Non-invertible selection rules in string theory and particle physics

Tatsuo Kobayashi

1. Introduction
2. Coupling selection rules from string compactifications

   Magnetized compactifications
3. Heterotic string on orbifolds
4. Summary

This talk based on collaborations with

J.Dong, T.Jeric, H.Mita, S.Miyamoto, R.Nishida, Y.Nishioka, H.Okada, H.Otsuka, M.Tanimoto, H.Uchida

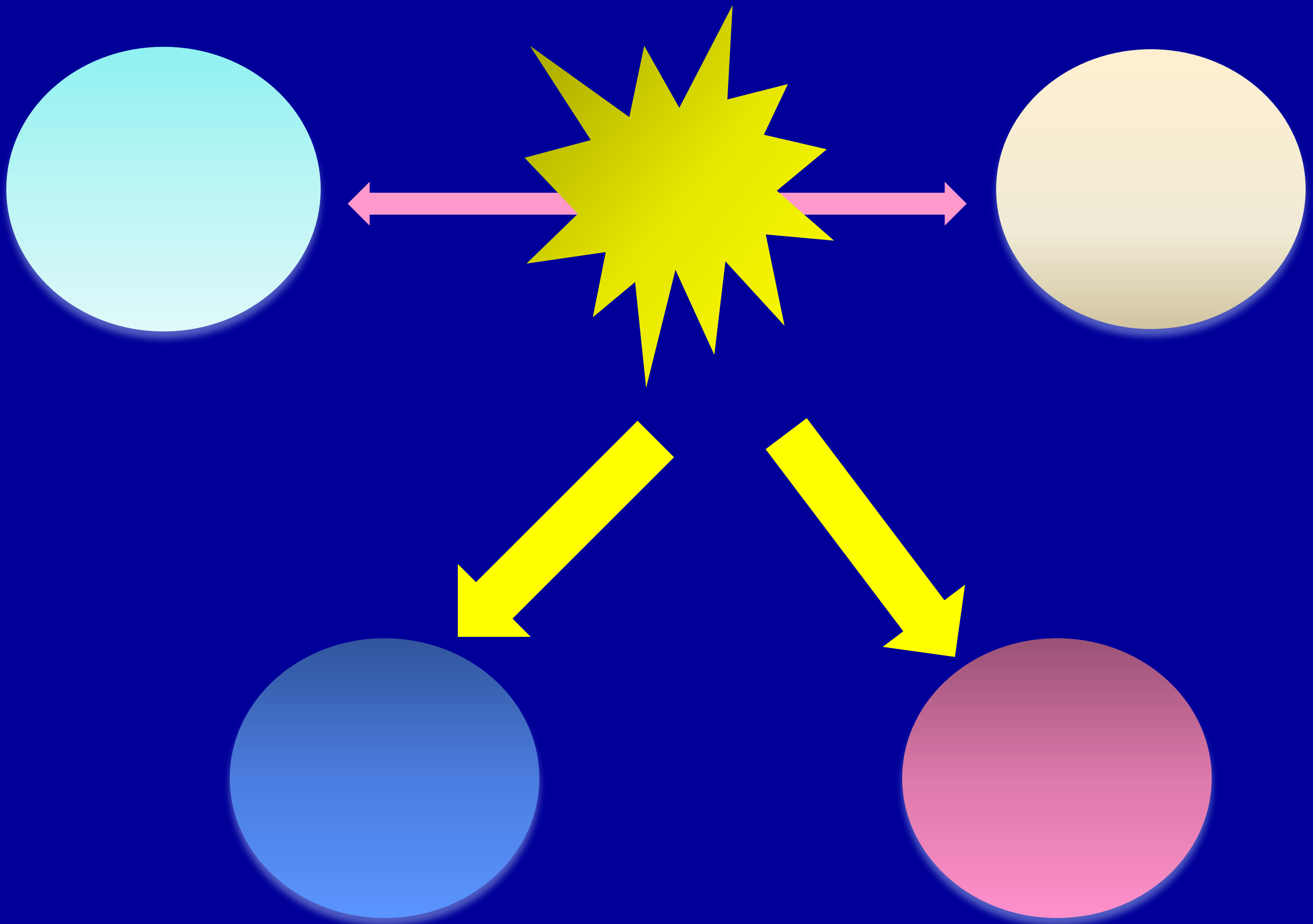
1. Introduction

Symmetries are important in physics.

