

# Effects of Chiral symmetry restoration

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“Chiral symmetry breaking, chiral partners, and the  $K_1$  and  $K^*$  in medium”

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# Symmetries in Quantum Chromodynamics (QCD)

- Symmetry of Lagrangian when  $m \rightarrow 0$ ;  $U(3)_R \times U(3)_L$

$q \rightarrow \exp[i\theta (1 \pm \gamma^5)]q$  where  $\theta = \theta^a \lambda^a$  including  $\lambda^0 = 1$

- Anomaly breaks the  $U_A(1)$  part  $SU(3)_R \times SU(3)_L \times U(1)$

broken:  $q \rightarrow \exp[i\theta (\gamma^5)]q$  where  $\theta = \theta^0 \lambda^0$

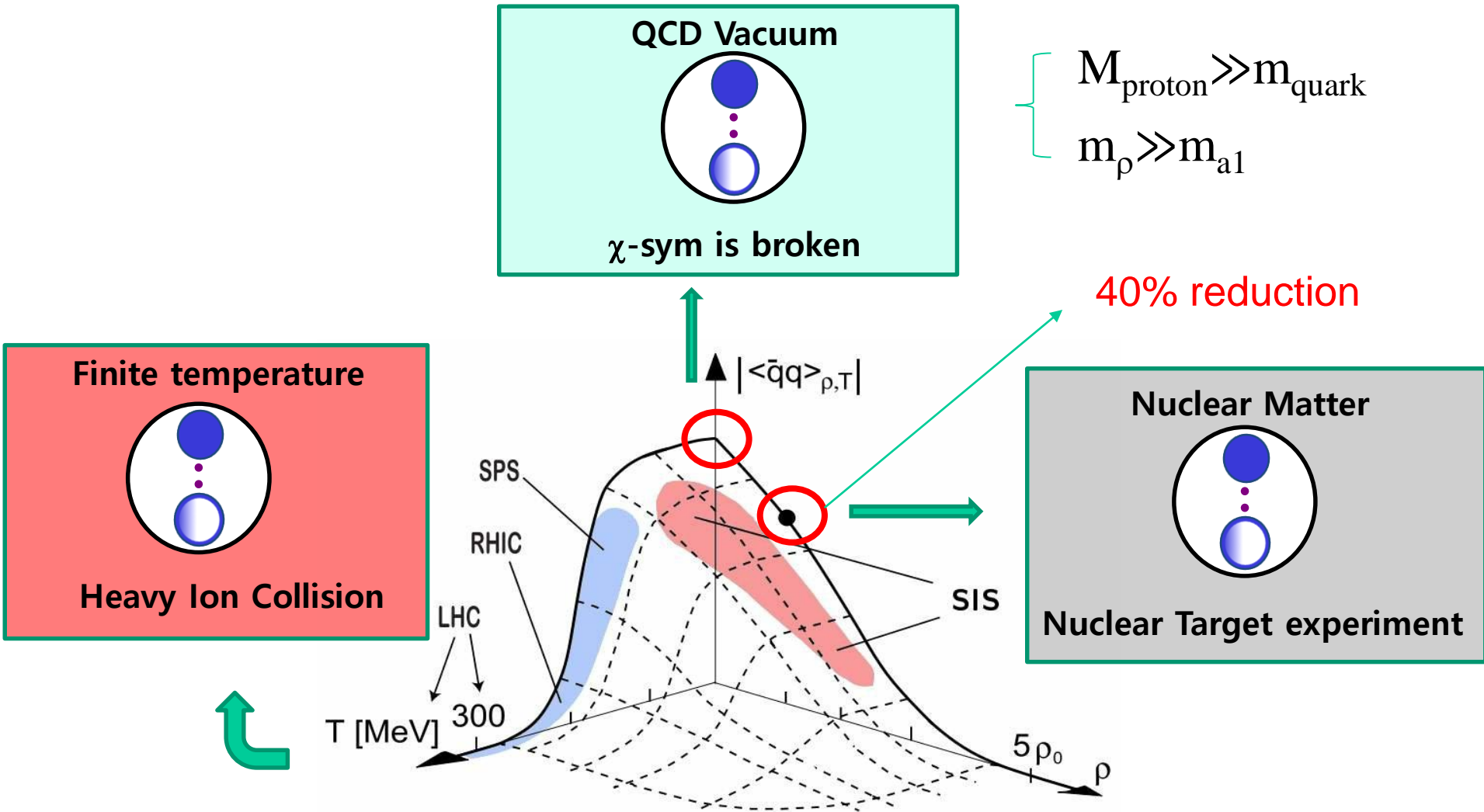
- Spontaneous breaking of Chiral symmetry  $SU(3)_R \times SU(3)_L \rightarrow SU(3)_V$

