arXiv:2603.22394

Kinetic Isocurvature Perturbation

A New Class of Primordial Fluctuations in Dark Matter

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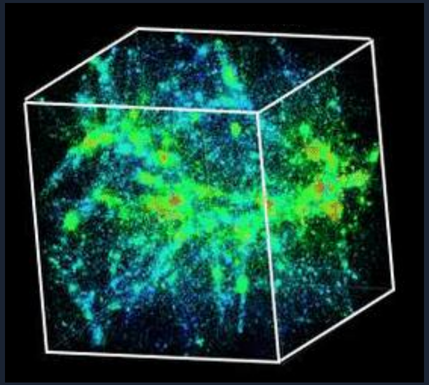
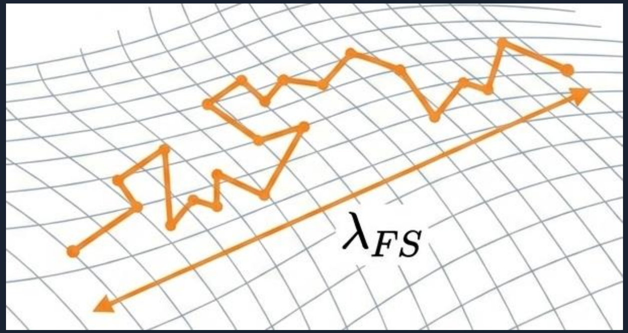
A question for the next generation of surveys

As small-scale observations improve — Lyman-α forests, strong lensing, galaxy surveys — we will measure quantities like the dark matter free-streaming scale λ_{FS} ($< O(\text{Mpc})$) with increasing precision.

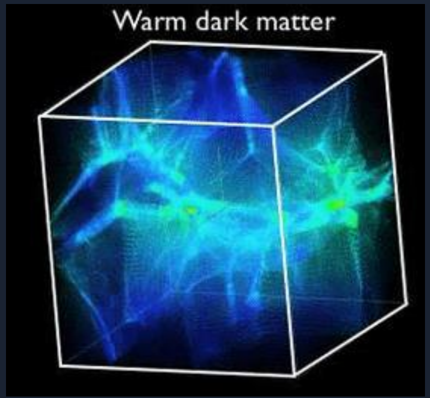
Could λ_{FS} measured in one patch of the sky differ from λ_{FS} in a distant patch?

Is there a large-scale spatial modulation of small-scale structure?





$$\lambda_{FS} = \int_0^{t_{eq}} dt \frac{v(t)}{a(t)}$$



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NAIVE EXPECTATION

Large-scale ($\gg$ Mpc) density perturbations are tiny: $\delta\rho/\rho \sim 10^{-5}$. In the standard picture, distant patches are not expected to carry systematically different free-streaming cutoffs.

But is this really the only possibility? Can we find a mechanism that allows O(1) differences?

The barrier — and the way around it

THE BARRIER

For λ_{FS} to differ across patches, a large-scale isocurvature perturbation must exist.

But standard DM isocurvature

$$\delta\left(\frac{\rho_{DM}}{s}\right)$$

is constrained by CMB to sub-sub-percent — forbidding significant variation.

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THE BREAKTHROUGH

What if the perturbation lives in DM kinetic energy, not number density?

As DM cools, $\delta\rho_{DM}$ from kinetic energy redshifts away — CMB sees nothing. But λ_{FS} modulation survives, allowing $O(1)$ patch-to-patch differences.

This opens the door to a new class of observables
— the long-range correlation of the small-scale power spectrum.

The paradigm shift: kinetic isocurvature perturbations

Instead of fluctuating particle number, let the momentum distribution vary from patch to patch.

The mass density stays constant — only the kinetic energy fluctuates.

STANDARD ISOCURVATURE

Patches A and B have different numbers of DM particles.

$$\delta n_{DM} \neq 0$$

→ **Highly constrained by Planck CMB**

KINETIC ISOCURVATURE · THIS WORK

Patches A and B have the same number but different momenta.

$$\delta n_{DM} = 0, \ \delta p_{DM} \neq 0$$

→ **Evades CMB bounds automatically**

DM can carry a qualitatively distinct isocurvature mode sourced entirely by initial kinetic-energy fluctuations.

Anatomy of the energy-density perturbation

The perturbation of the DM energy density $\rho_{DM} = E_{DM} n_{DM}$ $\left(E_{DM} = \sqrt{m_{DM}^2 + p_{DM}^2}\right)$ decomposes into two independent terms.

$$\frac{\delta \rho_{DM}}{\rho_{DM}} = \frac{\delta n_{DM}}{n_{DM}} + \frac{p_{DM}^2}{E_{DM}^2} \frac{\delta p_{DM}}{p_{DM}}$$

STANDARD TERM

Fluctuation in comoving number density. Set to zero in this framework (adiabatic).

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KINETIC TERM

Weighted by p^2/E^2. Dominant when DM is relativistic; vanishes as DM cools.

In warm-DM scenarios this momentum term becomes cosmologically significant.

