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TOPTIER CNS Summer School 2026

Nuclear radii of one-proton emitters

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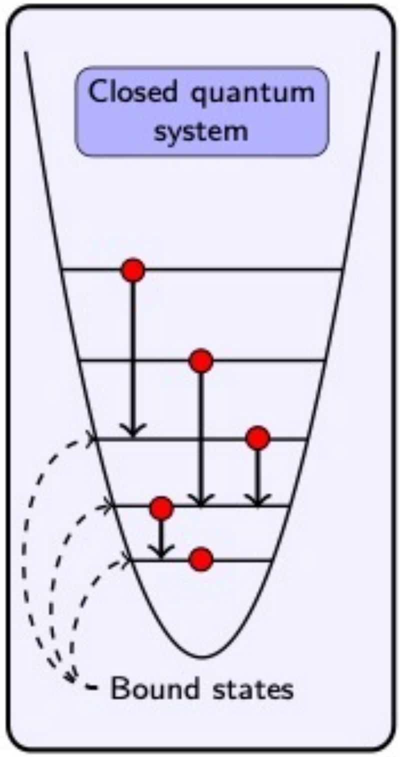
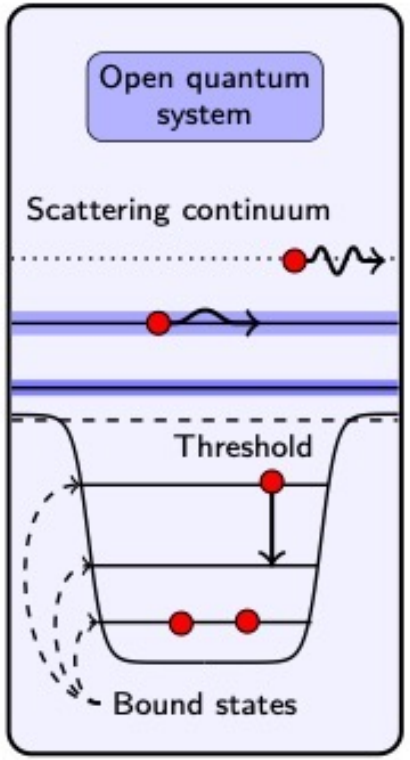
2026.8.24 in Daejeon



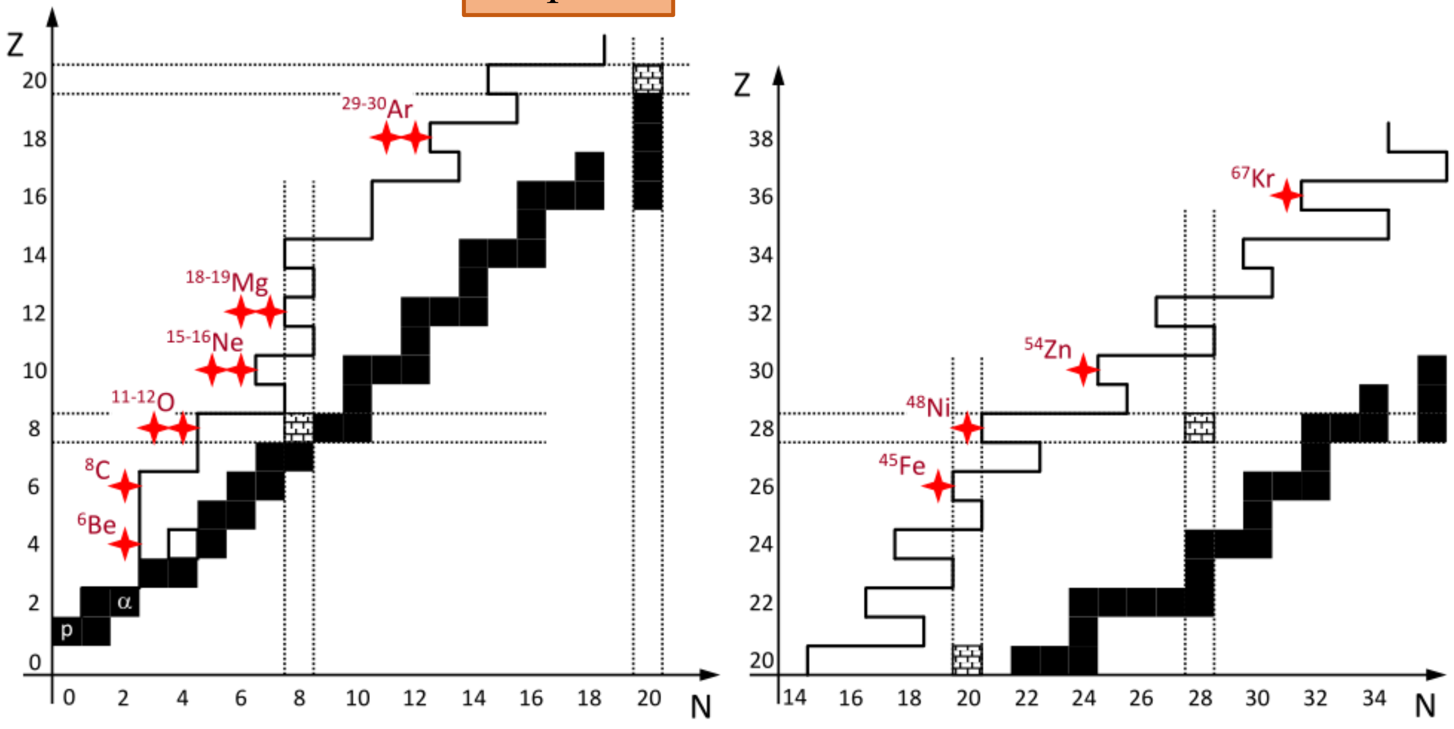


Adopted from K. Fossez's presentation.

Proton separation energy

$S_p < 0$

Proton dripline



M. Pfützner et al., Prog. Part. Nucl. Phys. 132, 104050 (2023)

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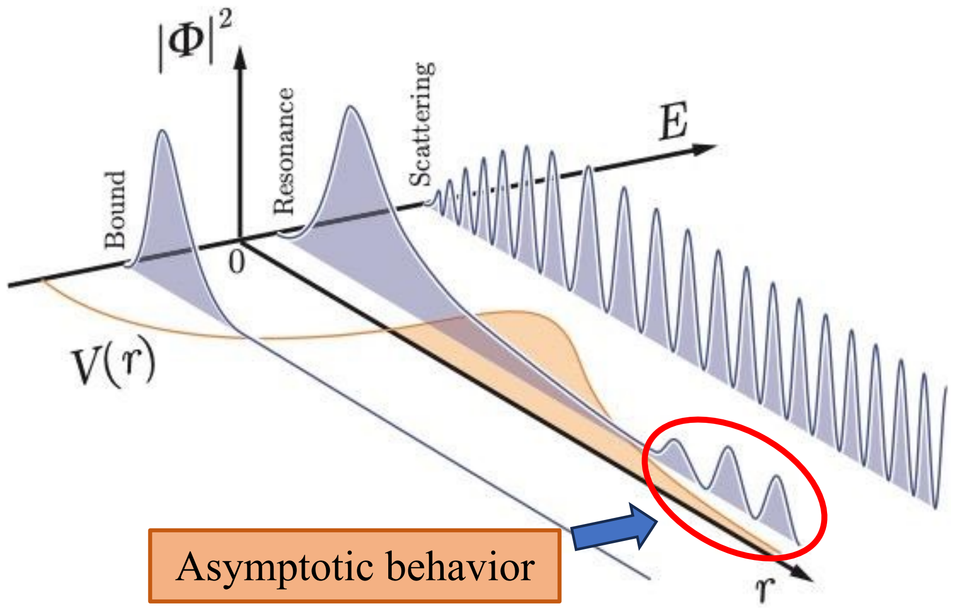


