Capturing the Schwinger Effect at One Loop: A Resummed In-In Formalism

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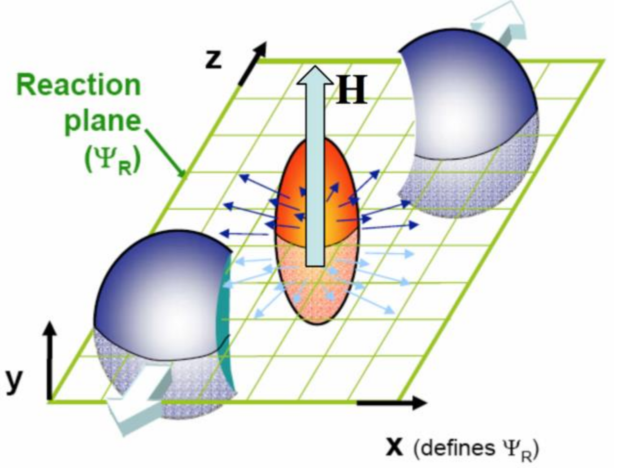
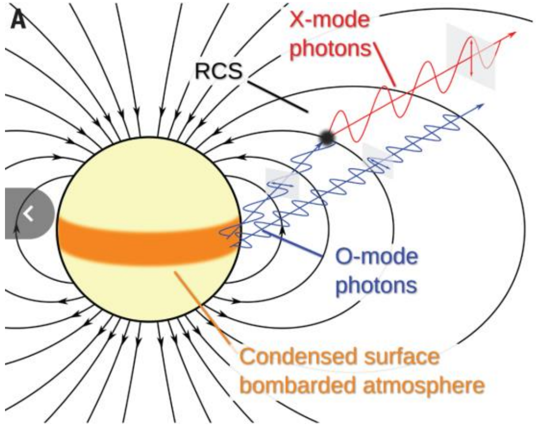
(Collaboration with Kenji Fukushima)

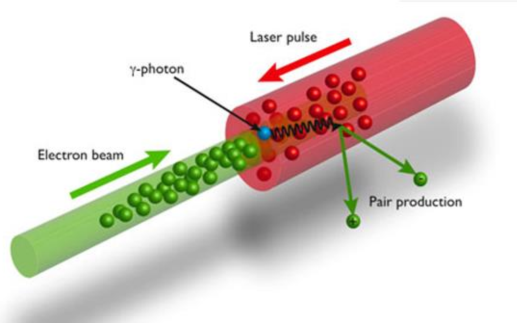
Based on: Fukushima, SM, JHEP 05 (2026) 139; arXiv: 2512.19337 [hep-ph]

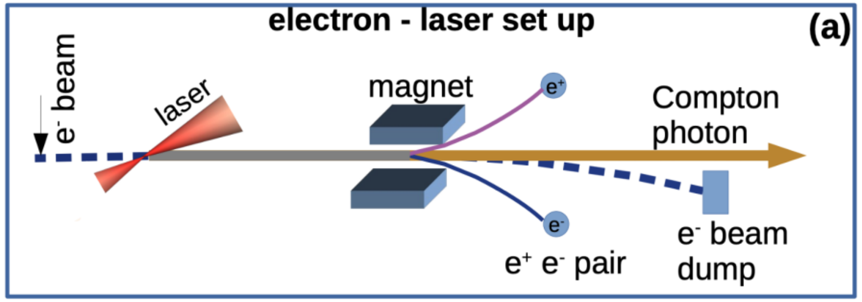
Opportunity for Strong EM fields ($eE, eB \gg m_e^2$):



