# Heavy-Quark Expansion Meets Experiment: Heavy-Hadron Lifetimes

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# A BIT OF HISTORY

First **FLAVOUR ANOMALIES** were **connected with lifetimes**:

- 80' - $\tau(D^+)/\tau(D_0) \sim 2.1$ — in 1980's
- 85' - $\tau(D_s)/\tau(D_0) \sim 1.5$ (when $D_s$ was called F ☺ ) — +/- in 1980's
- 90' - $\boldsymbol{\tau(\Lambda_b)/\tau(B) \sim 0.7\text{-}0.8}$ **BREAKING OF HQE??** — NICE EXAMPLE OF AN „ANOMALY" -> 2011 EXPERIMENTAL SOLUTION
- 2000 – WA large → influence on $V_{ub}$ inclusive — / nonperturbative?
- 2018/2020-22 – $\tau(\Omega_c) \sim$ 3-4 times bigger then previously measured — in 2023

# LIFETIMES OF HEAVY HADRONS

testing the heavy-quark expansion

on 24 different weakly decaying particles

- D ($\mathbf{c}q_i$) and B ($\mathbf{b}q_i$) mesons
- c-baryons ($\mathbf{c}q_1q_2$) & b-baryons ($\mathbf{b}q_1q_2$)
- doubly heavy baryons ($\mathbf{cc}q_i$ ,$\mathbf{bb}q_i$ ,$\mathbf{bc}q_i$)
- $B_c$ ($\mathbf{bc}$) meson

# WEAKLY DECAYING CHARMED HADRONS

## c-MESONS

$$D^+(\bar{c}d)$$
$$D^0(\bar{c}u)$$
$$D_s^0(\bar{c}s)$$

Guberina, Nussinov, Peccei, Rückl, PLB 89 (1979) 111
Bilić, Guberina, Trampetić, NPB 248 (1984) 261
Khoze, Shifman, Sov. Phys. Usp. 26 (1983) 387
Shifman, Voloshin, Sov. J. Nucl. Phys. 41 (1985) 120

King, Lenz, Piscopo, Rauh, Rusov, Vlahos, 2109.13219
Gratrex, Melic, Nisandzic, 2204.11935

## c-BARYONS

$$\Lambda_c^+(cud)$$
$$\Xi_c^+(cus)$$
$$\Xi_c^0(cds)$$

$$\Omega_c^0(css)$$

Guberina, Rückl, Trampetić, Z. Phys. C 33 (1986) 297
Shifman, Voloshin, Sov. Phys. JETP 64 (1986) 698
Guberina, Melic, 9704445
H-Y Cheng, 9704260
H-Y Cheng, 1807.00916
H-Y Cheng, C-W Liu, 2305.00665

Gratrex, Melic, Nisandzic, 2204.11935

## cc-BARYONS

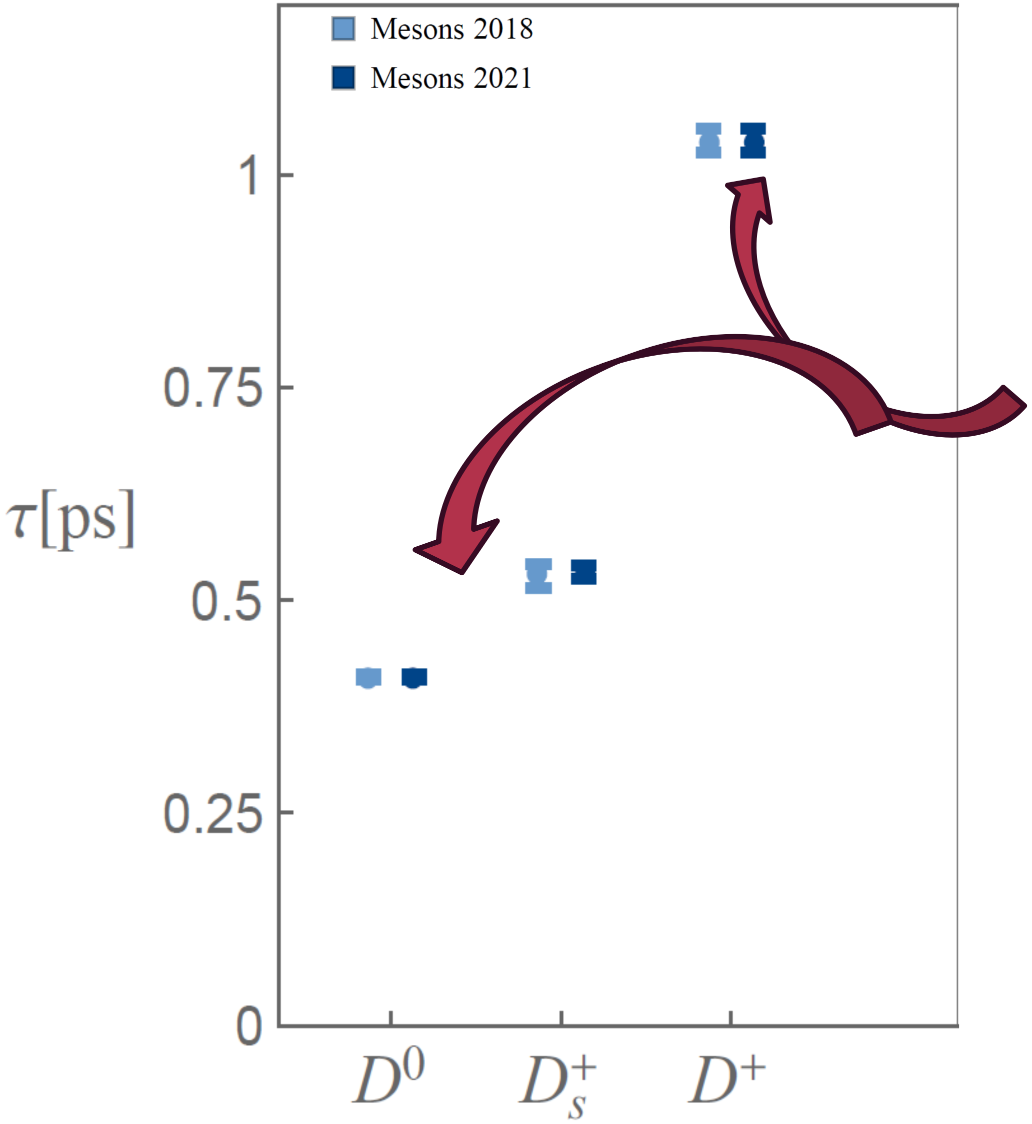
$$\Xi_{cc}^{++}(ccs)$$
$$\Xi_{cc}^+(ccd)$$
$$\Omega_{cc}^+(ccs)$$

Kiselev, Likhoded, Onishchenko, 9807354
Guberina, Melic, Stefancic, 9901323
H-Y Cheng, Y-L Shi, 1809.08102