M. Pfützner *et al.*, Prog. Part. Nucl. Phys. 132, 104050 (2023).

1

Hilbert space:

$$r_{\text{rms}}^2 = \langle\psi|r^2|\psi\rangle$$

- Bound state: finite value.
- Resonance: **exponential divergence.** $\rightarrow \infty$

2

Rigged Hilbert space:

$$\tilde{E} = \langle\tilde{\psi}|H|\psi\rangle = Q_p - i\Gamma/2$$

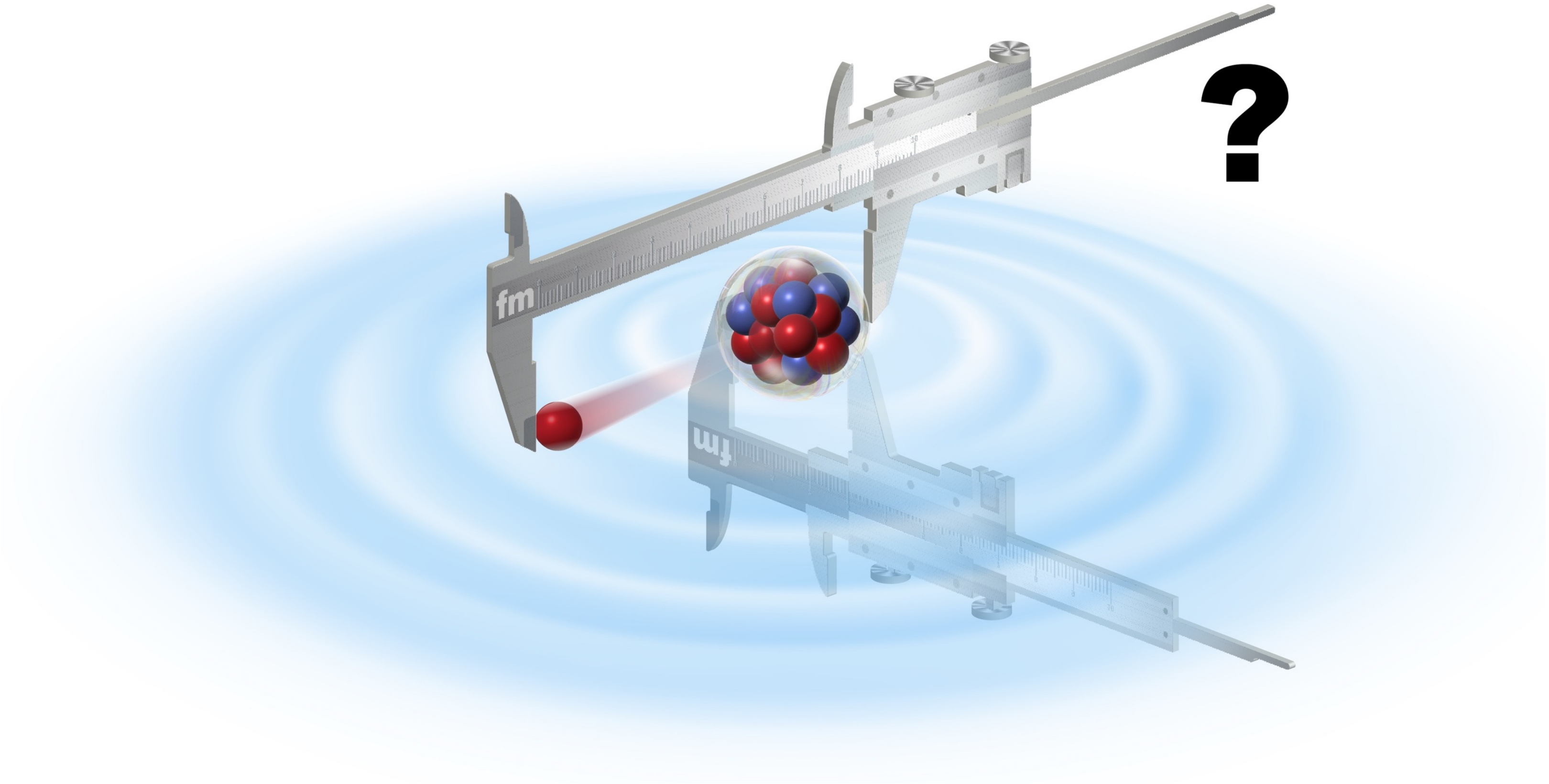
$$\tilde{r}_{\text{rms}}^2 \equiv \langle\tilde{\psi}|r^2|\psi\rangle = \int_0^\infty (r\psi)^2 r^2 dr$$

Complex expectation values

T. Berggren, Phys. Lett. B 373, 1 (1996).

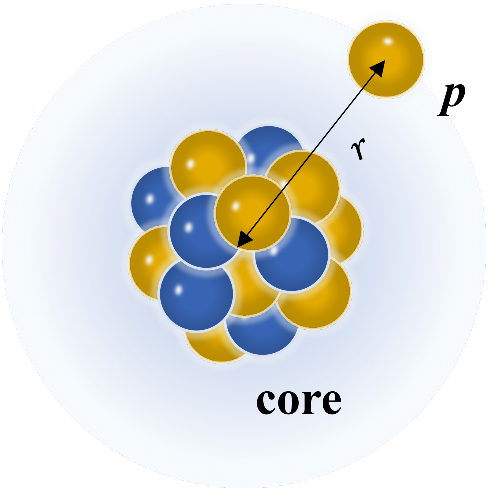
T. Myo *et al.*, Prog. Theor. Exp. Phys. 2014, 083D01 (2014).

YL, S. M. Wang, W. Nazarewicz, Phys. Rev. Lett. 136, 122501 (2026).

