

On the Origins of Varying Gauge Couplings

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Based on 2608.(SUBMITTED THIS MORNING)
with C. de Lima · D. McKeen · D. Morrissey



Flavor Symmetries and Consequences in
Accelerators and Cosmology 2026

Motivation

- * Several constructions utilize the following effective Lagrangian:

$$\mathcal{L} \supset -\frac{1}{2} \left(\frac{1}{g^2} - \frac{\phi}{\Lambda} \right) \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right)$$

- * Before BBN/EWPT little is known, dynamics may have differed drastically from standard cosmological evolution
- * Gauge couplings drive early-universe dynamics and since $g \rightarrow g(\mu)$, **different cosmological epochs map to different energy scales**
- * **Varying fine-structure** ([Lee, et. al., '22](#), [Chluba & Hart, '23](#), [Ghosh, et. al., '25](#)), **electroweak confinement** ([Berger, et. al., '19](#), [Howard, et. al., '21](#), [Bhalla-Ladd, et. al., '25](#)), **ultralight scalars** ([Brzeminski, et. al., '20](#), [Banerjee, et. al., '22](#), [Becker, et. al., '25](#)), **early QCD confinement** ([Ipek & Tait, '18](#), [Lu, et. al., '22](#))...
- * RGE-driven evolution depends only on heavy thresholds, only **slow (logarithmic!)** variations in energy are possible

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- * Gauge couplings drive early universe dynamics, map to different energy scales
- * Varying 1 vacuum shifts induce large variations? How fast can gauge couplings change at low energies?
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Is this interpretation theoretically consistent? Can vacuum shifts induce large variations? How fast can gauge couplings change at low energies?

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- * Varying **ultralight** vacuum shifts induce large variations? How fast can gauge couplings change at low energies? **weak confinement** (Berger, et. al., '19, Howard, et. al., '21, Bhalla-Ladd, et. al., '25), **early QCD confinement** (Ipek & Tait, '18, Lu, et. al., '22)...

- * RGE-driven evolution depends only on heavy thresholds, only **slow (logarithmic!)** variations in energy are possible

- * Drastic changes of couplings implies $\langle \phi \rangle \sim \Lambda$... e.g. confined EW requires $\frac{1}{g^2} \approx \frac{\langle \phi \rangle}{\Lambda} \sim 2.37$

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Varying Gauge Coupling Induced by Singlet Scalar...

- * The **EFT description breaks down** in this regime!

$$\mathcal{L} \supset -\frac{1}{2} \left[\frac{1}{g^2} - \sum_n c_n \left(\frac{\phi}{\Lambda} \right)^n \right] \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right)$$

- * Require: Low energy containing **only singlet ϕ** , described by **4D QFT**, including the approach to the UV
- * Generically can classify possible UV completions into **Higgs-like** or **Extra dimensions**

...and the Charged-Matter Portal

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- * Simplest mechanism: matter **charged under the gauge group** whose mass tracks a gauge-singlet scalar ϕ

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- * As ϕ shifts, $m(\phi)$ **moves the RG threshold** where ψ enters the $\beta(\mu)$, inducing a field-dependent low-energy coupling

The Charged-Matter Portal

* Leading term in the **one-loop** effective action (Shifman, et. al., '79): $\mathcal{L} \supset -\frac{1}{2g^2(\mu)} \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right)$

* $g(\mu)$ determined by RGE, well known solution: $\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b} \ln \left(\frac{\mu}{\Lambda} \right)$

* The slope is fixed by **all light charged degrees of freedom** present at that scale:

$$\tilde{b} = \frac{1}{4\pi^2} \left[\frac{11}{3} C_2(G) - \frac{2}{3} \sum_f S_2(r_f) - \frac{1}{3} \sum_s S_2(r_s) \right]$$

* Distinct coefficients $\tilde{b}_>$ ($m(\phi) < \mu$) and $\tilde{b}_<$ ($m(\phi) > \mu$), $g(\mu)$ **continuous** across thresholds at leading order

