Mixed Wave Dark Matter and Implications in Neutrino Physics

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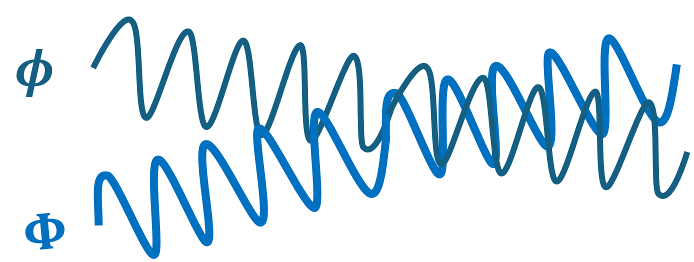
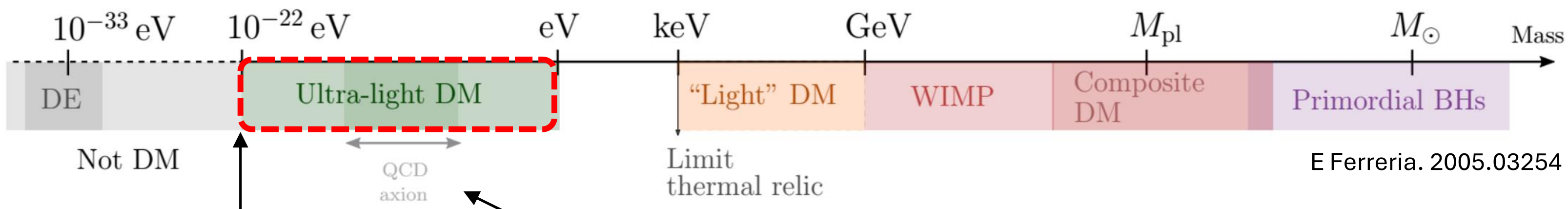
Based on arXiv: 2604.14480 (Accepted in PRD)

Collaborated with Hye-Sung Lee (KAIST)



FLASY 2026

Wave Dark Matter with Ultra-light Mass Scale



E Ferreria. 2005.03254

$$\mathcal{O}(r_s) \sim \lambda_{\rm dB} \sim 1\,{\rm kpc}\left(\frac{10^{-22}\,{\rm eV}}{m_\phi}\right)\left(\frac{10^{-4}c}{v}\right)$$

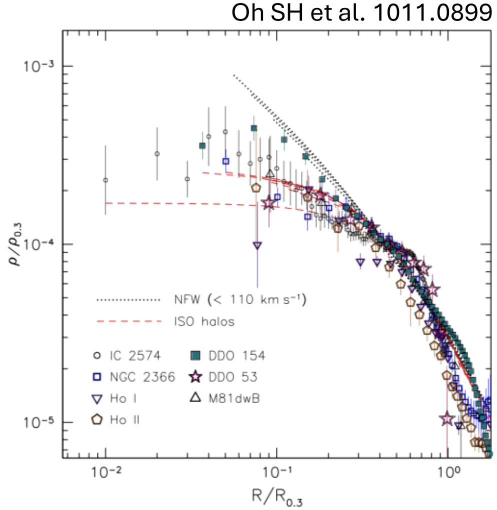
Solitonic core size of Dwarf galaxy

$$m_a \sim \frac{\Lambda^2}{f_a} \sim \frac{m_\pi f_\pi}{f_a}$$

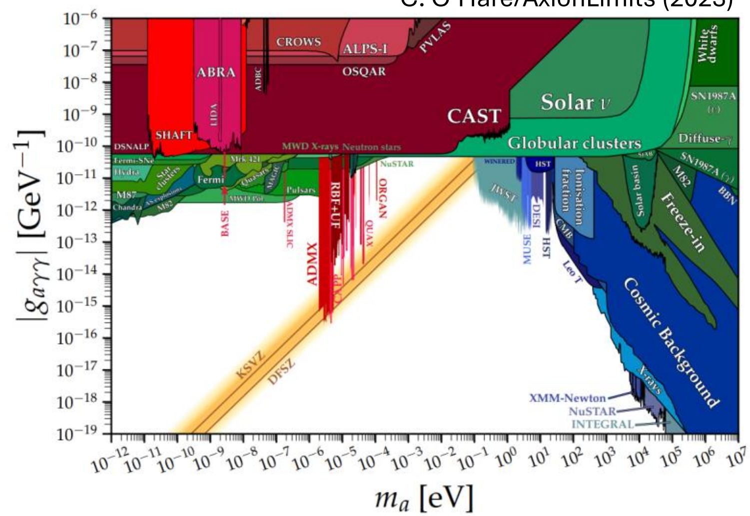
QCD axion or ALP

: Motivations of ultra-light mass of DM

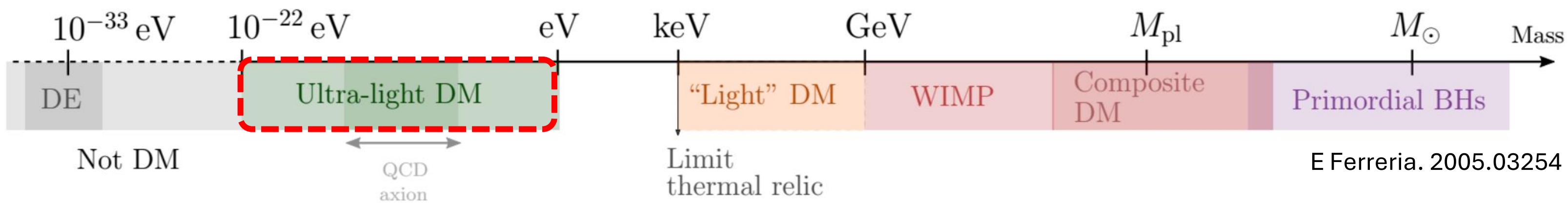
Oh SH et al. 1011.0899



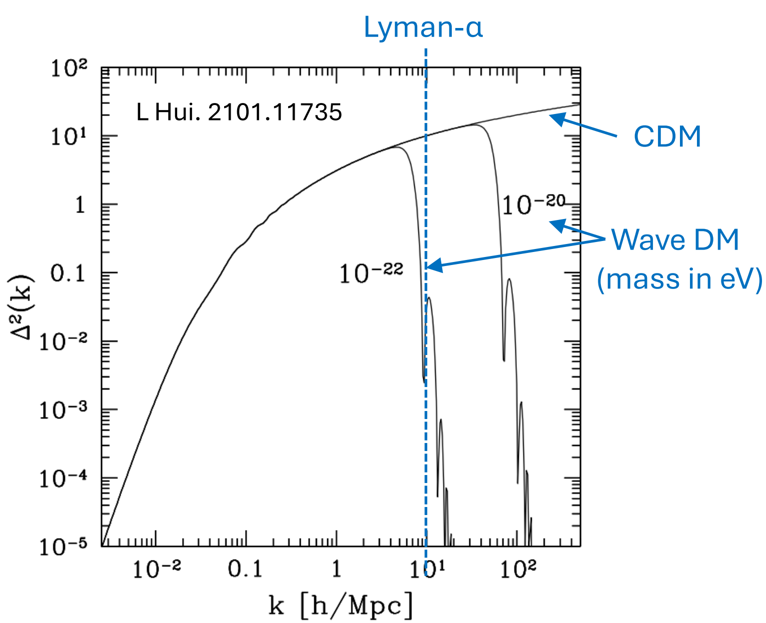
C. O'Hare/AxionLimits (2023)