In particular physics, symmetries determine allowed couplings and forbidden couplings, coupling selection rules.

That is symmetry.

Selection rules



charge conservation, etc.

Strangeness

interactions

$$K^- + p^+ \to \pi^0 + \Lambda^0$$
$$\to \pi^- + \Sigma^+$$
$$\pi^- + p^+ \to \Lambda^0 + K^0$$

It is conserved in strong and electromagnetic interactions, but not in weak interaction.

1. Introduction

In particle physics,

symmetries due to group theory are often considered.

In group theory, multiplication rules are

a,b,c ∈ G,

ab = c → $$\phi_a + \phi_b \to \phi_c$$

charge conservation

abc...x = identify,

the corresponding term appears in Lagrangian,

which is G-invariant.

$$\phi_a \phi_b \phi_c \cdots \phi_x$$

1. Introduction

Recently, selection rules without group actions have been studied.

In sec. 2, I show an examples of string compactifications, magnetized orbifold models.

Discuss its implications, Z2 gauging of ZM.

In sec. 3, I show more examples,

heterotic non-Abelian orbifold models,

multiplication rules of conjugacy classes.

2. Coupling selection rules from string compactifictions

Some coupling selection rules
derived from string compactifications
are understood by group theory,

but not all.

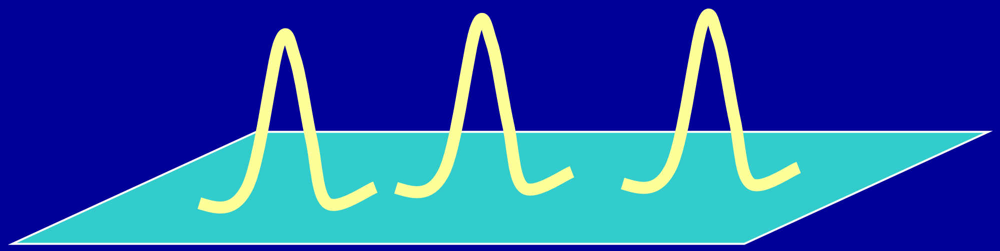
Magnetized compactification

Zero-mode equation on T^2 with flux M

$$\boxed{i\gamma^m D_m \psi = 0}$$

$\Rightarrow$ non-trivial zero-mode profile

the number of zero-modes, M



Magnetized T^2 models

Certain compactifications lead to ZM symmetries

$$\varphi_j \to g^{k_j} \varphi_j .$$

g: ZN phase

Intersecting/magnetized D-brane models on torus

K may be momenta, winding, their mixture.

Abe, Choi, T.K., Ohki, 0904.26331

Berasaluce-Gonzalez, et.al. 1206.2383

Marchesano, et.al., 1306.1284

Orbifold compactifications

T2/Z2 orbifold

$$\phi_j = \varphi_j + \varphi_{M-j}.$$

Abe, T.K., Ohki, 0806.4748

ZM symmetry is broken, but certain coupling

Selection rules remain.

In Z2 basis, they have both j and M-j (ZM) charges.

That is, Z2 gauging of ZM symmetry.

T.K., Otsuka, 2408.13984

Z2 gauging of ZM symmetries

ZM symmetry $g^k,$ g; ZM phase

We define class including k and –k charges.

$$[g^k] = \{g^k, g^{-k}\}$$

Z2 gauging

automorphism

$$ege^{-1} = g, \qquad rgr^{-1} = g^{-1}.$$

We define class

$$[g^k] = \{hg^k h^{-1} \,|\, h = e, r\},$$

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T.K., Otsuka, Tanimoto, 2409.05270

Z2 gauging of ZM symmetries

Multiplication rules

$$g^{k_1} g^{k_2} = g^{k_1+k_2}, \qquad g^{k_1} g^{-k_2} = g^{k_1-k_2},$$
$$g^{-k_1} g^{k_2} = g^{-k_1+k_2}, \; g^{-k_1} g^{-k_2} = g^{-k_1-k_2}.$$

Class

$$[g^{k_1}][g^{k_2}] = [g^{k_1+k_2}] + [g^{M-k_1+k_2}].$$

Each field corresponds to one class,

And they have both k and M-k charges.