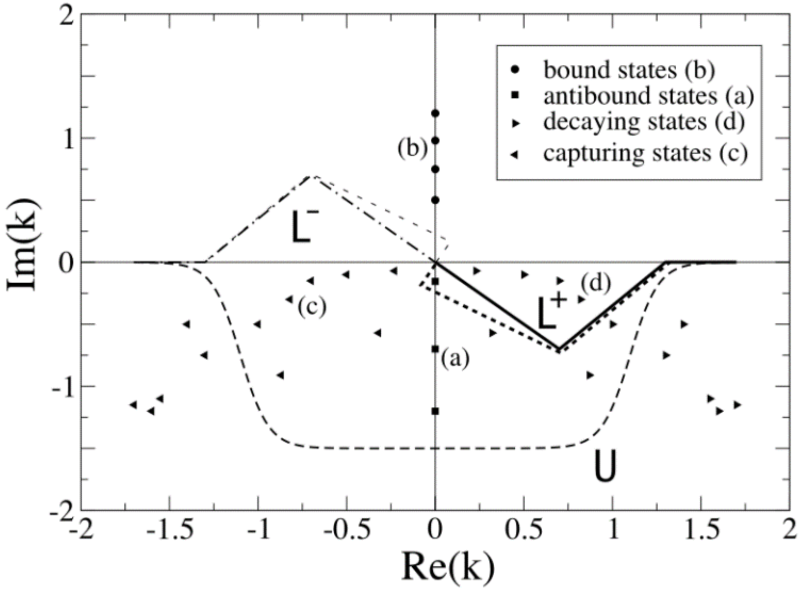


- The concept of complex expectation values has been widely used in atomic, hadronic, and nuclear physics.
- However, a direct link between the complex radius and experimental observables is still lacking.

One-proton emitter = core + *p*

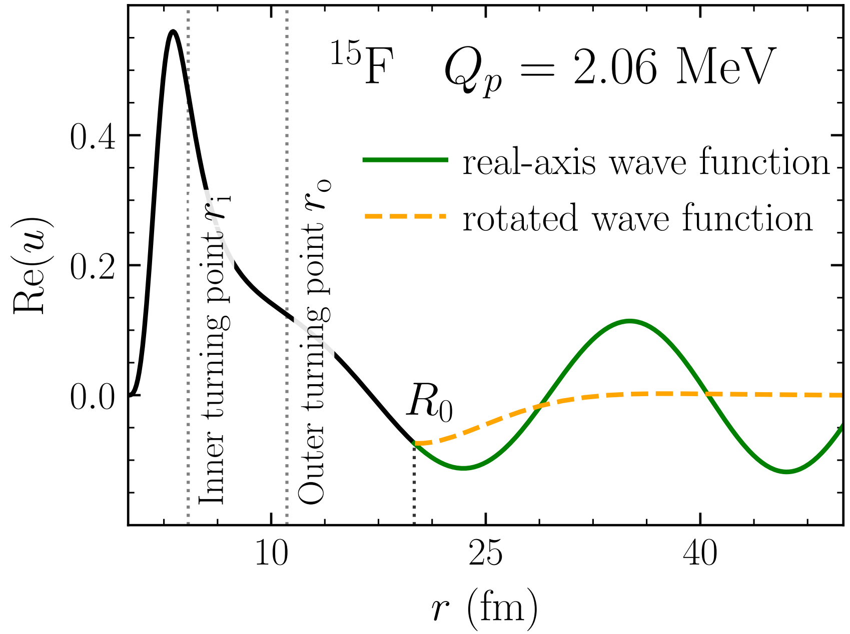


- Berggren basis



- Exterior complex scaling

B. Gyarmati *et al.*, Nucl. Phys. A 160, 523 (1971).

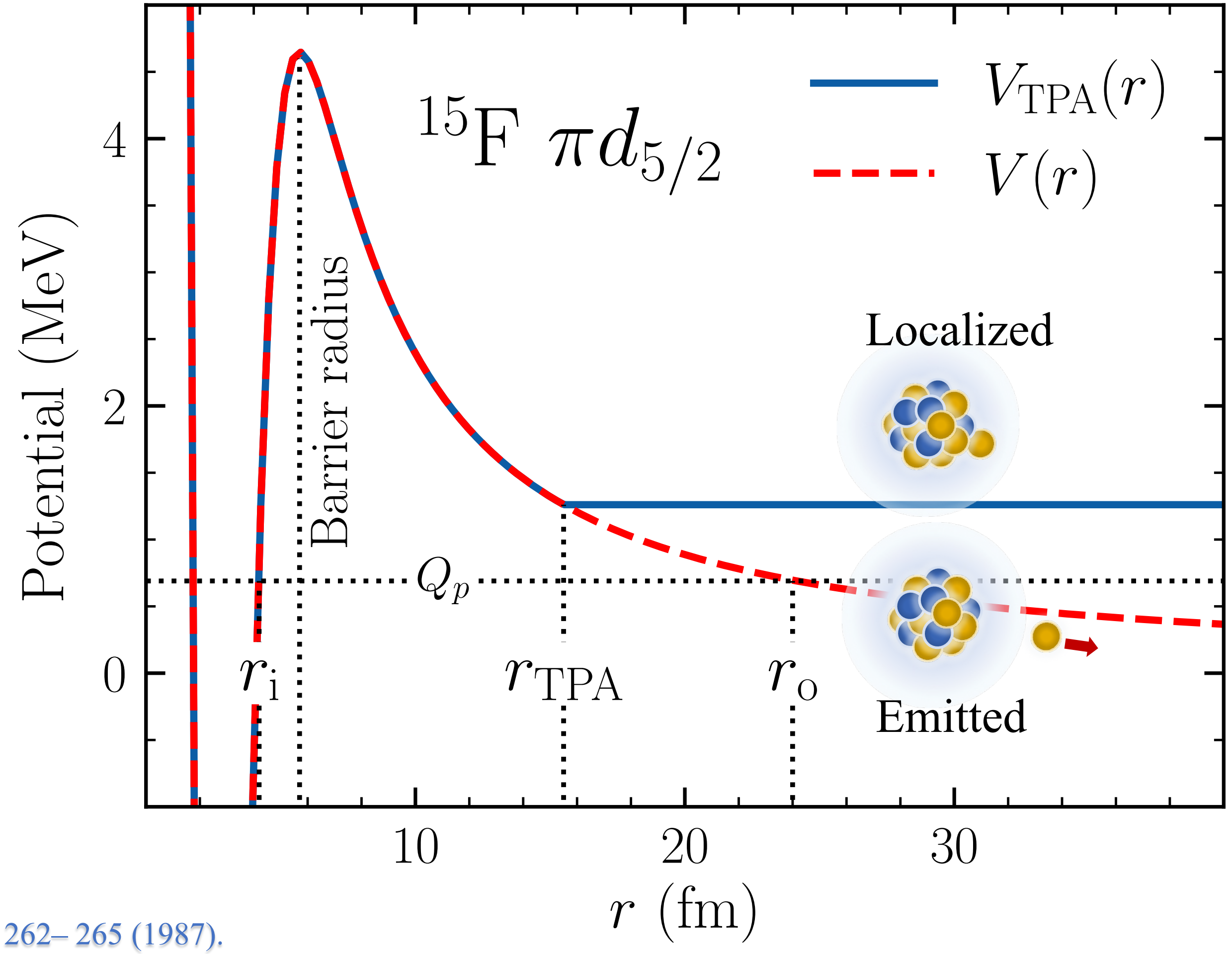


B. Buck *et al.*, Phys. Rev. C 45, 1688 (1992).
S. Åberg *et al.*, Phys. Rev. C 56, 1762 (1997).

T. Berggren, Nucl. Phys. A 109, 265 (1968).
N. Michel *et al.*, J. Phys. G: Nucl. Part. Phys. 36, 013101 (2009).

$$\tilde{r}_{\rm rms} = \left(\int_0^{R_0} r^4 \psi^2 dr - \int_{C_3} \tilde{r}^2 a_{J\ell}^2 \left(H_{\ell,\eta}^+\right)^2 d\tilde{r}\right)^{1/2}$$

