Electromagnetic signatures of axion dark matter halos around primordial black holes

Ongoing work (2609.xxxx)

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12th Workshop on Flavor Symmetries and Consequences in Accelerators and Cosmology, 2026-08-21

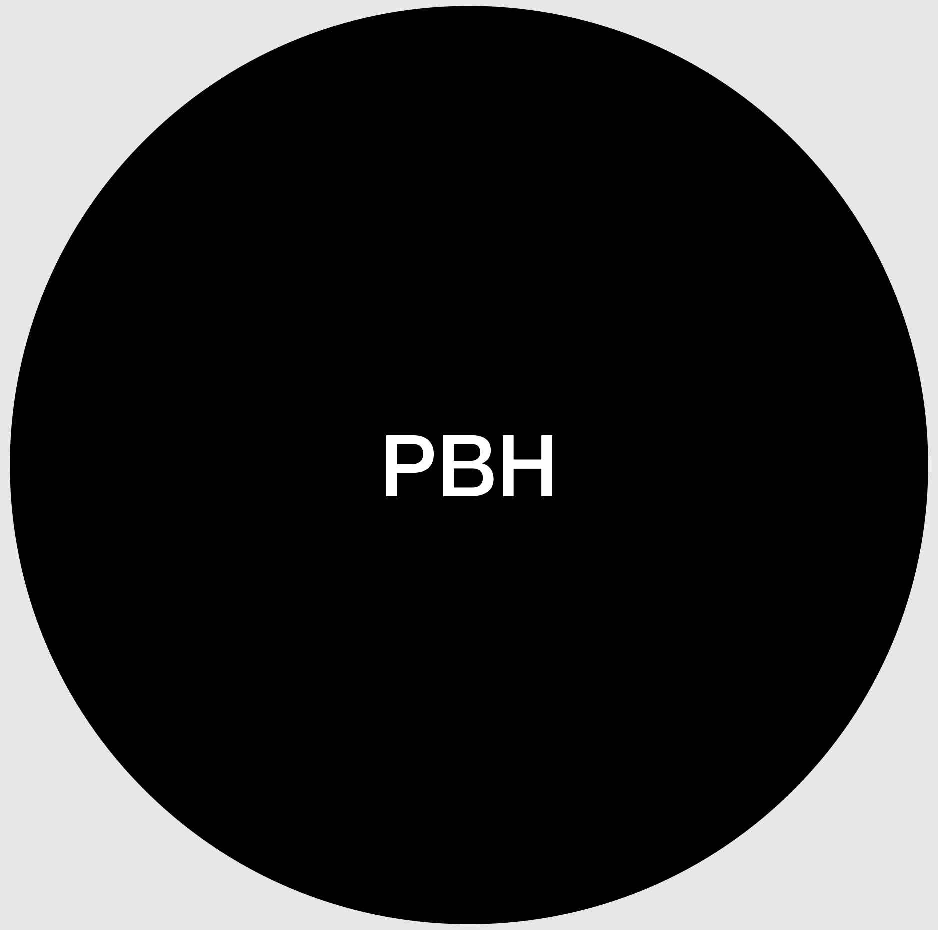
Motivation

The search for PBH-ALP DM co-existence

- **Coexistence:** The dark matter (DM) can be composed of both *Primordial Black Holes (PBHs)* and *Axion-Like Particles (ALPs)*.
- **ALP Halo:** Around PBHs, ALPs can form halos due to gravitational attraction. It is denser than the NFW profile, which is formed in MD era.
- **Photon Enhancement:** The high ALP density can *stimulate photon production* from ALP decay.
- **Observable Flux:** This enhancement produces a *detectable photon flux* that reaches Earth, providing a signature for searching for or constraining both PBHs and ALPs.

ALP Halo around PBH

Dark Matter: PBH + ALP



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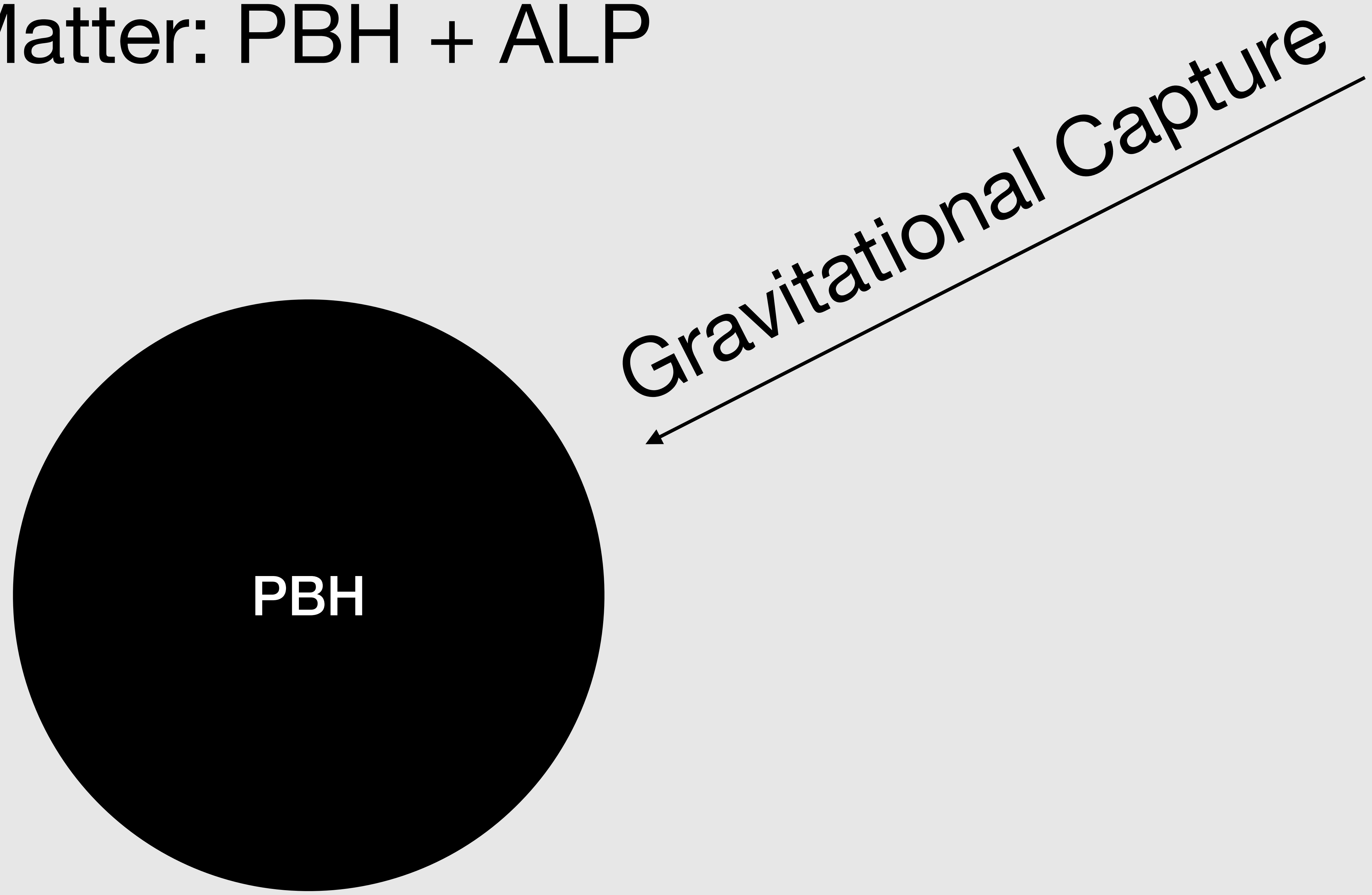
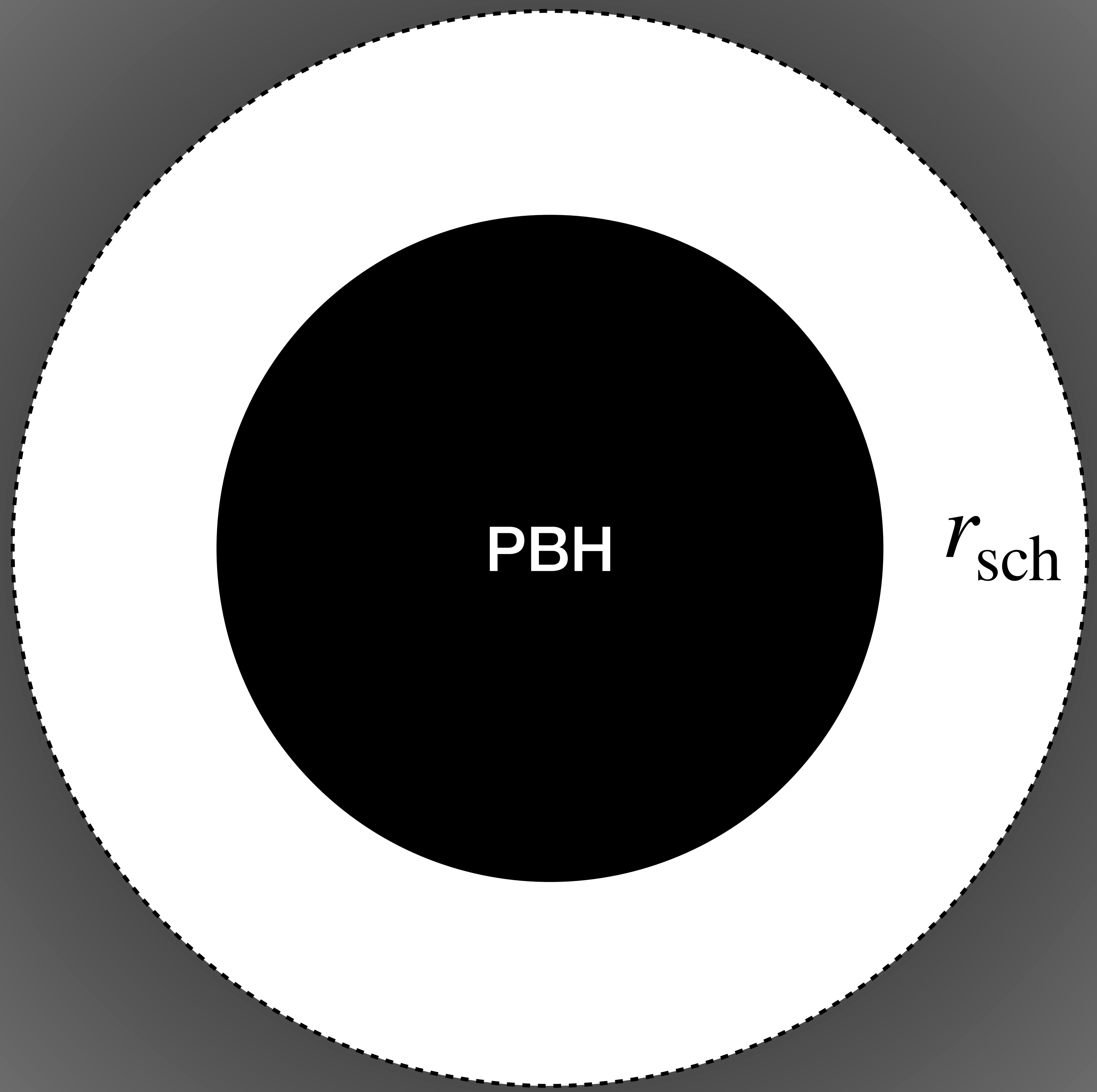


ALP Halo around PBH

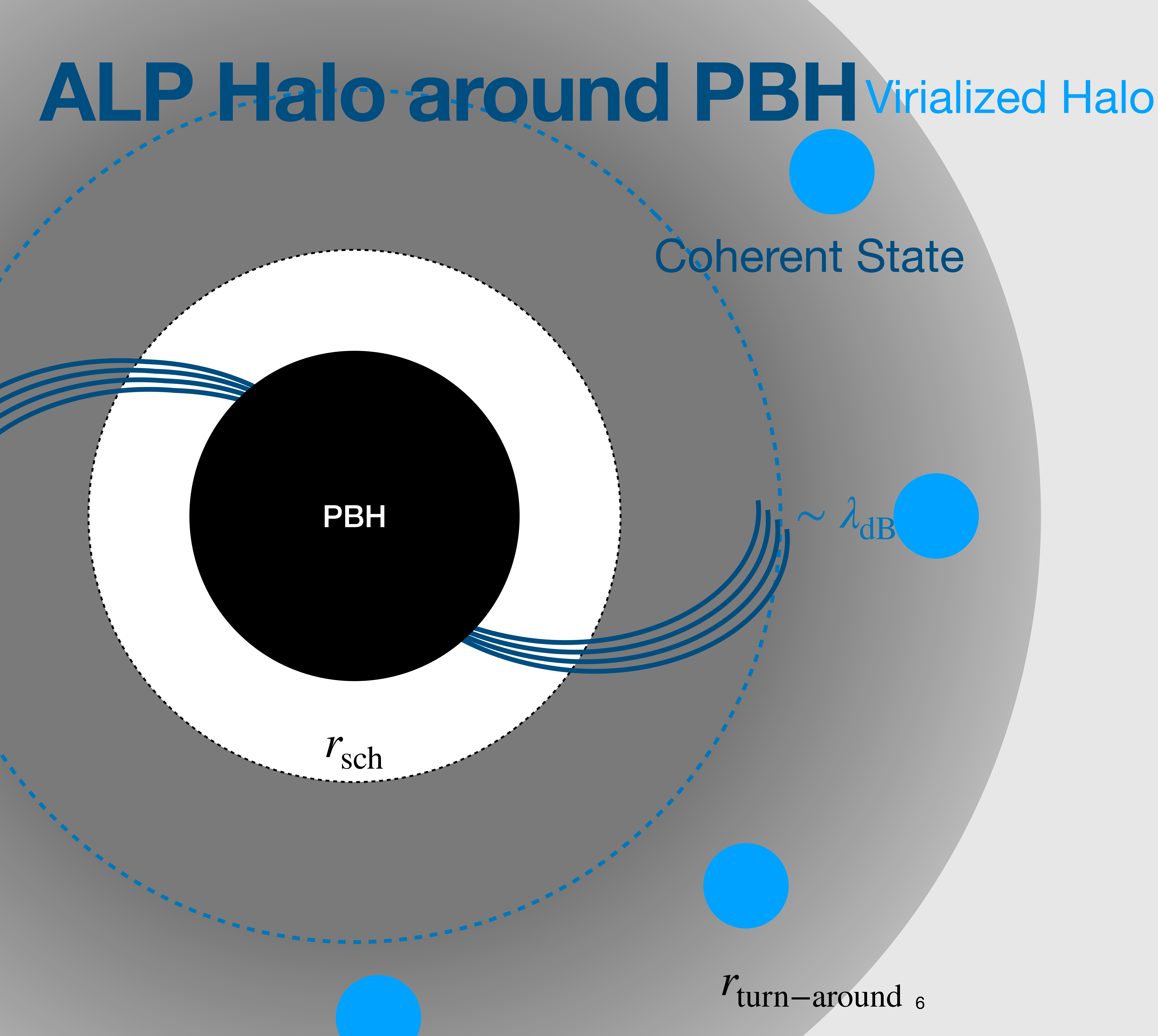


$$r_{\rm turn-around} \simeq (r_{\rm sch} t^2)^{1/3}$$

: Gravity = Expansion

Adamek et al., Phys.Rev.D 100 (2019) 2, 023506

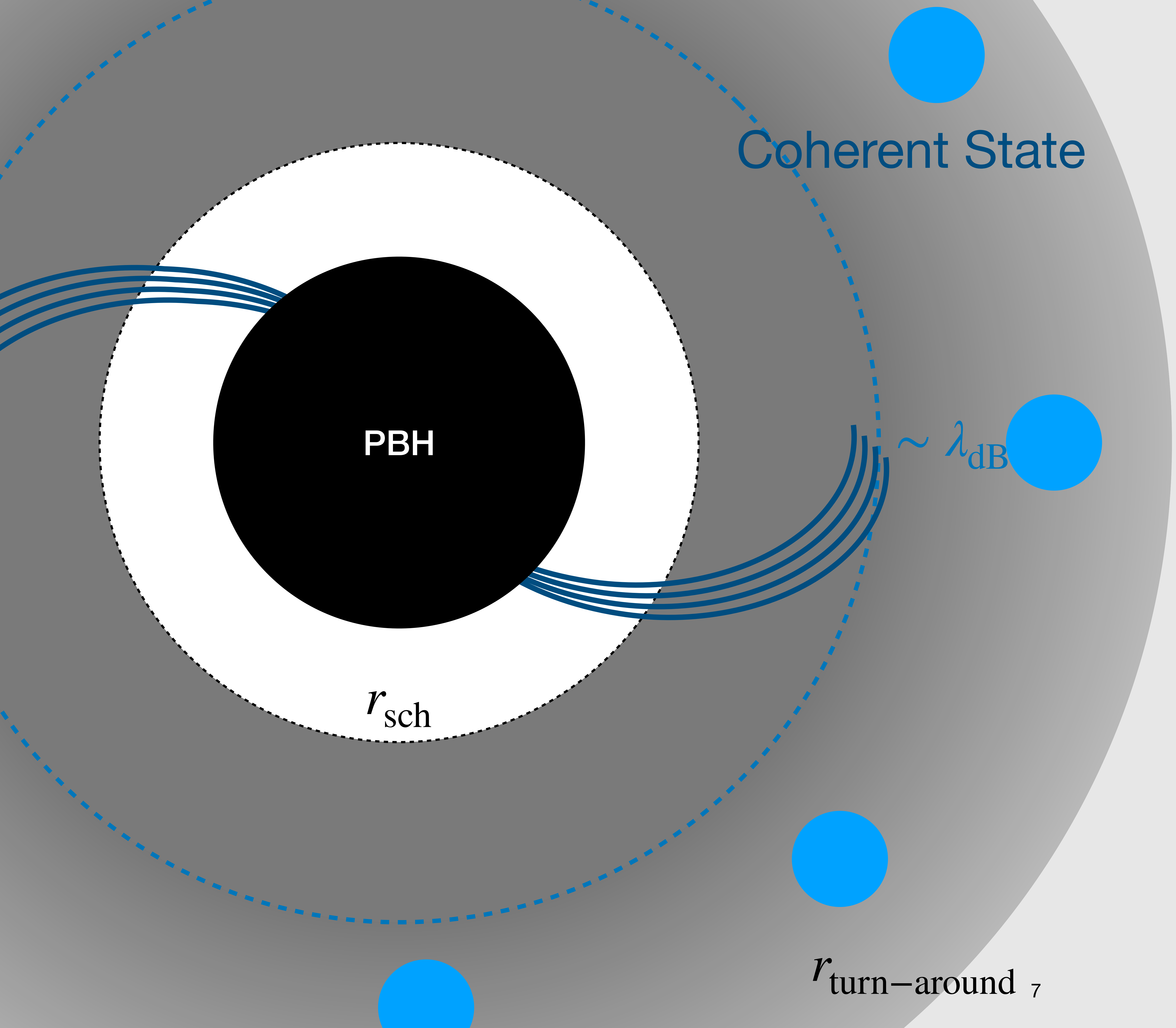
ALP Halo around PBH



ALP Halo around PBH

Virialized Halo → Spike density profile $\propto r^{-9/4}$

Quinlan, et al.,Astrophys.J. 440 (1995), 554-564
Gondolo, et al., Nucl.Phys.B Proc.Suppl. 87 (2000), 87-89