broken  $q \rightarrow \exp[i\theta (\gamma^5)]q$  where  $\theta = \theta^a \lambda^a$  including  $a = 1, \dots, 8$

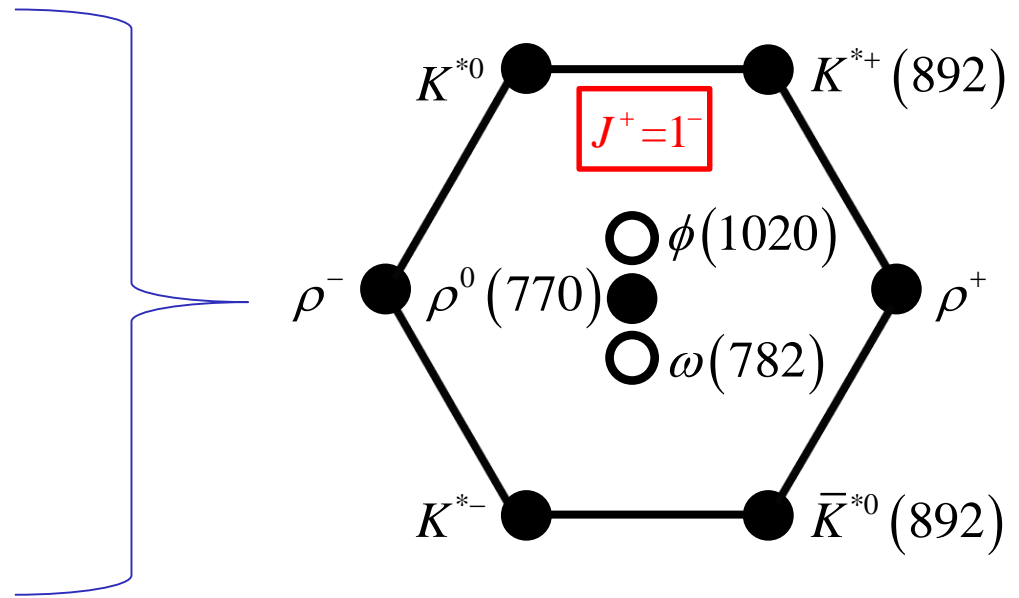
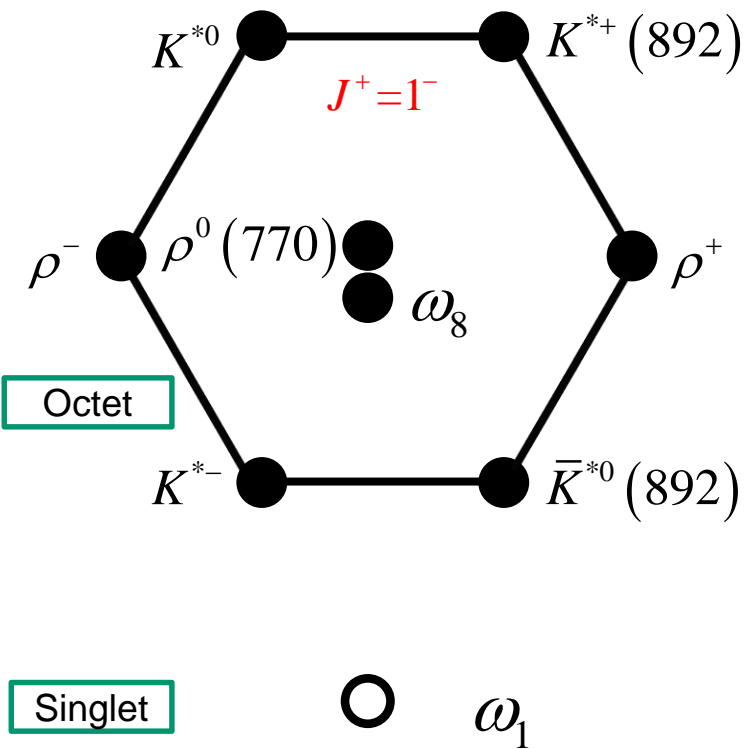
But we have  $SU(3)$  Flavor symmetry (u, d, s quark)