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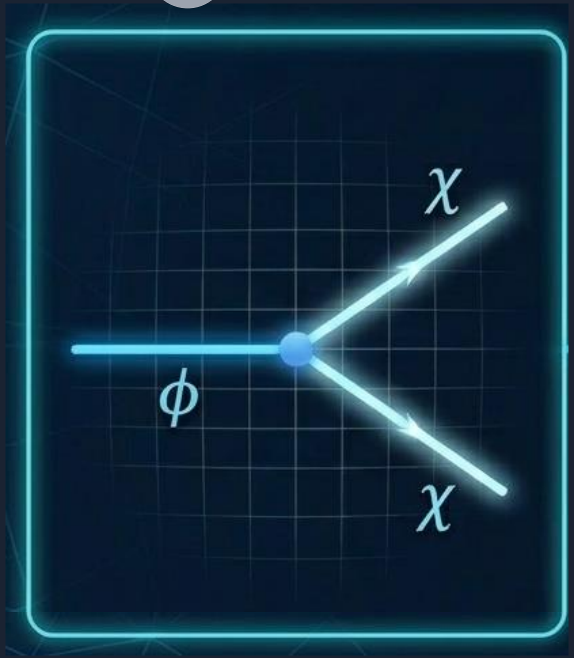
A remarkably simple realization (modulated decay)

Exotic new observables often require exotic models — but not here. A subdominant scalar ϕ decays into light DM χ through a coupling modulated by a field σ. This minimal setup naturally delivers $\delta n_\chi = 0$ with $\delta p_\chi \neq 0$.

1 Source

$\mathcal{L} = A\,\phi\chi^2$
(χ: boson)

$\mathcal{L} = y\,\phi\bar{\chi}\chi$
(χ: fermion)



Heavy scalar ϕ
(subdominant, nonrelativistic)

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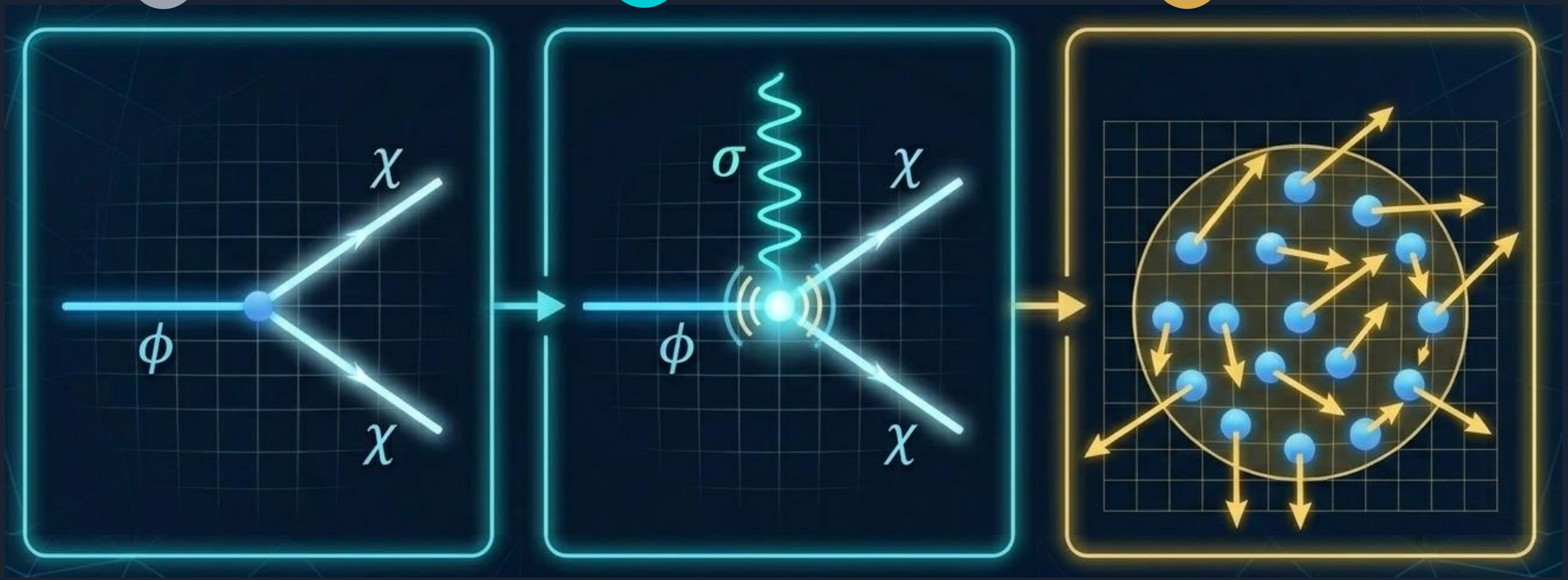

Heavy scalar ϕ
(subdominant, nonrelativistic)

Field σ modulates the coupling constant so that

$$\Gamma \to \Gamma + \delta\Gamma$$

χ produced relativistic with patch-dependent momentum.

$\delta n_\chi = 0, \qquad \delta p_\chi \neq 0$

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The redshift mechanism and effects on small scales

$T(a_i) \gg \text{keV}$

STAGE 1 — RELATIVISTIC

$$p_i = p_\chi(a_i) \simeq \frac{m_\phi}{2} \gg m_\chi$$

a_i is determined by Γ (ϕ decay rate)

$$1\phi \to 2\chi \;\Rightarrow\; N_\chi = 2N_\phi$$

$T(a_*) \sim \text{keV}$

STAGE 2 — EXPANSION/WARM

$$p_\chi(a_*) = p_i \left(\frac{a_i}{a_*}\right) \sim m_\chi$$

Momentum redshifts with the scale factor, and becomes warm

$T(a_f) \sim \text{eV}$

STAGE 3 — COLD

$$p_\chi(a_f) \ll m_\chi$$

When CMB scales enter , kinetic perturbation of DM is negligible

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When CMB scales enter , kinetic perturbation of DM is negligible

While the energy-density contrast vanishes at late times, the momentum of DM around a_* dictates how far each DM particle travels — leaving a surviving spatial variation of the free-streaming scale

$$\frac{\delta\Gamma}{\Gamma} \Rightarrow \frac{\delta a_i}{a_i} \Rightarrow \frac{\delta p_\chi(a_*)}{p_\chi(a_*)} \Rightarrow \frac{\delta\lambda_{FS}}{\lambda_{FS}}, \qquad \frac{\delta\rho_\chi(a_f)}{\rho_\chi(a_f)} < 10^{-5}$$

Generous on CMB scales, sensitive on small scales — the fingerprint of kinetic isocurvature.