Dulibic, Gratrex, Melic, Nisandzic, 2305.02243

# EXPERIMENTAL SITUATION – **CHARMED MESONS**

practically unchanged lifetime pattern since 1980‘s



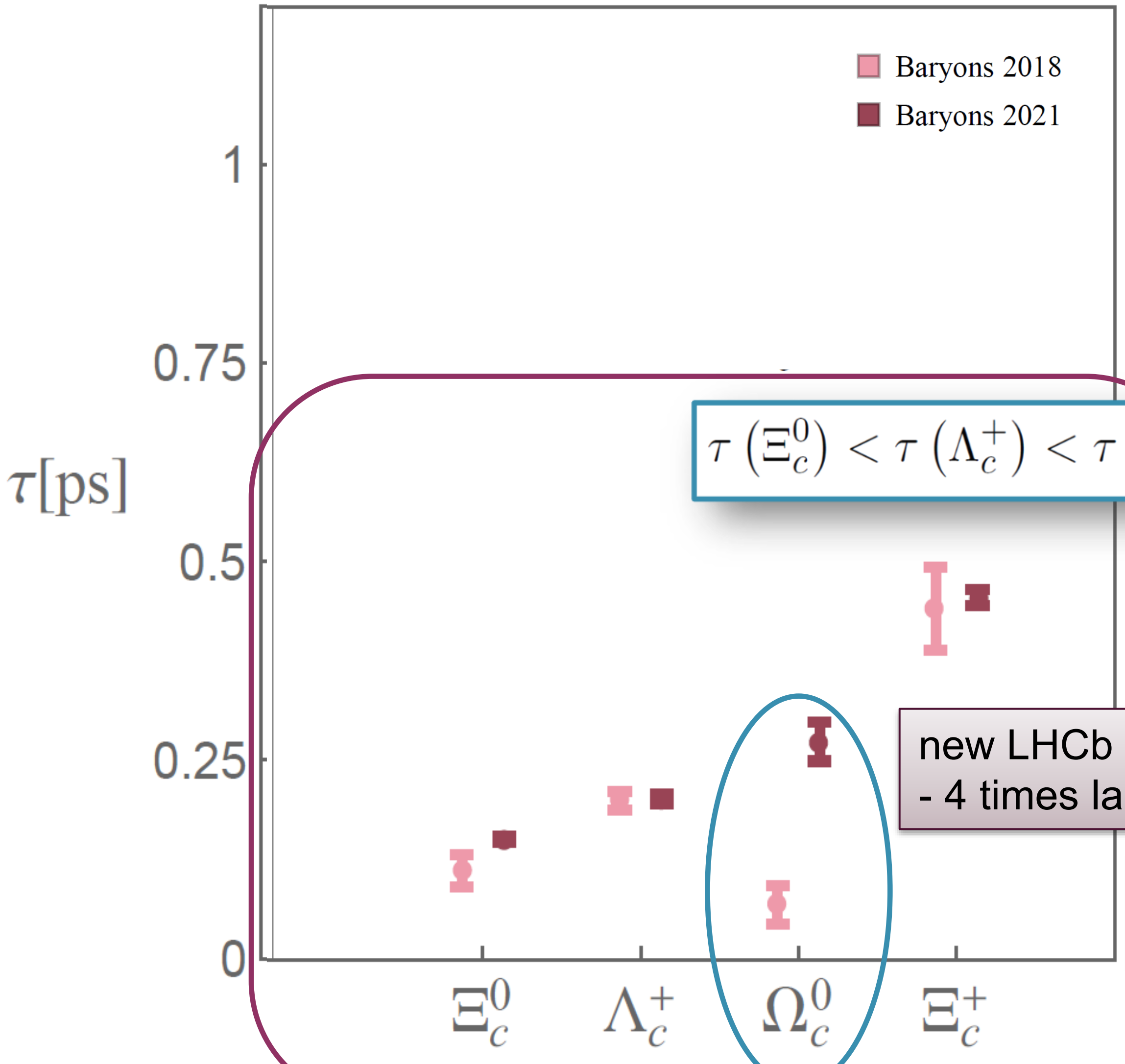

**broad spread of lifetimes of singly CHARMED MESONS:**

$$\boxed{\frac{\tau(D^+)}{\tau(D^0)} = 2.54 \pm 0.02}$$

$$\frac{\tau(D_s^+)}{\tau(D^0)} = 1.23 \pm 0.01$$

# EXPERIMENTAL SITUATION – **CHARMED BARYONS**

lifetime pattern changed 2018/2020

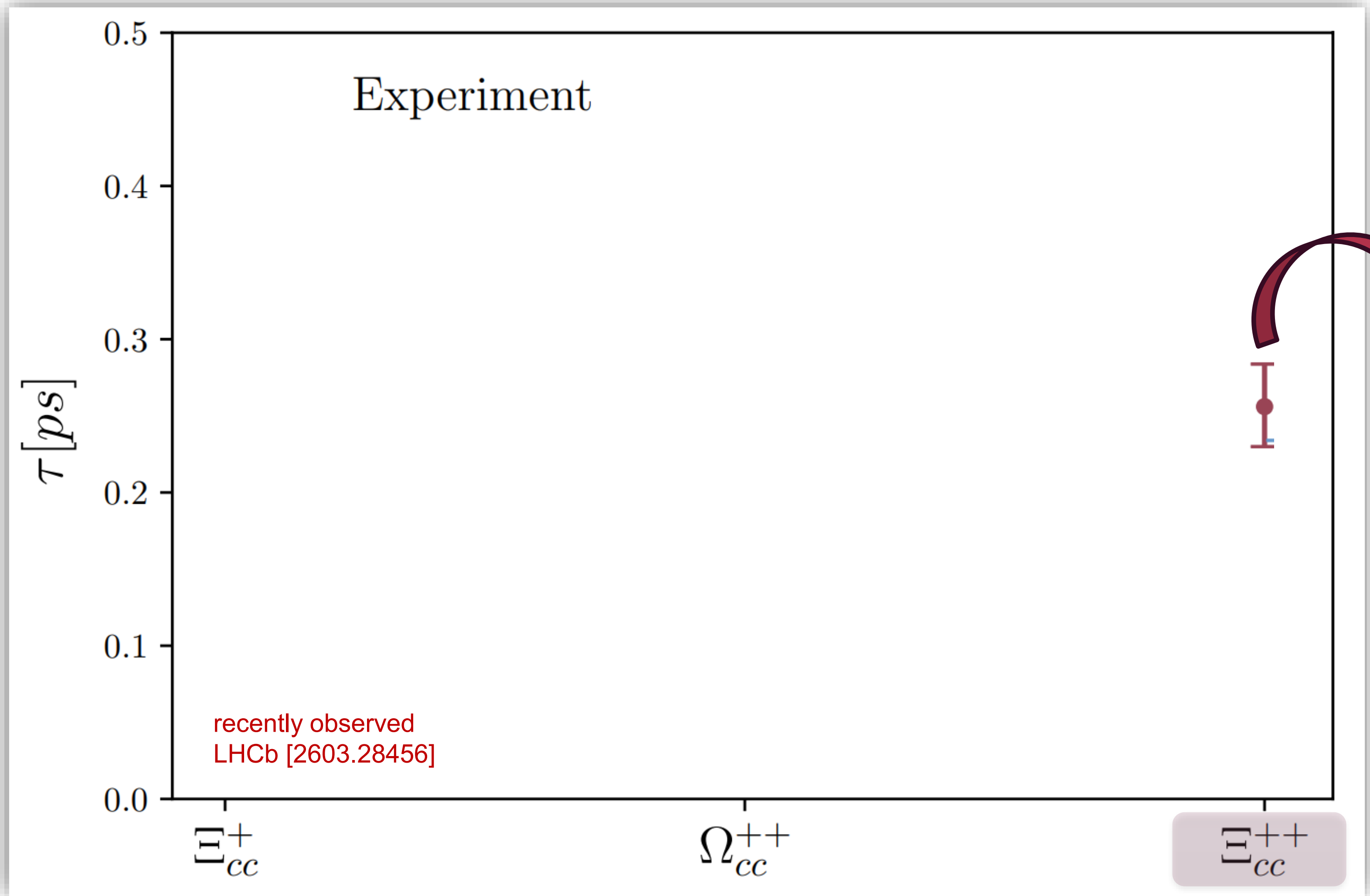



$\tau\left(\Xi_c^0\right) < \tau\left(\Lambda_c^+\right) < \tau\left(\Omega_c^0\right) < \tau\left(\Xi_c^+\right)$

new LHCb measurement 2018/2020
- 4 times larger $\tau(\Omega_c)$ lifetime !