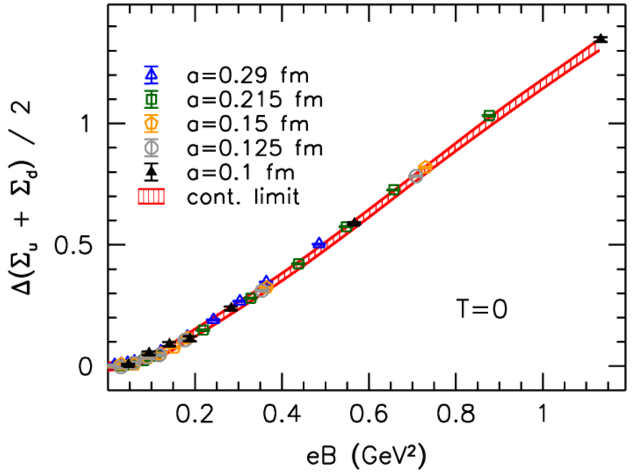
- heavy ions Strong $B \sim 10^{18-19}$G, short lifetime
- Magnetars Strong $B \sim 10^{15-16}$G, long lifetime
- intense lasers approaching $eE \sim m_e^2$, long lifetime

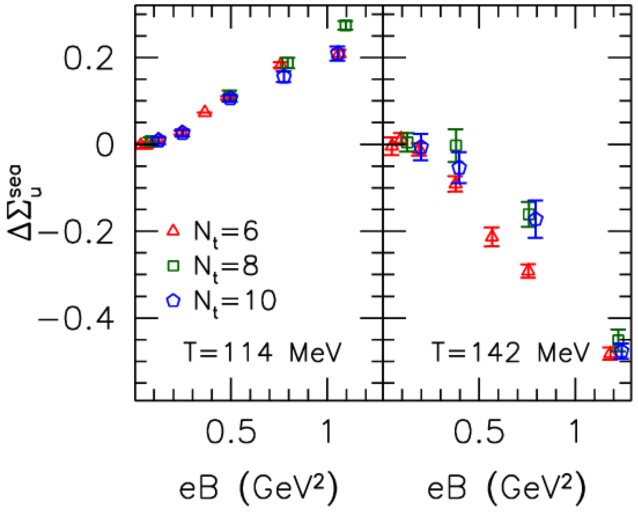


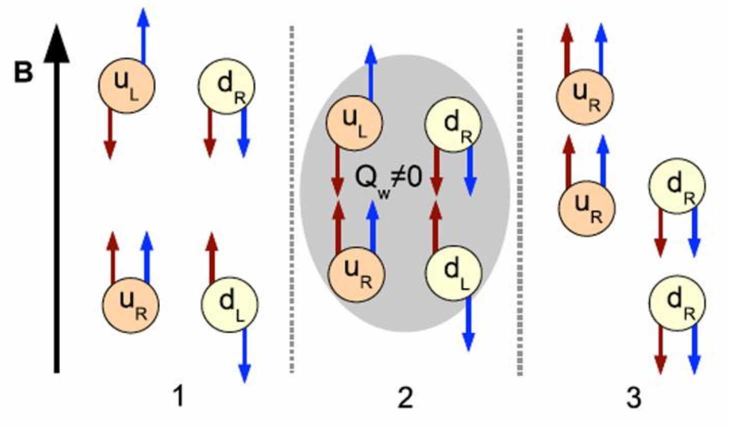




- magnetic catalysis of chiral condensate (low T)
  Bali+(2012), Shovkovy (2013)
- inverse magnetic catalysis in lattice QCD ($T \sim T_c$)
  Bruckmann-Endrodi-Kovacs (2013)
- Chiral Magnetic Effect
  Fukushima-Kharzeev-Warringa (2008)
  Kharzeev-Mclerran-Warringa (2008)







- Schwinger mechanism

  Schwinger (1951)

- Kinetic approaches

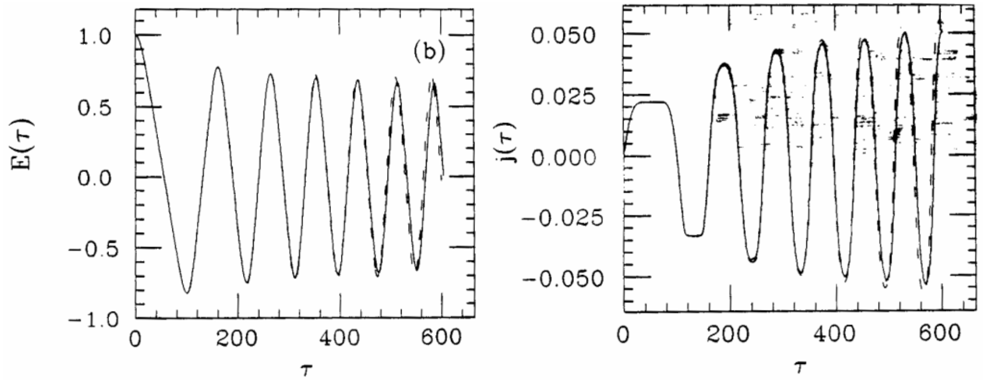
  Stephanov-Yin (2012), Hidaka-Pu-Wang-Yang (2022)

