What can we learn about QCD and related physics from astrophysical observations?

DAY 2

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— Ph.D School at XQCD 2026 —

Lecture Plan

[Part 1]

Basics of Compact Stars
Mass, Radius
Equation of State (EoS)
Lane-Emden eq. / Stability*

👉 **Blackboard**

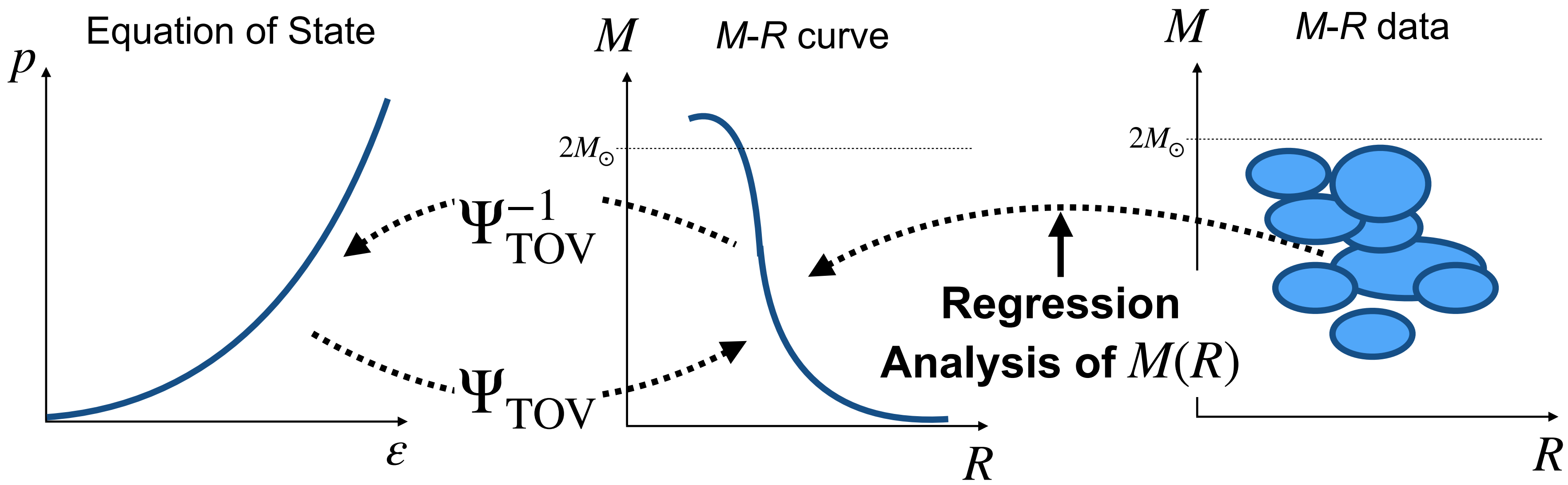
[Part 2]

EoS Inferences, Trace Anomaly
Gravitational Wave Implications
Recent Interpretation — Quarkyonic*

EoS Inferences

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation



Once one *M-R* curve is identified, one EoS is concluded.

The best we can do is to find the "likely" *M-R* curve.

EoS Inferences

Model Independent Approach

EoS ← Solving TOV ← *M*-*R* Curve ← Observation

$\boxed{\text{Bayesian Analysis}}$ **Ozel et al., Steiner et al. (2015~)**

A : EoS Parameters B : M-R Observation

(Bayes' theorem)

Normalization

$$\underbrace{P(A|B)}_{\text{Want to know}}\cancel{P(B)} = \underbrace{P(B|A)}_{\text{Likelihood}}\underbrace{P(A)}_{\text{prior}}$$

Model

EoS Inferences

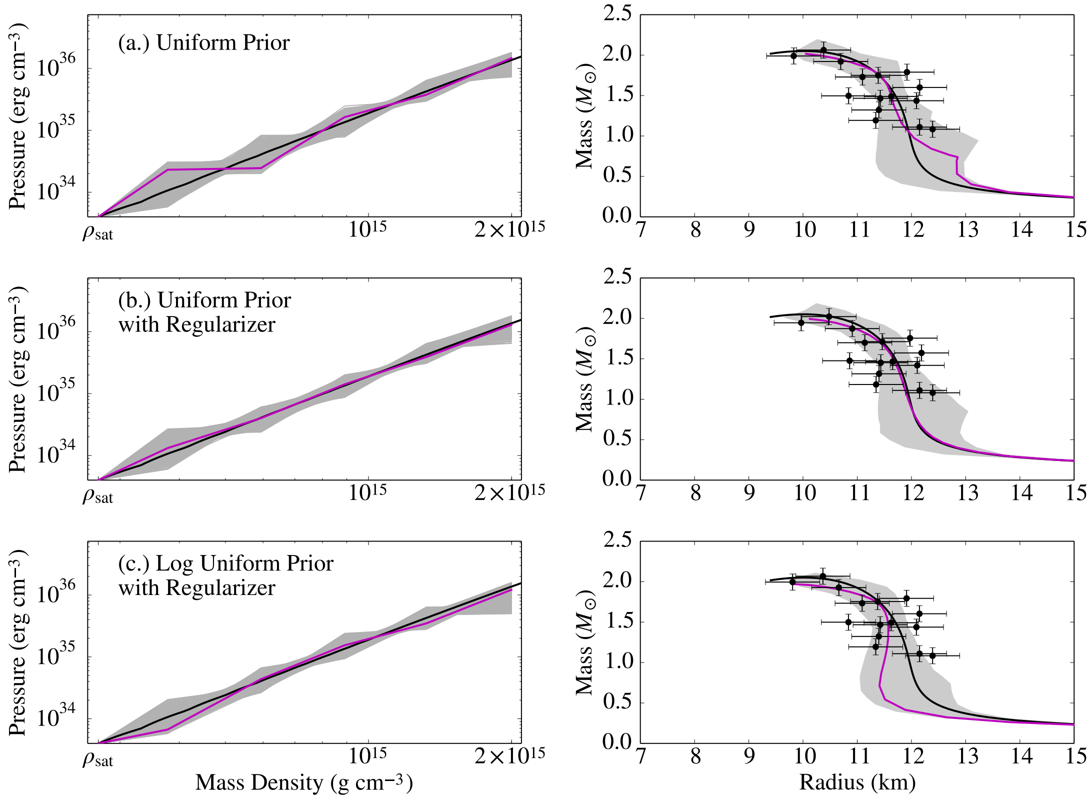
Bayesian Analysis **Ozel et al., Steiner et al. (2015~)**

If *B* (observation data) is abundant, the likelihood would become sharper → the prior dependence can be reduced… but…

Raithel-Ozel-Psaltis (2017)

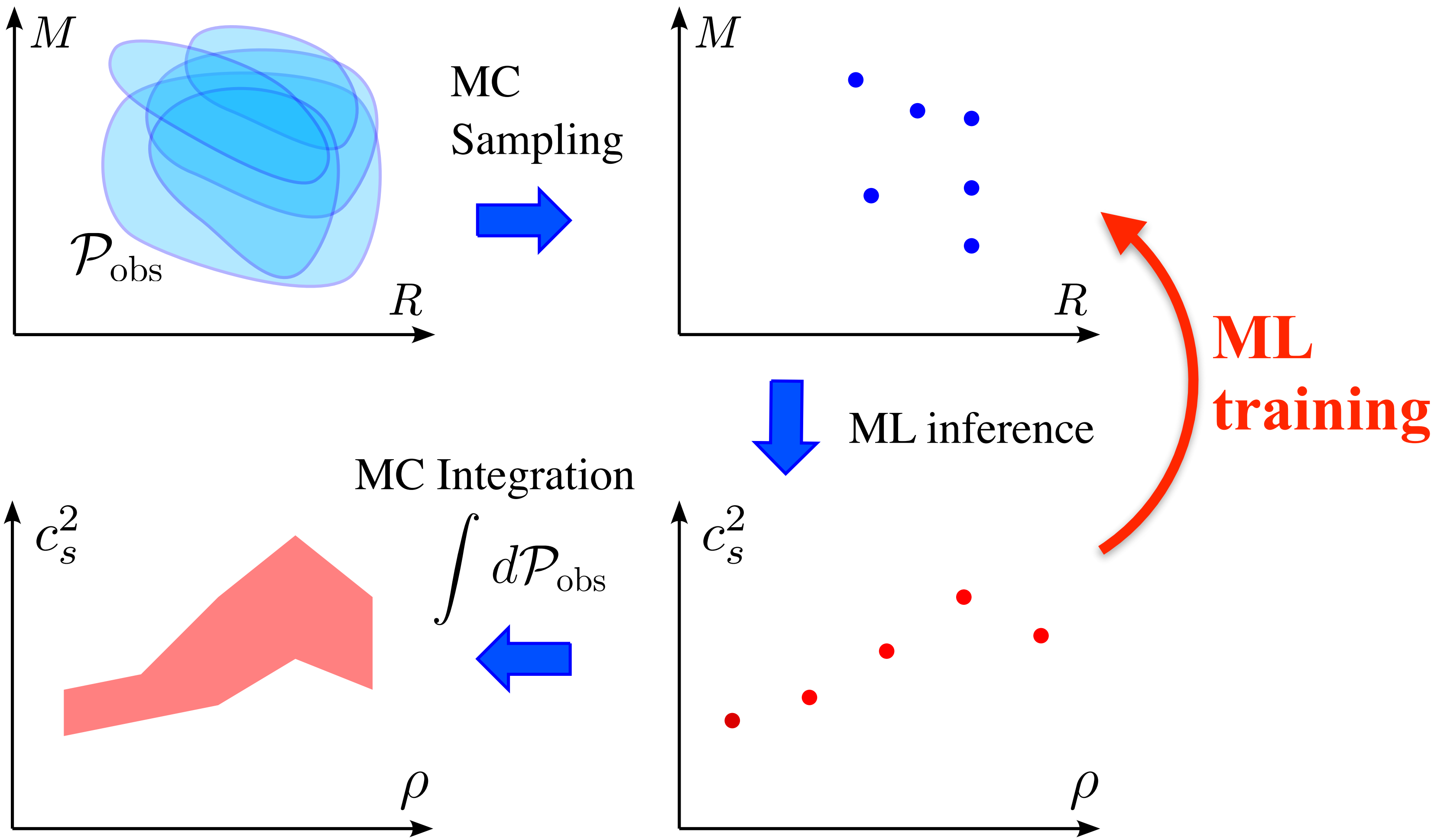
Mock data (SLy + Noises)