**confirmed** by Belle-II, arXiv: 2208.08573

# EXPERIMENTAL SITUATION – **DOUBLY CHARMED BARYONS**




Naively:

$$\tau(H_{cc}) = \frac{1}{2}\tau(H_c)$$

But it is somewhat larger, since the lifetimes of singly-charmed baryons spans as

$$\tau^{\exp}(H_c) = 0.15 - 0.45\ ps$$

# THEORY : TOTAL DECAY WIDTH ➡ LIFETIMES

$$\frac{1}{\tau(H)} = \Gamma(H) = \frac{1}{2m_H}\langle H|\mathcal{T}|H\rangle$$

Shifman, Voloshin 85

$$\mathcal{T} = \mathrm{Im}\, i \int d^4x\, T\left[\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\right]$$

forward-scattering amplitude

$\mathcal{H}_{eff}$ = weak effective hamiltonian for a heavy Q decay

Buchalla, Buras, Lauternbacher 96

$$\mathcal{H} = \frac{G_F}{\sqrt{2}}\Bigg[\sum_{q,q'=d,s} V_{cq}V^*_{uq'}\left(C_1(\mu)Q_1^{(qq')} + C_2(\mu)Q_2^{(qq')}\right) - V_{ub}V^*_{cb}\sum_{k=3}^{6} C_k(\mu)Q_k$$

$$+ \sum_{\substack{q=d,s \\ \ell=e,\mu}} V_{cq}Q^{(q\ell)}\Bigg]$$

neglected for charm decays

non-leptonic(NL) and semileptonic (SL) decays included

# WEAK HAMILTONIAN DIM-6 and DIM-7 OPERATORS

**Dim 6 operators:**

$$Q_1^{(qq')} = (\bar{c}^i \gamma_\mu (1-\gamma_5) q^j)(\bar{q}'^j \gamma^\mu (1-\gamma_5) u^i)\,,$$

$$Q_2^{(qq')} = (\bar{c}^i \gamma_\mu (1-\gamma_5) q^i)(\bar{q}'^j \gamma^\mu (1-\gamma_5) u^j)\,,$$

$$Q_{\rm SL}^{(q\ell)} = (\bar{c} \gamma_\mu (1-\gamma_5) q)(\bar{\ell} \gamma^\mu (1-\gamma_5) \nu_\ell)\,,$$

+ color-octet operators

+ penguin operators

+ μ-running and mixing

---

**Dim 7 operators:**

$$P_1^q = m_q (\bar{c}_i (1-\gamma_5) q_i)(\bar{q}_j (1-\gamma_5) c_j)\,,$$

$$P_2^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu (1-\gamma_5) D^\rho q_i)(\bar{q}_j \gamma^\mu (1-\gamma_5) c_j)\,,$$

$$P_3^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho q_i)(\bar{q}_j (1+\gamma_5) c_j)\,,$$

$$S_1^q = m_q (\bar{c}_i (1-\gamma_5) t^a_{ij} q_j)(\bar{q}_k (1-\gamma_5) t^a_{kl} c_l)\,,$$

$$S_2^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu (1-\gamma_5) t^a_{ij} D^\rho q_j)(\bar{q}_k \gamma^\mu (1-\gamma_5) t^a_{kl} c_l)\,,$$

$$S_3^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho (1-\gamma_5) t^a_{ij} D^\rho q_j)(\bar{q}_k (1+\gamma_5) t^a_{kl} c_l)\,.$$

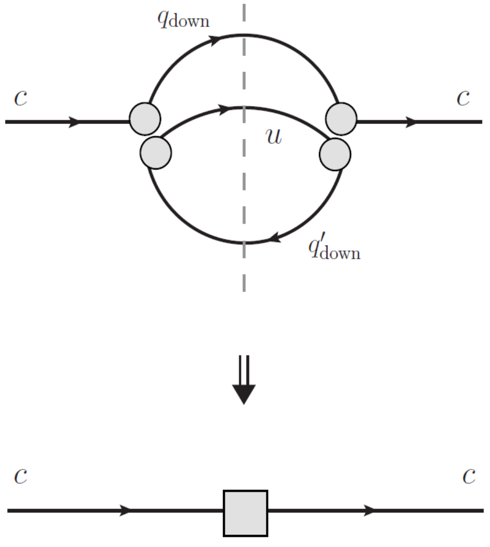
+ color-octet operators

**+ non-local operators** - *reabsorbed into* dim6 matrix elements (proven for mesons)

# HEAVY QUARK EXPANSION (HQE) – systematic expansion in $\Lambda_{QCD}/m_Q$ and $\alpha_S$

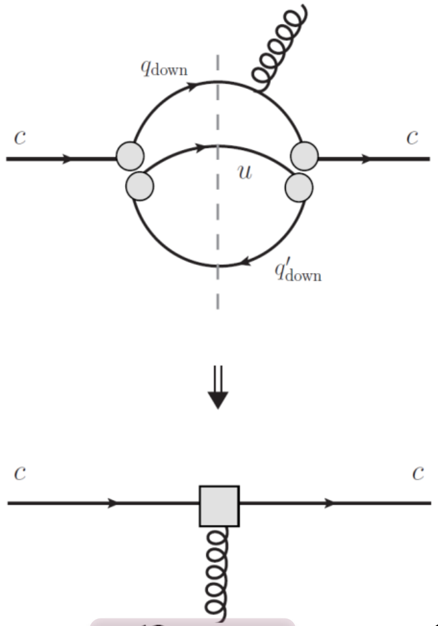
$$\mathcal{T} = \left( \mathcal{C}_3 \mathcal{O}_3 + \underbrace{\frac{\mathcal{C}_5}{m_Q^2}\mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3}\mathcal{O}_6}_{} + \ldots \right) + 16\pi^2 \left( \underbrace{\frac{\tilde{\mathcal{C}}_6}{m_Q^3}\tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4}\tilde{\mathcal{O}}_7}_{} + \ldots \right)$$

**TWO-QUARK LEADING**
NON-SPECTATOR
CONTRIBUTION



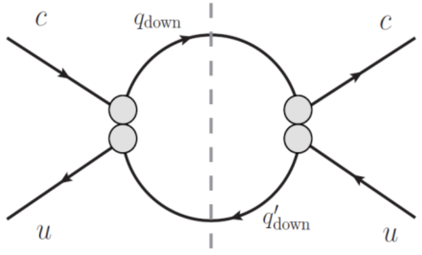

$\mathcal{O}_3$

**TWO-QUARK NON-LEADING**
NON-SPECTATOR
CONTRIBUTION **- from 90-ies; HQE**



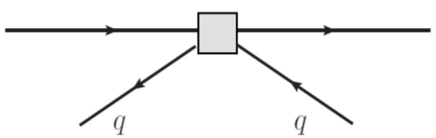

$\mathcal{O}_5$; $\mathcal{O}_6$ etc

**FOUR-QUARK SPECTATOR (w light quarks)**
CONTRIBUTIONS **– ONE –LOOP ENHANCED**




**light-quarks come into game**

$\Downarrow$




$\tilde{\mathcal{O}}_6$ ; $\tilde{\mathcal{O}}_7$ etc

# CALCULATION

$$\mathcal{T} = \boxed{\left(\mathcal{C}_3\mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2}\mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3}\mathcal{O}_6 + \ldots\right)} + \boxed{16\pi^2\left(\frac{\tilde{\mathcal{C}}_6}{m_Q^3}\tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4}\tilde{\mathcal{O}}_7 + \ldots\right)}$$

$$\Gamma(H) = \frac{1}{2m_H}\langle H|\mathcal{T}|H\rangle$$

**WILSON COEFFICIENTS** – perturbative part: $\mathcal{C}_i = \mathcal{C}_i^{(0)}(\mu,\mu_0) + \mathcal{C}_i^{(1)}(\mu,\mu_0)\alpha_s(\mu) + \mathcal{C}_i^{(2)}(\mu,\mu_0)\alpha_s(\mu)^2 + \ldots,$

**MATRIX ELEMENTS** OF VARIOUS $\mathcal{O}$ OPERATORS ARE NEEDED – nonperturbative part:

NON-SPECTATOR PART:

SPECTATOR PART:

$$\Gamma(H) = \Gamma_0\left[c_3 + \frac{c_\pi \boxed{\mu_\pi^2} + c_G \boxed{\mu_G^2}}{m_Q^2} + \frac{c_\rho \boxed{\rho_D^3}}{m_Q^3} + \ldots\right] + \frac{16\pi^2}{2m_H}\left(\sum_{i,q}\frac{c_{6,i}^q \boxed{\langle H|O_i^q|H\rangle}}{m_Q^3} + \sum_i \frac{c_{7,i}^q \boxed{\langle H|P_i^q|H\rangle}}{m_Q^4} + \ldots\right.$$

