FLASY 2026 · GYEONGJU, SOUTH KOREA

BEYOND SU(N)

$U(3) \times U(2)$ as the underlying symmetry of the strong and electroweak interactions

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PRD 114 (2026) 3, 035009 [arXiv:2512.14839]

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EXCELENCIA SEVERO OCHOA

HOW IT STARTED

It began with a mistake (mine)

My student, **Antonio Herrero-Brocal**, came to me and pointed out an arXiv paper that claimed to be based on a $U(2)$ **flavor symmetry** when, according to him, what it actually wrote down was $SU(2) \times U(1)$.

I told him, confidently, that these are *the same thing* because they share the **same algebra**. But equal algebras do not mean equal groups: they are *not* the same thing, and that error is exactly where this project began.

So we set out to take that difference seriously.

THE STARTING POINT

The Standard Model: successful, yet ad hoc

A Yang–Mills theory on $SU(3)_c \times SU(2)_L \times U(1)_Y$

Remarkably accurate, yet built on ad hoc empirical inputs rather than an organizing principle.

Hypercharge by hand

The Y assignments have no rationale inside the SM.

They are simply assumed.

Charge unquantized

$|Q_e| = |Q_p|$ exactly.

A coincidence that follows from no deeper rule.

Missing representations

Some minimal irreps are absent from the SM (and not observed in Nature).

Example: an $SU(2)_L$ doublet with $Y = 0$.

THE MOTIVATION

A redundancy we forget to gauge

Gauge symmetries are **redundancies** in how we describe a system. In QED it is $U(1)$, acting on the vector potential. Non-Abelian theories put it in $SU(N)$.

And **most $SU(N)$ theories carry an extra global $U(1)$** on top since fermions enter the Lagrangian through bilinear combinations.

THE QUESTION

Why gauge only the $SU(N)$, and leave the $U(1)$ phase as a *global* symmetry?

QCD

The quark phase of $SU(3)_c$ survives as baryon number $U(1)_B$. Accidental, and never gauged.

Electroweak

The same happens to the lepton phase of $SU(2)_L$, which survives as lepton number $U(1)_L$. An exact symmetry of the renormalizable Lagrangian, again left global by convention rather than by argument.

THE IDEA

Taking the gauge principle to its limit

01 · THE PRINCIPLE

Every $SU(N)$ should be accompanied by its global phase made local:

$$SU(N) \to U(N)$$

A proposal about how gauge theories ought to be built.

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02 · THIS WORK

$$\mathrm{U}(3) \times \mathrm{U}(2)$$

Applied to the strong and electroweak interactions, the principle gives one concrete model. Our hypothesis: this is their genuine underlying symmetry.

BACK TO THE MISTAKE

Why U(N) is not SU(N) × U(1)

Same generators, same algebra... but a different **group**.

01 · THE GROUPS DIFFER

$$U(N) \cong \frac{SU(N) \times U(1)}{\mathbb{Z}_N}$$

$SU(N) \times U(1)$ is an N-fold cover of $U(N)$, as $SU(2)$ double-covers $SO(3)$.

$U(2)$ and $SU(2) \times U(1)$

Under $(g, e^{i\alpha}) \mapsto e^{i\alpha} g$, both elements of $SU(2) \times U(1)$

$$(\mathbb{1}, 1) \quad \text{and} \quad (-\mathbb{1}, -1)$$

map to the same $u = \mathbb{1} \in U(2)$, differing by the kernel $\mathbb{Z}_2$.

02 · SO DO THEIR REPRESENTATIONS

Every $U(N)$ irrep is an irrep of $SU(N) \times U(1)$, **but not conversely**. The quotient keeps only those where the $U(1)$ charge is compatible with the $SU(N)$ irrep, so some charges simply do not survive.

A historical caveat. Glashow's 1961 paper is often credited with identifying the electroweak *group* $SU(2)_L \times U(1)_Y$. Strictly speaking, he identified the *algebra* $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$. Even today the SM gauge group is not fully fixed [O'Raifeartaigh '88, Hucks '91, Tong '17].

SPECIALIZING TO THE $U(3) \times U(2)$ MODEL

The U(3) × U(2) model: the charge constraints

From here on: $U(3) \times U(2)$, i.e. $SU(3)_c \to U(3)$ and $SU(2)_L \to U(2)$.
The two $U(1)$ charges, q_3 for $U(1)_3$ and q_2 for $U(1)_2$, must satisfy **consistency conditions** set by their representation.

THE CHARGE CONSTRAINTS

$$q_2 = 2j + 2n_2 \qquad q_3 = p - q + 3n_3$$

Notation: $2j$ is the $SU(2)$ Dynkin label (j the isospin), (p, q) the $SU(3)$ labels, and $n_2, n_3 \in \mathbb{Z}$. The charge is fixed only up to a multiple of N (its N-ality), which is exactly what quantizes it.