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\boldsymbol{x}}\cdot\nabla_x f + \sqrt{G}\dot{\boldsymbol{p}}\cdot\nabla_p f = C[f], \qquad \sqrt{G} = 1 + \hbar \boldsymbol{B}\cdot\boldsymbol{\Omega}$$

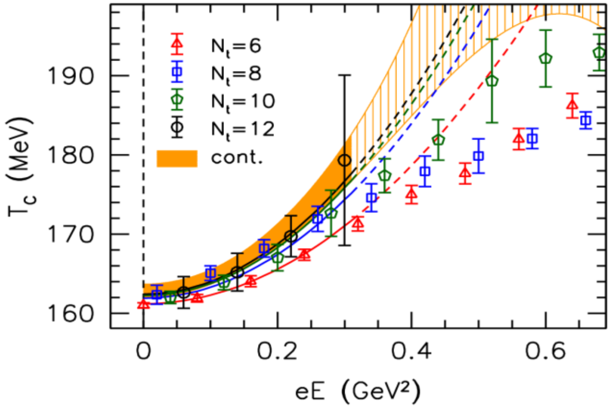
$$\sqrt{G}\dot{\boldsymbol{x}} = \boldsymbol{v} + \hbar\boldsymbol{E}\times\boldsymbol{\Omega} + \hbar(\boldsymbol{v}\cdot\boldsymbol{\Omega})\boldsymbol{B}$$

- Lattice QCD simulations

  Yamamoto (2013), Endrodi (2022)



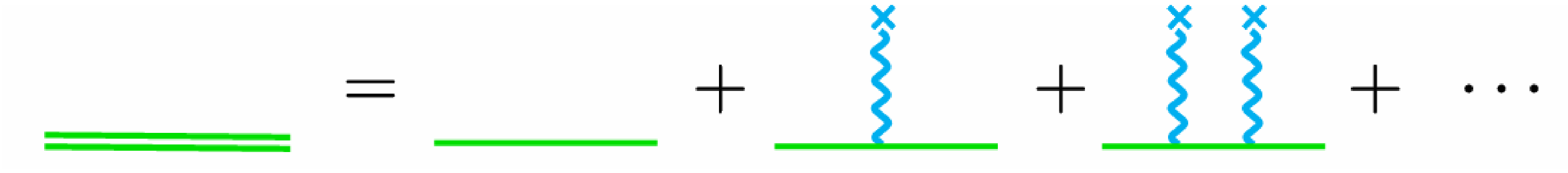
Tuchin (2013)



A magnetic field leaves the vacuum **stable** — static methods are enough. An electric field makes the vacuum **unstable**: the vacuum state evolves, and **time-dependent expectation values** are needed.

Introduction (Perturbative Treatment 1/3)