$\Gamma_0 \sim \boxed{m_Q^5}$

$$\mu_\pi^2(H) = \frac{-1}{2m_H}\langle H|\bar{c}_v(iD)^2 c_v|H\rangle\,,$$

kinetic parameter

$$\mu_G^2(H) = \frac{1}{2m_H}\langle H|\bar{c}_v\frac{1}{2}\sigma\cdot(g_sG)c_v|H\rangle\,,$$

chromomagnetic parameter

$$\rho_D^3(H) = \frac{1}{2m_H}\langle H|\bar{c}_v(iD_\mu)(iv\cdot D)(iD^\mu)c_v|H\rangle$$

Darwin term

$$\langle H|O_i^q|H\rangle$$

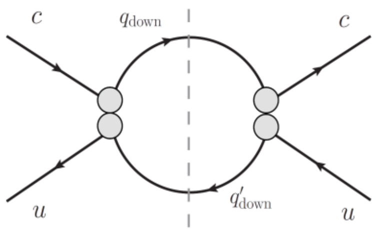
$$\langle H|P_i^q|H\rangle$$

four-quark matrix elements

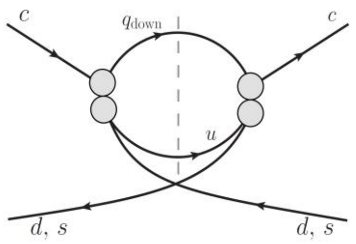
# FOUR-QUARK SPECTATOR (u,d,s) CONTRIBUTIONS ARE LEADING!

one-loop i.e $16\pi^2$ enhanced, although $1/m^3$ (dim6), $1/m^4$ (dim7) suppressed

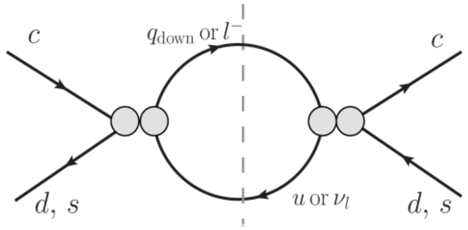
## MESONS




**WE**
weak exchenge

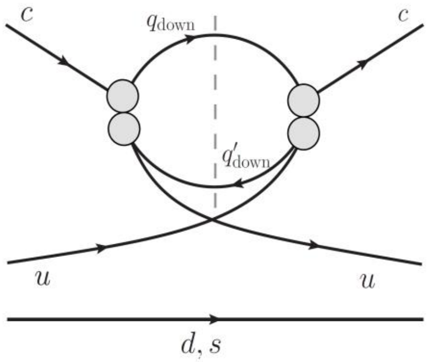



**PI**
Pauli interference

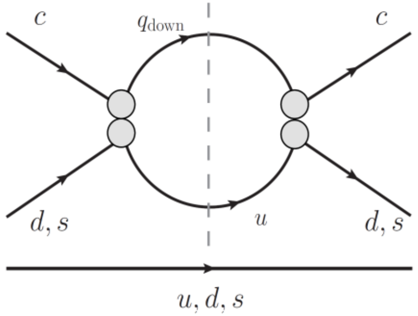



**WA**
weak annihilation

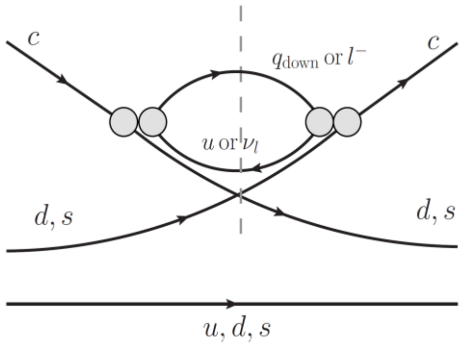
## BARYONS




**int-**
negative interference




**exc**
exchange




**int+**
positive interference

| decay \ $H_c$ | CE NL $c \to s\bar{d}u$ | CE SL $c \to s\bar{l}\nu_l$ |
|---|---|---|
| $\bar{D}^0(u\bar{c})$ | $\tilde{\Gamma}_{\rm WE}$ | - |
| $D^-(d\bar{c})$ | $\tilde{\Gamma}_{\rm PI}$ | - |
| $D_s^-(s\bar{c})$ | $\tilde{\Gamma}_{\rm WA}$ | $\tilde{\Gamma}^{\rm SL}_{\rm WA}$ |
| $\Lambda_c^+(udc)$ | $\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int-}$ | - |
| $\Xi_c^+(usc)$ | $\tilde{\Gamma}_{\rm int-} + \tilde{\Gamma}_{\rm int+}$ | $\tilde{\Gamma}^{\rm SL}_{\rm int+}$ |
| $\Xi_c^0(dsc)$ | $\tilde{\Gamma}_{\rm exc} + \tilde{\Gamma}_{\rm int+}$ | $\tilde{\Gamma}^{\rm SL}_{\rm int+}$ |
| $\Omega_c^0(ssc)$ | $\tilde{\Gamma}_{\rm int+}$ | $\tilde{\Gamma}^{\rm SL}_{\rm int+}$ |

CE = leading; Cabibbo enhanced

* effects are different in different mesons
* effects are different in different baryons
* no helicity suppression for baryons – **different lifetimes**
* effects in SL decays – **different SL BRs**

# PECULARITIES OF **DOUBLY-CHARMED BARYONS**

**Difference to singly-charmed baryons:**

- two c quarks decaying

- diquark system – **cc-pair** $\mathcal{B}_{cc} \sim (cc)q$
  (instead the light diquark $q_1q_2$-pair in singly-charmed baryons)

Additional contributions to some of the matrix elements, e.g :

$$\mu_G^2(\mathcal{B}_{cc}) \rightarrow \mu^2_{G\boxed{(D-q)}}(\mathcal{B}_{cc}) + \mu^2_{G\boxed{(c-c)}}(\mathcal{B}_{cc})$$

, similarily for $\mu_\pi^2(\mathcal{B}_{cc})$

Additional contributions accessed by NRQCD expansion (up to O($v^7$), $\Psi_c$ = 2-component NR spinor):

$$\bar{c}c = \bar{\Psi}_c\Psi_c - \frac{1}{2m_c^2}\bar{\Psi}_c(i\vec{D})^2\Psi_c + \frac{3}{8m_c^4}\bar{\Psi}_c(i\vec{D})^4\Psi_c - \frac{1}{2m_c^2}\bar{\Psi}_c\, g_s\vec{\sigma}\cdot\vec{B}\Psi_c - \frac{1}{4m_c^3}\bar{\Psi}_c\, g_s(\vec{\mathcal{D}}\cdot\vec{E})\Psi_c + \ldots$$

matrix elements, e.g.:

$$\frac{\langle\mathcal{B}_{cc}|\Psi_c^\dagger g_s\vec{\sigma}\cdot\vec{B}\Psi_c|\mathcal{B}_{cc}\rangle}{2M_{\mathcal{B}_{cc}}}\Big|_{c-c} = \frac{4}{9}\frac{g_s^2}{m_c}|\Psi_{cc}(0)|^2$$

where $|\Psi_{cc}(0)|^2 \neq \left(|\Psi_{\bar{c}c}(0)|^2 \equiv |\Psi_{J/\psi}(0)|^2\right)$

# CHARM QUARK MASS

$m_c$ – major input parameter; the decay rate starts as $m_c^5$

| $\overline{m}_c(\overline{m}_c) = 1.28$ GeV | 1-loop | 2-loop | 3-loop | 4-loop |
|---|---|---|---|---|
| $m_c^{\text{pole}}$ | 1.49 | 1.68 | 1.95 | 2.43 |
| $m_c^{\text{kin}}$ | 1.36 | 1.39 | 1.40 | - |
| $m_c^{\text{MSR}}$ | 1.33 | 1.35 | 1.36 | 1.36 |

bad convergence for $m_c$ / much better for for $m_b$
however pole mass is non-physical - renormalons

we provide results for different mass schemes... at the end no large differences in the final results – rearrangements among $1/m_c$ and $\alpha_S$-expansion !

