Flavor Alignment and Mass Hierarchy

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Contents

Two Standard Model Puzzles

particle	$SU(3)$	$SU(2)$	$U(1)$
$(u,d)_L$	3	2	1/6
u_R, d_R	3	1	$2/3, -1/3$
$(\nu, e)_L$	1	2	$-1/2$
e_R	1	1	-1
(ϕ^+, ϕ^0)	1	2	1/2

Quarks and leptons come in three families.

$$\bar{u}_L u_R \bar{\phi}^0 \Rightarrow \mathcal{M}_u = U_L^\dagger \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_R.$$

$$\bar{d}_L d_R \phi^0 \Rightarrow \mathcal{M}_d = D_L^\dagger \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} D_R.$$

$$\text{Observation} \Rightarrow V_{CKM} = U_L^\dagger D_L \sim 1, \ \text{and}$$

$$m_d, m_u < m_s, m_c < m_b, m_t,$$

but how and why?

To understand these two puzzles, physics beyond the standard model (SM) is required. Traditionally, new symmetries are introduced. To support them, new scalars (and fermions) are added, some with vacuum expectation values. Furthermore, nonrenormalizable higher-dimensional terms are often assumed.

In this presentation (arXiv:2606.26319), it will be shown how it is accomplished in a renormalizable field theory with only additions in the dark sector, and just the one SM Higgs doublet [MonoHiggsology].

Dark Matter

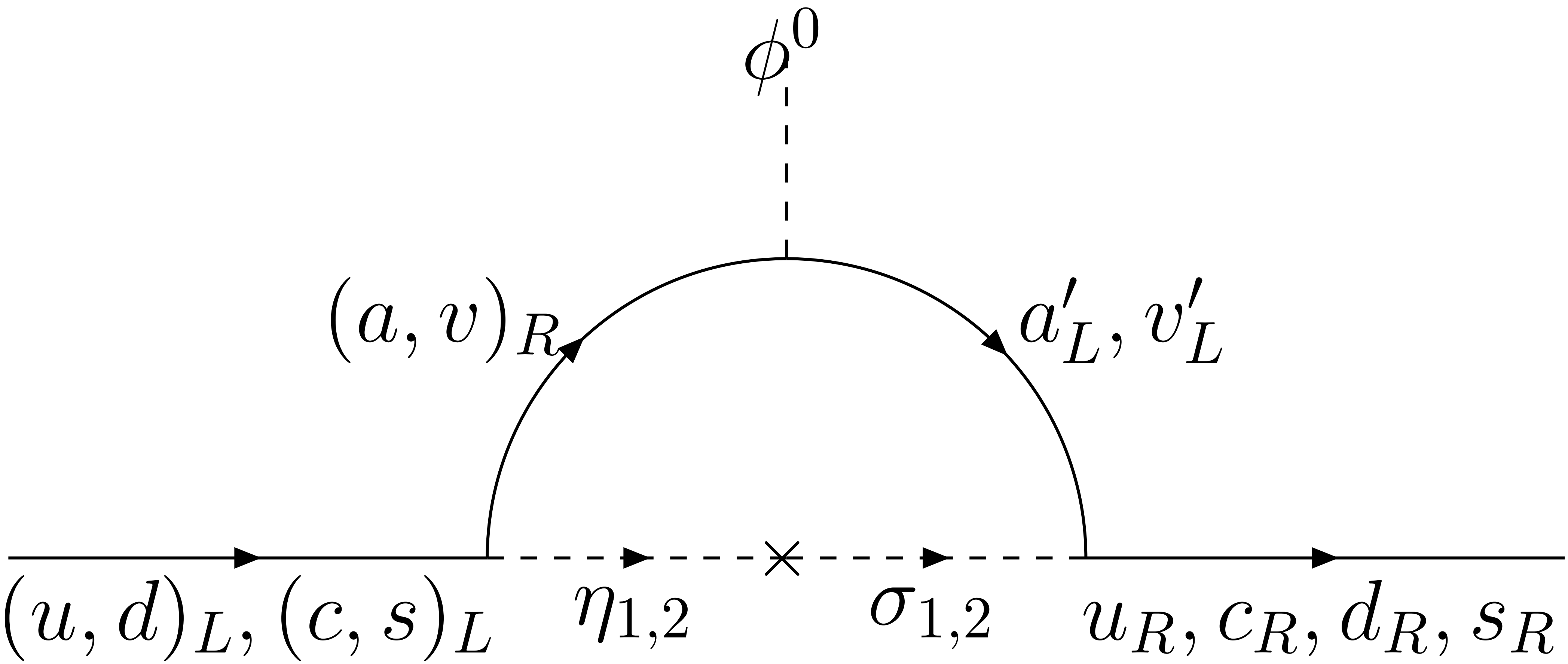
The key is to take advantage of the dark sector. We already know that dark matter exists. Perhaps it is also the answer to these two puzzles.

darkon	$SU(3)$	$SU(2)$	$U(1)$
$(a,v)_{L,R}$	3	2	1/6
$a'_{L,R}$	3	1	$2/3$
$v'_{L,R}$	3	1	$-1/3$
$\eta_{1,2}, \sigma_{1,2,3}$	1	1	0

Bit of history: Some 35 years ago, there were many studies of a possible 4th family. Instead of calling it (t', b'), the Wisconsin group (Barger et al.) chose (a, v) from the pattern

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} a \\ v \end{pmatrix}.$$

I now reassign a=angelic, v=volcanic to the dark sector.