- Two potential approach (TPA)



S. A. Gurvitz *et al.*, Phys. Rev. Lett. 59, 262– 265 (1987).
S. M. Wang *et al.*, Phys. Rev. Lett. 126, 142501 (2021).

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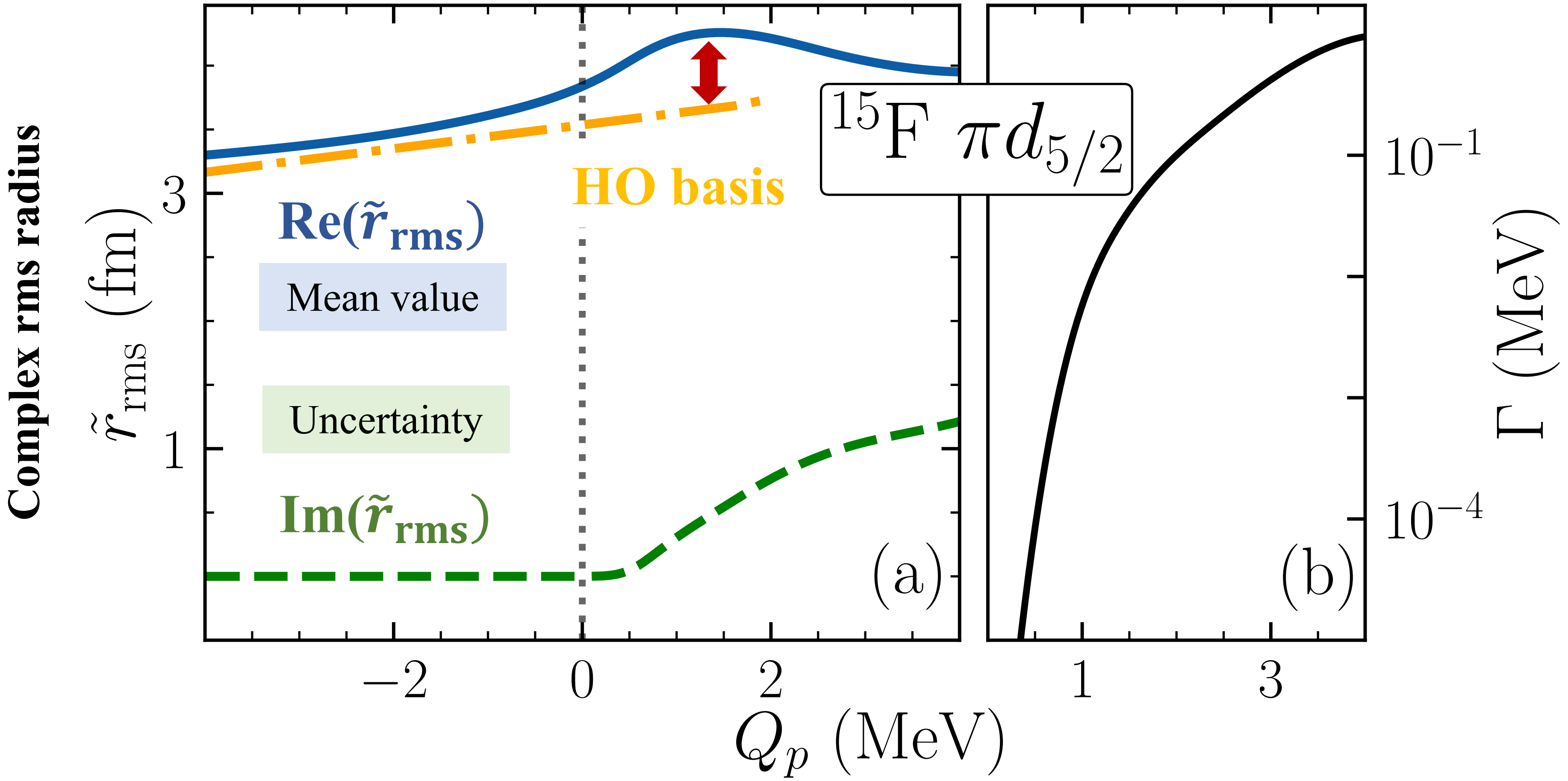
Halolike enhancement



- **Enhancement of charge radius (**Berggren–HO difference) reflects continuum coupling.