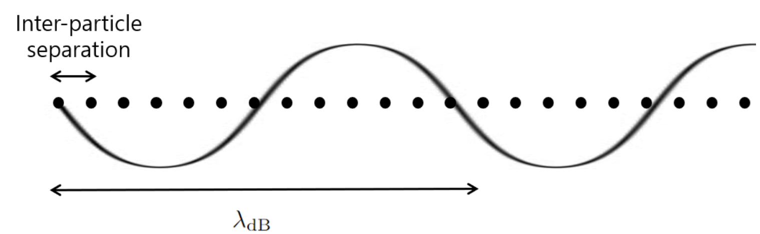
Wave Dark Matter with Ultra-light Mass Scale



Lower bound from structure formation



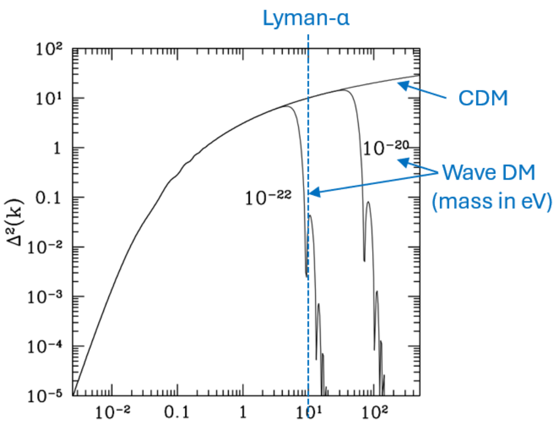
Upper bound from wave-like behavior



$$\mathcal{N} = n\,\lambda_{\rm dB}^3 = \left(\frac{30\,\mathrm{eV}}{m}\right)^4 \left(\frac{10^{-4}}{v}\right)^3 \gg 1$$

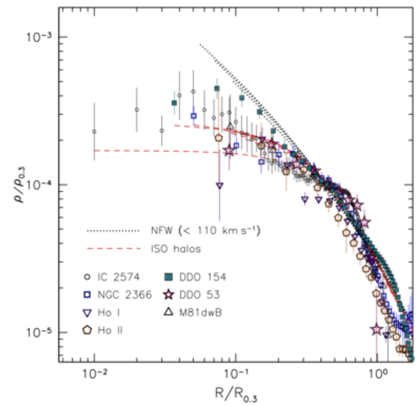
Observational / Experimental Probes of Wave Dark Matter

Matter power spectrum



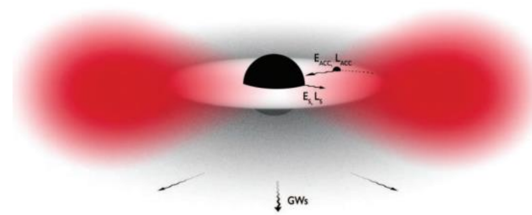
L Hui. 2101.11735

Galactic structure



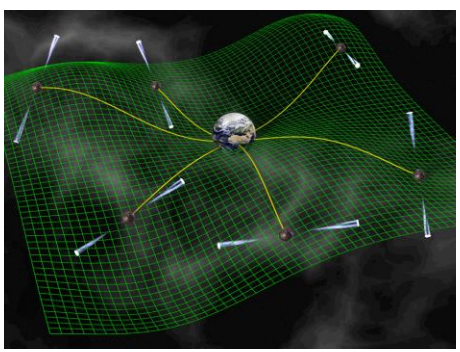
Oh SH et al. 1011.0899

BH superradiance



R. Brito et al. 1501.06570

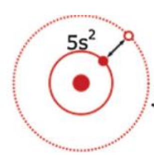
Pulsar time array



D. Champion/Max Planck Institute

Modulation signal

$$\alpha(t) = \alpha\left[1 + d_e\, \kappa\, \phi_0 \cos(m_\phi t)\right]$$



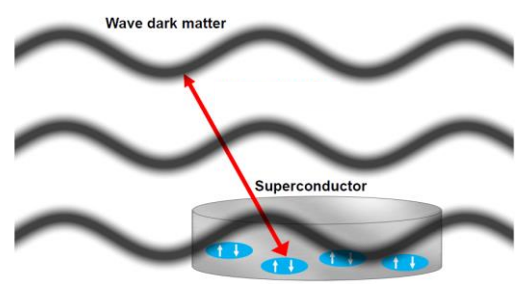
Sr Clock

$f_{Sr} \propto \alpha^{2+\epsilon} m_e c^2$

$\epsilon = 0.06$

$$\frac{\delta(f_{Sr}/f_c)}{f_{Sr}/f_c} = 1.06\,\frac{\delta\alpha}{\alpha}$$

CJ. Kennedy et al. 2008.08773



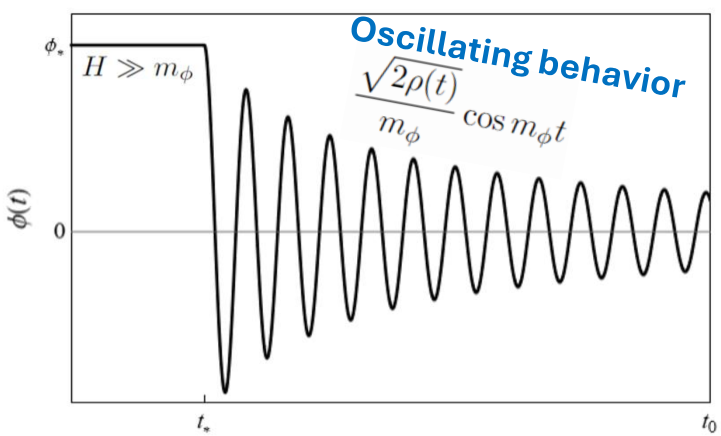
YL ChoeJo, YK, HS Lee, J Lee, J Yi. 2509.22892

Oscillating Wave Dark Matter to other sectors

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2} m_\phi^2\,\phi^2\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0$$

$$\phi(t) = \begin{cases} \phi_* & H > m_\phi \\ \dfrac{\sqrt{2\rho(t)}}{m_\phi}\cos m_\phi t & H < m_\phi \end{cases}$$



Coupled to gauge boson

A Arvanitaki et al. 1405.2925
K Van Tilburg et al. 1503.06886
J Guo et al. 2206.14221

$$\mathcal{L} \supset -\frac{1}{4} d_e\,\kappa\,\phi\,F_{\mu\nu}F^{\mu\nu}$$

$$\alpha(t) = \alpha\Big[1 + d_e\,\kappa\,\phi_0\cos(m_\phi t)\Big]$$

$$\mathcal{L} \supset -\frac{1}{4} d_g\,\kappa\,\phi\,G^a_{\mu\nu}G^{a\,\mu\nu}$$

$$\Lambda_{\rm QCD}(t) = \Lambda_{\rm QCD}\Big[1 + d_g\,\kappa\,\phi_0\cos(m_\phi t)\Big]$$