Very powerful approach but a complementary is desirable…



EoS Inferences

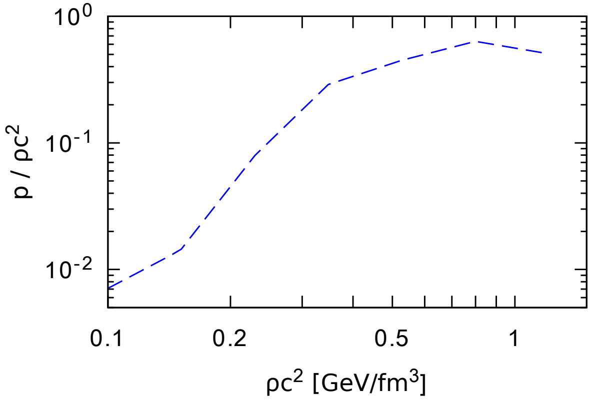
Fujimoto-Fukushima-Kamata-Murase (2018-2024)



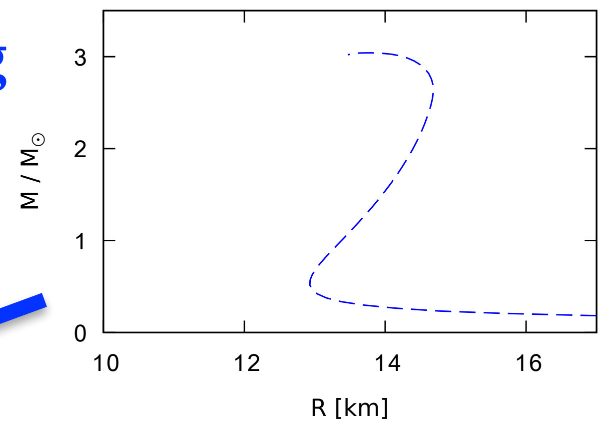
EoS Inferences

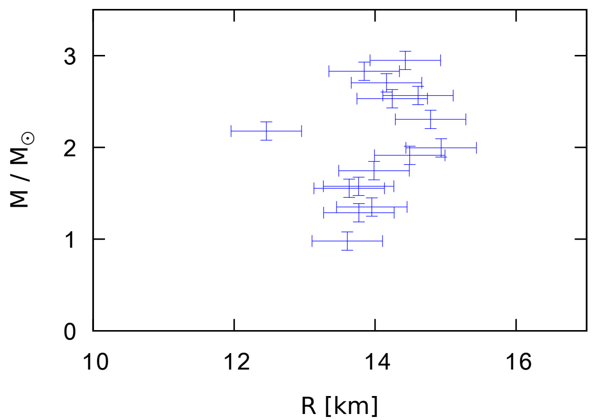
Fujimoto-Fukushima-Kamata-Murase (2018-2024)

Proof of principle



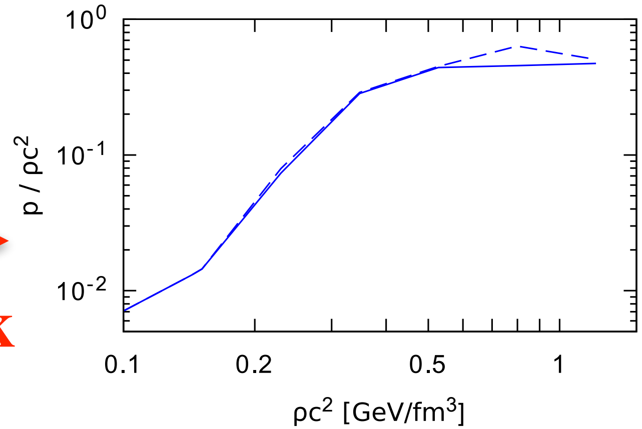
Corresponding M-R relation



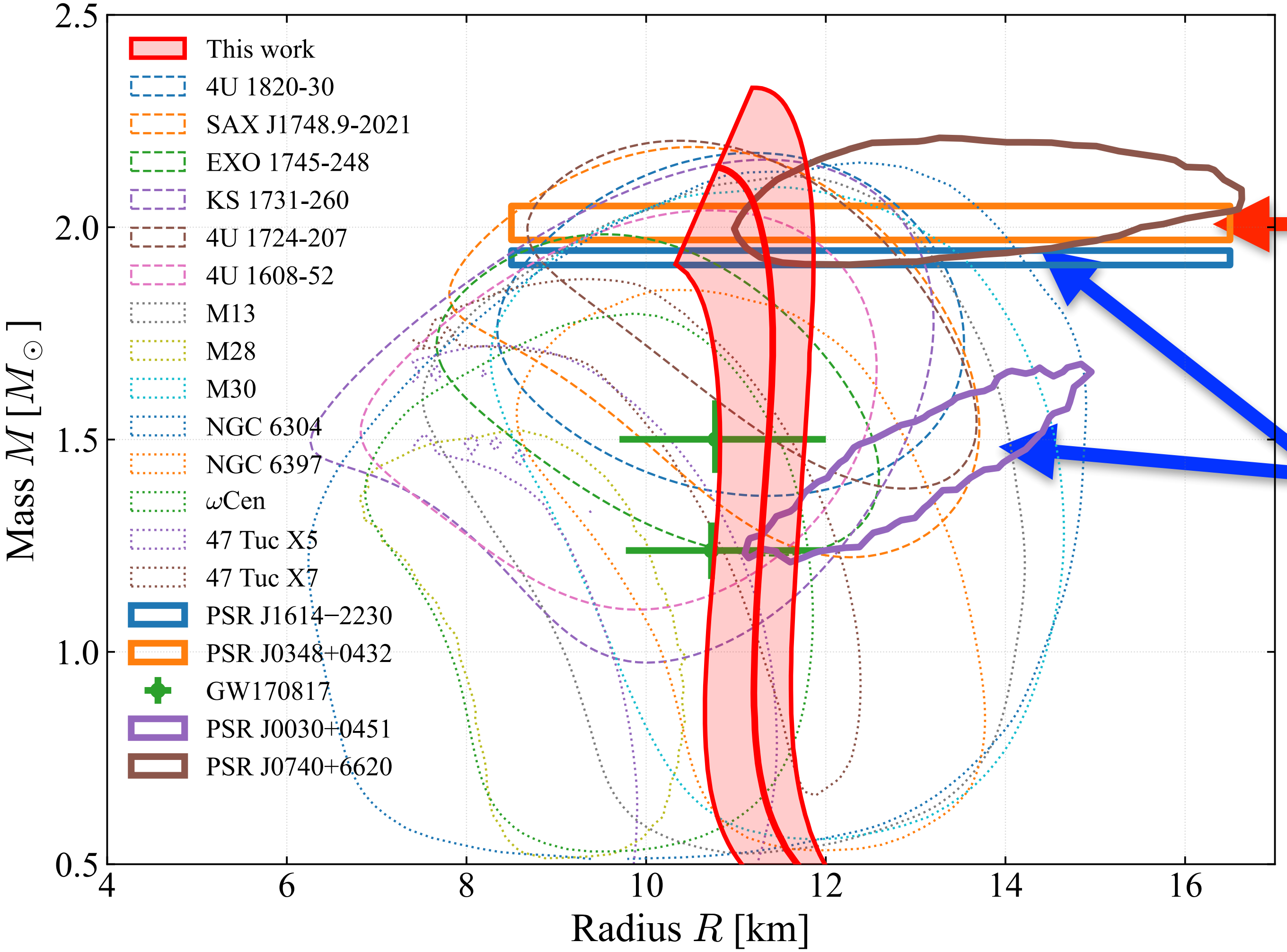


Mimic the astro data

Validity check



EoS Inferences

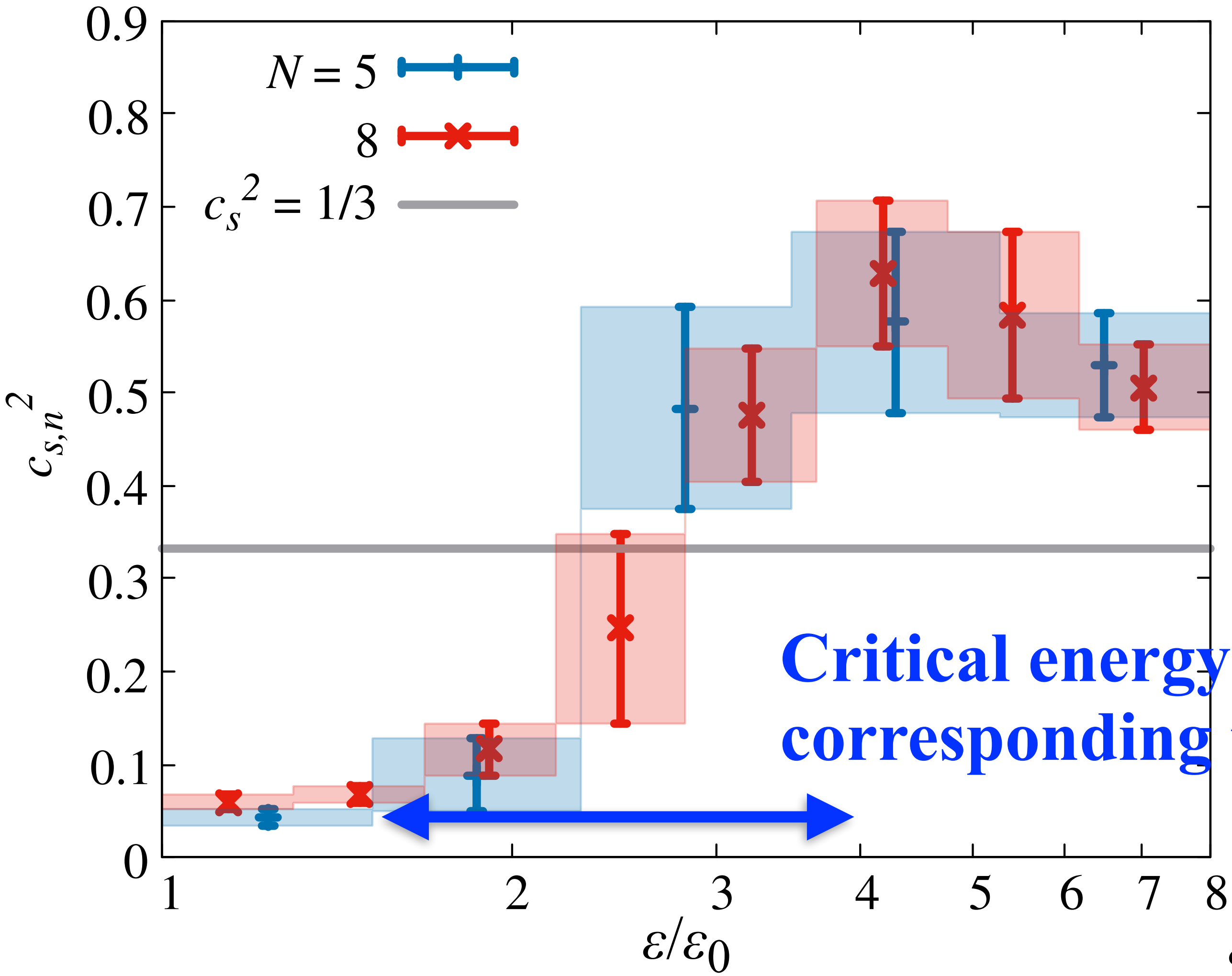


EOS must be "stiff" enough to reach here.