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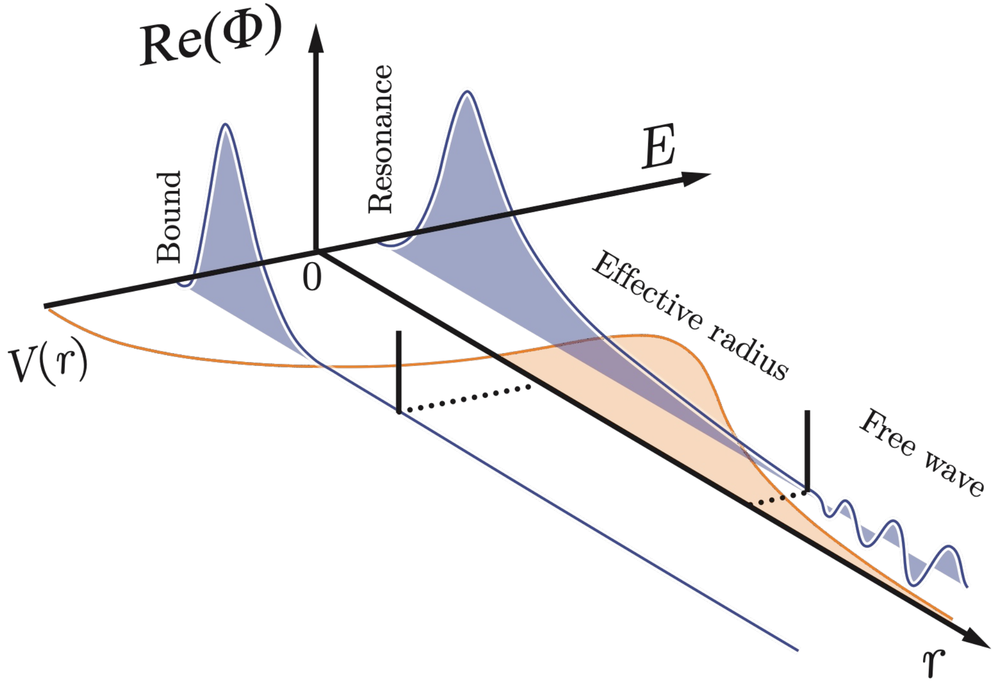
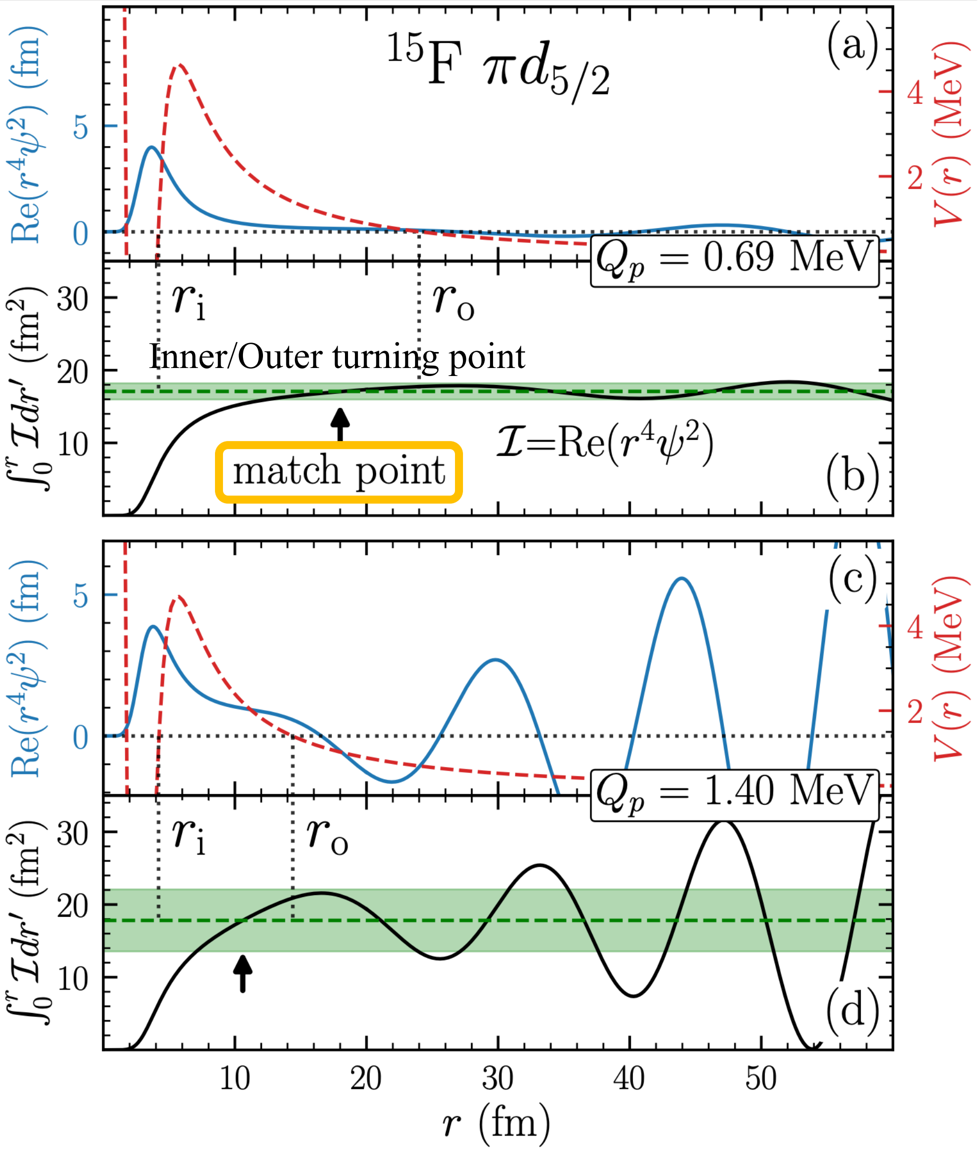
Which region provides the dominant contribution to the radius?





- The contributions from the **asymptotic region** to the complex radius essentially cancel out.
- The complex radius is determined by the integral over the **interior region**.

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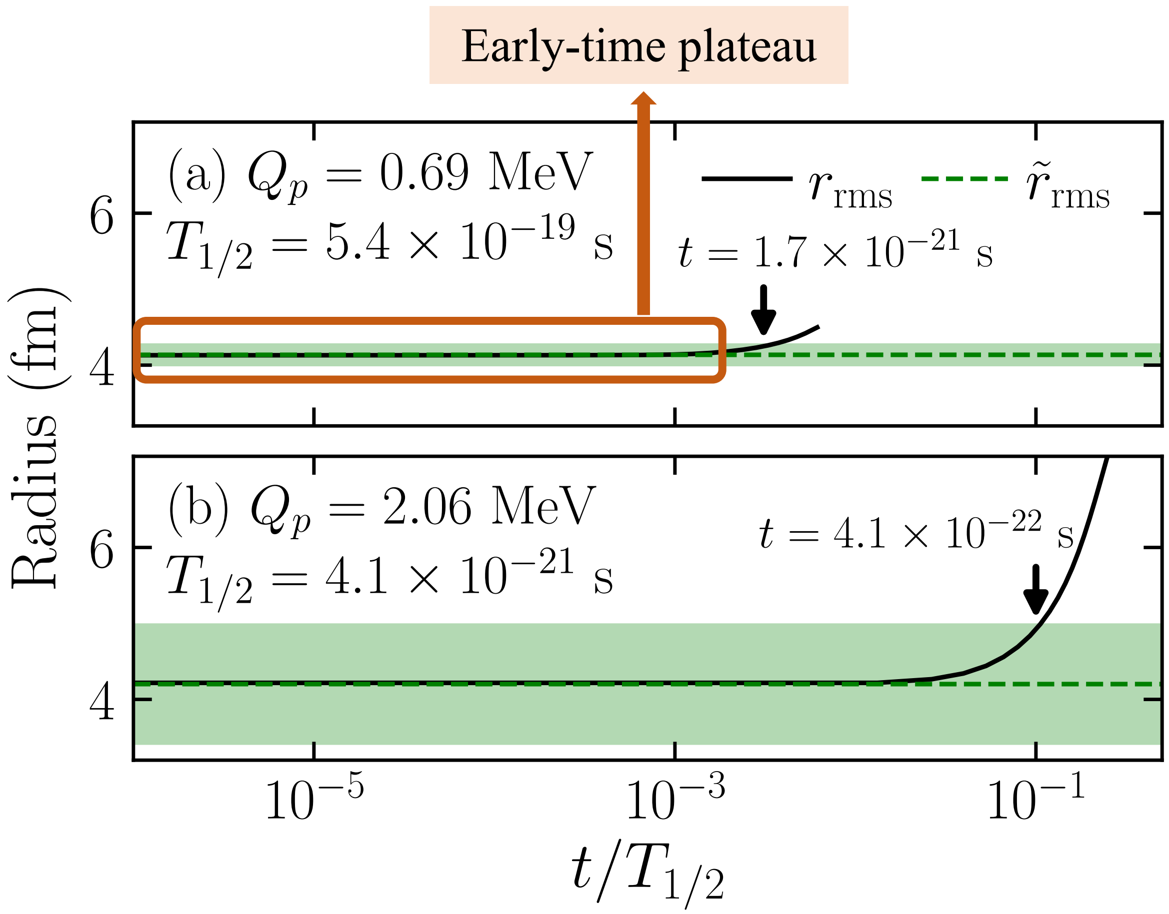