Coupled to SM fermion

A Arvanitaki et al. 1405.2925
M Filzinger et al. 2312.13723
YL ChoeJo, YK, HS Lee, J Lee, J Yi. 2509.22892

$$\mathcal{L} \supset -d_{m_e}\,\kappa\,\phi\,m_e\,\bar{e}e$$

$$m_e(t) = m_e\Big[1 + d_{m_e}\,\kappa\,\phi_0\cos(m_\phi t)\Big]$$

$$\mathcal{L} \supset -d_{m_q}\,\kappa\,\phi\,m_q\,\bar{q}q$$

$$m_q(t) = m_q\Big[1 + d_{m_q}\,\kappa\,\phi_0\cos(m_\phi t)\Big]$$

Coupled to neutrino

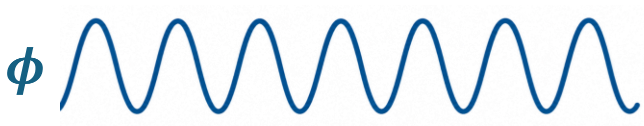
A Berlin. 1608.01307
G Krnjaic et al. 1705.06740
YL ChoeJo, YK, HS Lee. 2305.16900

$$\mathcal{L} \supset -\,g\,\phi\,\overline{N}N$$

$$M_N(t) = M_N + g\,\phi_0\cos(m_\phi t)$$

Single vs. Mixed Wave Dark Matter

Single wave dark matter

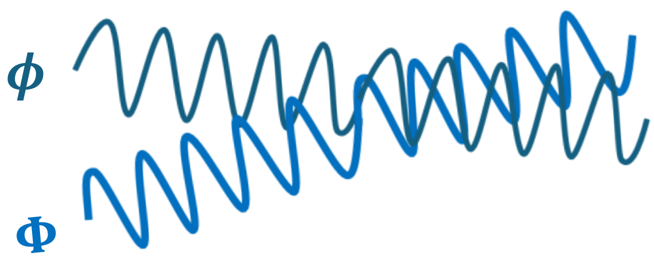


$\longrightarrow$

Harmonic oscillation

$$\phi(t) = \phi_a \cos m_\phi t$$

Mixed wave dark matter



$\longrightarrow$

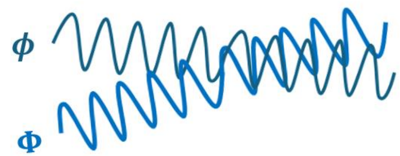
Non-trivial time-dependent structure
(non-monochromatic, symmetry-broken…)

$$\phi(t) = \phi_0(t) + \phi_a(t) f(m_\phi t)$$

Mixed Wave Dark Matter and parametric structure

$$V(\phi,\Phi)=\frac{1}{2}m^2\phi^2+\frac{1}{2}M^2\Phi^2+\boxed{\frac{1}{2}\kappa\phi^2\Phi^2}$$

Wave DM mixing



For $M^2 \gg \kappa\phi^2$

$$\Phi(t)\;\simeq\;\Phi_0\sin(Mt)$$

EoM of ϕ

$$\ddot{\phi}+3H\dot{\phi}+(m^2+\kappa\Phi^2)\phi=0$$

$$\boxed{\phi''(z)+\left[A-2q\cos(2z)\right]\phi(z)=0}$$

Mathieu-type equation

where $z\;\equiv\;Mt \quad {}'=d/dz$

$$q\equiv\frac{\kappa\Phi_0^2}{4M^2},\quad A\equiv\frac{m^2}{M^2}+2q$$

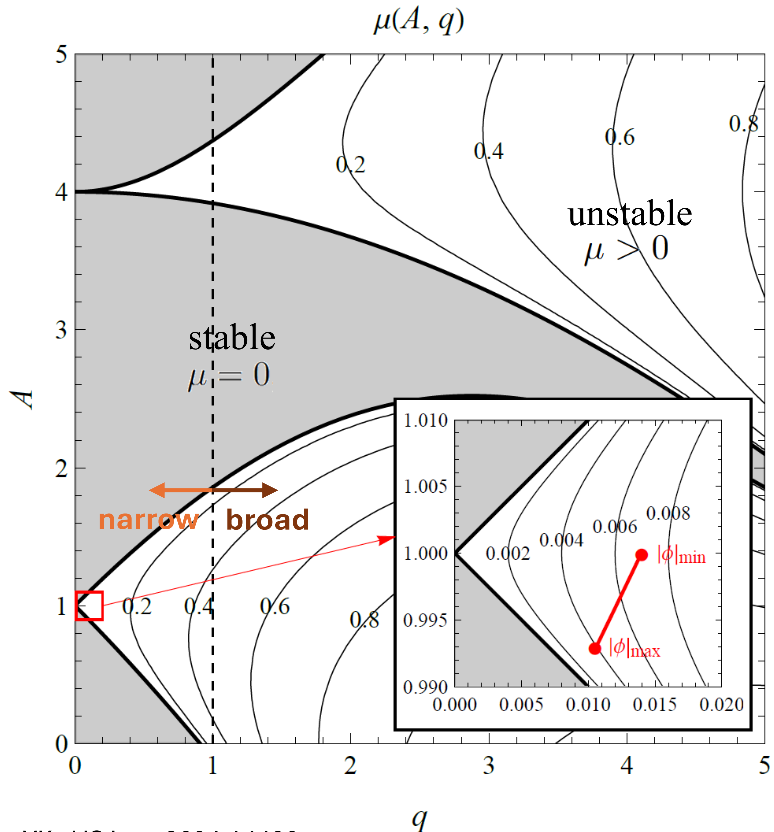
→ Solution

$$\phi(z)=e^{\tilde{\mu}(A,q)\,z}\,P(z)$$

$\tilde{\mu}=\mu+i\nu$: Mathieu characteristic

$P(z)$: Periodic function

Mixed Wave Dark Matter and parametric structure



YK, HS Lee. 2604.14480

EoM $\phi''(z) + [A - 2q\cos(2z)]\phi(z) = 0$

Sol $\phi(z) = e^{\tilde{\mu}(A,q)\,z}\,P(z) \qquad \tilde{\mu} = \mu + i\nu$

Broad resonance $q \gg 1$

- Instability regions are wide
- μ is large
- **Exponential growth** $|\phi(z)| \propto \exp(\mu\, z)$

Narrow resonance $q \ll 1$

- Instability regions are narrow

  $A \simeq n^2 \;(n = 1, 2, \cdots)$

  $|A - 1| \lesssim q$ for $n = 1$

- μ is small

  $\mu \simeq \frac{1}{2}\sqrt{q^2 - (A-1)^2}$ for $n = 1$

- **Growth is suppressed**

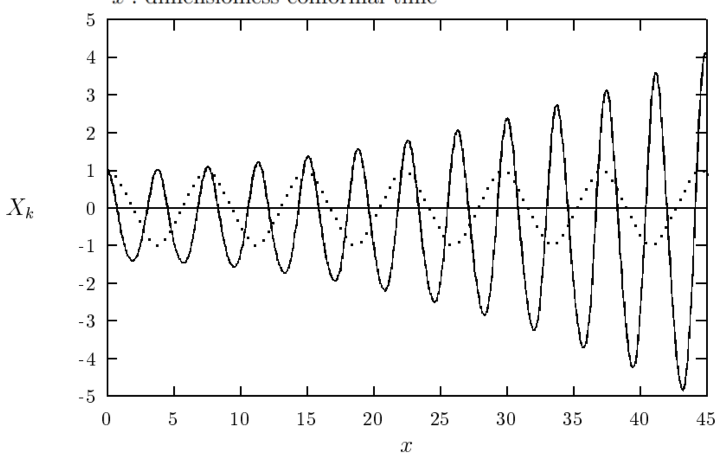
Mixed Wave Dark Matter and parametric structure

Broad resonance $q \gg 1$

- Parametric resonance
- i.e., Preheating after inflation
- **Amplified production** of particle

