Higgs Mass Is Natural

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The Hierarchy Problem

- Higgs mass measured at LHC (ATLAS, CMS 2012):

$$m_h \approx 125 \text{ GeV}$$

- Standard Model expected to be completed at $M_{\text{GUT}} \sim 10^{16}$ GeV

$$\frac{m_h^2}{M_{\text{GUT}}^2} \sim 10^{-28}$$

- One-loop correction from a field of mass M_{GUT}:

$$\delta m_h^2 \sim g^2 M_{\text{GUT}}^2$$

- This overwhelms the observed mass — requires a cancellation to 1 part in 10^{28}
- **Why is the Higgs so light compared to the GUT scale?**

Formulation

- One-loop self-energy integrals diverge $\delta m_h^2 \sim \int^{\Lambda} d^4k \frac{1}{k^2}$:

$$\delta m_f \propto m_f \log \Lambda^2 \qquad \text{fermion}$$
$$\delta m_h^2 \propto \Lambda^2 \qquad \text{scalar}$$

- Λ is **not physical** — it is merely the upper limit of the loop momentum integral
- Renormalization absorbs Λ-dependent terms; all observables are Λ-independent
- Old “hierarchy problem” appears as an artifact of this identification

- Modern formulation: a new physics
- e.g. a UV field with mass M coupling to the Higgs mass.
- $\frac{1}{2}M^2X^2 + \frac{1}{4}\kappa h^2X^2$.

’t Hooft’s Symmetry Argument

- ’t Hooft (1980): a mass parameter is *natural* if setting it to zero increases the symmetry
- **Fermion:** $m_f \to 0$ restores chiral symmetry $\Rightarrow$ protected

$$\delta m_f \propto m_f \log M^2$$

- **Scalar:** $m_h \to 0$ restores no symmetry $\Rightarrow$ unprotected

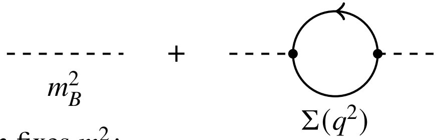
$$\delta m_h^2 \propto M^2 (?)$$

  - M is the mass of the heavy field in the loop
- Traditional conclusion: a light Higgs requires a new symmetry to stabilize it
  - Formally the same as Λ^2, and the mass is super-renormalizable.
  - This is naive dimensional analysis. **Precise calculation required.**

Higgs Self-Energy

- The mass is defined as the parameter m_B^2 in the Lagrangian, but not observable
- The **observable is the combination** of the bare mass plus the self-energy:

$$m^2(q^2) = m_B^2 + \Sigma(q^2) + (\text{field strength renormalization})$$



- The pole mass condition fixes m_B^2:

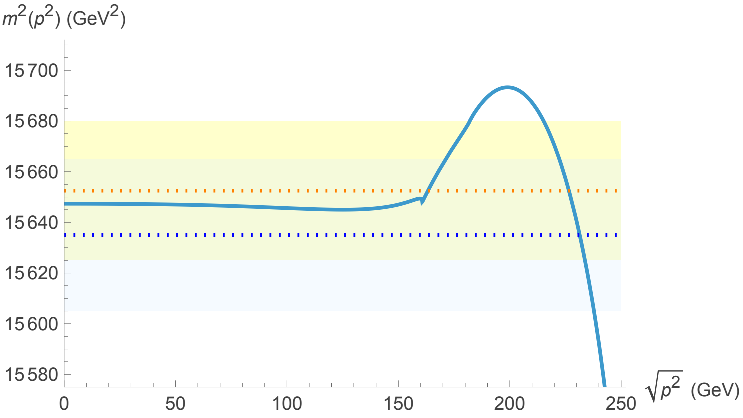
$$m_h^2 = m_B^2 + \Sigma(m_h^2) \quad \Longrightarrow \quad m_B^2 = m_h^2 - \Sigma(m_h^2)$$

- Substituting, the bare mass drops out:

$$m^2(q^2) = m_h^2 + \Sigma(q^2) - \Sigma(m_h^2) - (q^2 - m_h^2)\frac{d\Sigma}{dq^2}(m_h^2)$$

One-loop correction within the Standard Model

- Off-shell Higgs $m^2(q^2)$ away from the pole [KSC 2506.18667].
- Dominant contributions $W \approx Z > t > h$.



The running mass $m^2(q^2)$

- On-shell renormalization defines a mass at every external momentum q:

$$m^2(q^2) = m_h^2 + \Sigma(q^2) - \Sigma(m_h^2) - (q^2 - m_h^2)\frac{d\Sigma}{dq^2}(m_h^2) \equiv m_h^2 + \Sigma_{\text{ren}}(q^2)$$

- Finite up to $\mathcal{O}\left(\frac{(p^2-m_h^2)^2}{\Lambda^2}\right)$
  - $m^2(q^2)$ is the **difference of the same function** at two arguments q^2 and m_h^2
  - The **divergent** parts of Σ cancels automatically — no miracle
  - Natural interpretation: $m^2(q^2)$ is the RG running of the mass from the pole m_h^2 up to the probe scale q^2, or $m_h^2 \to \Lambda^2 \to q^2$. cancellation
- Analogous to the running coupling $\alpha_s(q^2)$ — physically measurable at every scale
- The pole mass m_h is a fixed number; the *function* $m^2(q^2)$ runs

Regularization Independence

- The one-loop fermion self-energy, after Feynman parameterization:

$$\int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - \Delta(q^2)}$$

with $\Delta(q^2) \equiv M_f^2 - x(1-x)q^2$.

