Tidings from Domain Walls:
Flavour Hierarchies are making (gravitational) Waves!

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CHEF

Outline

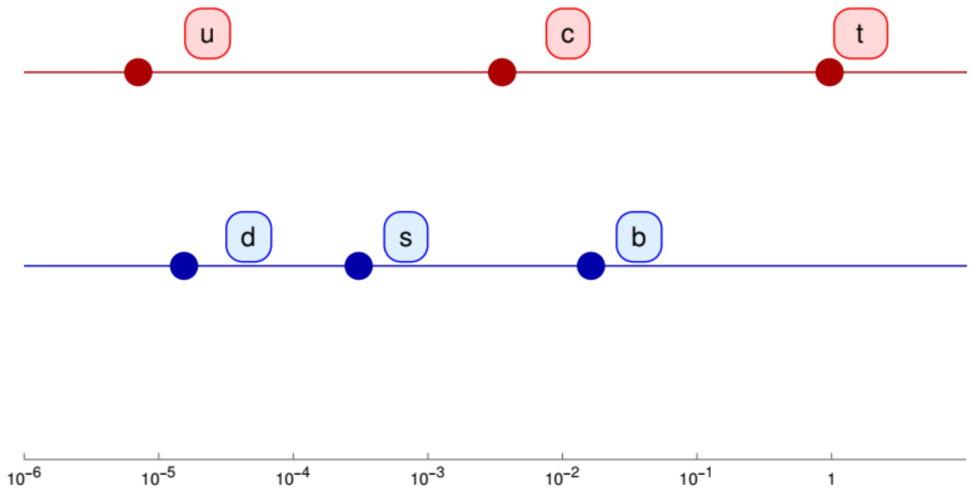
Flavour Puzzle
meets
Cosmological Consequences?

Outline

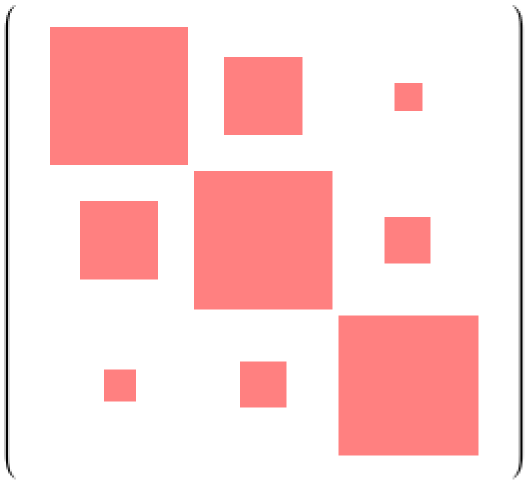
- Discrete Froggatt-Nielsen Models
- Domain Walls and Assumptions
- SUSY
  - Scalar Sector
  - Resulting Gravitational Waves
- Conclusions

Mandatory Introduction

- (Quark) Flavour Puzzle



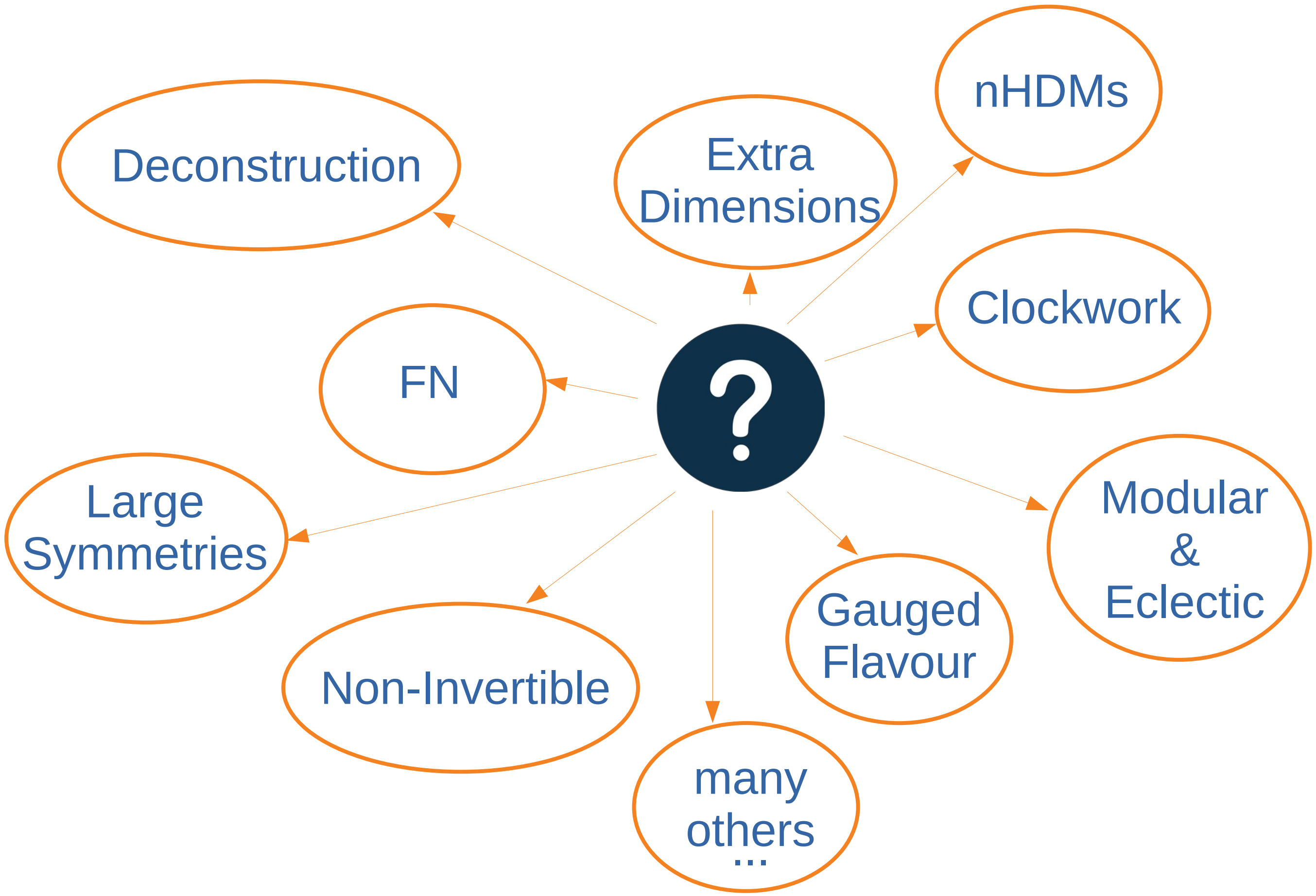
$$V_{\mathrm{CKM}} =$$



Is There an Underlying Reason?

