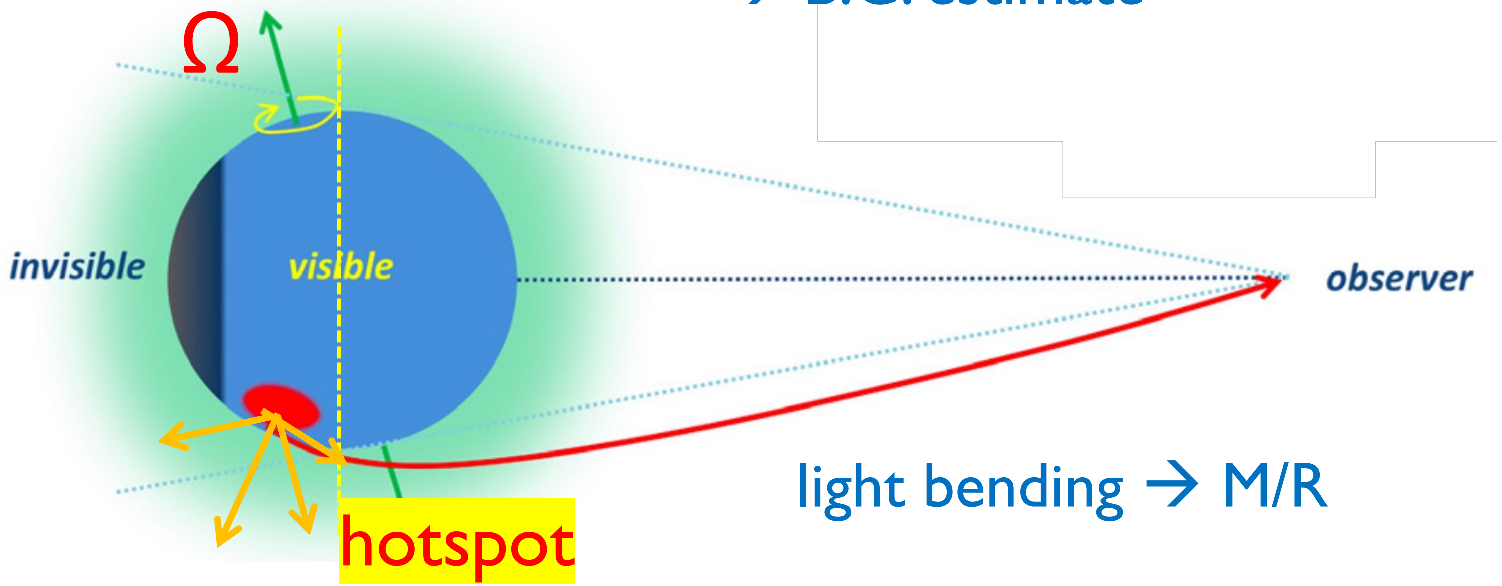


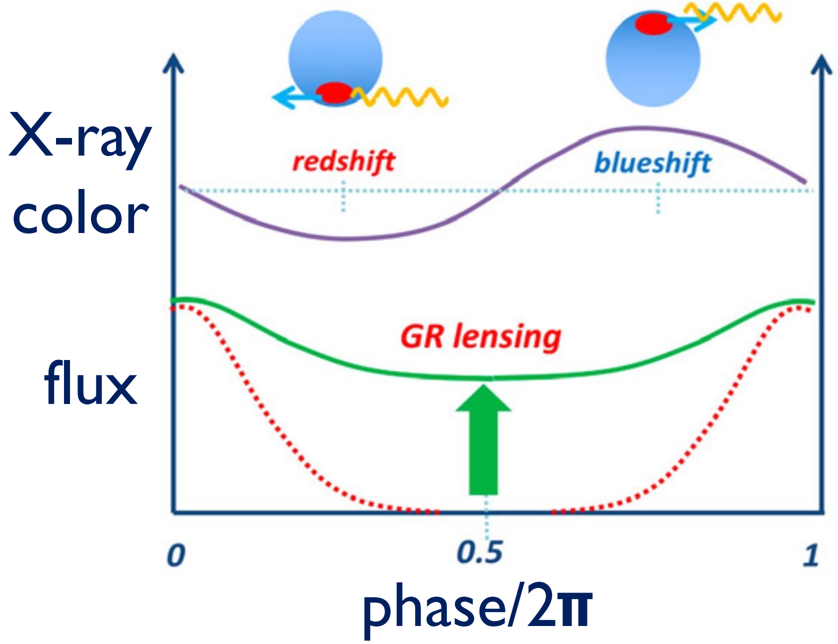
GW170817 + GRB170817A + AT2017gfo



NICER (+XMM-Newton) pulse profile → M, R

→ B.G. estimate



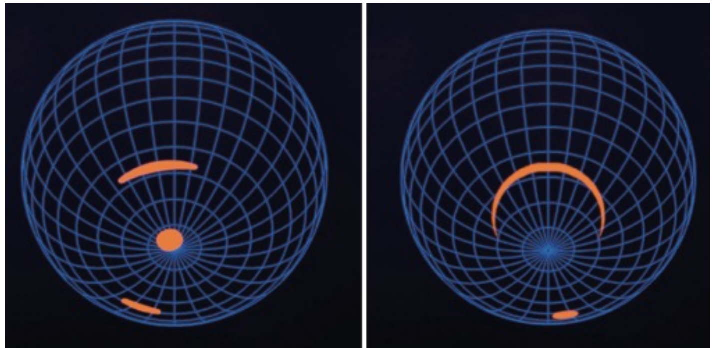


basic idea: keep track of rotating hot spots

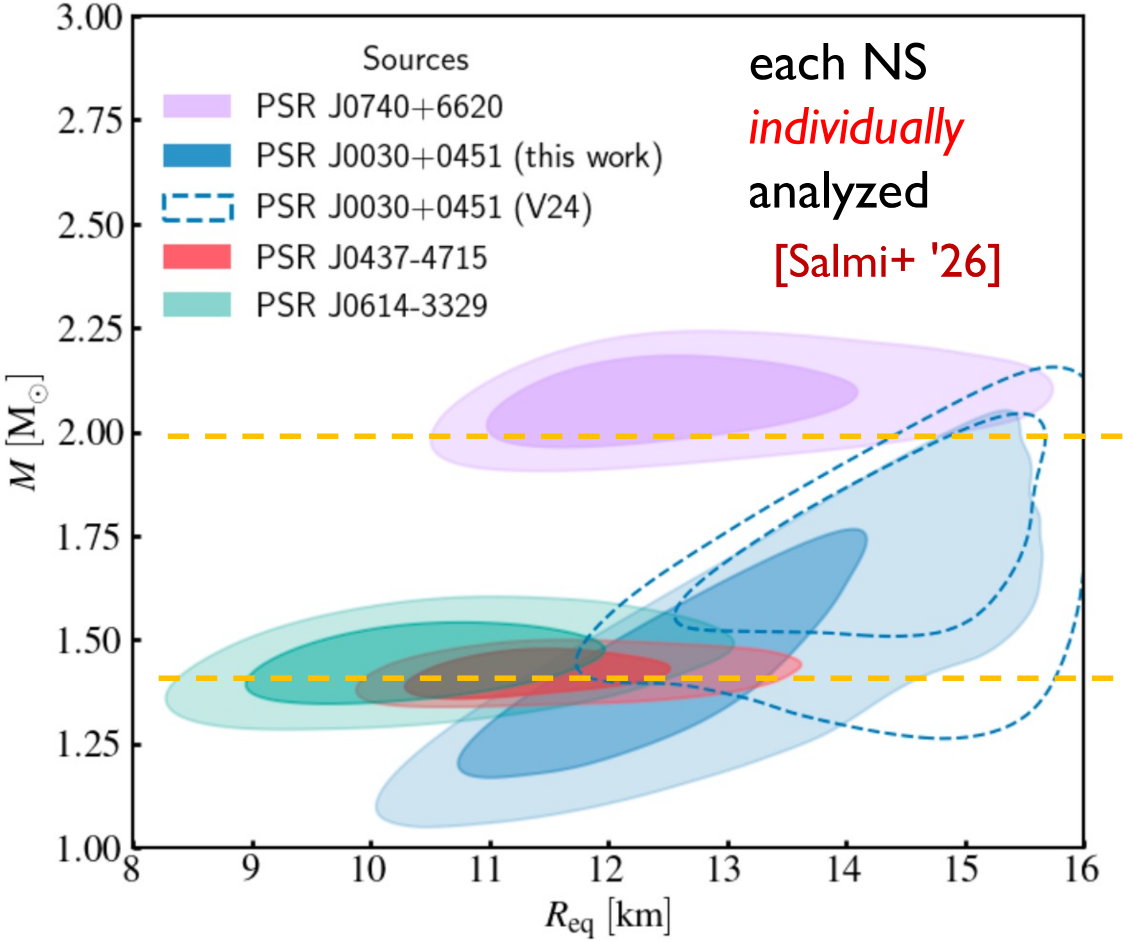
period		Ω	(pulse period)
(Doppler shifted) spectra	→	$R\Omega$	(surface velocity)
GR lensing		M/R	(mass/radius)

in reality...

