Two-Pion HBT Signatures of Spatial Inhomogeneity in Finite-Density QCD

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Intro: HBT began with a practical question: How large is a star?

- **A distant star may be too small to resolve in an ordinary image.**
- **Hanbury Brown and Twiss correlated intensity fluctuations measured by two separated detectors.**
- **As the distance between the two detectors increases, the correlation gradually decreases.**

  The rate of this decrease tells us the star’s angular size.
- **Conventional optical interferometry requires precise phase stability between the two detectors.**

  HBT intensity interferometry instead measures correlations between intensity fluctuations.

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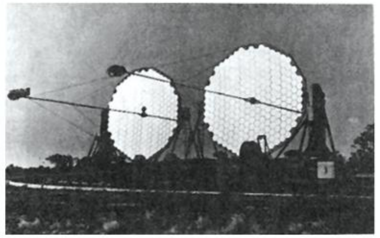
A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN
Jodrell Bank Experimental Station, University of Manchester
AND
DR. R. Q. TWISS
Services Electronics Research Laboratory, Baldock



Pict. from Padula, BJP35 (2004)

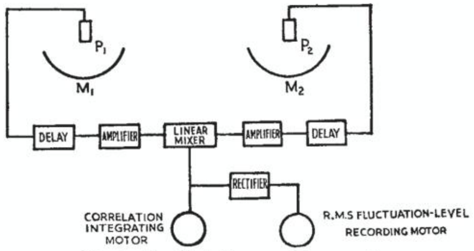


Fig. 1. Simplified diagram of the apparatus

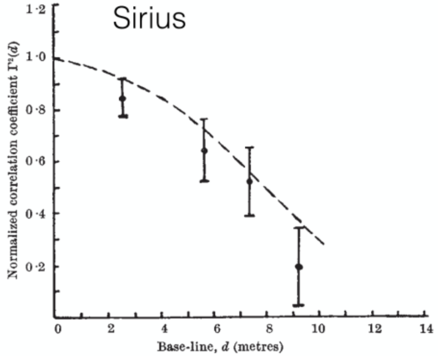
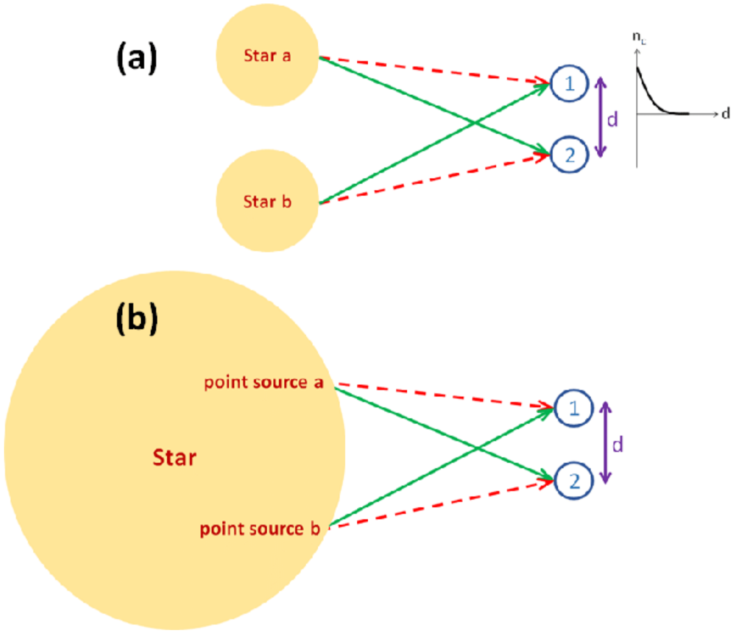


Fig. 2. Comparison between the values of the normalized correlation coefficient $\Gamma^2(d)$ observed from Sirius and the theoretical values for a star of angular diameter 0·0063″. The errors shown are the probable errors of the observations



The same correlation principle works for identical pions

- For identical bosons, exchanged two-particle amplitudes interfere constructively at small relative momentum q.
- This enhancement was first observed for identical pion pairs produced in $p\bar{p}$ annihilation.
- In collider physics,
  this phenomenon is called Bose–Einstein correlations (BEC),
  not a Bose–Einstein condensate.
  When used to study the emission source,
  the method is called HBT femtoscopy.

PION-PION CORRELATIONS IN ANTIPROTON ANNIHILATION EVENTS

Gerson Goldhaber, William B. Fowler, Sulamith Goldhaber, T. F. Hoang, Theodore E. Kalogeropoulos, and Wilson M. Powell

July 14, 1959

PHYSICAL REVIEW VOLUME 120, NUMBER 1 OCTOBER 1, 1960

Influence of Bose-Einstein Statistics on the Antiproton-Proton Annihilation Process*

Gerson Goldhaber, Sulamith Goldhaber, Wonyong Lee, and Abraham Pais†
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California
(Received May 16. 1960)

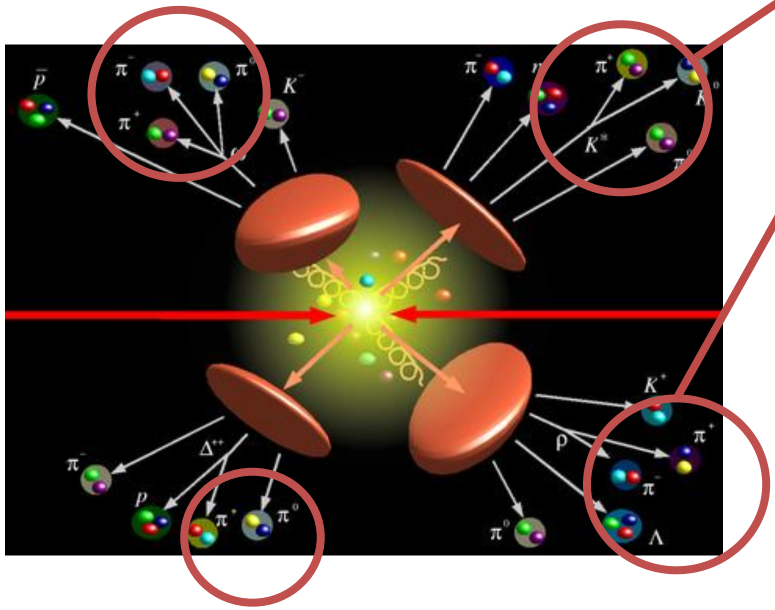
$$\psi(12)=\int\int|\phi^S(1,2)|^2\exp[-(r_1^2+r_2^2)/2\lambda]d\mathbf{r}_1d\mathbf{r}_2$$

$$\approx 1+\exp(-s^2),\quad s=|\mathbf{p}_1-\mathbf{p}_2|\lambda^{\frac{1}{2}},\quad \text{(Gaussian)},\qquad \rho=2.15\lambda^{\frac{1}{2}}.$$