- Strong QED needs resummation



All diagrams contributes the same order when $m_e \leq \sqrt{eE}, \sqrt{eB}$

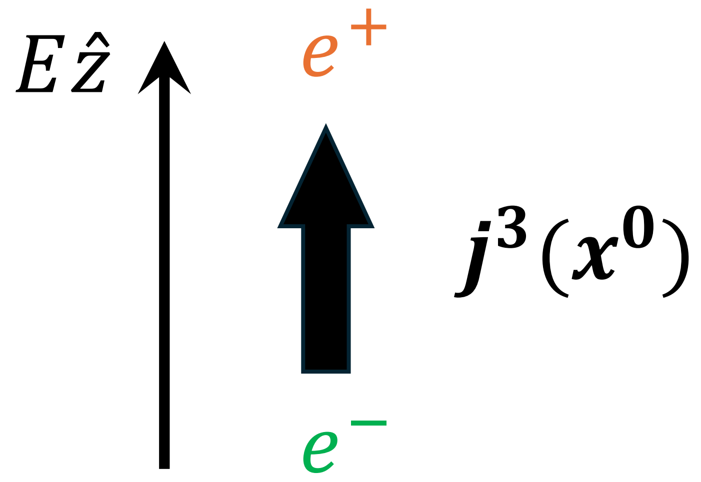
$$
\begin{aligned}
S(p|B,E) \;=\; & \int_0^\infty ds \Big[\frac{1-\gamma^1\gamma^2 \tan(q_f Bs)}{\cosh^2(q_f Es)} \not{p}_\parallel + \frac{1+\gamma^0\gamma^3 \tanh(q_f Es)}{\cos^2(q_f Bs)} \not{p}_\perp \\
& \qquad + m\big(1-\gamma^1\gamma^2 \tan(q_f Bs)\big)\big(1+\gamma^0\gamma^3 \tanh(q_f Es)\big) \Big] \\
& \qquad \times \exp\Big(-im^2 s + i\frac{p_\perp^2}{q_f B} \tan(q_f Bs) + i\frac{p_\parallel^2}{q_f E} \tanh(q_f Es) \Big)
\end{aligned}
$$

Introduction (Perturbative Treatment 2/3)

- Strong QED needs resummation

- Electric field requires real-time treatment = in-in formalism

e.g., Pair production induces the current



but $\langle out|j^3(x^0)|in\rangle = 0$

The in–out (S-matrix) formalism computes transition amplitudes — not the time-dependent expectation values. Real-time observables require the in–in formalism

- Every real-time observable induced by produced pair carries the Schwinger factor = nonperturbative:

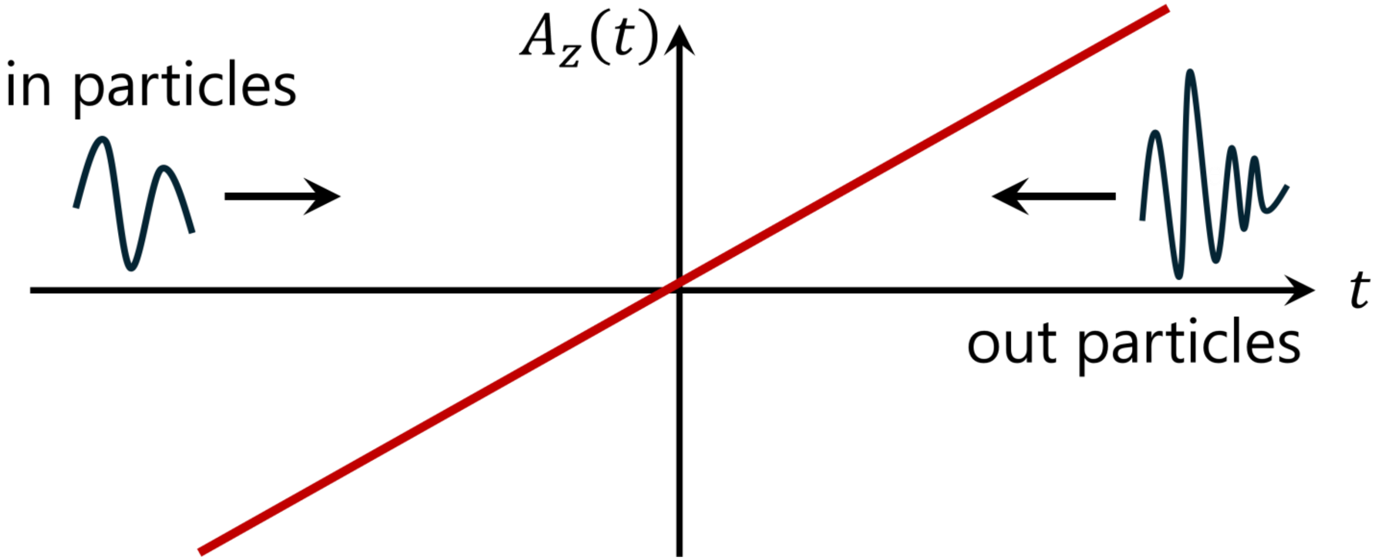
$$\langle in|j^3(x^0)|in\rangle = O(e^{-\pi m_e^2/eE})$$