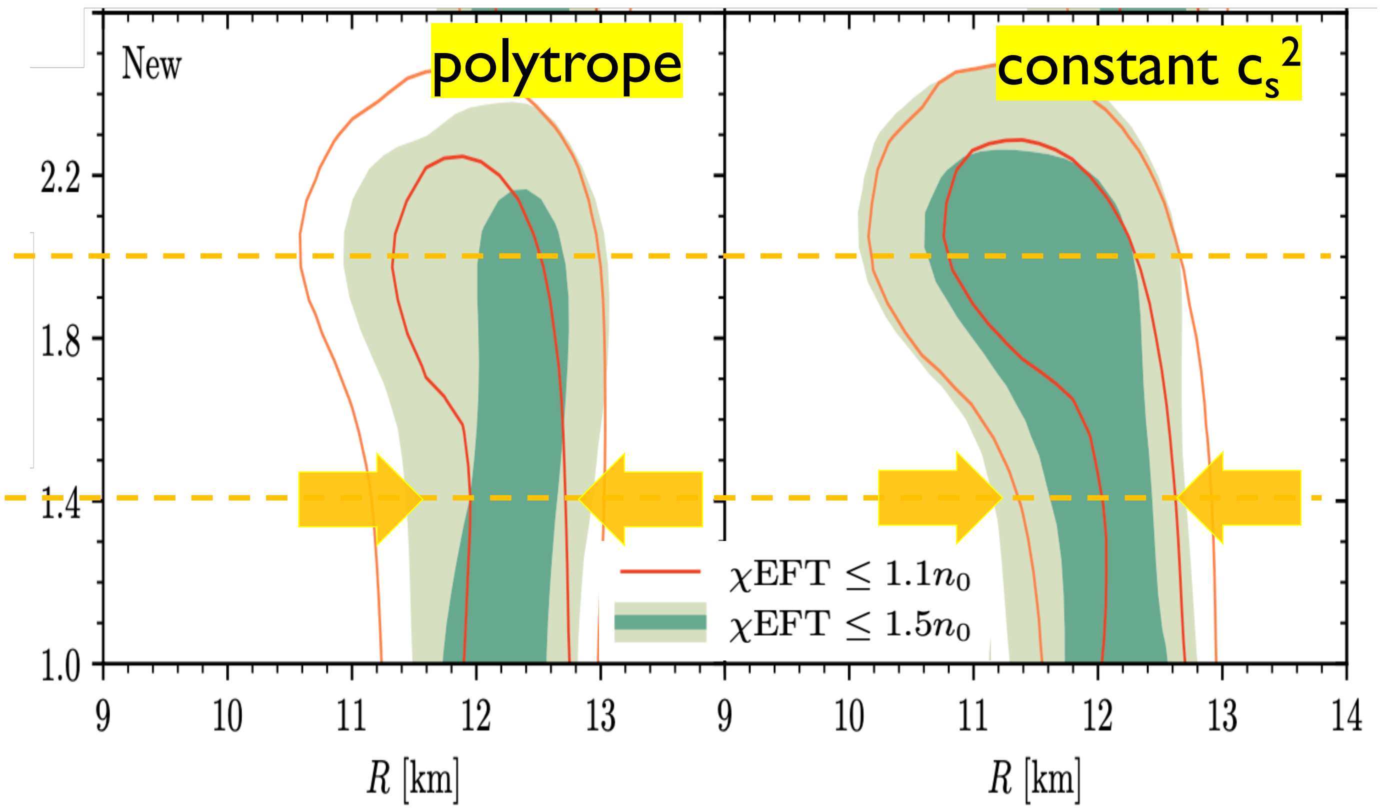
- distributions complicated
- B.G. mimics the GR lensing,.. etc.



NICER only vs EOS based correlations



EOS based joint analyses [e.g., Rutherford+ '24]



Physical EOS constraints correlates radii at different masses

yielding tighter constraints, robust against reasonable EOS model parametrization

Bayesian combined analyses

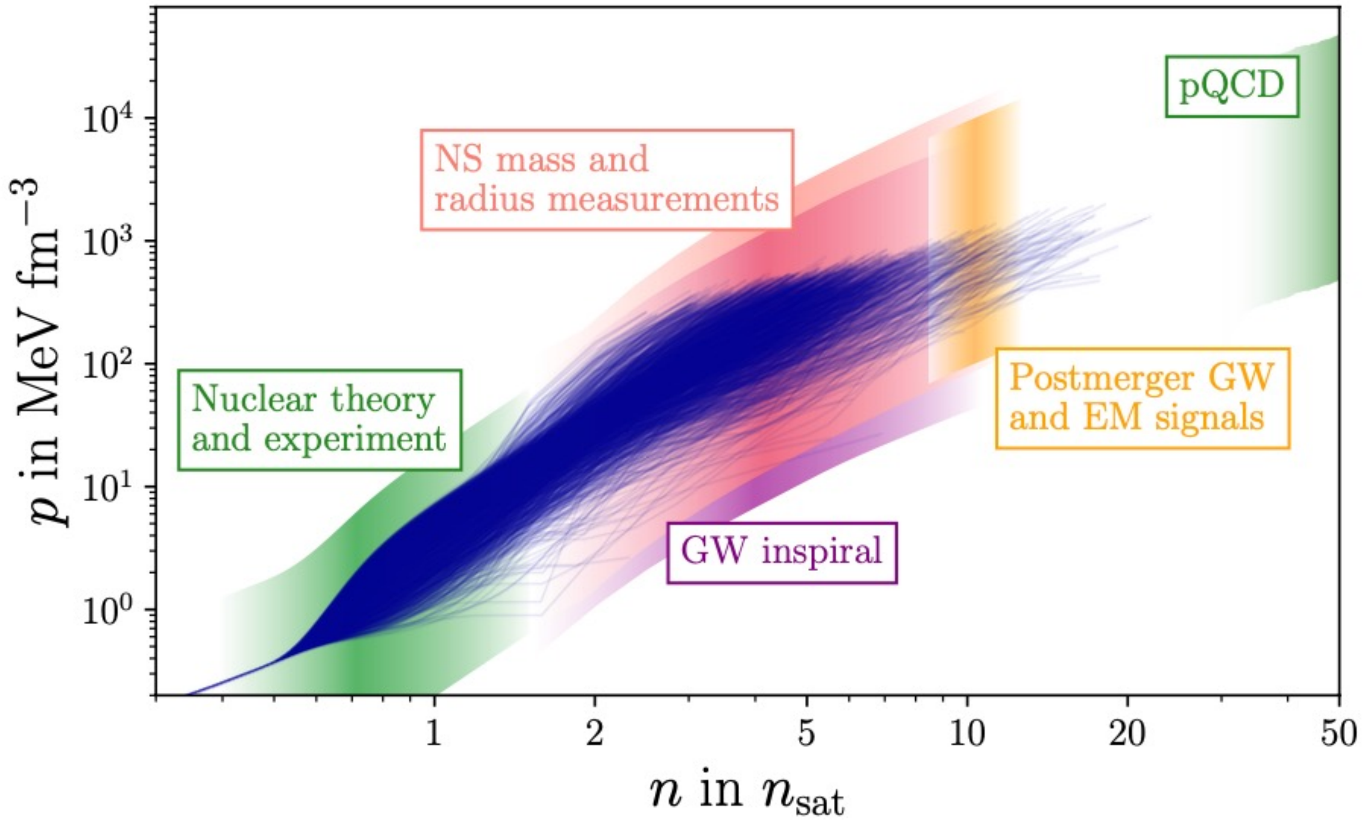
[e.g., Koehn+ '25, PRX]

conservative ———————— less conservative

Set	A	B	C
Description	χEFT	Set A	Set B
	pQCD	HIC	PREX-II
	Heavy pulsars	Black Widow J0952-0607	CREX
	NICER J0030+0451	qLMXBs	^{208}Pb dipole
	NICER J0740+6620	GW170817+ KN+GRB	Burster 4U 1702-429
	GW170817		Burster J1808.8-3658
			HESS J1731-347
			GW190425
			GRB211211A
			GW170817 postmerger
$R_{1.4}$ (km)	$12.26^{+0.80}_{-0.91}$	$12.41^{+0.57}_{-0.78}$	$12.17^{+0.46}_{-0.51}$
$M_{\rm TOV}$ ($M_\odot$)	$2.25^{+0.42}_{-0.22}$	$2.33^{+0.35}_{-0.22}$	$2.30^{+0.07}_{-0.20}$
$p_{3n_{\rm sat}}$ (MeV fm^{-3})	90^{+71}_{-31}	99^{+65}_{-29}	93^{+33}_{-20}
$n_{\rm TOV}$ ($n_{\rm sat}$)	$5.92^{+1.34}_{-1.38}$	$5.67^{+1.09}_{-1.09}$	$5.73^{+0.95}_{-0.83}$

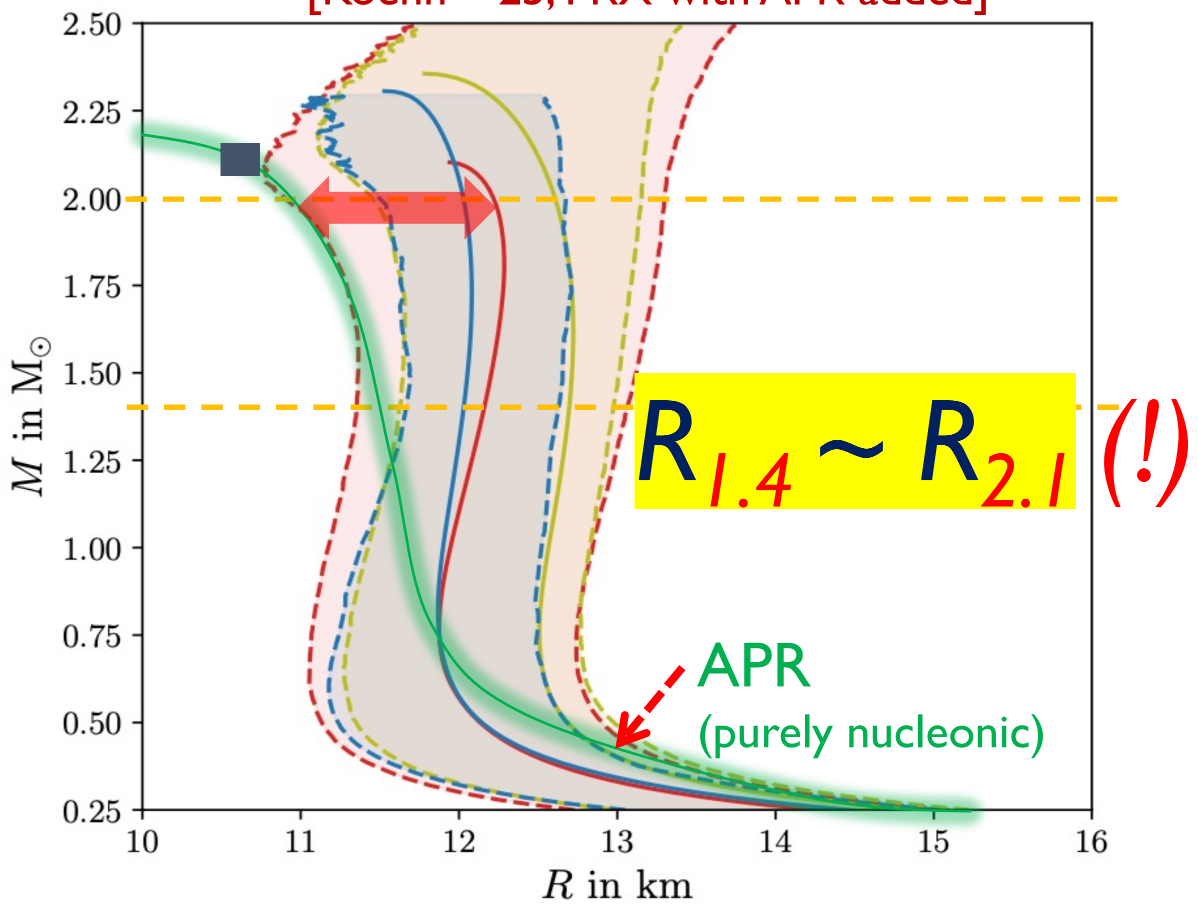
delineate the impacts of each constraint one by one