HBT probes the extended, evolving source created in HIC

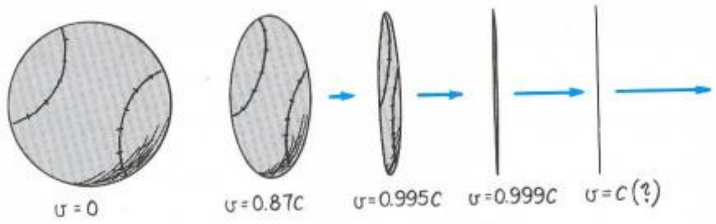
- Bose–Einstein correlations are measured in $e^+e^-, pp, p\overline{p}$ and nucleus–nucleus collisions.
- In elementary collisions they mainly constrain hadronization, fragmentation, and multiplicity-dependent source scales.
- Heavy-ion collisions create an extended, collectively expanding source with a well-characterized collision geometry.
- The extracted HBT radius is a pair-dependent homogeneity length, not automatically the full fireball radius.
- Our question is whether HBT can also retain a smaller internal spatial scale.

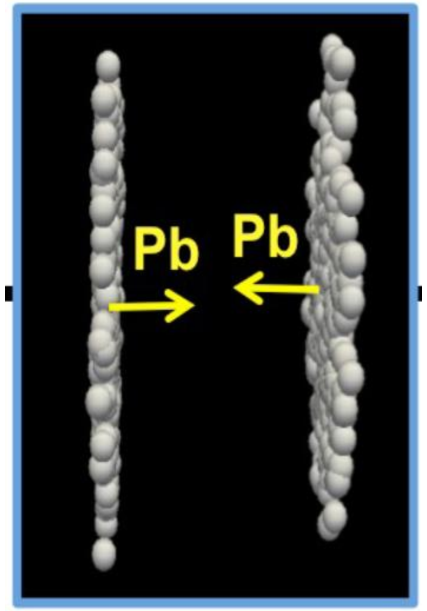
$\pi^+\pi^+$ pairs

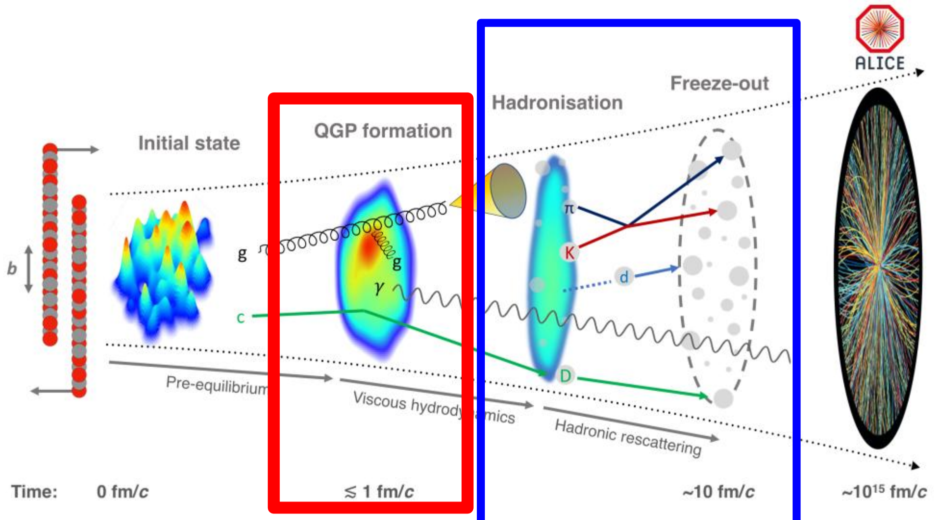


Final pions are the endpoint of a dynamical QCD evolution

- Lorentz-contracted nuclei arrive as thin “pancakes” and deposit energy in a very short time.
- Initial energy deposition and pre-equilibrium dynamics form a hot, dense quark–gluon plasma (QGP).
- The QGP expands, cools, and hadronizes. Hadronic rescattering continues until kinetic freeze-out.
- Detectors record only the final hadrons; the QGP is observed indirectly.
- An early spatial pattern matters only if a remnant survives to the pion-emission source.



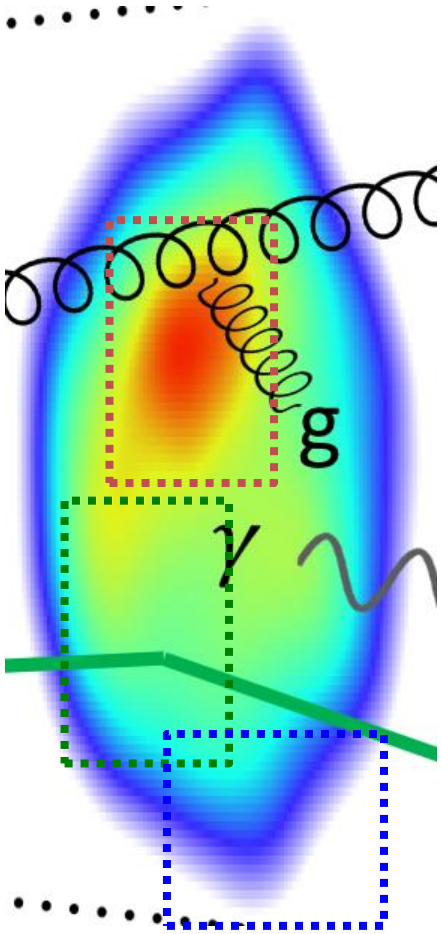




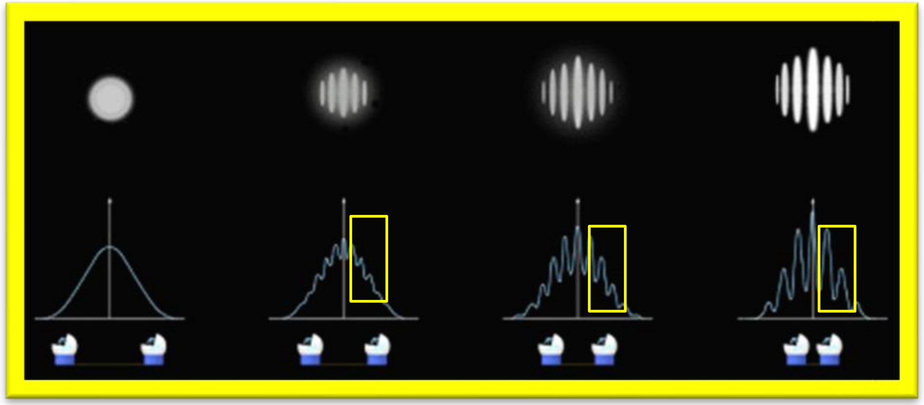
Primordial inhomogeneity: an early spatial pattern

- Nucleon and subnucleon positions already fluctuate from event to event.
- Here the target is more specific:

  an early clustered or modulated structure with a characteristic spatial scale.
- Expansion and rescattering can wash out the pattern.
- Here, we call such a metastable remnant primordial inhomogeneity..
- This is a hypothesis to test—not an established equilibrium phase of the QGP.