Selection rules are different from group theory.

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Example: M=3

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M=3 (Z3 symmetry)   2 class

[0], [1]   [1] includes charges 1 and 2

Multiplication rules

[0] [0]=[0],   [0][1]=[1]

[1] [1] = [0]+[1]

Fibonacci fusion rules

$$Y_{ab} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix} .$$

Couplings are allowed if [x][y][z]=[0]+...

Two-generations model [0],[1]

Yukawa texture when Higgs = [1]

Weinberg texture

Texture

Example

One can not derive by group theory.

Examine Abelian group.

no solution for U(1) and Zn symmetries

$$Y_{ab} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix} .$$

$$\begin{aligned} Q_{L1} + Q_{R1} &\neq Q_H , \\ Q_{L1} + Q_{R2} &= Q_H , \\ Q_{L2} + Q_{R1} &= Q_H , \\ Q_{L2} + Q_{R2} &= Q_H , \end{aligned}$$

Example: 4

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M=4 (Z4 symmetries)

3 class

[0], [1], [2]   [1] includes charges 1 and 3

Multiplication rules

[0] [0]=[0], [0][1]=[1], [0][2]=[2]

[1][2]=[1] , [2][2]=[1]

[1] [1] = [0]+[1]+[2]

Ising fusion rules

Example: 5

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M=5 (Z5 symmetry)

3 class

[0], [1], [2]   3 generations

[1] includes charges 1 and 4,

[2] includes charges 2 and 3

Multiplication rules

[0] [0]=[0],   [0][1]=[1], [0][2]=[2]

[1][1]=[0]+[2] ,   [2][2]=[0]+[1]

[1] [2] = [1]+[2]

Quark Yukawa textures for M=5

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T.K., Nishioka, Otsuka, Tanimoto, 2503.09966

various quark Yukawa texture

Example

$$Y_u, Y_d = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}.$$

NNI pattern

$$(f) \qquad Y_u = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & 0 & * \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

new one

More Quark Yukawa textures

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Example

$$(c) \qquad Y_u, Y_d = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix} .$$

$$(d) \qquad Y_u = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix} , \qquad Y_d = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix} .$$

Lepton sector

T.K., Otsuka, Tanimoto, Uchida 2505.07262

Textures in the charged lepton sector is similar.

Neutrino mass from Weinberg operators

$$\frac{{C_W}^{ij}}{\Lambda}(L_i \varepsilon H) C (L_j \varepsilon H),$$

Diagonal entries are always non-zero by our rules.

$$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \quad \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix},$$