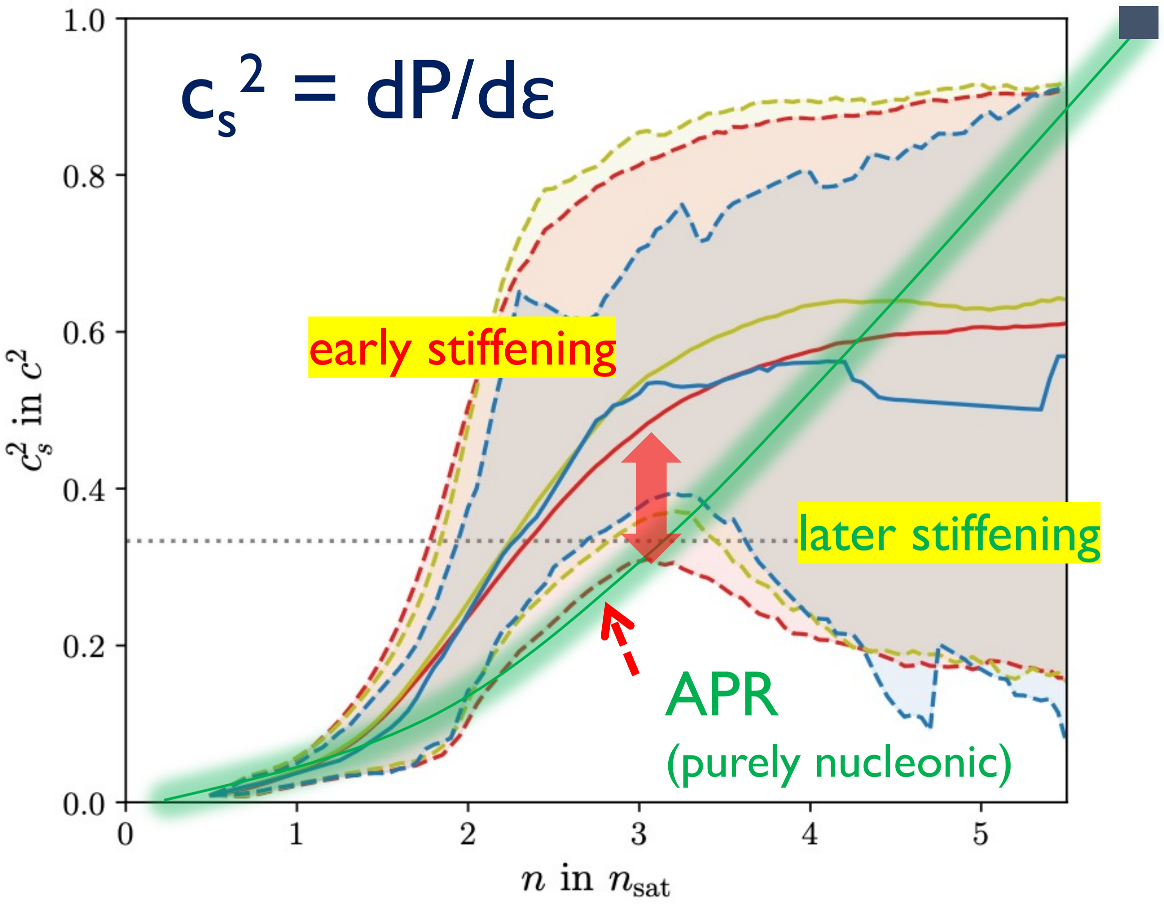
prioritize constraints; set A (conservative) to C (less conservative)



Early vs later stiffening: when does c_s^2 go beyond 1/3 ?

[Koehn+ '25, PRX with APR added]





soft EOS at $n_B < \sim 1.5 n_0$

$R_{1.4} \sim R_{2.1}$

early stiffening at $2\text{-}3 n_0$

Microphysics

how can achieve rapid stiffening at 2-3n_0 ?

Stiffening of nuclear EOS is slow

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

$$\varepsilon(n_B) = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^{\alpha} \quad \Rightarrow \quad P = \frac{2}{3}a\frac{n_B^{5/3}}{m_N} + b(\alpha-1)n_B^{\alpha}$$

soft (!)

Achieving a stiff nuclear EOS by many-body repulsion?

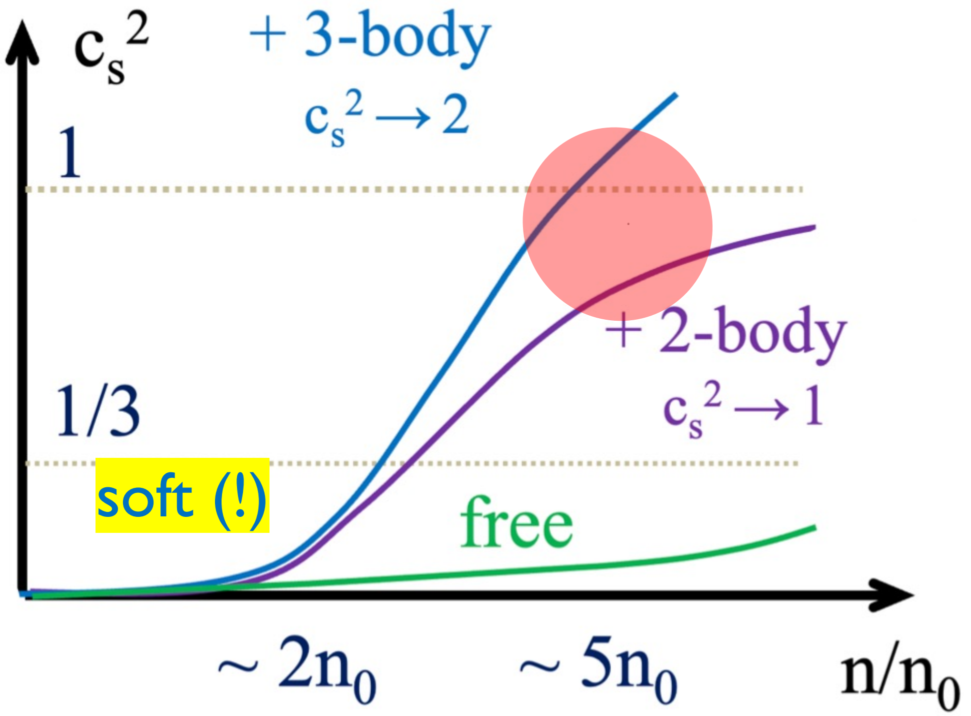
problems

1) Convergence of many-body expansion?

2) Stiffening is slow: P increases as ~ n_B^{α}

3) P ~ $(\alpha - 1)\varepsilon$ at large n_B

(danger to violate causality near M_{max})



