Flavor Physics of quarks and leptons in non-invertible selection rules

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Plan of my talk

1 Introduction

2 Non-invertible selection rules

3 Textures of dim.6 operators in SMEFT

4 Summary and Outlook

1 Introduction

Mass hierarchy

Flavor mixing

CP violation

of quarks and leptons

In order to challenge these questions, one can discuss them in the framework of "texture zeros approach".

Many works in Quarks and Leptons

Texture zeros approach

Two generations of quarks

- In the basis in which the up-type quark mass matrix is diagonal,

S. Weinberg, Trans. New York Acad. Sci. 38 (1977) 185

$$m_{\text{down}} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

2 parameters

$$\xrightarrow{M \gg m} \begin{pmatrix} m^2/M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}$$

one prediction

The Cabibbo angle is successfully predicted to be

$$\sin\theta_c \sim \tan\theta_c = m/M = \sqrt{m_d/m_s}$$

S. Weinberg (1977)

GST relation (1968)

R. Gatto, G. Sartori and M. Tonin, Phys. Lett. B 28 (1968) 128.

- How do we derive Yukawa interaction $Y_{ij}\psi_i^L\psi_j^R H$ with

$$Y_{ij} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

- Ex., let us consider $U(1)$ or $\mathbb{Z}_N$ symmetry
- U(1) or $\mathbb{Z}_N$ charges for fermions and Higgs should satisfy

5 charges

Q_1^L Q_2^L Q_1^R Q_2^R Q_H

$$\boxed{\begin{aligned} Q_1^L + Q_1^R + Q_H &\neq 0 \\ Q_1^L + Q_2^R + Q_H &= 0 \\ Q_2^L + Q_1^R + Q_H &= 0 \\ Q_2^L + Q_2^R + Q_H &= 0 \end{aligned}}$$

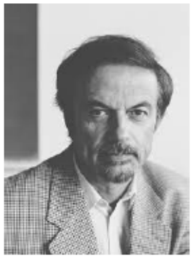
No solutions!

H.Fritzsch and Z.z.Xing, Mass and flavor mixing schemes of quarks and leptons
Prog. Part. Nucl. Phys. 45 (2000)

Four zero texture of Hermitian quark mass matrices and current experimental tests
Phys. Lett. B 555(2003)

Three generations of quarks

- Fritzsch extended this approach to the three family case (1978)



$$m_{u,d}^{(\mathrm{Fritzsch})} = \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix}$$

leading to various relations between masses and mixing angles

See for systematic approach with four or five zeros for symmetric or hermitian quark mass matrices by Ramond-Roberts-Ross (1993))

These textures are not viable today under the precise experimental data

Texture zeros approach

Three generations of quarks

The Fritzsch texture belongs to the more generic
Nearest-neighbor-interaction (NNI) form of quark mass matrices :

G.C.Branco, L.Lavoura, F.Mota (89),

Both up and down are NNI type

$$m_{u,d}^{(\mathrm{NNI})} = \begin{pmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & e \end{pmatrix}$$

12 parameters = 10 real ones and 2 phases

consistent with observed CKM matrices and quark masses

(also for leptons)

(obtained by choosing a weak-basis transformation
from general 3×3 matrices of Yukawa matrices)

Q : Origin of these textures ?

A : They cannot be derived by the conventional symmetry

"Occam's Razor" approach for quark mass matrices

For quark masses and CKM, Tanimoto and Yanagida arXiv:1601.04459

Three zeros of down-type quark mass matrix

Diagonal basis of up-type quarks Mu

$$M_D = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}, \qquad M_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

9 real parameters and one phase = CKM phase

Lepton sector

Successful neutrino mass matrix with 2 zeros

P.H. Frampton, S.L. Glashow and D. Marfatia, Phys. Lett. B536 (2002) 79.