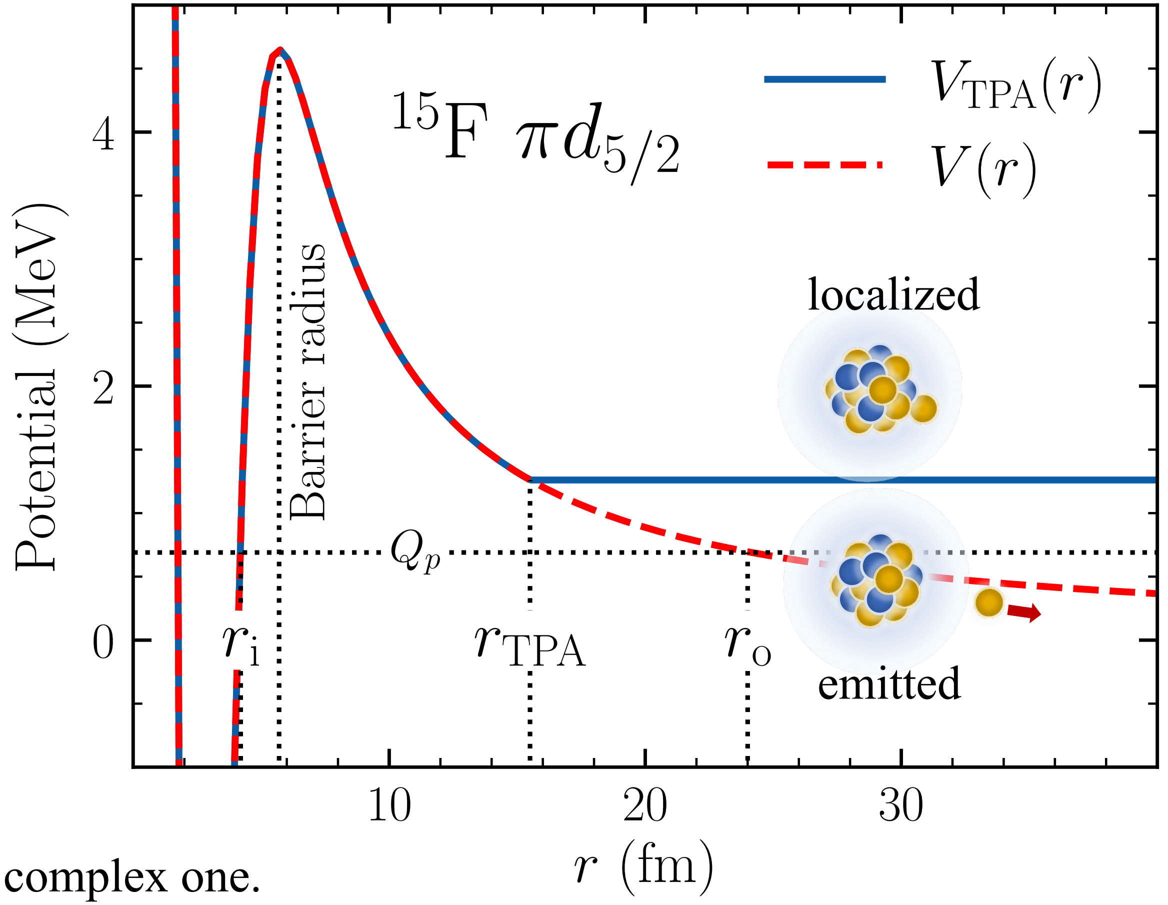

- Early-time plateau: the evolved radius matches with the complex one.
- Complex radius is experimentally accessible in early-time plateau.

Results | Time evolution of radius & Survival probability

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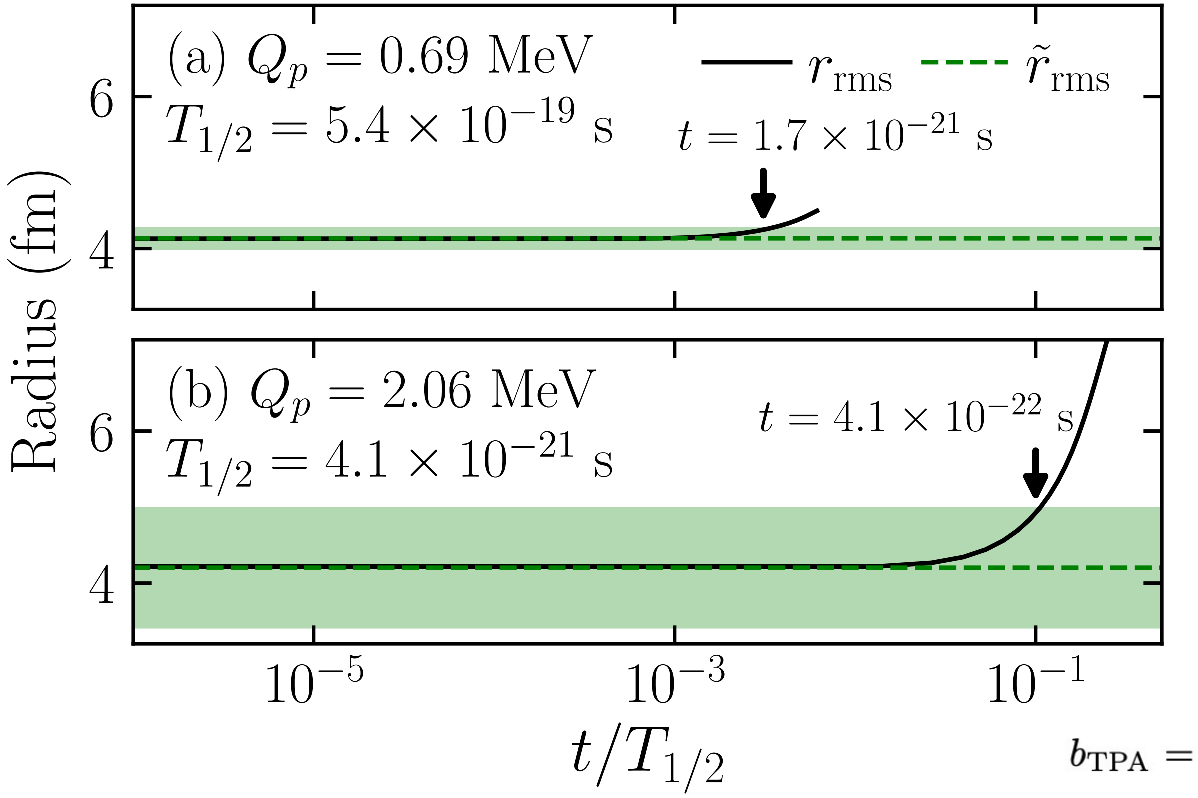
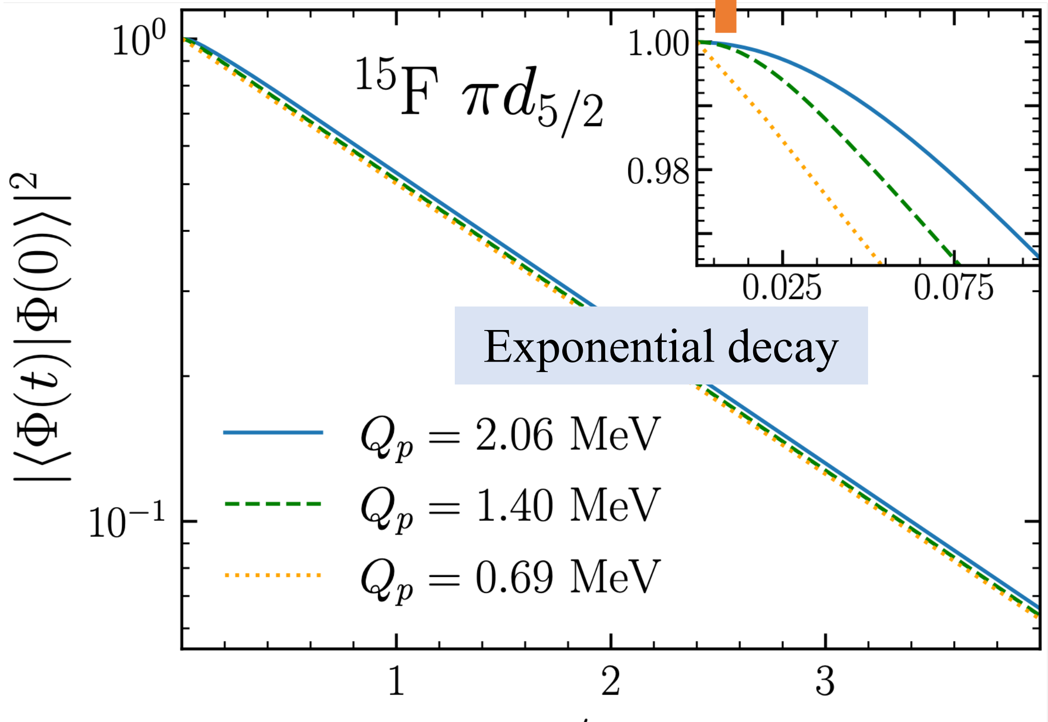