Quark EOS can be stiff

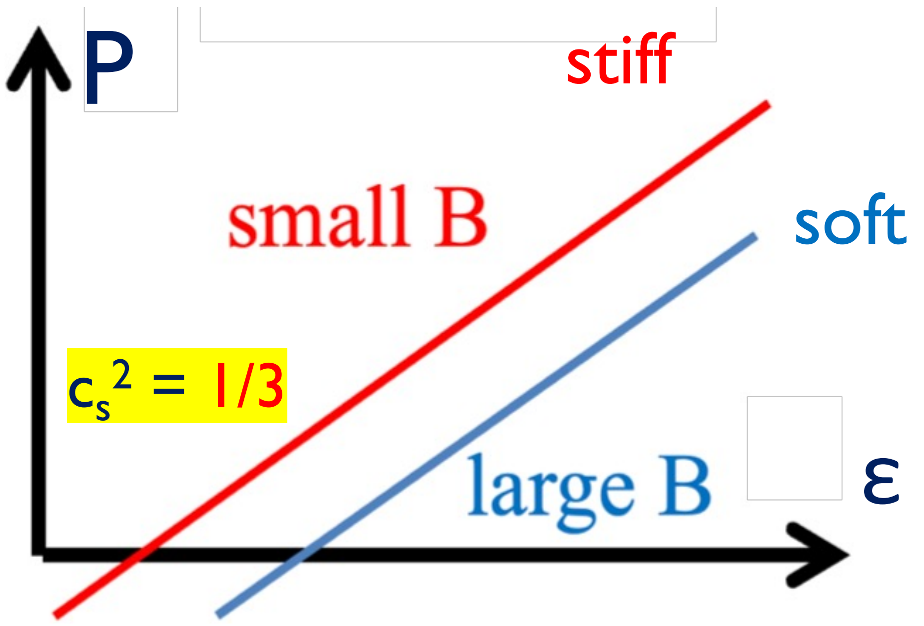
e.g.) free massless quarks

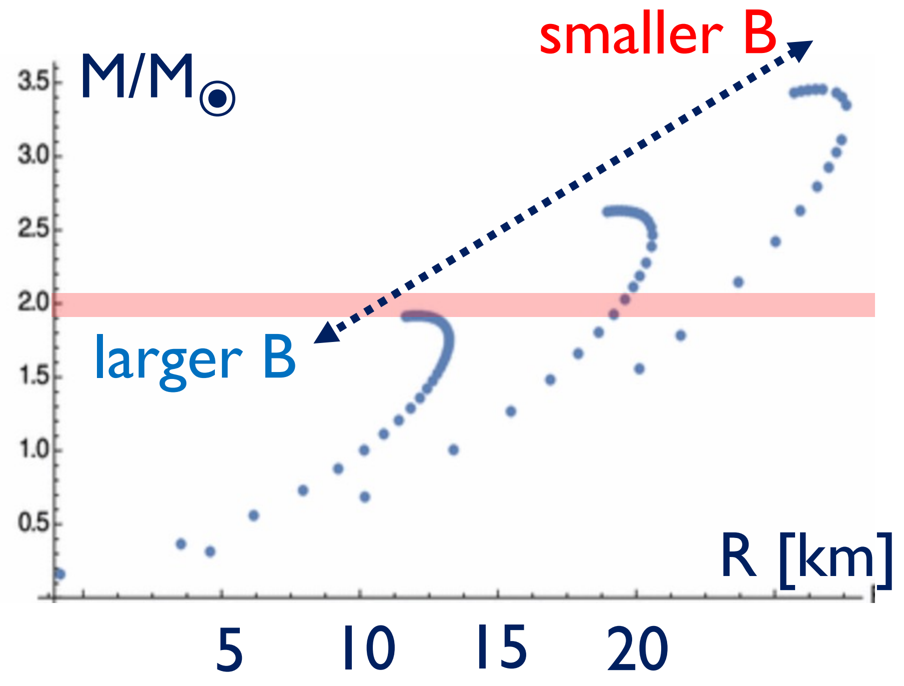
$$P = \frac{\varepsilon}{3} - B'$$

normalization

quark kin. pressure >> baryon kin. pressure

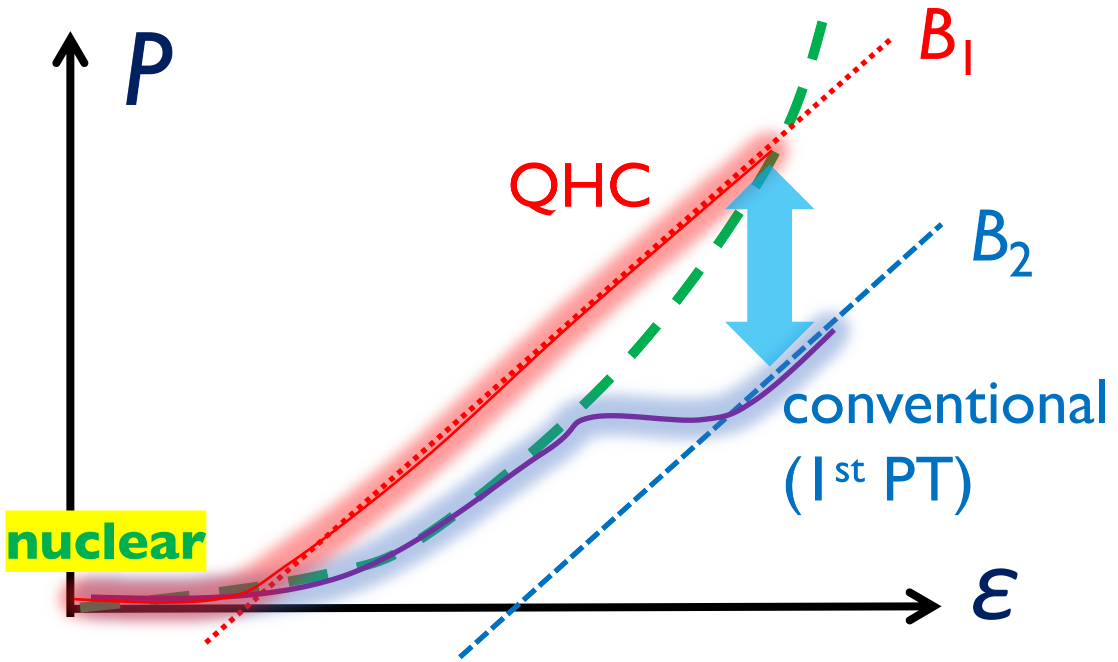
O(N_c) O($1/N_c$)





Conformal c_s^2 = 1/3 can be **stiff**

Hybrid EOS: normalization of EOS ??



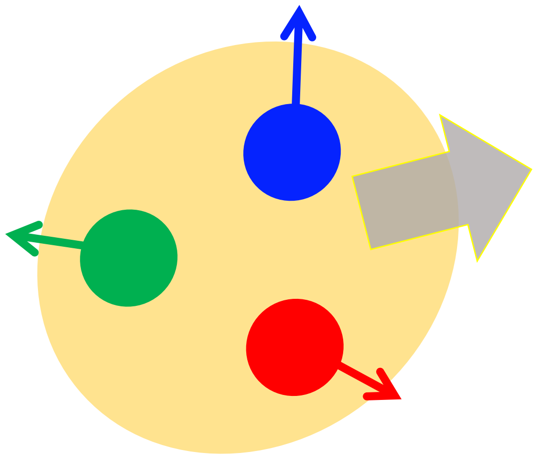
which B should be taken???

difficult to fix the relative normalization B of different models...

Need: *follow quark states from nuclear to quark matter*

Role of confinement

size ~1fm → p_q ~ 200-300 MeV



inner kin. energy $N_c\left(M_q + \frac{\langle \vec{p}^2\rangle}{2M_q} + \cdots\right)$ ➡ M_B

center of mass energy $\frac{P_B^2}{2N_c M_q} \sim \frac{P_B^2}{2M_B}$ << M_B

➡ $\varepsilon \sim M_B n_B >> P$ soft baryonic matter

confinement allows quarks to appear in energy density

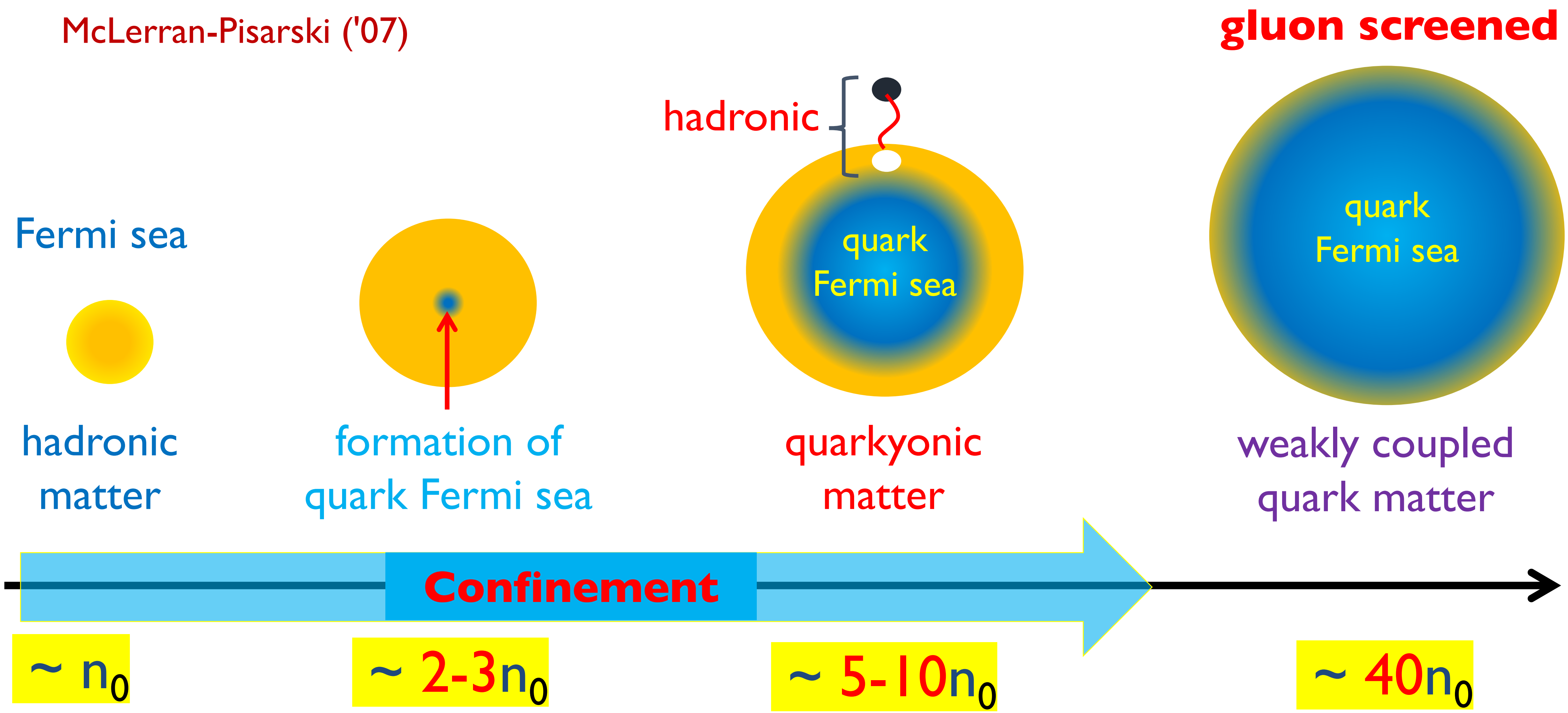
but NOT in pressure (!)

When do quarks begin to contribute to the thermodynamic pressure???