- **Cutoff regularization** ($k_E < \Lambda$):

$$\frac{i}{16\pi^2} \int_0^1 dx \left[\Lambda^2 - \Delta(q^2) \log \frac{\Lambda^2}{\Delta(q^2)}\right] + \mathcal{O}\left(\frac{(p^2-m_h^2)^2}{\Lambda^2}\right)$$

- **Dimensional regularization** ($d = 4 - \epsilon$):

$$\frac{i}{16\pi^2} \int_0^1 dx \left[-\frac{2}{\epsilon} + \gamma_E - \log 4\pi + \log \frac{\Delta(q^2)}{\mu^2} + \mathcal{O}(\epsilon)\right] \Delta(q^2)$$

- The divergent pieces (Λ^2, $1/\epsilon$) differ between schemes

- **Statement:** After applying the on-shell subtraction, the renormalized mass $m^2(q^2)$ is finite [BPHZ].

Decoupling

A careful calculation of $m^2(q^2)$

- A heavy field with $M^2 \gg q^2$ contributes:

$$\Sigma^{\text{heavy}}_{\text{ren}}(q^2) = \mathcal{O}\left(\frac{(q^2 - m_h^2)^2}{M^2}\right)$$

- *Not* M^2 — extends the Appelquist–Carazzone theorem to super-renormalizable masses
- Expressed entirely in terms of physical pole masses; scheme-independent
- q^2 is the scale at which the mass is probed — a physical external
- At $q^2 \sim m_h^2$, **we cannot see UV physics.**

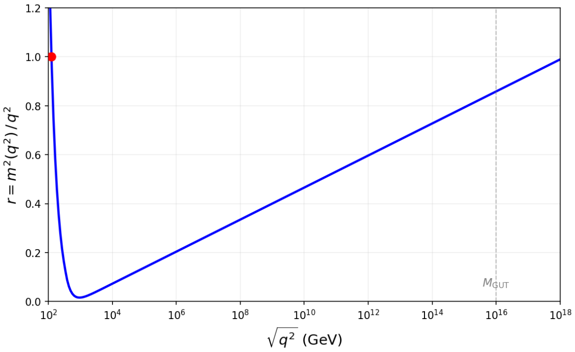
Power-law running

- The renormalized self-energy yields the same scaling:

$$\Sigma_{\text{ren}} \propto m_f \log q^2 \qquad \text{fermion}$$
$$\Sigma_{\text{ren}} \propto q^2 \log q^2 \qquad \text{scalar}$$

- These are the same equations as 't Hooft's — but the interpretation is different
- 't Hooft: corrections estimated *at the UV scale* M_{GUT}, with M treated as the source
- This work: the equations hold **at every energy scale**; q^2 is not a cutoff but the physical probe scale
- **No symmetry protection gives rise to the power-law running.**

The Running Ratio $r = m^2(q^2)/q^2$



- Red dot: measured pole mass $m_h = 125$ GeV
- Dashed line: $M_{\text{GUT}} \sim 10^{16}$ GeV
- $m^2(q^2)/q^2$ is $\mathcal{O}(1)$ at every scale
- **Prediction:** $r \to 1$ near 10^{18} GeV: upper bound on the SM's perturbative domain

$\mathcal{O}(1)$ at the GUT Scale

- Complete one-loop SM result at $\sqrt{q^2} = M_{\text{GUT}} = 10^{16}$ GeV:

$$\frac{m^2(M_{\text{GUT}}^2)}{M_{\text{GUT}}^2} = \underbrace{+1.135}_{\text{top}} \underbrace{-0.169}_{W} \underbrace{-0.109}_{Z} \underbrace{+0.002}_{h} = +0.858$$

- Order one — no fine-tuning
  - Dominant contribution from the top quark Yukawa coupling $y_t \approx 1$
  - W and Z partially cancel the top (spin-statistics: fermion loops carry (-1))
  - This partial cancellation is structural, not accidental — persists at two loops
- Net slope of $\Sigma_{\text{ren}}^{\text{SM}}/q^2$:

$$\frac{\Sigma_{\text{ren}}^{\text{SM}}}{q^2} \approx 0.0142 \log \frac{q^2}{m_{\text{EW}}^2} - 0.045, \qquad c \equiv \frac{N_c m_t^2}{8\pi^2 v^2} \approx 0.0142$$