More applications

Radiative neutrino masses

T.K., Okada, Otsuka, 2505.14878

Matter symmetries to avoid the fast proton decay in SUSY

T.K., Mita, Otsuka, Sakuma, 2506.10241

SMEFT

T.K., Otsuka, Tanimoto, Yamamoto, 2608.XXXXX

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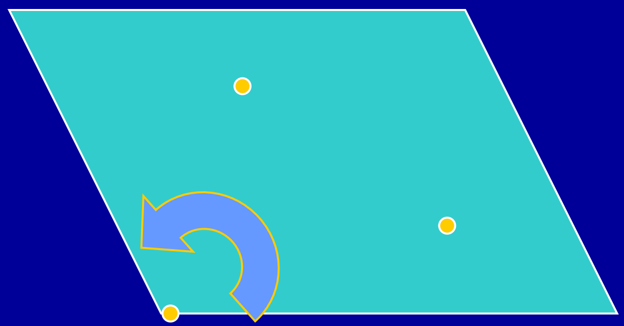
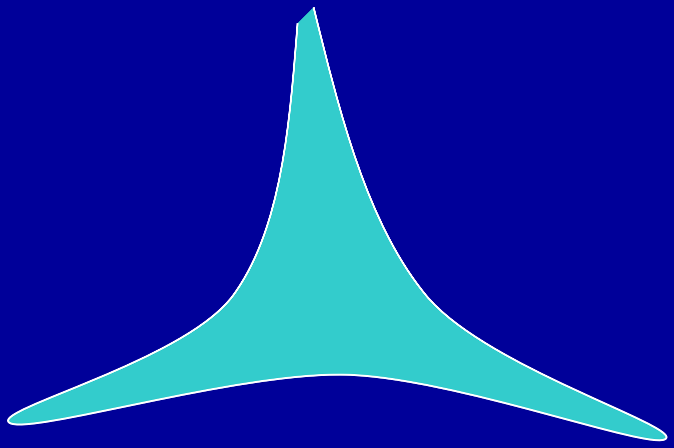
3. Heterotic string on orbifolds

Abelian orbifold $T^g/\mathbb{Z}_N$

T2/Z3 Orbifold



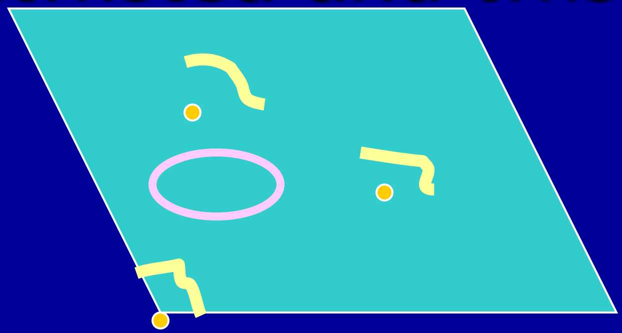


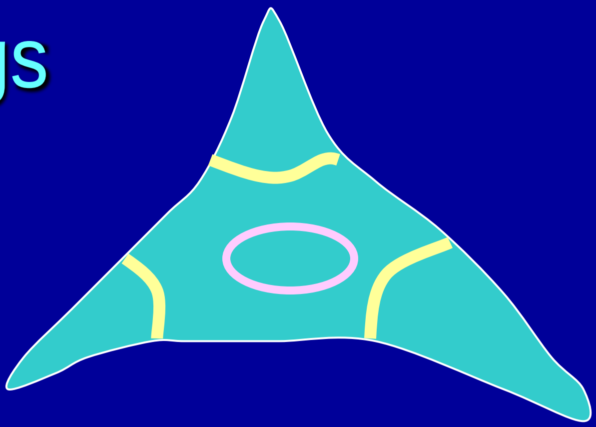
There are three fixed points on Z3 orbifold

Heterotic string on orbifold

T2/Z3 closed string on orbifold

Untwisted and twisted strings





B.C. $X(\sigma=\pi)=\theta^k X(\sigma=0)+e$ $e=n_a e_a^i$

e_a^i lattice vector (θ, e) space group

In what follows, we focus on e=0 for simplicity.

Non-Abelian orbifolds

Abelian orbifold $T^g/\mathbb{Z}_N$

-> Non-Abelian orbifold

$$T^g/G$$

G is a non-Abelian discrete group.

String on non-Abelian orbifolds

B.C.

$$X(\sigma = \pi) = gX(\sigma = 0)$$

another string hX satisfies the same B.C.

$$hX(\sigma = \pi) = ghX(\sigma = 0)$$

$$g,\ h \in G$$

That is,

$$X(\sigma = \pi) = h^{-1}ghX(\sigma = 0)$$

String is classified not by group elements, but by conjugacy classes.

String on non-Abelian orbifolds

String is classified by not group elements, but conjugacy classes.

$$X(\sigma=\pi) = h^{-1} g h X(\sigma=0) \qquad g,\ h \in G$$

String coupling selection rules are determined not by group theory, but by multiplication rules of conjugacy classes.

Example: T2/S3

There 3 conjugacy classes.