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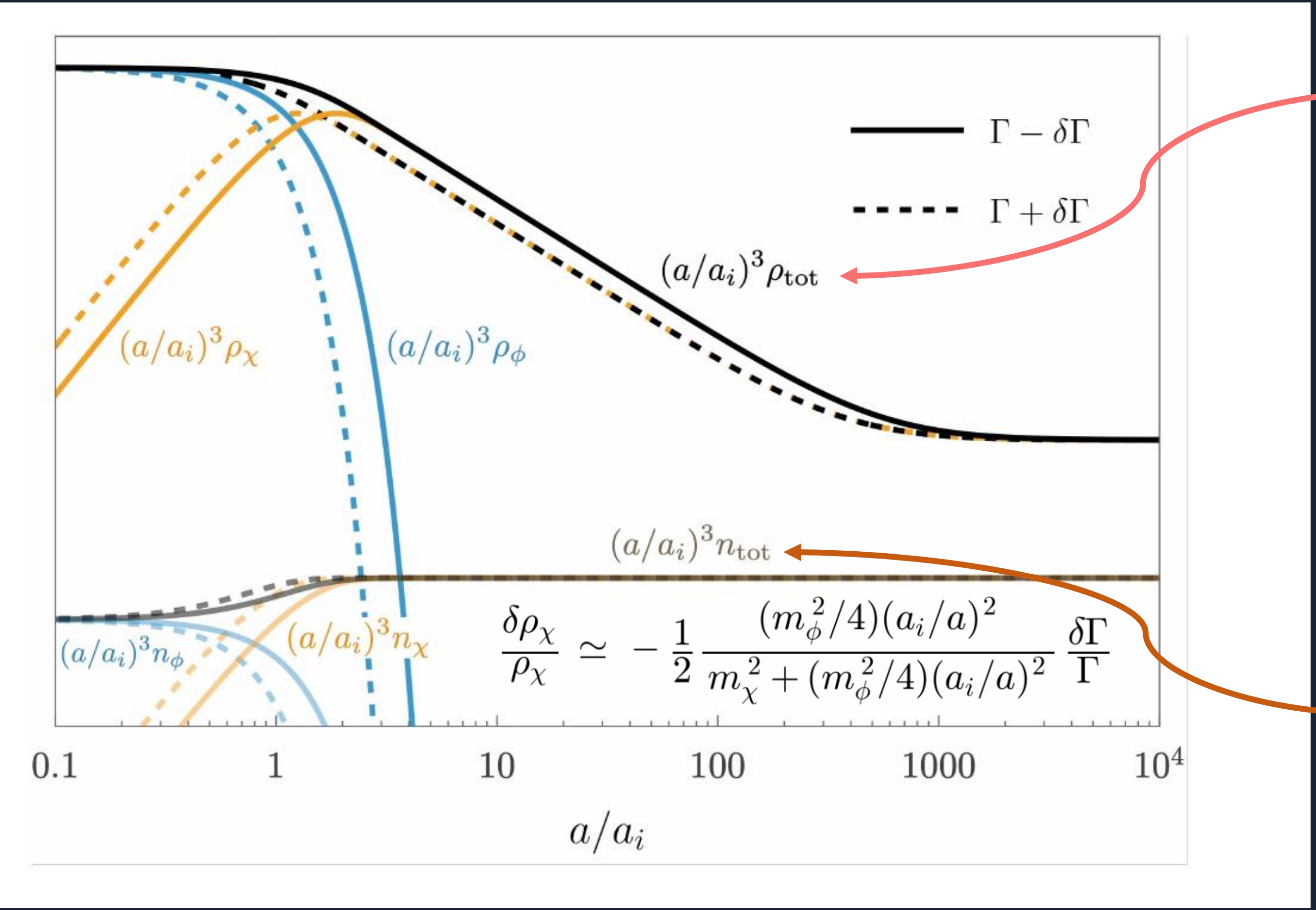
Cosmological evolution: Boltzmann equation analysis



Energy density fluctuates early on — $\delta\Gamma$ leaves a visible gap between curves.

The construction delivers exactly what is needed: number density adiabatic, kinetic energy modulated.

Comoving number density after production converges adiabatically — independent of decay timing.