* **ϕ dependence** of the gauge coupling arises when $\mu < m(\phi)$... Running and matching gives

$$\frac{1}{g^2(\mu, \phi)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}_> \ln \left(\frac{\mu}{\Lambda} \right) + 2\tilde{b}_\Delta \ln \left[\frac{\mu}{m(\phi)} \right] \Theta [m(\phi) - \mu]$$

Logarithmic, Not Linear

* Re-expressed via the coupling measured today, $g(\mu_0, \phi_0)$:

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- * The dependence on the scalar is **logarithmic, not linear**, re-summing the EFT tower keeps it valid for $|\Delta m|/m \gtrsim 1$
- * Applies for any **charged matter** whose mass depends on ϕ ; **multiple fields summed!**
- * Small excursions recover the dimension-5 form, but the variation is **far slower than linear**

$$\mathcal{L} \supset -\frac{1}{2} \left[\frac{1}{g^2(\mu, \phi_0)} - 2\tilde{b}_\Delta \frac{1}{m} \frac{dm}{d\phi} \right]_0 (\phi - \phi_0) + \dots \text{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

Couplings in the Early Universe

- * Toy abelian model: N_f vector-like fermions + ϕ ; a **second-order transition** profile

$$V(\phi, T) = \frac{\lambda}{2} (\phi^2 - \langle \phi \rangle^2)^2 + f(\lambda, y, \dots) \phi^2 T^2$$

$$\Rightarrow \langle \phi(T) \rangle = \Theta(T_c - T) \langle \phi \rangle \sqrt{1 - \left(\frac{T}{T_c} \right)^2} \quad m(\phi) = |M \pm y \langle \phi(T) \rangle|$$

- * The probed energy scale **is the temperature** ($\mu \rightarrow T$) of the bath; as the universe cools, thresholds shift
- * With $N_f = 100$, many distinct UV theories flow to the **same IR coupling**
- * The naïve linear EFT estimate drastically **overestimates** the achievable variation.

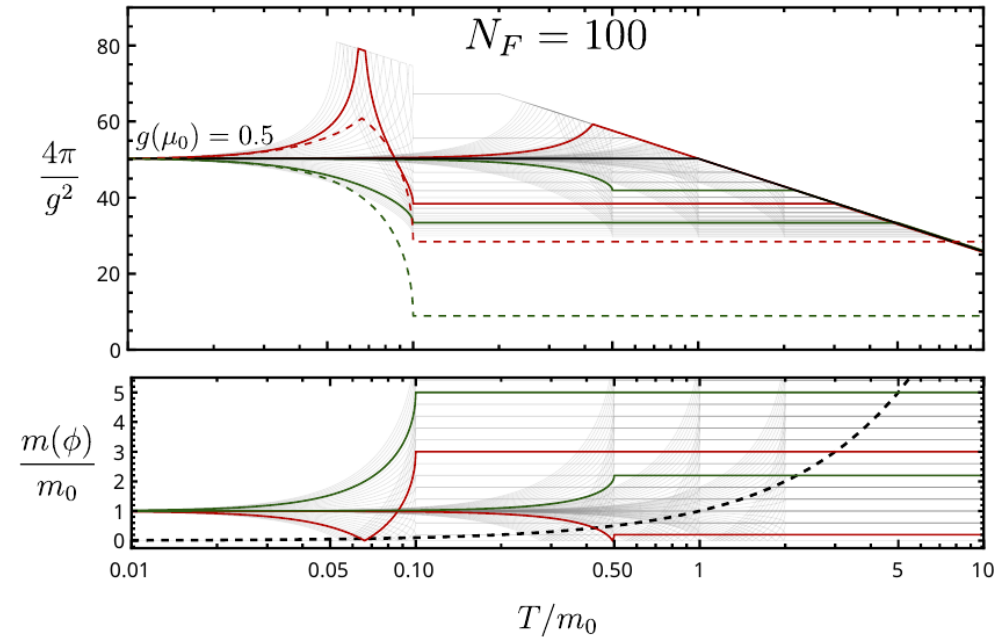


Fig. 1 Coupling fixed in the IR; dashed = naïve linear

Symmetries on the Scalar

- * When ϕ transforms non-trivially under a symmetry, dependence of the effective coupling can be modified
- * Consider theory with ϕ and N_f fermions ψ_n , $n = 0, 1, 2, \dots, N_f - 1$ and invariant under $\mathbb{Z}_{N_f}$ symmetry
 $\phi \rightarrow e^{2i\pi/N_f} \phi$, $\psi_n \rightarrow e^{-2i\pi n/N_f} \psi_n$