A model of crossover

Quarkyonic matter := quark matter with confining gluons

McLerran-Pisarski ('07)



IdylliQ model

= **Ideal dual** Quarkyonic model

Describe **single** physics in **two** languages (baryon/quark)

Powerful in transient regimes (2-5n_0)

Sum rules for occupation probabilities

cf) [TK '21]

occupation **probability** of **quark** state with p

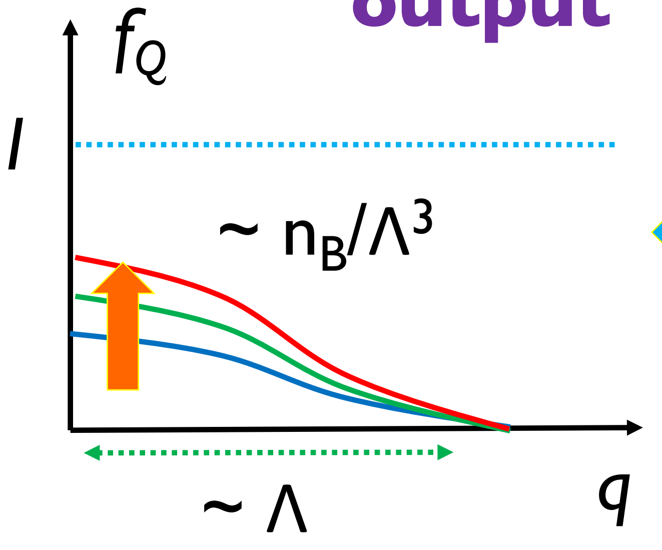
occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_Q(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B)\,\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_c)$$

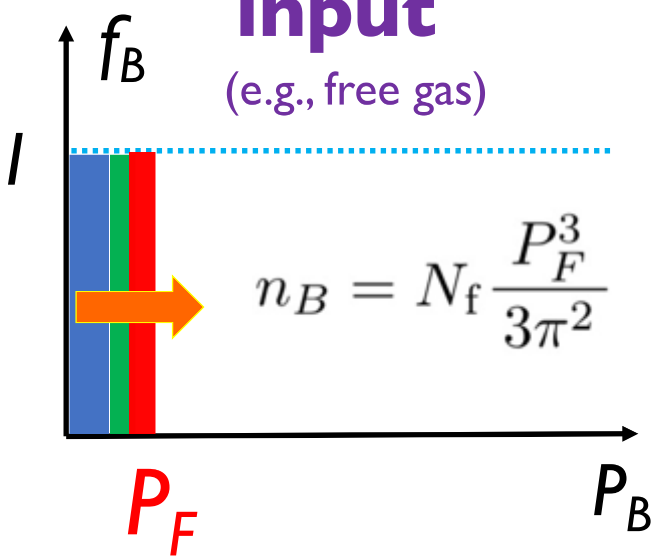
e.g.) in **ideal** baryonic matter

output



input

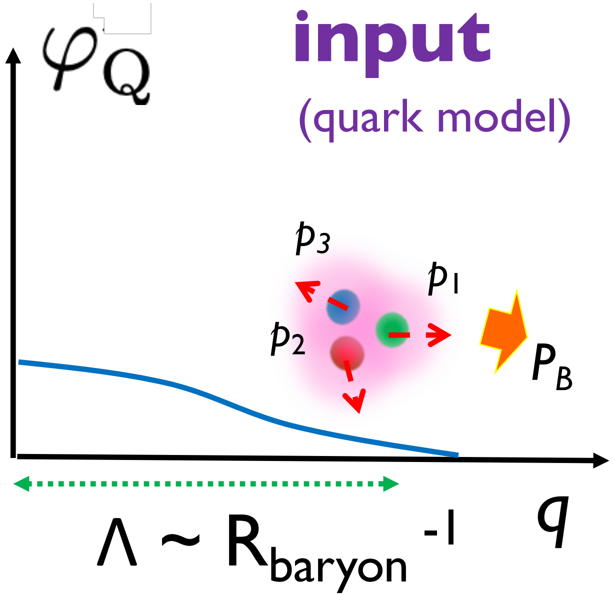
(e.g., free gas)



$\otimes$

input

(quark model)



An ideal model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $$\varepsilon_{\mathrm{B}}[f_{\mathrm{B}}] = 4\int_k E_{\mathrm{B}}(k) f_{\mathrm{B}}(k)$$

2) keep using the same φ_{Q} (quarkyonic)

3) use a special quark distribution → sum rules analytically *invertible*

$$\varphi_{3\mathrm{d}}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda} \qquad \hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2}\delta(\boldsymbol{p}-\boldsymbol{q})$$

nontrivial output

$$f_{\mathrm{Q}}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\mathrm{B}}(\boldsymbol{P}_B)\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\mathrm{c}})$$

natural at ***low*** *density*

⇄

nontrivial output

$$f_{\mathrm{B}}(N_{\mathrm{c}}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\mathrm{c}}^3}\hat{L}[f_{\mathrm{Q}}(\boldsymbol{q})]$$

natural at ***high*** *density*

An **ideal** model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $$\varepsilon_{\rm B}[f_{\rm B}] = 4\int_k E_{\rm B}(k) f_{\rm B}(k)$$

2) keep using the ... (quarkyonic)

3)

To Do: solve variational problem of $f_B(k)$

with *quark substructure constraints* !

nontrivial output

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B)\varphi_Q^B(\mathbf{q}-\boldsymbol{P}_B/N_{\rm c})$$

natural at **low** density

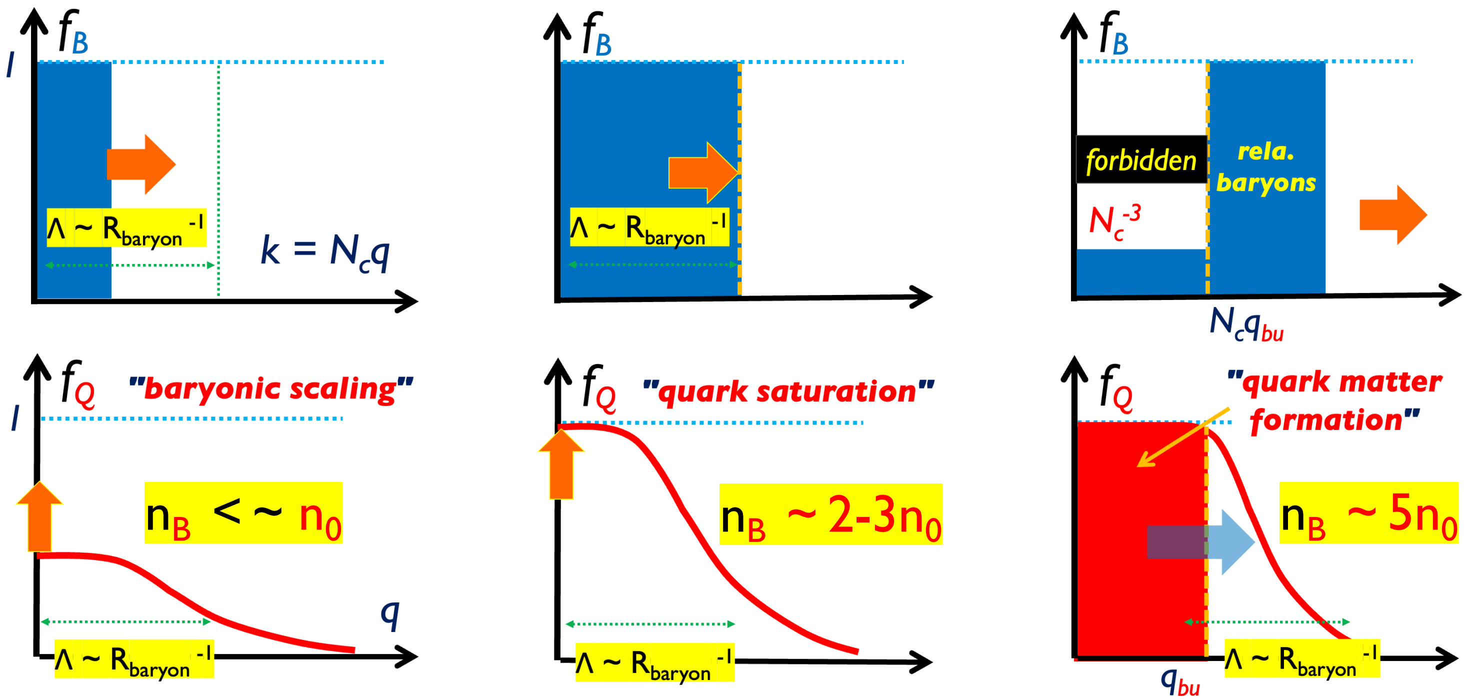
nontrivial output

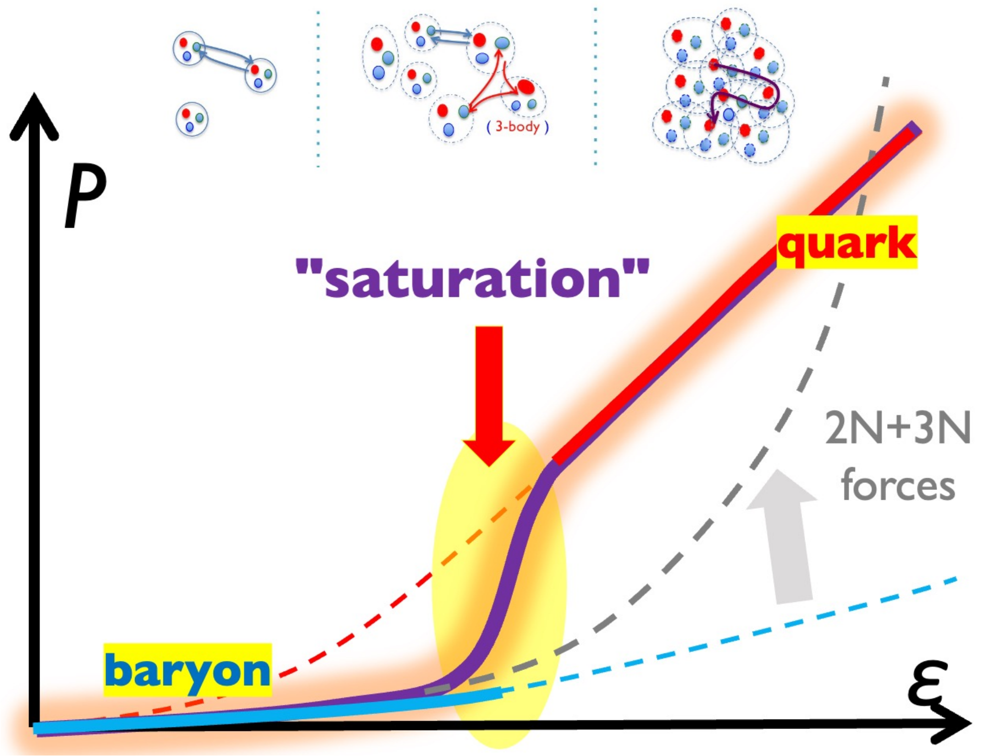
$$f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3}\hat{L}[f_{\rm Q}(\boldsymbol{q})]$$