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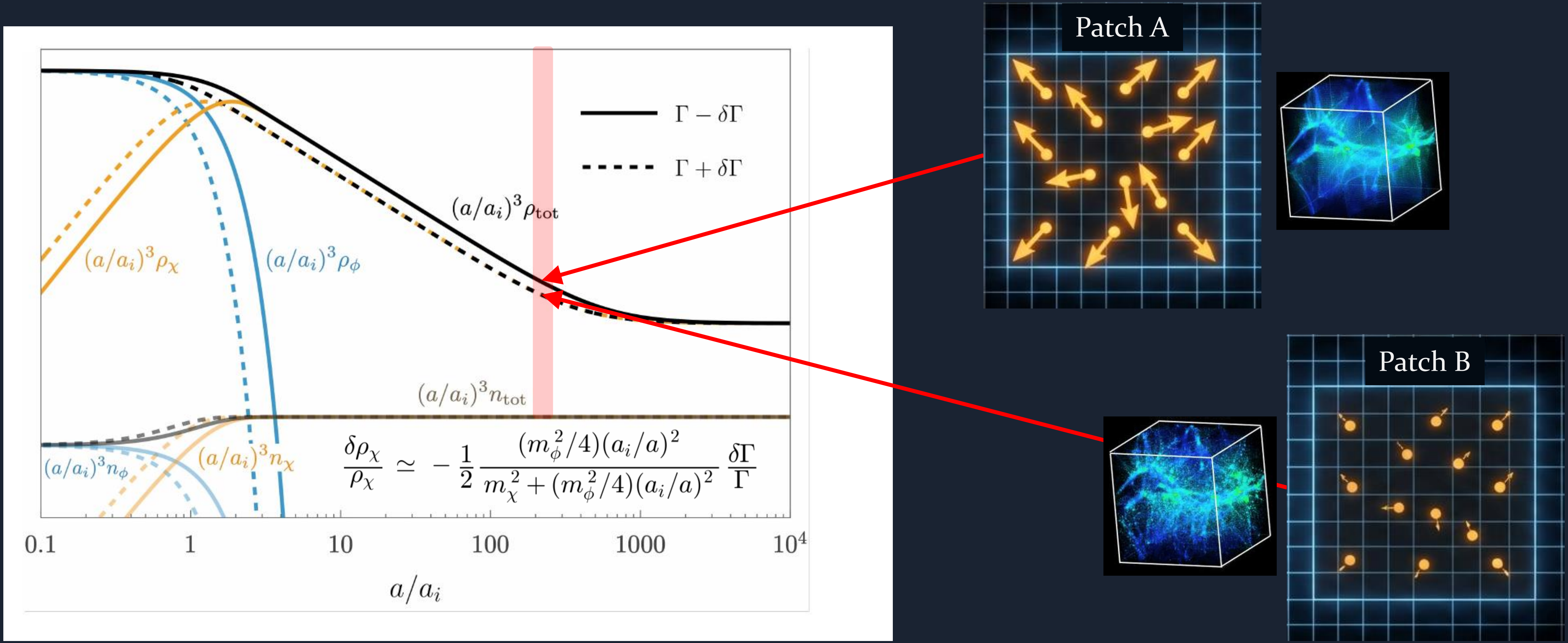
Small-scale suppression varies from patch to patch



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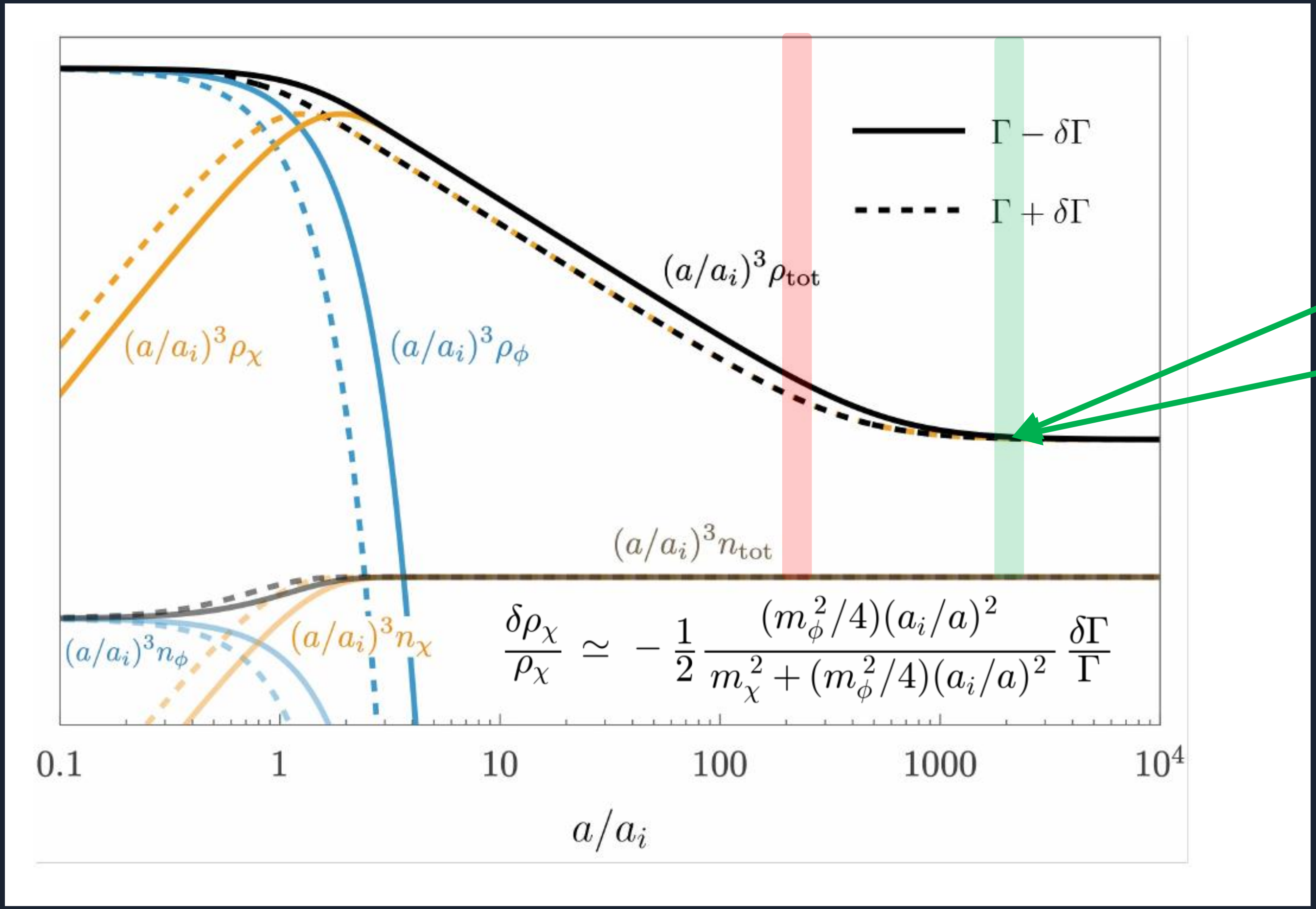
The density contrast fades on CMB scales



$\delta\Gamma$ initially produces a sizable $\delta\rho_\chi/\rho_\chi$. But as the universe expands, the kinetic weight $(a_i/a)^2$ shrinks. By the time CMB-scale modes enter the horizon, $\delta\rho_\chi/\rho_\chi$ is negligible.