2 Non-invertible selection rules

Possible Origin of Texture zeros

"Modular Zeros" talk by Michale Ratz

Group theory

Closure

If a and b are elements of group G, c=a $\circ$ b is also its element.

c is unique

each field corresponds to a (representation of) element of G

$$\phi_a + \phi_b \to \phi_c$$

Inverse process is realized

a b=1

Fusion Rule

Consider set of elements $\{U_i\}$

$$U_i U_j = \sum_k c_{ijk} U_k$$

Let us consider an example, $U_1 U_2 = U_3 + U_5$

$$\phi_1 + \phi_2 \to \phi_3 \qquad \phi_1 + \phi_2 \to \phi_5$$

Right-handed side is not unique.

Inverse operation cannot be defined !

These fusion rules may be originated from string compactification.

Ex: Type IIB magnetized D-brane models on toroidal orbifolds T.Kobayashi, H.Otsuka

Many examples in 2D CFT
E.P.Verlinde (1988),
G.W.Moore and N. Seiberg (1989),

- **Ordinary symmetry :**

  - Fusion rule of symmetry operator :

$$U_{g_1} U_{g_2} = U_{g_1 g_2} \qquad g_1, g_2 \in G$$

Ex., $U(1) : e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$

$$\boxed{U_\theta U_{-\theta} = 1}$$

- **Non-invertible symmetry :**

  - Fusion rule of symmetry operator :

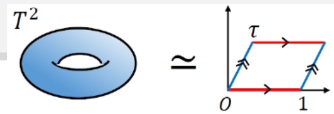
Ex., $U_{\theta_1} U_{\theta_2} = U_{\theta_1+\theta_2} + U_{\theta_1-\theta_2}$

$U_\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ except for $\theta = 0, \pi$

For $\theta_2 = -\theta_1 = \theta$ $\quad \boxed{U_\theta U_{-\theta} = U_0 + U_{2\theta} = 1 + U_{2\theta}}$

Non-invertible symmetries in type IIB string theory

T^2



H.Otuka

- Operators: $e^{\frac{i}{M}\hat{D}_1}$ $e^{\frac{i}{M}\hat{D}_2}$

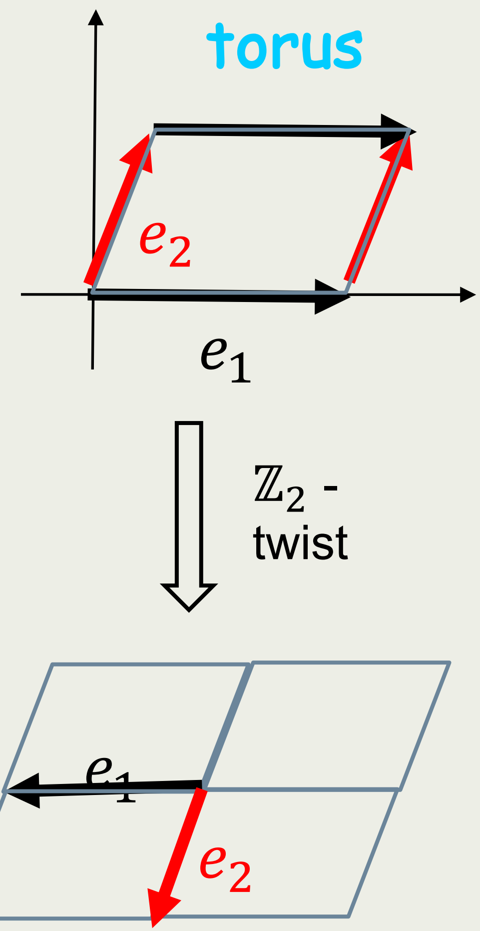
- Chiral matters: $|\psi^{j,M}\rangle$

- Representations:

$$\mathbb{Z}_M^{(D_1)} : e^{\frac{i}{M}\hat{D}_1}|\psi^{j,M}\rangle = |\psi^{j+1,M}\rangle$$

$$\mathbb{Z}_M^{(D_2)} : e^{\frac{i}{M}\hat{D}_2}|\psi^{j,M}\rangle = e^{2\pi i\frac{j}{M}}|\psi^{j,M}\rangle$$