Lensing and timing (NICER) constraining the gravity strength and the radius!

EoS Inferences

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



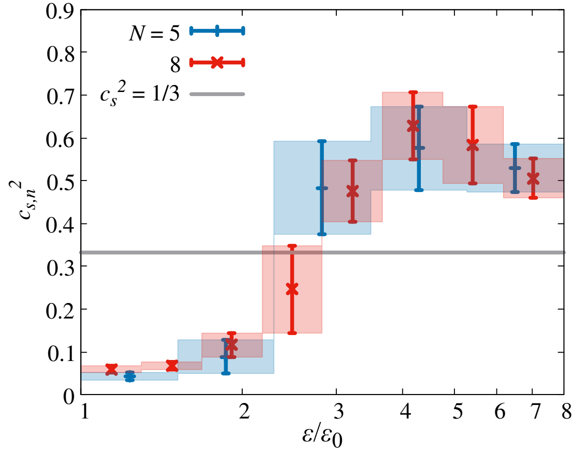
Not minimum, but maximum!?

Critical energy density corresponding to a QGP

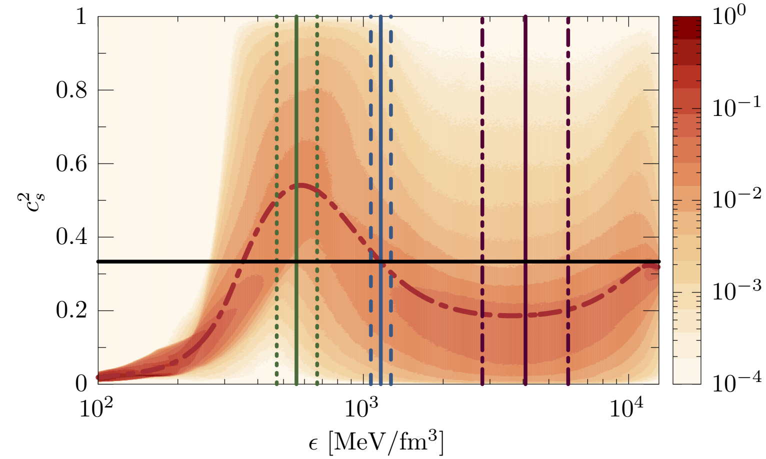
$\varepsilon_0 = 150\,\mathrm{MeV\ fm}^{-3}$

Sound Speed Peak

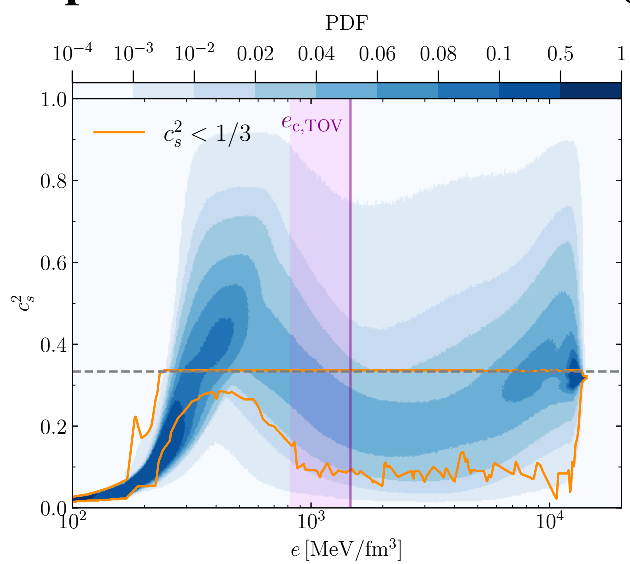
Fujimoto-Fukushima-Kamata-Murase (2018-2024)



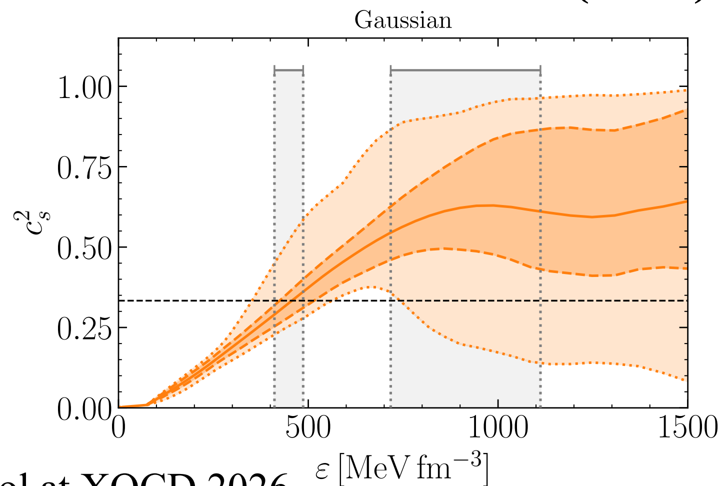
Marczenko-McLerran-Redlich-Sasaki (2022)



Altiparmak-Ecker-Rezzolla (2022)

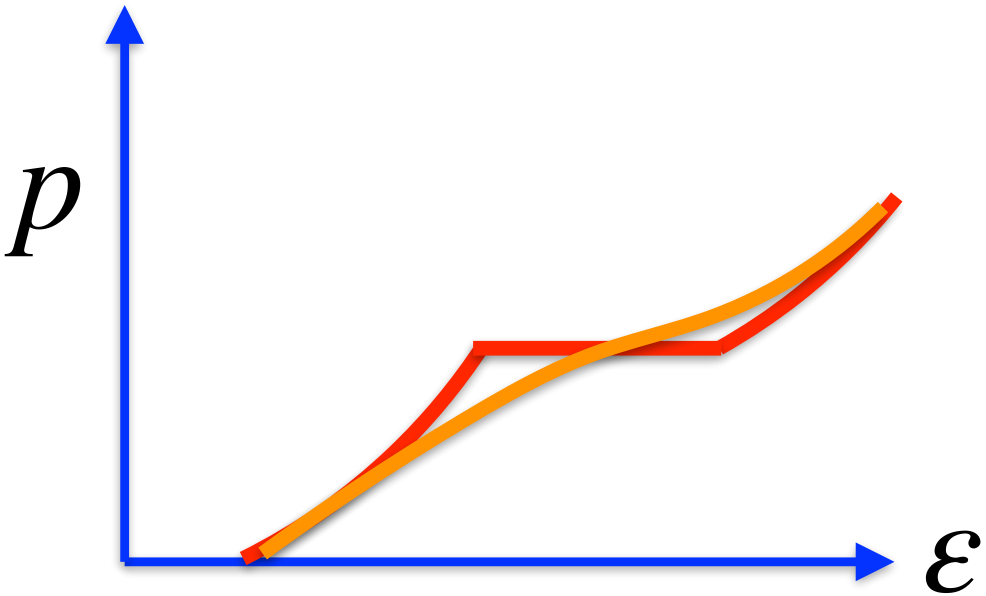


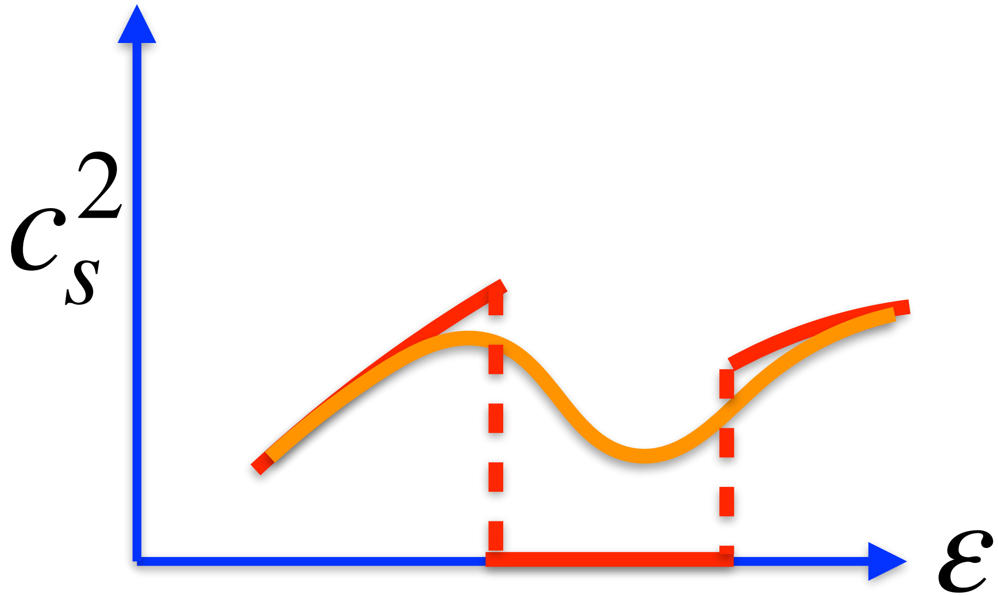
Brandes-Weise-Kaiser (2022)



Sound Speed Peak

[1st-order-like EoS]