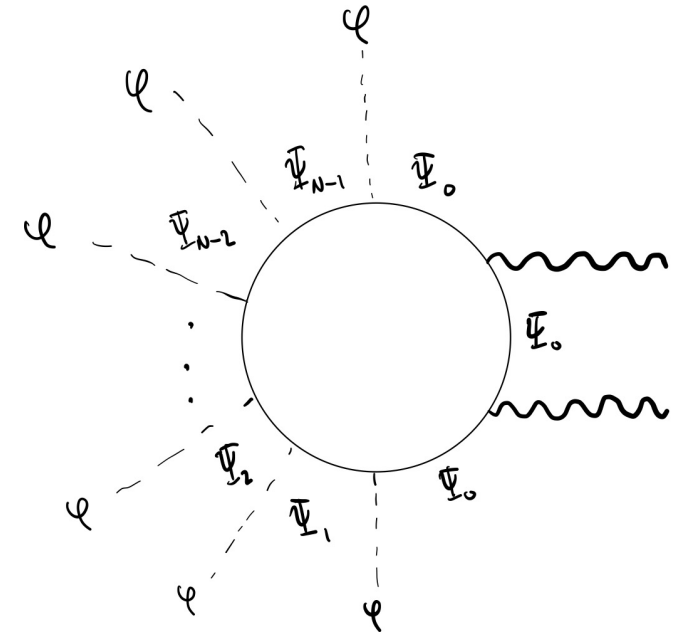
$$\mathcal{L} = - \sum_{k=0}^{N_f-1} \left[M_k \bar{\psi}_k \psi_k + y_k \phi \bar{\psi}_k \psi_{k+1} + Y \bar{\psi}_0 \phi \psi_{N_f} + \text{h.c.} \right]$$

$$\Rightarrow \frac{1}{g^2(\mu, \phi)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}_> \ln \left(\frac{\mu}{\Lambda} \right) + 2\tilde{b}_\Delta \ln \left[\frac{\mu^{N_f}}{|\det m(\phi)|} \right]$$

- * The symmetry is encoded in $|\det m(\phi)| \dots$ e.g. $N_f = 3$:

$$\det m(\phi) = M_0 M_1 M_2 - M_0 |y_1 \phi|^2 - M_1 |y_2 \phi|^2 - M_2 |y_0 \phi|^2 + (y_0 y_1 Y \phi^3 + \text{h.c.})$$

- * ϕ^3 term $\propto Y \dots$ $Y \rightarrow 0$ restores a larger $U(1)$ symmetry.



Enhanced Symmetry $\rightarrow$ Higher Operators

* Generalizing... $\det m(\phi) \supset \prod_i M_i |y_{i+1}\phi|^2 + Y\phi^{N_f}$

* Expand $\phi = \phi_0 + \Delta\phi$...

$$\ln |\det m(\phi)| = \ln |\det m(\phi_0)| + \text{Re} \left[\text{Tr} \left(m^{-1} \frac{dm}{d\phi} \right)_{\phi_0} \right] \Delta\phi + \frac{1}{2} \text{Re} \left[\text{Tr} \left(m^{-1} \frac{d^2 m}{d\phi^2} \right)_{\phi_0} - \text{Tr} \left(m^{-1} \frac{dm}{d\phi} \right)_{\phi_0}^2 \right] (\Delta\phi)^2$$

* Generic point: the trace is non-zero $\rightarrow$ leading dependence is **linear** (a dimension-5 operator).