$X_k(t) = a(t)\chi_k(t)$

x : dimensionless conformal time



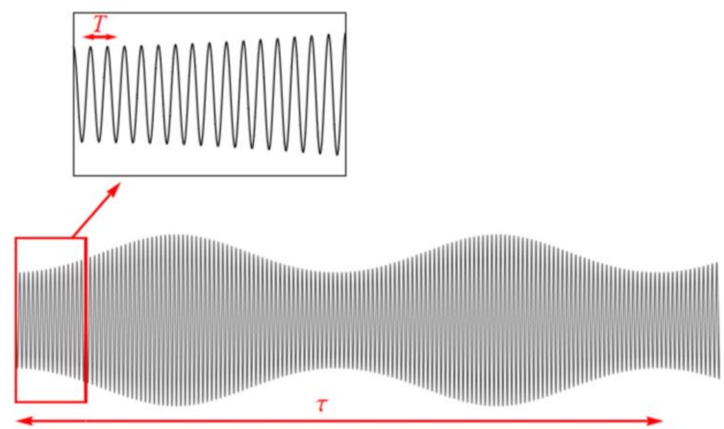
P Greene et al. (1997)

Narrow resonance $q \ll 1$

$$V(\phi, \Phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\Phi^2 + \frac{1}{2}\kappa\phi^2\Phi^2 \qquad q \equiv \frac{\kappa\Phi_0^2}{4M^2}$$

Subdominant mixing term

Amplification is insufficient → Growth stops



YK, HS Lee. 2604.14480

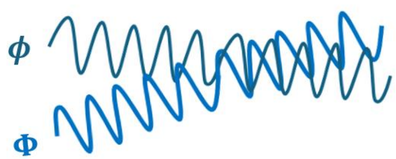
Mixed Wave Dark Matter to Wave-envelope DM

$e^{\mu z}$ with $z = Mt$

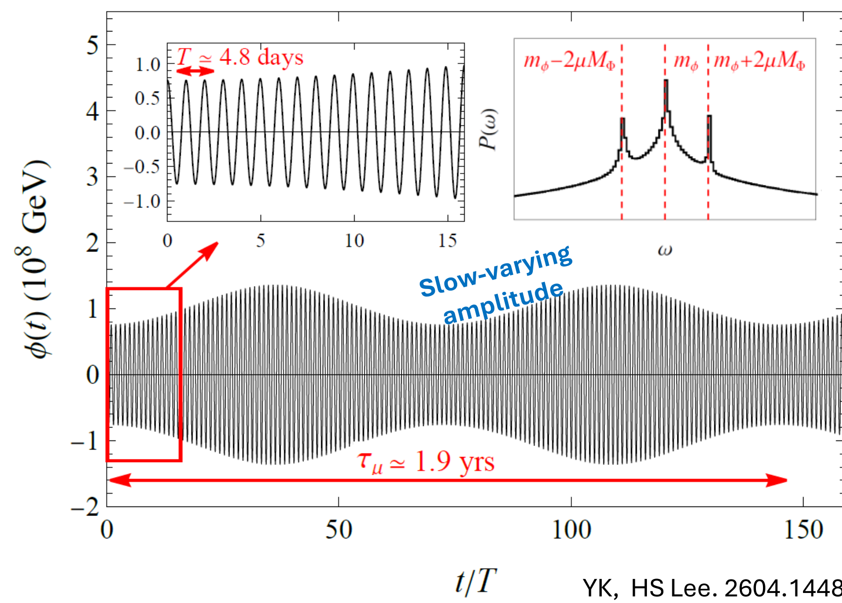
New time scale $\underbrace{\tau_\mu \simeq \frac{2\pi}{\mu M}}_{\text{Slow oscillation from mixing}} \gg \underbrace{T \simeq \frac{2\pi}{M}}_{\text{Wave DM oscillation}}$ for $\mu \ll 1$

Narrow resonance

Mixed wave DM



"Wave-envelope DM"