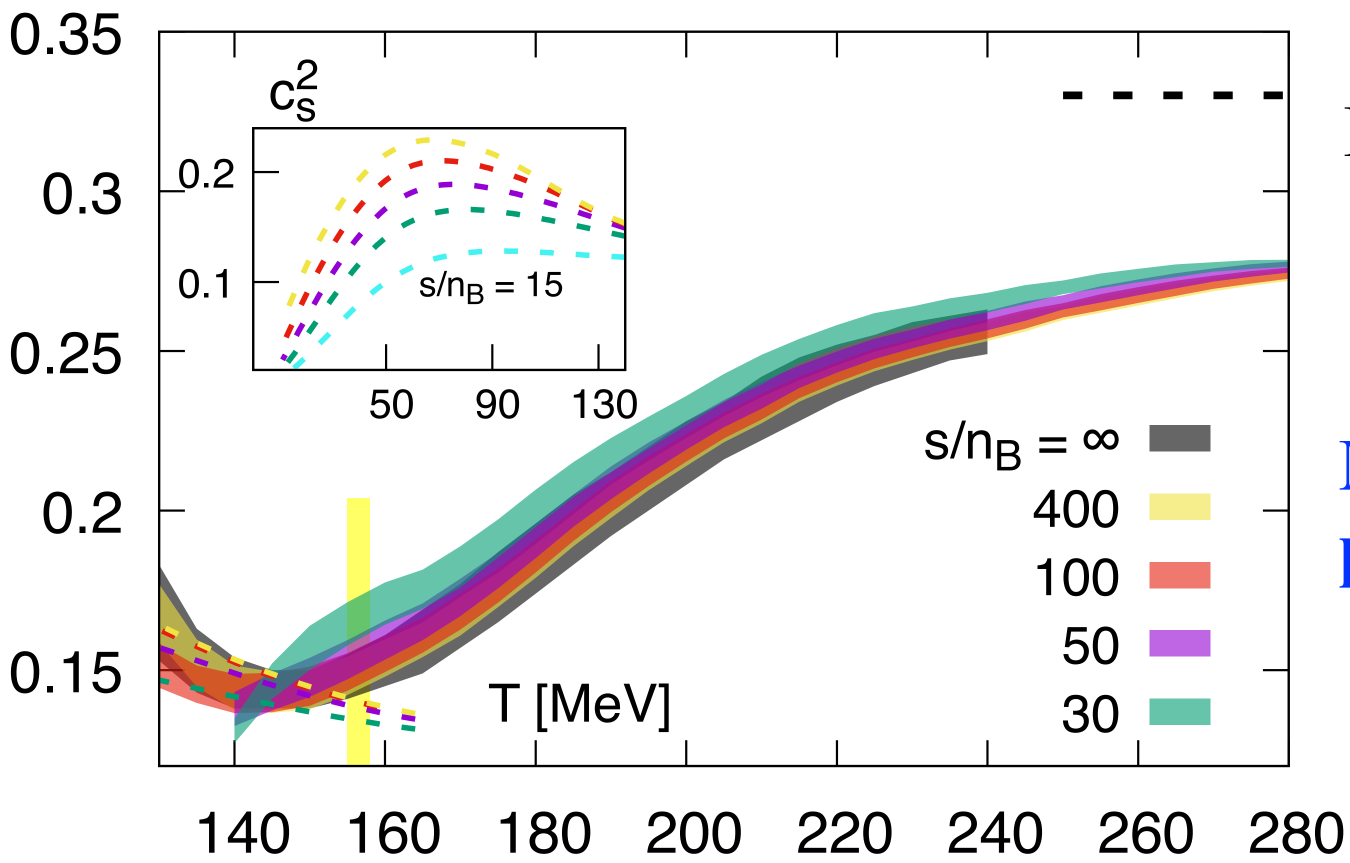




Phase transition is manifested by a *minimum* in the speed of sound, not a maximum…?

Sound Speed Peak

[High-Temperature QCD — QGP Crossover]



HotQCD Collab. (2212.09043)

Minimum around phase transition

Sound Speed Peak

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

Trace anomaly measure $\boxed{\Delta = \frac{1}{3} - \frac{p}{\varepsilon}}$

Gavai-Gupta-Mukherjee (2004)

$$c_s^2 = \frac{dp}{d\varepsilon} = c^2_{s,\,\mathrm{deriv}} + c^2_{s,\,\mathrm{non-deriv}}$$

$$c^2_{s,\,\mathrm{deriv}} = -\,\varepsilon \frac{d\Delta}{d\varepsilon} \qquad c^2_{s,\,\mathrm{non-deriv}} = \frac{1}{3} - \Delta$$

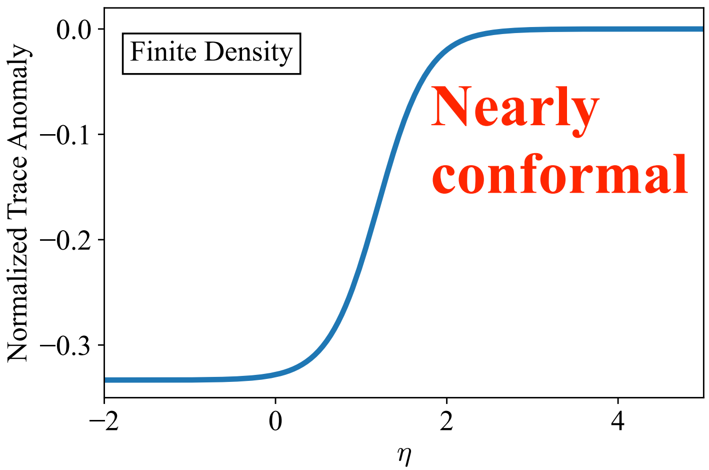
Derivative **Non-Derivative**

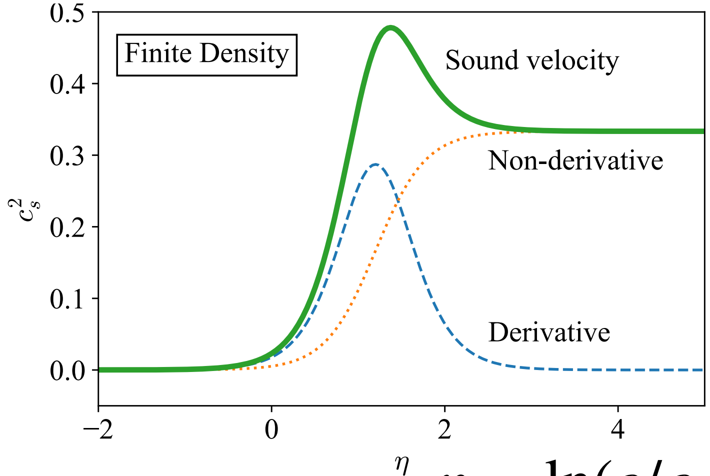
Dominant at high density making a peak!

Sound Speed Peak

Derivative contribution makes a peak structure!

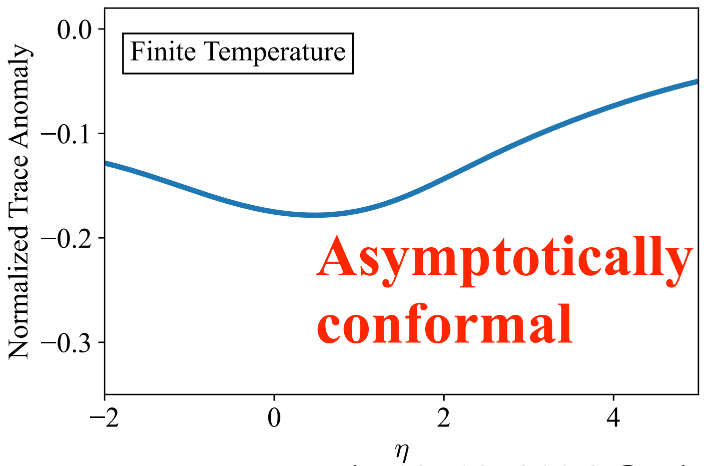
Sign Flipped $-\Delta$

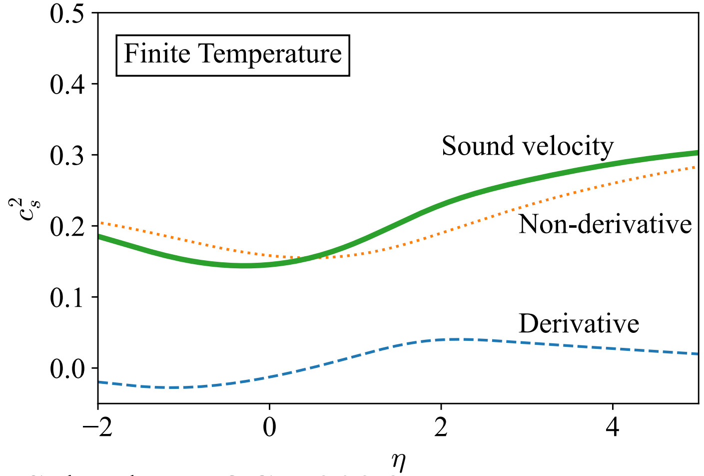




$$\eta = \ln(\varepsilon/\varepsilon_0)$$

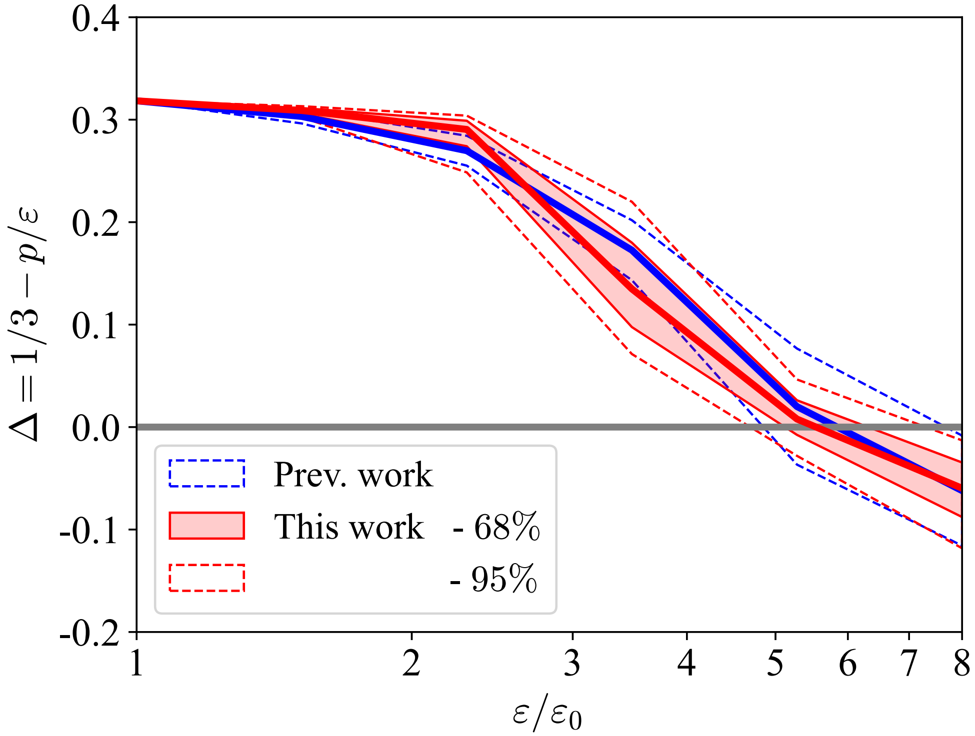
Speed of Sound





Sound Speed Peak

Interesting question… $\Delta < 0$???



$$\Delta \propto \varepsilon - 3p$$

$$\propto \frac{d}{d\mu}\left(\frac{p}{\mu^4}\right)$$

Thermodynamic degrees of freedom

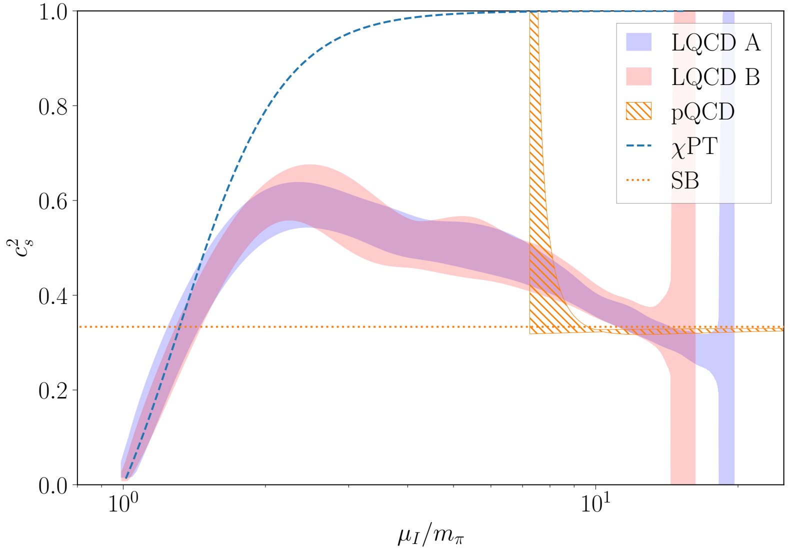
Negative trace anomaly implies the presence of "condensates"!?