Multiplication rules

	C_1	C_2^1	B_1
C_1	C_1	C_2^1	B_1
C_2^1	C_2^1	$2C_1 + C_2^1$	$2B_1$
B_1	B_1	$2B_1$	$3C_1 + 3C_2^1$

Similarly, DN, A4, S4, T7, Δ(27), Δ(54) orbifolds,etc.

Dong, Jeric, T.K., Nishida, Otsuka, 2507.02375

Dong, T.K., Miyamoto, Nishida, Otsuka, 2603.14836

Example: T2/S3

There 3 conjugacy classes.

Multiplication rules Z2 gauging of Z3

	C_1	$C_{\frac{1}{2}}^{1}$	B_1
C_1	C_1	$C_{\frac{1}{2}}^{1}$	B_1
$C_{\frac{1}{2}}^{1}$	$C_{\frac{1}{2}}^{1}$	$2C_1 + C_{\frac{1}{2}}^{1}$	$2B_1$
B_1	B_1	$2B_1$	$3C_1 + 3C_{\frac{1}{2}}^{1}$

Similarly, DN, A4, S4, T7, Δ(27), Δ(54) orbifolds,etc.

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More discrete gauging

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Z2 gauging of ZM,

Z2 invariant classes of DN

Z3 gauging of ZN, ZN x ZN, ZNxZNxZN

Z3 invariant classes of TN, $\Delta(3N^2)$,

S3 gauging of ZN x ZN

S3 invariant classes of $\Delta(6N^2)$

………………………………….

More discrete gauging

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Z3 gauging of ZN   N=7

	C_1	C_3^1	C_3^2
C_1	C_1	C_3^1	C_3^2
C_3^1	C_3^1	$C_3^1 + 2C_3^2$	$3C_1 + C_3^1 + C_3^2$
C_3^2	C_3^2	$3C_1 + C_3^1 + C_3^2$	$2C_3^1 + C_3^2$

Z3 gauging has complex representations, while representations of Z2 gauging are self-conjugate, i.e., real representations.

Loop effects

Loop effects violate selection rules at tree level,

But are calculable.

No new parameters.

Example: S3

There 3 conjugacy classes.   Multiplication rules

	C_1	C_2^1	B_1
C_1	C_1	C_2^1	B_1
C_2^1	C_2^1	$2C_1 + C_2^1$	$2B_1$
B_1	B_1	$2B_1$	$3C_1 + 3C_2^1$

$$0 = C_1 = \{e\}, \quad \dot{1} = C_2^1 = \{a, a^2\}, \quad \dot{2} = B_1 = \{b, ab, a^2b\}.$$

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Example: S3

There 3 conjugacy classes. Multiplication rules

$$0 = C_1 = \{e\}, \quad \dot{1} = C_2^1 = \{a, a^2\}, \quad \dot{2} = B_1 = \{b, ab, a^2 b\}.$$

Allowed couplings at tree level

$$\lambda^{(0)}_{000}, \quad \lambda^{(0)}_{0\dot{1}\dot{1}}, \quad \lambda^{(0)}_{0\dot{2}\dot{2}}, \quad \lambda^{(0)}_{\dot{1}\dot{1}\dot{1}}, \quad \lambda^{(0)}_{\dot{1}\dot{2}\dot{2}}.$$

