

RG Running of Multiple Neutrino Mixing Parameters at Oscillation Experiments

Chui-Fan Kong

kongcf@ibs.re.kr
IBS-CTPU-PTC

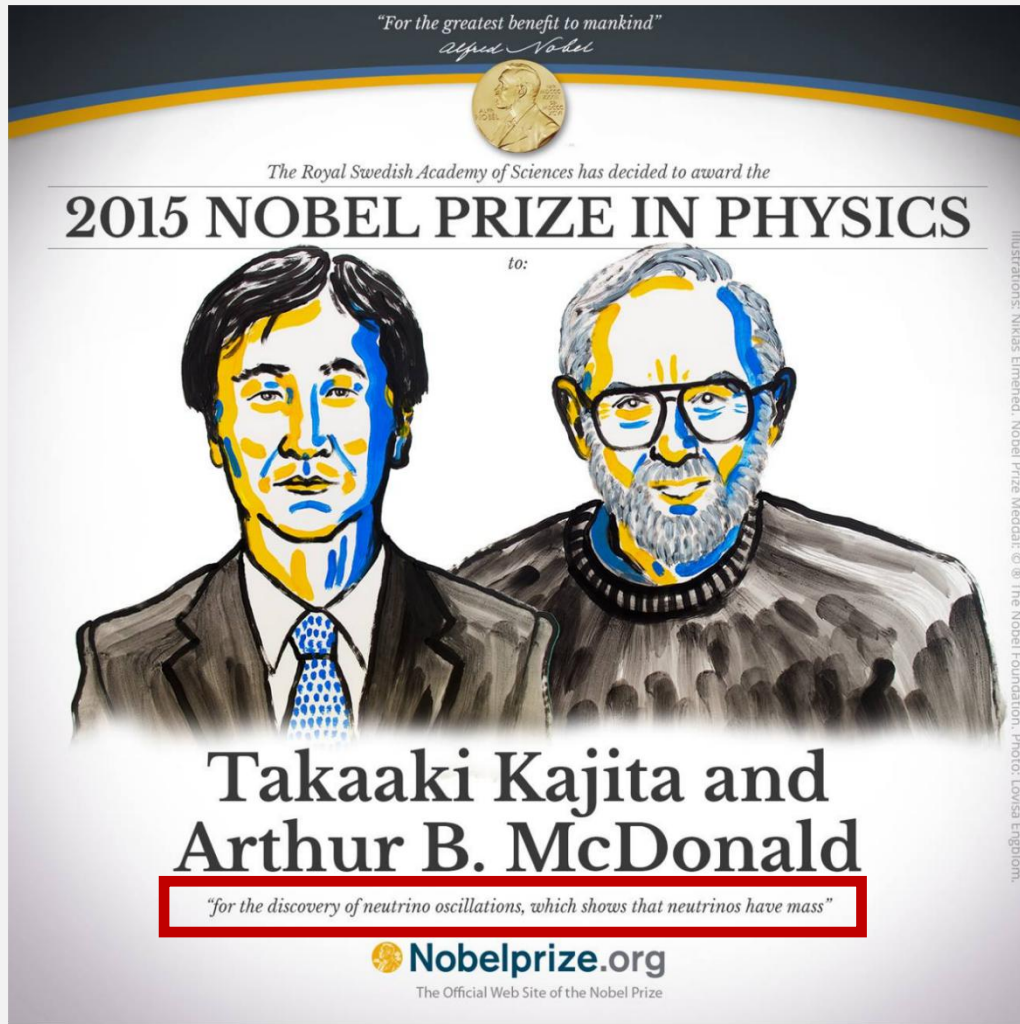
Based on arXiv:2606.26720 [hep-ph]

In collaboration with:

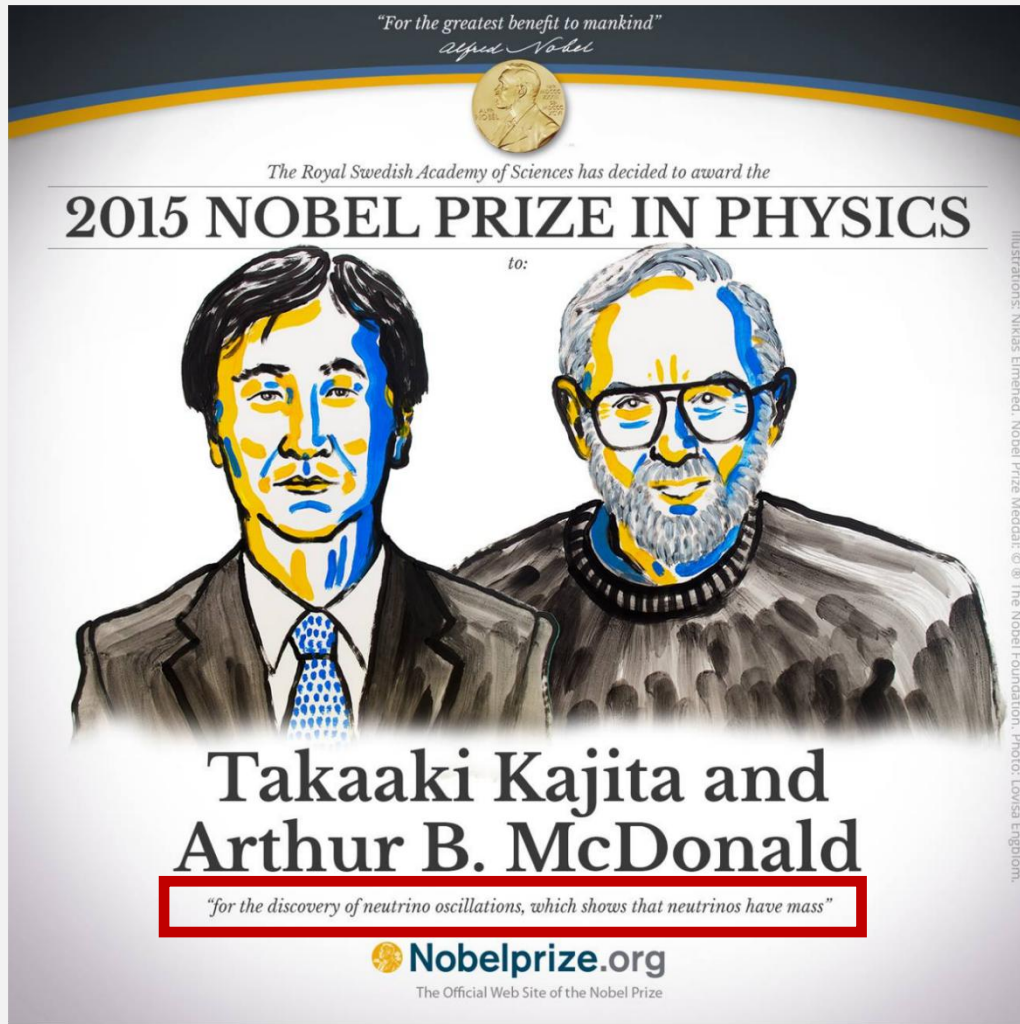
Peter B. Denton (BNL), Shao-Feng Ge (TDLI & SJTU)

Pedro Pasquini (UNICAMP)





“for the discovery of neutrino oscillations, which shows that neutrinos have mass”



“for the discovery of neutrino oscillations, which shows that neutrinos have mass”

Neutrino oscillation $\xrightarrow{\text{indicates}}$ New physics?!
 $\xleftarrow{\text{can be tested}}$

- Neutrino oscillation in a nutshell
- RG running effect on neutrino oscillation
- Experimental sensitivities

Neutrino oscillation in a nutshell

See also talks by Jose Valle and Byeongsu Yang

Mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



Neutrino Oscillation in A Nutshell

Mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavor eigenstates $\longleftrightarrow$ mixing matrix $\longleftrightarrow$ mass eigenstates