Smooth source
A single central correlation peak



Spatially patterned (inhomogeneous) source
Additional side structure in the correlation signal

From a smooth source to an inhomogeneous source: a spatial pattern produces additional structure in the correlation signal.

Outline

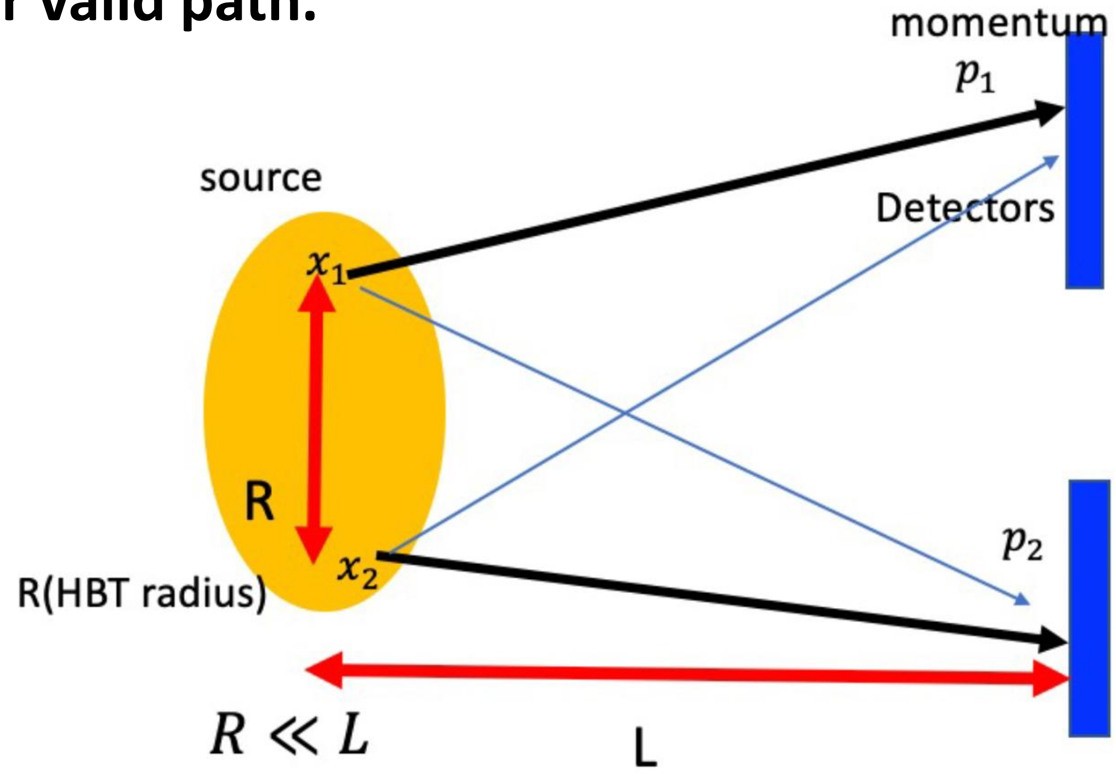
1. Intro
2. HBT Femtoscopy
3. QGP Substructure
4. Simulation
5. Conclusion

HBT Femtoscopy

Basic idea: Identical pions have two exchange paths

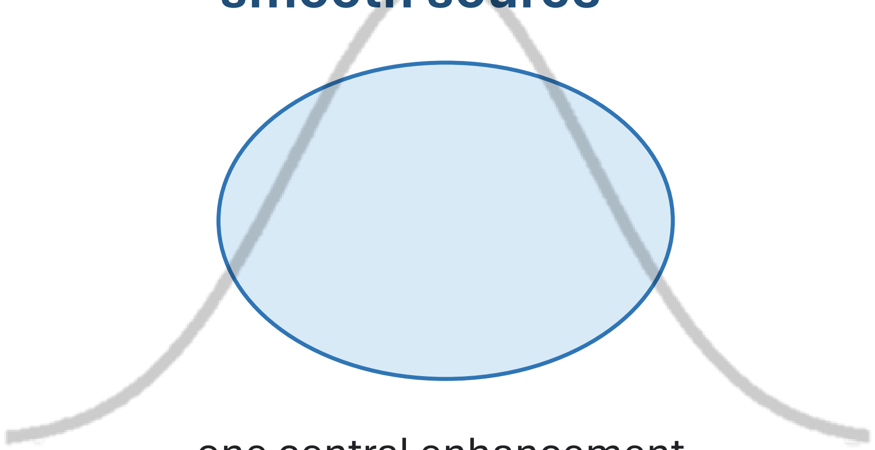
- **Pion 1 and pion 2 may be assigned to two source points.**
- **Exchanging the assignments gives another valid path.**
- **The two paths cannot be distinguished.**
- **Their interference enhances pairs**

  with similar momenta.