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The surviving imprint

Kinetic energy history inscribed in the free-streaming scale

INITIAL STATE (fades)

$$\delta p_{DM}$$

Large initial momentum fluctuation.
Washed out by redshift.

CUMULATIVE INTEGRAL (transforms)

$$\lambda_{FS} = \int \frac{dt}{a(t)} \frac{p_{DM}(a)}{E_{DM}(a)}$$

Kinetic energy vanishes, but the comoving distance traveled is accumulated.

PERMANENT IMPRINT (preserved)

$$\frac{\delta \lambda_{FS}}{\lambda_{FS}}$$

Spatial fluctuation of λ_{FS}.
Permanently imprinted as the small-scale cutoff.

The kinetic energy is washed away by cosmic expansion — but λ_{FS}, set by the initial dispersion, remains different from patch to patch.

The initial δp_{DM} is permanently transformed into $\delta \lambda_{FS}$.

Free-streaming length and Isocurvature perturbation

$$\lambda_{FS} \simeq 1\ \mathrm{Mpc}\left(\frac{p_i^2}{m_\chi^2}\frac{t_i}{10^6\ \mathrm{sec}}\right)\left[1 - 0.07\ln\left(\frac{p_i^2}{m_\chi^2}\frac{t_i}{10^6\ \mathrm{sec}}\right)\right]$$

$$\delta_\lambda = \frac{\delta\lambda_{FS}}{\lambda_{FS}} = -\frac{1}{2}\frac{\delta\Gamma}{\Gamma}\left(1 + \frac{2K_\chi}{\ln\left(\frac{1+K_\chi}{1-K_\chi}\right)}\right)$$

$$K_\chi = \frac{m_\chi}{\sqrt{m_\chi^2 + p_\chi^2(a_{eq})}} \simeq 1$$

$$\delta_\chi = \left(\frac{\delta\rho_\chi}{\rho_\chi}\right)_k \simeq \frac{\delta_\lambda}{1 + \frac{1.56}{\sqrt{g_*}}\left[1 + 0.27\ln\left(\frac{\mathrm{Mpc}}{\lambda_{FS}}\right)\right]\left(\frac{\mathrm{Mpc}}{\lambda_{FS}}\right)^2\left(\frac{6.7\ \mathrm{Mpc}^{-1}}{k}\right)^2}$$

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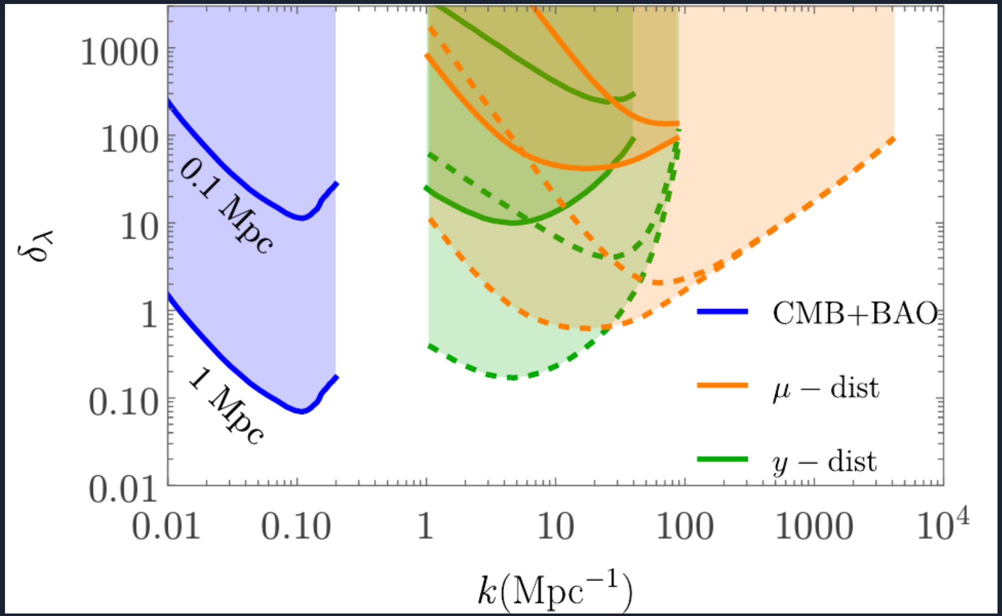
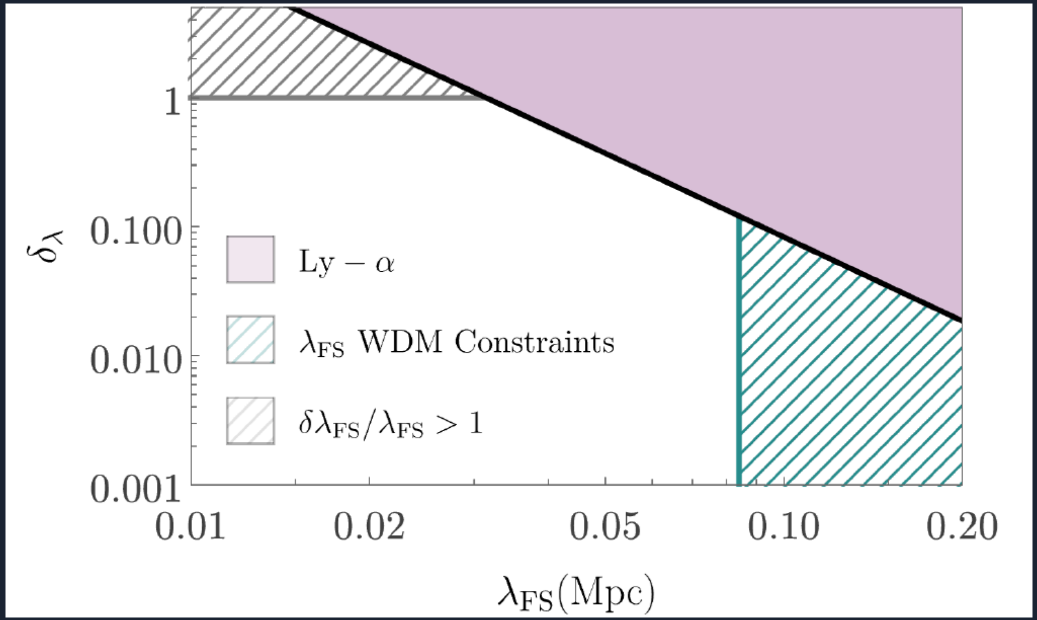
Isocurvature constraints from CMB+BAO and Lyman-α