SU(2) doublet

Only **odd** q_2 is allowed → $\mathbf{2}_1$ permitted, $\mathbf{2}_0$ **forbidden.**

SU(3) triplet

q_3 cannot be a multiple of 3 and the singlet and the triplet must differ in charge.

KEY RESULT

Charge is quantized, and not every representation is allowed.

THE NEXT STEP

The U(3) × U(2) model: anomaly cancellation

Anomaly cancellation is **a new, independent step**. Before charge quantization, it reduces to the familiar anomaly conditions of $SU(3) \times SU(2) \times U(1) \times U(1)$.

HOW THE ANOMALIES SPLIT

$$U(N)^3 \longrightarrow SU(N)^3,\ \ SU(N)^2\,U(1),\ \ U(1)^3$$
$$\mathrm{grav}^2\,U(N) \longrightarrow \mathrm{grav}^2\,U(1)$$

Each $U(N)$ anomaly decomposes into its $SU(N)$ and $U(1)$ parts

We assume every fermion sits in the **fundamental** or **singlet** of $SU(N)$. Combined with the **charge constraints**, this leads to the conditions below.

$SU(3)^2\,U(1)_3$

$$N_{3L} = N_{3R} + 3k_3$$

Counts $SU(3)$ fundamentals, with $k_3 \in \mathbb{Z}$.

$SU(2)^2\,U(1)_2$

$$N_{2L} = 2k_2$$

Cancels for an even number of doublets, $k_2 \in \mathbb{Z}$. This coincides with the global Witten condition. [Witten '82]

$U(1)_i\,\mathrm{grav}^2$

$$2\ SU(N)\ \text{singlets}$$

Complete $SU(N)$ singlets must cancel the fundamentals' residual. Gravity alone allows one, but the cubic and mixed anomalies force **two**.

The minimal non-trivial and irreducible solution is $k_3 = 0$ and $k_2 = 2$.

The minimal anomaly-free content is exactly **one Standard Model generation plus an additional singlet.**

THE FERMIONS

The U(3) × U(2) model: Minimal fermion content

The minimal content is one SM generation plus one new singlet. Every charge follows from those of Q_L (q_i) and u_R (u_i).

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_3$	$U(1)_2$
Q_L	$\mathbf{3}$	$\mathbf{2}$	q_3	q_2
u_R	$\mathbf{3}$	$\mathbf{1}$	u_3	u_2
d_R	$\mathbf{3}$	$\mathbf{1}$	$2q_3 - u_3$	$2q_2 - u_2$
L_L	$\mathbf{1}$	$\mathbf{2}$	$-3q_3$	$-3q_2$
e_R	$\mathbf{1}$	$\mathbf{1}$	$-2q_3 - u_3$	$-2q_2 - u_2$
ν_R	$\mathbf{1}$	$\mathbf{1}$	$-4q_3 + u_3$	$-4q_2 + u_2$

All charges are written in terms of q_i, u_i. The remaining charge constraints are then satisfied automatically.

PREDICTION — I

Quantized charge & hypercharge

PREDICTED HYPERCHARGE

$$Y = \alpha\, Q_3 + \beta\, Q_2$$

$$\alpha = \frac{4q_2 - u_2}{6\,\Delta}, \qquad \beta = -\frac{4q_3 - u_3}{6\,\Delta}$$

Such a combination always exists provided $\Delta \equiv q_2 u_3 - q_3 u_2 \neq 0$. Otherwise the two $U(1)$s would coincide.

01 Charge is quantized

Anomaly cancellation alone fixes only the *relations* among charges, leaving q_i, u_i as continuous real parameters.

The compact $U(N)$ structure (the charge constraints) then **discretizes** them, so the SM's rational charge ratios emerge rather than being fitted.

02 Hypercharge is unique

The charge constraints rule out the other assignments the SM would tolerate:

Trivial $Y_i = 0 \;\; \forall i$

Vector-like $Y_Q = Y_L = Y_e = 0, \;\; Y_u = -Y_d$

Only the observed Y survives.

THE CORRECT SM HYPERCHARGES COME OUT ON THEIR OWN, INDEPENDENTLY OF q_i, u_i

Q_L	u_R	d_R
$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$
L_L	e_R	ν_R
$-\frac{1}{2}$	-1	0

PREDICTION — II

Right-handed neutrinos & B–L

ν_R

Must exist

Internal consistency requires right-handed neutrinos as part of the minimal content.

It is the singlet **neutral under hypercharge**.