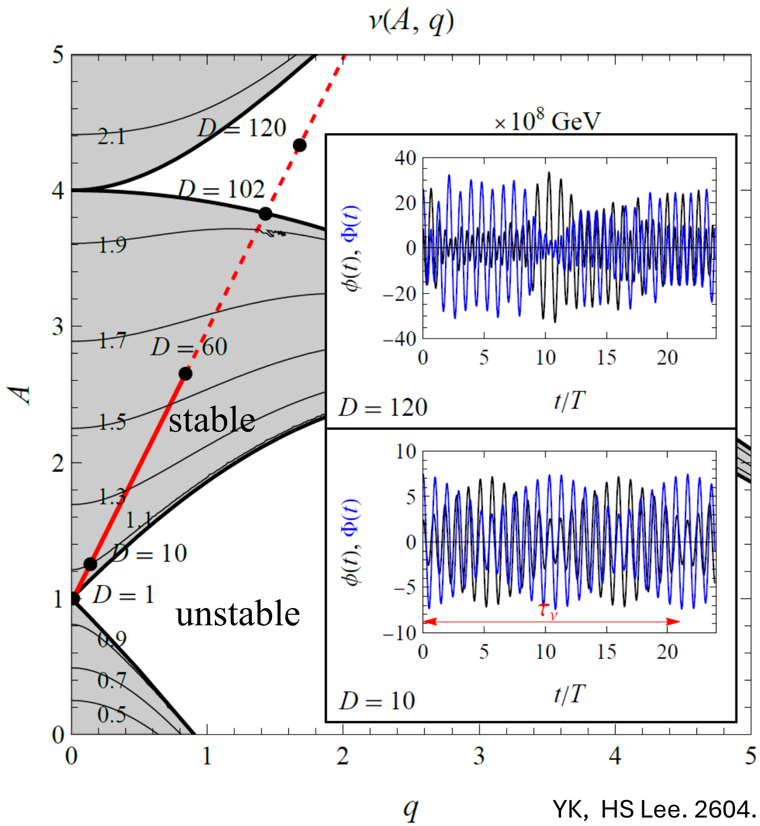


YK, HS Lee. 2604.14480

Mixed Wave Dark Matter to Wave-envelope DM

EoM $\phi''(z) + \left[A - 2q\cos(2z)\right]\phi(z) = 0$

Sol $\phi(z) = e^{\tilde{\mu}(A,q)\,z}\,P(z)$ $\quad \tilde{\mu} = \mu + i\nu$



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Stable region

$\mu = 0,\ \nu = n + \delta$

$$\tau_\nu \simeq \frac{4\pi}{|\nu - n|M}$$

Unstable region

$\mu > 0,\ \nu = 0$

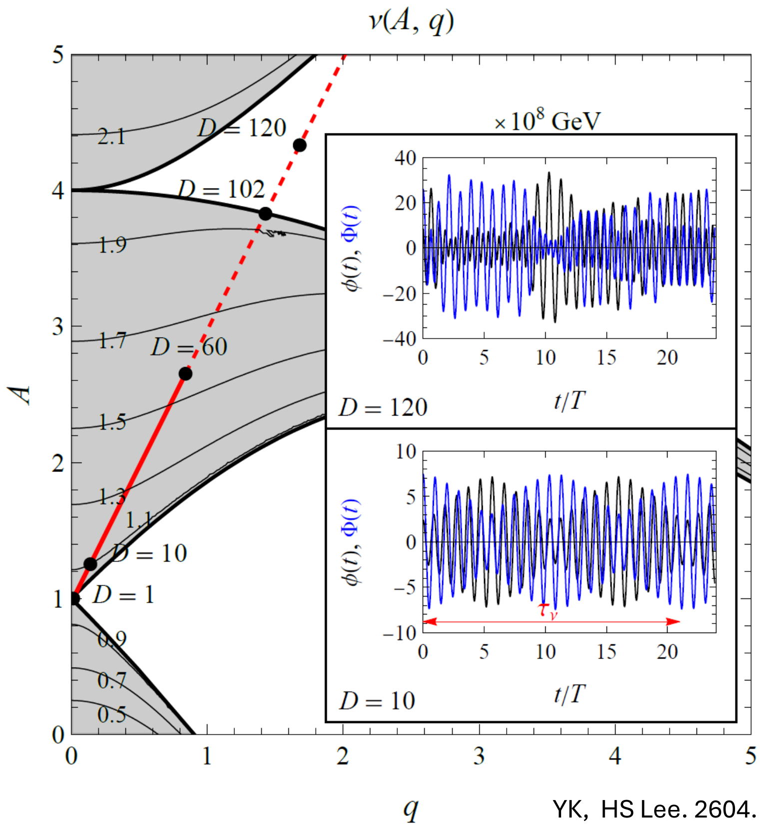
$$\tau_\mu \simeq \frac{2\pi}{\mu M}$$

Mixed Wave Dark Matter to Wave-envelope DM

Dense environment with enhancement factor D

$$q \equiv \frac{\kappa \Phi_0^2}{4M^2}, \quad A \equiv \frac{m^2}{M^2} + 2q$$

$$q(D) = Dq_0, \; A(D) = A_0 + 2q_0(D-1)$$

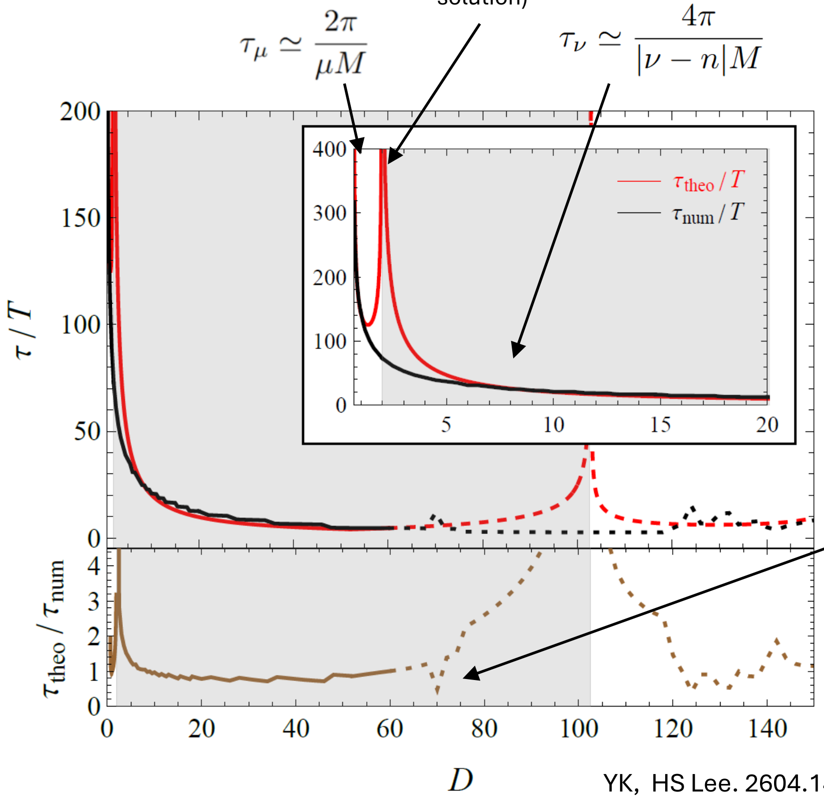


YK, HS Lee. 2604.14480

$D = 1$	1st instability band, narrow resonance
$D = 2$	Enter stable region
$D = 10$	Example plot 1 (envelope in stable region)
$D = 60$	Break down of Mathieu approximation
	Due to backreaction
$D = 102$	Get out stable region
$D = 120$	Example plot 2 (irregular pattern)

Mixed Wave Dark Matter to Wave-envelope DM

Mismatch at boundary (due to secular solution)



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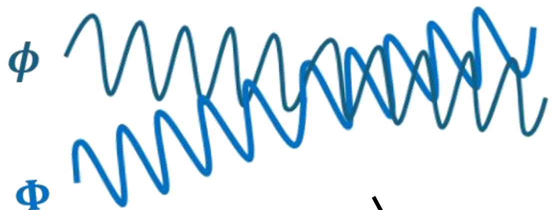
Theoretical vs **Numerical** envelope timescales

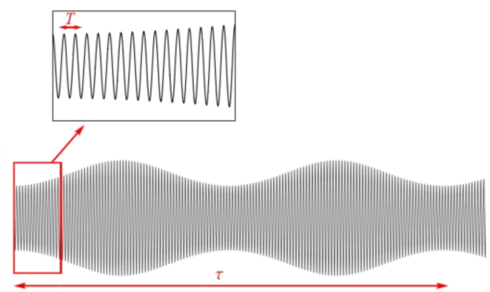
$$\tau_{\rm theo} \begin{cases} \tau_\mu \simeq \dfrac{2\pi}{\mu M} \\ \tau_\nu \simeq \dfrac{4\pi}{|\nu - n|M} \end{cases}$$

$$\ddot{\phi} + 3H\dot{\phi} + (m^2 + \kappa\Phi^2)\phi = 0$$
$$\ddot{\Phi} + 3H\dot{\Phi} + (M^2 + \kappa\phi^2)\Phi = 0$$
$$\downarrow$$
$$\tau_{\rm num}$$

- $D = 1$ 1st instability band, narrow resonance
- $D = 2$ Enter stable region
- $D = 10$ Example plot 1 (envelope in stable region)
- $D = 60$ Break down of Mathieu approximation
  Due to backreaction
- $D = 102$ Get out stable region
- $D = 120$ Example plot 2 (irregular pattern)

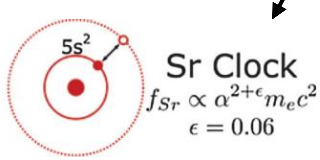
Mixed Wave Dark Matter

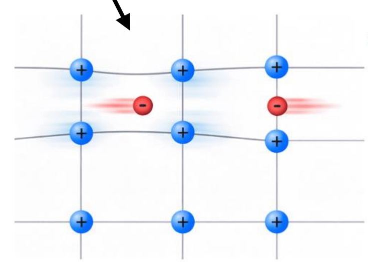


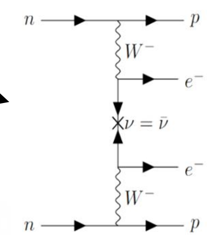


“Wave-envelope DM”

Beyond harmonic modulation

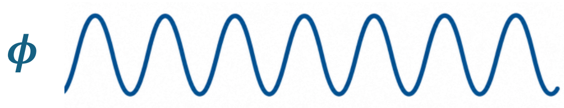






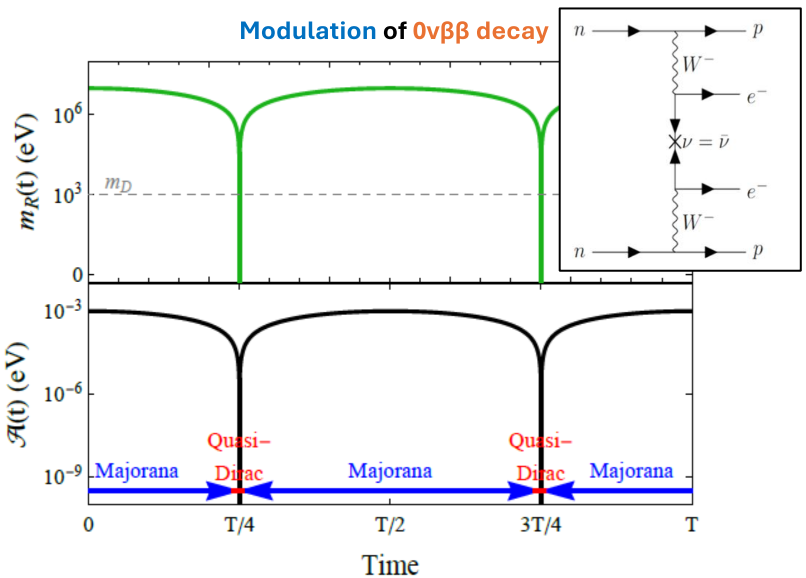
Implications in Neutrino Physics $\mathcal{L} \supset -\frac{1}{2} g\phi\bar{\nu}_M \nu_M$