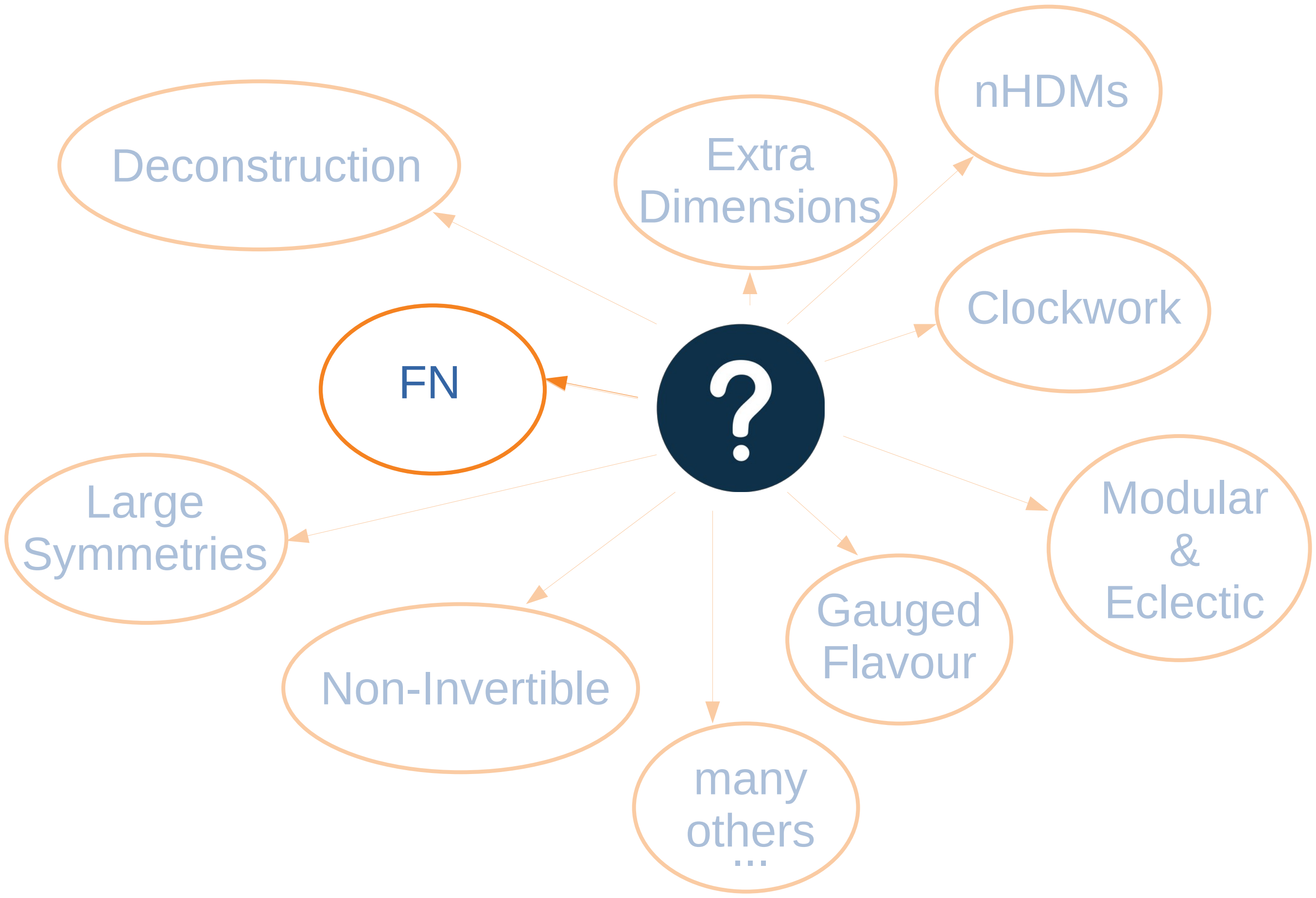
Mandatory Introduction

- One Question → Many Answers



Mandatory Introduction

- One Question → Many Answers



Froggatt-Nielsen (Quarks)

- No Yukawas $\longrightarrow$ $U(3)^5$ Symmetry
- All Yukawas $\longrightarrow$ obviously not

$U(3)^5$ breaking from G_F SSB

- (Some) Yukawas forbidden @ tree-level
- FN Chains generate low-scale Yukawas

EFT $\longrightarrow$ Spurion: $\boxed{\varepsilon = \frac{v_0}{\Lambda}}$

Froggatt-Nielsen (Quarks)

FN Chains generate low-scale Yukawas

$$Y_u \sim \begin{pmatrix} \varepsilon^{n^u_{11}} & \varepsilon^{n^u_{12}} & \varepsilon^{n^u_{13}} \\ \varepsilon^{n^u_{21}} & \varepsilon^{n^u_{22}} & \varepsilon^{n^u_{23}} \\ \varepsilon^{n^u_{31}} & \varepsilon^{n^u_{32}} & \varepsilon^{n^u_{33}} \end{pmatrix}$$

$$Y_d \sim \begin{pmatrix} \varepsilon^{n^d_{11}} & \varepsilon^{n^d_{12}} & \varepsilon^{n^d_{13}} \\ \varepsilon^{n^d_{21}} & \varepsilon^{n^d_{22}} & \varepsilon^{n^d_{23}} \\ \varepsilon^{n^u_{31}} & \varepsilon^{n^d_{32}} & \varepsilon^{n^d_{33}} \end{pmatrix}$$

- Goal: Choose n^f_{ij}

$\downarrow$

Well-behaved PT

$\downarrow$

upper bounds on m^f_i (and θ_{ij})

"Good" n^f_{ij} choices $\longrightarrow$ $\mathcal{O}(1)$ Correct masses & mixings

Froggatt-Nielsen (Quarks)

- Standard Formulation: $\boxed{G_F = U(1)}$

Choose n_{ij}^f

$$Y^f = \cdots$$

But:

large enough n and $\varepsilon \to 1$ → $\mathcal{O}(1)$ becomes trivial

Preferably low powers of ε

Froggatt-Nielsen (Quarks)

- Discrete Versions:

$$\boxed{G_F = Z_N}$$

We only care about the textures...

not much changes

$\boxed{\text{Why not?}}$

Froggatt-Nielsen (Quarks)

- Discrete Versions:

$$\boxed{G_F = Z_N}$$

Smallest Hierarchy $\longrightarrow$ defines ε (usually $\varepsilon \sim \lambda$)

Largest Hierarchy $\longrightarrow$ defines Z_N (usually $y_u \sim \varepsilon^8$)

Froggatt-Nielsen (Quarks)

- Discrete Versions: $\boxed{G_F = Z_N}$

Smallest Hierarchy $\longrightarrow$ defines ε (usually $\varepsilon \sim \lambda$)

λ not too small & $\lambda \sim \theta_{12}^u + \theta_{12}^d$ $\longrightarrow$ $\varepsilon \sim \lambda^2$

Largest Hierarchy $\longrightarrow$ defines Z_N (usually $y_u \sim \varepsilon^8$)

$$y_u \sim \varepsilon^4 \longrightarrow \begin{cases} \text{SM:} & Z_8 \\ \text{MSSM:} & Z_5 \end{cases}$$

Froggatt-Nielsen (Quarks)

- One example

$$Y_u \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}$$

$$Y_d \sim \varepsilon \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & \varepsilon & \varepsilon \\ 1 & 1 & 1 \end{pmatrix}$$

t_β

Upsides

- large t_β
- SU(5) embeddable
- no PMNS selection rules
- Z_N not too large

Downsides (common to FNs)

- Not “predictive”
- No information on scales

Froggatt-Nielsen (Quarks)

- One example

$$Y_u \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^3 & \varepsilon \\ \varepsilon^4 & \varepsilon^3 & 1 \end{pmatrix}$$

$$Y_d \sim \underset{\displaystyle\downarrow \atop \displaystyle t_\beta}{\varepsilon} \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & \varepsilon & \varepsilon \\ 1 & 1 & 1 \end{pmatrix}$$

Upsides

- large t_β
- SU(5) embeddable
- no PMNS selection rules
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Downsides (common to FNs)

- Not “predictive”
- $\boxed{\text{No information on scales}}$

Froggatt-Nielsen (Quarks)

- No info on scales
  - Extra pheno w/ FN sector (FCNCs)

$G_F = Z_N$ → Axial mode is massive

decoupling $v_0 \, , \, \Lambda \nearrow\nearrow$

Invisible

Can we get any information on upper bounds?

Froggatt-Nielsen (Quarks)

$$G_F = Z_N$$

Gravitational

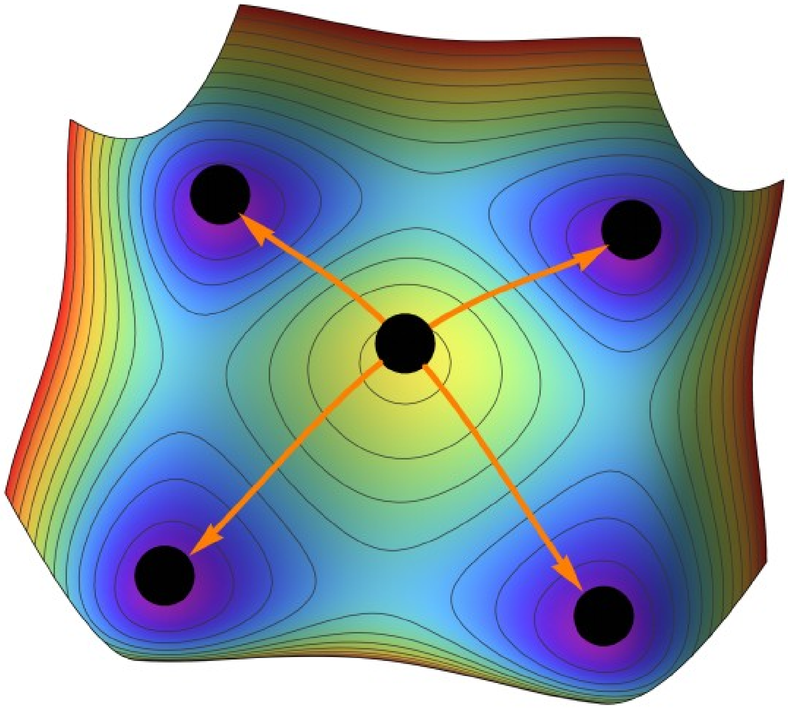
Waves!