# LIFETIMES OF **SINGLE BEAUTY HADRONS**

## DIFFERENCES CHARM vs BEAUTY:

Lifetimes of hadrons containing a *b* quark provide stringent tests of the heavy quark expansion (HQE)

- convergence in $1/m_b$ is better known (HQE is working better)
- matrix elements are better known (HQET is applicable)
- penguin operators need to be included
- in the bottom sector, the achieved precision of lifetime observables is impressive, at percentage level, in particular in theoretically cleaner quantities such as lifetime ratios

# LIFETIMES OF **DOUBLY HEAVY HADRONS**

## DIFFERENCES DOUBLY CHARMED cc-BARYONS vs DOUBLY BEAUTY bb- AND bc-HADRONS:

- **DOUBLY HEAVY HADRONS** provide an additional important testing ground for the HQE, as their lifetimes are gradually becoming experimentally accessible.

$$\mathcal{B}_{cc} = (\Xi_{cc}^{++}, \Xi_{cc}^{+}, \Omega_{cc}^{+}) \longrightarrow \mathcal{B}_{bb} = (\Xi_{bb}^{0}, \Xi_{bb}^{-}, \Omega_{bb}^{-})$$ bb - diquark $S_{QQ} = 1$

$$\left.\begin{array}{l} \mathcal{B}_{bc} = (\Xi_{bc}^{+}, \Xi_{bc}^{0}, \Omega_{bc}^{0}) \\ \mathcal{B}_{bc}' \equiv (\Xi_{bc}'^{+}, \Xi_{bc}'^{0}, \Omega_{bc}'^{0}) \end{array}\right\}$$ bc- diqark $S_{bc} = 0 \text{ or } S_{bc} = 1$

(scalar or vector)
still undetermined

lifetime predictions may help distinguish the nature of the (bcq) ground states

- **$B_c$ MESON** is a peculiar meson, built of two heavy quarks which both decay

# RESULTS

# **CHARMED MESON** LIFETIMES

Lifetime ratios :

$$\frac{\tau(D^+_{(s)})}{\tau(D^0)} = 1 + \left(\Gamma^{\rm th}(D^0) - \Gamma^{\rm th}(D^+_{(s)})\right)\tau^{\rm exp}(D^+_{(s)})$$

- some uncertainties cancel in the ratios

Inclusive SL branching ratios ( $e$ only ) :

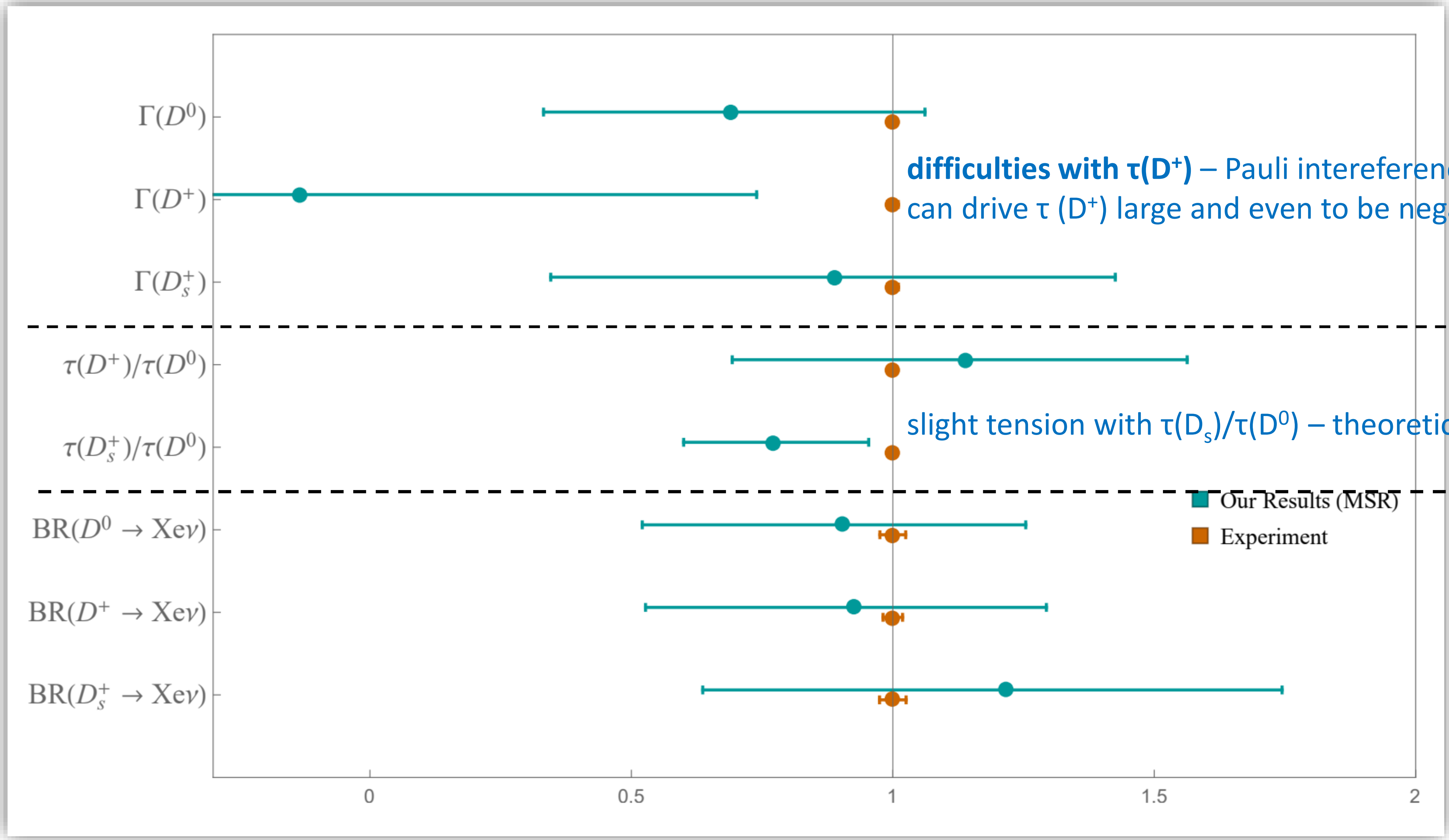
$$BR^{(e)}(D) = \Gamma^{(e)}(D)\tau^{\rm exp}(D)$$

$$\frac{\Gamma^{(e)}(D^+_{(s)})}{\Gamma^{(e)}(D^0)} = 1 + (\Gamma^{(e)\,\rm th}(D^+_{(s)}) - \Gamma^{(e)\,\rm th}(D^0))\left(\frac{\tau(D^0)}{BR^{(e)}(D^0)}\right)^{\rm exp}$$

Our Results (MSR)
Experiment

Gratrex, Melic,Nisandzic, 2204.11935
King, Lenz, Piscopo, Rauh, Rusov, 2109.13219