Single wave dark matter



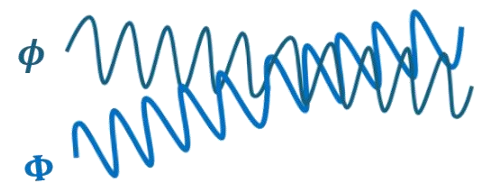
$$m_R(t) = m_{R,0} \cos m_\phi t$$

Modulation of 0νββ decay



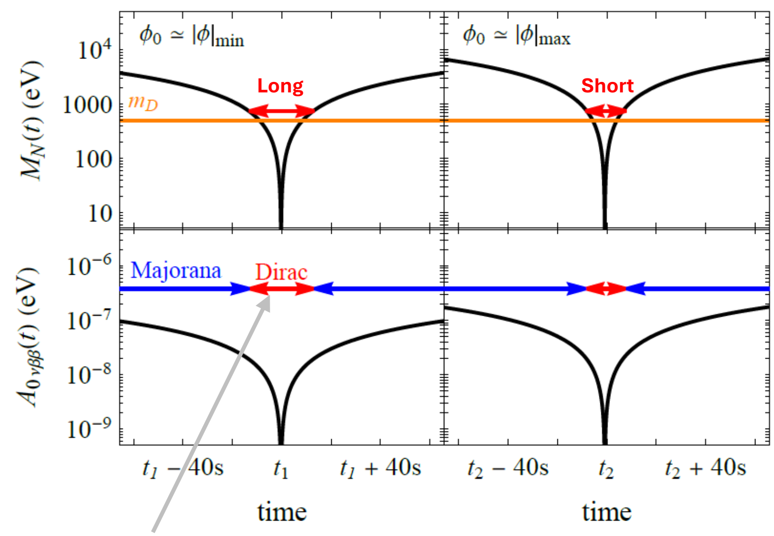
YL ChoeJo, YK, HS Lee. 2305.16900

Mixed wave dark matter



$$m_R(t) = m_{R,0}(t) \cos m_\phi(t) t$$

Modulation of turn-off time of 0νββ decay



$$\Delta T_{\rm Dirac} \simeq \frac{T}{\pi} \sin^{-1}\left(\frac{m_D}{g\phi_0}\right)$$

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Implications in Neutrino Physics $\mathcal{L} \supset -\frac{1}{2} g \phi \bar{\nu}_M \nu_M$

effective μ^2 term

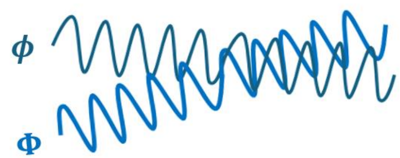
$$\mu_{\text{eff}}^2(t) \equiv \mu_0^2 + \kappa\, \Phi(t)^2$$

$$\mathcal{L} = -\frac{1}{2}\boxed{(\mu_0^2 + \kappa\, \Phi^2)}\phi^2 - \frac{\lambda}{4}\phi^4 - \frac{1}{2}M^2\Phi^2$$

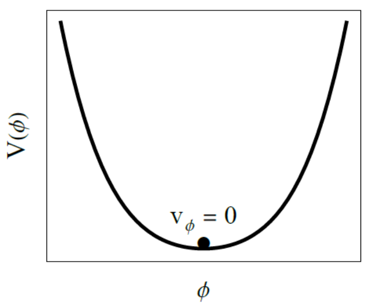
$\mu_0^2 < 0$

tachyonic bare term

Mixed Wave DM

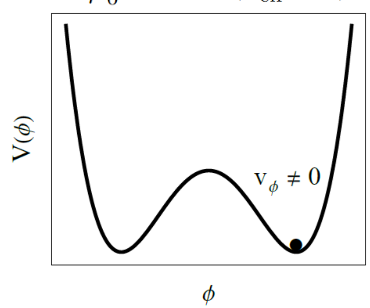


$-\mu_0^2 < \kappa\Phi^2 \ (\mu_{\text{eff}}^2 > 0)$



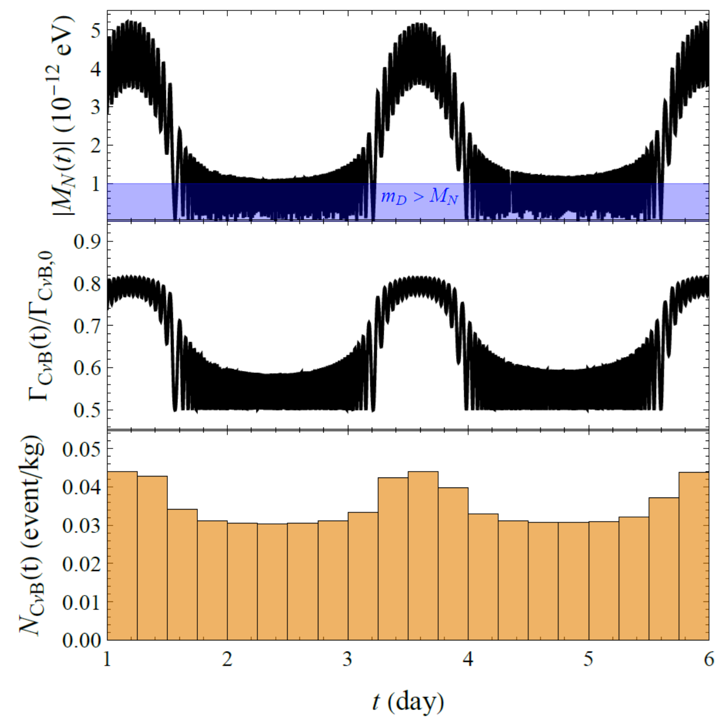
Symmetry-restored phase

$-\mu_0^2 > \kappa\Phi^2 \ (\mu_{\text{eff}}^2 < 0)$



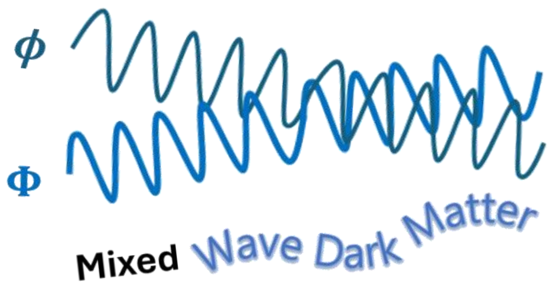
Symmetry-broken phase

$\Gamma_{\text{C}\nu\text{B}} \propto (1 - P(\nu_a \to \nu_s)) \sim (1 - \frac{1}{2}\sin^2 2\theta_{as})$

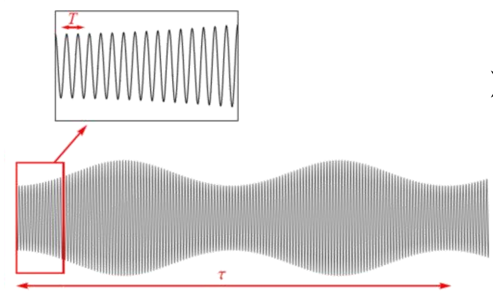


YK, HS Lee. 2503.04949

Summary



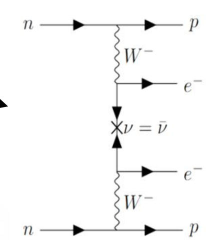
- **Mixed wave DM** can lead to the non-trivial time-dependent structure.