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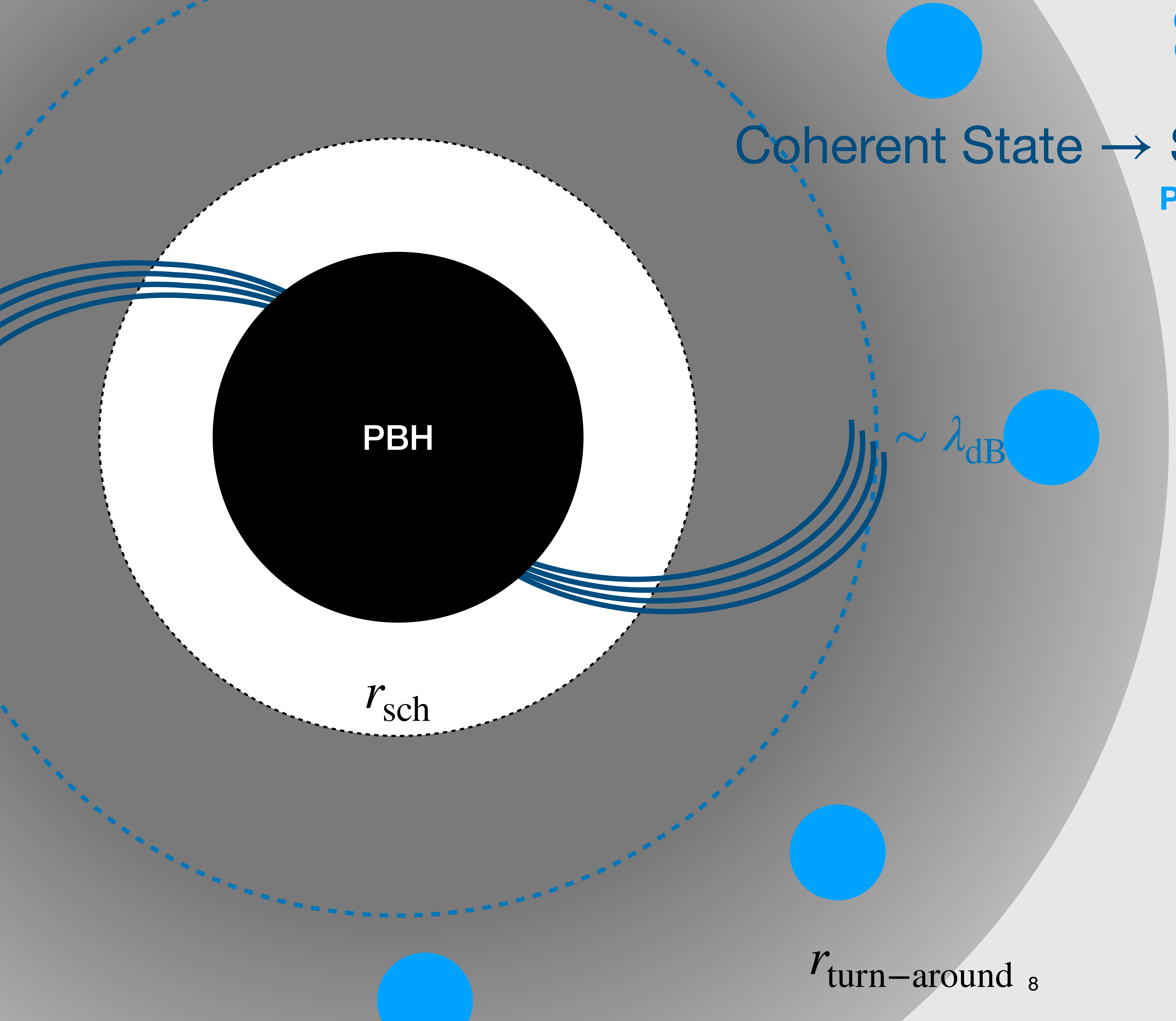
Coherent State → Soliton-like Halo $\propto e^{-r/r_0}$

Palomares-Chavez, et al., Phys.Rev.D 112 (2025) 2, 023545



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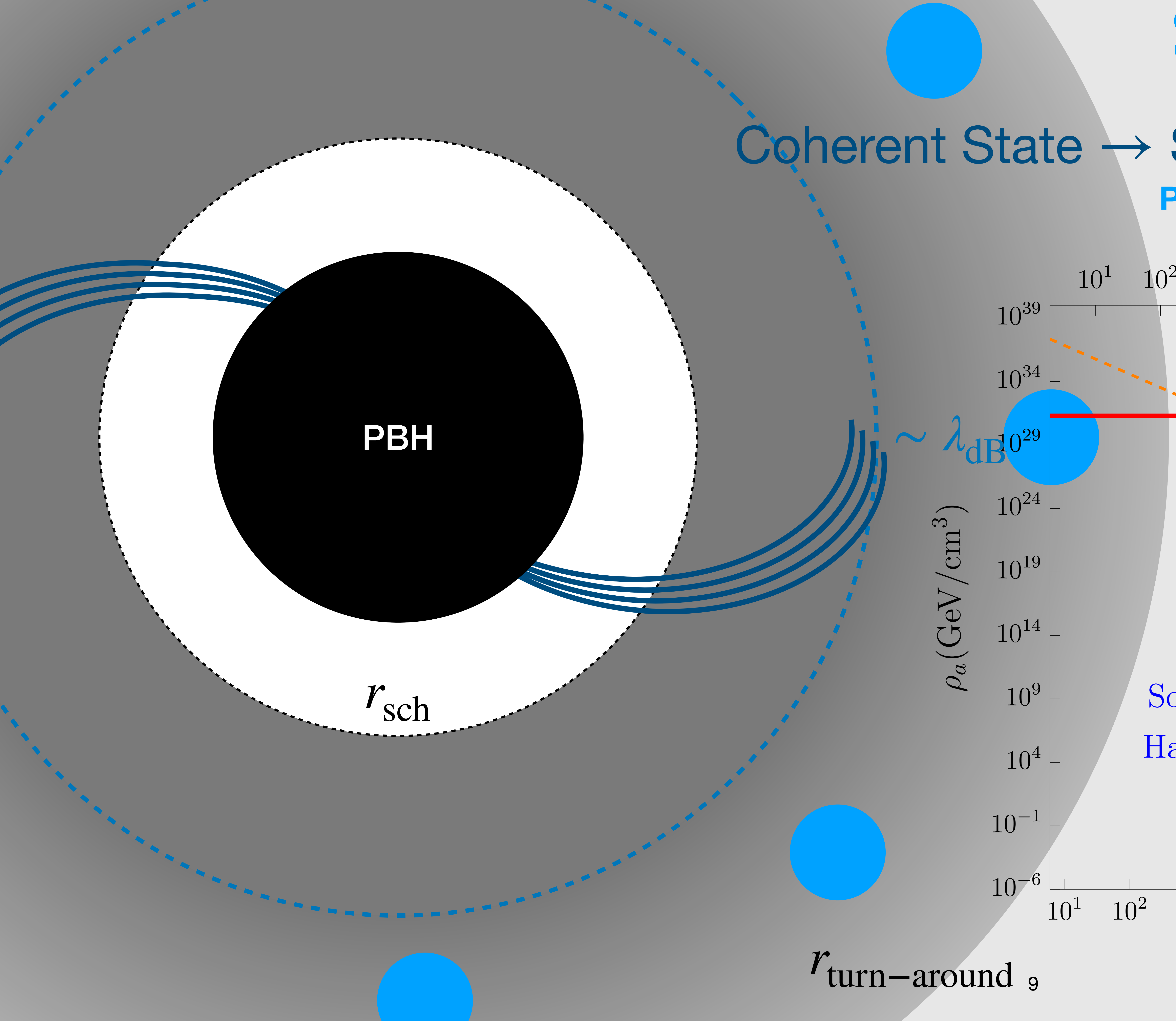

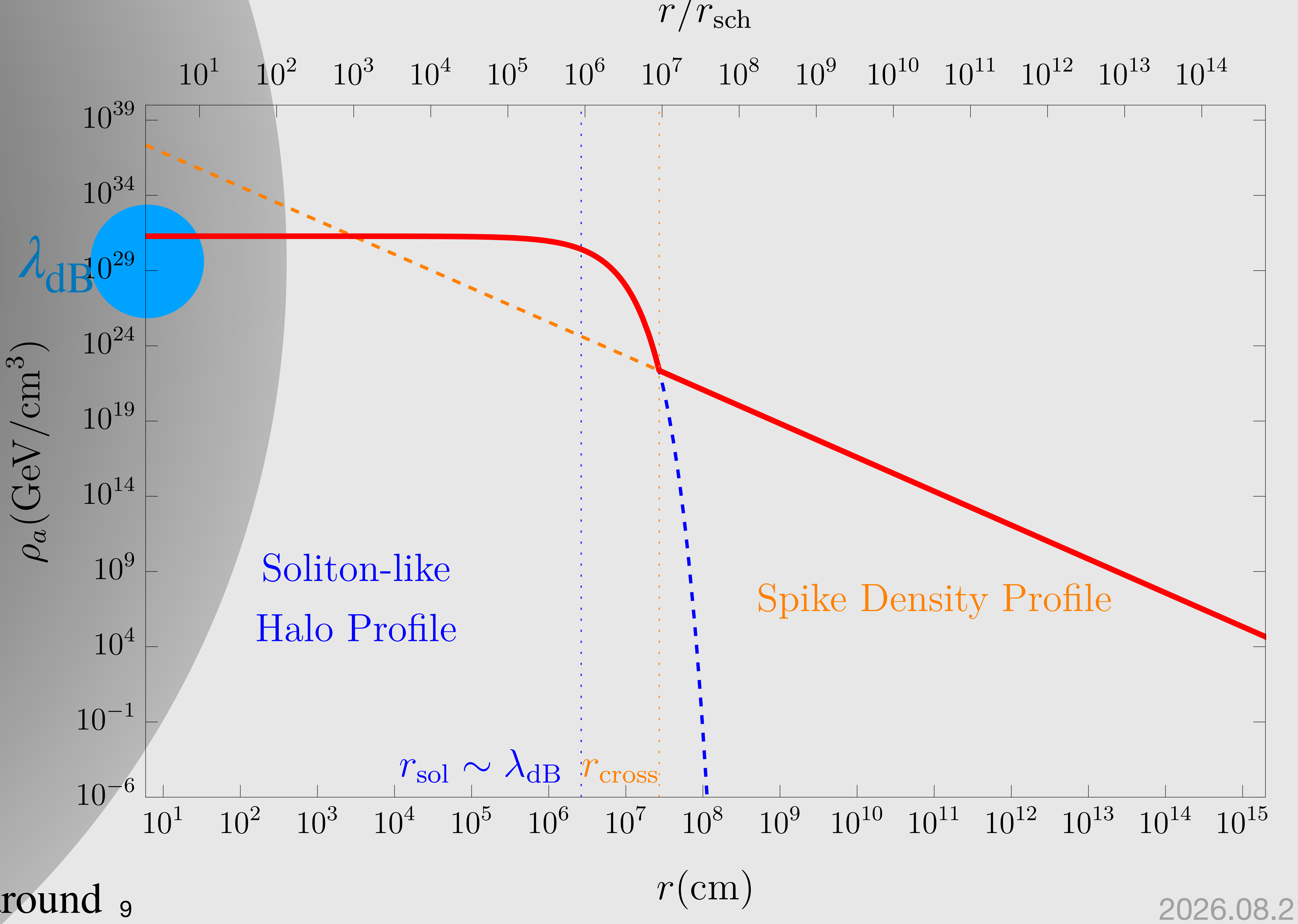

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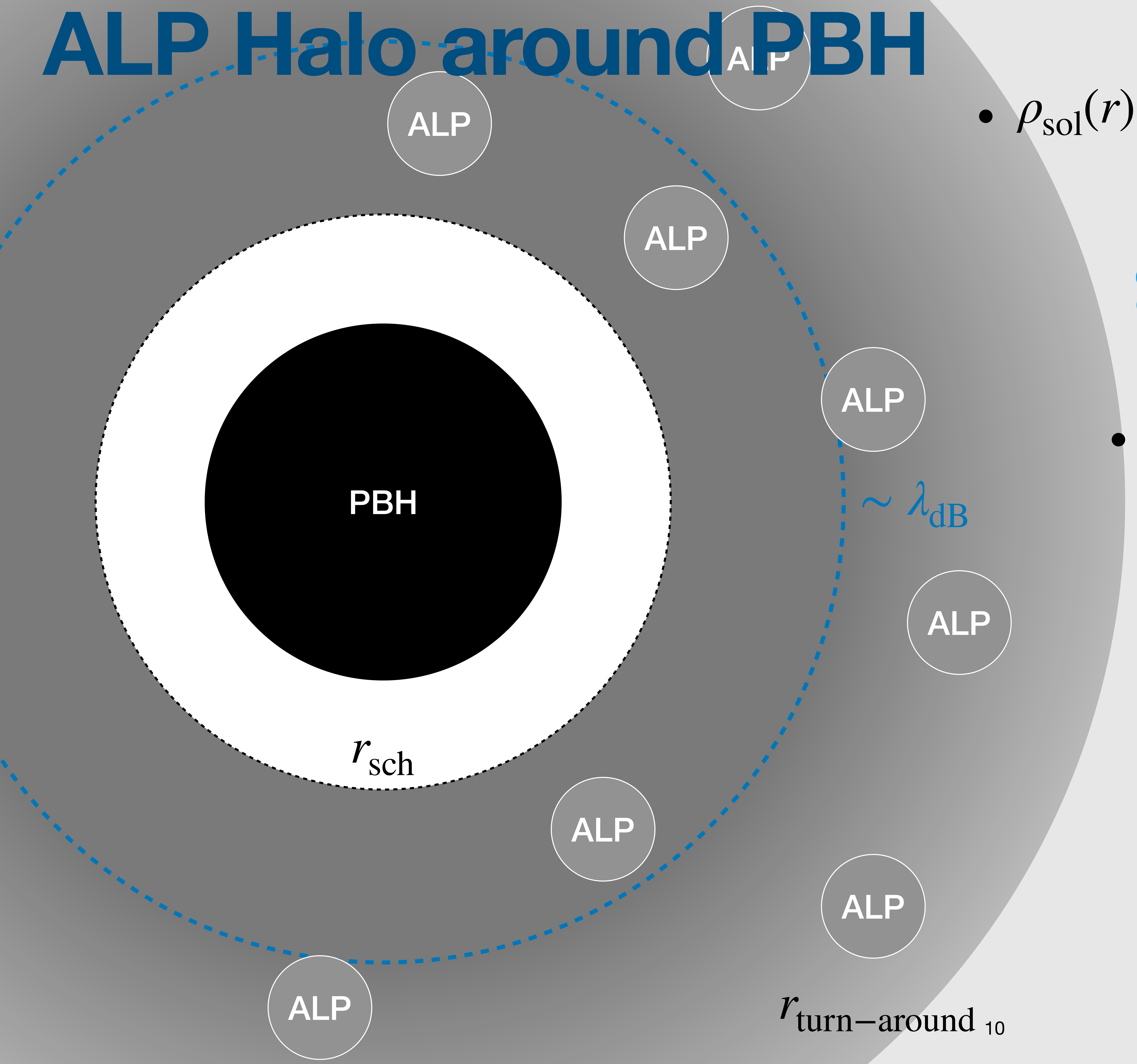

Aghaie, et al., Phys.Rev.D 109 (2024) 10, 103030

- $$\rho_{\rm sol}(r) = \frac{\epsilon\, m_a^2}{\pi G} \left(GM_{\rm PBH} m_a\right)^4 \exp\left[-2GM_{\rm PBH} m_a^2 r\right]$$
  where $r_{\rm sch} \leq r \lesssim \lambda_{\rm dB}$

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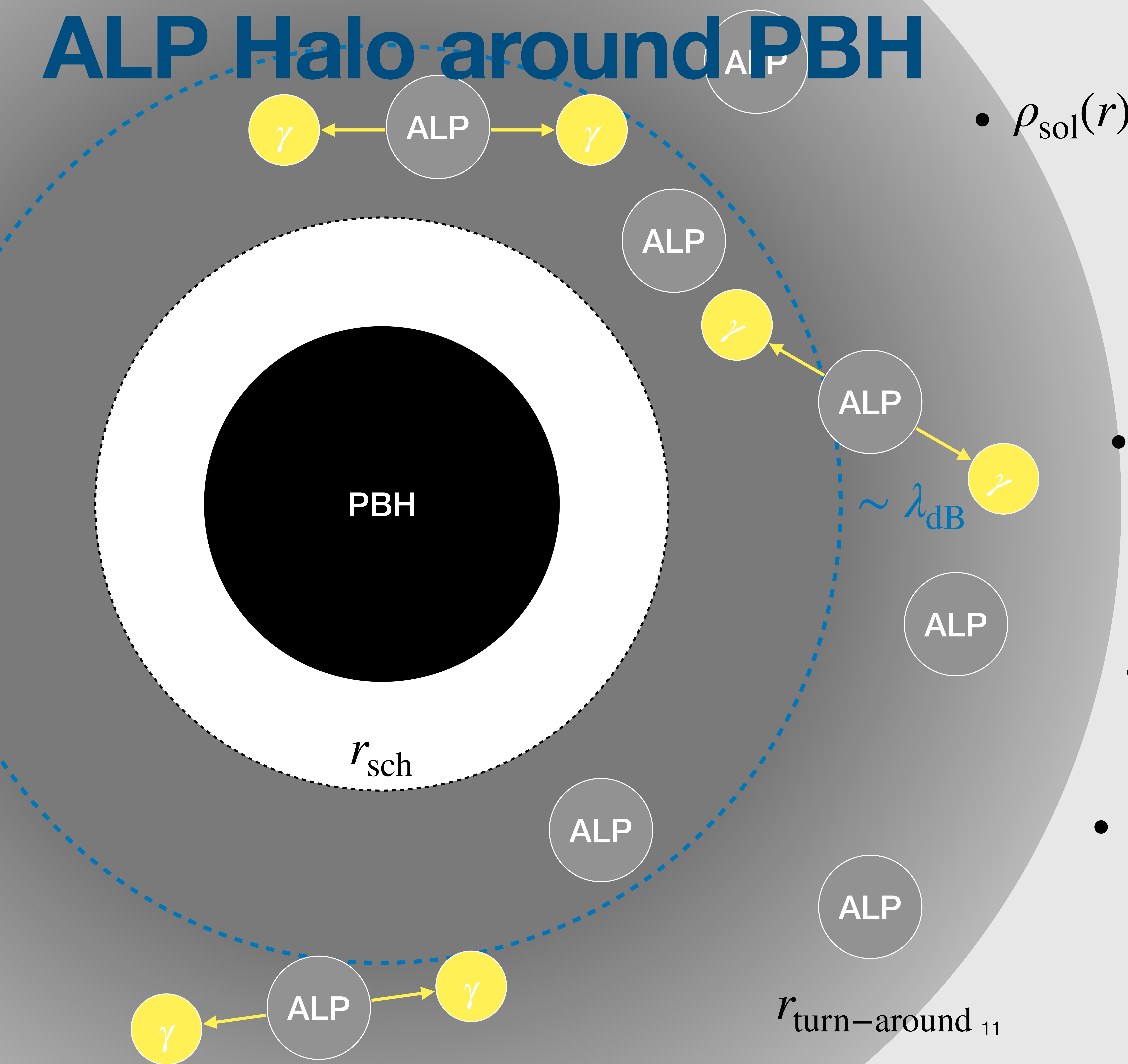

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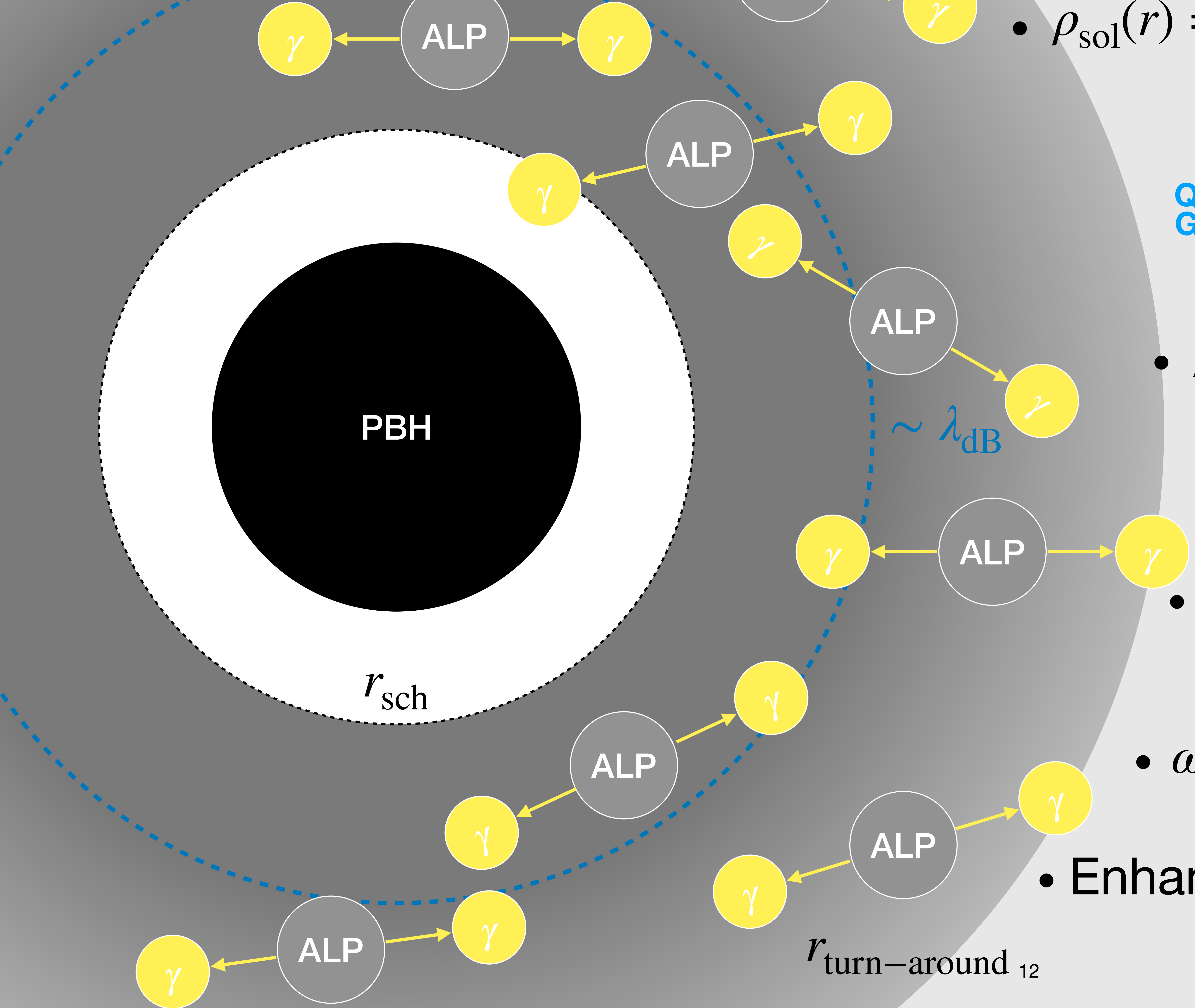
- $$\mathscr{L}_{a\gamma\gamma} \supset \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- $\omega_\gamma \simeq m_a/2$