results are largely compatible with the experiment




**difficulties with $\tau(D^+)$** – Pauli intereference term can drive $\tau(D^+)$ large and even to be negative!

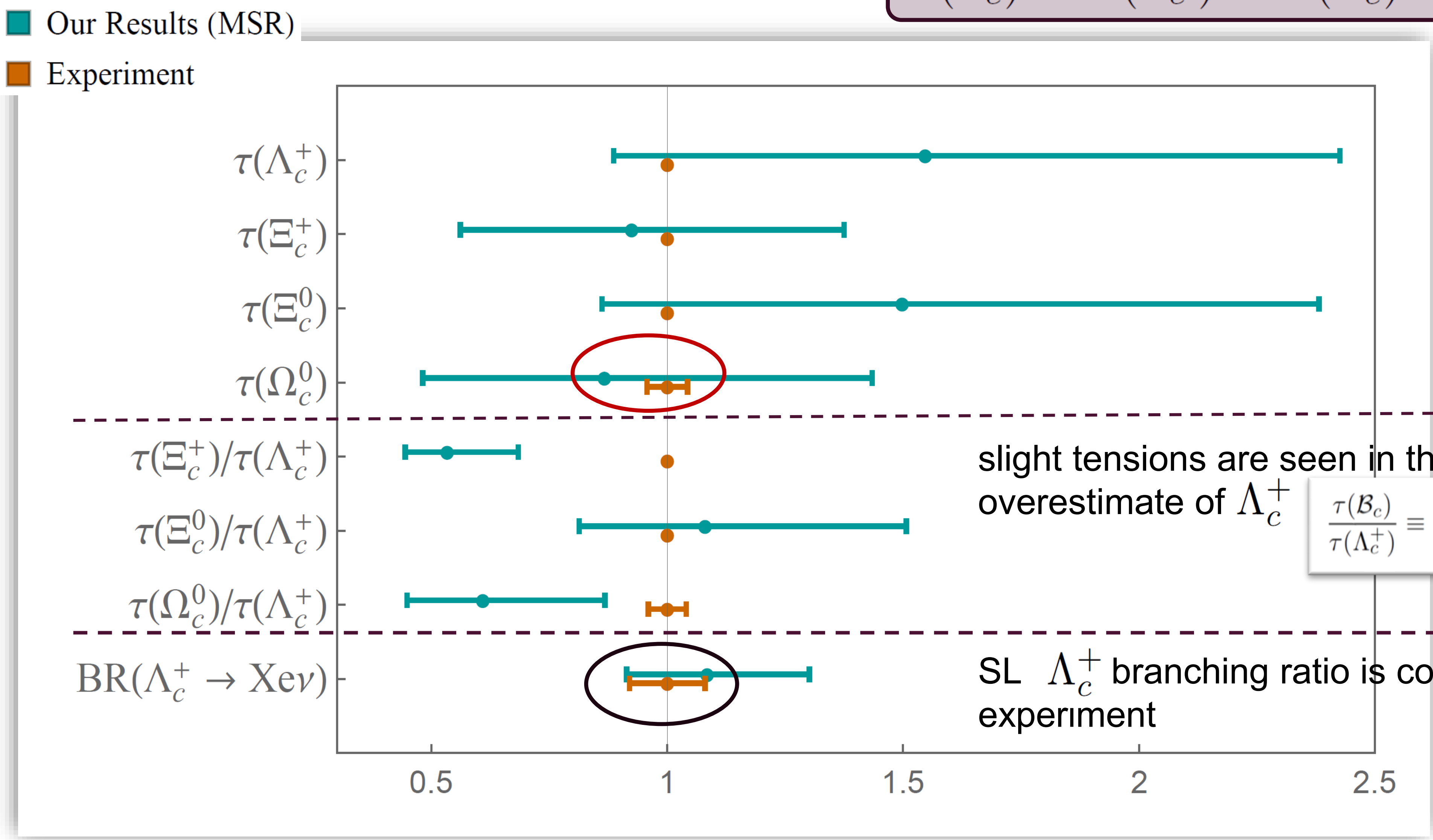
slight tension with $\tau(D_s)/\tau(D^0)$ – theoretically closer to 1

# CHARMED BARYON LIFETIMES

Gratrex, Melic, Nisandzic, 2204.11935

$$\tau\left(\Xi_c^0\right) < \tau\left(\Lambda_c^+\right) < \tau\left(\Omega_c^0\right) < \tau\left(\Xi_c^+\right)$$

In agreement with Belle-II, aug.2022




individual lifetimes compatible with the experiment, even $\Omega_c^0$ !

slight tensions are seen in the ratios due to our overestimate of $\Lambda_c^+$

$$\frac{\tau(\mathcal{B}_c)}{\tau(\Lambda_c^+)} \equiv \frac{1}{1+(\Gamma^{\rm th}(\mathcal{B}_c)-\Gamma^{\rm th}(\Lambda_c^+))\tau^{\rm exp}(\Lambda_c^+)}$$

SL $\Lambda_c^+$ branching ratio is compatible with the experiment

# CHARMED BARYON SL BRs

$$BR(\mathcal{B}_c \to Xe\nu) \equiv \Gamma(\mathcal{B}_c \to Xe\nu)\, \tau^{\exp}(\mathcal{B}_c)$$

MSR $m_c$ mass scheme:

| | |
|---|---|
| $BR(\Lambda_c^+ \to Xe\nu)/\%$ | $4.28^{+0.47+0.39}_{-0.37-0.30}$ |
| $BR(\Xi_c^+ \to Xe\nu)/\%$ | $14.95^{+2.66+1.59}_{-2.45-1.50}$ |
| $BR(\Xi_c^0 \to Xe\nu)/\%$ | $5.06^{+0.91+0.54}_{-0.84-0.51}$ |
| $BR(\Omega_c^0 \to Xe\nu)/\%$ | $11.19^{+3.01+1.94}_{-2.89-2.09}$ |

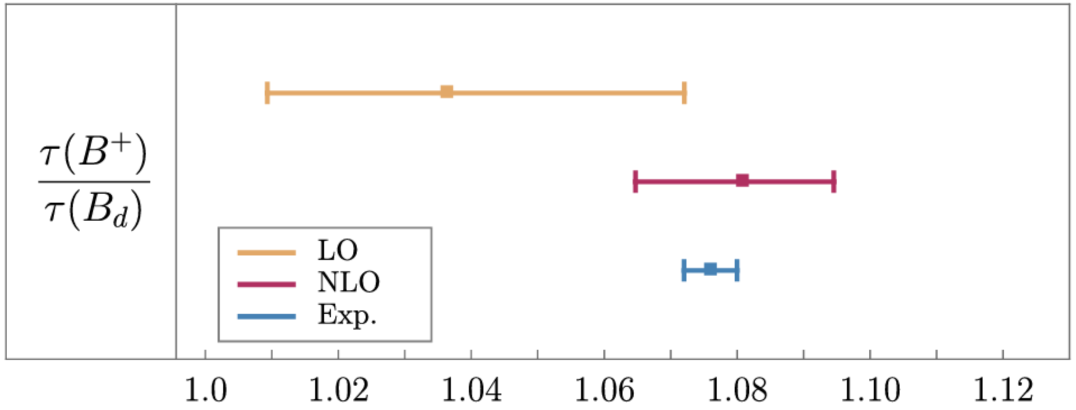
SL decays are important to assess the validity of HQE in charmed baryons

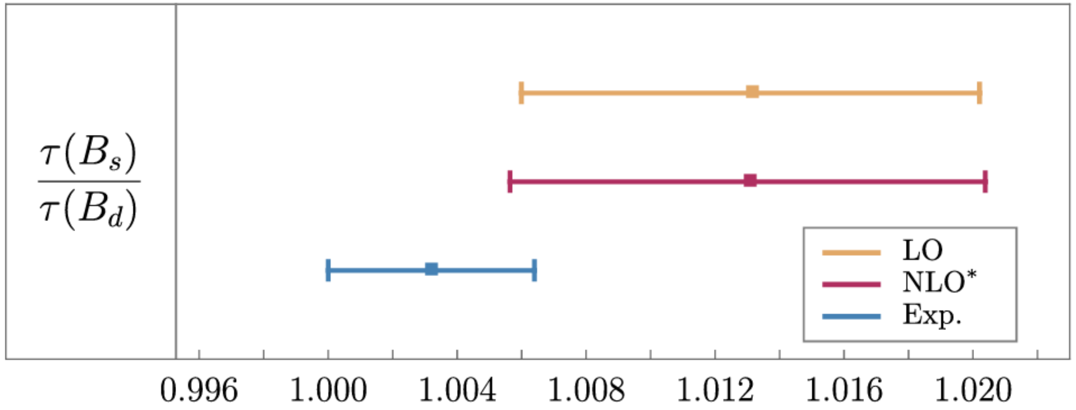
- experimental measurements of $BR_{SL}(\Xi_c^+)$, $BR_{SL}(\Xi_c^0)$ and $BR_{SL}(\Omega_c^0)$ are needed