$$P_{\delta_\lambda}^{(\text{delta})} = A_{\delta_\lambda}\delta(\ln(k/k_0))$$



Recasting neutrino density isocurvature (NDI) bounds (arXiv:2502.20434) gives conservative estimates. Low $k \to$ CMB+BAO; high $k \to$ spectral distortions.

$$P_{\delta_\lambda}^{(\text{flat})} = A_{\delta_\lambda}$$



For a flat δ_λ spectrum the dominant constraint comes from Lyman-α at $k \sim 1$ Mpc^{-1}. Still, $\delta_\lambda = O(0.1\text{–}1)$ survives over the warmness-allowed range.

$$P_{\delta_\chi}(k) \simeq g_*(0.1\,\lambda_{FS}k)^4 P_{\delta_\lambda}(k) \quad \text{for} \quad \lambda_{FS}k \ll 1\,, \qquad P_{\delta_\chi}(k) = P_{\delta_\lambda}(k) \quad \text{for} \quad \lambda_{FS}k > O(1)$$

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Isocurvature constraints from CMB+BAO and Lyman-α

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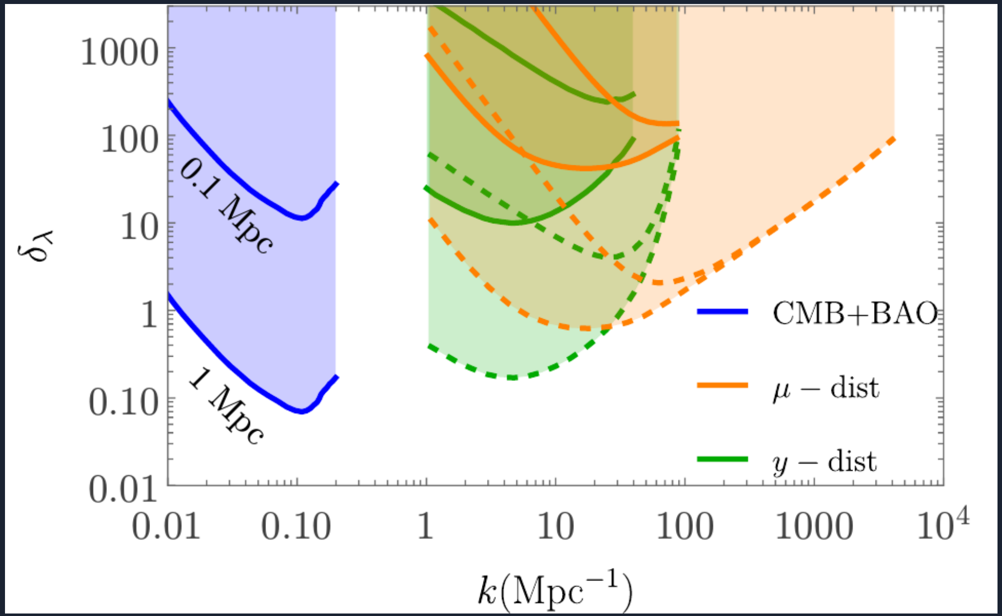