2026.08.21

ALP Halo around PBH



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- $\mathscr{L}_{a\gamma\gamma} \supset \dfrac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

- $\omega_\gamma \simeq m_a/2$

- Enhancement condition: $f_\gamma > 1$

Boltzmann Equation in ALP Halo

- Boltzmann equations for photons:

$$\frac{dn_\gamma}{dt} = 2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma\right) - \frac{8\pi^2\Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3}\right) - \frac{n_\gamma}{t_{\mathrm{esc}}}$$

- Boltzmann equations for axions:

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- Boltzmann equations for sterile axions ($E_a > E_{a,\max}$):

$$\frac{dn_s}{dt} = \frac{4\pi^2\Gamma_a}{m_a^3} n_\gamma^2$$

* Momentum distribution is flat
* Axion energy range: $m_a \leq E_a \leq E_{a,\max}$
* Photon energy range: $\omega_{\gamma,\min} \leq \omega_\gamma \leq \omega_{\gamma,\max}$

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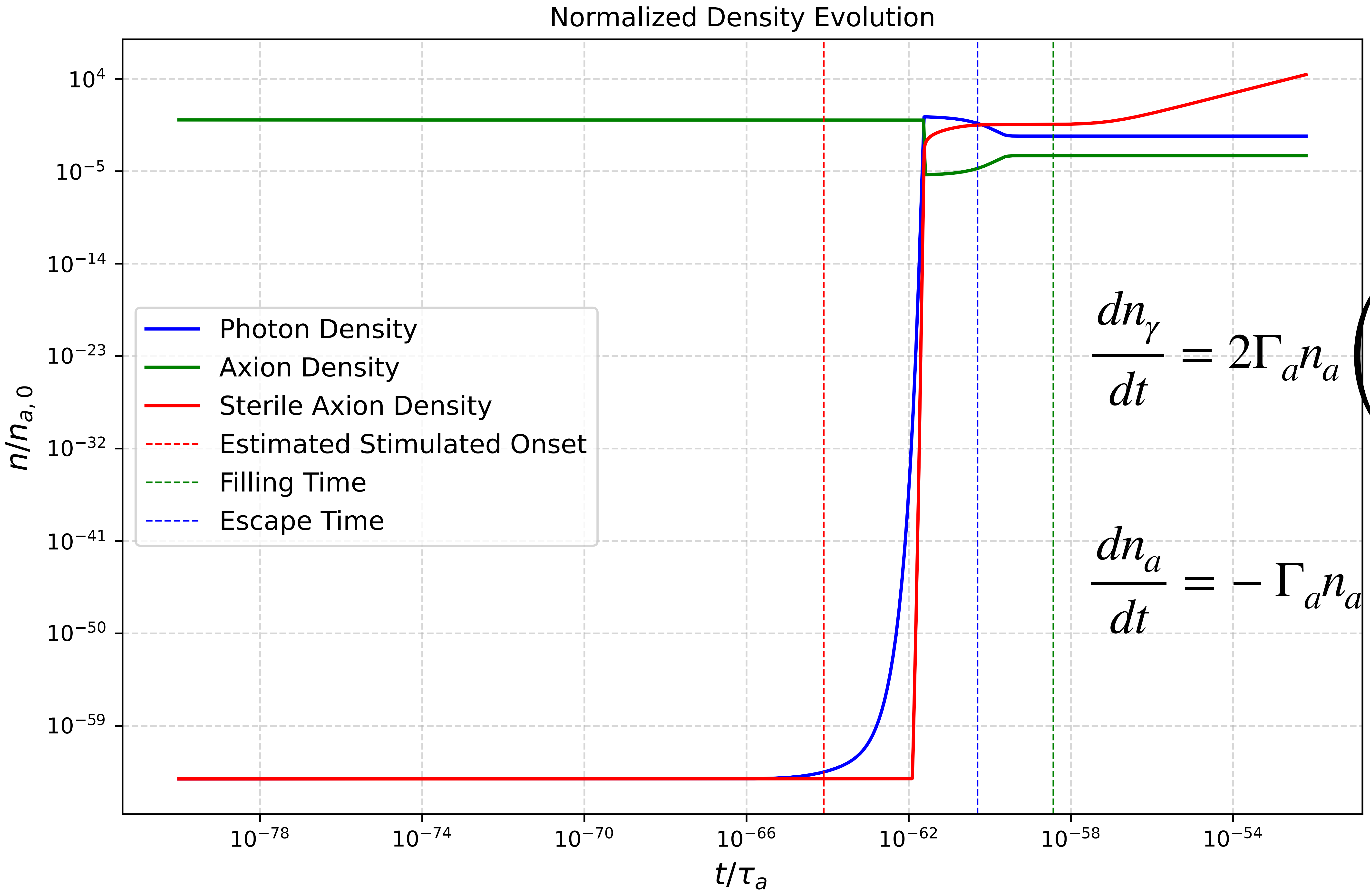
* Production of photons → Spontaneous decay / Stimulated decay

* Annihilation of photons → Axion outside halo / Axion inside halo

* Escape of photons

* Filling of axions from background

Example Result of Enhancement Case



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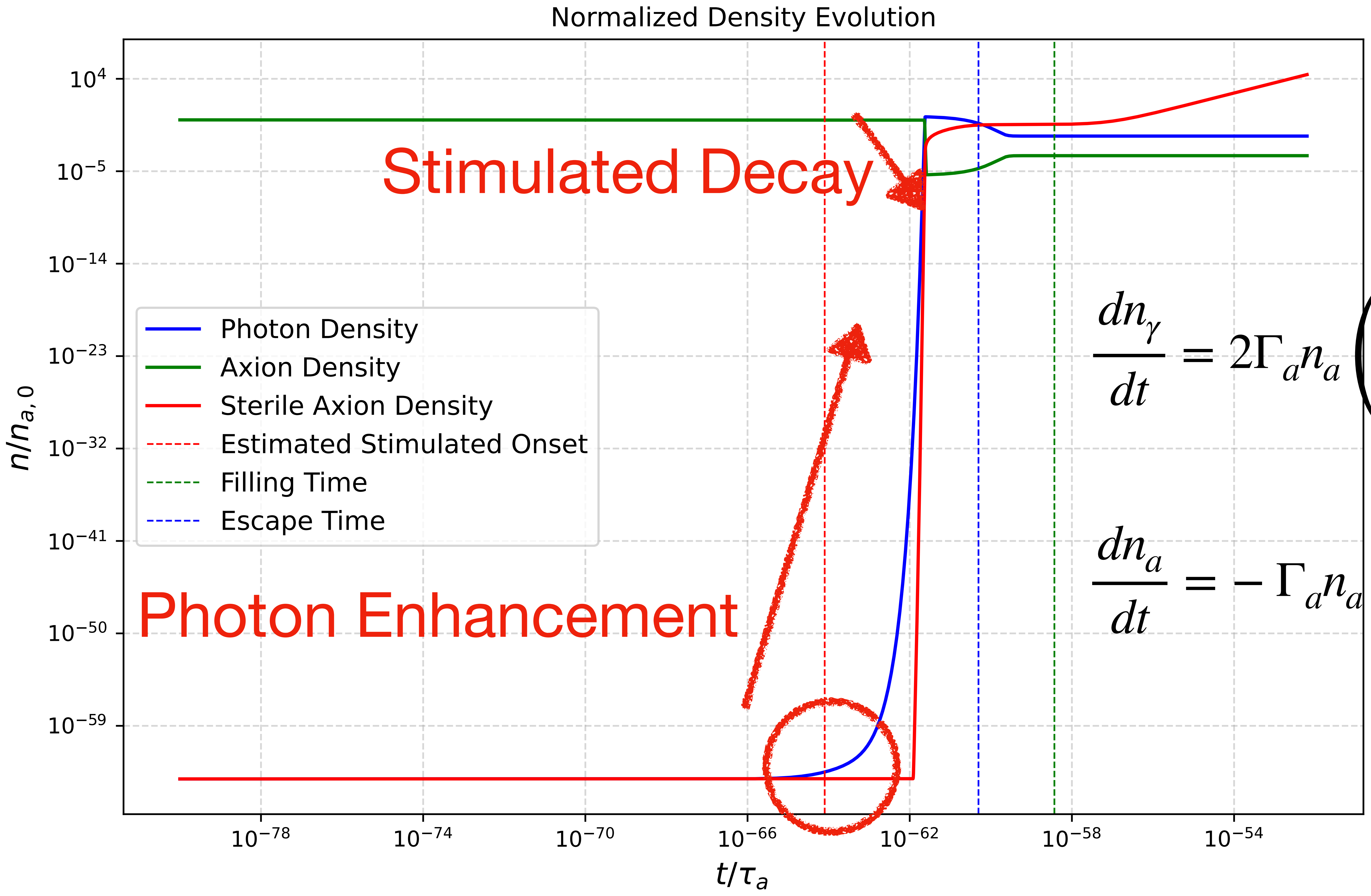

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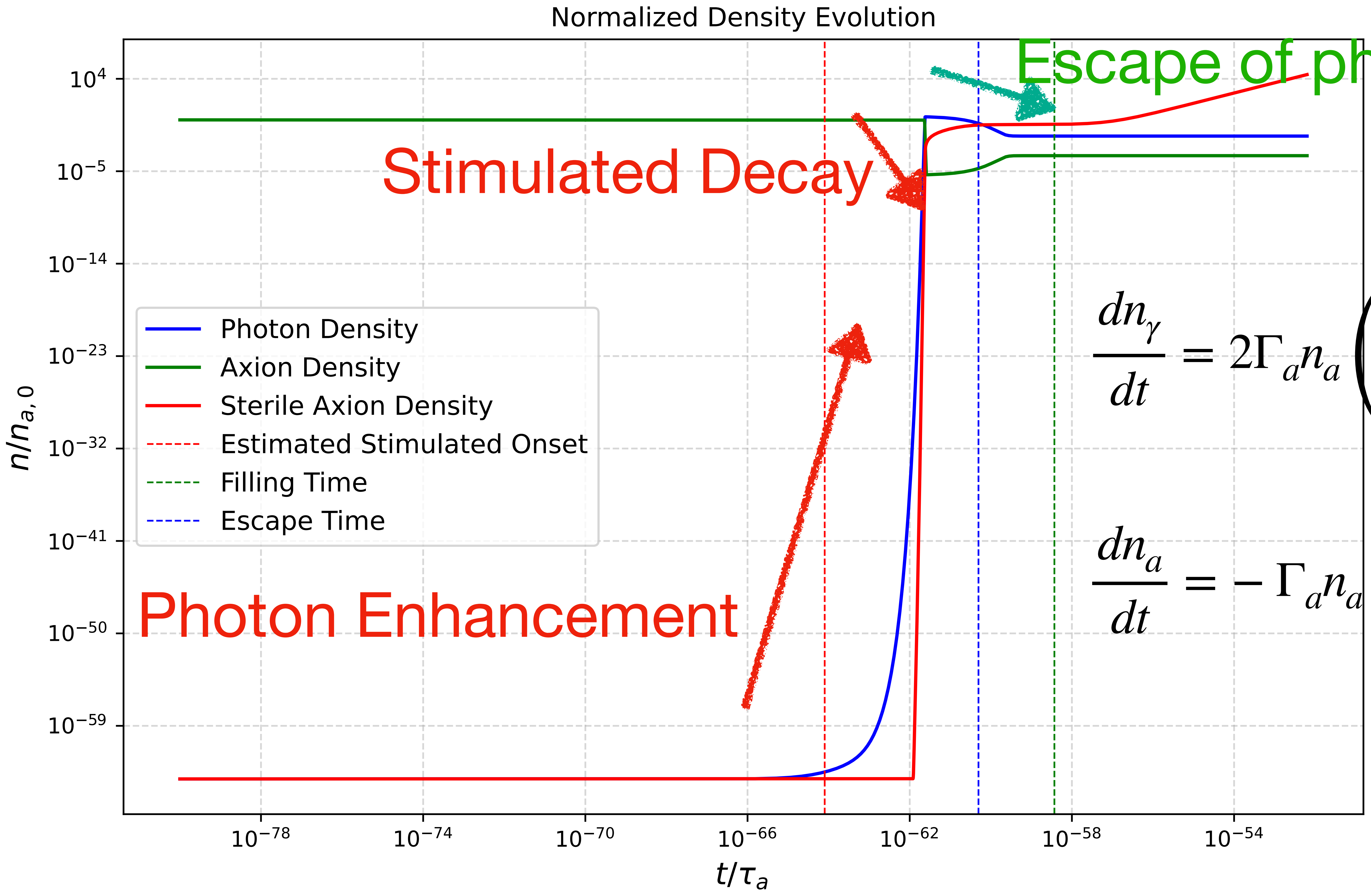

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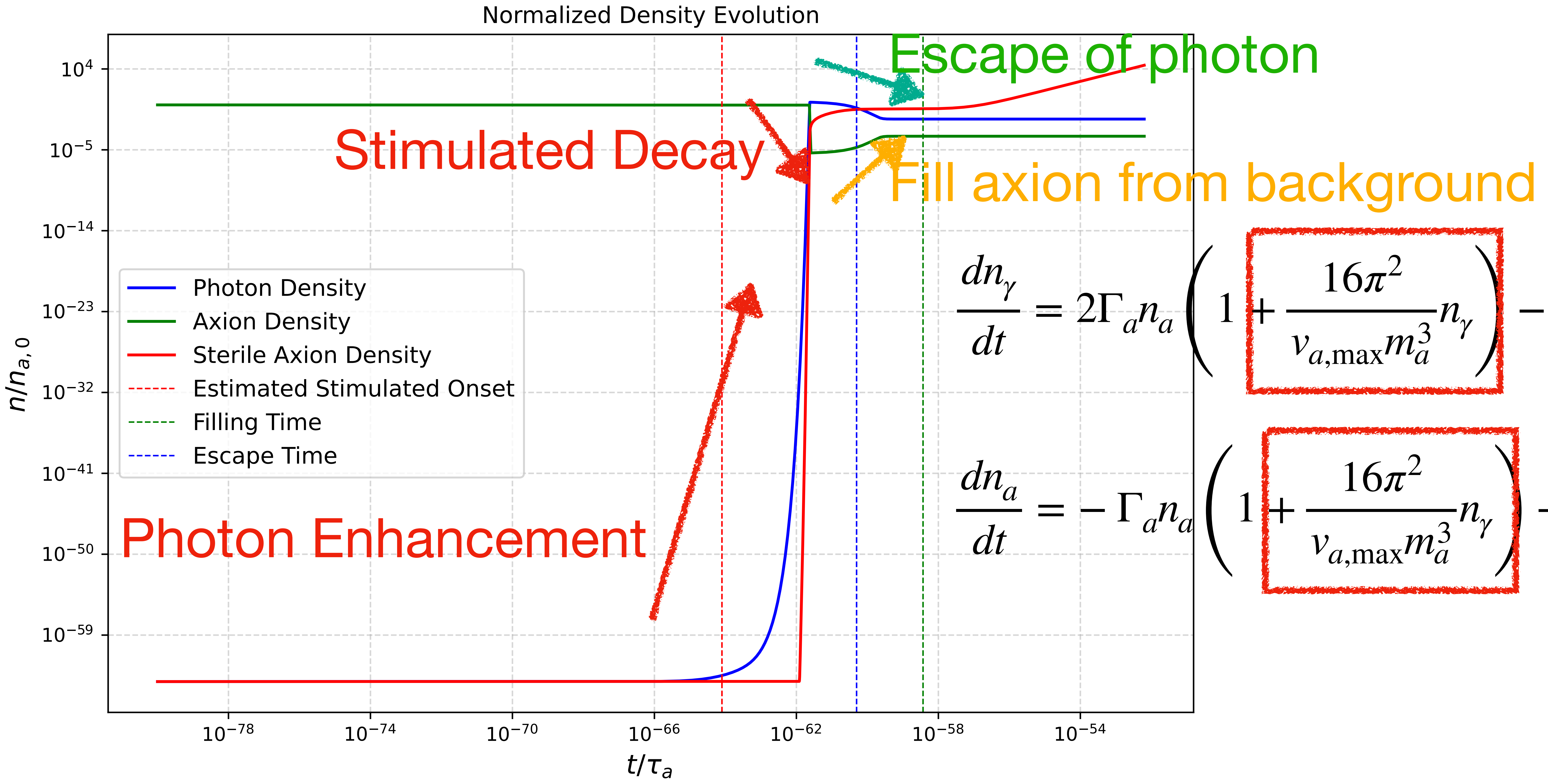

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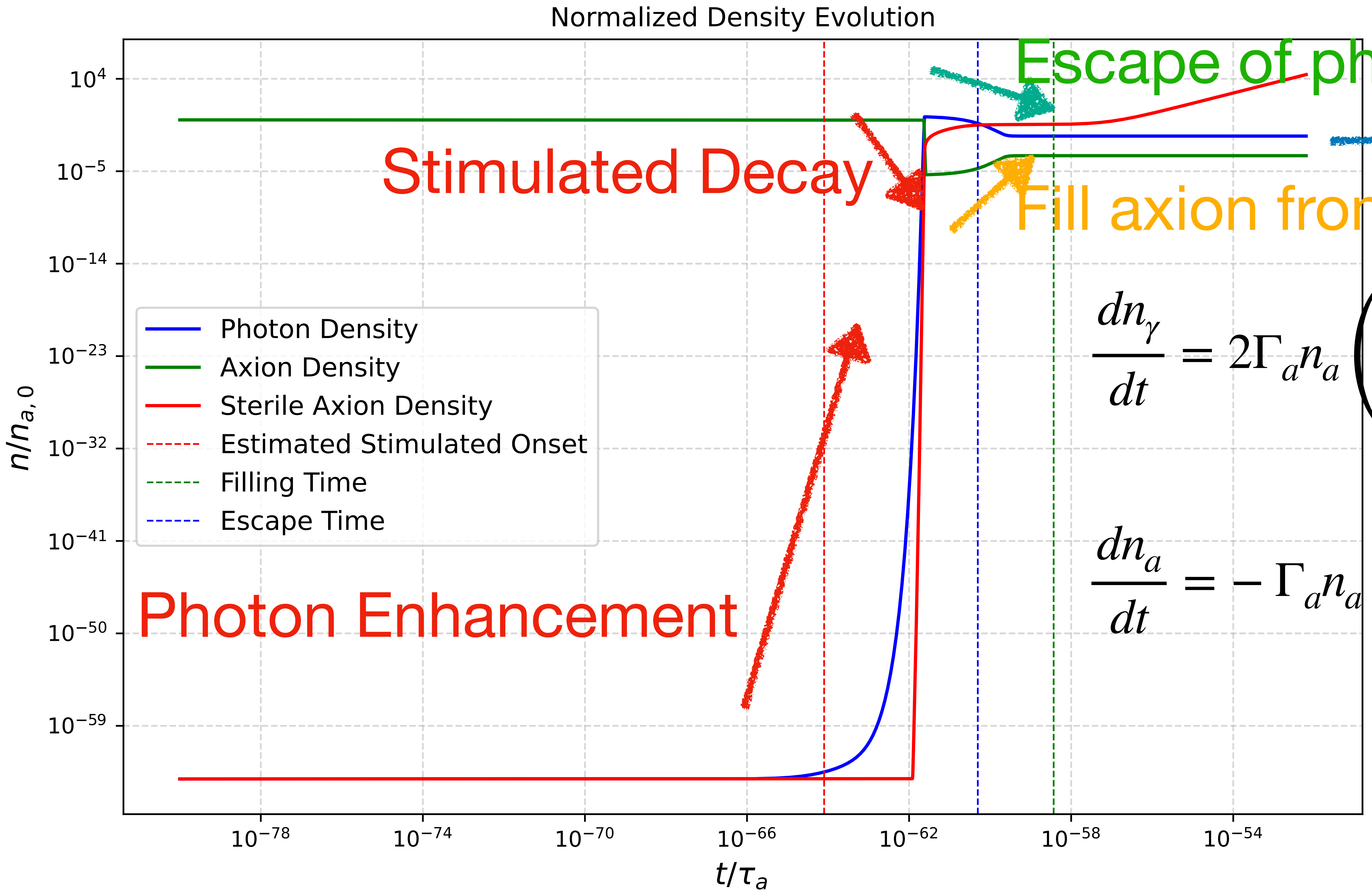