natural at **high** density

Quark saturation & inevitable stiffening

$$f_{\mathrm{Q}}(\boldsymbol{q};n_{\mathrm{B}}) = \int_{\boldsymbol{P}_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{P}_{\mathrm{B}};n_{\mathrm{B}})\varphi_{\mathrm{Q}}^{\mathrm{B}}(\boldsymbol{q};\boldsymbol{P}_{\mathrm{B}})$$





low p baryons quenched

dual

quark matter scaling

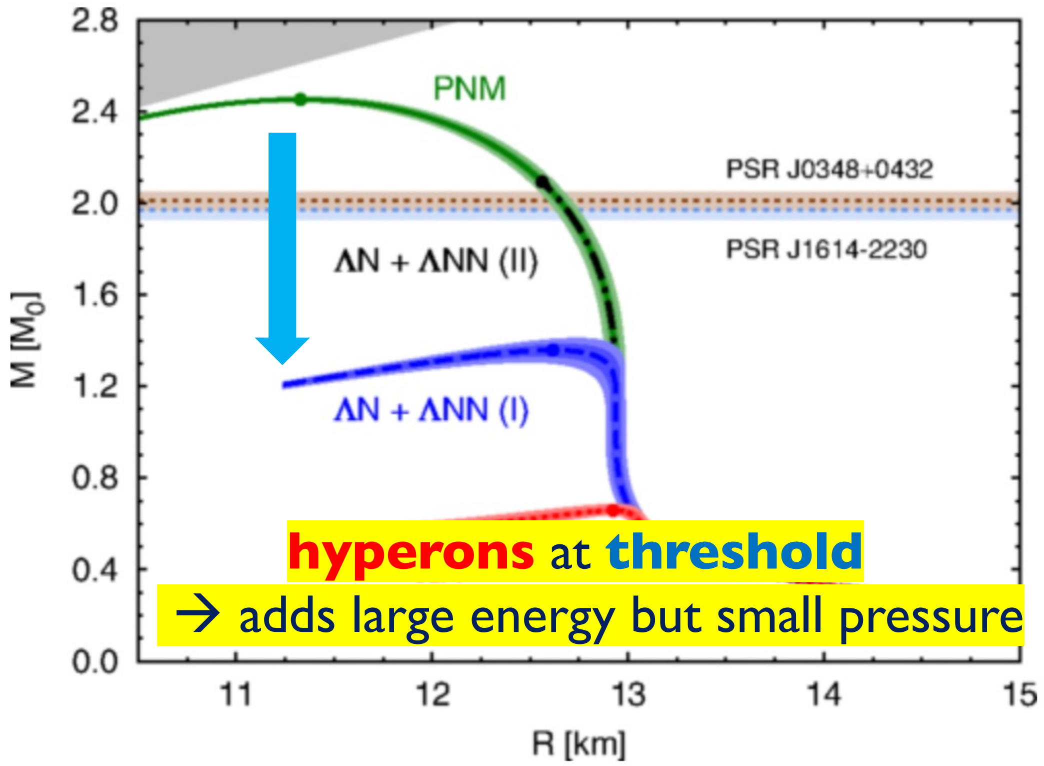
stiff EOS

[see also McLerran,-Reddy PRL'19]

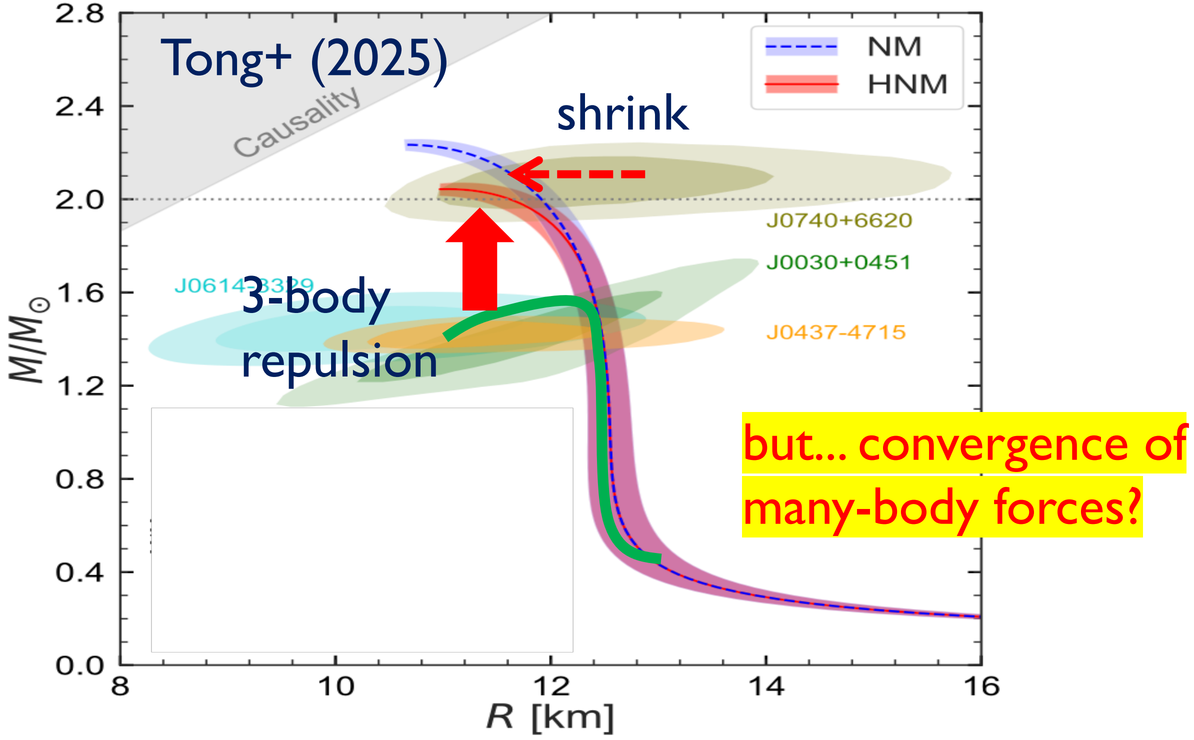
$n_B \sim 2\text{-}3 n_0$

"*inevitable*" *stiffening*

Hyperon Puzzle ?



state-of-art, with empirical YN & YNN forces



supplemental idea:

quark saturation

1) statistical repulsion (Pauli blocking)

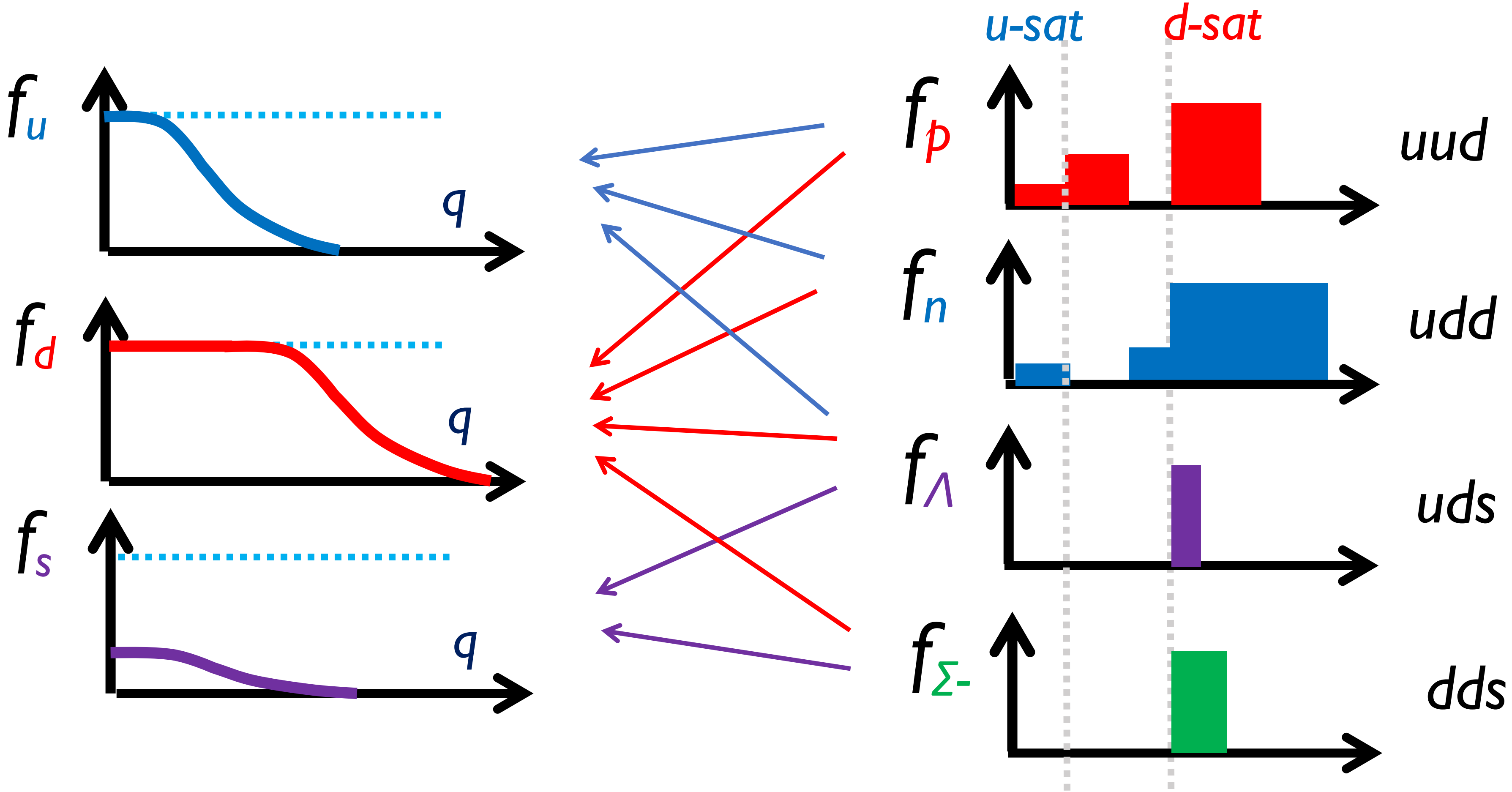
2) stronger repulsion at higher density

3) no double counting of quarks

Multi-flavor extension

$$f_{\mathrm{Q}}(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_{\mathrm{Q}}^{\mathrm{B}} \int_{\boldsymbol{k}} f_{\mathrm{B}}(\boldsymbol{k})\varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right)$$