Forbidden coupling $\lambda^{(1)}_{00\dot{1}}$

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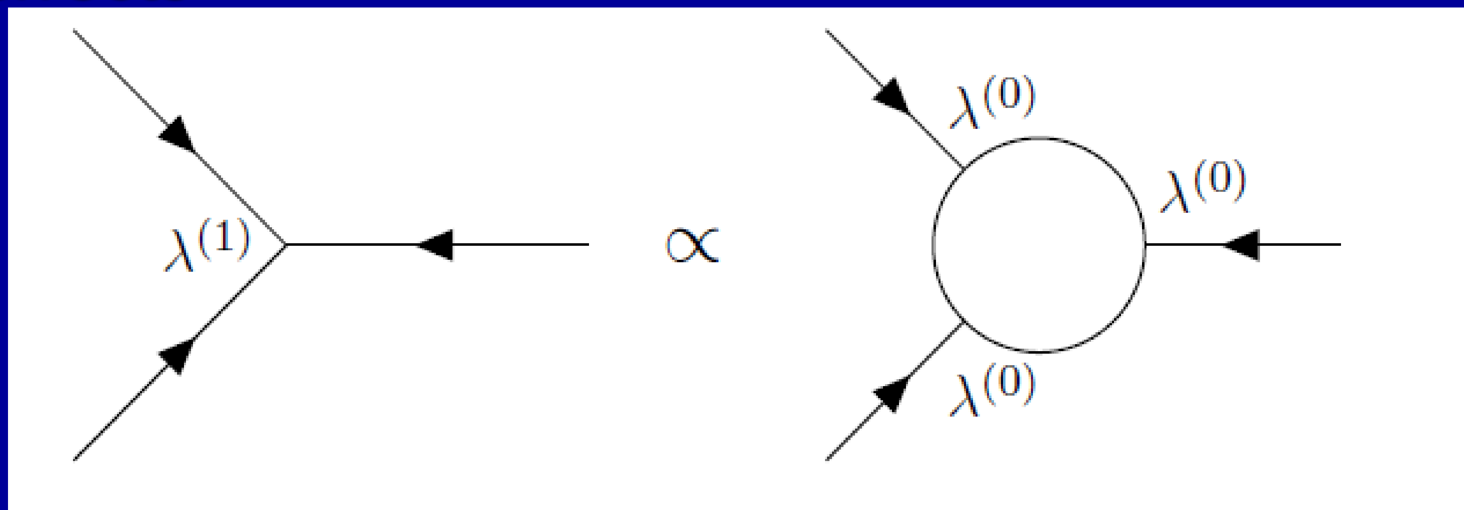
Example: S3

Loop effects



$$\lambda^{(1)}_{00\dot{1}} \propto \lambda^{(0)}_{0\dot{1}\dot{1}}\lambda^{(0)}_{0\dot{1}\dot{1}}\lambda^{(0)}_{\dot{1}\dot{1}\dot{1}} + \lambda^{(0)}_{0\dot{2}\dot{2}}\lambda^{(0)}_{0\dot{2}\dot{2}}\lambda^{(0)}_{\dot{1}\dot{2}\dot{2}}.$$

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Example: S3

Loop effects

$$\lambda^{(1)}_{00\dot{1}} \propto \lambda^{(0)}_{0\dot{1}\dot{1}}\lambda^{(0)}_{0\dot{1}\dot{1}}\lambda^{(0)}_{1\dot{1}\dot{1}} + \lambda^{(0)}_{0\dot{2}\dot{2}}\lambda^{(0)}_{0\dot{2}\dot{2}}\lambda^{(0)}_{1\dot{2}\dot{2}}.$$

We need certain tree-level couplings to induce $\lambda^{(1)}_{00\dot{1}}$

We introduce approximate Z2' symmetry.

0,2 Z2 even, 1, Z2 odd

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Example: S3

Loop effects

$$\lambda^{(1)}_{00\dot{1}} \propto \lambda^{(0)}_{0\dot{1}\dot{1}}\lambda^{(0)}_{0\dot{1}\dot{1}}\lambda^{(0)}_{\dot{1}\dot{1}\dot{1}} + \lambda^{(0)}_{0\dot{2}\dot{2}}\lambda^{(0)}_{0\dot{2}\dot{2}}\lambda^{(0)}_{\dot{1}\dot{2}\dot{2}}.$$

We need certain tree-level couplings to induce $\lambda^{(1)}_{00\dot{1}}$

We introduce approximate Z2' symmetry.
0,2 Z2 even, 1, Z2 odd

That is violated by some couplings.

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Loop effects

Our selection rules are violated by loop effects.

One can think that
tree level Lagrangian is obtained
at the compactification scale.

Summary

We have studied non-invertible coupling
selection rules
and their implications in particle physics.
That provides us with a new scenario
in particle physics

Summary

Theoretical side:

we will study which non-invertible symmetries can appear

e.g., from string compactifications.

Phenomenological side:

We will study their implications

in particle physics including symmetries other

than known ones from string theory.

It is important to study both complementary.