Dynamical Equalibrium

$$\frac{dn_\gamma}{dt} = 2\Gamma_a n_a \left(1 \boxed{+ \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma}\right) - \frac{8\pi^2\Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3}\right) \boxed{- \frac{n_\gamma}{t_{\rm esc}}}$$

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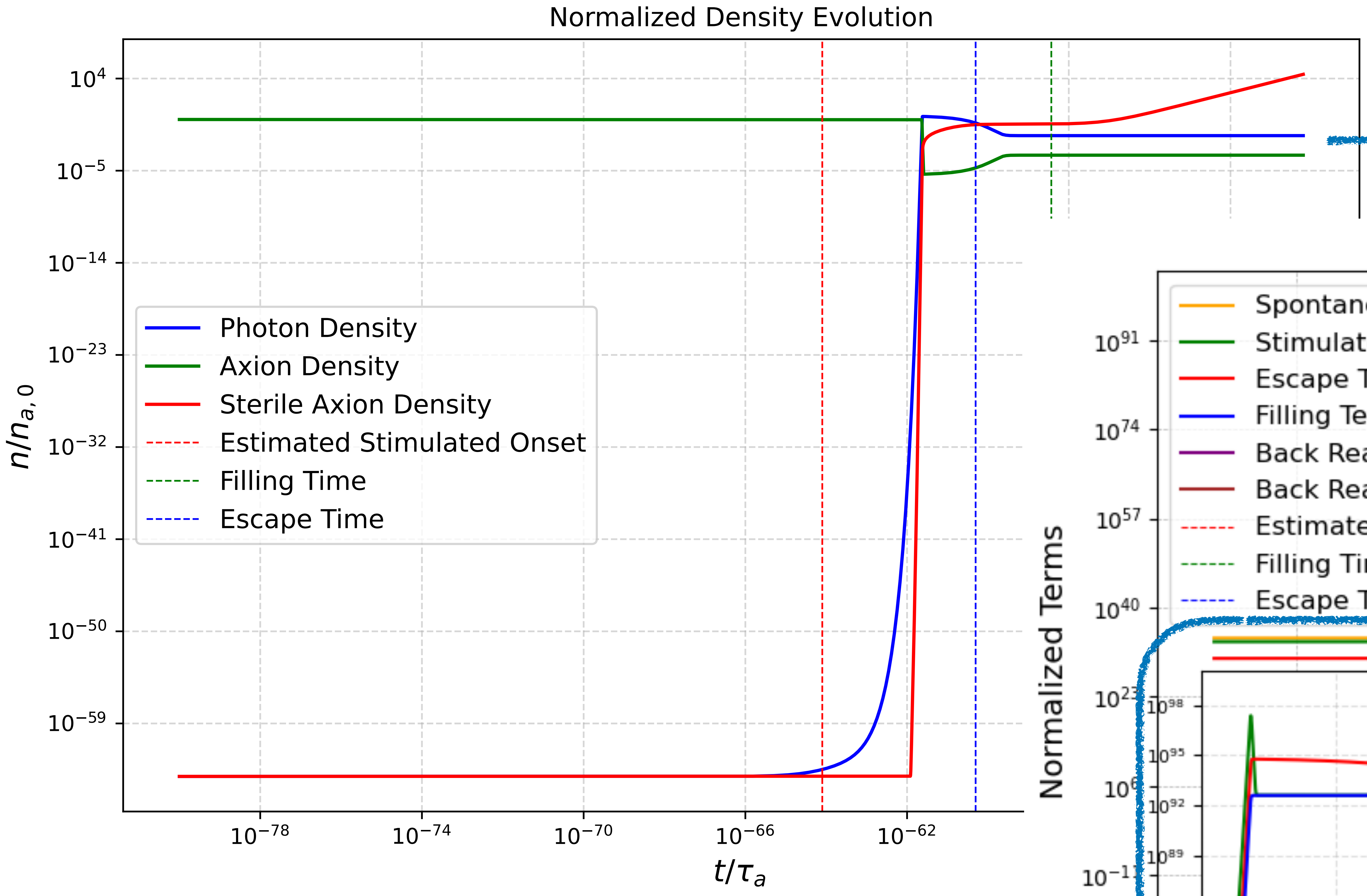

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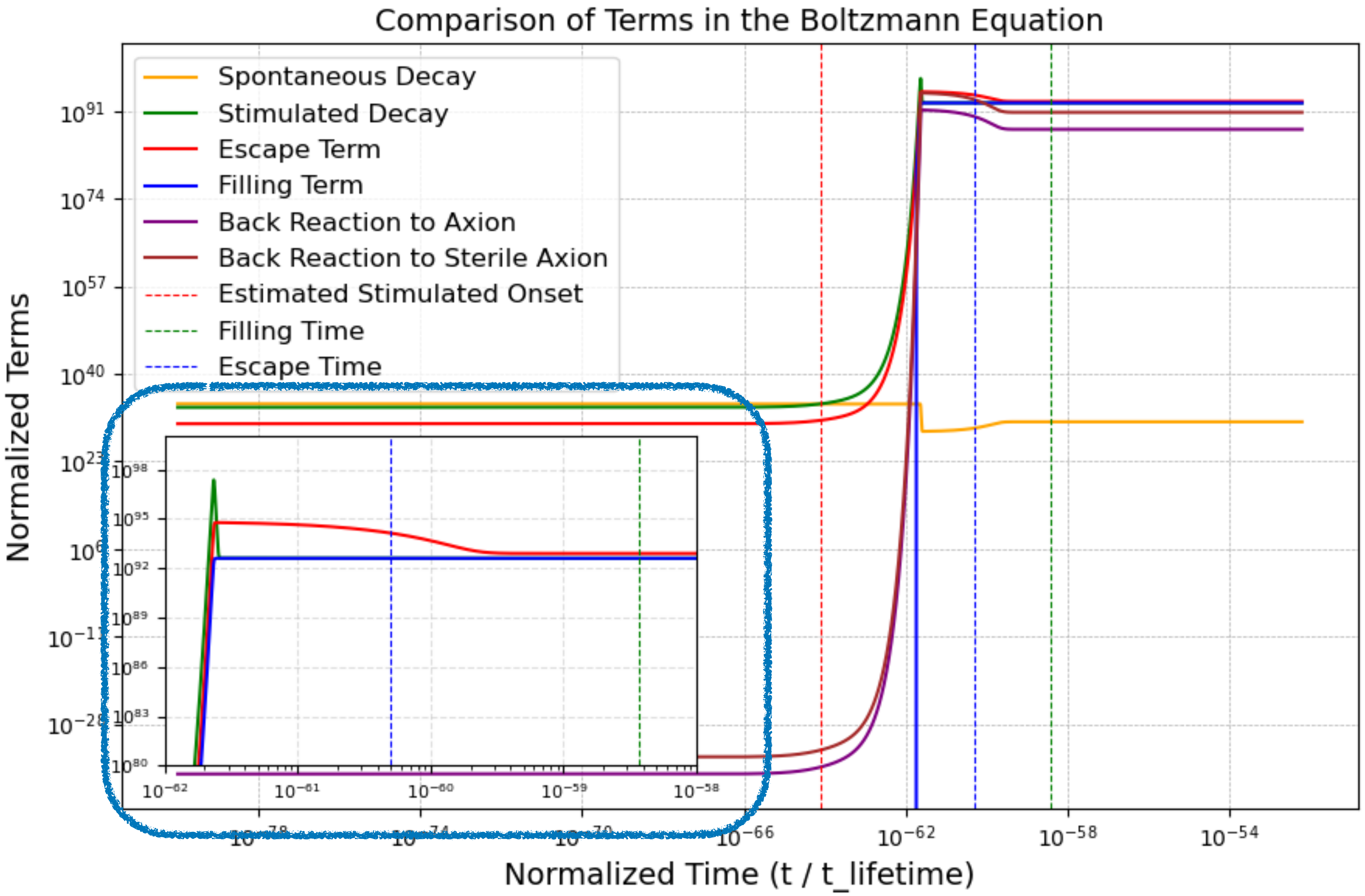
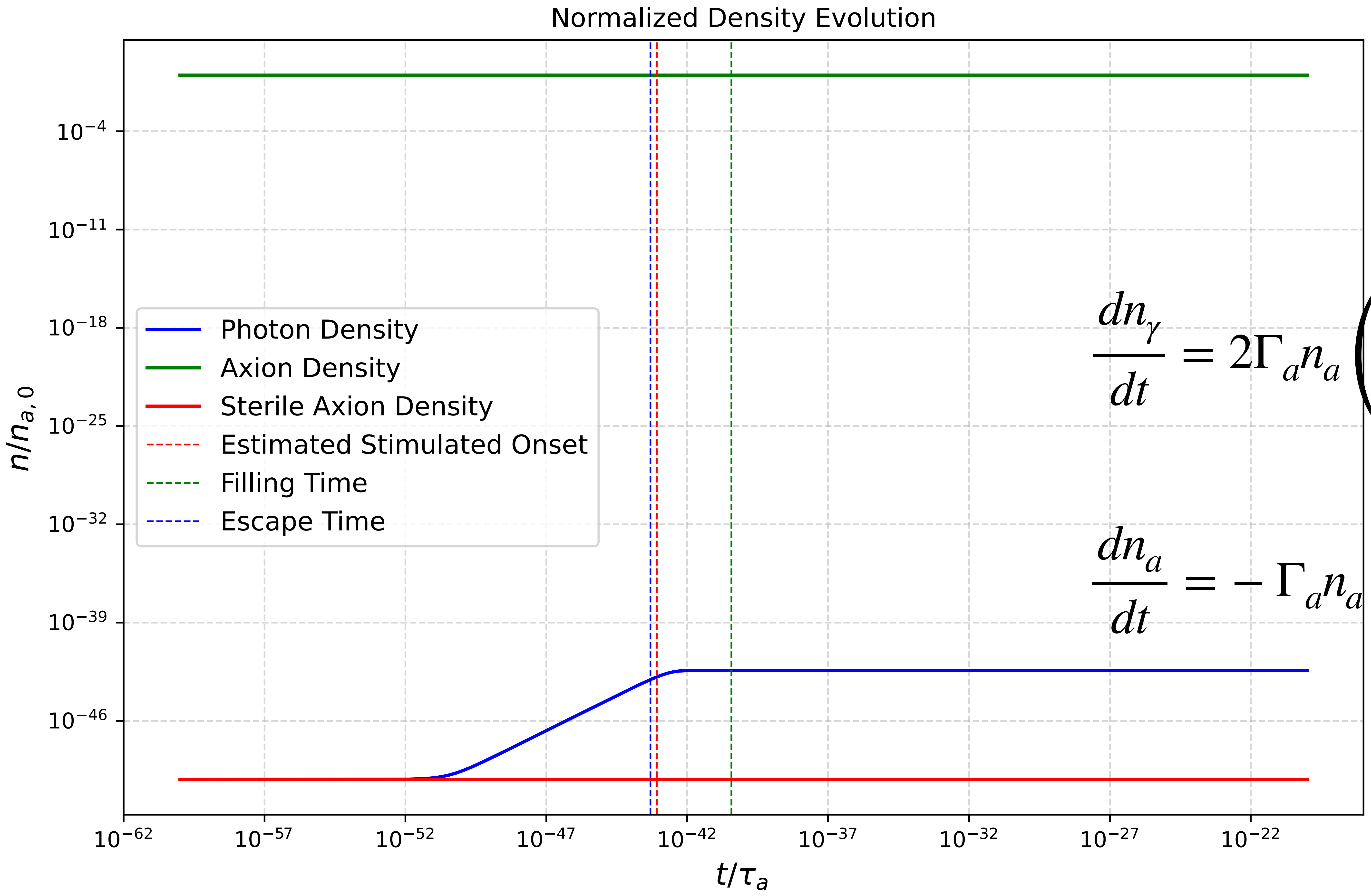


Example Result of Non-Enhancement Case



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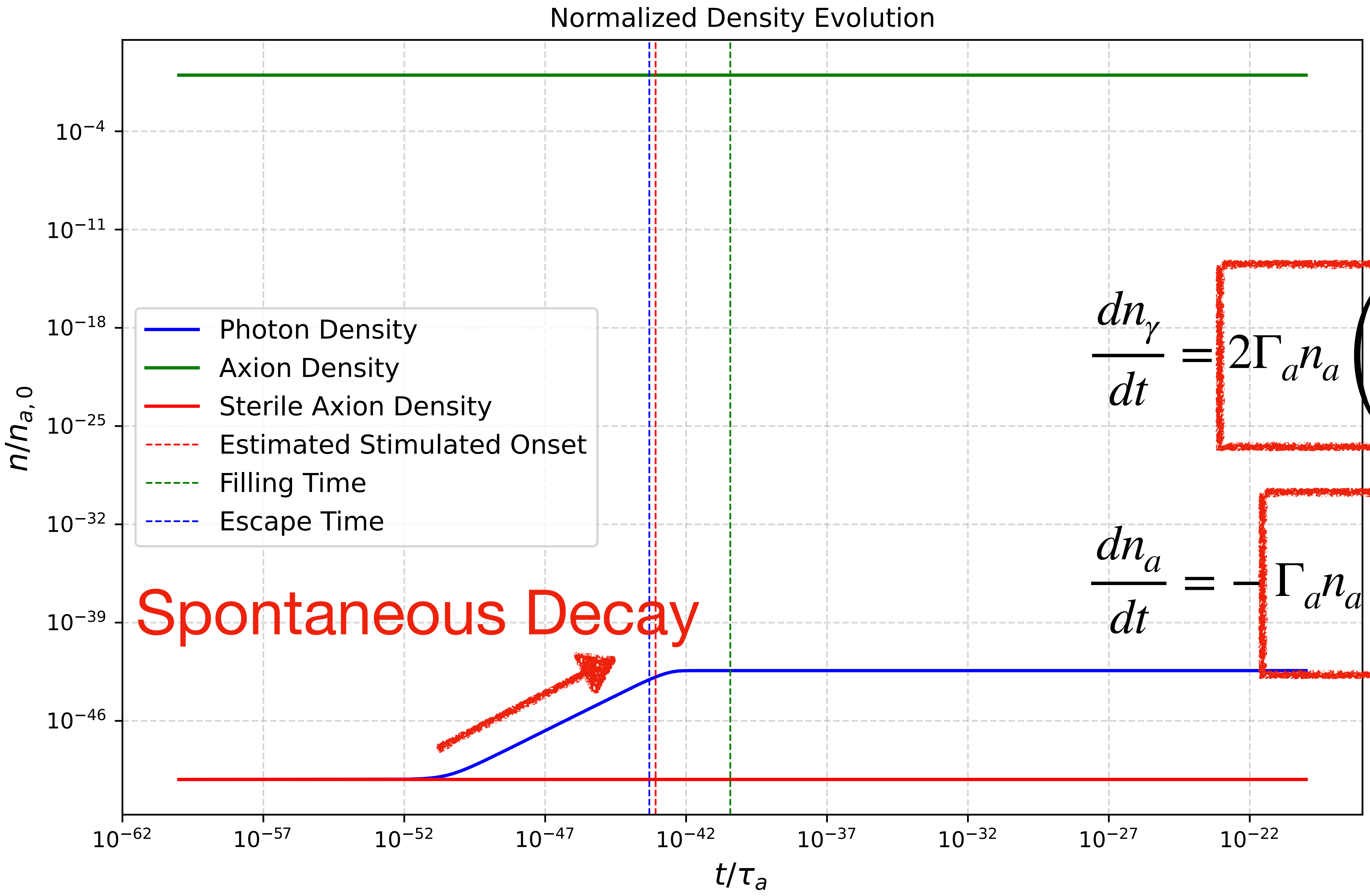

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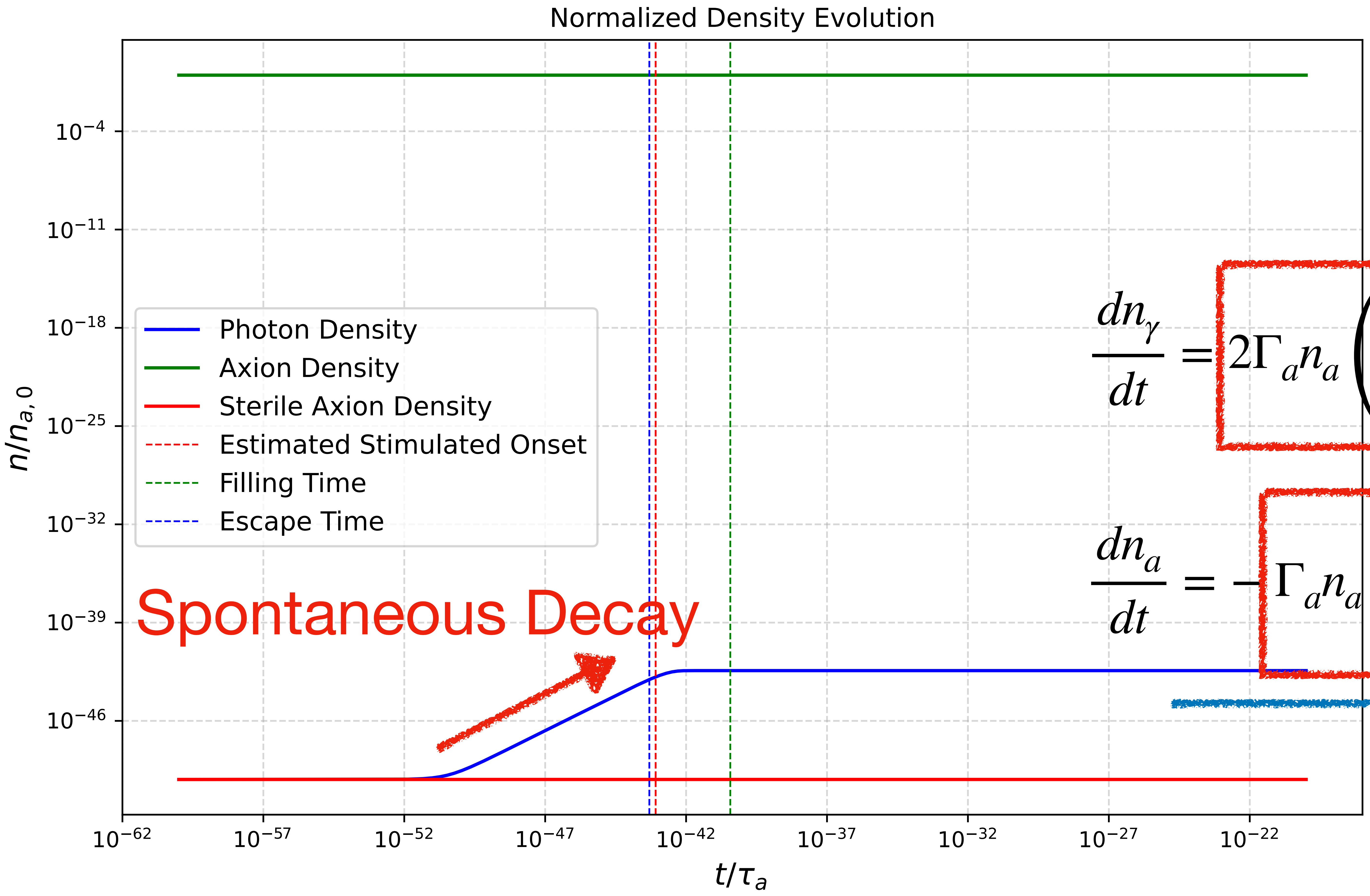