- Essential singularity at $eE = 0$: every coefficients of the expansion vanishes

- So any finite-order loop expansion gives exactly zero

Goal of this talk: controlled **resumed in-in expansion** for observables of the Schwinger effect

Subtlety:



- The Hamiltonian is time-dependent: many inequivalent vacua
- Loop expansion needs a reference vacuum — for $H(t)$ there is no canonical choice

Around which vacuum should in-in perturbation theory be organized?

Fukushima-Minato (2026)

This work: expand in-in expectation value around the out-vacuum

1. Rewrite the in-vacuum in terms of the out vacuum (Bogoliubov transformation)
2. The mismatch of two vacua introduces a new quadratic terms in generating function on the closed time path
3. Quadratic → self energy→ resummed propagator (only propagators are modified; vertices are untouched)

Fukushima-Minato (2026)

Why expand around out vacuum?

In systems with an unstable vacuum, the reference vacuum for the loop expansion is not unique

1. An expansion around $|0_{\mathrm{in}}\rangle$ requires the infinite summation of diagrams
2. An expansion around $|0_{\mathrm{out}}\rangle$ **already resums** the effects of the background electromagnetic field

Intuitive understanding:

The observed quantities are written by out-particles defined on $|0_{\mathrm{out}}\rangle$, so the loop expansion around $|0_{\mathrm{out}}\rangle$ makes things simpler

Fukushima-Minato (2026)

- The in vacuum ー the out vacuum plus a condensate of out pairs

$$|0_{\rm in}\rangle = \mathcal{N}\hat{\mathscr{U}}_{\rm D}^{\dagger}|0_{\rm out}\rangle \qquad \hat{\mathscr{U}}_{\rm D}^{\dagger} := \exp\left[\sum_s \int_{\boldsymbol{p}} \sigma_{\boldsymbol{p}}^{*}\, \hat{a}_{{\rm out},\boldsymbol{p}}^{(s)\dagger}\, \hat{b}_{{\rm out},-\boldsymbol{p}}^{(s)\dagger}\right]$$

- The in-in expectation value can be computed around $|0_{out}\rangle$

$$\langle 0_{\rm in}|\hat{O}(x^0)|0_{\rm in}\rangle = |\mathcal{N}|^2 \langle 0_{\rm out}|\hat{\mathscr{U}}_{\rm D}\hat{O}(x^0)\hat{\mathscr{U}}_{\rm D}^{\dagger}|0_{\rm out}\rangle$$

- Exactly keeping $\hat{\mathscr{U}}$

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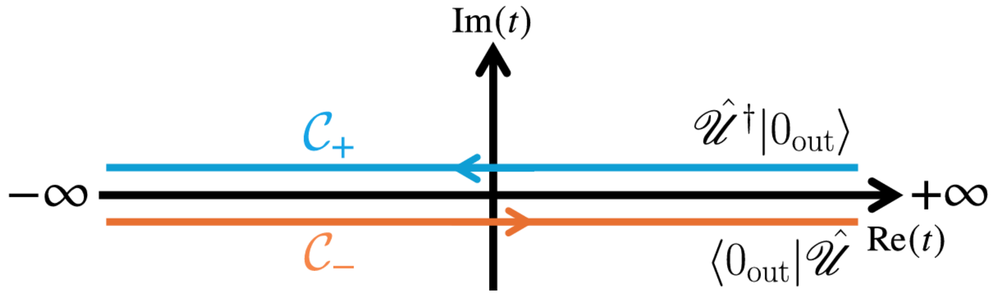
- On the closed time path, the squeeze factor $\hat{\mathscr{U}}$ becomes a quadratic kernel localized at the late-time boundary; a **self-energy**-like insertion Σ_D in **generating functional**.