B – L

Now gauged

$B - L$ is predicted as a local gauge symmetry, no longer accidental and global.

$$B - L = \gamma Q_3 + \delta Q_2$$

$$\gamma = \frac{q_2 - u_2}{3\Delta}, \quad \delta = -\frac{q_3 - u_3}{3\Delta}$$

Z'

New heavy boson

Recovering the SM keeps Y at low energy, so the physical Z' aligns with the **orthogonal** combination, not necessarily $B - L$.

$Y \perp B - L$ only if:

$$3(4q_2 - u_2)(q_2 - u_2) + 2(4q_3 - u_3)(q_3 - u_3) = 0$$

RECOVERING THE SM

The scalar sector & symmetry breaking

A singlet Φ breaks the extra $U(1)$ while **leaving Y unbroken**; a Higgs-like H then breaks the electroweak symmetry. Choosing $h_i = u_i - q_i$ recovers the full SM Yukawa sector:

$$\mathcal{L}_Y = y_d\,\bar{Q}Hd_R + y_u\,\bar{Q}\tilde{H}u_R + y_e\,\bar{L}He_R + y_\nu\,\bar{L}\tilde{H}\nu_R + \text{h.c.}$$

Any scalar compensating the charges of $\bar{Q}u_R$ does so for $\bar{Q}d_R$, $\bar{L}e_R$ and $\bar{L}\nu_R$ too.

Neutrality of Φ under Y fixes $(4q_3 - u_3)\,\phi_2 = (4q_2 - u_2)\,\phi_3$.

SYMMETRY BREAKING

$$U(3) \times U(2)$$

$$\downarrow \langle\Phi\rangle$$

$$\big(SU(3)_c \times SU(2)_L \times U(1)_Y\big)/\Gamma$$

$$\downarrow \langle H\rangle$$

$$\big(SU(3)_c \times U(1)_{\text{em}}\big)/\Gamma'$$

Exactly the SM Lagrangian, plus one complex singlet fermion and scalar.

A NATURAL CONSEQUENCE

Neutrino masses

ν_R is required and is **neutral under hypercharge**, carrying only the extra-$U(1)$ charge. So both the nature of neutrino masses and the origin of their smallness are decided by *how that* $U(1)$ *is broken*, i.e. by the charge of Φ.

(i) $q_\Phi = 2\,q_{\nu_R}$

Type-I seesaw

A Majorana mass $M_R = \lambda\langle\Phi\rangle$ is induced.

$$\tfrac{\lambda}{2}\,\overline{\nu_R^c}\,\Phi^*\,\nu_R$$

(ii) $q_\Phi = q_{\nu_R}$

Inverse seesaw

Adding a pure singlet ψ opens the inverse-seesaw structure.

$$y_\psi\,\overline{\nu_R^c}\,\Phi\,\psi + \tfrac{\mu}{2}\,\overline{\psi^c}\psi$$

(iii) any other q_Φ

Dirac neutrinos

No Majorana term is allowed; neutrinos remain purely Dirac.

All three follow from a single Yukawa, $y_\nu\,\bar{L}\tilde{H}\nu_R$, already present in the minimal model, the **seesaw scale** is then set by $\langle\Phi\rangle$.

WHERE IT LEADS

Beyond the minimal model

Dark matter

Breaking the new $U(1)$ can leave a residual $\mathbb{Z}_m$ discrete symmetry that **stabilizes a dark-matter candidate** with no extra symmetry imposed by hand. The order m is fixed by the charge of Φ [Krauss–Wilczek '89].

Unification

Promoting $SU(5) \to U(5)$ gives

$$U(5) \cong \frac{SU(5)\times U(1)}{\mathbb{Z}_5}$$

Which coincides with **flipped** $SU(5)$. And $U(3) \times U(2) \subset U(5)$.

The message generalizes: any GUT on $SU(N)$ should be promoted to $U(N)$, unifying all interactions in a single group with two distinct couplings.

A curiosity: the naive product $SU(3)_c \times SU(2)_L \times U(1)_Y$ does *not* fit inside $SU(5)$, only $\big(SU(3) \times SU(2) \times U(1)\big)/\mathbb{Z}_6$ fits. Global structure, once again.

Plenty of novel model-building directions still to explore.

HOW WE WOULD TELL THEM APART

Perturbatively identical, physically not

$$U(3) \times U(2) \overset{\text{PERTURBATIVELY}}{\simeq} SU(3)_c \times SU(2)_L \times U(1)_a \times U(1)_b$$

Same algebra, same Feynman rules.

The familiar Z' searches and neutral gauge-boson mixing carry over unchanged.

But non-perturbative effects know about the **global structure** of the group, not just its algebra: that is where the two part ways.

01

Axions & topology

Add an axion for strong CP: the quantized axion–gauge-boson couplings and the formation of domain walls are fixed by the global structure.

02

Forbidden representations

Reps allowed by the algebra but absent from $U(3) \times U(2)$. Observe one and the hypothesis is dead.