Sound Speed Peak

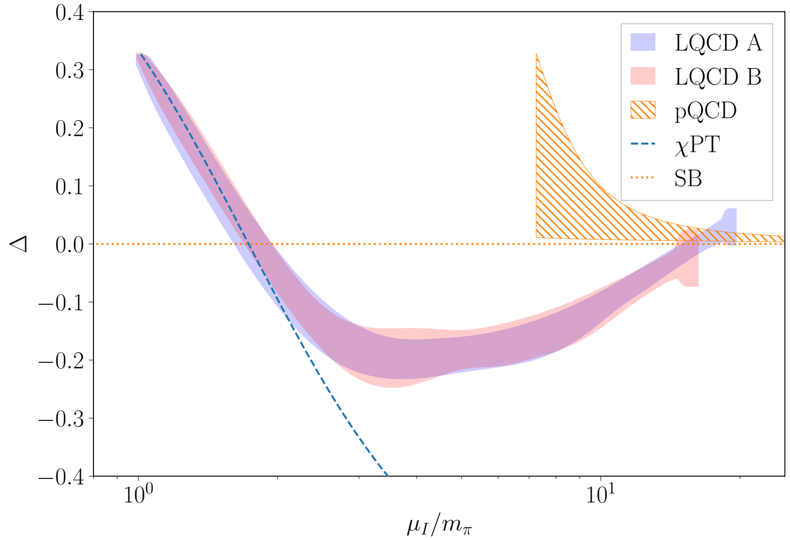
Lesson from high-isospin matter (lattice QCD)

Abbott+ (2023)

[Speed of sound peak]



[Negative trace anomaly]



$$p_{\chi\mathrm{PT}} = \frac{f_\pi^2 \mu_I^2}{2}\left(1 - \frac{m_\pi^2}{\mu_I^2}\right)^2$$

No loops, just condensates

Sound Speed Peak

Quick derivation

Son-Stephanov (2001)

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}\mathrm{Tr}\nabla_\nu\Sigma\nabla_\nu\Sigma^\dagger - \frac{m_\pi^2 f_\pi^2}{2}\mathrm{ReTr}\Sigma$$

$$\nabla_0\Sigma = \partial_0\Sigma - \frac{\mu_I}{2}(\tau_3\Sigma - \Sigma\tau_3)$$

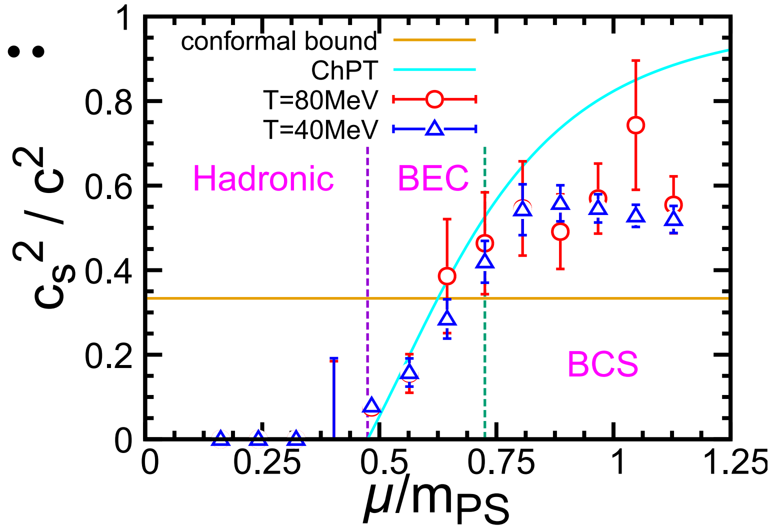
$$\pi^0 = 0,\quad \pi^\pm \neq 0 \longrightarrow \bar{\Sigma} = \cos\alpha + i(\tau_1\cos\phi + \tau_2\sin\phi)\sin\alpha$$

$$F = -\frac{f_\pi^2}{2}\mu_I^2\sin^2\alpha - m_\pi^2 f_\pi^2\cos\alpha \longrightarrow \cos\alpha = m_\pi^2/\mu_I^2$$

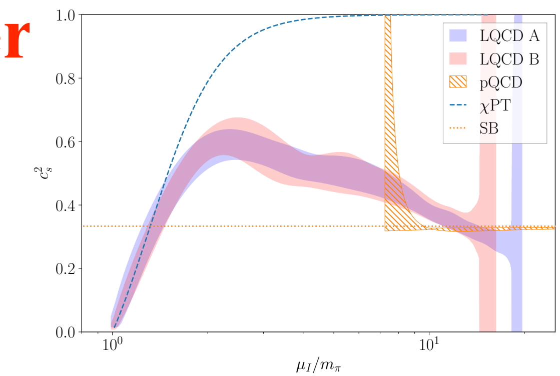
$$\longrightarrow F = -\frac{f_\pi^2}{2}\mu_I^2\left(1 + \frac{m_\pi^4}{\mu_I^4}\right)$$

Sound Speed Peak

We should perform extensive studies of the condensation effects on the speed of sound for…



* **Diquark superfluid in QC_2D**
To be compared with
Lattice: Itou+ (2023-2024)



* **Pion-condensed high-isospin matter**
To be compared with
Lattice: Abbott+ (2023)

* **Two-flavor color-superconducting (2SC) matter?**

GW Implications

GW170817 **Phys. Rev. Lett. 119, 161101 (2017)**

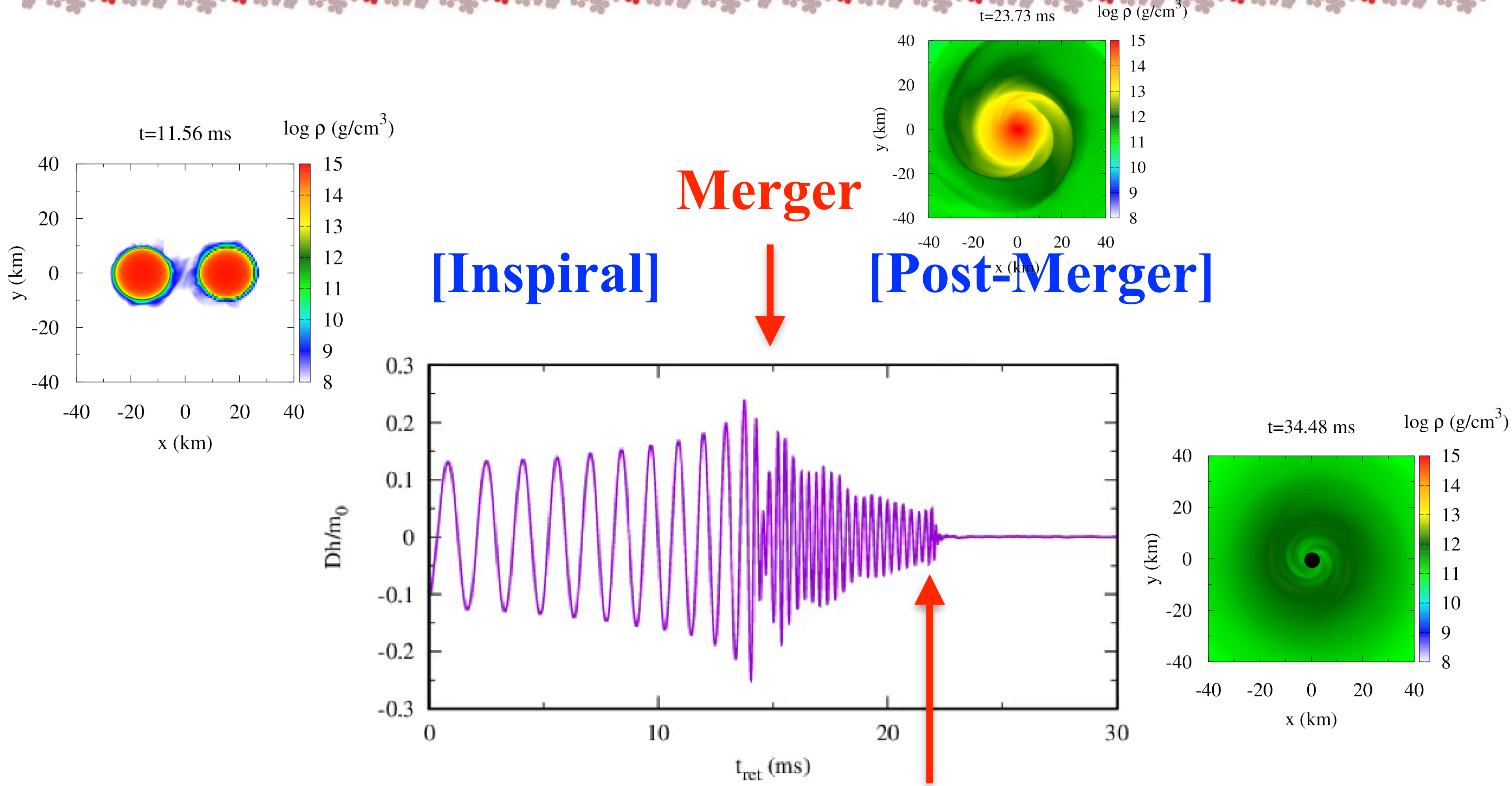
Advanced LIGO/Virgo reported the gravitational waves from the binary NS merger on August 17, 2017.