$$\hat{\mathscr{U}}_{\mathrm{D}}^\dagger := \exp\left[\sum_s \int_{\boldsymbol{p}} \sigma_{\boldsymbol{p}}^*\, \hat{a}_{\text{out},\boldsymbol{p}}^{(s)\dagger}\, \hat{b}_{\text{out},-\boldsymbol{p}}^{(s)\dagger}\right]$$

$$\langle 0_{\text{out}}|\hat{\mathscr{U}}_{\mathrm{D}}\, \hat{O}(x^0)\, \hat{\mathscr{U}}_{\mathrm{D}}^\dagger|0_{\text{out}}\rangle \longrightarrow \Psi_{\mathrm{D}}(\psi_+(+\infty)) = \langle \psi_+(+\infty), \bar{\psi}_+(+\infty); +\infty|\hat{\mathscr{U}}_{\mathrm{D}}^\dagger|0;\text{out}\rangle$$

Functional quantization

Quadratic kernel

$$\longrightarrow \bar{\Sigma}_{\mathrm{D}}(x,y) := \sum_s \int_{\boldsymbol{p}} \sigma_{\boldsymbol{p}}^*\, \gamma^0\, U_{\boldsymbol{p}s}^{\text{out}}(x)\, V_{\boldsymbol{p}s}^{\text{out}}(y)^\dagger\, \delta(x^0-\infty)\, \delta(y^0-\infty)$$

Self energy

Bogoliubov coefficient

Mode functions at out vacuum

- Vertices are untouched: only propagators are reorganized

Fukushima-Minato (2026)

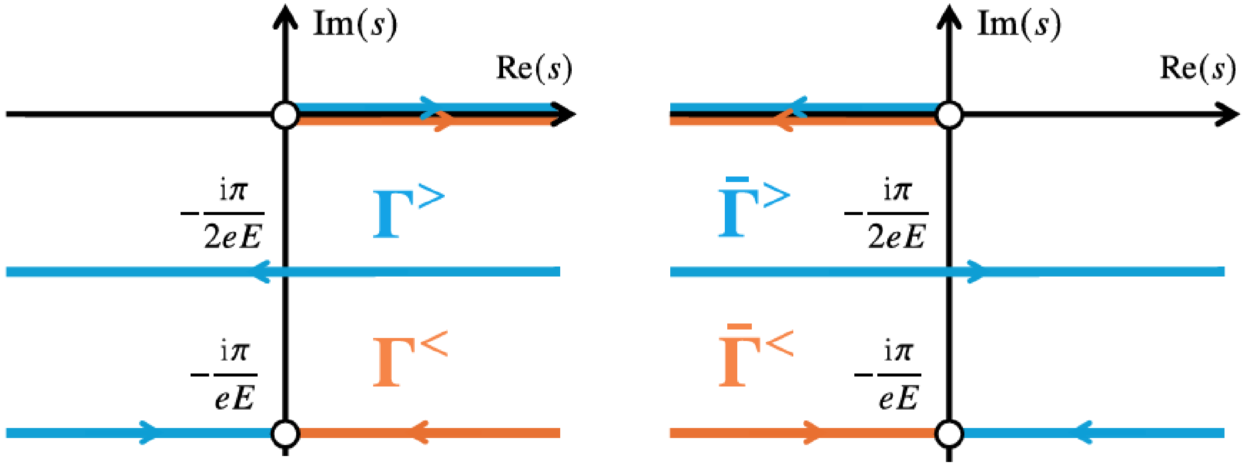
- The Dyson series of a quadratic boundary insertion is geometric — it closes exactly and reorganize the diagonal elements:

$$
\begin{aligned}
S^{--}(x,y)^{-1} &= S_0^{--}(x,y)^{-1} + \Sigma_{\mathrm{D}}(x,y)\,, \\
S^{++}(x,y)^{-1} &= S_0^{++}(x,y)^{-1} + \bar{\Sigma}_{\mathrm{D}}(x,y)\,.
\end{aligned}
$$

- Off-diagonal propagators are also reorganized, but technically much more involved (see paper)

Fukushima-Minato (2026)