03

Beyond phenomenology

A symmetry-based reason for $U(1)$-extension features that would otherwise be entirely ad hoc.

TAKEAWAYS

Take the gauge principle to its limit: promote every $SU(N)$ to $U(N)$.

THE IDEA

- **Take the gauge principle to its limit**: promote every $SU(N)$ factor to $U(N)$, so each carries its own $U(1)$ phase.

THE $U(3) \times U(2)$ MODEL

- **Explains** charge quantization, the observed hypercharge, and the forbidden representations.
- The single anomaly-free generation (one SM family plus ν_R) is forced, nothing added by hand.
- **Predicts** right-handed neutrinos, gauged $B - L$ and a new heavy Z'.

THE EXTENSIONS

- Variants of the seesaw naturally implemented, with the scale set by $B - L$ breaking.
- **Extends** naturally: a residual $\mathbb{Z}_m$ can stabilize dark matter, and $U(3) \times U(2) \subset U(5)$, namely flipped $SU(5)$.

Every $SU(N)$ should carry its phase: embracing the gauge principle in full opens new avenues for model building.

APPENDIX

Backup

Anomaly cancellation · the center & N-ality · the covariant derivative

BACKUP — I

Anomaly cancellation

Each anomaly forces the next piece of matter. Nothing is put in by hand. The whole generation is dictated by consistency.

01 · $SU(3)^2U(1)_3$

$$N_{3L} = N_{3R} + 3k_3$$

Its counting, together with the $SU(3)^3$ triplet–antitriplet balance, needs the quarks Q, u_R, d_R: $k_3 = 0$.

⇒ quarks

02 · WITTEN

$$N_{2L} = 2k_2$$

Q is three colored doublets: odd. A lepton doublet L restores parity: $k_2 = 2$.

⇒ lepton doublet

03 · $SU(N)^2U(1)$

$$l_i = -3q_i,\ d_i = 2q_i - u_i$$

With the quarks and L present, the mixed anomalies lock every charge to the hypercharge pattern.

⇒ charges fixed

04 · GRAV² & $U(1)^3$

$$e_R,\ \nu_R$$

Two $U(1)$ factors demand two singlets — e_R completes the SM, ν_R is new.

⇒ two singlets

THE FULL SOLUTION

$$l_i = -3q_i, \qquad d_i = 2q_i - u_i,$$
$$\nu_i = -4q_i + u_i, \qquad e_i = -2q_i - u_i$$

A TWOFOLD DEGENERACY

The system is invariant under $\nu_R \leftrightarrow e_R$: a second solution describes the same physics under a relabeling of the two $SU(2)$ singlets. Removing any field breaks one of the four conditions.

The center of $SU(N)$ and N-ality

The center

The center of $SU(N)$ is $\mathbb{Z}_N$, the matrices $z_k = e^{2\pi i k/N}\,\mathbf{1}$ with $k = 0, \ldots, N-1$, all commuting with every group element.

- $SU(2)$: $\{\pm 1\}$
- $SU(3)$: $\{1,\ e^{2\pi i/3},\ e^{4\pi i/3}\}$

The N-ality

On an irrep R the center acts as a single phase, $R(z_k) = e^{2\pi i k q/N}\,\mathbf{1}$. The integer $q \in \{0, \ldots, N-1\}$ is the **N-ality**, the phase a representation picks up under the center.

$SU(2)$ IRREPS · 2-ALITY

spin	dim	name	2-ality
$j = 0$	$\mathbf{1}$	singlet	0
$j = \frac{1}{2}$	$\mathbf{2}$	doublet	**1**
$j = 1$	$\mathbf{3}$	triplet	0
$j = \frac{3}{2}$	$\mathbf{4}$	—	**1**

A spin-j irrep carries the phase $(-1)^{2j}$: even-dimensional reps are odd, odd-dimensional even.

> An irrep of $SU(N)$ lifts to a rep of $U(N)$ only if its $U(1)$ charge matches the N-ality, $q_{U(1)} \equiv q \pmod{N}$.

Two couplings, one group

$$D_\mu = \partial_\mu - i\, g_a A^a_\mu T^a$$

A single $U(N)$ admits **two independent gauge couplings**: one for the $SU(N)$ generators, one for the $U(1)$. Different couplings are allowed for any generators that commute; the $U(1)$ generator $\propto \mathbb{1}$ commutes with all others. The physics is fixed by the Lie algebra.

$$A^a_\mu\, \alpha^b\, (g_a - g_c)\, f^{abc}\, T^c\, \phi = 0$$

Covariance $D_\mu\psi \to U D_\mu\psi$ needs equal couplings only for generators linked by a non-zero structure constant f^{abc}. For the $U(1)$ generator all $f^{abc} = 0$, so its coupling is free.