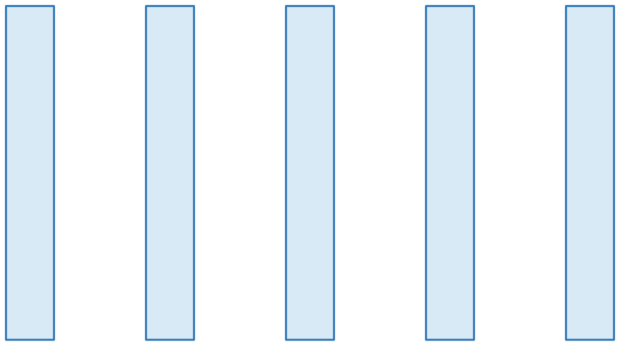
A characteristic spatial scale can generate finite-q HBT structure

smooth source



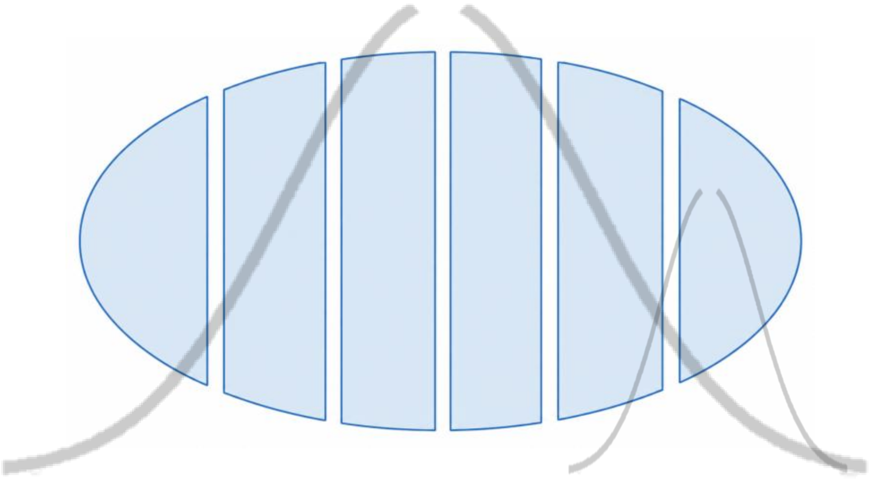
one central enhancement

spatial pattern



central + finite-q structure

- **HBT is Fourier-sensitive to the relative emission source.**
- **An additional source scale can move part of the correlation strength away from q = 0.**



C_2 connects measured pion pairs to the relative emission source

- **What is measured?** $C_2(\mathbf{p}_1, \mathbf{p}_2) = \frac{P_2(\mathbf{p}_1, \mathbf{p}_2)}{P_1(\mathbf{p}_1) P_1(\mathbf{p}_2)} \equiv \frac{\text{two-particle yield}}{\text{uncorrelated reference yield}}$
    - The numerator contains pion pairs from the same event.
    - The denominator estimates the yield without pair correlations.
    - $C_2 = 1$ means no correlation; $C_2 > 1$ indicates an enhancement.
- **How is it related to the source?**
    - For identical, noninteracting bosons in the plane-wave approximation.
    - $|\psi_{rel}(q, r)|^2 = 1 + \cos(\mathbf{q} \cdot \mathbf{r})$, $\quad C_2(\mathbf{q}) = \int d^3\mathbf{r}\ S(\mathbf{r})\, |\psi_{rel}(\mathbf{q}, \mathbf{r})|^2 = 1 + \langle \cos(\mathbf{q} \cdot \mathbf{r}) \rangle_S$
    - $S(r)$ describes the distribution of relative emission points.
    - A smooth source produces one central peak around $q = 0$.
    - Therefore, $C_2(q)$ contains Fourier information about the source.
    - A spatial pattern—that is, a *modulation* of $S(r)$ —can produce additional structure at finite q in $C_2(q)$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$$
$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

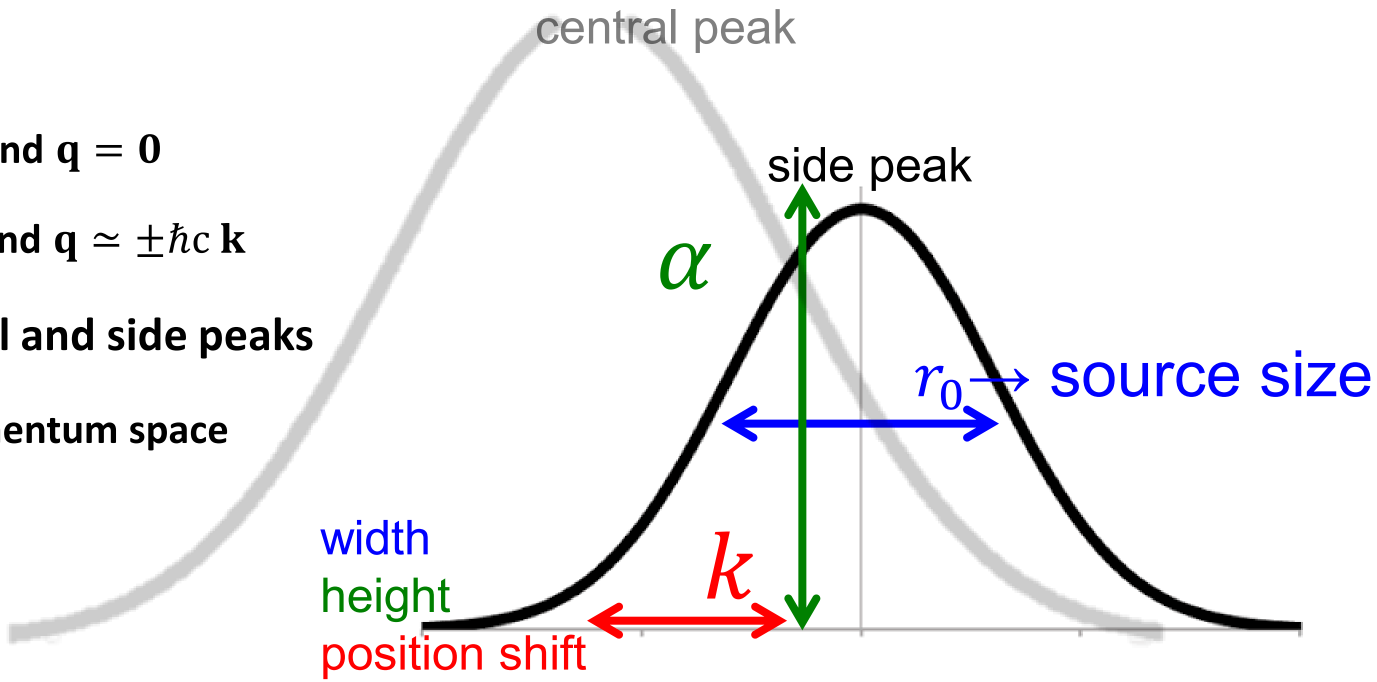
$$\boxed{\text{modulation in } S(r) \xrightarrow{\text{Fourier transform}} \text{side peak in } C_2(q)}$$