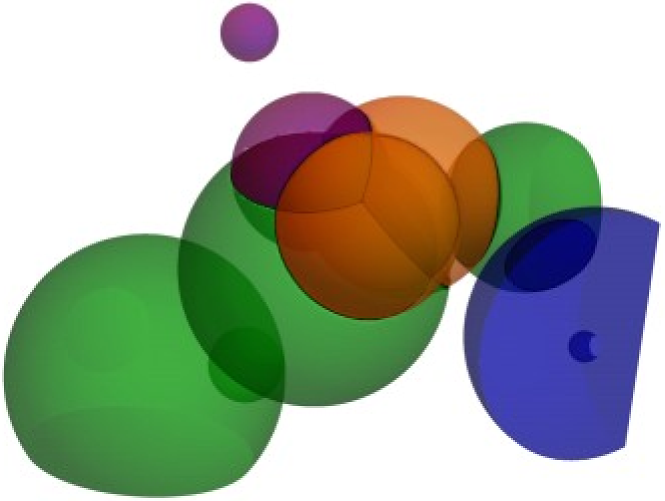
(maybe)

Can we get any information on upper bounds?

Scalar Sector: Domain Walls

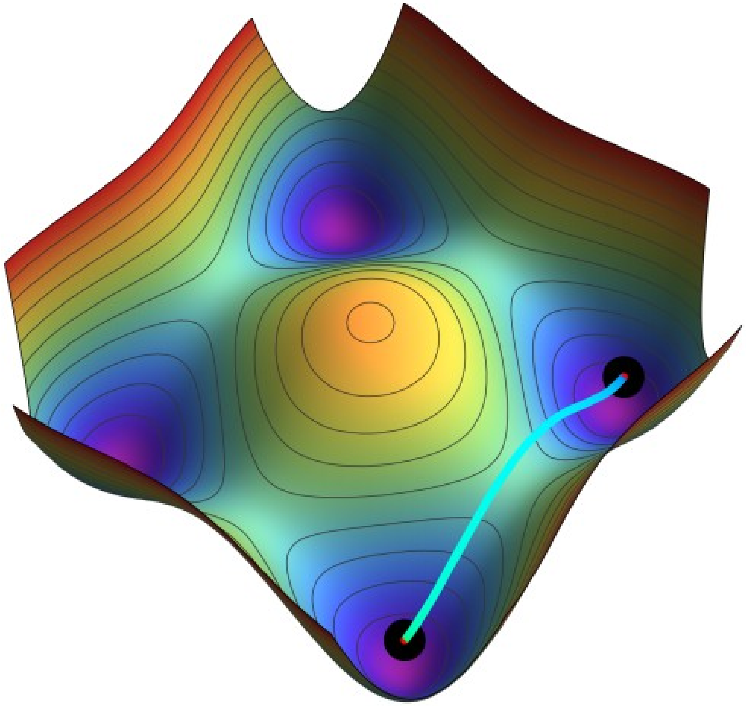
$$G_F = Z_N$$

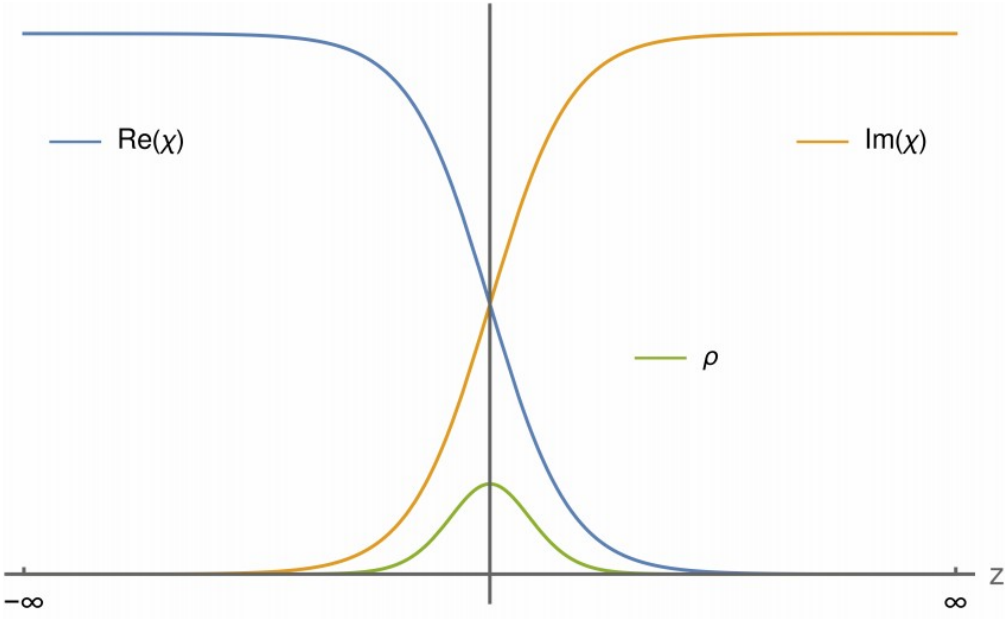




Scalar Sector: Domain Walls

$$G_F = Z_N$$





Scalar Sector: Domain Walls

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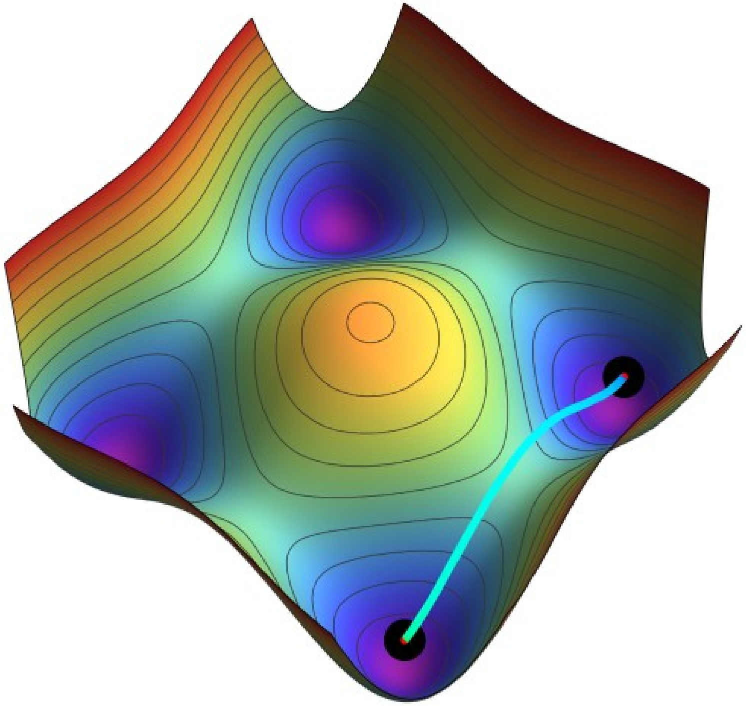
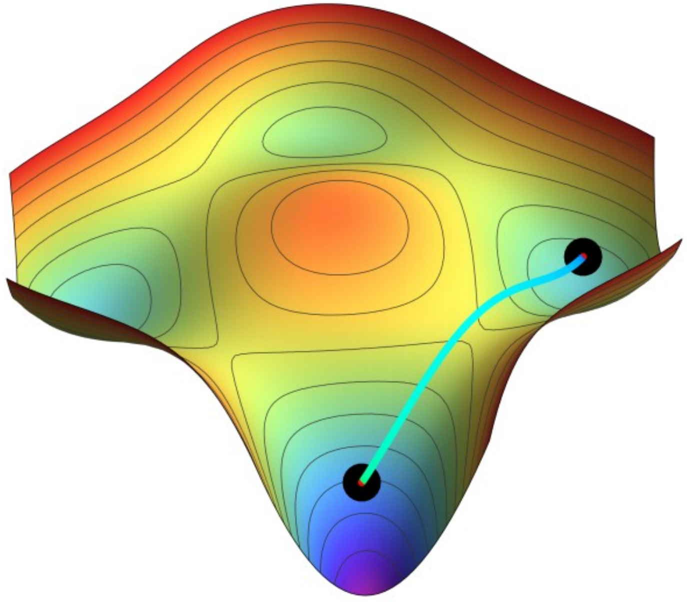