* **Enhanced-symmetry point** (e.g. $\phi_0 = 0$): leading dependence is **quadratic or higher** (dimension-6+).

* **Qualitatively different** low-energy EFT... significant **phenomenological implications** ([Brzeminski, et. al., '20](#), [Banerjee, et. al., '22](#), [Becker, et. al., '25](#), [Ghosh, et. al., '25](#))

Why Logarithms? Gauge Invariance

- * Gauge invariance forbids mass/kinetic mixing of gauge bosons with neutral fields (exception: Abelian kinetic mixing)
- * Up to dimension four, gauge bosons couple **only to charged matter**, which feeds back at one loop
- * To change the coupling we must **change charged matter...** only masses and non-gauge couplings may vary
- * At one loop, **only masses** reach the gauge running
- * **How does a moving mass threshold enter the coupling?**

The Logarithm Is Generic

- * $G_{>} \supseteq G_{<}$ matched at massive threshold m ... arises from scale-invariant interactions
- * **Spontaneous breaking of scale invariance** by ϕ
- * **Above m** have renormalizable theory, **no ϕ dependence**; scale anomaly coefficient at one-loop: $\partial_\mu D^\mu = \tilde{b}_{>} \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right)$
- * **Below $m(\phi)$** expand in powers of g :

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{2} \left[\frac{1}{g^2(\mu)} + f(\phi) + \mathcal{O}(g^2) \right] \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right)$$

- * Scale anomaly coefficient **below threshold**:

$$\partial_\mu D^\mu = \left(\tilde{b}_{<} - \frac{1}{2} \frac{df}{d \ln \phi} \right) \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right)$$

- * **Matching** the light-field anomaly across the threshold **fixes $f(\phi)$** :

$$\frac{df}{d(\ln \phi)} = -2(\tilde{b}_{<} - \tilde{b}_{>}) = -2\tilde{b}_\Delta \implies f(\phi) = \text{constant} - 2\tilde{b}_\Delta \ln \phi$$

Yukawas Vary at Tree Level

- * Yukawas are **not protected by gauge invariance**

- * Froggatt–Nielsen-style UV completion ([Froggatt & Nielsen, '79](#), [Leurer, et. al., '92](#), [Leurer, et. al., '93](#)):

$$-\mathcal{L} \supset \xi^c \Phi (y\eta + Y\psi) + (m + f\phi)\psi^c \eta + (M + F\phi)\psi^c \psi$$

	η	ξ^c	ψ	ψ^c	Φ	ϕ
$SU(N)$	$\mathbb{1}$	$\bar{\mathbf{N}}$	$\mathbb{1}$	$\mathbb{1}$	$\mathbf{N}$	$\mathbb{1}$
$U(1)$	1	-1	1	-1	0	0

- * Integrate out ψ and **re-sum ϕ insertions**:

$$-\mathcal{L}_{\text{eff}} \supset \xi^c \Phi \eta \left(y + \frac{Yf\phi}{M + F\phi} \right) \equiv y(\phi) \xi^c \Phi \eta \quad y(\phi) = y + \frac{fY\phi}{M + F\phi}$$

- * A genuine **linear** dependence (EFT restructuring once $F\phi \gtrsim M$)

Faster-than-Linear Yukawas

- * Add a second heavy pair protected by a $\mathbb{Z}_2$; the leading insertion is now ϕ^2 :

$$-\mathcal{L}_{\text{eff}} \supset \xi^c \Phi \eta \left(y + \frac{Y f F_{12} \phi^2}{M_1 M_2 - F_{12} F_{21} \phi^2} \right) \equiv y(\phi) \xi^c \Phi \eta \quad y(\phi) = y + \frac{Y f F_{12} \phi^2}{M_1 M_2 - F_{12} F_{21} \phi^2}$$

- * Larger discrete symmetries push the leading power higher, giving

$$y(\phi) \simeq Y \left[\frac{\phi}{\Lambda(\phi)} \right]^p$$

- * Exponent **controlled by symmetry...** power-law sensitivity, not from any logarithm!