→ Can distinguish all effect when looking at correct observables

# Chiral symmetry restoration and hadron mass



# Vector meson in Flavor SU(3) symmetry and broken symmetry



$$\omega_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$$

$$\omega_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

Mixing due to  $m_s \gg m_d \sim m_u$

$$\phi(1020) = \bar{s}s$$

$$\omega(782) = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

# Chiral transformation of quark bilinears

- Chiral symmetry  $SU(3)_R \times SU(3)_L$

$$q_{R,L} \rightarrow \exp\left(i\vec{\theta}\left(1 \pm \gamma^5\right)\right) q_{R,L} \quad \text{where} \quad \vec{\theta} = \theta^a \lambda^a$$

- Spontaneous breaking of Chiral symmetry  $SU(3)_R \times SU(3)_L \rightarrow SU(3)_V$

☞ Broken part is  $q \rightarrow \exp\left(i\vec{\theta}\gamma^5\right) q, \quad \bar{q} \rightarrow \bar{q} \exp\left(i\vec{\theta}\gamma^5\right)$

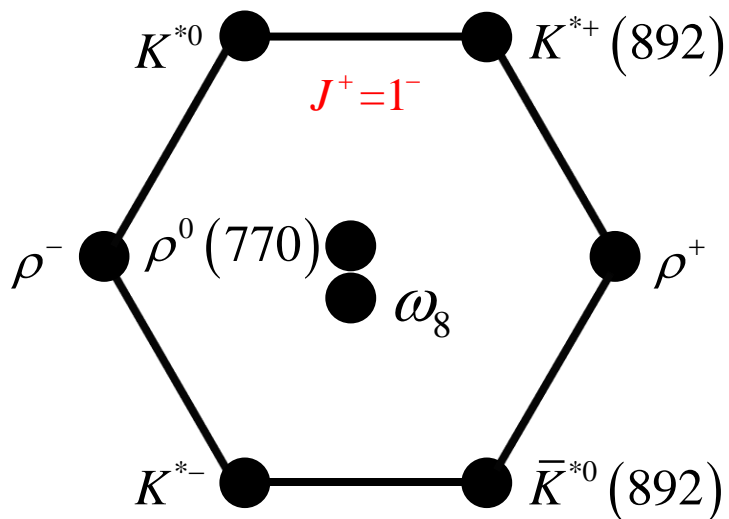
- Quark bilinear  $\bar{q}\Gamma q \rightarrow \bar{q}\left(1+i\vec{\theta}\gamma^5\right)\Gamma\left(1+i\vec{\theta}\gamma^5\right)q \rightarrow \bar{q}\Gamma q + \bar{q}\left(i\vec{\theta}\gamma^5\Gamma + \Gamma i\vec{\theta}\gamma^5\right)q$