Exact



Broken

Scalar Sector: Gravitational Waves

$$G_F = Z_N$$

$$\frac{f_p}{10^{-9}\ \mathrm{Hz}} = 1.1 \left(\frac{g_*(T_{\rm ann})}{10}\right)^{\frac{1}{6}} \left(\frac{T_{\rm ann}}{10^{-2}\ \mathrm{GeV}}\right),$$

$$\frac{\Omega_p h^2}{7.2\times 10^{-18}} = \tilde{\epsilon}_{\rm gw}\mathcal{A}^2 \left(\frac{g_*(T_{\rm ann})}{10}\right)^{-\frac{4}{3}} \boxed{\left(\frac{\sigma}{\mathrm{TeV}^3}\right)^2} \left(\frac{T_{\rm ann}}{10^{-2}\ \mathrm{GeV}}\right)^{-4},$$

$$T_{\rm ann} = 3.41\times 10^{-2}\ \mathrm{GeV}(C_{\rm ann}\,\mathcal{A})^{-\frac{1}{2}} \left(\frac{g_*(T_{\rm ann})}{10}\right)^{-\frac{1}{4}} \boxed{\left(\frac{\sigma}{\mathrm{TeV}^3}\right)^{-\frac{1}{2}} \left(\frac{\Delta V}{\mathrm{MeV}^4}\right)^{\frac{1}{2}}},$$

Scalar Sector: Planck-Breaking

$$G_F = Z_N$$

Explicit-Breaking Assumption

$$\cancel{Z_N} \sim \boxed{\frac{\lambda}{M_P}} \times \mathcal{O}_{\text{Gauge Invariant}} \quad \Rightarrow \quad \begin{cases} V_{\cancel{Z_N}} \sim \lambda \dfrac{\chi^5}{M_P} \\ Y^f_{\cancel{Z_N}} \sim y \; \overline{f}_L H f_R \dfrac{v_0}{M_P} \end{cases}$$

$$M_P \to \infty \quad \Rightarrow \quad Z_N \text{ exact}$$

SUSY

$$G_F = Z_N$$

SUSY

SUSY: Superpotential & SSB

$$G_F = Z_N$$

$$U(1)_R$$

SUSY: Specifications

$$R(W) = 2\,, \qquad R(S) = 2\,, \qquad R(\chi) = 0$$

$$W = S\left(\frac{\chi^N}{\Lambda^{N-2}} - M^2\right)$$

$$\left[Y^f f f^c H \frac{\chi^n}{\Lambda^n}\right]$$

SUSY: Superpotential & SSB

$$G_F = Z_N$$

- Scalar Potential

$$V = |F_S|^2 + |F_\chi|^2 \,, \qquad F_\phi = \frac{\partial W}{\partial \phi}$$

- Minima

$$F_\phi = 0 \qquad \Rightarrow \qquad \langle S \rangle = 0 \qquad \text{and} \qquad \langle \chi \rangle \approx \sqrt{\frac{M^2}{\varepsilon^{N-2}}}$$

SUSY: Superpotential & SSB

$$G_F = Z_N$$

$$V(\chi)\Big|_{\langle S\rangle} = \left(\frac{|\chi|^{2N}}{\Lambda^{2N-4}} - 2M^2\frac{\mathrm{Re}(\chi^N)}{\Lambda^{N-2}} + M^4\right)$$

$$M^2 = v_0^2\varepsilon^{N-2}, \quad \text{with} \quad \varepsilon = \frac{v_0}{\Lambda}$$

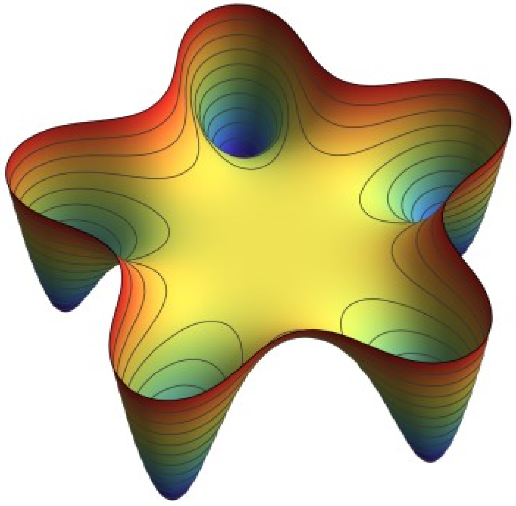
$$V(\chi)\big|_{\langle S\rangle} = v_0^4\varepsilon^{2N-4}\left(V'_{U(1)} + V'_{Z_N} + 1\right)$$

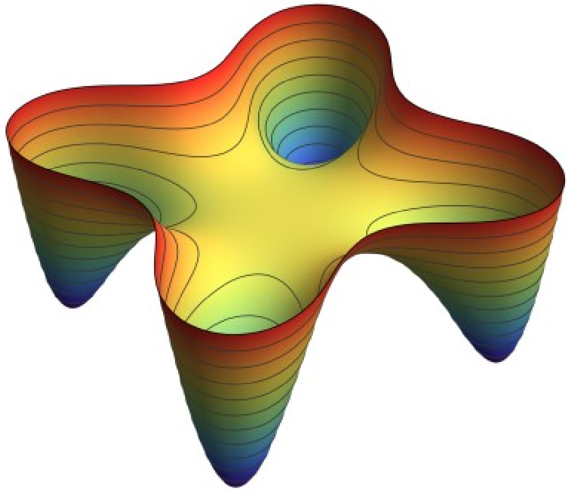
no U(1) !

SUSY: Superpotential & SSB

$$G_F = Z_N$$

$$V(\chi)\big|_{\langle S\rangle} = v_0^4 \varepsilon^{2N-4}\left(V'_{U(1)} + V'_{Z_N} + 1\right)$$





SUSY: Planck-Breaking

$$G_F = Z_N$$

$$\boxed{\cancel{Z_N} \sim \frac{\lambda}{M_P} \times \mathcal{O}}$$

- Superpotential

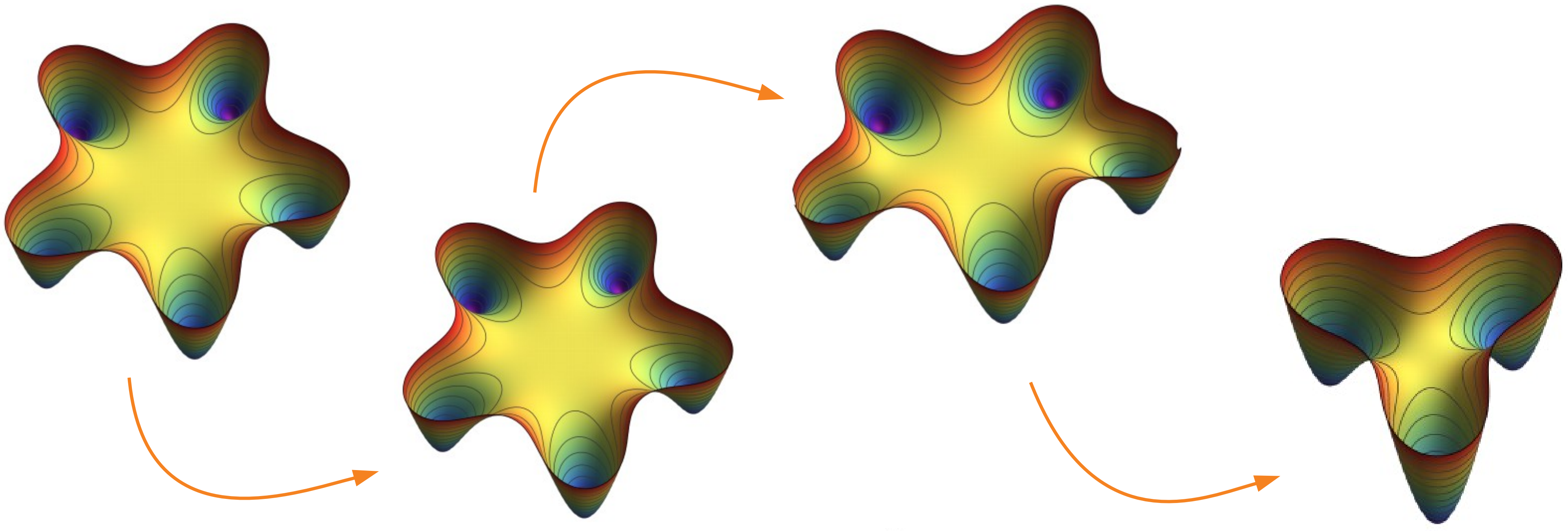
$$W = S\left(\frac{\chi^N}{\Lambda^{N-2}} + \lambda e^{i\alpha}\frac{\chi^3}{M_P} - M^2\right) \equiv S f(\chi)$$