A spatial pattern adds finite-q side peaks to C_2

- **For a Gaussian source with this spatial modulation, $C_2(\boldsymbol{q})$ is given by**

$$C_2(\boldsymbol{q}) = 1 + \langle \cos(\mathrm{q}\cdot\mathrm{r})\rangle = 1 + \frac{e^{-r_0^2 q^2} + \frac{\alpha}{2}\left[e^{-r_0^2|\mathbf{q}+k\mathbf{n}|^2} + e^{-r_0^2|\mathbf{q}-k\mathbf{n}|^2}\right]}{1+\alpha e^{-r_0^2 k^2}}$$

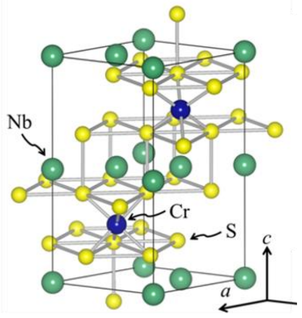
- $\boldsymbol{k}$ **: <u>sets the side-peak position</u>**
- $\boldsymbol{\alpha}$ **: controls the side-peak strength**
  - $\boldsymbol{\alpha = 0}$ **: one central HBT peak around $\mathbf{q} = \mathbf{0}$**
  - $\boldsymbol{\alpha \neq 0}$ **: additional side peaks around $\mathbf{q} \simeq \pm\hbar c\,\mathbf{k}$**
- $\boldsymbol{r_0}$**: sets the width of both the central and side peaks**
  - **larger r_0 → narrower peaks in momentum space**



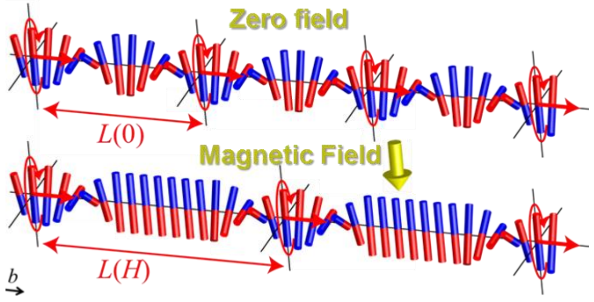
QGP Substructure

Possible origins of inhomogeneity in dense QCD matter

- Dense QCD matter may not always be spatially uniform.
- $\mathrm{CrNb_3S_6}$ is a monoaxial chiral magnet. Under an external magnetic field, it can form a periodic spin structure called a Chiral Soliton Lattice (CSL).
- Low-energy QCD at finite density under a strong magnetic field has a mathematically analogous effective theory.
- Noncentral heavy-ion collisions generate extremely strong but short-lived magnetic fields, approximately 10^{18}–10^{19} G.
- This theoretical analogy motivates a possible transient QCD CSL and a CSL-like inhomogeneity in the evolving QCD matter.



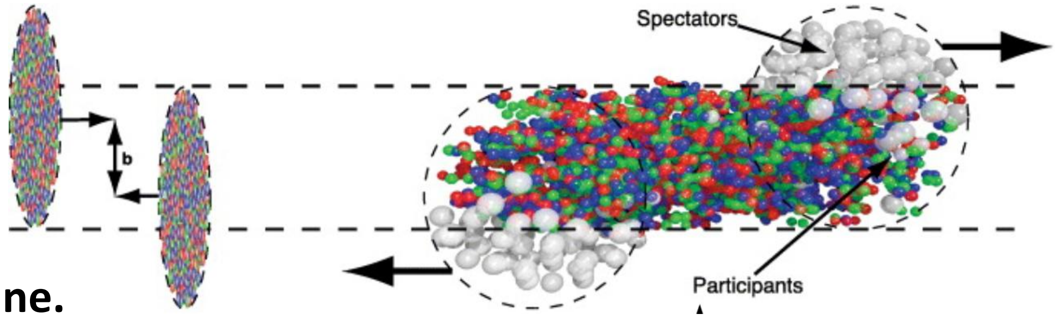
(a)



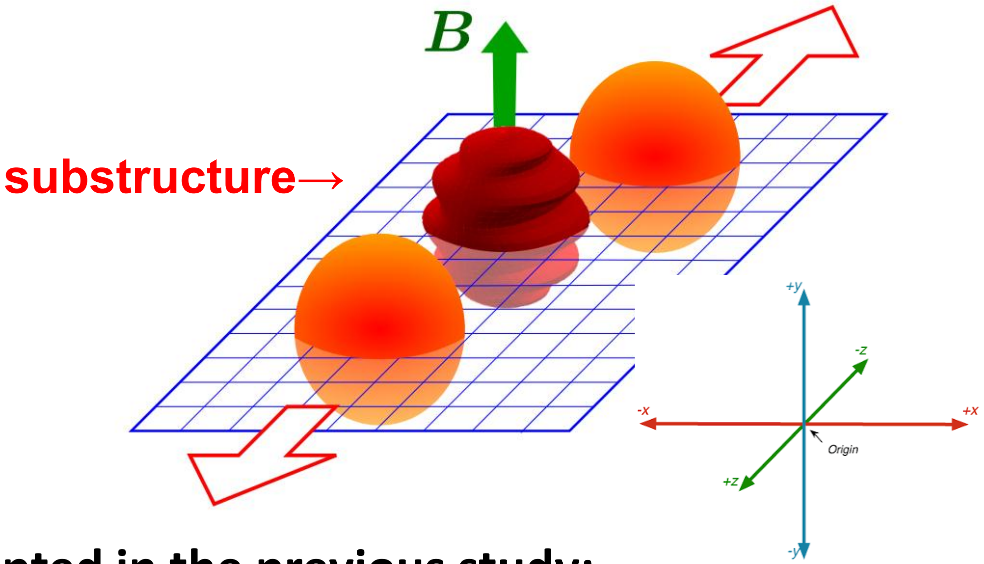
(b)

Chiral Soliton Lattice

Strong B selects the direction; μ_B sets the modulation scale



- Collision geometry
  - z : beam direction, x : impact-parameter direction
  - $x - z$: reaction plane
  - y : magnetic-field direction, perpendicular to the reaction plane. Spectator protons move along $\pm z$ on opposite sides of the collision, and their magnetic-field contributions add mainly along $\pm y$.



- Strong-field effect
  - When $\sqrt{eB}$ exceeds the relevant QCD scale, transverse quark motion is strongly suppressed, favoring pseudo-one-dimensional modulation along B.
- The QCD CSL consists of periodic neutral-pion domain walls, with baryon density localized around the layers.
- For the beam-energy scan, we follow the benchmark relation adopted in the previous study:

$$\boxed{k = \frac{2\mu_B}{3}, \qquad k\,[\mathrm{fm}^{-1}] \simeq \frac{\mu_B\,[\mathrm{MeV}]}{300}}$$

  - This benchmark is motivated by an analogous one-dimensional chiral modulation; it is not a universal CSL prediction.