$T^2/\mathbb{Z}_2$



Non-invertible symmetries in type IIB string theory

T^2	$T^2/\mathbb{Z}_2$
- Operators: $e^{\frac{i}{M}\hat{D}_1}$, $e^{\frac{i}{M}\hat{D}_2}$	$\hat{U}_1 \equiv e^{\frac{i}{M}\hat{D}_1} + e^{-\frac{i}{M}\hat{D}_1}$, $\hat{U}_2 \equiv e^{\frac{i}{M}\hat{D}_2} + e^{-\frac{i}{M}\hat{D}_2}$
- Chiral matters: $\lvert\psi^{j,M}\rangle$	$\mathbb{Z}_2$ -even : $\lvert\psi^{j,M}\rangle_+ = \frac{1}{2}\left(\lvert\psi^{j,M}\rangle + \lvert\psi^{M-j,M}\rangle\right)$
- Representations: $\mathbb{Z}_M^{(D_1)} : e^{\frac{i}{M}\hat{D}_1}\lvert\psi^{j,M}\rangle = \lvert\psi^{j+1,M}\rangle$, $\mathbb{Z}_M^{(D_2)} : e^{\frac{i}{M}\hat{D}_2}\lvert\psi^{j,M}\rangle = e^{2\pi i\frac{j}{M}}\lvert\psi^{j,M}\rangle$	$\hat{U}_1 \lvert\psi^{j,M}\rangle_+ = \lvert\psi^{j+1,M}\rangle_+ + \lvert\psi^{j-1,M}\rangle_+$, $\hat{U}_2 \lvert\psi^{j,M}\rangle_+ = 2\cos\left(\frac{2\pi j}{M}\right)\lvert\psi^{j,M}\rangle_+$

Corresponding to the gauging the (outer) automorphism of each $\mathbb{Z}_M^{(D_1)}$ and $\mathbb{Z}_M^{(D_2)}$

$$j \quad \rightarrow \quad [j] \ : \ j \text{ and } M-j$$

Selection rules without group actions

T. Kobayashi, H. Otuka, M. Tanimoto, 2409.05270

- Let us start with $\mathbb{Z}_M$ *symmetry (generators :* $g = e^{2\pi i/M}$*)*

  Transformation of fields ϕ_j *in 4D QFT :*

$$\phi_j \to g^{k_j}\phi_j$$

  Charge k_j under $\mathbb{Z}_M$

- Let us consider the automorphism of $\mathbb{Z}_M$, *i.e.,* $\boxed{D_M \cong \mathbb{Z}_M \rtimes \mathbb{Z}_2}$

D_M group

$$D_M \cong \mathbb{Z}_M \rtimes \mathbb{Z}_2$$

$g^M=e \quad r^2=e \quad rgr^{-1}=g^{-1}$

"class"

$$ege^{-1} = g \qquad rgr^{-1} = g^{-1}$$

$$[g^k] = \boxed{\{hg^k h^{-1} \mid h = e, r\} = \{g^k, g^{M-k}\}} \qquad g^{-k} = g^{M-k}$$

Example: D_5 case **There are 4 classes**

Z_2 invariant

$C_1=\{e\} \quad C_2^{(1)}=\{g, g^4\} \quad C_2^{(2)}=\{g^2, g^3\} \quad C_5=\{r, gr, g^2r, g^3r\ g^4r\}$

Labelling fields by class

$$\boldsymbol{\Phi_j} = \phi_j + \phi_{M-j}$$

In $\mathbb{Z}_2$ basis, they have both j and M-j $\mathbb{Z}_\mathbf{M}$ charges.