SUSY: Planck-Breaking

$$G_F = Z_N$$

SUSY
Consistency Condition

$$\frac{\chi^3}{M_P} \gg \frac{\chi^N}{\Lambda^{N-2}} \quad \Rightarrow v_0 \ll M_P \varepsilon^{N-2} \sim 10^{14}\ \mathrm{GeV}$$



$v_0 \nearrow$

SUSY: Planck-Breaking

$$G_F = Z_N$$

$$\boxed{\cancel{Z_N} \sim \frac{\lambda}{M_P} \times \mathcal{O}}$$

- Superpotential

$$W = S\left(\frac{\chi^N}{\Lambda^{N-2}} + \lambda e^{i\alpha}\frac{\chi^3}{M_P} - M^2\right) \equiv S f(\chi)$$

- Scalar Potential and Minima

$$V = |F_S|^2 + |F_\chi|^2 \geq 0 \qquad F_\phi = 0$$

- But still:

$$V\left(\langle S\rangle, \langle\chi\rangle\right) = 0$$

SUSY: SUSY and Planck-Breaking

$$G_F = Z_N$$

SUSY-breaking

- Including Soft-terms

$$V = |f(\chi)|^2 + |S|^2|f'(\chi)|^2 + m_S^2|S|^2 + m_\chi^2|\chi|^2 + m_{\rm SUSY}\left[S\left(\mathbf{c}_1 M^2 + \frac{\mathbf{c}_2\chi^N}{\Lambda^{N-2}} + \frac{\mathbf{c}_3\chi^3}{M_P}\right) + \text{h.c.}\right]$$

$$\equiv |f(\chi)|^2 + m_{S,\rm eff}^2|S|^2 + m_{\rm SUSY}\left[SN(\chi) + \text{h.c.}\right]$$

- Driving field at Minimum

$$\frac{\partial V}{\partial S} = 0 \Leftrightarrow \langle S\rangle = -m_{\rm SUSY}\frac{N^*(\chi^\dagger)}{m_{S,\rm eff}^2}$$

- Flavon Potential:

$$V(\chi)\Big|_{\langle S\rangle} = |f(\chi)|^2 + \boxed{m_\chi^2|\chi|^2 - m_{\rm SUSY}^2\frac{|N(\chi)|^2}{m_{S,\rm eff}^2}}$$

SUSY: Domain Walls

$$G_F = Z_N$$

SUSY: DW estimates

- Symmetric Part (axial mode)

$$M_P \to \infty \qquad \left|f(v_0 e^{i\theta})\right|^2 = v_0^4 \varepsilon^{2N-4}\left[1-\cos(N\theta)\right] \quad \Rightarrow \quad \sigma \approx v_0^3 \varepsilon^{N-2}$$

- Explicit Breaking

$$|f(\chi)|^2 \supset \quad 2v_0^4\varepsilon^{2N-4}\,\lambda\frac{v_0}{M_P\varepsilon^{N-2}}\Big[\cos\big(\alpha-(N-3)\beta\big)-\cos\big(\alpha-3\beta\big)\Big] \quad \to \quad \Delta V \approx \frac{v_0^5}{M_P}$$

SUSY: Domain Walls

$$G_F = Z_N$$

SUSY: DW estimates

- Symmetric Part (axial mode)

$$M_P \to \infty \qquad \left|f(v_0 e^{i\theta})\right|^2 = v_0^4 \varepsilon^{2N-4}\left[1-\cos(N\theta)\right] \quad \Rightarrow \quad \boxed{\sigma \approx v_0^3 \varepsilon^{N-2}}$$

- Explicit Breaking

$$m_S \ll v_0 \varepsilon^{N-2} \qquad \boxed{\Delta V \approx \frac{v_0^2 m_{\rm SUSY}^2}{N^2}\left(\frac{v_0}{M_P \varepsilon^{N-2}}\right)} \left\{ |\mathbf{c}_1 + \mathbf{c}_2|^2 \frac{6\lambda}{N} \Delta \cos\left(2\beta + \alpha\right) - 2c_1 c_3 \Delta \cos\left(3\beta + \gamma_3 - \gamma_1\right) - 2c_2 c_3 \Delta \cos\left(3\beta + \gamma_3 - \gamma_2\right) \right\}$$

SUSY: Gravitational Waves

$$G_F = Z_N$$

SUSY: GW estimates

- Peak Frequency

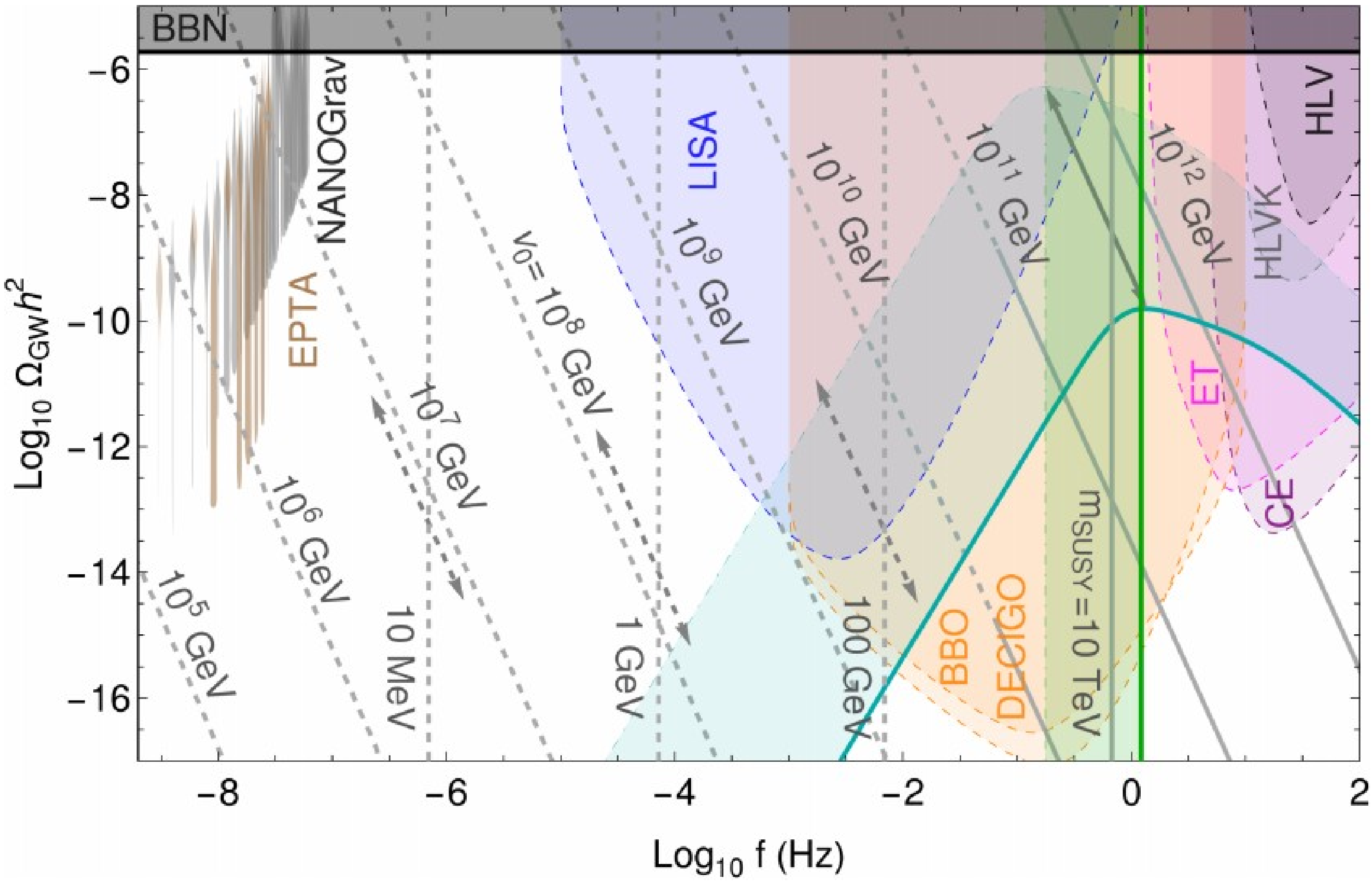
$$f_p \approx \frac{10^{-4}}{N\varepsilon^{N-2}} m_{\text{soft}}$$