A controlled model of periodic substructure

- **Phase-space density** as

$$\rho_a(\boldsymbol{p},\boldsymbol{r},t) = \sum_n \delta(\boldsymbol{p}-\boldsymbol{p}_n)\delta\left[\boldsymbol{r}-\boldsymbol{r}_n-\boldsymbol{e}_y a\cos(k y_n)\right]\delta(t-t_n)$$

$\boldsymbol{a}$: displacement amplitude

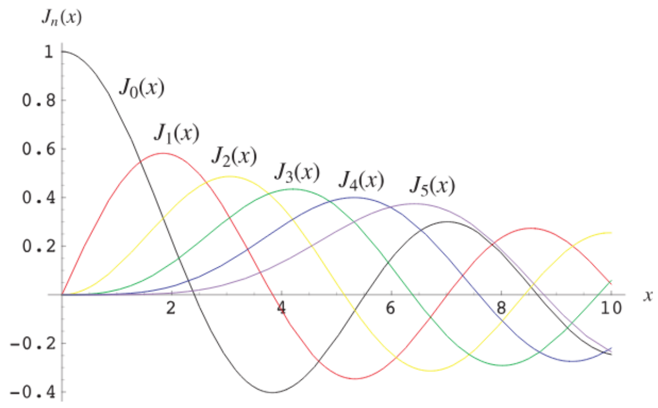
- **spatial Fourier transform**

$$\widetilde{\rho_a}(\boldsymbol{p},\boldsymbol{q},t) = \int d^3 r\, e^{i\boldsymbol{q}\boldsymbol{r}} \rho_a(\boldsymbol{p},\boldsymbol{r},t) = \sum_n \delta(\boldsymbol{p}-\boldsymbol{p}_n)\delta(t-t_n) e^{i\boldsymbol{q}\cdot\boldsymbol{r}_n} e^{iq_y a\cos(k y_n)}.$$

- **Using the Jacobi–Anger expansion,**

$$e^{iq_y a\cos(k y_n)} = \sum_{m=-\infty}^{\infty} i^m J_m(q_y a) e^{imk y_n}$$

Bessel Function of the First Kind

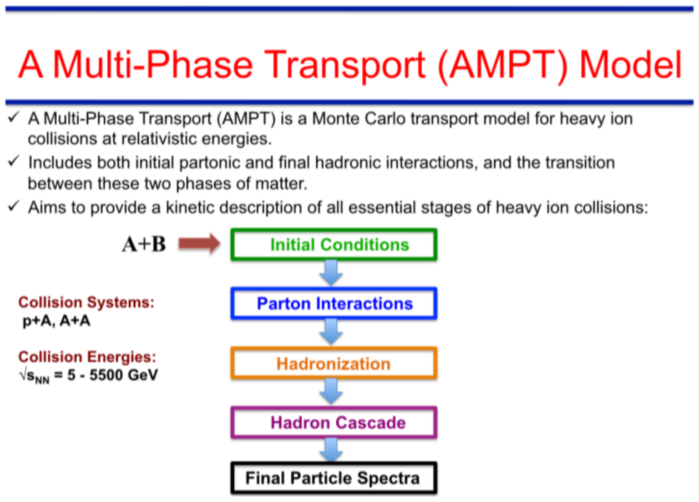


- **Fourier–Bessel scaling for the Gaussian source.**

Simulation

Testing the idea with *transport models*: AMPT and JAM2

- **We use simulations to test how a modulation changes the HBT correlation.**
- **A *transport model* follows individual particles and their interactions through time.**
- **It provides final particle positions and momenta for HBT analysis.**
- **The modulation is imposed after the transport evolution.**
- **1,000 Au-Au events per model and beam energy**
  - **Beam energies: 39, 27, 19.6, 11.5, 9.2, and 7.7 GeV**
  - **Noncentral collisions: 3 fm < b < 4 fm**
  - **Lower beam energies probe more baryon-rich QCD matter.**
  - **freeze-out particles → construct 10^8 pion pairs → impose the modulation while evaluating the pairs → calculate C_2**



JAM microscopic transport model

- spece-time propagation of particles based on cascade method
- Resonance (up to 2GeV) and string excitation and decays
- Re-scattering among all hadrons
- DPM type string excitation law as in HIJING.
- Use Pythia6 for string fragmentation
- Propagation by the hadronic mean-fields within RQMD/S formulation
- EoS controlled collision term

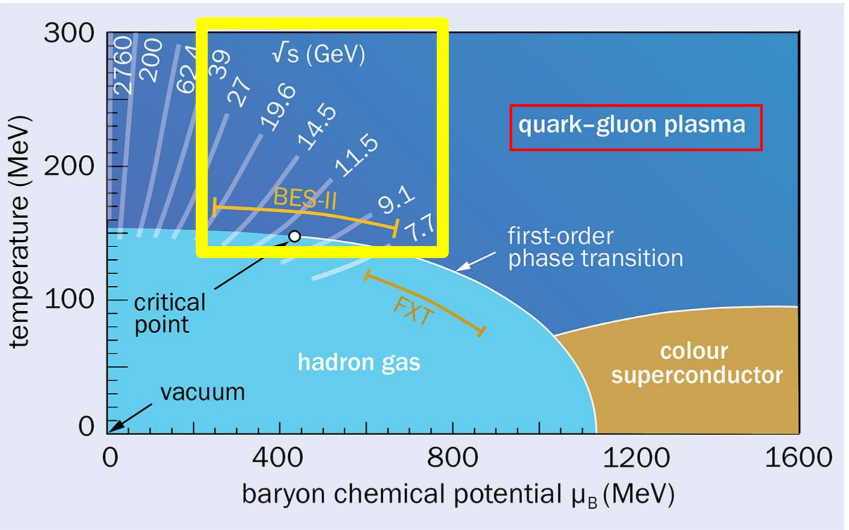
Changing k shifts the side peak across beam energies