corresponding to $\mathbb{Z}_2$ invariant modes : $\boldsymbol{\Phi_j} = \phi_j + \phi_{M-j}$

labelling fields by class

No definite Z_M charge

★ ϕ_j is not invariant under Z_2 orbifolding $\mathbf{T^{2n}}/\mathbb{Z}_2$

★ Φ_j has both charge j and M-j

That is $\mathbb{Z}_2$ gauging of $\mathbb{Z}_\mathrm{M}$ symmetry

Selection rules without group actions

Instead of labelling fields by **representations**, we label them by **class**:

$$[g^k] = \{\tilde{g},\ \tilde{g}^{M-k}\}$$

$$\tilde{g}^{k_1}\tilde{g}^{k_2} = \tilde{g}^{k_1+k_2}, \quad \tilde{g}^{k_1}\tilde{g}^{-k_2} = \tilde{g}^{k_1-k_2},$$
$$\tilde{g}^{-k_1}\tilde{g}^{k_2} = \tilde{g}^{-k_1+k_2}, \quad \tilde{g}^{-k_1}\tilde{g}^{-k_2} = \tilde{g}^{-k_1-k_2}.$$

$$[g^{k_1}][g^{k_2}] = [g^{k_1+k_2}] + [g^{M-k_1+k_2}]$$

Suppose that Φ_j is labeld by a class $[g^{k_j}]$.
An interaction in the classical Lagrangian

$$\mathcal{L} \supset \Phi_1 \cdots \Phi_n$$

is allowed when $\tilde{g}^{k_1} \cdots \tilde{g}^{k_n} = e$ (identity) $\quad \tilde{g}^{k_j} \in [g^{k_j}]$

How do we understand the Yukawa texture without group-like structure?

$$Y_{ij} = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

$M = 3$ ($\mathbb{Z}_2$ *gauging of* $\mathbb{Z}_3 \rtimes \mathbb{Z}_2$)

T. Kobayashi, H.Otsuka, M. Tanimoto, 2409.05270

- Two different classes :

$$[g^0], \qquad [g^1] = \{\tilde{g}^1, \tilde{g}^2\}$$

$$[\boldsymbol{g}^{k}] \supset \tilde{\boldsymbol{g}}^{k}, \tilde{\boldsymbol{g}}^{3-k}$$

- *Fusion rule :*

$$[\boldsymbol{g}^{k_1}][\boldsymbol{g}^{k_2}] = [\boldsymbol{g}^{k_1+k_2}] + [\boldsymbol{g}^{M-k_1+k_2}]$$

$$[g^0][g^0] = [g^0]$$
$$[g^0][g^1] = [g^1]$$
$$[g^1][g^1] = [g^0] + [g^1]$$

- *Previous example for two generations of fermions:*

Left : ($[g^0], [g^1]$)
Right : ($[g^0], [g^1]$)
Higgs : $[g^1]$

$$Y = \begin{pmatrix} 0 & * \\ * & * \end{pmatrix}$$

$M = 5$ ($\mathbb{Z}_2$ gauging of $\mathbb{Z}_5 \rtimes \mathbb{Z}_2$)

- Three different classes :

$$[g^0], \quad [g^1], \quad [g^2]$$

$$[\boldsymbol{g}^{\boldsymbol{k}}] \supset \tilde{\boldsymbol{g}}^{\boldsymbol{k}}, \tilde{\boldsymbol{g}}^{\boldsymbol{5-k}}$$

- *Yukawa couplings for 3 generations of fermions :*

$$[\boldsymbol{g}^{\boldsymbol{k_1}}][\boldsymbol{g}^{\boldsymbol{k_2}}] = [\boldsymbol{g}^{\boldsymbol{k_1+k_2}}] + [\boldsymbol{g}^{\boldsymbol{M-k_1+k_2}}]$$

Fusion rules :

$[g^0][g^0] = [g^0]$

$[g^1][g^1] = [g^0] + [g^2]$

$[g^2][g^2] = [g^0] + [g^1]$

$[g^1][g^2] = [g^1] + [g^2]$

Left : ($[\boldsymbol{g}^{\boldsymbol{0}}], [\boldsymbol{g}^{\boldsymbol{1}}], [\boldsymbol{g}^{\boldsymbol{2}}]$***)***