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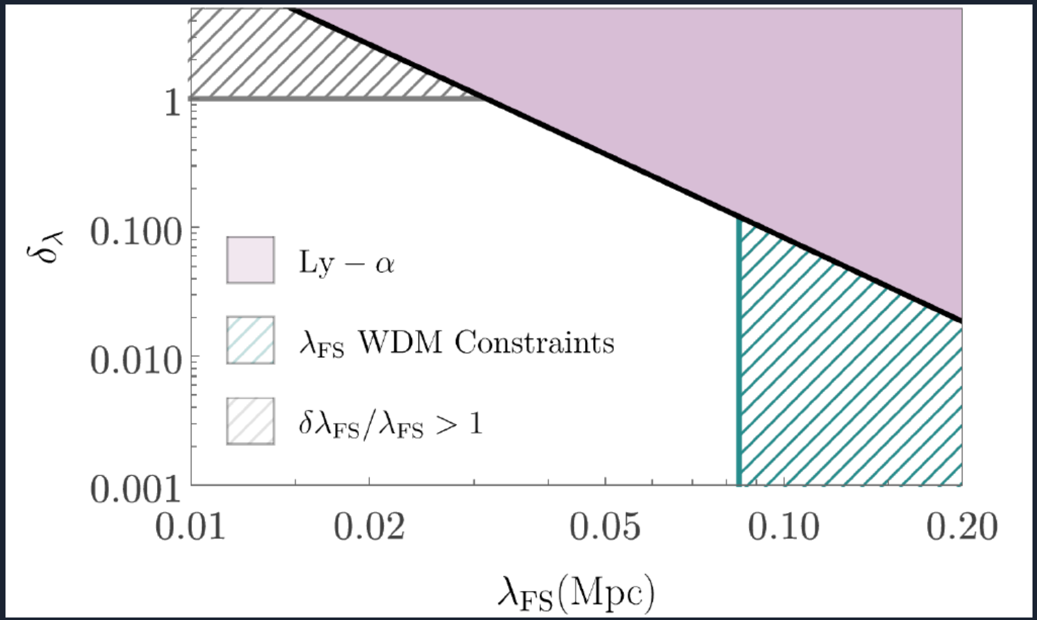

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Bottom line: $\boldsymbol{\delta_\lambda \sim \mathcal{O}(0.1 - 1)}$ ***survives all current bounds.***

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Comparisons

Model	δn_{DM}	δp_{DM}	$\delta\lambda_{FS}$	CMB safe?
Thermal WDM	Adiabatic	Adiabatic	0	Yes (no fluctuation)
Field-dependent WDM (e.g. axion)	$\neq 0$	$\neq 0$	$\neq 0$	No (violated)
Kinetic Isocurvature	$= 0$	$\neq 0$	$\neq 0$	**Yes**

The only mechanism that independently generates free-streaming modulation while preserving number density.

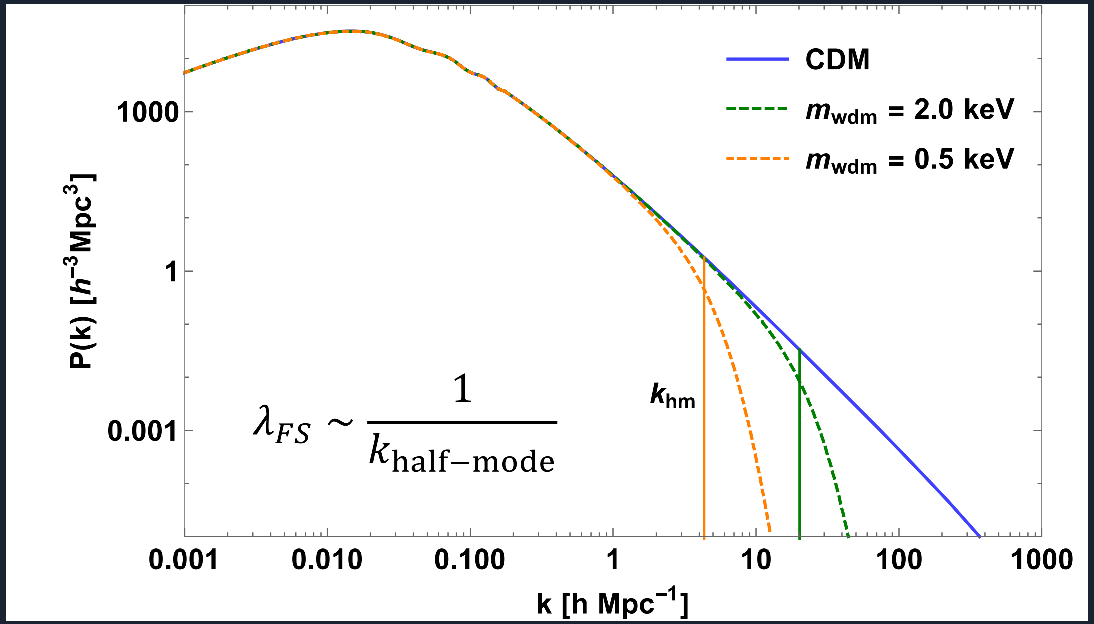
Matter power spectrum with a spatially modulated small-scale cut-off

Warmness of dark matter is encoded in the linear matter power spectrum $P^{\text{lin}}_{\text{WDM}}(k)$, through a suppression of power below the free-streaming scale λ_{FS}. This suppression can be described by the transfer function

$$T_{\text{FS}}(k, \lambda_{FS}) = \left[\frac{P^{\text{lin}}_{\text{WDM}}(k, \lambda_{FS})}{P^{\text{lin}}_{\text{CDM}}(k)}\right]^{1/2}$$