$$\bar{q}q \rightarrow \bar{q}q + 2\bar{q}\left(i\vec{\theta}\gamma^5\right)q$$

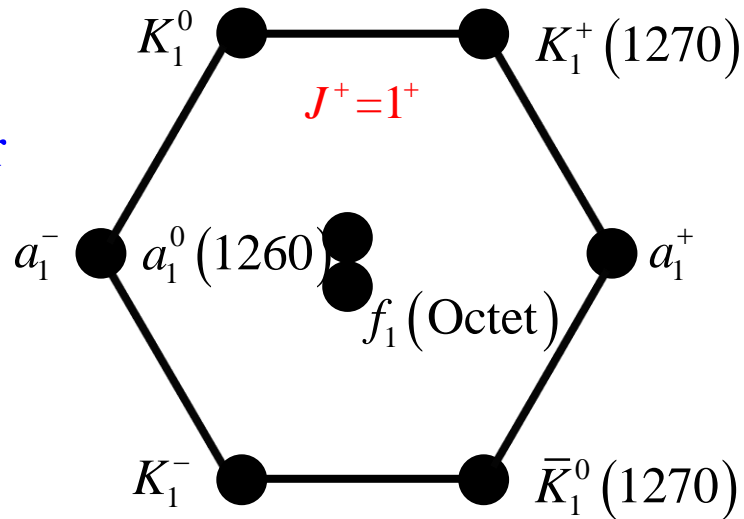
$$\bar{q}\gamma^\mu\tau^a q \rightarrow \bar{q}\gamma^\mu\tau^a q + \bar{q}i\gamma^5\gamma^\mu\left[\vec{\theta},\tau^a\right]q$$

$$\bar{q}\gamma^\mu q \rightarrow \bar{q}\gamma^\mu q + \bar{q}i\gamma^5\gamma^\mu\left[\vec{\theta},1\right]q = \bar{q}\gamma^\mu q$$