Right : ($[\boldsymbol{g}^{\boldsymbol{0}}], [\boldsymbol{g}^{\boldsymbol{1}}], [\boldsymbol{g}^{\boldsymbol{2}}]$***)***

Higgs : $[\boldsymbol{g}^{\boldsymbol{1}}]$

$$\boxed{Y_{[g^1]} = \begin{pmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & e \end{pmatrix}}$$

Consider Direct product group theory

$\mathbb{Z}_2$ gaugings of $\mathbb{Z}_{\mathbf{M}} \times \mathbb{Z}_{\mathbf{M}'}$ symmetries

$$\mathbb{Z}_{\mathbf{M}} \times \mathbb{Z}_{\mathbf{M}'} \rtimes \mathbb{Z}_{\mathbf{2}}$$

Two multiplication rules

$$U_i U_j = \sum_k c_{ijk} U_k \,, \qquad U'_{i'} U'_{j'} = \sum_{k'} c'_{i'j'k'} U'_{k'} \,.$$

$$(U_i, U'_{i'})(U_j, U'_{j'}) = \sum_{k,k'} c_{ijk} c'_{i',j',k'} (U_k, U'_{k'}) \,.$$

$\mathbb{Z}_5 \times \mathbb{Z}_5 \rtimes \mathbb{Z}_2$ 3 classes $\boxed{\{[g^0],[g^1],[g^2]\}}$

$$
\begin{array}{lcl}
Q_L & : & ([g_1^0][g_2^0],\ [g_1^1][g_2^1],\ [g_1^0][g_2^1]) \\
U_R & : & ([g_1^0][g_2^0],\ [g_1^1][g_2^1],\ [g_1^0][g_2^1]) \\
D_R & : & ([g_1^1][g_2^1],\ [g_1^2][g_2^2],\ [g_1^2][g_2^1])
\end{array}
$$

Hu : $[g^0]\ [g^0]$, Hd : $[g^2]\,[g^2]$

Fusion rules :

$[g^0][g^0]=[g^0]$

$[g^1][g^1]=[g^0]+[g^2]$

$[g^2][g^2]=[g^0]+[g^1]$

$[g^1][g^2]=[g^1]+[g^2]$

$$
M_D = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}, \qquad M_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}
$$

which are obtained in “Occam’s Razor” approach

3 Textures of dim.6 operators in SMEFT

T. Kobayashi, H. Otuka, M. Tanimoto, K. Yamamoto, arXiv xxxx

$\mathbb{Z}_5 \times \mathbb{Z}_5 \rtimes \mathbb{Z}_2$ 3 classes $\boxed{\{[g^0],[g^1],[g^2]\}}$

$$[g^0]\otimes[g^0]=[g^0]\,,\qquad [g^0]\otimes[g^1]=[g^1]\otimes[g^0]=[g^1]\,,$$
$$[g^0]\otimes[g^2]=[g^2]\otimes[g^0]=[g^2]\,,\qquad [g^1]\otimes[g^1]=[g^0]+[g^2]\,,$$
$$[g^1]\otimes[g^2]=[g^2]\otimes[g^1]=[g^1]+[g^2]\,,\qquad [g^2]\otimes[g^2]=[g^0]+[g^1]\,.$$

Let us take a simple set of quark and lepton mass matrices.

$$-\mathcal{L}_{\text{Yukawa}} = Y_u \bar{q}_i \widetilde{\Phi}_u u_j + Y_d \bar{q}_i \Phi_d d_j + Y_e \bar{\ell}_i \Phi_d e_j + Y_{\mathcal{W}} \frac{\ell\widetilde{\Phi}_u \ell\widetilde{\Phi}_u}{\Lambda}$$

$$Y_u = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix},\quad Y_d = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix},\quad Y_e = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix},\quad Y_{\mathcal{W}} = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