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$\rightarrow$ Dynamical Equalibrium

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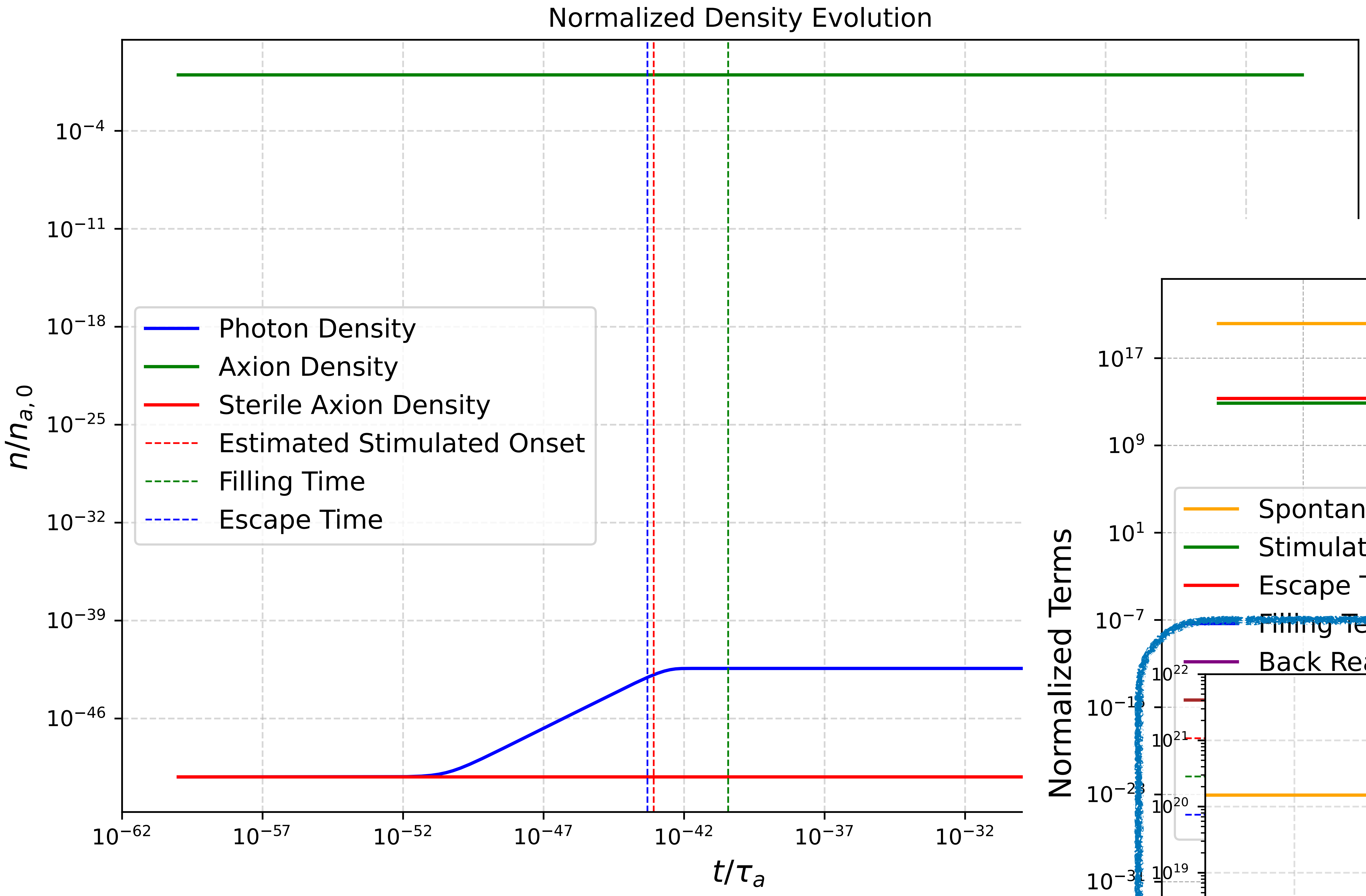
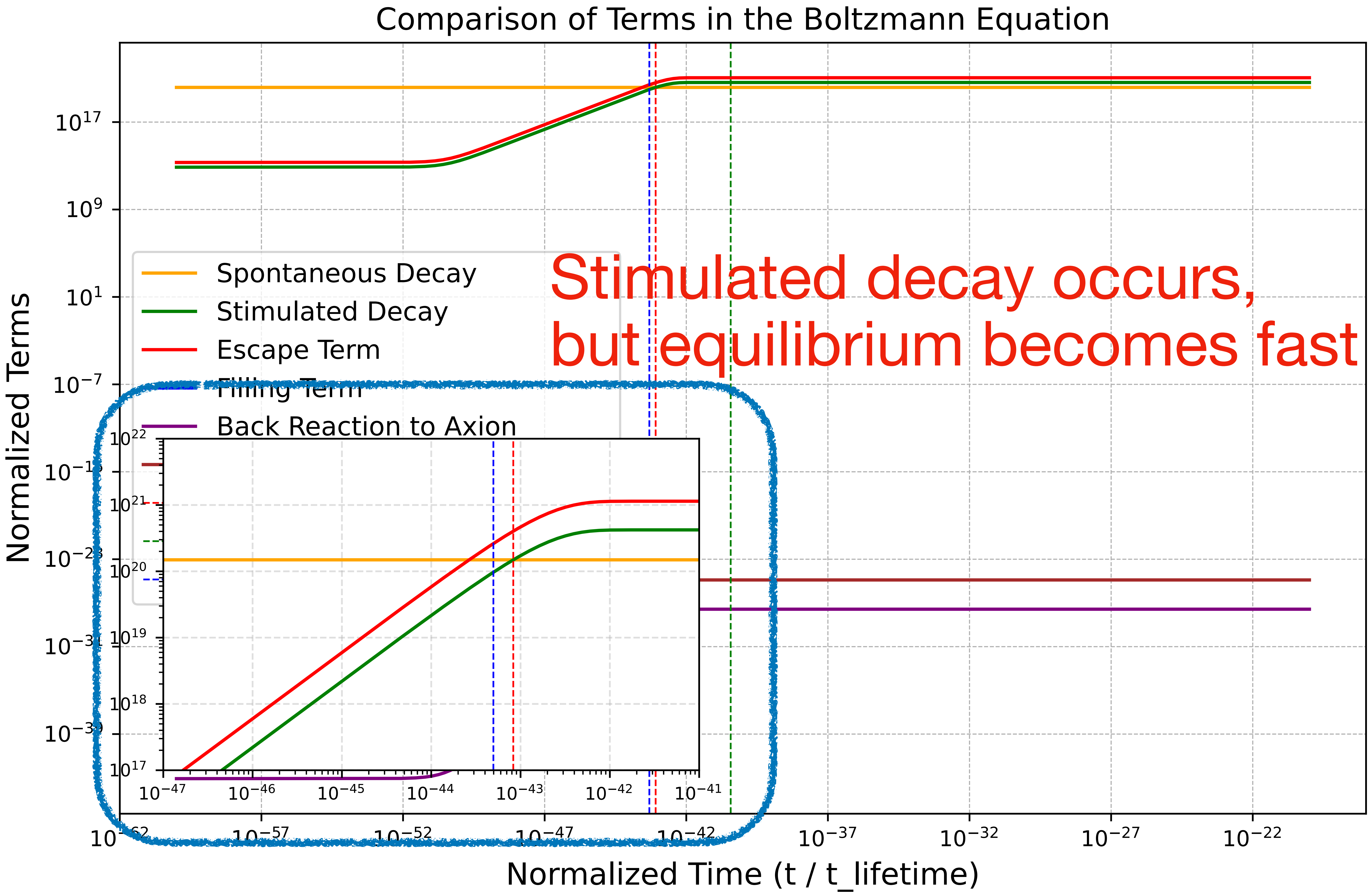
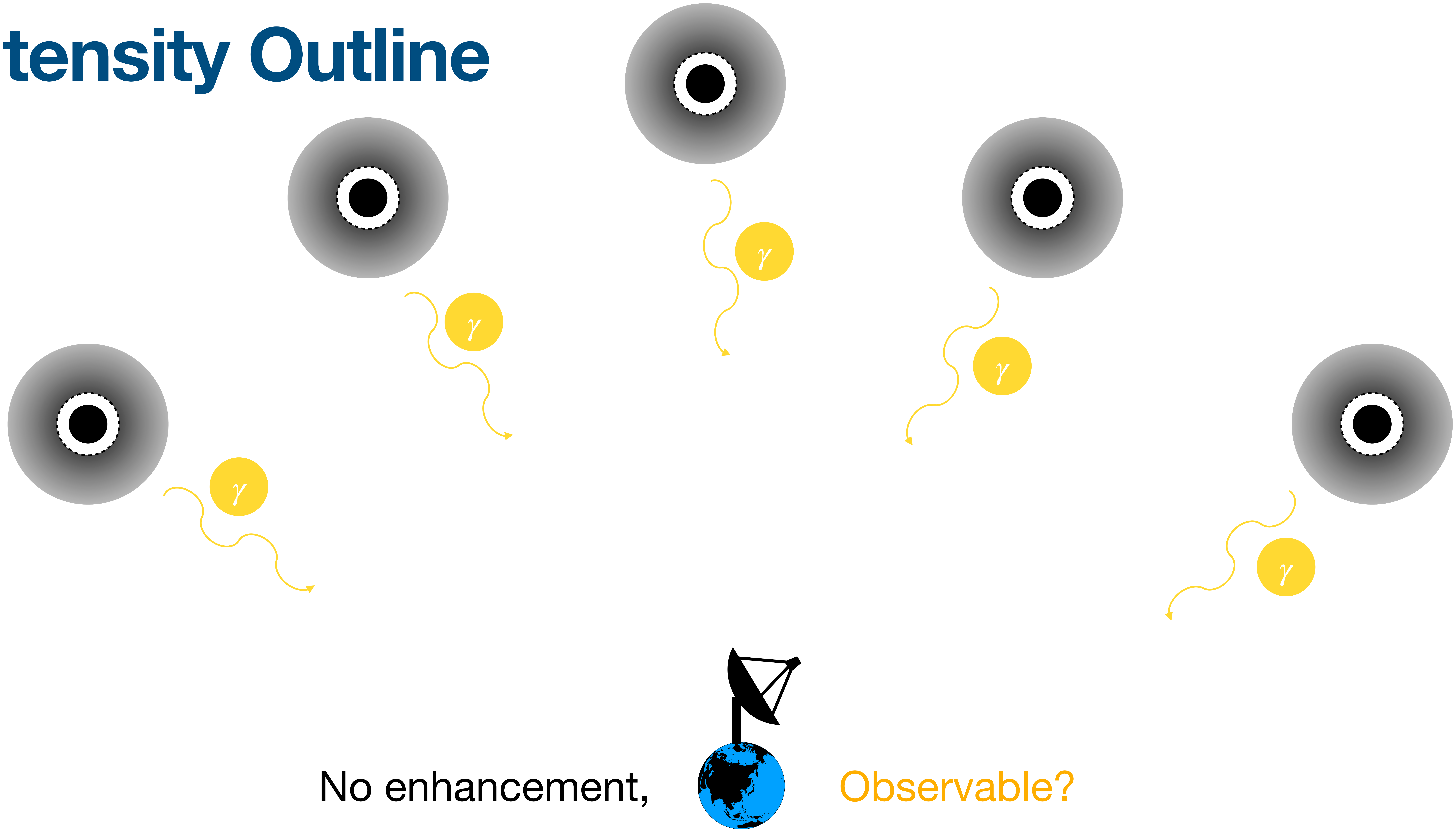




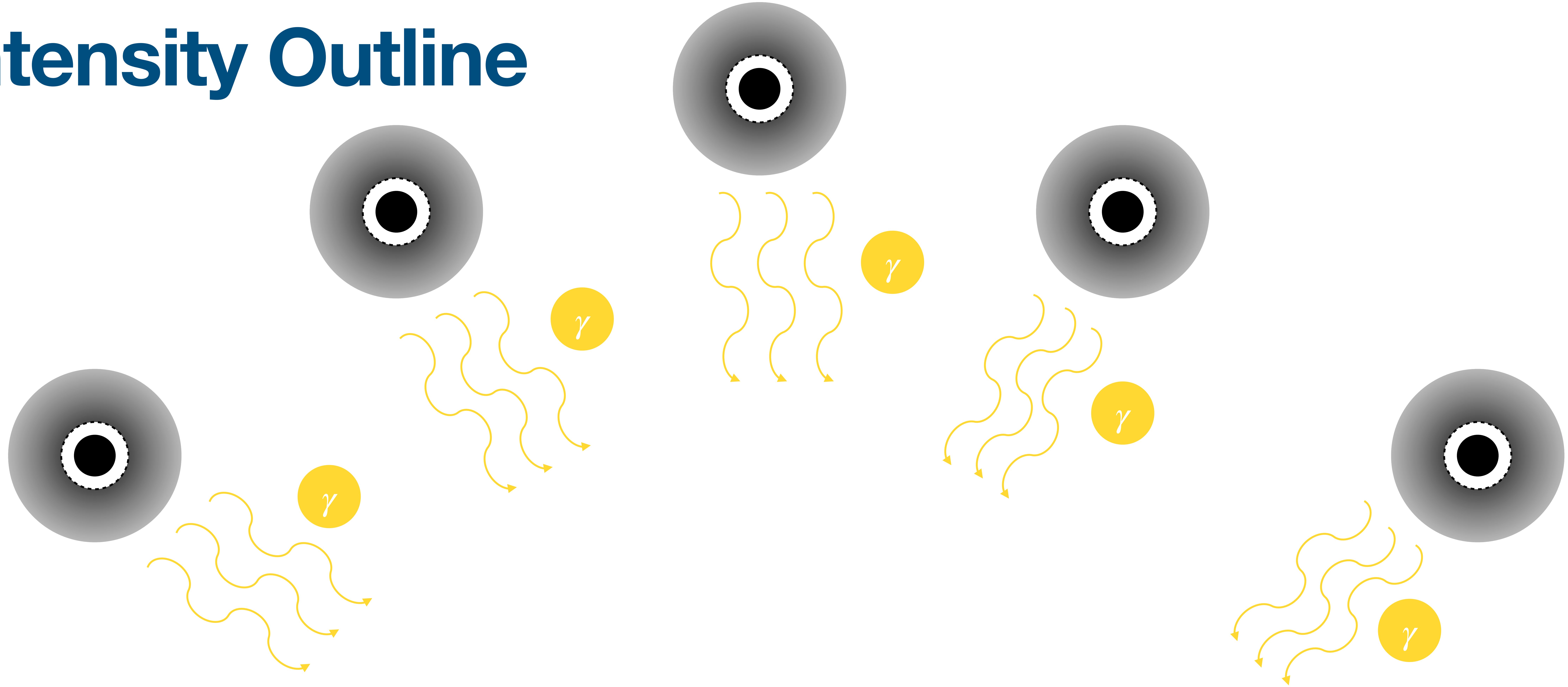
Stimulated decay occurs, but equilibrium becomes fast

Intensity Outline

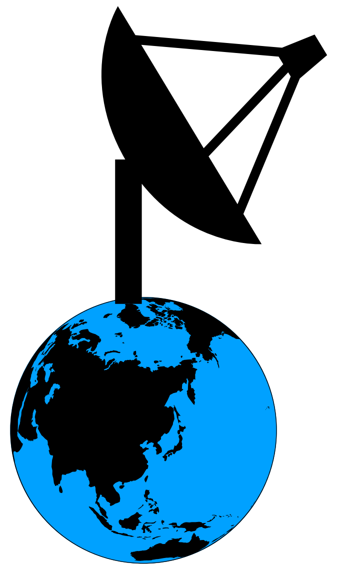


No enhancement, Observable?

Intensity Outline



Enhancement,



Observable!

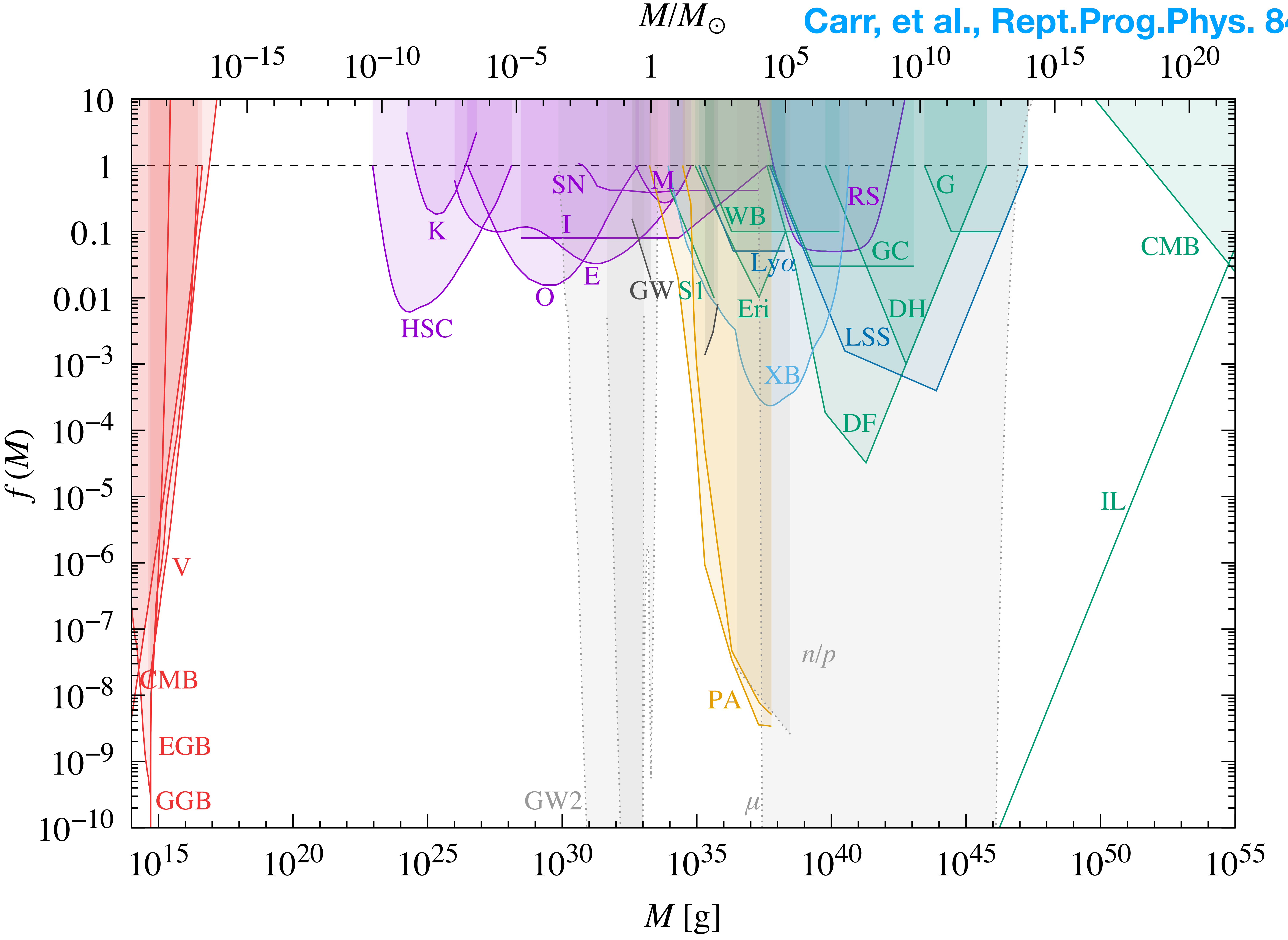
Summary

- **Verified Mechanism:** Using the *Boltzmann Equation*, we confirmed that ALP halos surrounding PBHs lead to significant *photon enhancement*.
- **Future Work:**
    - **PBH and ALP Phase Space:** Identifying the PBH–ALP parameter space where photon production is enhanced.
    - **Observational Signature:** Predicting detectable radio signals from PBH halos and deriving constraints on PBHs and ALPs.

Back-Up

PBH constraints

Carr, et al., Rept.Prog.Phys. 84 (2021) 11, 116902



Boltzmann Equation momentum distribution. ver.