- Amplitude

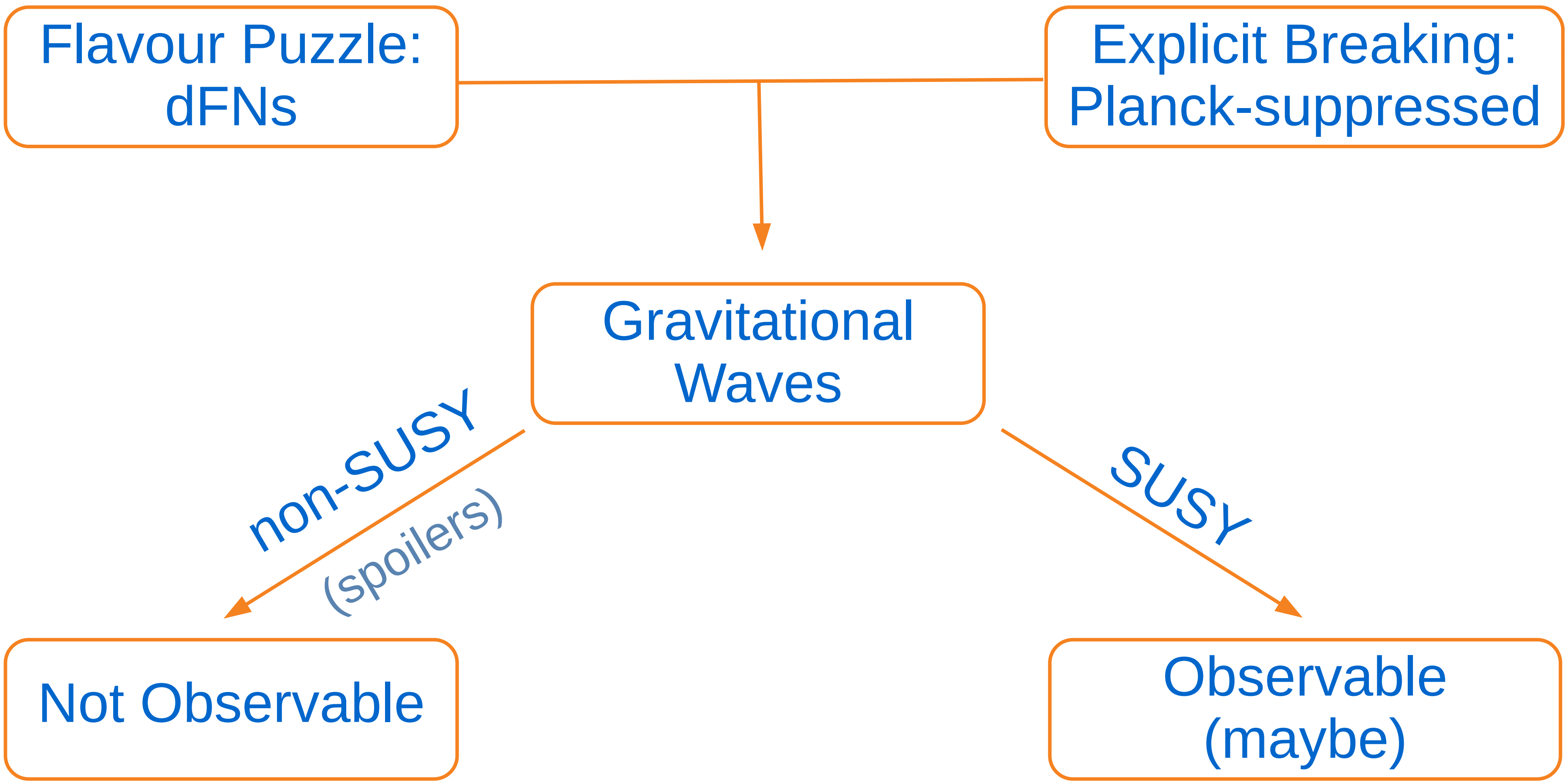
$$\Omega_p h^2 \approx 10^{-38} \frac{N^4 v_0^6 \varepsilon^{6(N-2)}}{m_{\text{soft}}^4}$$

SUSY: Gravitational Waves

$$G_F = Z_N$$



Conclusions



Thank You

non-SUSY

$$G_F = Z_N$$

non-SUSY

Scalar Sector: Orange-Squeezers

$$G_F = Z_N$$

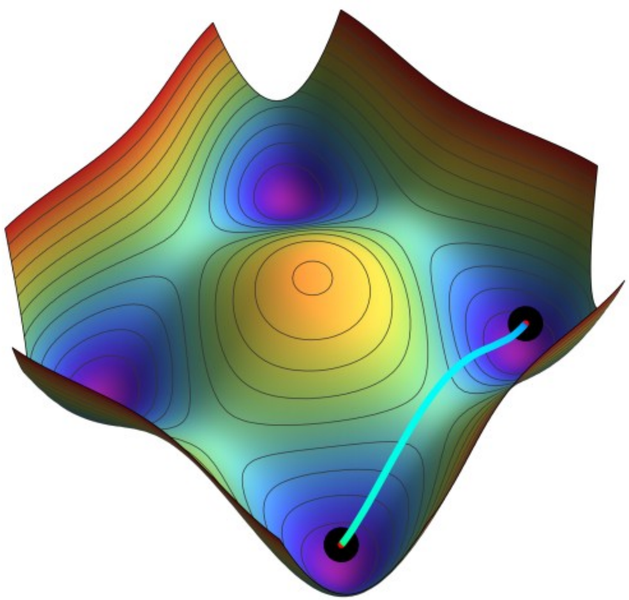
Large N non-SUSY
and the
Orange-Squeezer Shape

$$N \leq 4\,, \quad \frac{V(\chi)}{v_0^4} = V'_{Z_N} + \frac{v_0}{M_P} V'_{\cancel{Z_N}}\,,$$
$$N = 5\,, \quad \frac{V(\chi)}{v_0^4} = V'_{U(1)} + \frac{v_0}{M_P} V'_{\cancel{Z_N}} + \frac{v_0}{\Lambda} V'_{Z_N}\,,$$
$$N \geq 6\,, \quad \frac{V(\chi)}{v_0^4} = V'_{U(1)} + \frac{v_0}{M_P} V'_{\cancel{Z_N}} + \frac{v_0^{N-4}}{\Lambda^{N-4}} V'_{Z_N}\,.$$