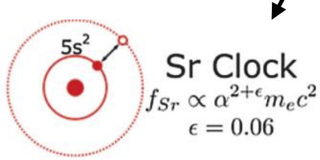
"Wave-envelope DM"

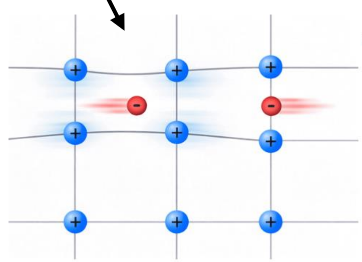
- **Wave-envelope DM** can arise with slow-varying amplitude from parametric dynamics.

Beyond harmonic modulation



- This can induce **modulation signals** in various observables.





Thank you!

Backup slides

Mixed Wave Dark Matter to Wave-envelope DM

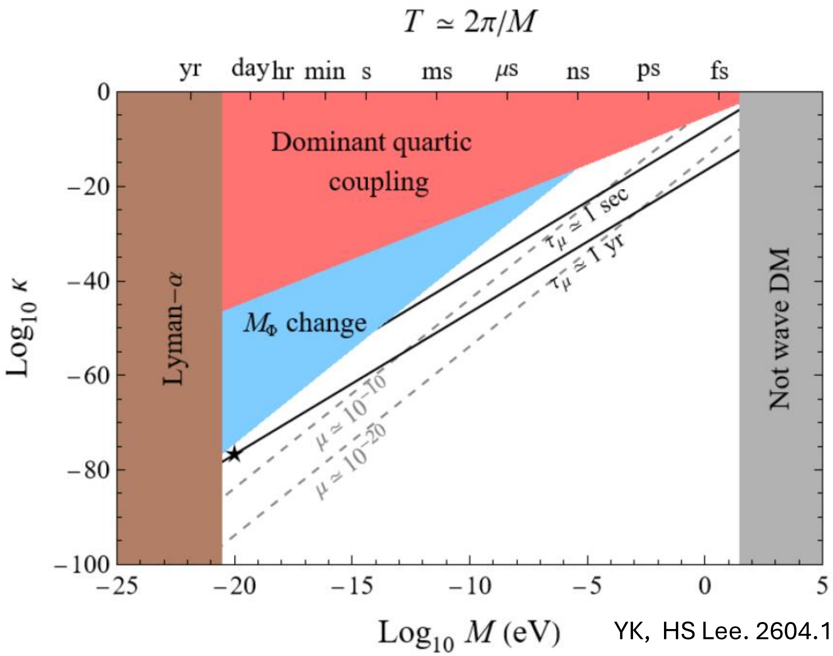
$$V(\phi,\Phi)=\frac{1}{2}m^2\phi^2+\frac{1}{2}M^2\Phi^2+\frac{1}{2}\kappa\phi^2\Phi^2$$

$$M^2 \gg \kappa\phi^2$$

$$q\equiv\frac{\kappa\Phi_0^2}{4M^2},\quad A\equiv\frac{m^2}{M^2}+2q$$

$\mu \simeq \frac{1}{2}\sqrt{q^2-(A-1)^2}$ for $A\simeq 1$

$$\underbrace{\tau_\mu \simeq \frac{2\pi}{\mu M}}_{\text{Slow oscillation from mixing}} \gg \underbrace{T\simeq\frac{2\pi}{M}}_{\text{Wave DM oscillation}}$$



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Mixed Wave Dark Matter to Wave-envelope DM

EoM $\phi''(z) + \big[A - 2q\cos(2z)\big]\phi(z) = 0$

Sol $\phi(z) = e^{\tilde{\mu}(A,q)\,z} \underbrace{P(z)}_{\text{period } \pi}$

Stable region

$\mu = 0,\ \nu = n + \delta \quad |\delta| \ll 1$

$$\phi = e^{i\nu z} \sum_{l=-\infty}^{\infty} c_l e^{2ilz}$$

$$-(\nu + 2l)^2 c_l + A c_l - q(c_{l-1} + c_{l+1}) = 0$$

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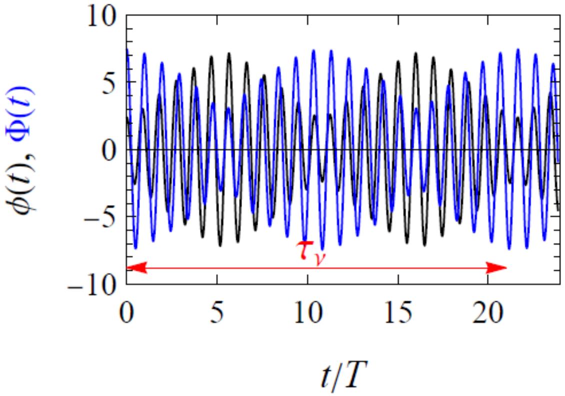