Impose $[q_i]=[\ell_i],\quad [d_i]=[e_i]$ for assingnment of Class

$\mathbb{Z}_5 \times \mathbb{Z}_5 \rtimes \mathbb{Z}_2$

$$
\begin{array}{lcl}
Q_L = L_L & : & ([g_1^0][g_2^0],\ [g_1^1][g_2^1],\ [g_1^0][g_2^1]) \\
U_R & : & ([g_1^0][g_2^0],\ [g_1^1][g_2^1],\ [g_1^0][g_2^1]) \\
D_R = e_R & : & ([g_1^1][g_2^1],\ [g_1^2][g_2^2],\ [g_1^2][g_2^1]) \\
H_u & : & [g_1^0][g_2^0]\,, \qquad\qquad H_d : [g_1^2][g_2^2]
\end{array}
$$

Case (a)

There are other 4 independent assignments.

4 fermi Semi-leptonic operator $(L\bar{L})\,(L\bar{L})$

$$C_{ijkl} Q_{\ell q}^{(1)} = C_{ijkl} (\bar{\ell}_L^i \gamma_\mu \ell_L^j)(\bar{q}_L^k \gamma^\mu q_L^l),$$
$$C_{ijkl} Q_{\ell q}^{(3)} = C_{ijkl} (\bar{\ell}_L^i \gamma_\mu \tau^I \ell_L^j)(\bar{q}_L^k \gamma^\mu \tau^I q_L^l).$$

Flavor basis

$$C_{ijkl}^{(a)} = \begin{pmatrix} [0]_1^2[0]_2^2 & [0]_1[1]_1[0]_2[1]_2 & [0]_1^2[0]_2[1]_2 \\ [0]_1[1]_1[0]_2[1]_2 & [1]_1^2[1]_2^2 & [0]_1[1]_1[1]_2^2 \\ [0]_1^2[0]_2[1]_2 & [0]_1[1]_1[1]_2^2 & [0]_1^2[1]_2^2 \end{pmatrix}_{\ell\ell} \begin{pmatrix} [0]_1^2[0]_2^2 & [0]_1[1]_1[0]_2[1]_2 & [0]_1^2[0]_2[1]_2 \\ [0]_1[1]_1[0]_2[1]_2 & [1]_1^2[1]_2^2 & [0]_1[1]_1[1]_2^2 \\ [0]_1^2[0]_2[1]_2 & [0]_1[1]_1[1]_2^2 & [0]_1^2[1]_2^2 \end{pmatrix}_{qq}$$

$$= \begin{array}{c} \\ k=1 \\ \\ k=2 \\ \\ k=3 \end{array} \begin{array}{c} \quad l=1 \qquad\quad l=2 \qquad\quad l=3 \\ \left(\begin{array}{ccc|ccc|ccc} \checkmark & 0 & 0 & 0 & \checkmark & 0 & 0 & 0 & \checkmark \\ 0 & \checkmark & 0 & \checkmark & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \checkmark & 0 & 0 & 0 & \checkmark & 0 & 0 \\ \hline 0 & \checkmark & 0 & \checkmark & 0 & 0 & 0 & 0 & 0 \\ \checkmark & 0 & 0 & 0 & \checkmark & 0 & 0 & 0 & \checkmark \\ 0 & 0 & 0 & 0 & 0 & \checkmark & 0 & \checkmark & 0 \\ \hline 0 & 0 & \checkmark & 0 & 0 & 0 & \checkmark & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \checkmark & 0 & \checkmark & 0 \\ \checkmark & 0 & 0 & 0 & \checkmark & 0 & 0 & 0 & \checkmark \end{array}\right) \end{array}$$