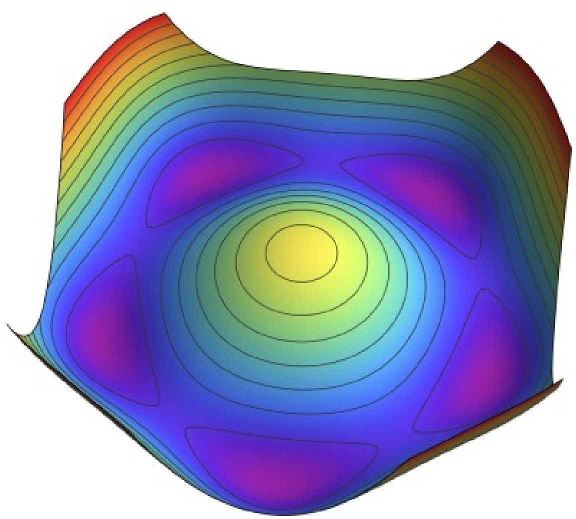
Scalar Sector: Orange-Squeezers

$$G_F = Z_N$$

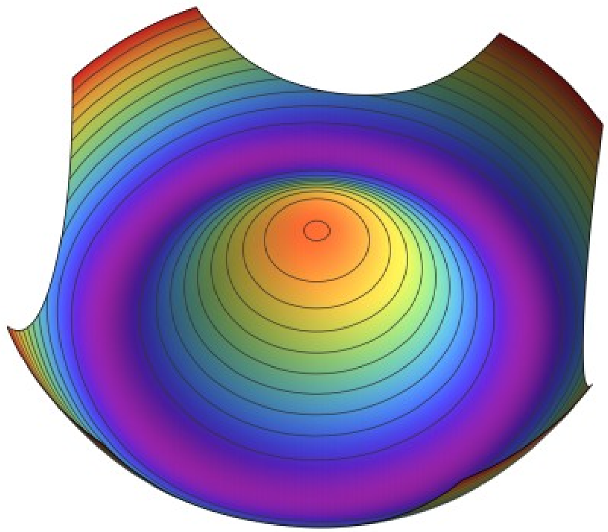
Z_4



Z_5



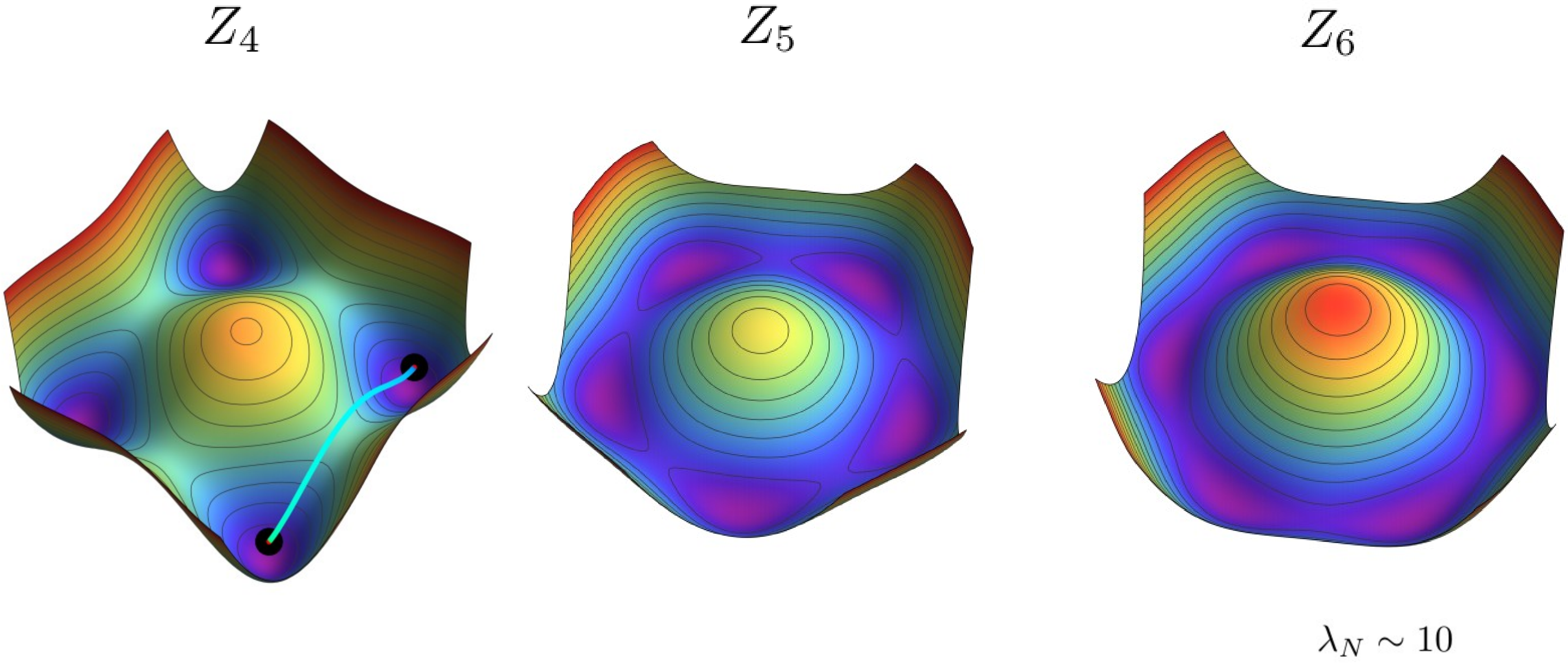
Z_6



U(1) @ order-4

Scalar Sector: Orange-Squeezers

$$G_F = Z_N$$



Scalar Sector: Orange-Squeezers

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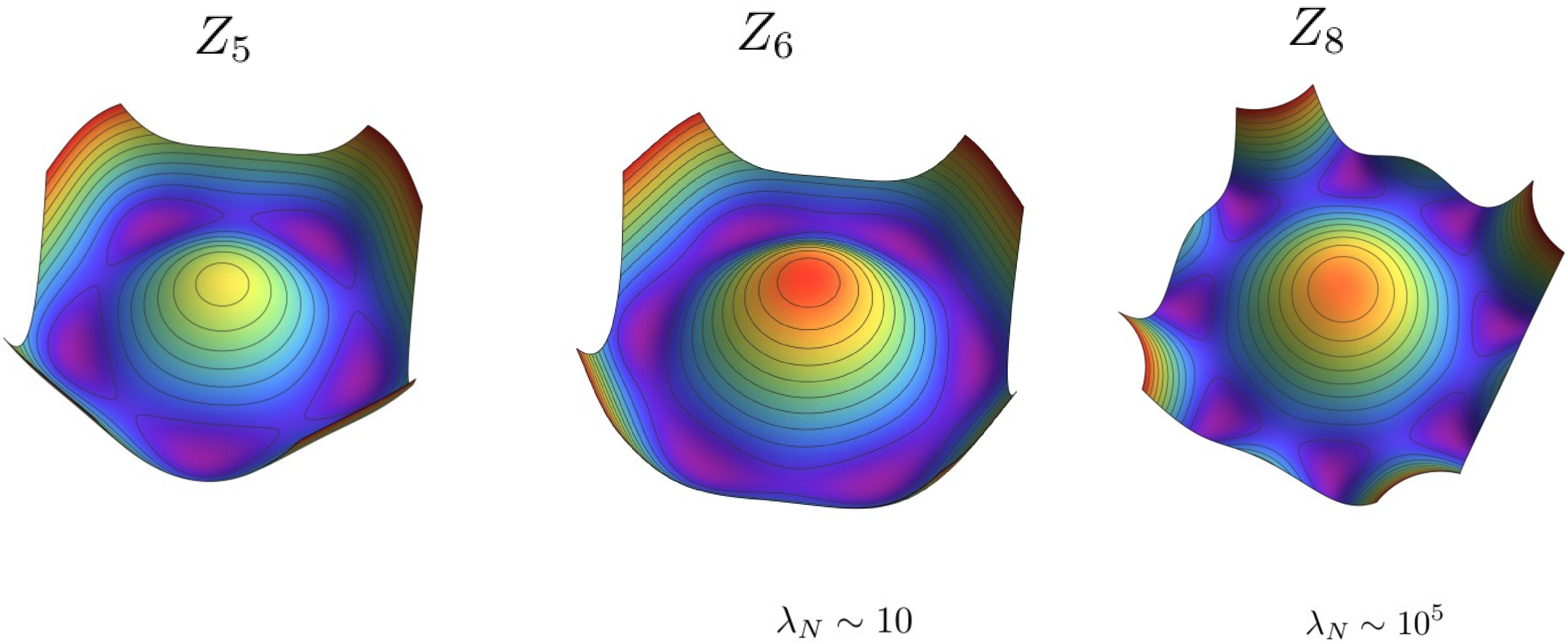