- These propagators have the proper time representations. But with additional contours.
  Fradkin-Gitman-Shvartsman (1991)



$$S^{--}(x,y) = \mathrm{i}(\mathrm{i}\not{D}_x + m)\left[\theta(z_3)\int_{\Gamma^{>}} \mathrm{d}s\, g(s;x,y) + \theta(-z_3)\int_{\Gamma^{<}} \mathrm{d}s\, g(s;x,y)\right]$$

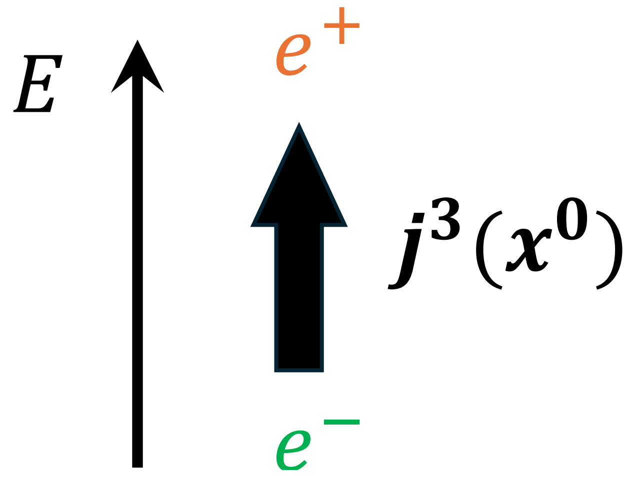
$$g(s;x,y) := \langle x|\mathrm{e}^{\mathrm{i}(-\not{D}^2 - m^2 + \mathrm{i}\epsilon)s}|y\rangle$$

- Additional integral contour comes from the nonlocal self-energy from boundary wavefunctionals
- Resummation introduces a new pole in the proper-time contour, whose residue carries the Schwinger factor

Fukushima-Minato (2026)

Example calculation:

This propagator correctly includes the pair-production effect



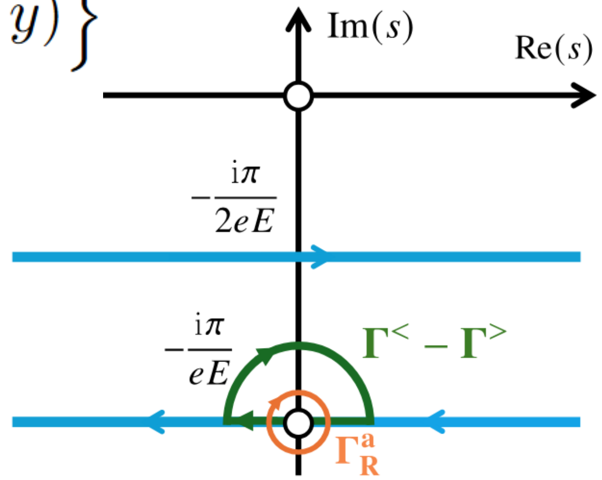
$$j^3(x^0) = \bar{\psi}\gamma^3\psi$$

$$\langle j^3(x^0)\rangle = \lim_{x\to y} \mathrm{Tr}_{\mathrm{s}}\left[\gamma^3 S^{--}(x,y)\right]$$

$$= -4\lim_{x\to y} D_x^3\left\{\theta(-z_3)\theta^+(z_3^2 - z_0^2)\int_{\Gamma_{\mathrm{R}}^{\mathrm{a}}} \mathrm{d}s\,\cosh(eEs)\,f(s;x,y)\right\}$$

$$= \Lambda\frac{eE\,x^0}{4\pi^3}\mathrm{e}^{-\pi m^2/(eE)}$$

Pair created current



See also Copinger-Fukushima-Pu (2018)

Limitations and Applicability

Assumptions:

- Spatially homogeneous background
- in/out vacua are well defined, and a complete set of mode functions is available

Applicability(under the above limitations):

- time dependent electric field adiabatically turned-off
- magnetic and electric background
- Non-abelian field

Conclusion

- A loop expansion depends on the choice of reference vacuum
- In an electric field, the vacuum instability makes the choice of reference vacuum important
- We recast the in-in generating functional as a loop expansion around the out vacuum, which builds pair-production effects into propagators.
- Applications are left for future work.