Distance was only **40Mpc** (many theory calculations were around 100Mpc but thought to be too optimistic…)

Total mass ~ 2.75$M_\odot$ **Mass ratio *q* is hard to know…**

Kilonova (AT 2017gfo) was confirmed: thermal radiation of light — an important hint for the *r*-process
Ejecta mass ~ 0.05$M_\odot$

GW Implications



Gravitational Collapse
Black Hole Formation

[Figures from K. Kyutoku]

GW Implications

Three possible final states:

Supramassive Neutron Star (SMNS)

Uniform rotation exceeding M_{TOV} (up to $\sim 1.2 M_{\mathrm{TOV}}$)

Hypermassive Neutron Star (HMNS)

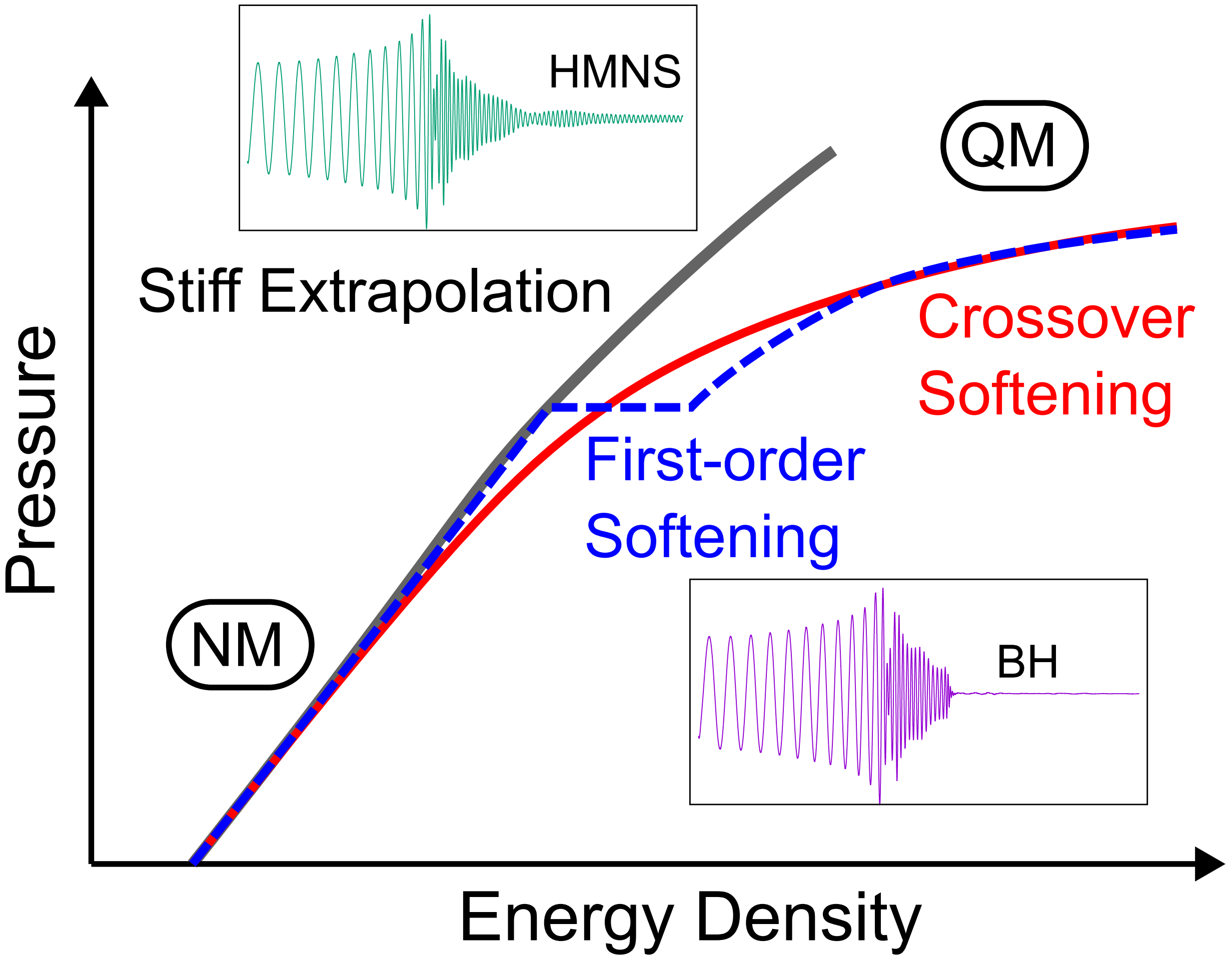
Non-uniform rotation exceeding the mass further

Black Hole

If the total mass of the binary system is too large,
the final state immediately collapses to BH.

GW Implications

Fukushima (2025)



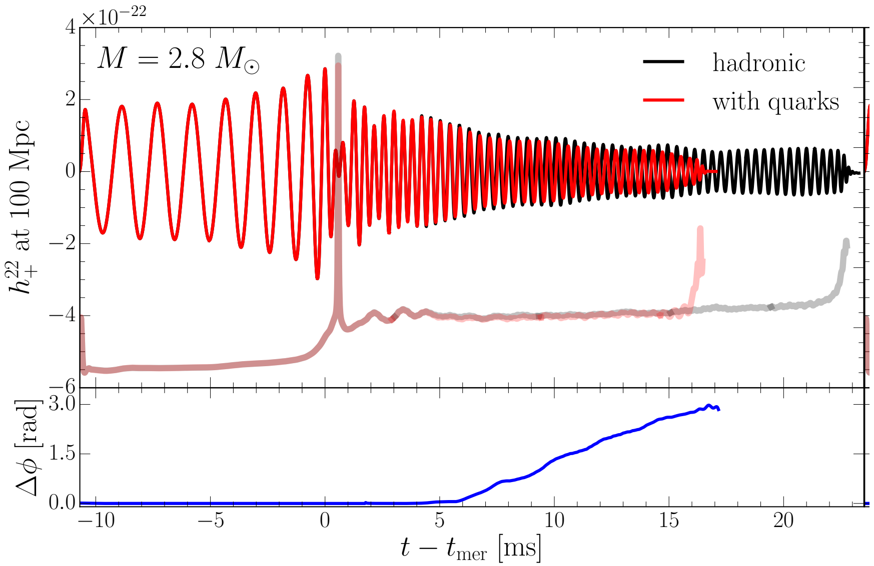
PT or softening affects the final state after the NS merger.
Post-merger GW study possible in the future?

GW Implications

Can we see the phase transition with the GW signal?

Most-Papenfort-Dexheimer-Hanauske-Schramm-Stocker-Rezzolla (2018)

$\mathrm{CMF_Q}$: EOS with a strong-1st PT to Quark Matter (3~4 times n_{sat})
$\mathrm{CMF_H}$: EOS without quarks

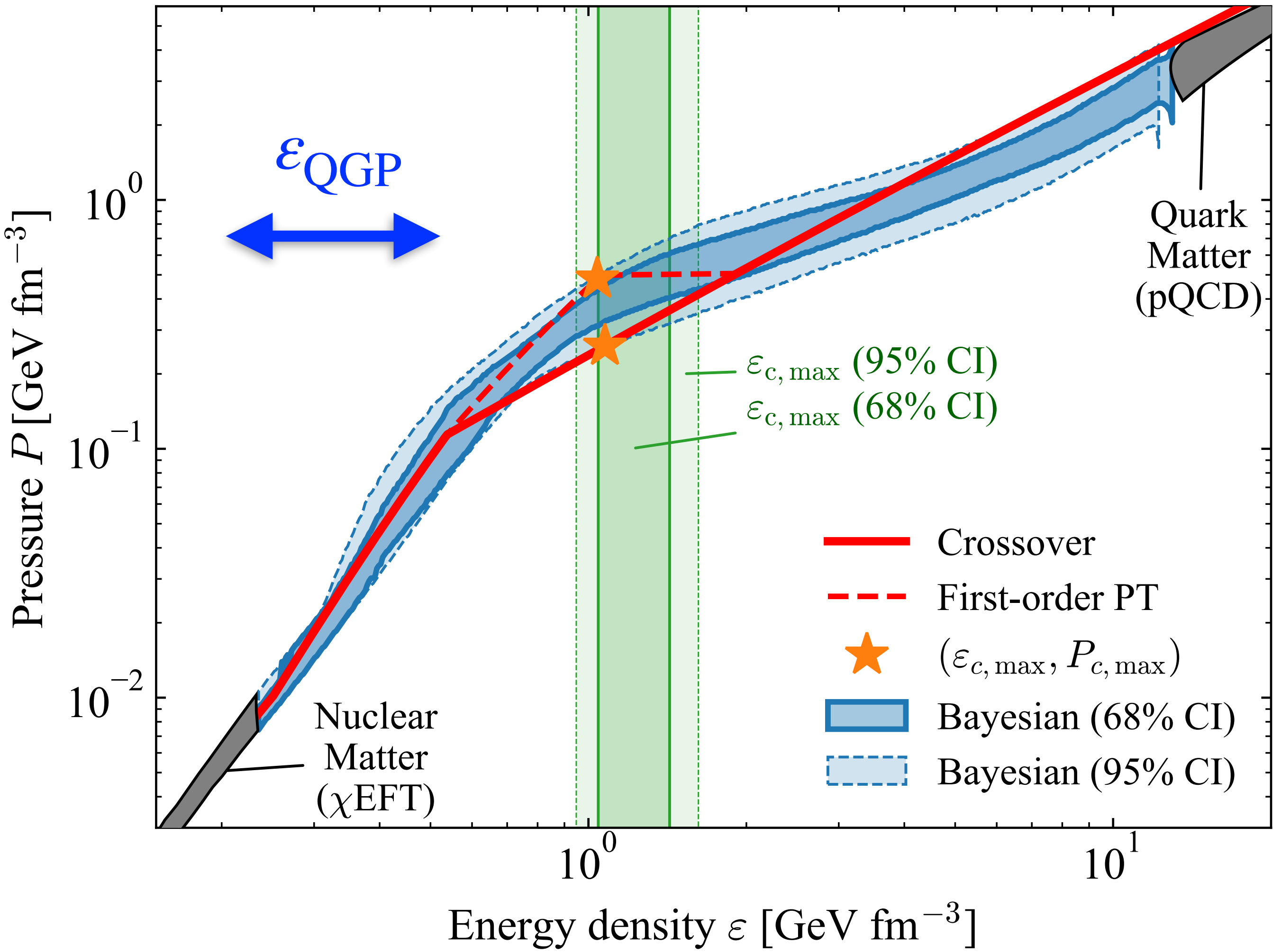


Quark matter shortens the lifetime of post-merger supramassive/hypermassive (uniform / differential) neutron star.

What if the transition is only a smooth crossover?