Q = u, d, s



and so on…

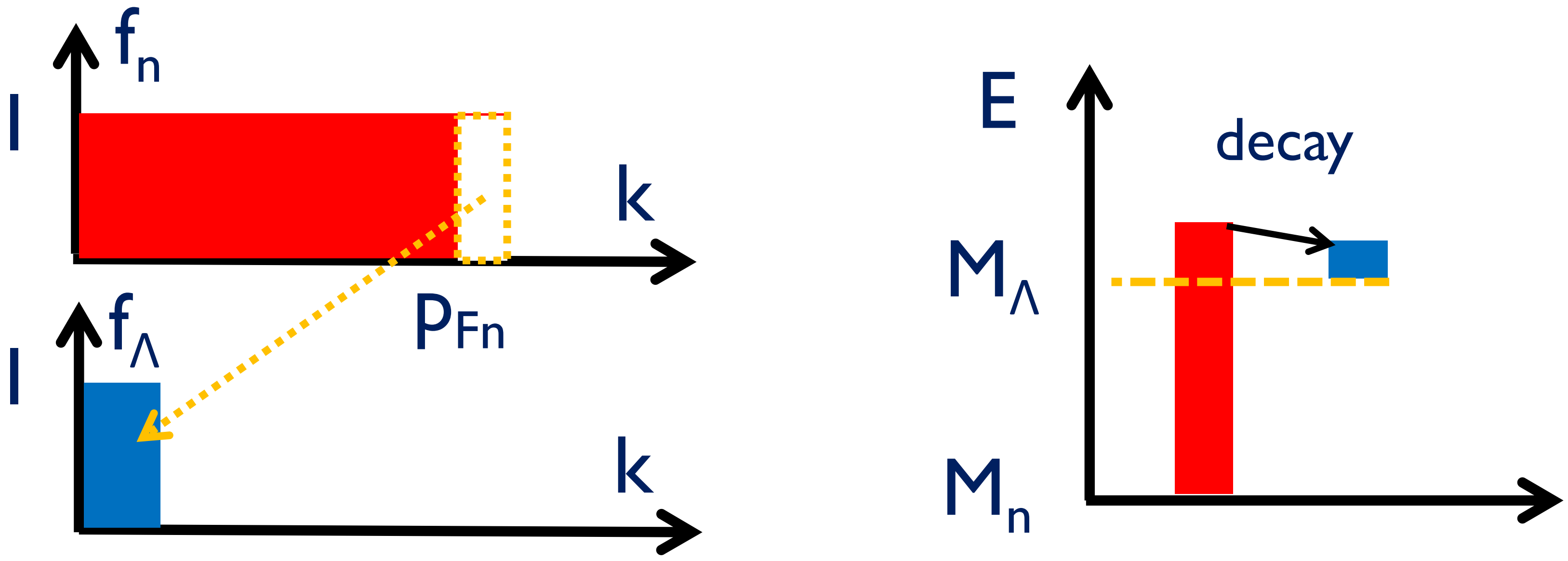
saturation constraints

flavor correlations

n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]

· **conventional**



at $n_B \sim 2\text{-}3n_0$

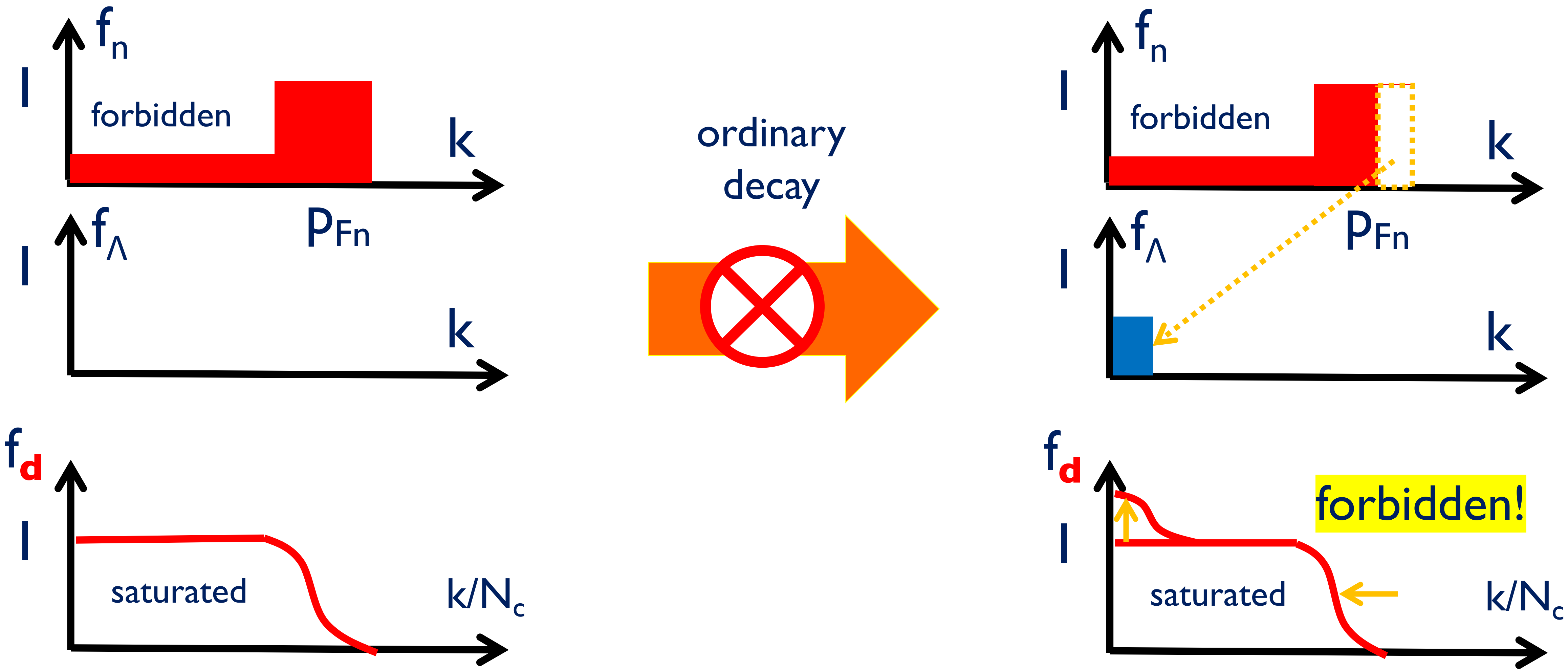
neutrons *at high momenta* → *non-relativistic* Λ

ε increases much but pressure does not (**softening**)

n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]

- **with d-quark saturation**

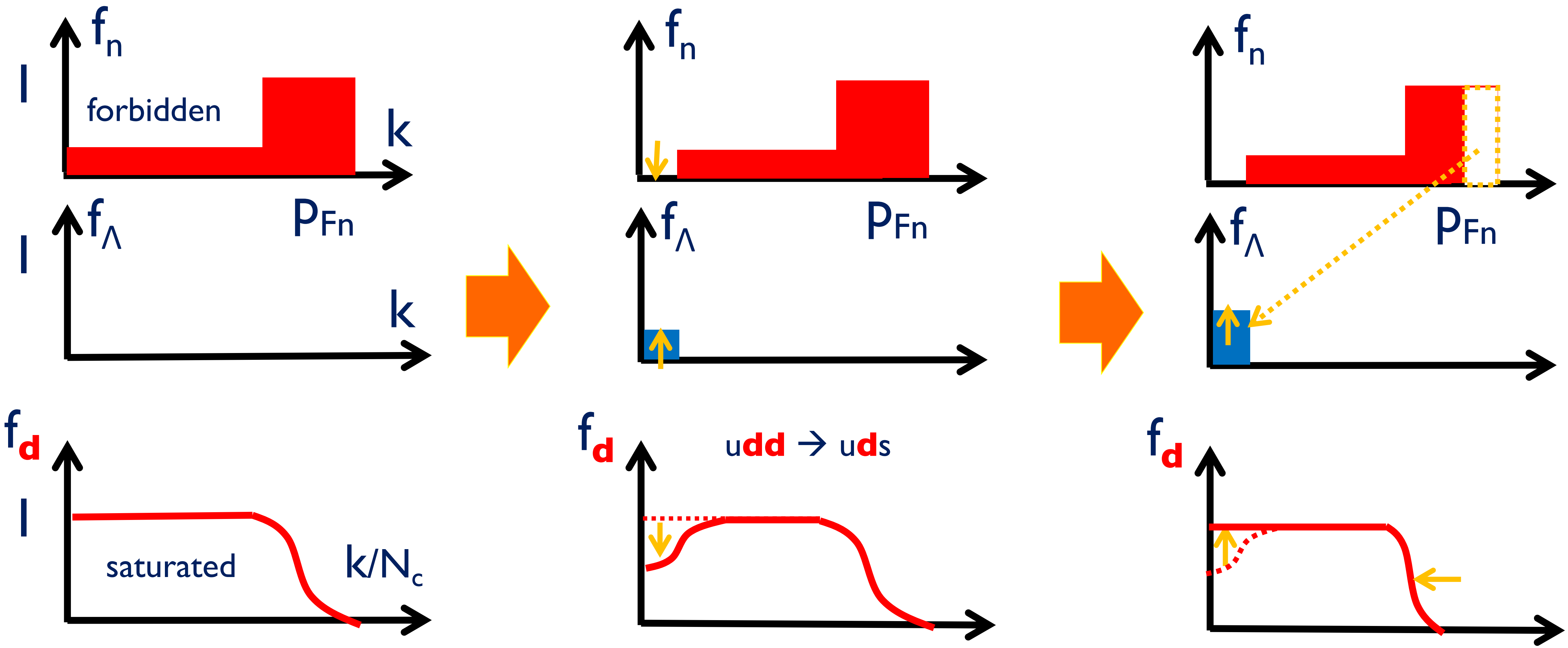


n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]