# Spin-1 Flavor Octet

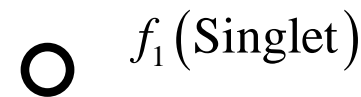


Chiral Partner



# Spin-1 Flavor Singlet

No Chiral Partner



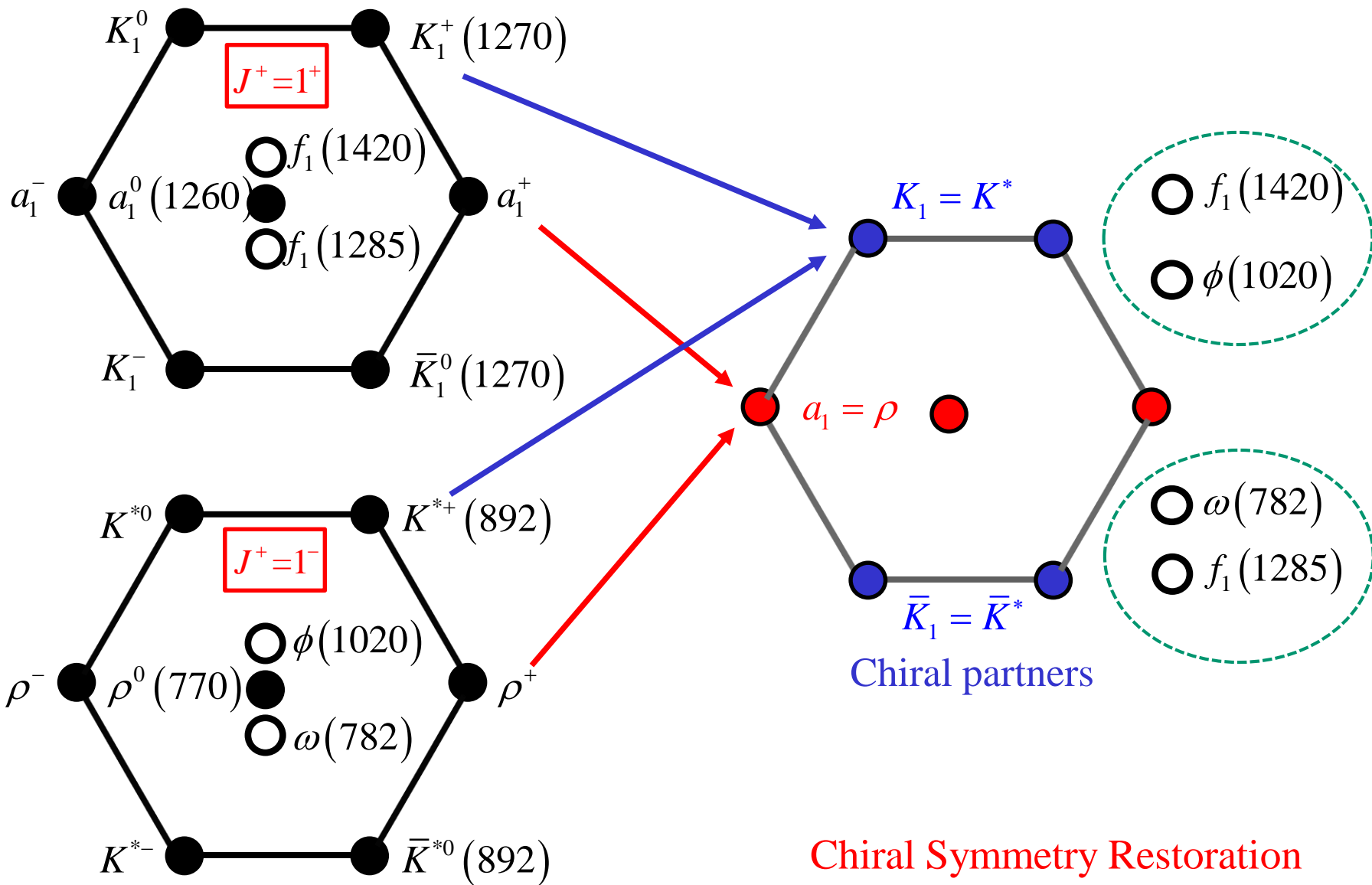
$$\phi(1020) = \bar{s}s$$

$$\omega(782) = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

$$m_s \gg m_d \sim m_u$$

$$\omega_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$$

$$\omega_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$



Broken Chiral Symmetry

Chiral Symmetry Restoration

# Chiral symmetry restoration

1) 40 % at nuclear matter

2) Order parameters of chiral symmetry

$$\Rightarrow \langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle = -\lim_{x \rightarrow 0} \langle \text{Tr} [S(x, 0)] \rangle = -\lim_{x \rightarrow 0} \left\langle \frac{1}{2} \text{Tr} [S(x, 0) - i\gamma^5 S(x, 0) i\gamma^5] \right\rangle$$

Through model calculation  $\rightarrow$  pionic atom,  $\pi$ -N production at threshold

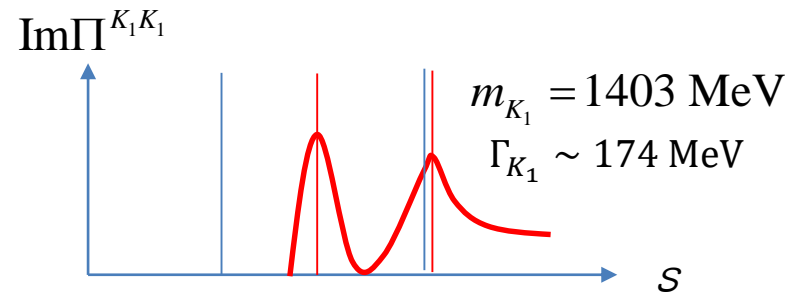
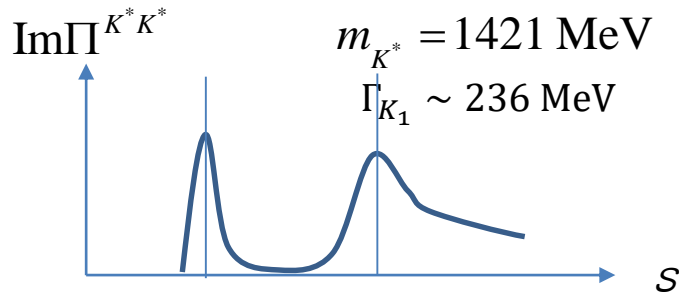
$$\begin{aligned} \Rightarrow \Pi^{VV} - \Pi^{AA} &= \frac{1}{V} \int d^4x \left[ \langle \bar{q} \gamma^\mu \tau^a q(x), \bar{q} \gamma^\mu \tau^a q(0) \rangle - \langle \bar{q} \tau^a i\gamma^5 \gamma^\mu q(x), \bar{q} \tau^a i\gamma^5 \gamma^\mu q(0) \rangle \right] \\ &= -\frac{1}{2} \text{Tr} \left[ \gamma^\mu (S(x, 0) - i\gamma^5 S(x, 0) i\gamma^5) \gamma^\mu (S(0, x) - i\gamma^5 S(0, x) i\gamma^5) \right] \end{aligned}$$