Scalar Sector: Orange-Squeezers

$$G_F = Z_N$$

Large N non-SUSY
and the
Orange-Squeezer Shape

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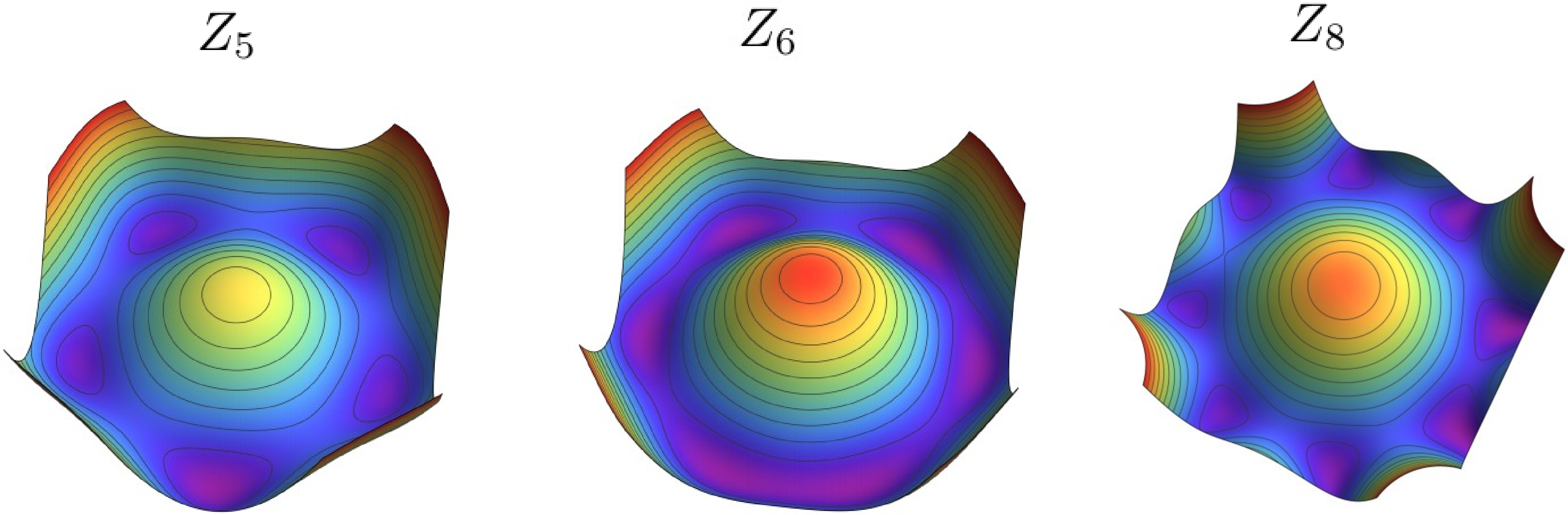
$$N \geq 6\,, \quad \frac{V(\chi)}{v_0^4} = V'_{U(1)} + \frac{v_0}{M_P} V'_{\cancel{Z_N}} + \frac{v_0^{N-4}}{\Lambda^{N-4}} V'_{Z_N}\,.$$

Scalar Sector: Orange-Squeezers

$$G_F = Z_N$$

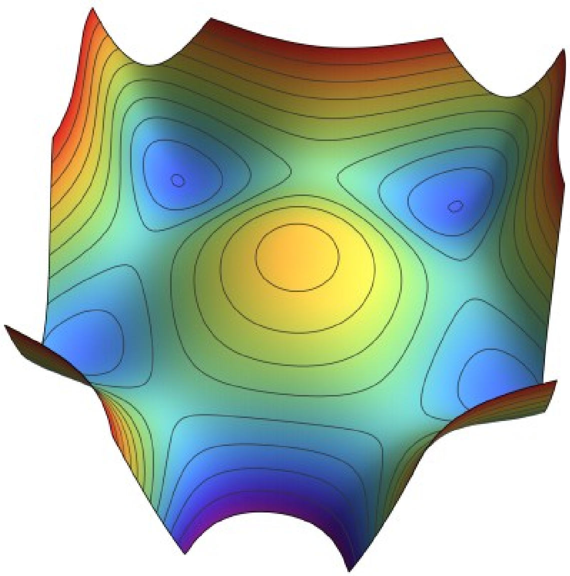


Scalar Sector: Orange-Squeezers

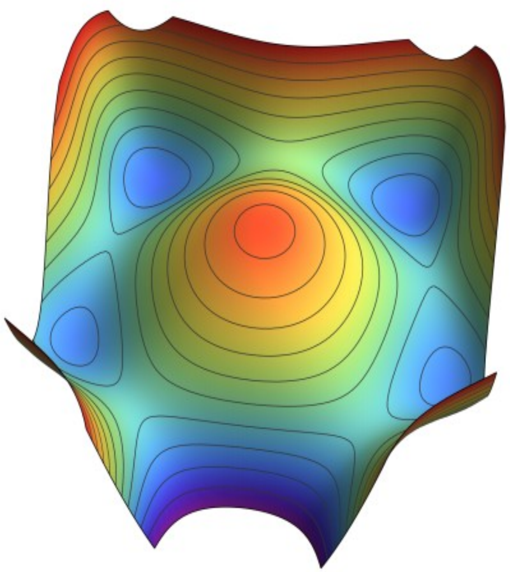
$$G_F = \mathbb{Z}_N$$

$$V_{\mathbb{Z}_N} \gg V_{\cancel{\mathbb{Z}_N}}$$

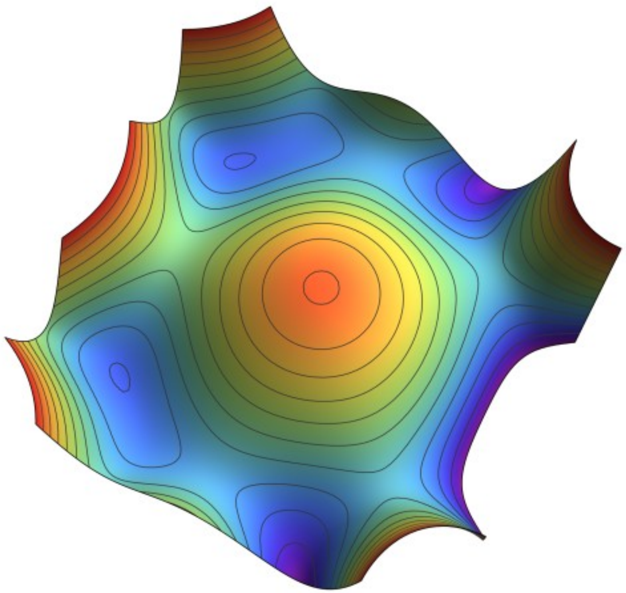
$\mathbb{Z}_5$



$\mathbb{Z}_6$



$\mathbb{Z}_8$



Scalar Sector: Orange-Squeezers

$$G_F = Z_N$$

non-SUSY
Consistency Condition

$$V(\chi): \qquad V_{Z_N} \gg V_{\cancel{Z_N}} \quad \Rightarrow \quad v_0 \ll M_P \varepsilon^{N-4}$$

$$\mathrm{FN}: \qquad y^f \varepsilon^{\max} \gtrsim \frac{v_0}{M_P} \quad \Rightarrow \quad v_0 \lesssim M_p \varepsilon^{N/2}$$

$$\downarrow$$

$$v_0 \ll 10^{12-13}\ \mathrm{GeV}$$

non-SUSY: Domain Walls

$$G_F = Z_N$$

non-SUSY: DW estimates

- Symmetric Part (axial mode)

$$V\left(v_0 e^{i\theta}\right) = 2\lambda_N \varepsilon^{N-4} v_0^4 \left[1 - \cos\left(N\theta\right)\right] \longrightarrow \sigma = \frac{8\sqrt{2}\sqrt{\lambda_N}\varepsilon^{\frac{N-4}{2}} v_0^3}{N}$$

- Explicit Breaking

$$V_{\cancel{Z_N}} \sim \lambda \frac{\chi^5}{M_P} \longrightarrow \Delta V \sim \frac{v_0^5}{M_P}$$

non-SUSY: Domain Walls

$$G_F = Z_N$$