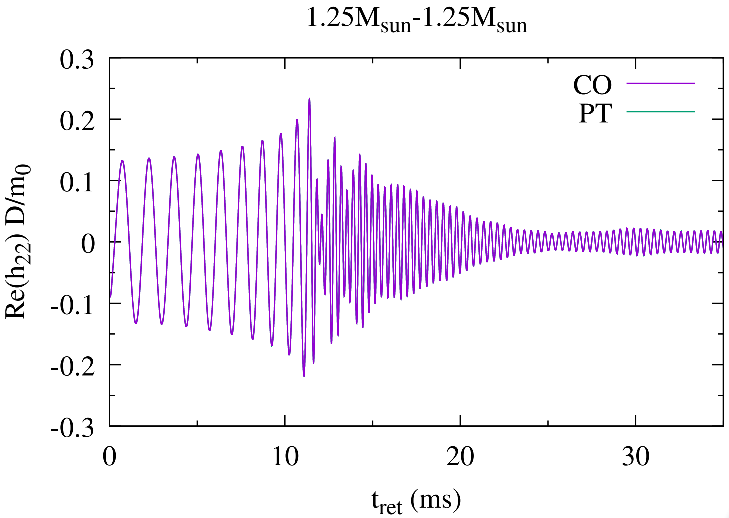
GW Implications

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



GW Implications

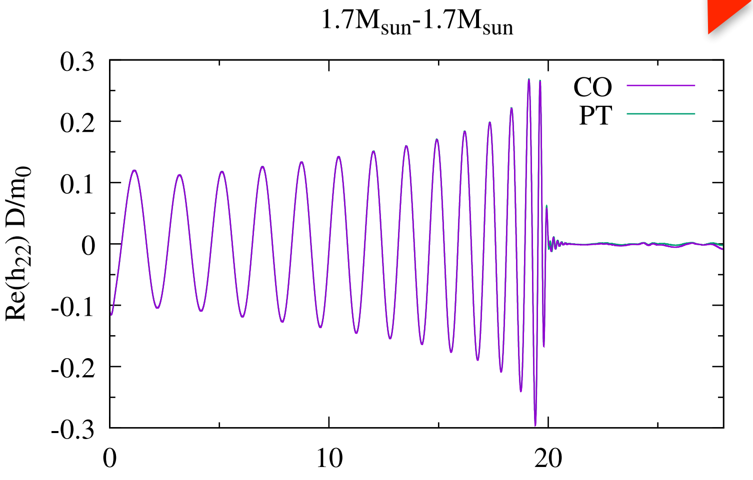
Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)

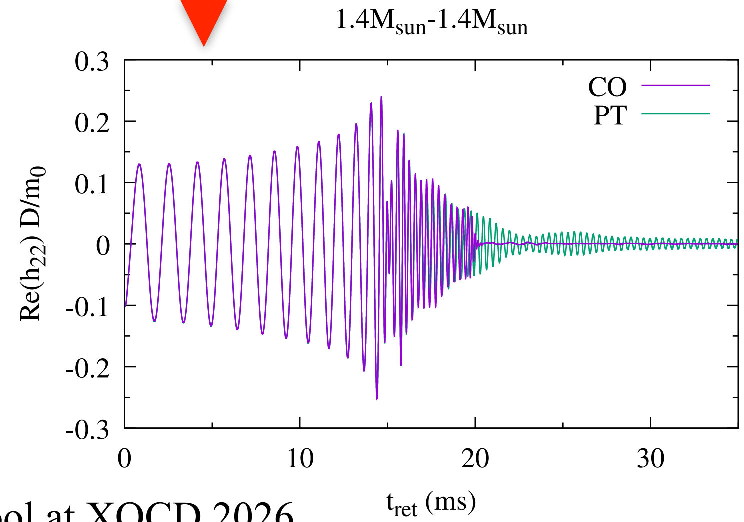


Light System (No BH)

Heavy System (Both BH)

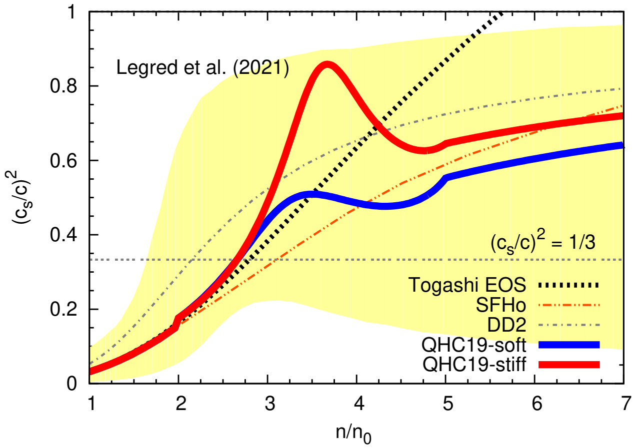
Like GW170817 (Discriminable!)

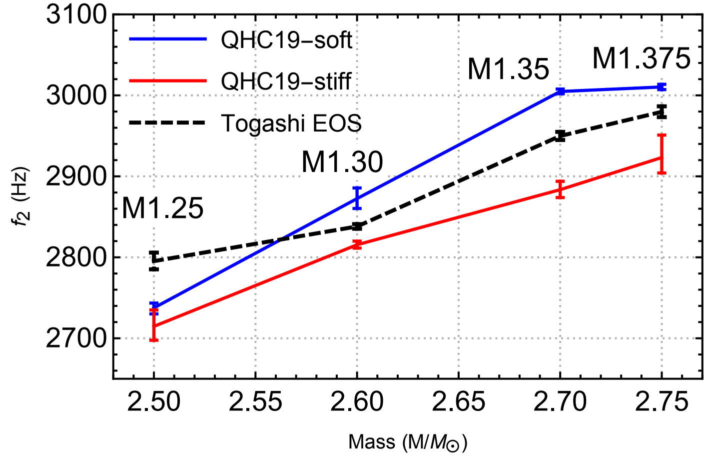




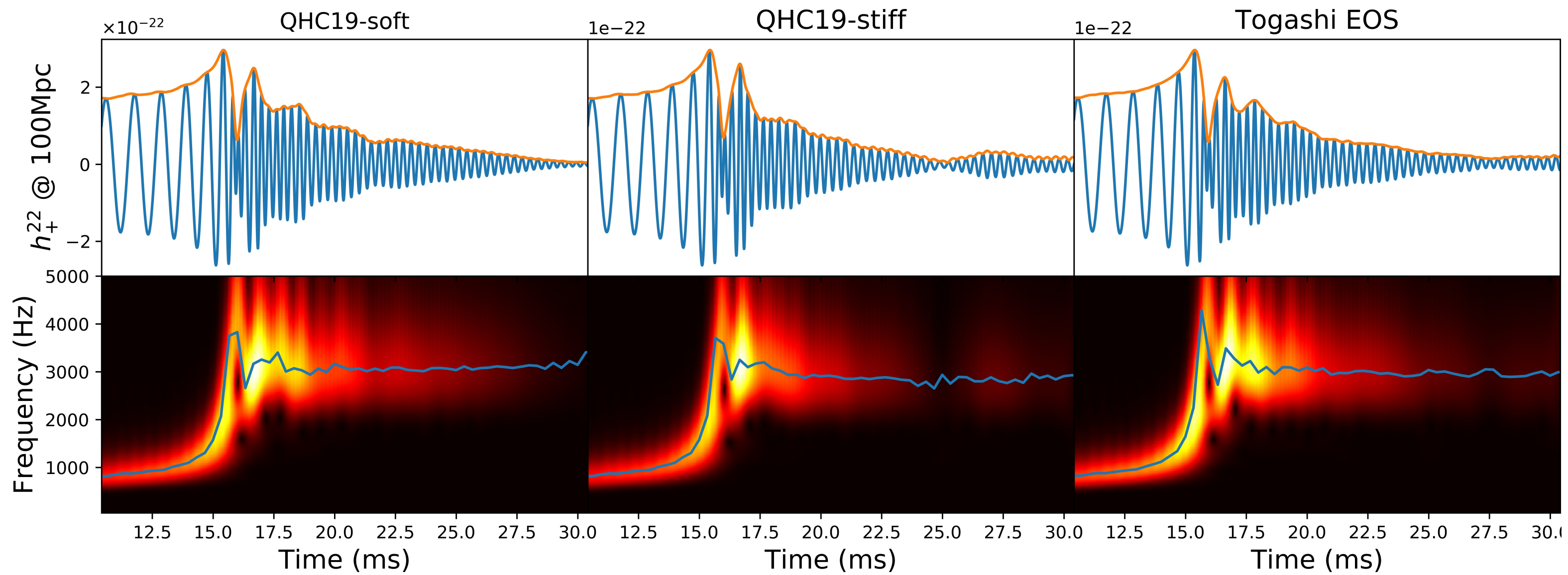
GW Implications

Huang-Baiotti-Kojo-Takami-Sotani-Togashi-Hatsuda-Nagataki-Fan (2022)





Peak frequency may change…



GW Implications

[Comment 1]

Why two simulations lead to different final states?

Thermal pressure parameterized by a thermal index:

$$p_{\mathrm{th}} = (\Gamma_{\mathrm{th}} - 1)\,\rho\,\varepsilon_{\mathrm{th}}$$

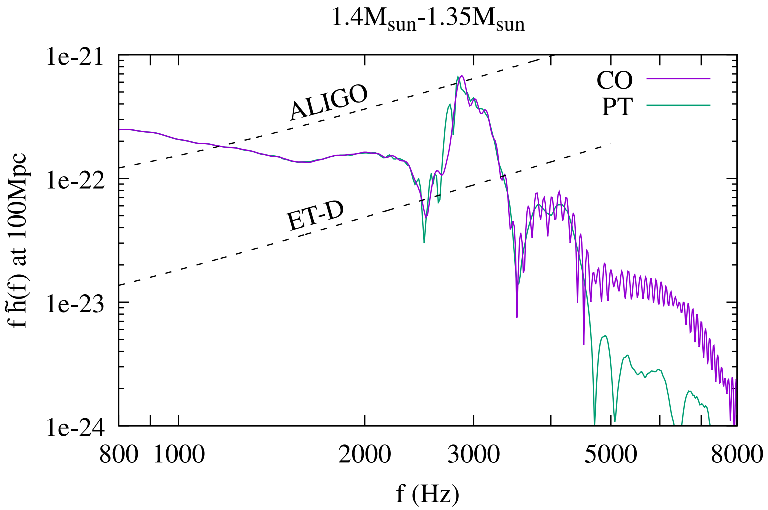
BH for $\Gamma_{\mathrm{th}} \lesssim 1.8$ but not for larger.
Huang et al. chose $\Gamma_{\mathrm{th}} = 2.0$, while $\Gamma_{\mathrm{th}} = 1.75$ in ours.