If λ_{FS} is modulated on large scales, the cut-off scale also becomes position dependent: spatially varying cut-off

$$\lambda_{FS} \to \lambda_{FS}(\boldsymbol{x})$$



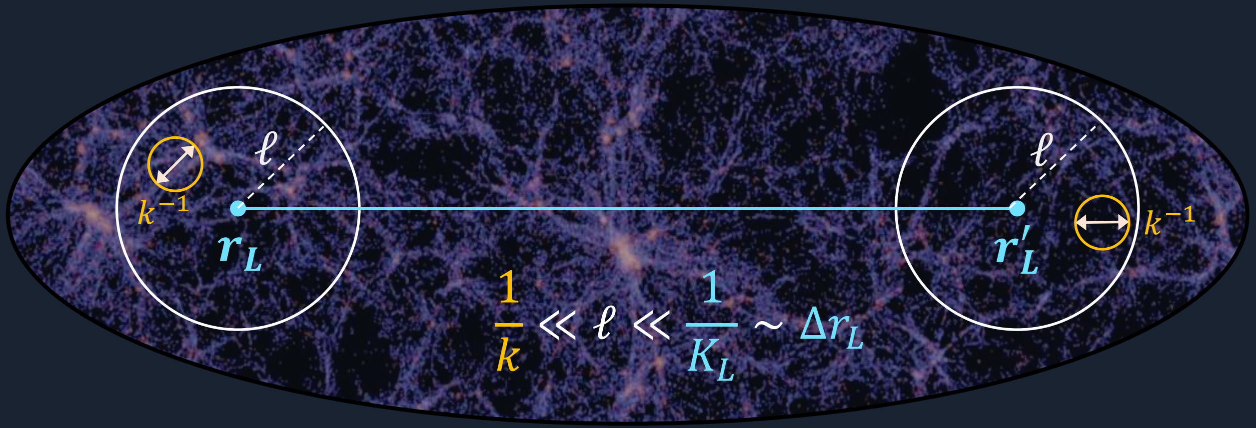
e.g. Matter power spectrum for different DM warmness
MNRAS 481, 1290-1299(2018)

Large-scale modulation of small-scale structure formation

arXiv:2603.22394

The new observable we propose

If $\delta_\lambda \sim O(0.1\text{–}1)$ is allowed, how do we actually detect it? By measuring the small-scale power spectrum in separate patches and asking whether they are correlated over long distances — effectively a **4-point function of the matter density field**.



Response of P_m to δ_λ

$$\Delta P_m(k, \lambda_{FS}(r_L)) \simeq \lambda_{FS} \frac{dP_m}{d\lambda_{FS}} \delta_\lambda(r_L)$$

Patch-to-Patch Correlation = 4-point function

$$P_{PP}(k, K_L) = \lambda_{FS}^2 \left(\frac{dP_m}{d\lambda_{FS}}\right)^2 P_{\delta_\lambda}(K_L)$$

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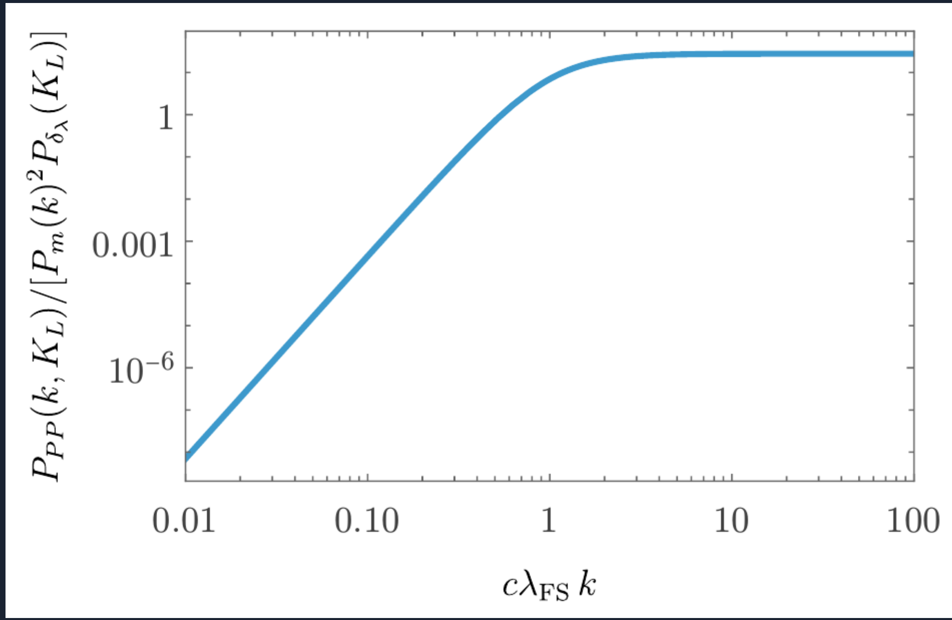
Main result: the 4-point correlation signal

Using a standard WDM transfer function:

$$T_{\rm FS}(k, \lambda_{FS}) \simeq \left(1 + (c\lambda_{FS}k)^{\beta}\right)^{\gamma}, \qquad \beta = 2.4, \gamma = -1.1$$

At the characteristic scale $k_{FS} = 1/(c\,\lambda_{FS})$, the patch-to-patch ratio simplifies to:

$$\frac{P_{PP}(k_{FS}, K_L)}{P_m(k_{FS})^2} \simeq 10\, P_{\delta_\lambda}(K_L)$$



An O(0.1-1) signal — potentially detectable

This is the central quantitative prediction of this work: kinetic isocurvature opens a detectable, O(0.1-1)-level signal in a new class of long-range correlations — providing a concrete observational target for future surveys.

Summary and outlook

01 · FRAMEWORK

A new paradigm

A distinct class of primordial fluctuations driven by momentum. $\delta p \neq 0$, $\delta n = 0$.

02 · MECHANISM

The natural detour

The kinetic contribution to $\delta\rho$ redshifts away, automatically evading standard CMB bounds.

03 · SIGNAL

The observable

Initial momentum dictates λ_{FS}, leaving an O(0.1-1) imprint as patch-to-patch variation in the small-scale power-spectrum.

OPEN FRONTIER

Galaxy surveys and Lyman-α forests offer a direct path to probing DM microphysics through a new class of 4-point statistics at $K_L < 0.01 - 0.1\ Mpc^{-1}$.