Zeno effect

Non-exponential decay



$$b_{\rm TPA} = \langle \boldsymbol{r} \cdot \boldsymbol{\nabla}(V_{\rm TPA} - V)/m \rangle_{t=0}$$

- Early non-exponential decay relates to the plateau.
- The plateau for the rms radius due to the quantum Zeno effect.
- The radius evolves quadratically in time at early times.

$$r^2_{\rm rms}(t) \simeq r^2_{\rm rms}(0) + b_{\rm TPA}\, t^2$$

When t is small

$$r_{\rm rms}(t) \simeq r_{\rm rms}(0) + \frac{1}{2}\frac{b_{\rm TPA}}{r_{\rm rms}(0)}\, t^2$$

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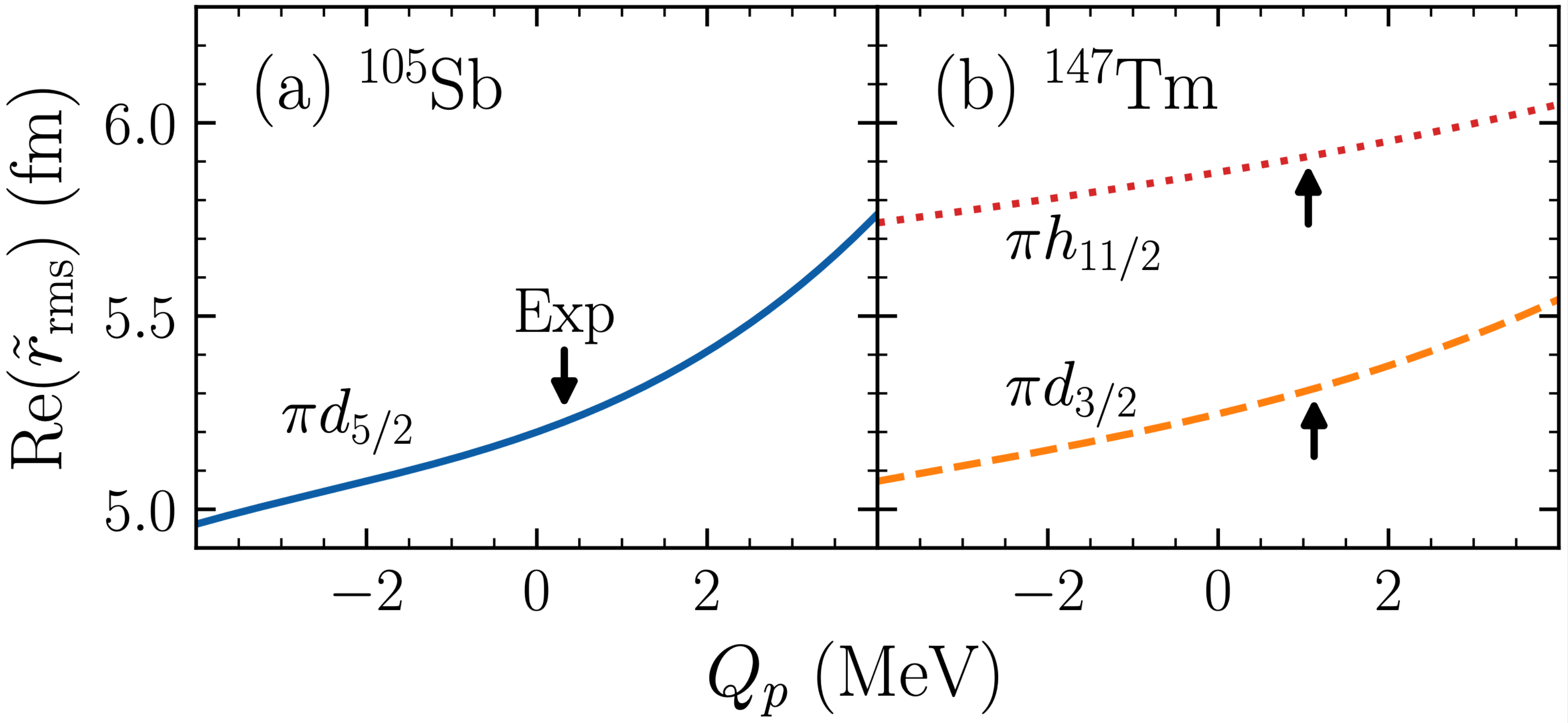


- For long-lived system, one can extract the radius experimentally.
- The enhancement of the charge radius is also shown in ^{105}Sb and ^{147}Tm.
- The total rms charge radius for experimental comparison requires combine the daughter-core radius with our computed $\tilde{r}_{\mathrm{rms}}$ including mass weighting, center-of-mass, and also nucleonic and relativistic corrections.