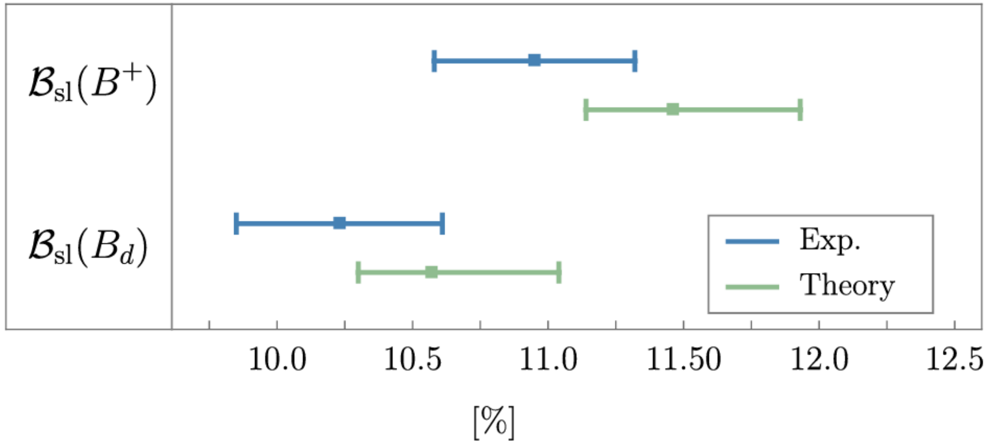
# BEAUTY MESON LIFETIMES

Egner, Fael, Lenz, Piscopo, Rusov, Schoenwald, Steinhauser :2412.14035 – NNLO improvements

impressive theoretical precision achieved:



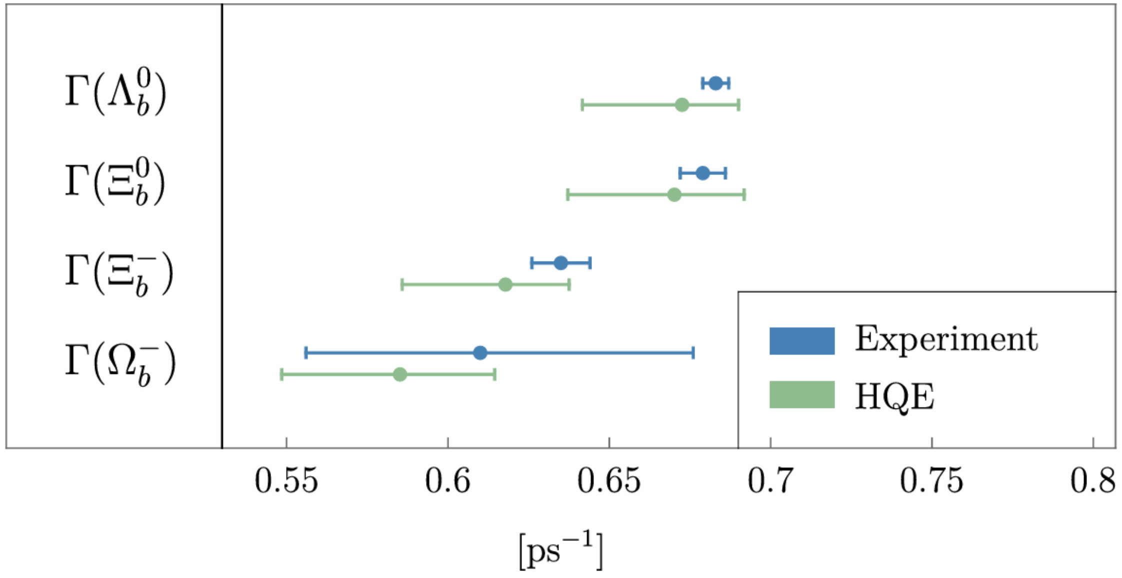



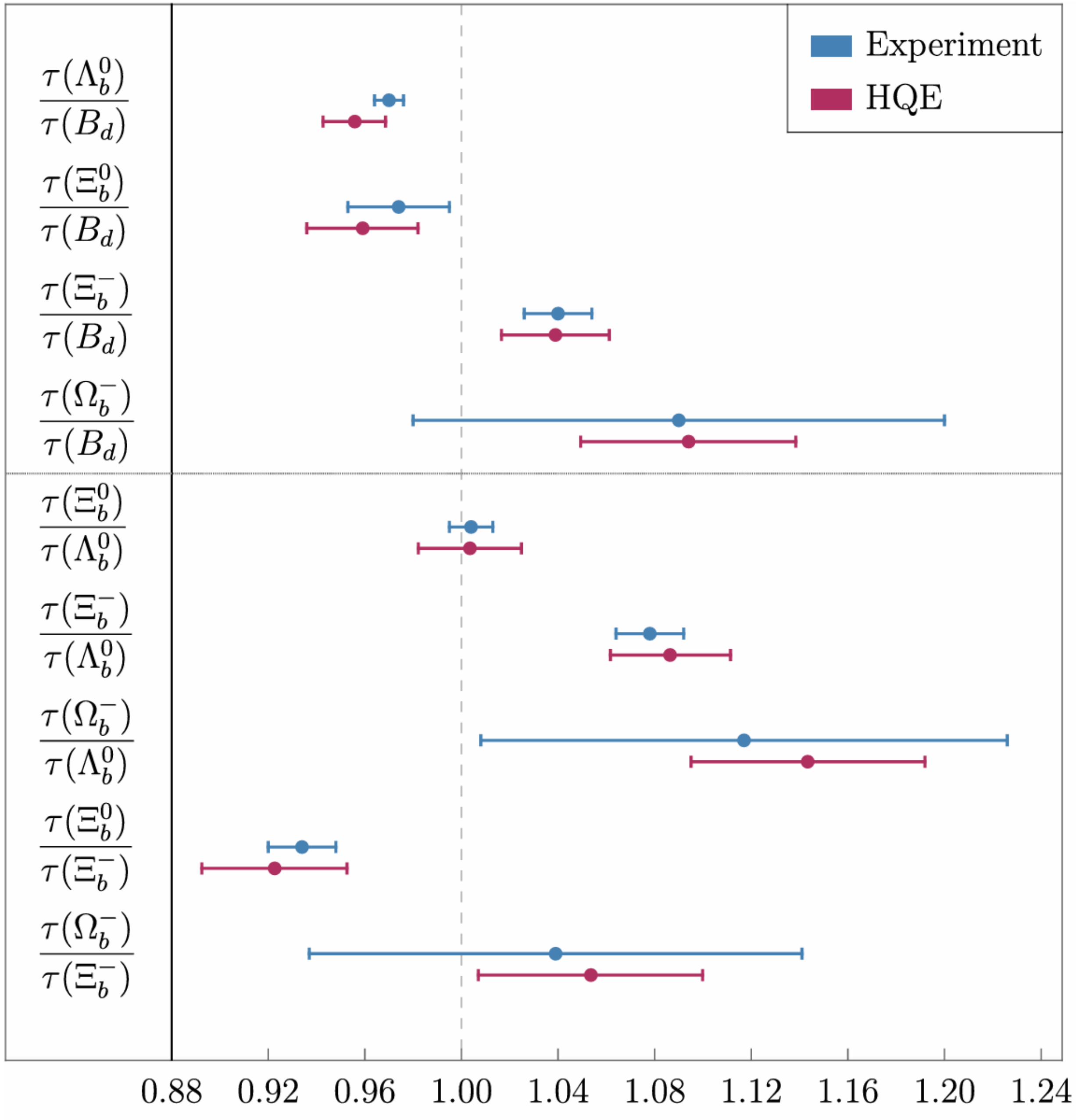



# BEAUTY BARYON LIFETIMES

Gratrex, Lenz, Melić, Nišandžić, Piscopo, Rusov, 2301.07698
Lenz, Piscopo, Rusov, 2607.06538 – NNLO improvements