Near the n-th band with $(A, q) \simeq (n^2, 0)$

Dominant modes $l = 0$ and $l = -n$

Frequencies ν and $\nu - 2n$

$\nu = n + \delta$ $\quad n + \delta$ and $n - \delta$

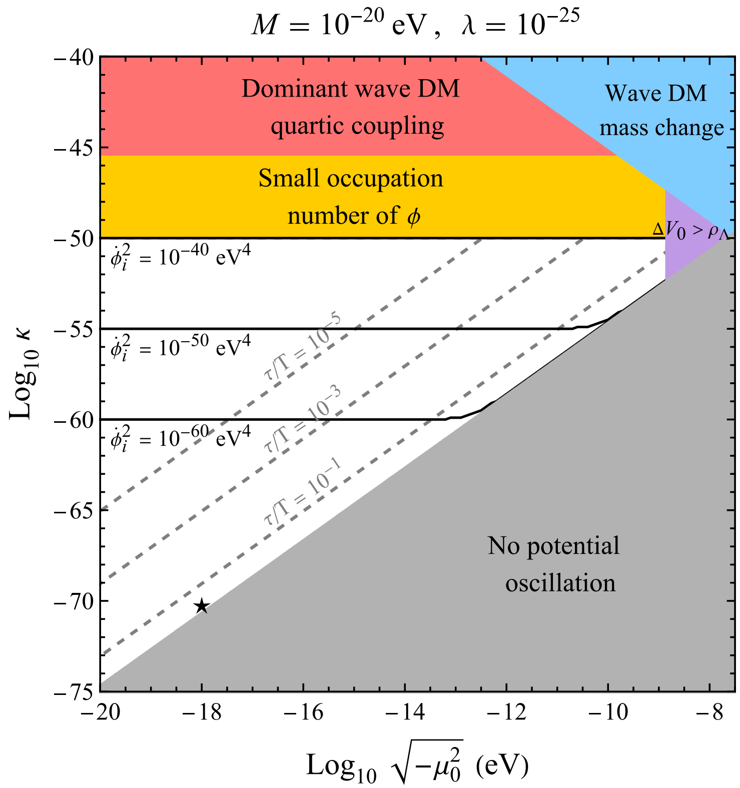
adjacent to the n-th band

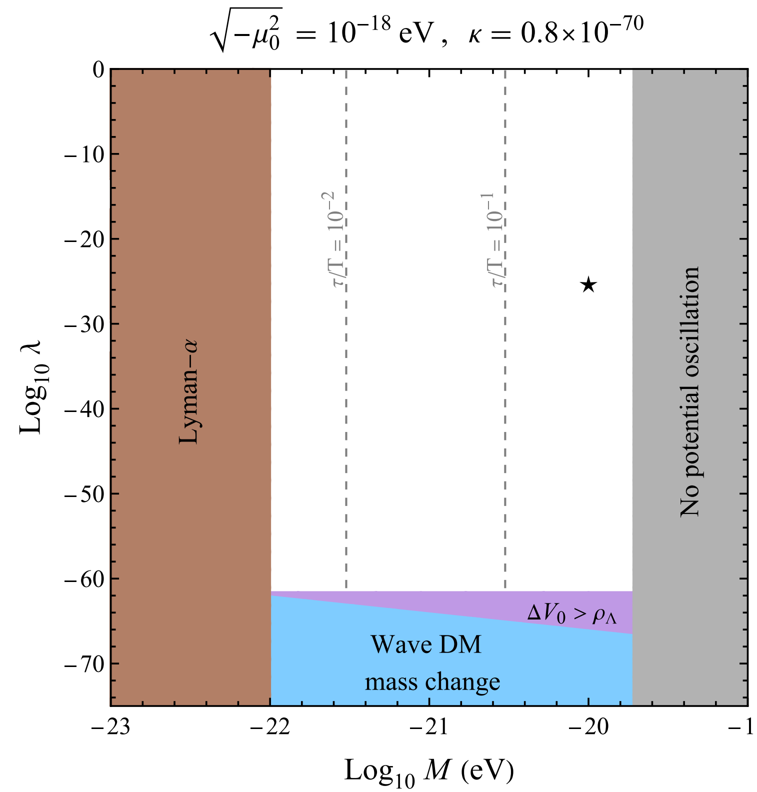
$$\boxed{\tau_\nu \simeq \frac{4\pi}{|\nu - n|M}} \gg T \simeq \frac{2\pi}{M}$$

$|\delta| = |\nu - n| \ll 1$

Potential oscillation constraint

$$\mathcal{L} = -\frac{1}{2}\left(\mu_0^2 + \kappa\,\Phi^2\right)\phi^2 - \frac{\lambda}{4}\phi^4 - \frac{1}{2}M^2\Phi^2$$





τ: time interval of symmetry-broken phase in one period

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Implications in Neutrino Physics

Dirac

vs

Majorana

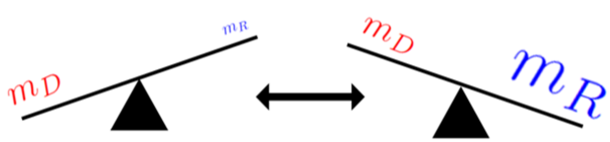
Dirac mass | Majorana mass

$$\mathcal{L} = \boxed{-\, m_D \bar{\nu}_D \nu_D} \boxed{-\, \frac{1}{2} m_R \bar{\nu}_M \nu_M}$$

$$\nu_D = \nu_L + \nu_R$$
$$\nu_M = \nu_R + \nu_R^c$$

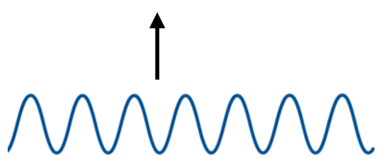
$$\mathcal{L} \supset -\frac{1}{2} g \phi \bar{\nu}_M \nu_M$$

$$m_R(t) = m_{R,0} \cos m_\phi t$$



Quasi-Dirac type

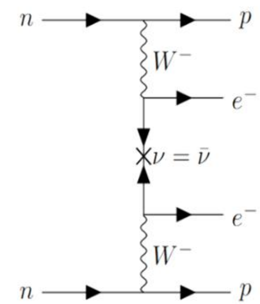
Majorana type

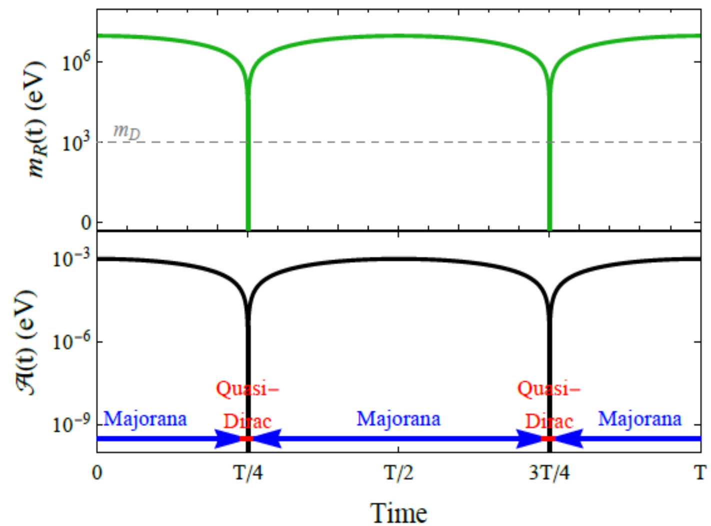


Single wave DM ϕ

0νββ decay

(LNV)





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Mixed Wave Dark Matter Scenario II: Potential Oscillation

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effective μ^2 term

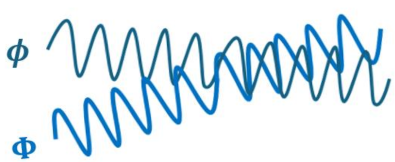
$$\mu_{\rm eff}^2(t) \equiv \mu_0^2 + \kappa\,\Phi(t)^2$$

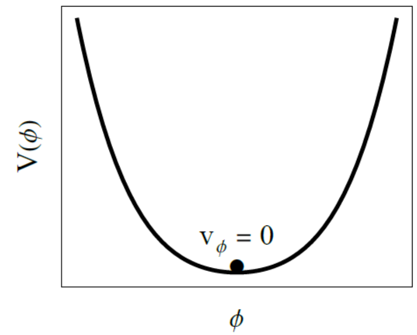
$$\mathcal{L} = -\frac{1}{2}\boxed{(\mu_0^2 + \kappa\,\Phi^2)}\phi^2 - \frac{\lambda}{4}\phi^4 - \frac{1}{2}M^2\Phi^2$$

$\mu_0^2 < 0$

tachyonic bare term

Mixed Wave DM

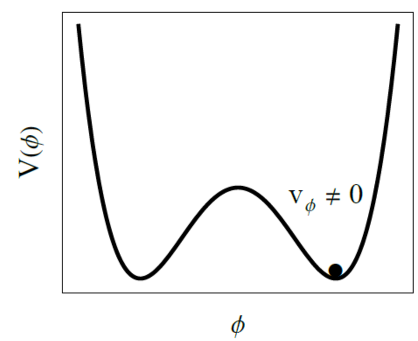




$-\mu_0^2 < \kappa\Phi^2 \quad (\mu_{\rm eff}^2 > 0)$

Symmetry-restored phase

w/ zero VEV



$-\mu_0^2 > \kappa\Phi^2 \quad (\mu_{\rm eff}^2 < 0)$

Symmetry-broken phase

w/ nonzero VEV