ϕ^0
$(a, v)_R$
a'_L, v'_L
$(u, d)_L, (c, s)_L$
$\eta_{1,2}$
$\sigma_{1,2}$
u_R, c_R, d_R, s_R

Quark Flavor Alignment

Consider the smallest non-Abelian discrete symmetry S_3 with three irreducible representations $1, 1', 2$, where

$$2 \times 2 = 1 + 1' + 2.$$

Let

$$[(u, d)_L, (c, s)_L] \sim 2, \quad (t, b)_L \sim 1,$$
$$u_R, d_R, c_R, s_R \sim 1', \quad t_R, b_R \sim 1,$$
$$(\eta_1, \eta_2) \sim 2, \quad \sigma_{1,2} \sim 1', \quad \sigma_3 \sim 1.$$

The basis of the 2-dimensional representation is arbitrary. It may be transformed for example by

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

The analog of this in N dimensions is

$$y_j = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} exp[(2\pi i/N)(j-1)(k-1)] x_k,$$

i.e. a discrete Fourier transform.

The 3-dimensional version is the well-known [Cabibbo/Wolfenstein(1978)]

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, $\omega^3 = 1$. It was used by Lagrange to reproduce the solution [Tartaglia(1535)] of the cubic equation $x^3 + ax = b$.

$$x = \left(\frac{b}{2} + \sqrt{\frac{a^3}{27} + \frac{b^2}{4}}\right)^{1/3} + \left(\frac{b}{2} - \sqrt{\frac{a^3}{27} + \frac{b^2}{4}}\right)^{1/3}.$$

Only t and b acquire tree-level masses from the one Higgs doublet. The other quark masses are scotogenic, with soft S_3 breaking from $\eta - \sigma$ mixing.

The invariant coupling

$$(\bar{a}_R u_L + \bar{v}_R d_L)\eta_1 + (\bar{a}_R c_L + \bar{v}_R s_L)\eta_2$$

then ensures the alignment of $(u, c)_L$ with $(d, s)_L$ in V_{CKM} as desired.

To connect the first two families with the third, $\sigma_3 \sim 1$ is needed, with again soft S_3 breaking.

Quark Mass Hierarchy

Whereas the scotogenic masses of the first two quark families are expected to be suppressed relative to the tree-level masses of the third family, the mass hierarchy between the first and second families is yet to be explained.

To do this, let u_R, d_R, σ_1 have an additional imposed Frogatt-Nielsen charge, then $\sigma_2 - \eta_2$ mixing (which defines η_2 as well as $(c,s)_L$) breaks only S_3, whereas $\sigma_1 - \eta_{1,2}$ mixing breaks also $U(1)_{FN}$.

This framework explains

$$m_{u,d} < m_{c,s} < m_{t,b}$$

Within each family, the variation comes from the different Yukawa couplings within an order of magnitude.

The structure of quark mass matrices is such that flavor alignment is determined on the left and mass hierarchy on the right. With soft breaking of S_3 and $U(1)_{FN}$, both are accomplished in a renormalizable field theory. All particles beyond those of the standard model are in the dark sector.

Observable Consequences

Even though there is only one Higgs boson, its coupling to SM fermions is now not $(m_f/v\sqrt{2})$ as in the SM. [Fraser/Ma(2014)]

In particular, there should be off-diagonal Higgs couplings which are absent in the SM. As an illustration, suppose the 2×2 mass matrix linking $(\bar{c}, \bar{t})_L$ to $(c, t)_R$ is

$$\mathcal{M}_{ct} = \begin{pmatrix} c_L & s_L \\ -s_L & c_L \end{pmatrix} \begin{pmatrix} m_c & 0 \\ 0 & m_t \end{pmatrix} = \begin{pmatrix} c_L m_c & s_L m_t \\ -s_L m_c & c_L m_t \end{pmatrix},$$

then the Higgs coupling matrix may be written as

$$\mathcal{Y}_{ct} = \frac{1}{v\sqrt{2}} \begin{pmatrix} (1+\epsilon_1) c_L m_c & (1+\epsilon_2) s_L m_t \\ -(1+\epsilon_3) s_L m_c & c_L m_t \end{pmatrix},$$

where $\epsilon_{1,2,3}$ mostly come from interactions other than the trilinear Higgs couplings which contribute only to the mass terms. In the mass-diagonal basis,

$$\begin{pmatrix} c_L & -s_L \\ s_L & c_L \end{pmatrix} \mathcal{Y}_{ct} = \frac{1}{v\sqrt{2}} \begin{pmatrix} (1+\epsilon_1 c_L^2 + \epsilon_3 s_L^2) m_c & \epsilon_2 s_L c_L m_t \\ (\epsilon_1 - \epsilon_3) s_L c_L m_c & (1+\epsilon_2 s_L^2) m_t \end{pmatrix}.$$

This means that the rare decay $t \to hc$, which proceeds in the SM only through a loop and is very much suppressed, is enhanced a great deal and could be experimentally observed. The most recent limit (CMS) on this branching fraction is 4.3×10^{-4}. Using the analysis of Gutierrez/Jain/Kao(2021), it translates to the bound

$$\epsilon_2 s_L c_L < 0.057.$$

The mixing parameter s_L is unknown but probably of order $|V_{cb}| \sim 0.04$ and ϵ_2 may be of order 1. Hence this decay is potentially observable in the future at the upgraded LHC.

Last Words

With apology to J. R. R. Tokien:

Three families of quarks and leptons,

one Higgs to rule them all,

and in the darkness bind them.

To conclude, the SM may be complete as it is.
The rest is in the dark sector.

MonoHiggsology is the future!