[Comment 2]

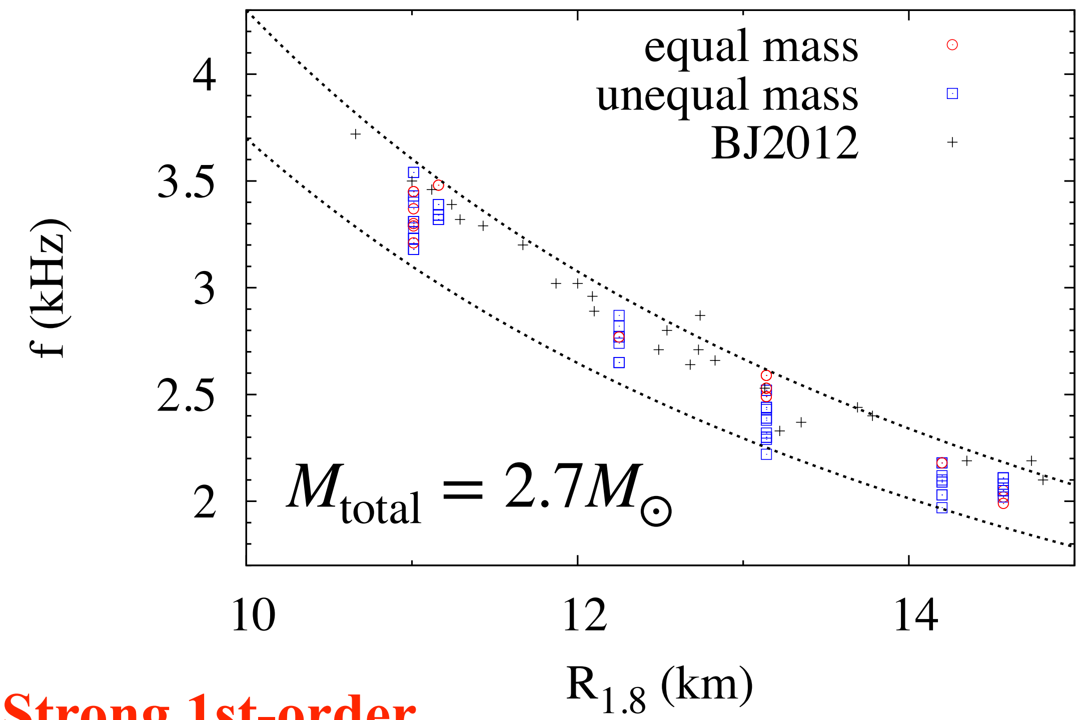
If the peak frequency so important? $\to$ Yes!

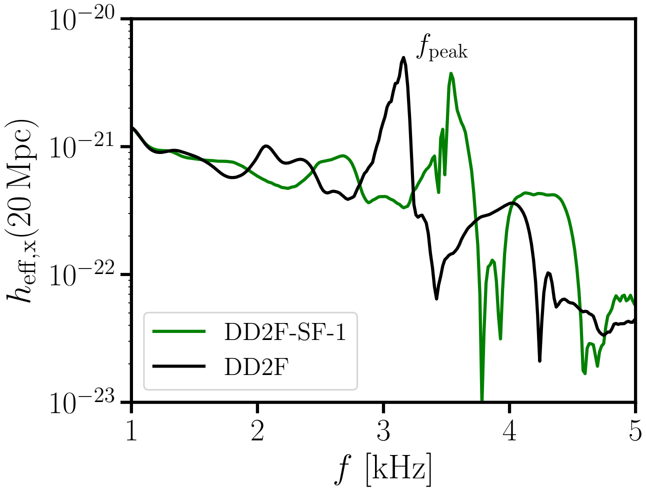
GW Implications

Post-merger stage is very challenging to see:



Hotokezaka+ (2013)



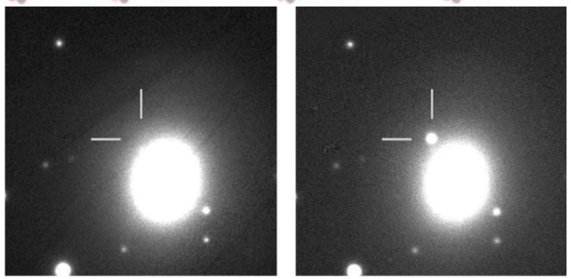


← Strong 1st-order PT scenario

Bausewein+ (2018)

GW Implications

Kilonova brightness:
ejected mass $> 0.05 M_\odot$



AT 2017 gfo

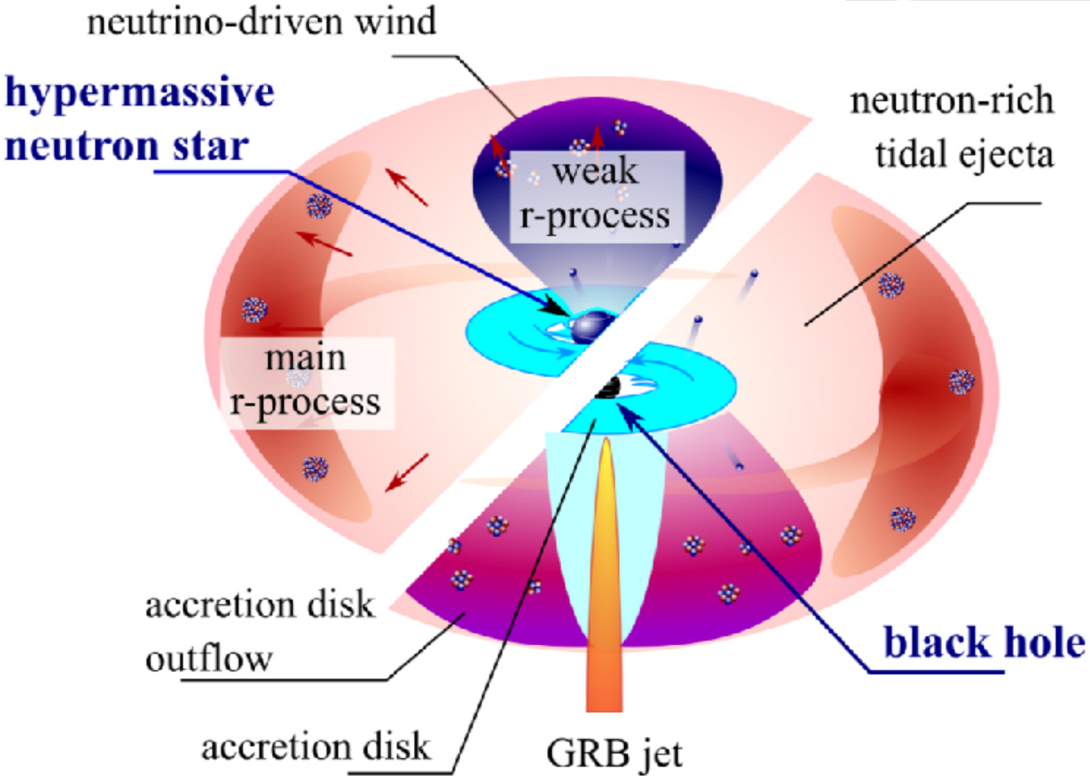


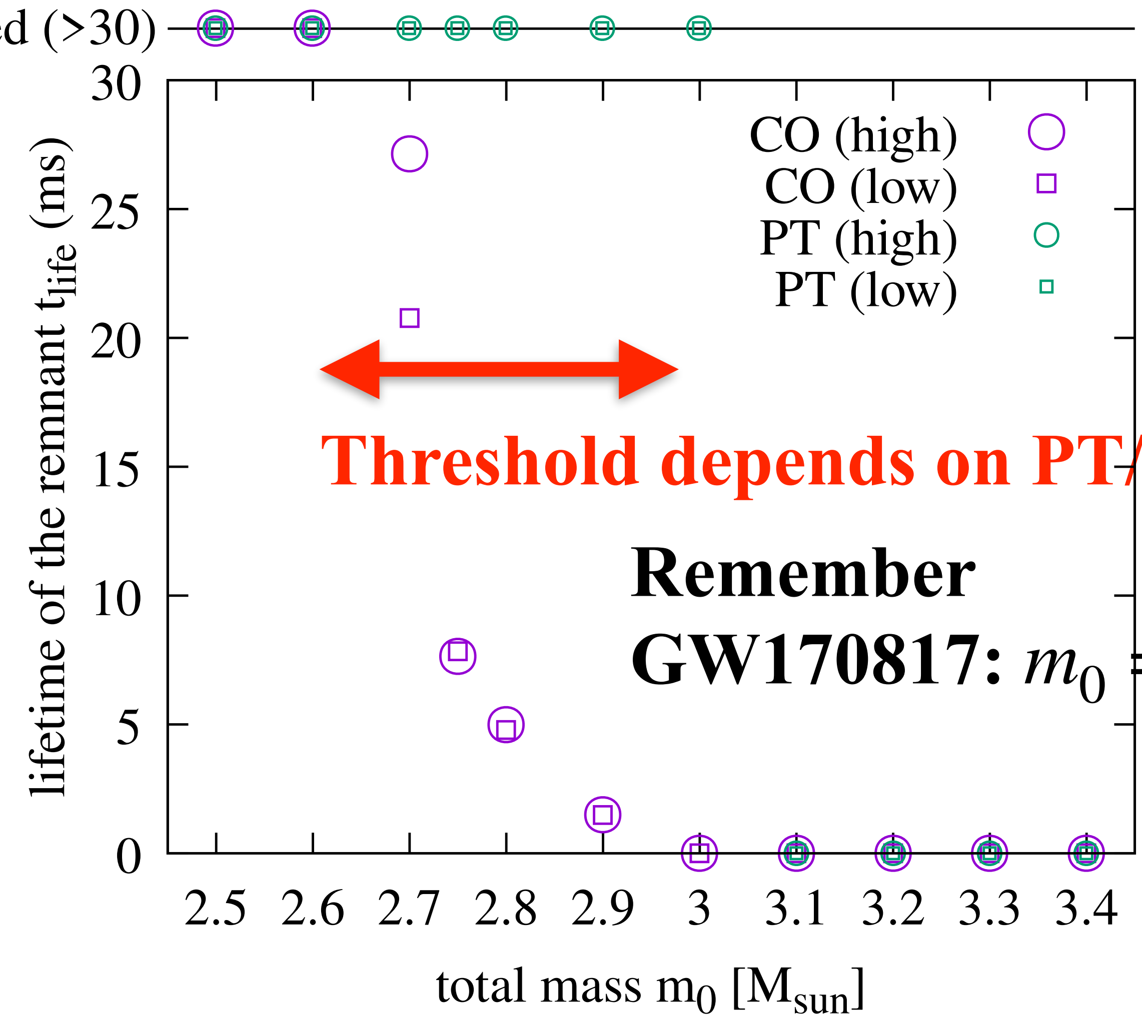
Illustration from Korobkin+ (2021)

Brightness and "color" depend on the EoS and the total mass.

GW Implications

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)

Kilonova could be formed.



Threshold depends on PT/EoS.

Remember
GW170817: $m_0 = 2.75 M_\odot$

Black Hole (no Kilonova)

Blackboard Plan

[Lecture about a recent idea of "quarkyonic"]

Large-N_c Counting

Phase Diagram in the Large-N_c Limit

Quarkyonic Interpretation

Stiffening Scenario with Momentum Shell