- ALP energy distribution:

$$f_a(E_a, r) = \begin{cases} (2\pi)^3 \mathcal{N}(v_{\rm esc}) \dfrac{\rho_a(r)}{m_a} \dfrac{1}{2\pi m_a^2 \sigma_v^2}^{3/2} e^{-\frac{E_a^2 - m_a^2}{2m_a^2\sigma_v^2(r)}} & \text{where } \mathrm{E_a} \lesssim \mathrm{E_{max}}, \\ 0 \quad \text{where } E_a > E_{\rm max} & \end{cases}$$

:Truncated Gaussian distribution

King, Astron.J. 71 (1966), 64
Eroshenko, Astron.Lett. 42 (2016) 6, 347-356

- ALP energy range:

$$|\vec{v}_a| \le v_{\rm esc} = \sqrt{\frac{2GM_{\rm PBH}}{r}} \quad \Rightarrow \quad E_a \le E_{a,\rm max} = \frac{m_a}{\sqrt{1 - v_{\rm esc}^2}}$$

- Photon energy range:

$$\omega_{\gamma,\rm min} \le \omega_\gamma \le \omega_{\gamma,\rm max} \quad \text{where } \omega_{\gamma,\rm max/\gamma,min} = \frac{E_{a,\rm max} \pm \sqrt{E_{a,\rm max}^2 - m_a^2}}{2} = \frac{m_a(1 \pm v_{\rm esc})}{2\sqrt{1 - v_{\rm esc}^2}}$$

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω:

$$\frac{df_\lambda(\omega)}{dt} = \underbrace{\frac{m_a\Gamma_a}{\omega^2}\int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a)\left(1+f_\lambda(\omega)+f_\lambda(E_a-\omega)\right)}_{\text{Production of photons}} \underbrace{- \frac{m_a\Gamma_a}{\omega^2}\int_{m_a-\omega}^{\infty} d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} f_\lambda(\omega)}_{\text{Loss of photons}}$$

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω:

$$\frac{df_\lambda(\omega)}{dt} = \underbrace{\frac{m_a\Gamma_a}{\omega^2}\int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a)\Big(\underbrace{1}_{\text{Spontaneous decay}} + \underbrace{f_\lambda(\omega) + f_\lambda(E_a-\omega)}_{\text{Stimulated decay}}\Big)}_{\text{Production of photons}} \underbrace{- \underbrace{\frac{m_a\Gamma_a}{\omega^2}\int_{m_a-\omega}^{\infty} d\omega'' f_\lambda(\omega) f_\lambda(\omega'')}_{\text{Back-reaction}} - \underbrace{\frac{3}{2r} f_\lambda(\omega)}_{\text{Photon escape}}}_{\text{Loss of photons}}$$

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω:

$$\frac{df_\lambda(\omega)}{dt} = \underbrace{\frac{m_a\Gamma_a}{\omega^2}\int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a)\Big(\underbrace{1}_{\text{Spontaneous decay}} + \underbrace{f_\lambda(\omega) + f_\lambda(E_a-\omega)}_{\text{Stimulated decay}}\Big)}_{\text{Production of photons}} \underbrace{- \underbrace{\frac{m_a\Gamma_a}{\omega^2}\int_{m_a-\omega}^{\infty} d\omega'' f_\lambda(\omega) f_\lambda(\omega'')}_{\text{Back-reaction}} - \underbrace{\frac{3}{2r} f_\lambda(\omega)}_{\text{Photon escape}}}_{\text{Loss of photons}}$$

Range of ALP energy to produce photon with momentum ω

Range of photon momentum to produce ALP by annihilating with a photon ω

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω:

$$\frac{df_\lambda(\omega)}{dt} = \underbrace{\frac{m_a\Gamma_a}{\omega^2}\int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a)\left(1 + f_\lambda(\omega) + f_\lambda(E_a - \omega)\right)}_{\text{Production of photons}} - \underbrace{\frac{m_a\Gamma_a}{\omega^2}\int_{m_a-\omega}^{\infty} d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} f_\lambda(\omega)}_{\text{Loss of photons}}$$

Stimulated decay

Spontaneous decay

Back-reaction

Photon escape

Range of ALP energy to produce photon with momentum ω

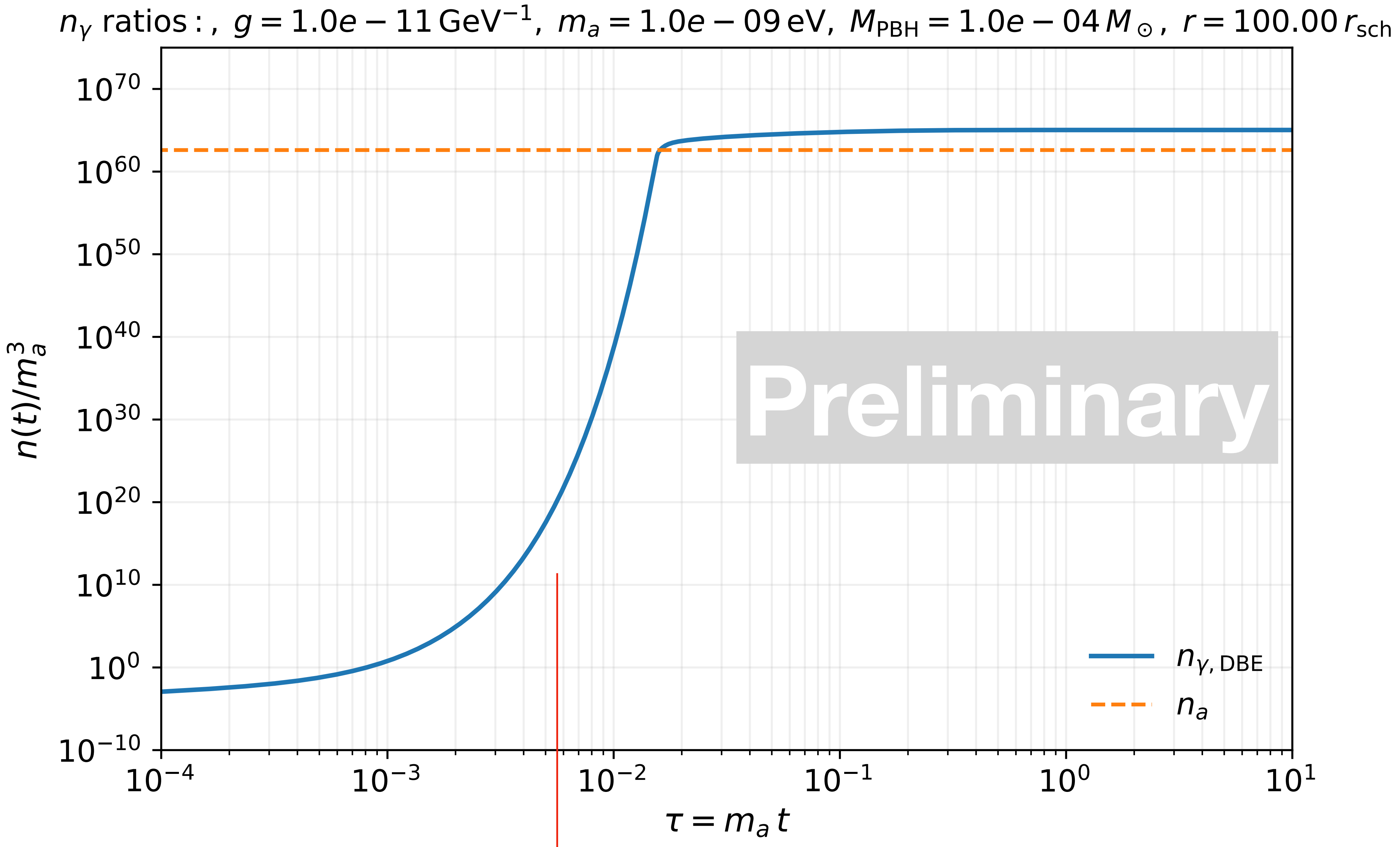
Range of photon momentum to produce ALP by annihilating with a photon ω

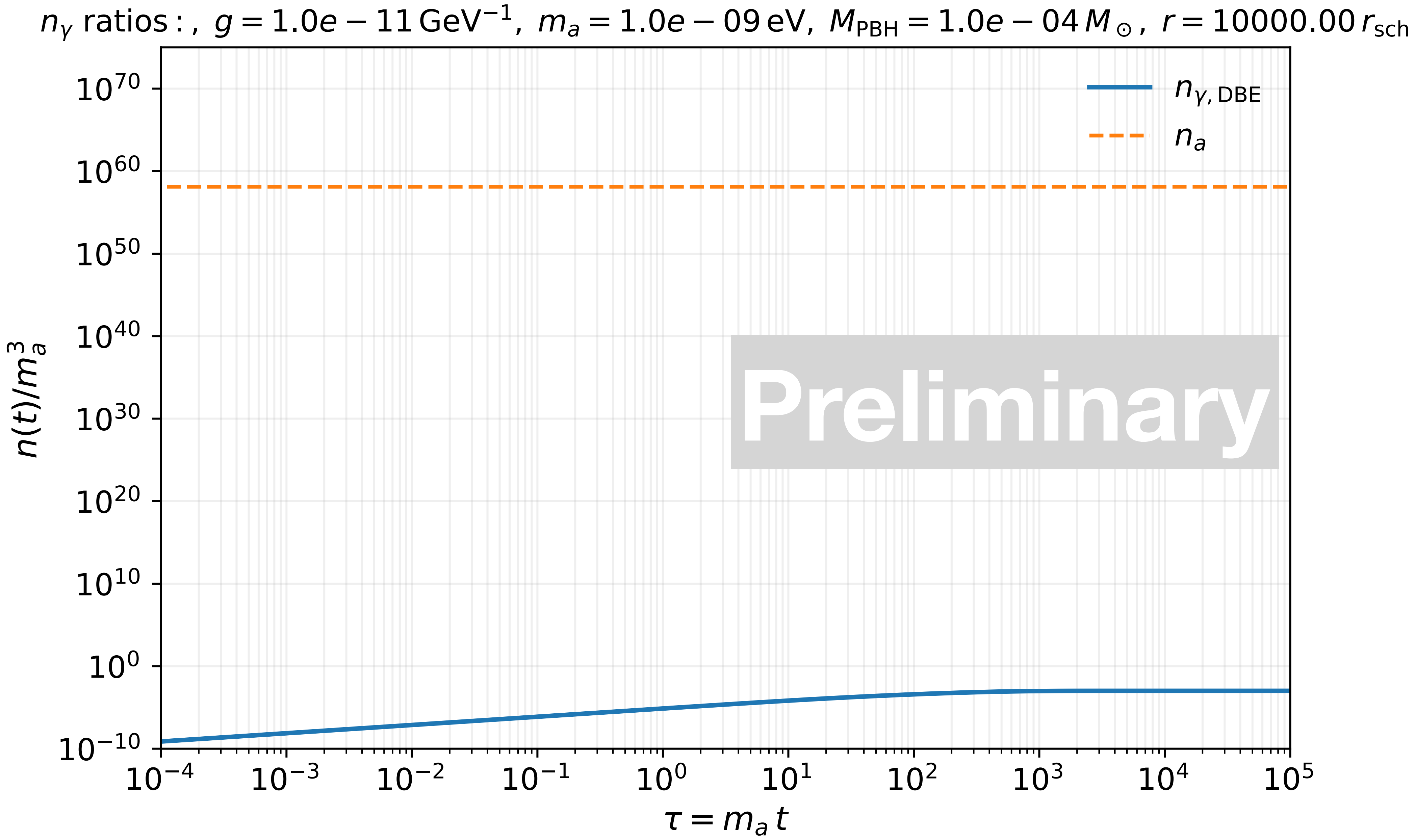
- Number density: $n_\gamma(r) = \sum_{\lambda=\pm} \frac{1}{(2\pi)^3}\int d^3\vec{\omega} f_\lambda(r, \vec{\omega})$, $\quad n_a(r) = \frac{\rho_a(r)}{m_a}$

Assumption:Constant on time

Results of Enhancement/Non-Enhancement (md ver.)

$$\frac{dn_\gamma}{dt} = \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{E_{\min}}^\infty dE_a f_a(E_a) \Big(1 \boxed{+ f_\lambda(\omega) + f_\lambda(E_a - \omega)\Big)} - \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{m_a - \omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} n_\gamma,$$

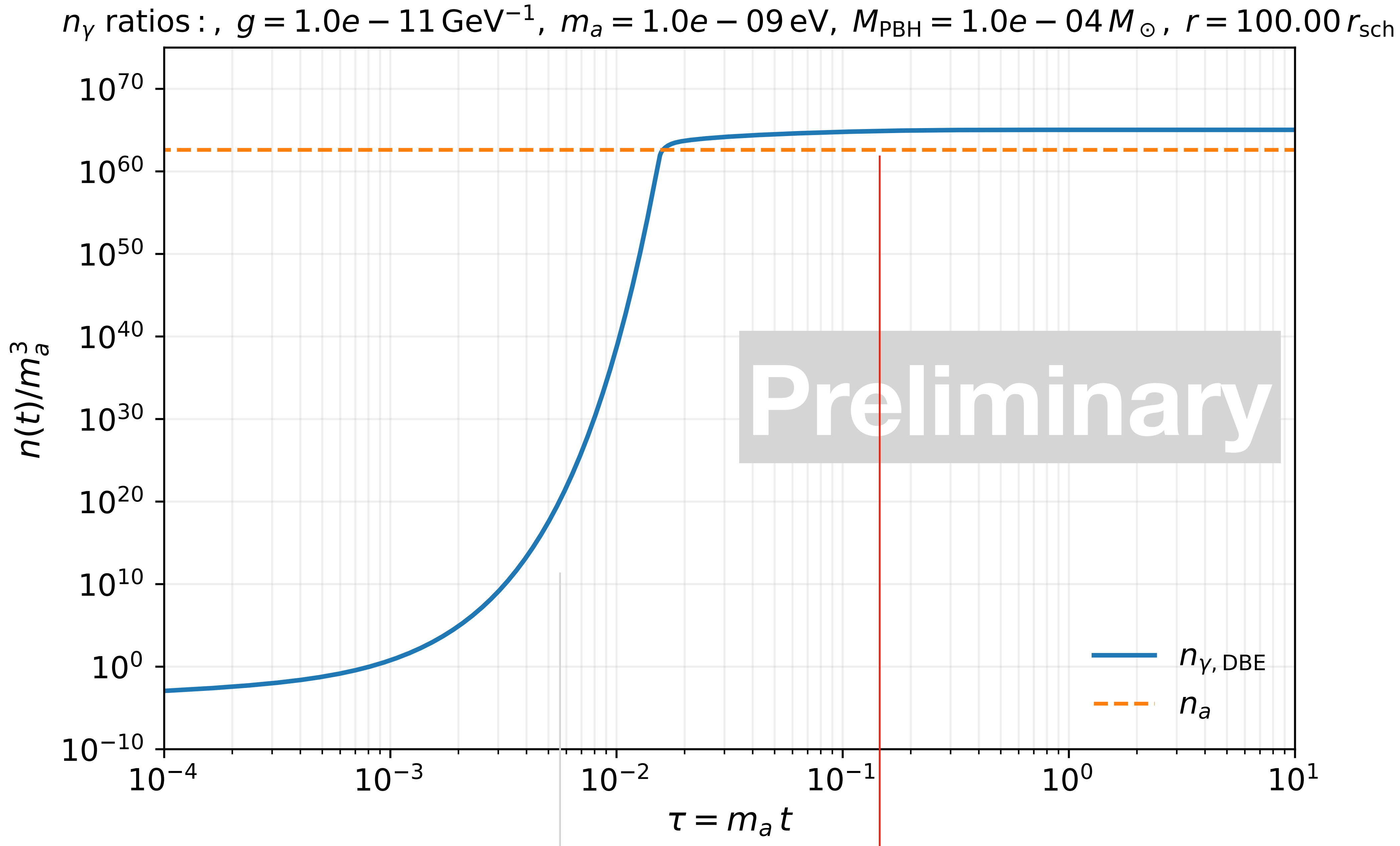


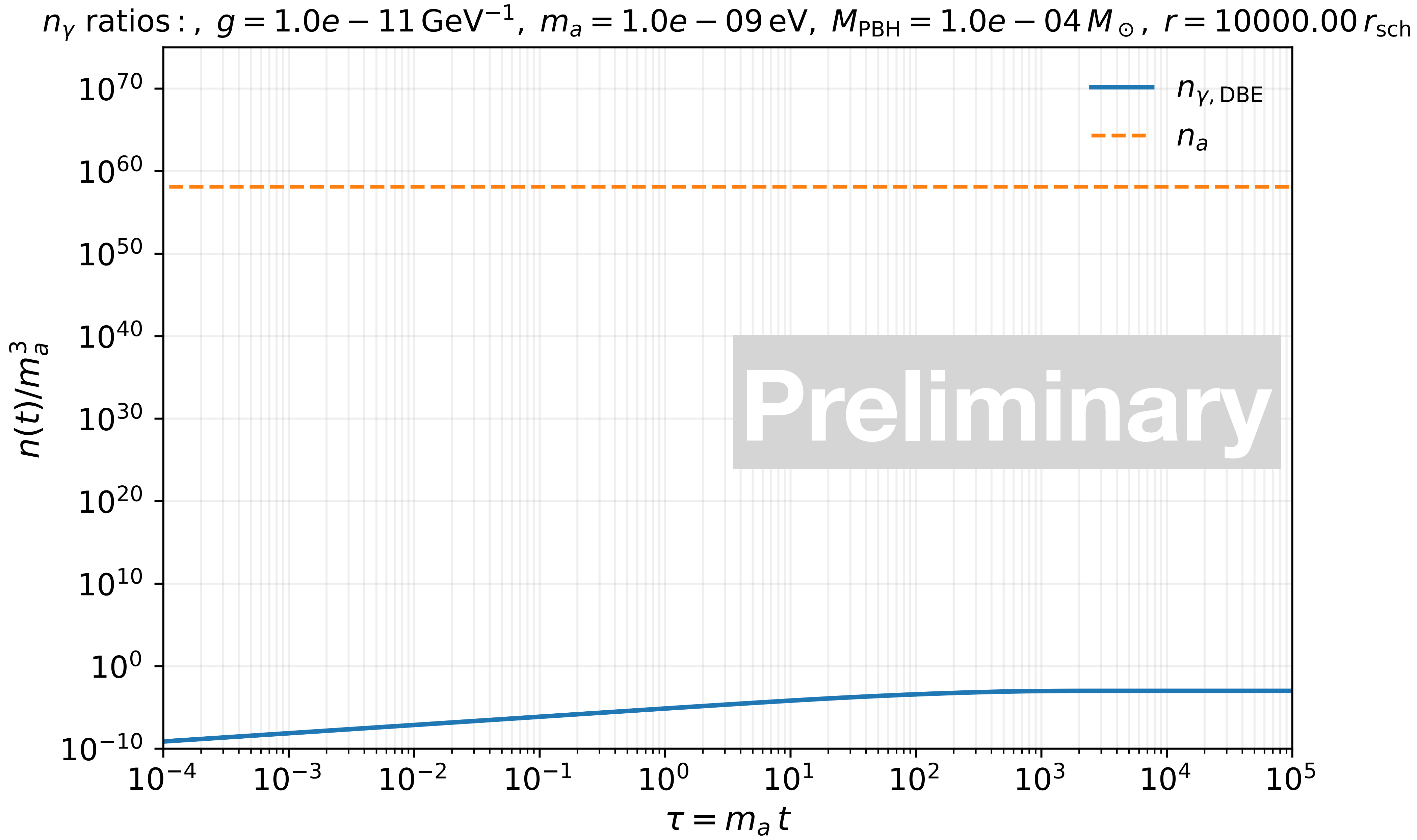


Stimulated decay is dominant → Enhancement occurs

Results of Enhancement/Non-Enhancement (md ver.)

$$\frac{dn_\gamma}{dt} = \frac{m_a\Gamma_a}{\pi^2}\int_0^\infty d\omega \int_{E_{\min}}^\infty dE_a f_a(E_a)\left(1 \boxed{+ f_\lambda(\omega) + f_\lambda(E_a-\omega))}\right) \boxed{- \frac{m_a\Gamma_a}{\pi^2}\int_0^\infty d\omega \int_{m_a-\omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'')} - \frac{3}{2r}n_\gamma,$$