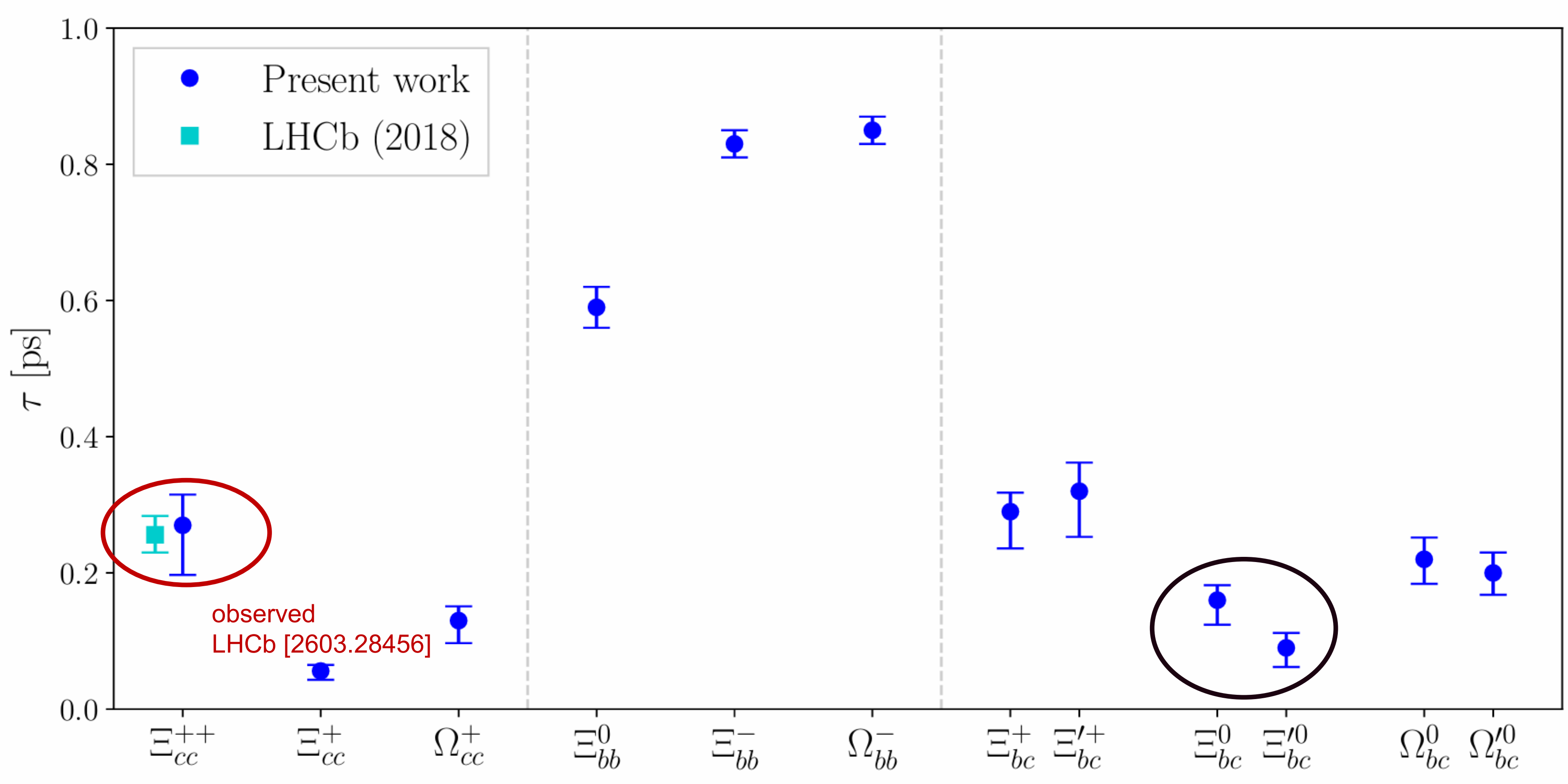
impressive theoretical precision achieved:



# **DOUBLY HEAVY** HADRON LIFETIMES

Dulibic, Gratrex, Melic, Nisandzic 2305.02243
Dulibic, Melic, Nisandzic 2605.04967




$\tau(\Xi_{cc}^{+}) < \tau(\Omega_{cc}^{+}) < \tau(\Xi_{cc}^{++})$

$\tau(\Xi_{bb}^{0}) < \tau(\Xi_{bb}^{-}) \simeq \tau(\Omega_{bb}^{-})$

$\tau(\Xi_{bc}^{0}) \lesssim \tau(\Omega_{bc}^{0}) < \tau(\Xi_{bc}^{+})$

$\tau(\Xi_{bc}^{\prime 0}) < \tau(\Omega_{bc}^{\prime 0}) < \tau(\Xi_{bc}^{\prime +})$

$\tau(\Xi_{cc}^{++})$ is the only measured doubly-heavy baryon lifetime (LHCb 2018) - good agreement

the predicted lifetime of $\Xi^{0}_{bc}$ is significantly larger than that of $\Xi'^{\,0}_{bc}$, with the difference exceeding the quoted theoretical uncertainties.

# $B_c$ MESON LIFETIME

EXP: $\tau(B_c) = 0.510 \pm 0.009$ ps

**Our predictions:**

| Decay widths | $\overline{\text{MS}}$ |
|---|---|
| $\bar{b} \to \bar{c}u(\bar{s}+\bar{d})$ | $0.31^{+0.25}_{-0.02}$ |
| $\bar{b} \to \bar{c}c(\bar{s}+\bar{d})$ | $0.15^{+0.07}_{-0.01}$ |
| $\bar{b} \to c(e+\mu+\tau)\nu$ | $0.12^{+0.10}_{-0.00}$ |
| $\boxed{\sum \bar{b} \to \bar{c}}$ | $\boxed{0.58^{+0.42}_{-0.02}}$ |
| $c \to (s+d)u(\bar{d}+\bar{s})$ | $0.55^{+0.56}_{-0.15} + (0.75^{+0.31}_{-0.10})\vert_{\rho^3_{D,c}}$ |
| $c \to (s+d)(e+\mu)\nu$ | $0.18^{+0.09}_{-0.03} + (0.29^{+0.03}_{-0.01})\vert_{\rho^3_{D,c}}$ |
| $\boxed{\sum c \to s}$ | $\boxed{0.72^{+0.65}_{-0.18} + (1.05^{+0.33}_{-0.12})\vert_{\rho^3_{D,c}}}$ |
| WA: $\bar{b}c \to c(\bar{s}+\bar{d})$ | $0.12^{+0.13}_{-0.03}$ |
| WA: $\bar{b}c \to \tau\nu$ | $0.03$ |
| PI | $-0.11^{+0.04}_{-0.17}$ |
| $\Gamma_{B_c}$ | $1.35 \pm 0.02^{+1.03}_{-0.19}$ $+(1.05 \pm 0.21^{+0.33}_{-0.12})\vert_{\rho^3_{D,c}}$ |

$$\tau^{\text{kin}}(B_c) = 0.293 \pm 0.022^{+0.019}_{-0.047} \text{ ps}$$

too small lifetime predicted

Interestingly, dropping the unusually large Darwin contribution results with the value comparable with the experiment:

$$\tau^{\text{kin}}_{\text{excl.}\,\rho_{D,c}}(B_c) = 0.463 \pm 0.009^{+0.029}_{-0.075}$$

# HEAVY HADRON LIFETIMES - CONCLUSIONS

Up-to-date results for lifetimes of ALL weakly decaying hadrons with a single charm/bottom quark and with two heavy quarks is provided

- results compatible with experiment
  - B-mesons and b-baryons with the inclusion of NNLO in the decay have achieved remarkable precision;
  - still large uncertainties for those particles involving charm quark decay

- recent LHCb (2018/20) and Belle-II (8/2022) result for $\tau(\Omega^0_c)$ lifetime (~ 4× bigger than old measurements) confirmed theoretically

- reasonably controlled description of the lifetimes of all doubly heavy hadrons; measurements of remaining cc-baryon lifetimes, $\tau(\Xi_{cc}^{+})$ and $\tau(\Omega_{cc}^{+})$ at LHCb Run3 are feasible

- predictions for unmeasured semileptonic $BR_{SL}(H)$ are important for complete assessment

- $B_c$-meson lifetime is not compatible with the experiment – need more theory investigation

conclusions above are largely independent of the charm/bottom-quark mass scheme

# FINAL REMARKS:

**HEAVY QUARK EXPANSION work for both, bottom and charm decays** and lifetimes provide valuable tests of the HQE, however,

- there is a need for inclusion of higher order corrections and even QED effects for b-decays
  / dim 7 operator ($1/m_c^4$) contributions were essential for showing compatibility of $\tau(\Omega^0_c)$ with experiment
  / Darwin term ($1/m_c^3$) appeared to be quite large, in the prediction of $B_c$ lifetime even unusually large

- improvement of the nonperturbative input/matrix elements is needed to reduce uncertainties even more

**HQE is a powerful framework for inclusive heavy hadron decays!**