Through model calculation  $\rightarrow$  pionic atom,  $\pi$ -N production at threshold

$$\begin{aligned} \Rightarrow \Pi^{K^*K^*} - \Pi^{K_1K_1} &= \dots - \frac{2\pi\alpha}{Q^6} \left[ \langle (\bar{u} \gamma_\mu \gamma^5 \lambda^a s) (\bar{s} \gamma_\mu \gamma^5 \lambda^a u) \rangle - \langle (\bar{u} \gamma_\mu \lambda^a s) (\bar{s} \gamma_\mu \lambda^a u) \rangle \right] \\ &= \dots + \frac{2\pi\alpha}{Q^6} \left[ \langle (\bar{s} \lambda^a \gamma_\mu [S_u(x, 0) - i\gamma^5 S_u(x, 0) i\gamma^5] \lambda^a \gamma_\mu s) \rangle - \langle u \leftrightarrow s \rangle \right] \end{aligned}$$

- Weinberg type sum rules

☞ 
$$\Pi^{K^*} - \Pi^{K_1} = \dots \frac{1}{Q^6} [2 \langle B_{su} \rangle] + \dots = \frac{1}{\pi} \int ds \frac{\text{Im}(\Pi^{K^*} - \Pi^{K_1})}{s - q^2}$$



☞ 
$$f_{K^*}^2 m_{K^*}^4 - f_{K_1}^2 m_{K_1}^4 = -2m_s \langle \bar{u}u \rangle \quad f_{K^*}^2 m_{K^*}^6 - f_{K_1}^2 m_{K_1}^6 = 2 \langle B_{su} \rangle = -64\pi\alpha_s \langle \bar{u}u \rangle \langle \bar{s}s \rangle$$

➔ 
$$f_{K^*}^2 m_{K^*}^4 (m_{K_1}^2 - m_{K^*}^2) = 2m_s \langle \bar{u}u \rangle m_{K_1}^2 + 64\pi\alpha_s \langle \bar{u}u \rangle \langle \bar{s}s \rangle$$

➔ 
$$m_{K_1}^2(T) = m_{K^*}^2 + \frac{\langle \bar{u}u \rangle_T}{\langle \bar{u}u \rangle_0} (m_{K_1}^2 - m_{K^*}^2)$$

## 2) Order parameters of chiral symmetry

$$\begin{aligned}\Pi^{\sigma\sigma} - \Pi^{\pi\pi} &= \frac{1}{V} \int d^4x \left[ \langle \bar{q}q(x), \bar{q}q(0) \rangle - \langle \bar{q}\tau^a i\gamma^5 q(x), \bar{q}\tau^a i\gamma^5 q(0) \rangle \right] \\ &= -\frac{1}{2} \text{Tr} \left[ (S(x,0) - i\gamma^5 S(x,0)i\gamma^5)(S(0,x) - i\gamma^5 S(0,x)i\gamma^5) \right]\end{aligned}$$

$\sigma(500) \rightarrow \pi\pi$  production at threshold a possible project at **RAON**

**3) Don't give up (work on it for 30 years)**