Explaining the Electroweak Scale

- GUT–SM matching at M_{GUT} defines the boundary condition:

$$r \equiv \frac{m^2(M_{\mathrm{GUT}}^2)}{M_{\mathrm{GUT}}^2} = \mathcal{O}(1), \qquad 0 < r \leq 1$$

- Inverting gives the Higgs pole mass as a prediction:

$$m_h^2 = r\, M_{\mathrm{GUT}}^2 - \Sigma_{\mathrm{ren}}^{\mathrm{SM}}(M_{\mathrm{GUT}}^2)$$

- Using the approximate closed form with loop coefficient $c \approx 0.0142$:

$$m_h^2 \sim e^{-r/c}\, M_{\mathrm{GUT}}^2$$

- The 28 orders of magnitude are generated by the **smallness of the SM loop factor** c, not by tuning
- Varying v over all scales: $r = \mathcal{O}(1)$ throughout (Fig. 2 in paper) — no value of the electroweak scale is preferred or fine-tuned

Is fine tuning hidden?

Higgs mass

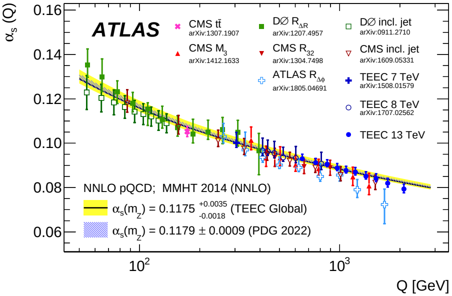
$$m_h^2 \sim M_{\rm GUT}^2 \, e^{-r/c}$$

- $r = 0.858, \quad c \approx N_c y_t^2/(16\pi^2) \approx 0.014$

QCD scale

$$\Lambda_{\rm QCD} \sim m_Z \, e^{-2\pi/(b_0 \, \alpha_s)}$$

- $\alpha_s(m_Z) = 0.1180$
- $\Delta\alpha_s = 0.001 \;\Rightarrow\; \Delta\Lambda/\Lambda \sim 10\%$



Both are $\mathcal{O}(1)$ couplings whose specific values determine exponential hierarchies.

No Naturalness Crisis

Traditional (top-down)

- Start with bare Lagrangian at M_{GUT}
- Propagate $\delta m_h^2 \sim M^2$ downward
- Arrive at a "crisis" — cancellation to 10^{-28}
- The problem is built from unobservable quantities

This work (bottom-up)

- Start from measured $m_h = 125$ GeV
- Run $m^2(q^2)$ upward
- $\mathcal{O}(1)$ at every scale — no crisis
- The 28 orders are natural scaling of q^2

- **Analogy:** $y_e \sim 10^{-6}$ vs. $y_t \approx 1$ is not called a naturalness crisis
- The specific value of m_h is determined by GUT couplings — an order-one parameter, not a fine-tuning
- *The absence of symmetry protection is not the disease.*
  It provides an explanation of the hierarchy.

Doublet-triplet splitting

Doublet-triplet splitting

- Both doublet and triplet mass functions are $\mathcal{O}(M_{\mathrm{GUT}}^2)$ at the GUT scale
- Below M_{GUT}, the triplet also runs as q^2 with model-independent coefficients fixed by its color-triplet quantum numbers
- If its running is slow enough, no special splitting mechanism is needed (calculation in progress)

Cosmological Constant

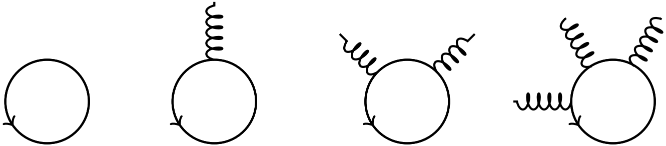
- The CC is not constant in the presence of the gravity

$$\mathcal{L} \supset \sqrt{\det g}\,\Lambda$$

- By the same 't Hooft symmetry argument, the renormalized CC is

$$\frac{\Lambda(q^2)}{q^4} = \mathcal{O}(1) \quad \text{at every scale}$$

- Requires a renormalizable formulation of quantum gravity



Summary

1. The renormalized Higgs mass function $m^2(q^2)$ is finite, physical and runs as q^2
2. Heavy fields decouple properly: $\Sigma_{\text{ren}}^{\text{heavy}} = \mathcal{O}((q^2 - m_h^2)^2/M^2)$, not M^2
3. $m^2(q^2)/q^2 = \mathcal{O}(1)$ at every scale from m_{EW} to M_{GUT}
4. The 28 orders of magnitude are generated by the SM loop factor:

$$m_h^2 \sim e^{-r/c}\, M_{\text{GUT}}^2, \qquad c = \frac{N_c m_t^2}{8\pi^2 v^2} \approx 0.0142$$

5. An order-one GUT boundary condition suffices to produce the electroweak scale — no new symmetry is needed

The absence of symmetry protection is not the disease.
It provides an explanation of the hierarchy.