21 non-zero entries

$$C_{ijkl}^{(a)} = f_{ijkl} \left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \right)$$

i, j = leptons
k, ℓ= quarks

59 dim.6 operators in Warsaw basis

MFV

Operator class	No symmetry, 3 Gen.		$U(3)^5$, $\mathcal{O}(Y^1_{e,d,u})$		$U(2)^5$, $\mathcal{O}(V^1, \Delta^1)$	
	CP-even	CP-odd	CP-even	CP-odd	CP-even	CP-odd
X^3, H^6, H^4D^2, X^2H^2	9	6	9	6	9	6
ψ^2H^3	27	27	3	3	9	9
ψ^2XH	72	72	8	8	24	24
ψ^2H^2D	51	30	7	–	19	5
$(\bar{L}L)(\bar{L}L)$	171	126	8	–	40	17
$(\bar{L}L)(\bar{R}R)$	360	288	8	–	53	21
$(\bar{R}R)(\bar{R}R)$	255	195	9	–	29	–
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	405	405	–	–	29	29
Total	1350	1149	52	17	212	111

Our case (a) 693 953

Introducing Suprions

Let us move to the mass basis

$$L_d Y_d R_d^\dagger = \mathrm{diag}\,(y_d,\, y_s,\, y_b)\,, \qquad L_e^\dagger Y_e R_e = \mathrm{diag}\,(y_e,\, y_\mu,\, y_\tau)$$

$$V_{CKM} \equiv L_d^\dagger\,, \qquad U_{PMNS} \equiv L_e^\dagger$$

$$R_d^\dagger \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & \sqrt{2}\lambda & \lambda^5 \\ -\lambda & 1 & 1 \\ \lambda & -1 & 1 \end{pmatrix} \qquad R_e \simeq \begin{pmatrix} 1 & \lambda^2/\zeta & \zeta\lambda^3 \\ -\lambda^2/\zeta & 1 & \zeta\lambda \\ \zeta\lambda^3 & -\zeta\lambda & 1 \end{pmatrix}$$

Consider a few examples of operators

not complehensive analyses

Charged current

$$C_{ijkl}Q^{(3)}_{\ell q} = C_{ijkl}(\bar{\ell}^i_L \gamma_\mu \tau^I \ell^j_L)(\bar{q}^k_L \gamma^\mu \tau^I q^l_L) \quad b \to c\, \tau\, \bar{\nu}$$

Flavor basis $C^{(a)}_{ijkl} = f_{ijkl}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$

Mass basis $\tilde{C}^{(a)}_{pqrs} = \sum_{i,\ell=1}^{3} U_{pi} V^*_{s\ell} f_{ijkl}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$

PMNS CKM

$$\tilde{C}^{(a)}_{pqrs} = f_{qqrr}\, U_{pq} V^*_{sr} + f_{rqrq}\, U_{pr} V^*_{sq} + \sum_{\ell=1}^{3} f_{\ell rr\ell} U_{p\ell} V^*_{s\ell}\, \delta_{qr}$$

$b \to c\, \tau\, \nu$ $\quad \tilde{C}^{(a)}_{3323} = \cancel{f_{3322}\, U_{33} V^*_{32}} + f_{2323}\, U_{32} V^*_{33}$

$b \to c\, \mu\, \nu$ $\quad \tilde{C}^{(a)}_{2223} = \cancel{f_{2222}\, U_{22} V^*_{32} + f_{2222}\, U_{22} V^*_{32}} + \sum_{\ell=1}^{3} f_{\ell 22\ell} U_{2\ell} V^*_{3\ell}$

$b \to c\, e\, \nu$ $\quad \tilde{C}^{(a)}_{1123} = f_{1122}\, U_{11} V^*_{32} + f_{2121}\, U_{12} V^*_{31}$ $\quad \lambda^2$ **suppressed**

Neutral current

$$b \to s\, \nu_i \bar{\nu}_j$$

$$C_{ijkl} Q_{\ell q}^{(1)} = C_{ijkl} (\bar{\ell}_L^i \gamma_\mu \ell_L^j)(\bar{q}_L^k \gamma^\mu q_L^l),$$
$$C_{ijkl} Q_{\ell q}^{(3)} = C_{ijkl} (\bar{\ell}_L^i \gamma_\mu \tau^I \ell_L^j)(\bar{q}_L^k \gamma^\mu \tau^I q_L^l).$$