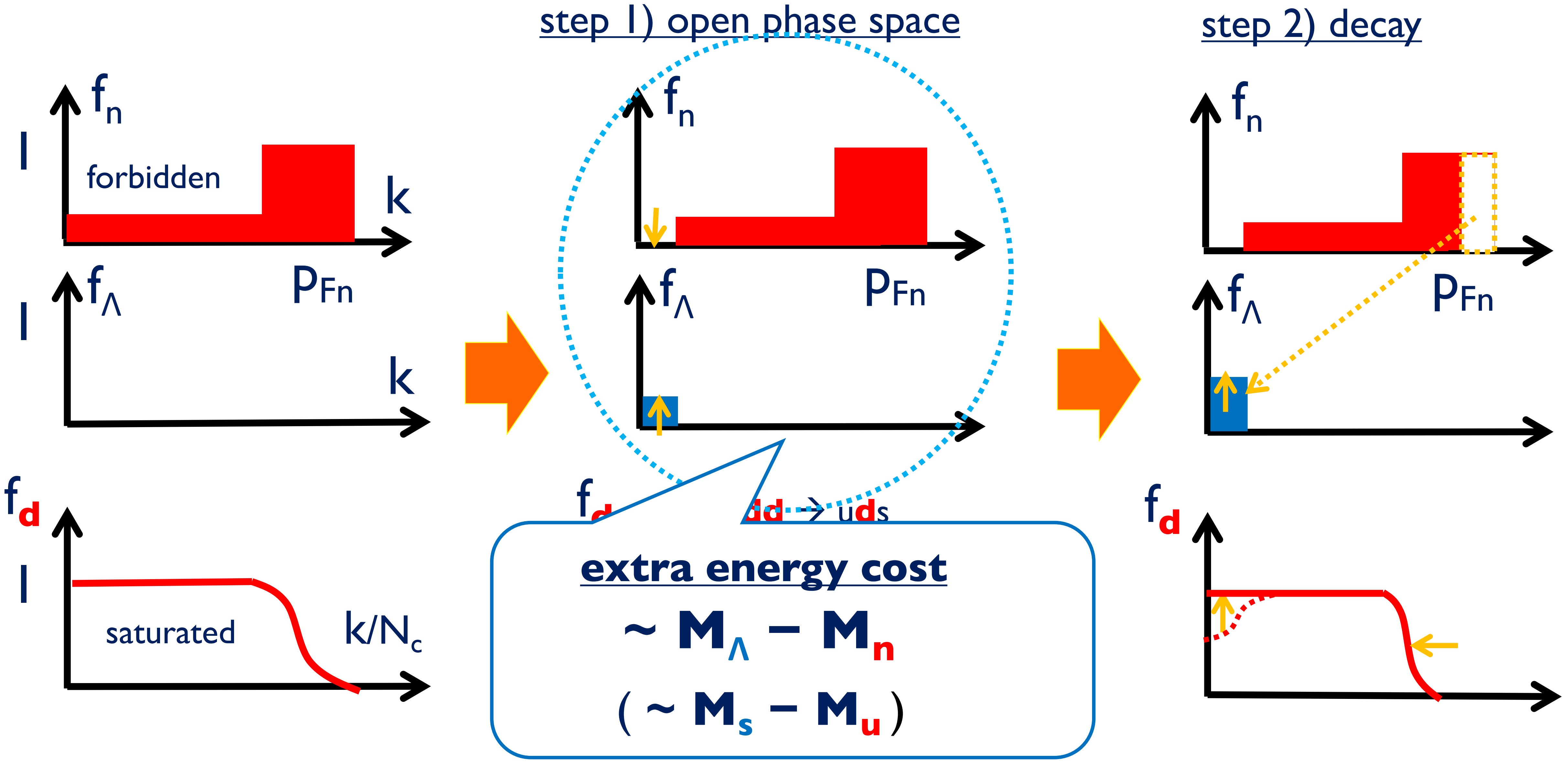
step 1) open phase space

step 2) decay



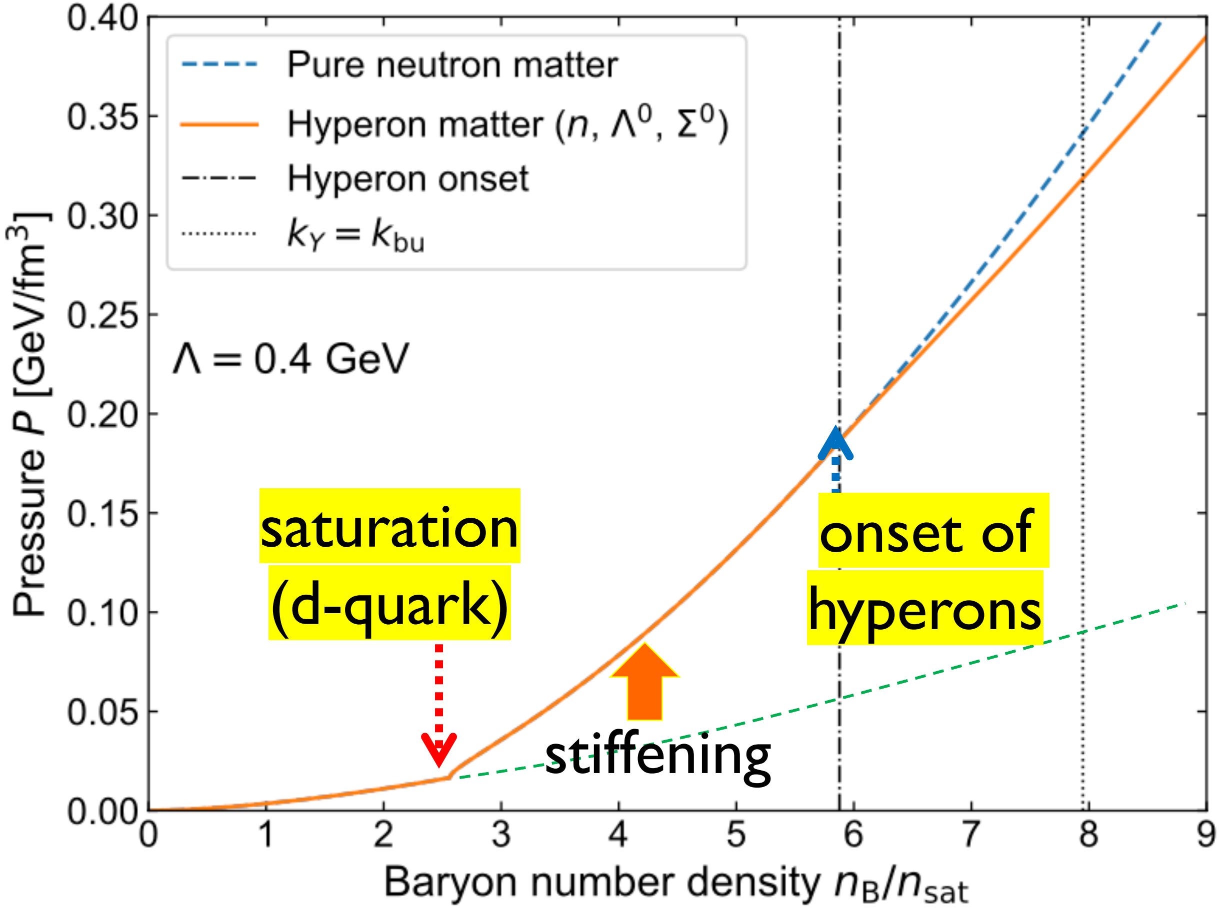
n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]



EOS: n-Λ matter

[Fujimoto-TK-McLerran, '24]



see also
Nagatsuka-TK '26
for 2-color, 2+**2** flavor QCD

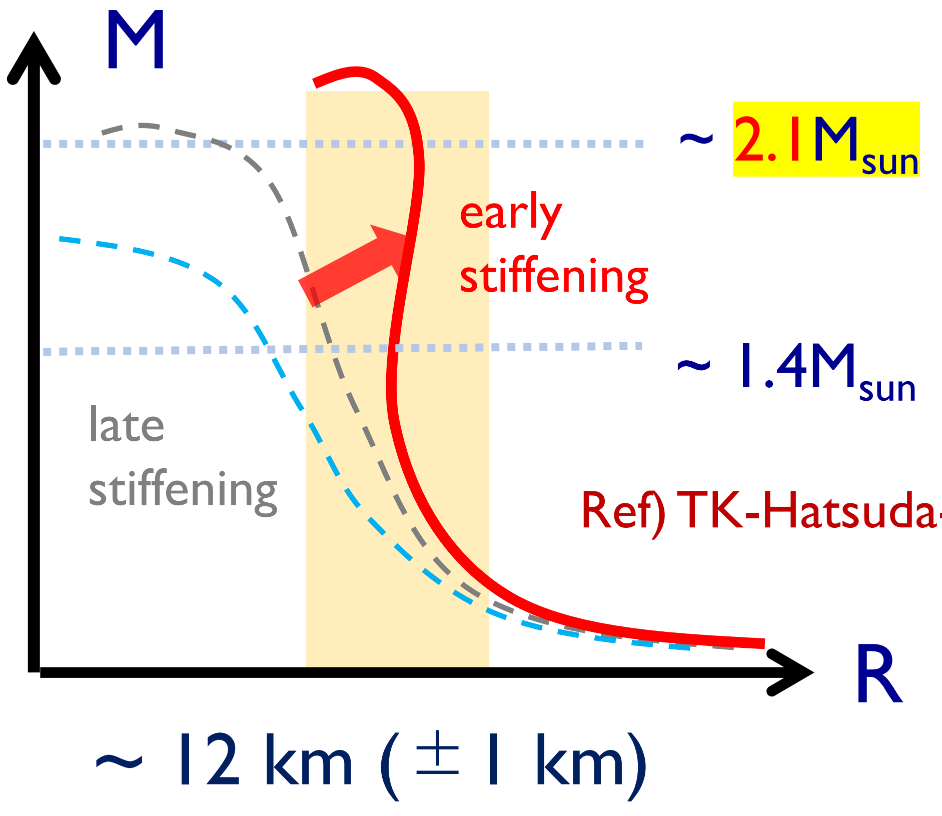
$n_B^{\Lambda\text{-onset}} \sim 2n_0 \rightarrow$ 5-6n_0

conventional → with saturation

Summary

NICER for 1.4 & 2.1 M_{sun} + **GW** + **nuclear** (< ~1.5n_0)

→ $R_{1.4} \sim R_{2.1}$ (!)



Ref) TK-Hatsuda-Baym ('21)