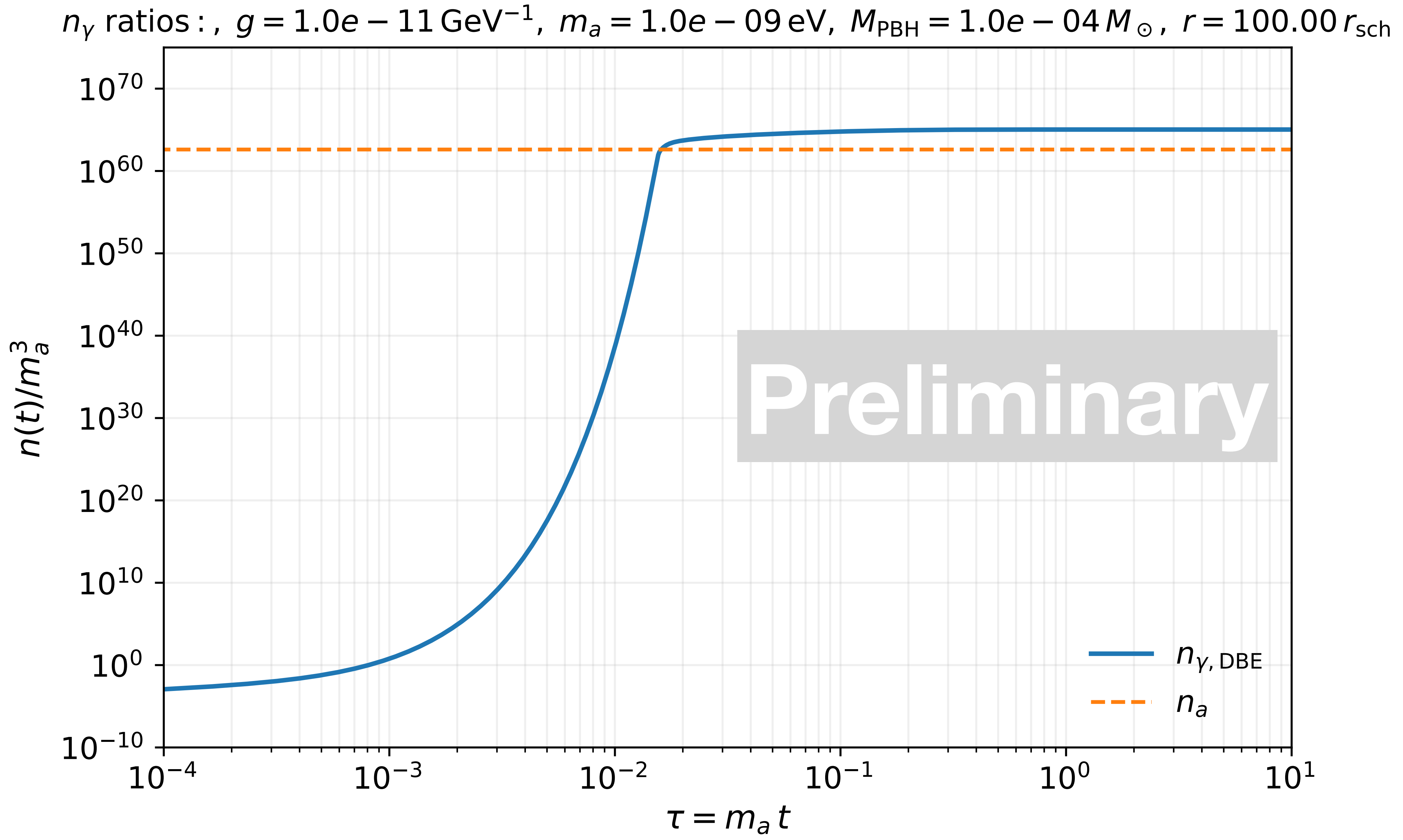


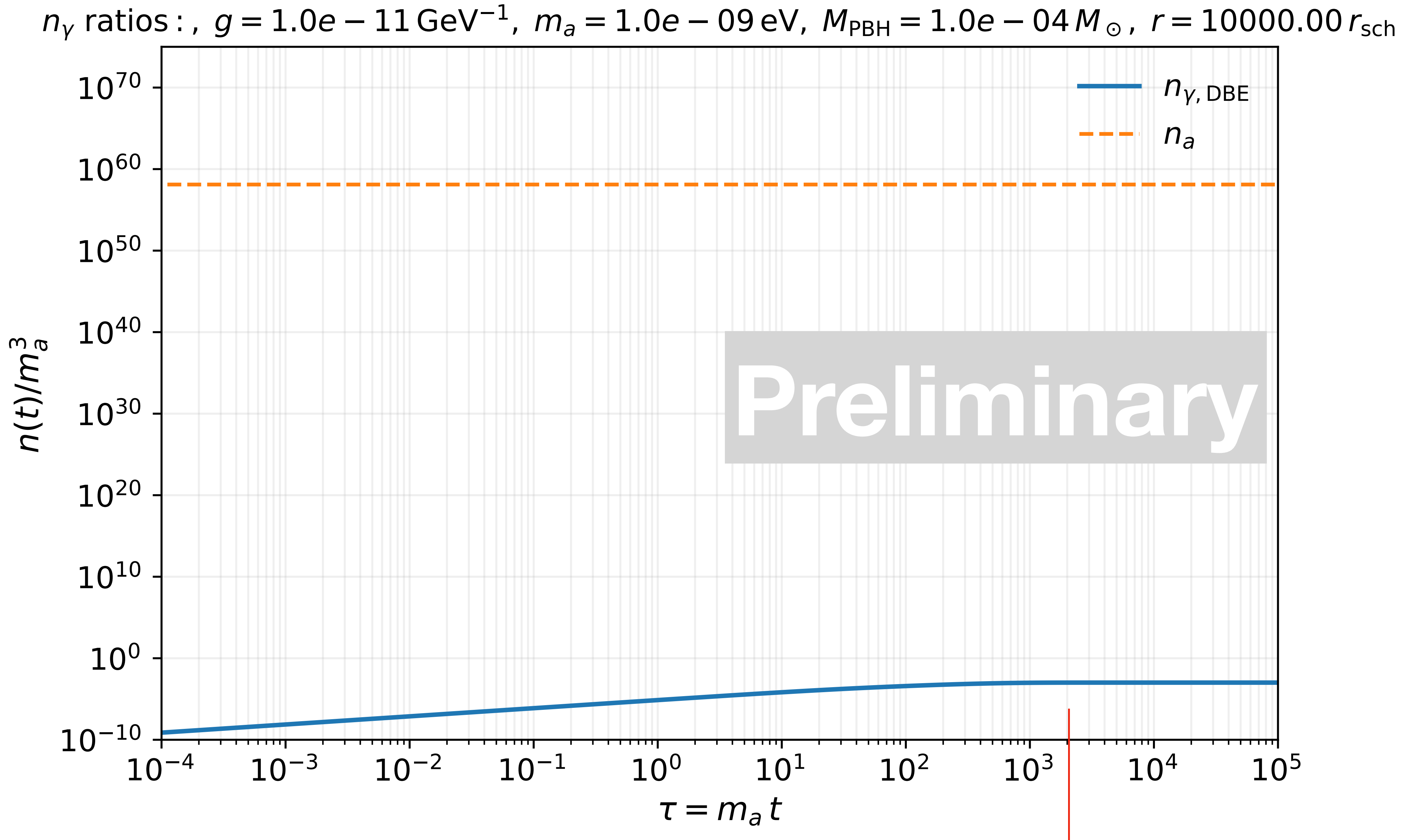
Stimulated decay is dominant → Enhancement occurs

Stimulated decay = Back-reaction → Number density reaches a constant

Results of Enhancement/Non-Enhancement (md ver.)

$$\frac{dn_\gamma}{dt} = \boxed{\frac{m_a\Gamma_a}{\pi^2}\int_0^\infty d\omega \int_{E_{\rm min}}^\infty dE_a f_a(E_a)\left(1 + f_\lambda(\omega) + f_\lambda(E_a-\omega)\right)} - \frac{m_a\Gamma_a}{\pi^2}\int_0^\infty d\omega \int_{m_a-\omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'') \boxed{- \frac{3}{2r} n_\gamma,}$$



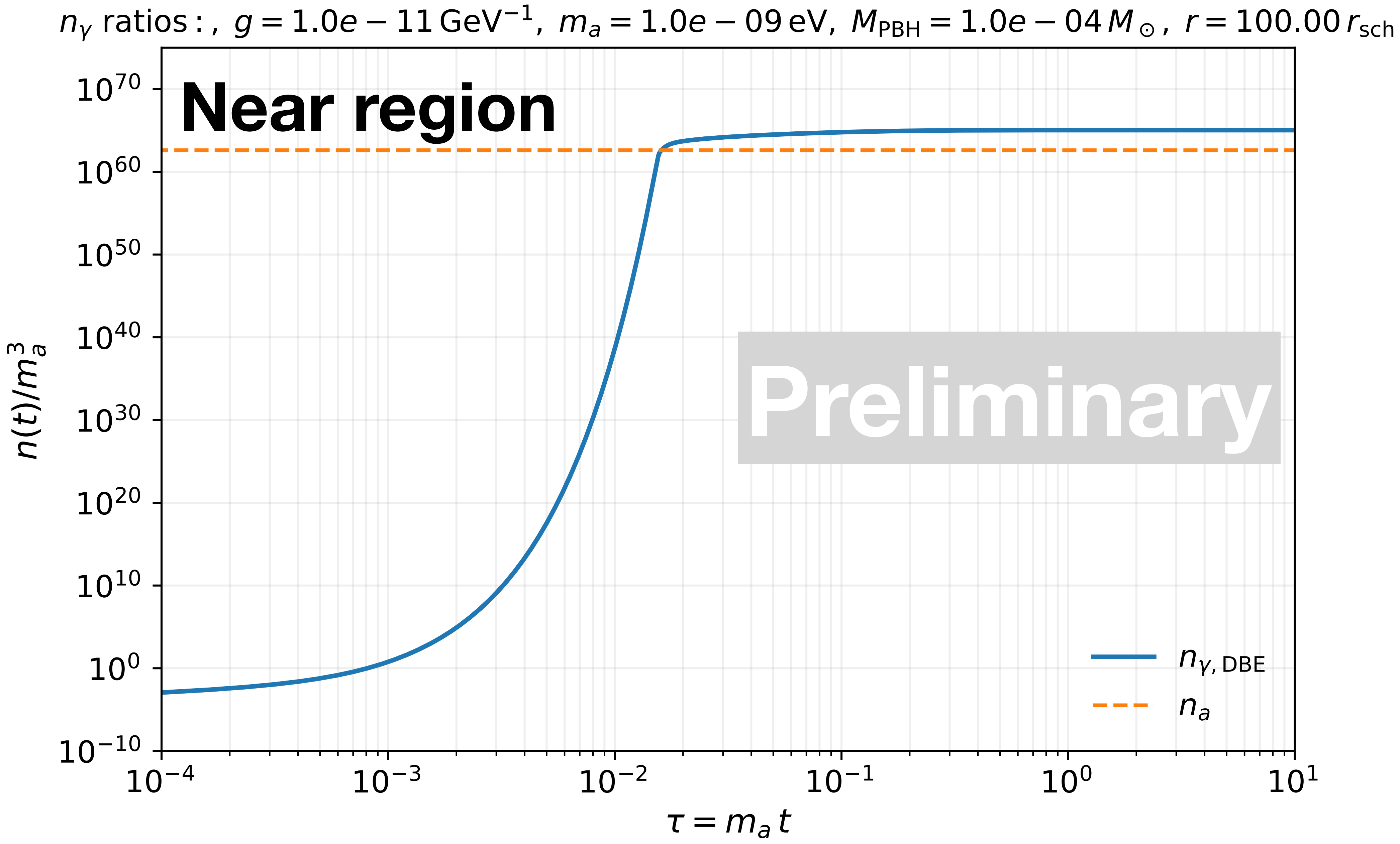


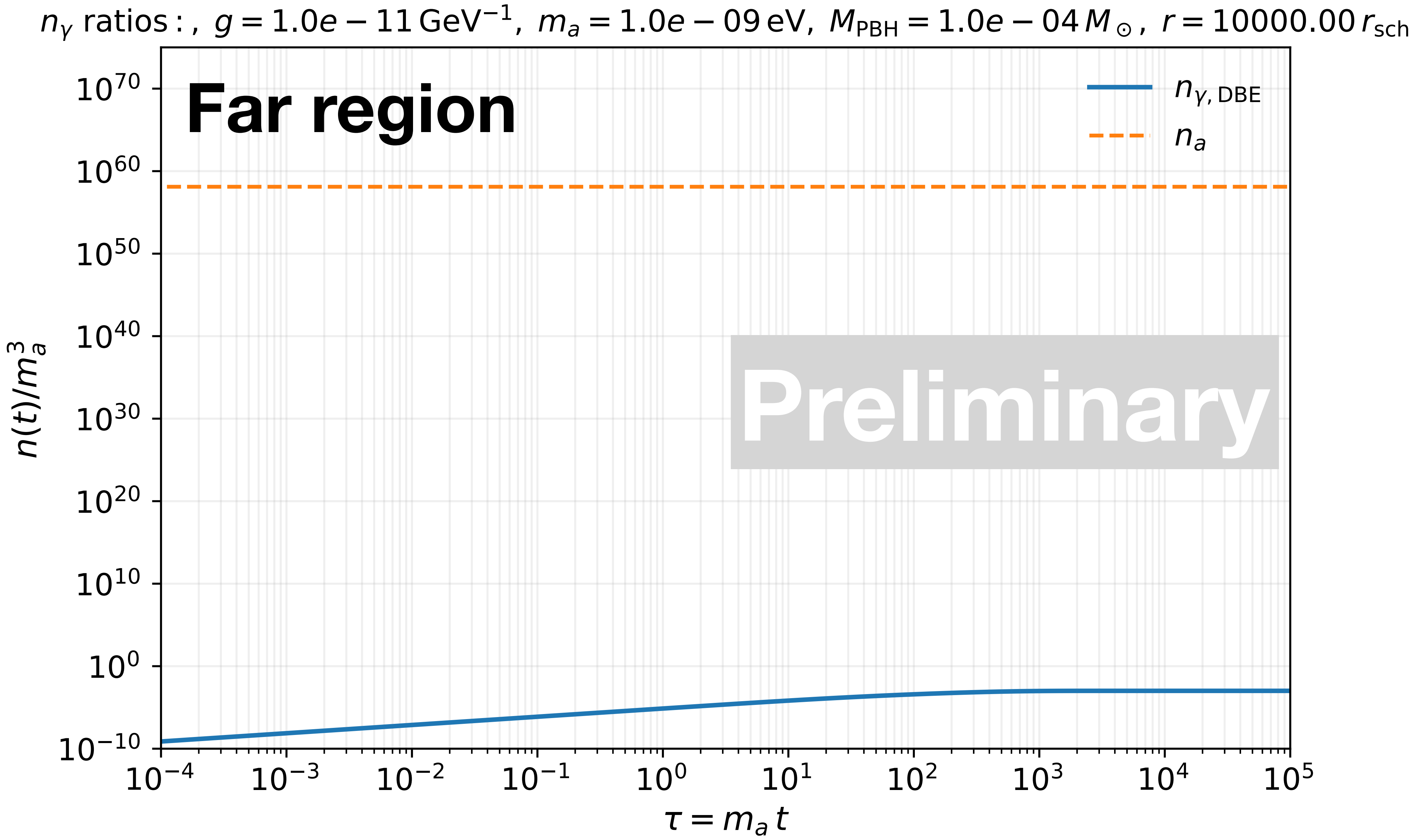
Escape is dominant → No enhancement occurs

Decay = Escape → Number density reaches a constant

Results of Enhancement/Non-Enhancement (md ver.)

$$\frac{dn_\gamma}{dt} = \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{E_{\min}}^\infty dE_a f_a(E_a) \left(1 \boxed{+ f_\lambda(\omega) + f_\lambda(E_a - \omega)}\right) - \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{m_a - \omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'') \boxed{- \frac{3}{2r} n_\gamma,}$$