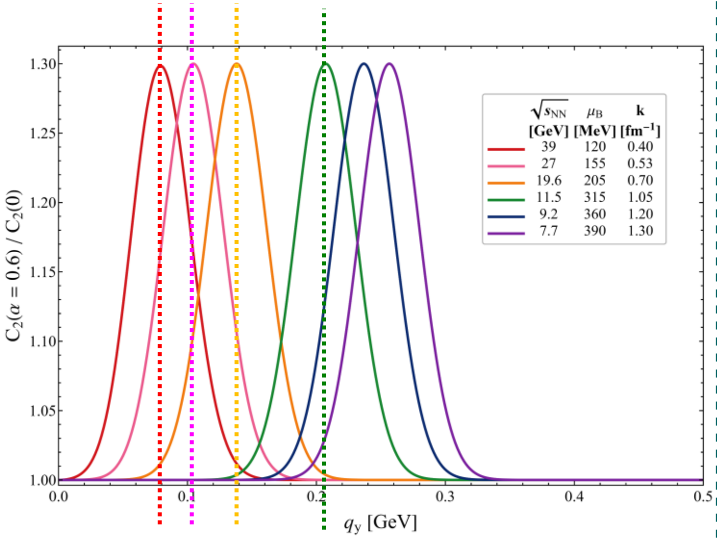
- **In the simulation, side peaks appear near the expected momentum positions at different beam energies.**

$$k\,[\mathrm{fm}^{-1}] \simeq \frac{\mu_B\,[\mathrm{MeV}]}{300}$$

The expected beam-energy dependence is therefore

$$\sqrt{s_{NN}}\downarrow \;\Longrightarrow\; \mu_B\uparrow \;\Longrightarrow\; k\uparrow \;\Longrightarrow\; q^{(1)}_{\mathrm{peak}}\uparrow$$

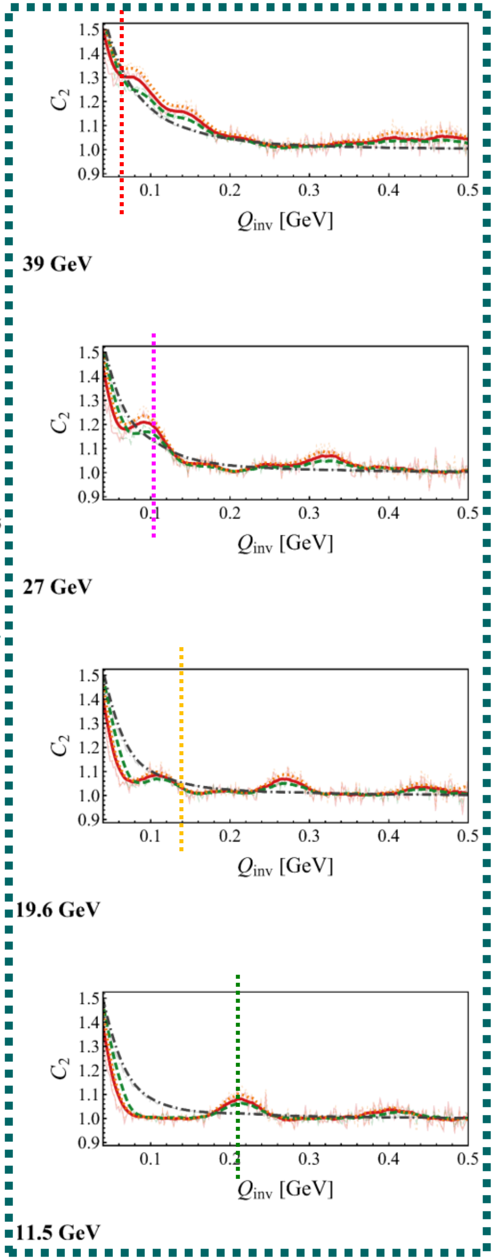




Simulation results →

TABLE I. Adopted baryon chemical potentials, corresponding modulation wave numbers, and expected leading Gaussian side-peak positions for the selected beam energies. The momentum values are obtained from $q^{(1)}_{\mathrm{peak}} \simeq \hbar c\, k$.

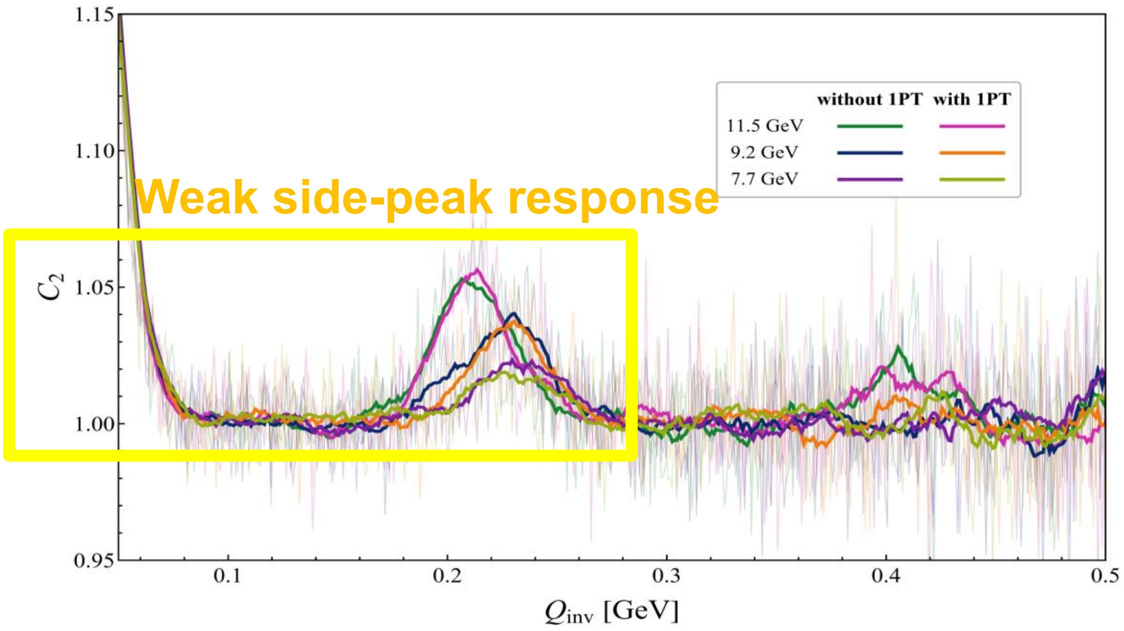
$\sqrt{s_{NN}}$ [GeV]	μ_B [MeV]	k [fm^{-1}]	$q^{(1)}_{\mathrm{peak}}$ [GeV]
39	120	0.40	0.079
27	155	0.53	0.105
19.6	205	0.70	0.138
11.5	315	1.05	0.207
9.2	360	1.20	0.237
7.7	390	1.30	0.257



Low-energy simulation response

- At fixed $a = 5$ fm, the side-peak excess above unity decrease to approximately 5%, 4%, and 2% at 11.5, 9.2, and 7.7 GeV, respectively.

- Does the HBT signal become too weak to observe as the beam energy decreases?