Flavor basis $\quad C_{ijkl}^{(a)} = f_{ijkl} \left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$

Mass basis $\quad \tilde{C}_{ijrs}^{(a)} = \sum_{k,\ell=1}^{3} V_{rk} V_{s\ell}^* f_{ijkl} \left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$

Taking i=j and r=2, s=3 $\quad b \to s\, \nu_i \bar{\nu}_j$

$$\sum_{i=1}^{3} \tilde{C}_{ii23}^{(a)} = \sum_{i,k=1}^{3} f_{iikk} V_{2k} V_{3k}^* + \sum_{k=1}^{3} f_{kkkk} V_{2k} V_{3k}^* + \sum_{\ell=1}^{3} f_{\ell\ell\ell\ell} V_{2\ell} V_{3\ell}^* \longrightarrow 0$$

For the case $i \neq j$

$b \to s\, \nu_2 \bar{\nu}_3$ and $b \to s\, \nu_3 \bar{\nu}_2$ are not Cabibbo suppressed.

4 Summary and Outlook

Non-invertible selection rules give us new aspects in flavor physics

These are obtained by the String (Orbifold) compactification

Applications of non-invertible selection rules are progressive in particle physics:
(Yukawa couplings, SMEFT .....)

◎ Desired texture zeros of Yukawas are realized simply.

◎ Texture zeros in coefficients of dim.6 operators can predict the flavor structure of the new physics, which will be testable in the rare deacy processes of quarks and leptons.

B anomaly, LFV, g-2, EDM, Higgs decays

Back up slide

$$M_D = \begin{pmatrix} 0 & 0 & a_D \\ 0 & b_D & c_D \\ a'_D & c'_D & d_D e^{-i\phi} \end{pmatrix} , \qquad M_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

“Occam’s Razor” approach for quark mass matrices   Tanimoto, Yanagida 2016

$$M_D = v_D \begin{pmatrix} 0 & 0 & a_D \\ 0 & b_D & c_D \\ d_D & e_D & f_D\, e^{-i\phi_D} \end{pmatrix}_{RL}, \qquad M_U = v_U \begin{pmatrix} a_U & 0 & 0 \\ 0 & b_U & 0 \\ 0 & 0 & d_U \end{pmatrix}_{RL}$$

$$\det[M_D] = -a_D b_D d_D\,, \qquad \det[M_U] = -a_U b_U d_U\,.$$

$\mathbb{Z}_5 \times \mathbb{Z}_5 \rtimes \mathbb{Z}_2$

	T **10**	$\overline{F}$ **5***	H_1	H_2	η
$\tilde{\mathbb{Z}}_5^{(1)}$	$([g^0],\,[g^1],\,[g^2])$	$([g^0],\,[g^1],\,[g^2])$	$[g^1]$	$[g^2]$	$[g^2]$
$\tilde{\mathbb{Z}}_5^{(2)}$	$([g^0],\,[g^1],\,[g^2])$	$([g^0],\,[g^1],\,[g^2])$	$[g^1]$	$[g^0]$	$[g^1]$

3 classes $\boxed{\{[g^0],[g^1],[g^2]\}}$

$$[g^0]\otimes[g^0] = [g^0]\,, \qquad [g^0]\otimes[g^1] = [g^1]\otimes[g^0] = [g^1]\,,$$
$$[g^0]\otimes[g^2] = [g^2]\otimes[g^0] = [g^2]\,, \qquad [g^1]\otimes[g^1] = [g^0]+[g^2]\,,$$
$$[g^1]\otimes[g^2] = [g^2]\otimes[g^1] = [g^1]+[g^2]\,, \qquad [g^2]\otimes[g^2] = [g^0]+[g^1]\,.$$