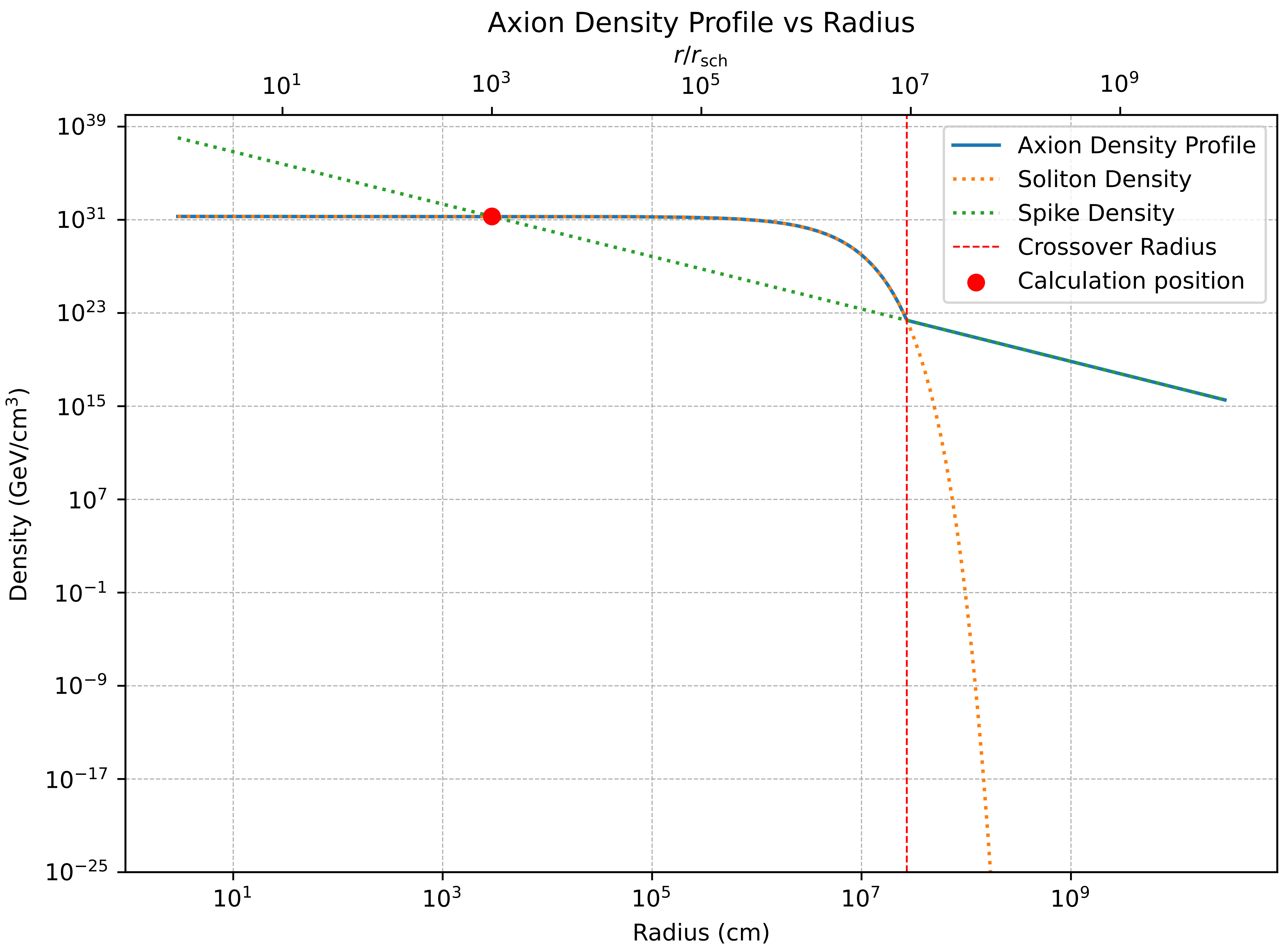


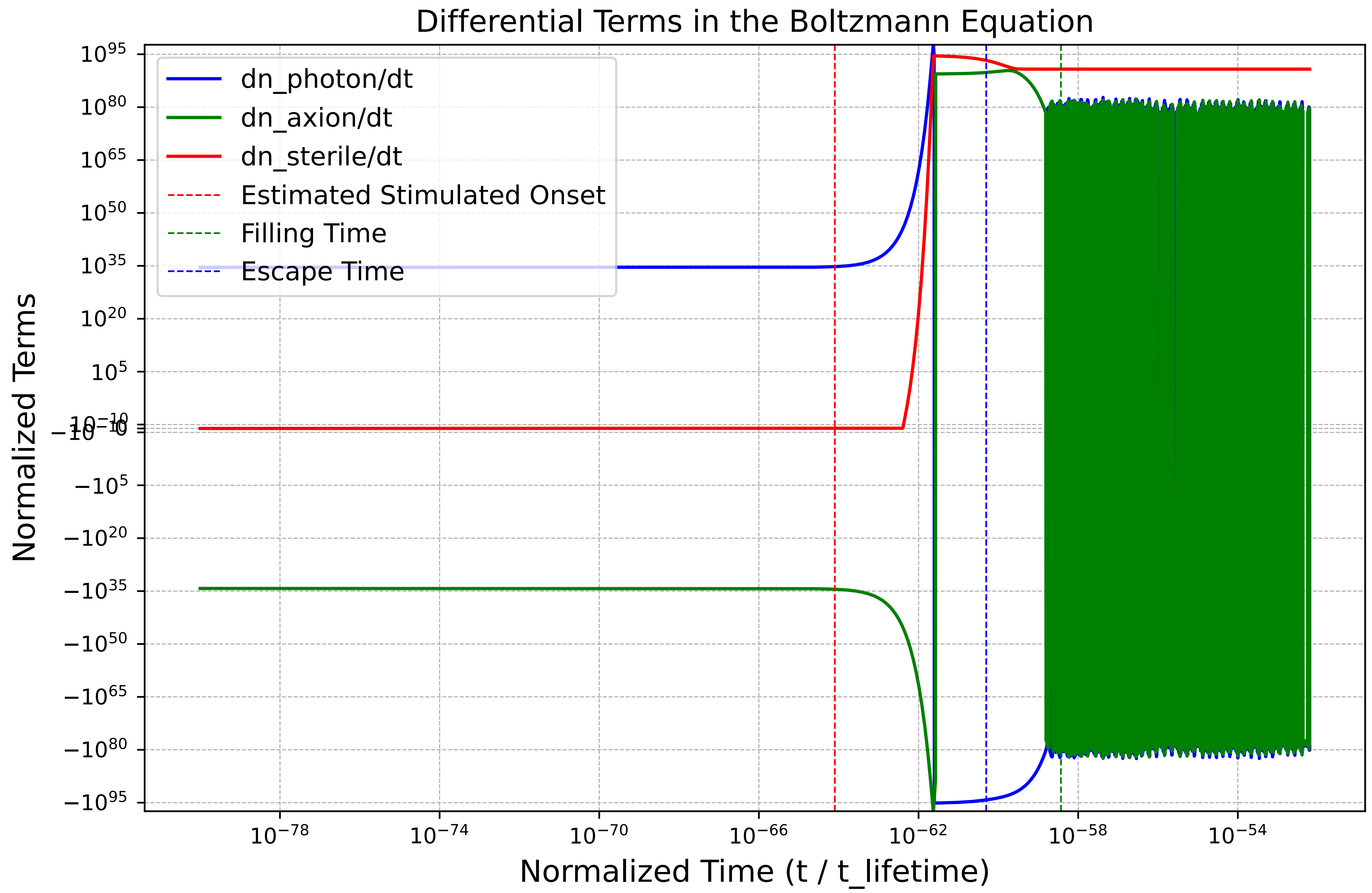
Stimulated decay > Photon escape

Stimulated decay < Photon escape

r_{enh}

Example Result of Enhancement Case

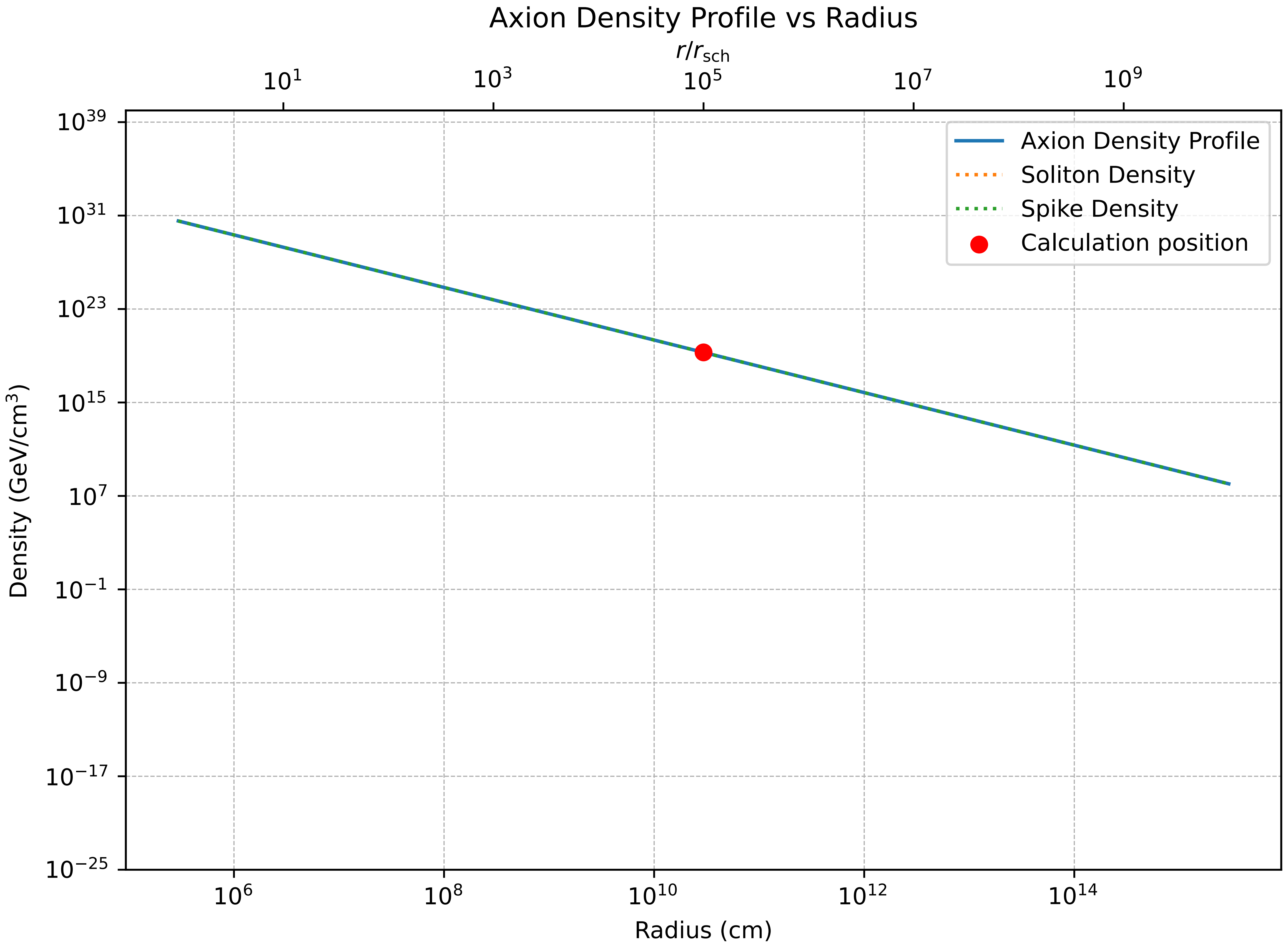


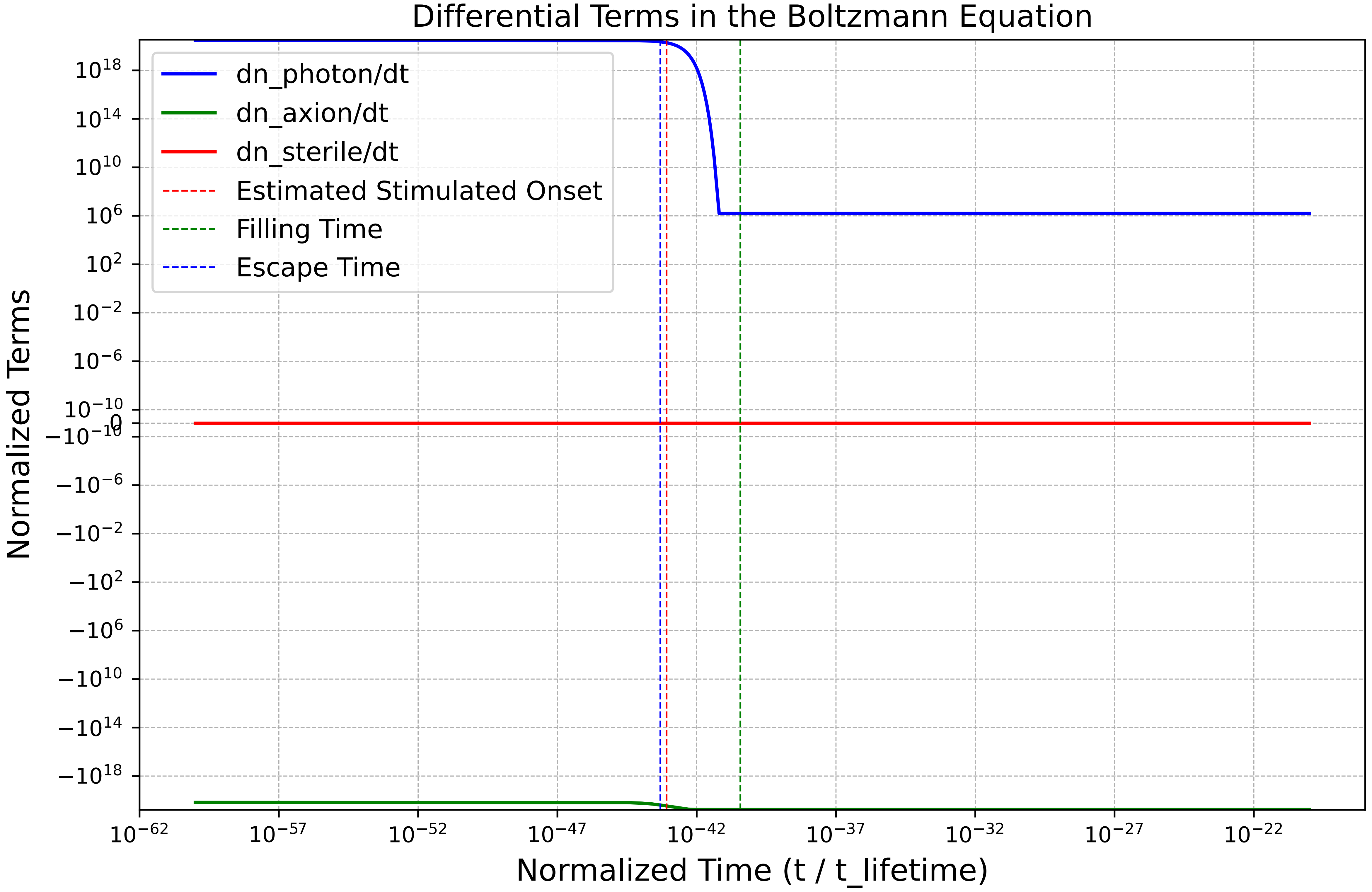


Axion energy density and calculation point

Differential of number densities

Example Result of Non-Enhancement Case





Axion energy density and calculation point

Differential of number densities

Intensity

Carr, et al., Mon.Not.Roy.Astron.Soc. 506 (2021) 3, 3648-3661
Carr, et al.,Phys.Rev.D 81 (2010), 104019
Ferraz, et al., JCAP 07 (2022) 07, 026

- Photon number density:

$$\frac{dn_\gamma}{dt}(E_\gamma, t) = n_{\mathrm{PBH}}(t)\frac{dN_\gamma}{dt}(E,t) \simeq n_{\mathrm{PBH}}(t)E_\gamma \underline{\frac{d^2N_\gamma}{dtdE_\gamma}(E,t)}$$

Logarithmic binning approximation $\Delta E \simeq E$

- Intensity:

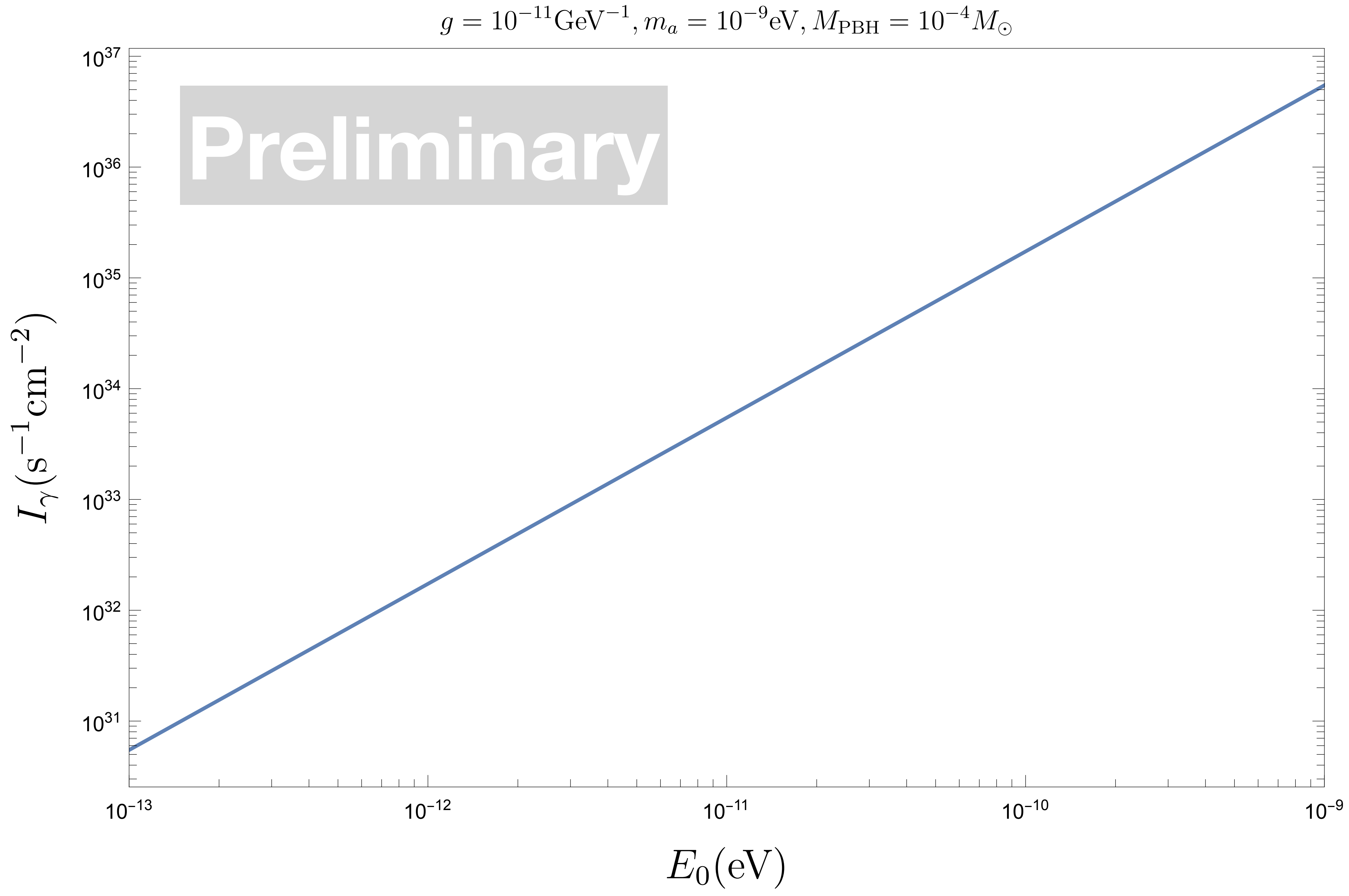
$$I \simeq \frac{dI}{dE}E = \frac{1}{4\pi}\rho_{c,0}\Omega_{\mathrm{PBH},0}E_0\int_{t_{\mathrm{rec}}}^{t_{\mathrm{cloud}}} dt\frac{(1+z)}{M(t)}\frac{d^2N_\gamma}{dEdt}(E_0(1+z))$$

$$\simeq \frac{3}{4\pi^2}\rho_{c,0}\Omega_{\mathrm{PBH},0}\frac{E_0^3}{M_{\mathrm{PBH}}}\int_{t_i}^{t_f} dt(1+z)^3\int_{100r_{\mathrm{sch}}}^{r_0(t)} drrf_\lambda(E_0(1+z),r)dr$$

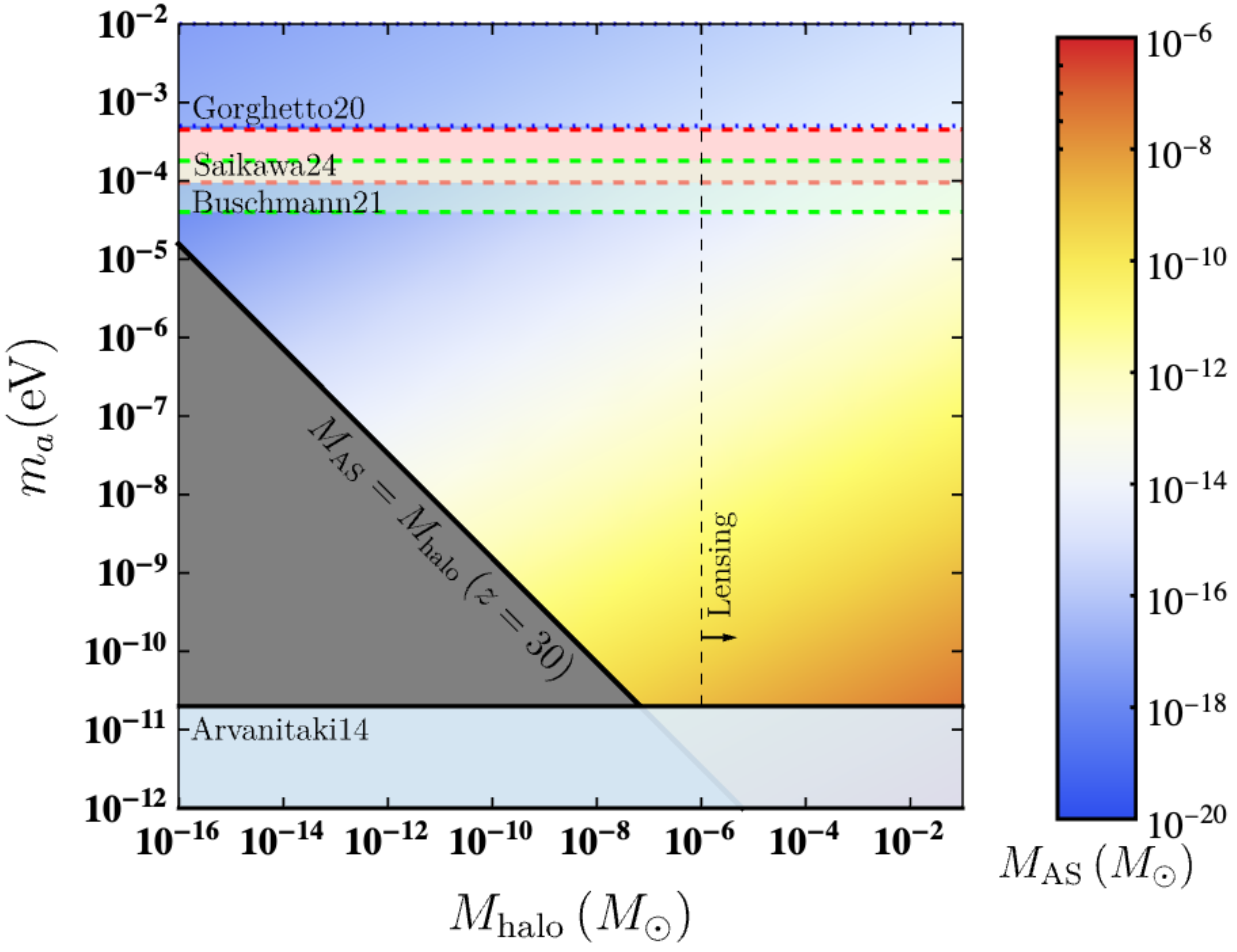
Intensity

$$\frac{df_\lambda(\omega)}{dt} = \frac{m_a\Gamma_a}{\omega^2}\int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a)\left(1 + f_\lambda(\omega) + f_\lambda(E_a-\omega)\right) - \frac{m_a\Gamma_a}{\omega^2}\int_{m_a-\omega}^{\infty} d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} f_\lambda(\omega)$$

$$I \simeq \frac{3}{4\pi^2}\rho_{c,0}\Omega_{\text{PBH},0}\frac{E_0^3}{M_{\text{PBH}}}\int_{t_i}^{t_f} dt(1+z)^3 \int_{100r_{\text{sch}}}^{r_0(t)} drrf_\lambda(E_0(1+z), r)dr$$



Axion Star in ALP Halo around PBH



Yin, et al., JCAP 10 (2024), 013