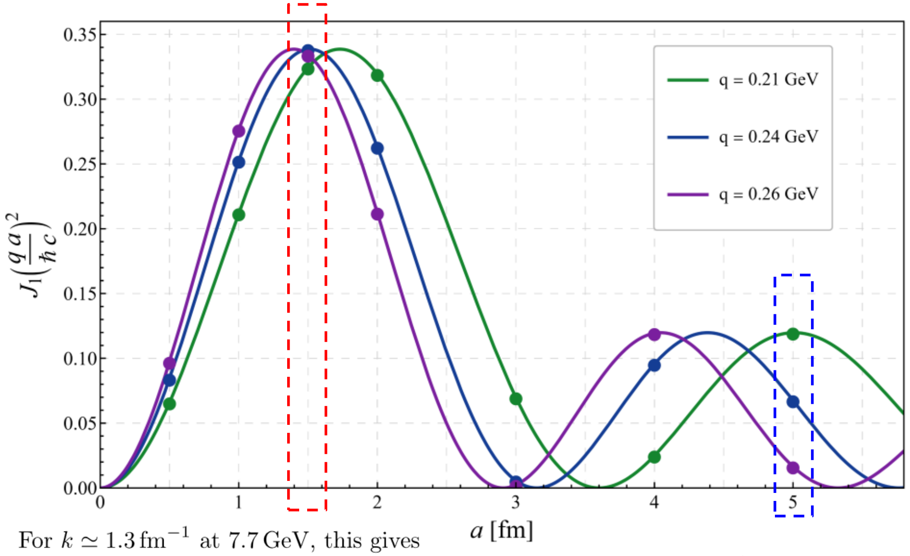
$a = 5$ fm fixed

The side-peak response is non-monotonic in a

- a is the amplitude of the imposed periodic coordinate displacement.
- The side-peak strength approximately follows the Fourier–Bessel response, $\Delta C_2^{(1)} \propto J_1^2(ka)$, so, it does not increase monotonically with a.
- At 7.7 GeV, $k \simeq$ 1.3 fm^{-1}.
  The first maximum of $J_1^2(ka)$ predicts $a_{\mathrm{opt}} \simeq \frac{1.84}{k} \simeq$ 1.4 fm
- The simulation shows the strongest side peak near a =1.5 fm, consistent with this scaling.
- For a = 5 fm, $ka \simeq$6.5, where the Bessel response is suppressed.

The first maximum of $J_1^2(x)$ occurs near

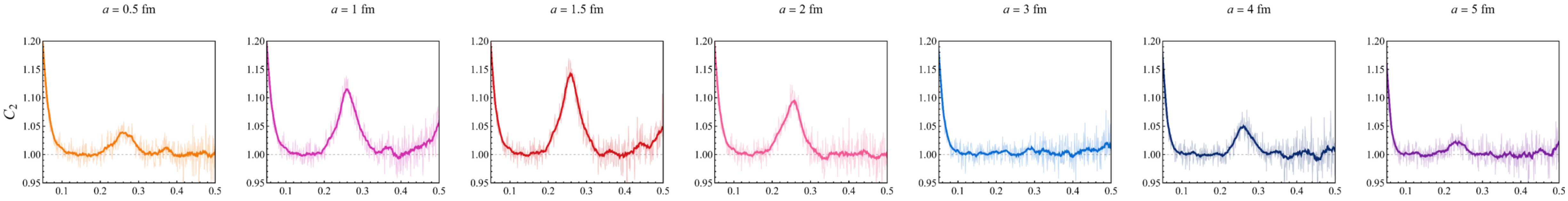
$x_{\max} \simeq 1.84.$



For $k \simeq 1.3\,\mathrm{fm}^{-1}$ at 7.7 GeV, this gives

$a_{\mathrm{opt}} \simeq 1.4\,\mathrm{fm},$

$ka \simeq (1.3\,\mathrm{fm}^{-1})(5\,\mathrm{fm}) \simeq 6.5,$

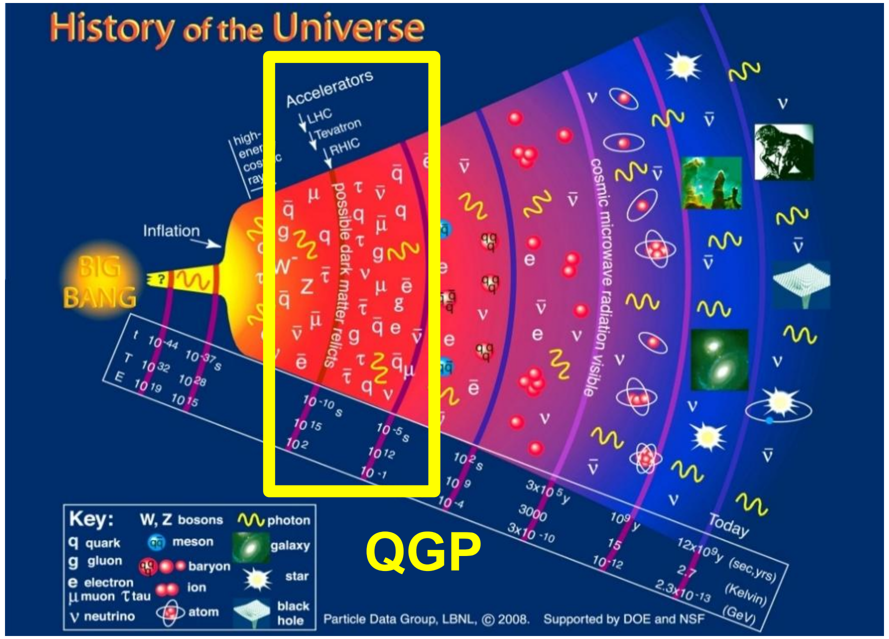


- The weak response at fixed a = 5 fm does not imply a loss of HBT sensitivity.

Conclusion

Conclusions: HBT tells us where to look

- **Heavy-ion collisions create QCD matter under conditions related to the early universe, but detectors observe only the final hadrons.**
  HBT is a Fourier probe of the final emission source, not only a measure of its overall radius.



- **If a pseudo-one-dimensional structure survives, it can produce a side peak at $q_{peak}^{(1)} \simeq \hbar c\, k$ with strength $\propto J_1^2(ka)$.**

- **Linking *k* to μ_B provides a beam-energy-dependent experimental search strategy.**

- **A side peak would be an indirect signature of spatial substructure, not proof of a QCD CSL.**

- **<u>Theory proposes a possible primordial scale; HBT tells experiments where its remnant may appear.</u>**