Two-Loop Caveat: Yukawa Couplings Augment the Running

- * At two loops the Yukawa enters the gauge β -function:

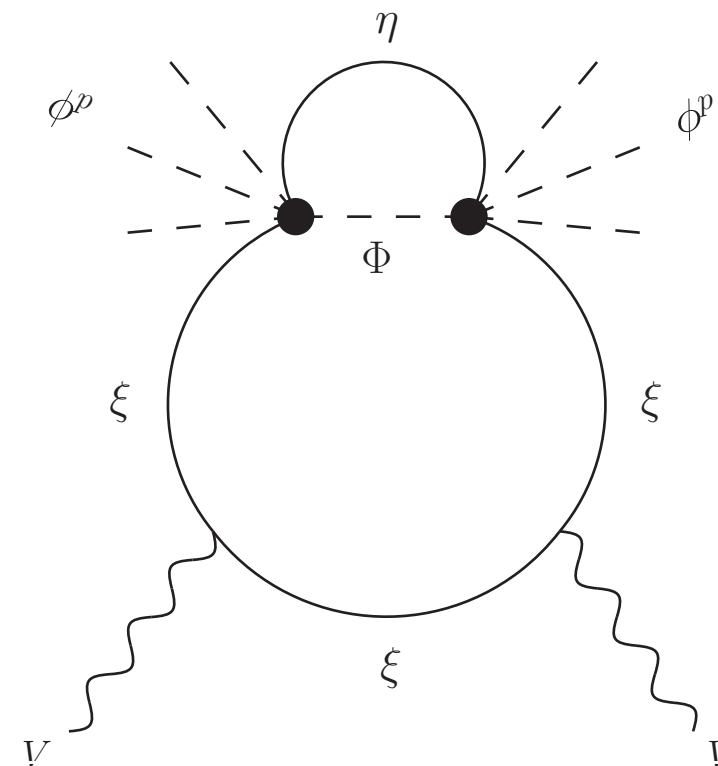
$$\frac{dg}{dt} \simeq -\frac{b_1}{(4\pi)^2} \left[1 + \frac{b_2^{(y)}}{b_1(4\pi)^2} y^2 \right] g^3$$

- * Integrating (with Yukawa as effective β coefficient):

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + \frac{2b_1}{(4\pi)^2} \left[1 + \frac{b_2^{(y)}}{b_1(4\pi)^2} y^2 \right] \ln \left(\frac{\mu}{\Lambda} \right)$$

- * With $y \rightarrow y(\phi) \simeq Y \left[\frac{\phi}{\Lambda(\phi)} \right]^p$, the gauge coupling inherits a **power-law** piece:

$$\left(\frac{1}{g^2} \right)_{2\text{-loop}} \propto y^2(\phi) \ln \left(\frac{\mu}{\Lambda} \right) \sim Y^2 \left[\frac{\phi}{\Lambda(\phi)} \right]^{2p} \ln \left(\frac{\mu}{\Lambda} \right)$$



Gauge Couplings vs. Yukawa Couplings

GAUGE COUPLINGS

$$\mathcal{L} \supset -\frac{1}{2} \left[\frac{1}{g^2(\mu)} - c \ln \left(1 + \frac{\Delta m(\phi)}{M} \right) \right]$$

- * Protected by gauge invariance
- * Enter only at **one loop**, only through **mass thresholds**
- * Sensitivity is **logarithmic** in ϕ
- * Large change needs many states/big excursions

YUKAWA COUPLINGS

$$y(\phi) \simeq Y \left[\frac{\phi}{\Lambda(\phi)} \right]^p$$

- * Unprotected can be modified at tree-level
- * **Symmetry** and **mass hierarchy** set power p
- * Sensitivity can be **power-law** in ϕ
- * Feed back into gauge coupling at two loops

Bottom line: in 4D, gauge couplings are logarithmically stiff; reaching **power-law variation** costs a **two-loop Yukawa channel** (or **Extra Dimensions**)

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Thank You!

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