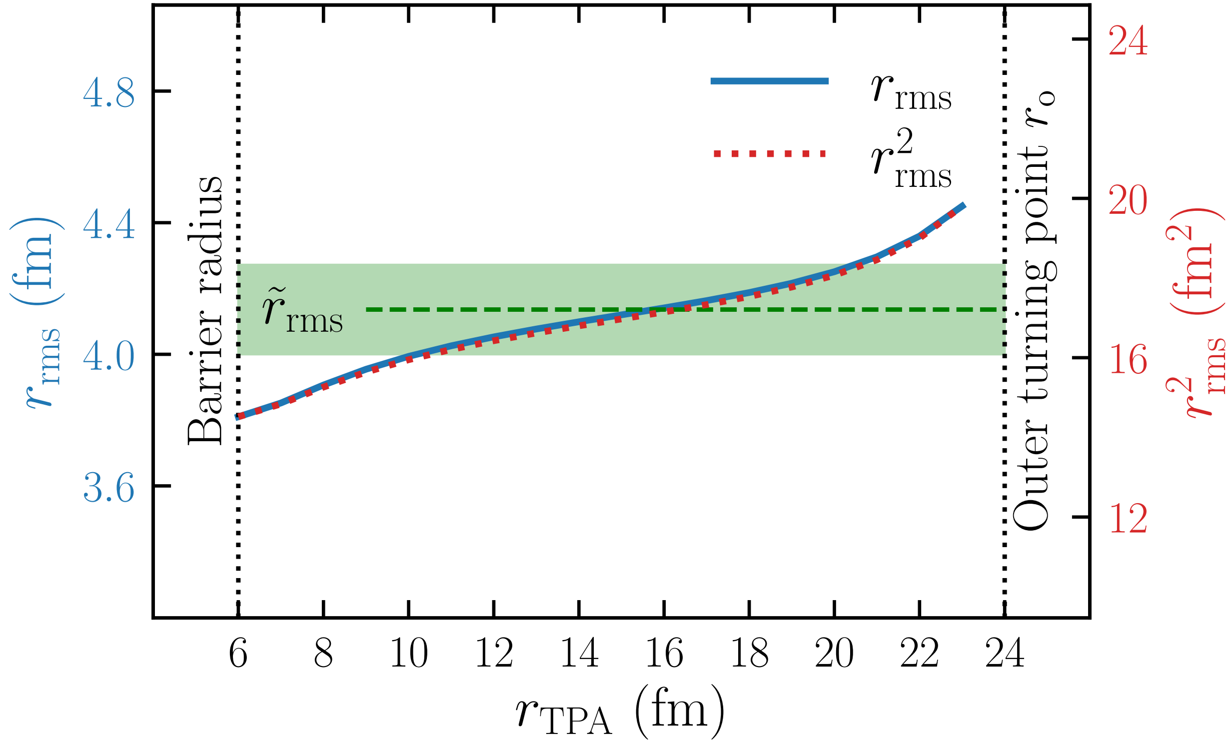
- The complex radius provides a good description of the resonance radius. Its real part corresponds to the spatial mean value, while its imaginary part is related to the uncertainty associated with the decay.

- The **early plateau** offers a temporal window for experimental observation.

- A **proton-halo effect** is predicted in proton emitters, characterized by an **enhanced charge radius**.

Thanks for your attention!

Q_p (MeV)	Γ (MeV)	$\tilde{r}_{\rm rms}$ (fm)		
		$R_0 = 20$ fm, $\theta = \pi/6$	$R_0 = 20$ fm, $\theta = \pi/4$	$R_0 = 30$ fm, $\theta = \pi/4$
0.69	8.40×10^{-4}	$4.136 + 0.136i$	$4.136 + 0.136i$	$4.136 + 0.136i$
1.40	2.48×10^{-2}	$4.251 + 0.503i$	$4.251 + 0.503i$	$4.247 + 0.504i$
2.06	1.12×10^{-1}	$4.203 + 0.784i$	$4.201 + 0.785i$	$4.212 + 0.782i$



Appendix | Formulation

Ehrenfest Theorem

Virial theorem

In this section, we provide an approximate estimation of the time-dependent evolution of the rms radius $r_{\rm rms}$. According to the Ehrenfest theorem, the time derivative of $r^2_{\rm rms}$ can be expressed as

$$\frac{d}{dt} r^2_{\rm rms} \equiv \frac{d}{dt}\langle r^2 \rangle = \frac{1}{i\hbar}\langle [\boldsymbol{r}^2, H] \rangle. \tag{S5}$$

Since $\boldsymbol{r}^2$ commutes with the potential V, and $[\boldsymbol{r}^2, \boldsymbol{p}^2] = 2i\hbar\,(\boldsymbol{r}\cdot\boldsymbol{p} + \boldsymbol{p}\cdot\boldsymbol{r})$, we obtain

$$\frac{d}{dt} r^2_{\rm rms} = \frac{1}{m}\,\langle \boldsymbol{r}\cdot\boldsymbol{p} + \boldsymbol{p}\cdot\boldsymbol{r} \rangle. \tag{S6}$$

At $t = 0$, the decay rate of the resonance vanishes [59–62], so the first derivative of $r^2_{\rm rms}$ is zero, and the second derivative can be evaluated as

$$\frac{m}{2}\frac{d^2}{dt^2} r^2_{\rm rms} = 2\langle T \rangle - \langle \boldsymbol{r}\cdot\boldsymbol{\nabla} V \rangle, \tag{S7}$$

where $T = \boldsymbol{p}^2/(2m)$ is the kinetic energy operator.

The initial wave function $\Phi(t = 0)$ is prepared using the Two Potential Approach (TPA) [57], in which the system is confined by an auxiliary potential $V_{\rm TPA}$ shown in Fig. S1 that coincides with the physical potential V inside a chosen matching radius $r_{\rm TPA}$, but is constant in the outer region to produce a quasi-bound state. According to the virial theorem, one has

$$\begin{aligned} 2\langle T \rangle_{t=0} &= \langle \boldsymbol{r}\cdot\boldsymbol{\nabla} V_{\rm TPA} \rangle_{t=0}, \\ \big[2\langle T \rangle - \langle \boldsymbol{r}\cdot\boldsymbol{\nabla} V \rangle \big]_{t=0} &= \langle \boldsymbol{r}\cdot\boldsymbol{\nabla}(V_{\rm TPA} - V) \rangle_{t=0}. \end{aligned} \tag{S8}$$

Thus, Eq. (S7) can be approximated at $t = 0$ as

$$\frac{m}{2}\frac{d^2}{dt^2} r^2_{\rm rms} \simeq \langle \boldsymbol{r}\cdot\boldsymbol{\nabla}(V_{\rm TPA} - V) \rangle_{t=0}. \tag{S9}$$

Integrating twice with respect to time gives the approximate expression

$$r^2_{\rm rms}(t) \simeq r^2_{\rm rms}(0) + b_{\rm TPA}\, t^2, \tag{S10}$$

where

$$b_{\rm TPA} = \frac{1}{m}\langle \boldsymbol{r}\cdot\boldsymbol{\nabla}(V_{\rm TPA} - V) \rangle_{t=0}. \tag{S11}$$

As $b_{\rm TPA}$ depends only on the asymptotic form of the wave function at $r > r_{\rm TPA}$, and the integrand $\boldsymbol{r}\cdot\boldsymbol{\nabla}(V_{\rm TPA} - V)$ decays as r^{-1}, $b_{\rm TPA}$ is exceedingly small for narrow resonances. For instance, for ^{15}F, we obtain $b_{\rm TPA} = 2.61 \times 10^{-6}\ c^2$