ν_α

source

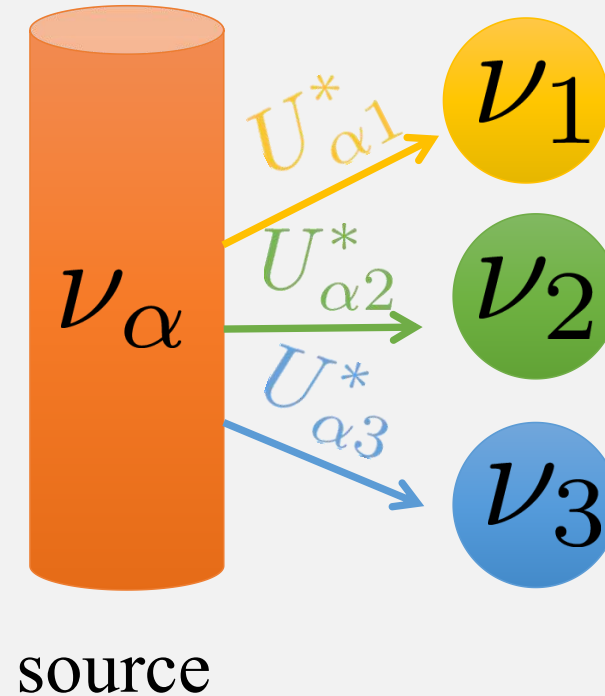
$$\alpha, \beta \in \{e, \mu, \tau\}$$

Neutrino Oscillation in A Nutshell

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Production:

$$|\nu_\alpha\rangle = U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle$$

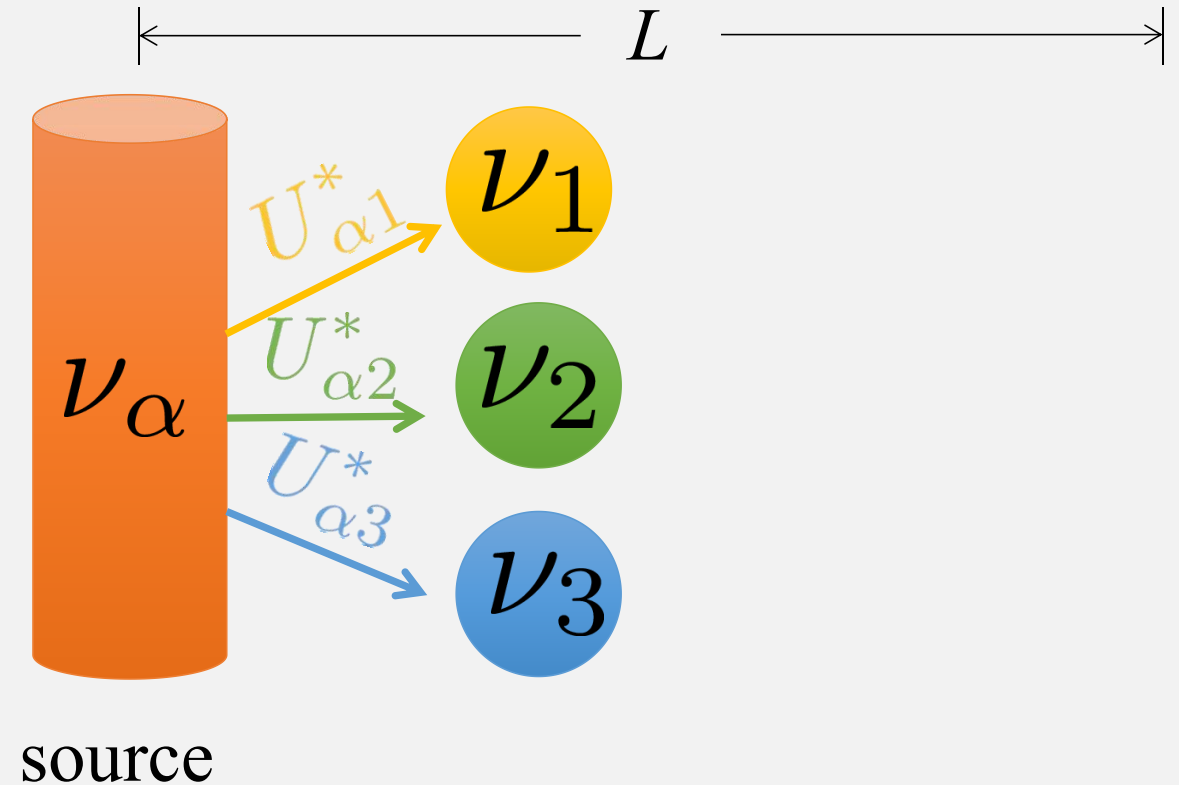


$$\alpha, \beta \in \{e, \mu, \tau\}$$

Neutrino Oscillation in A Nutshell

Propagation after a distance L :

$$\begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \\ e^{-i\frac{m_1^2}{2E}L} & e^{-i\frac{m_2^2}{2E}L} & e^{-i\frac{m_3^2}{2E}L} \end{array}$$



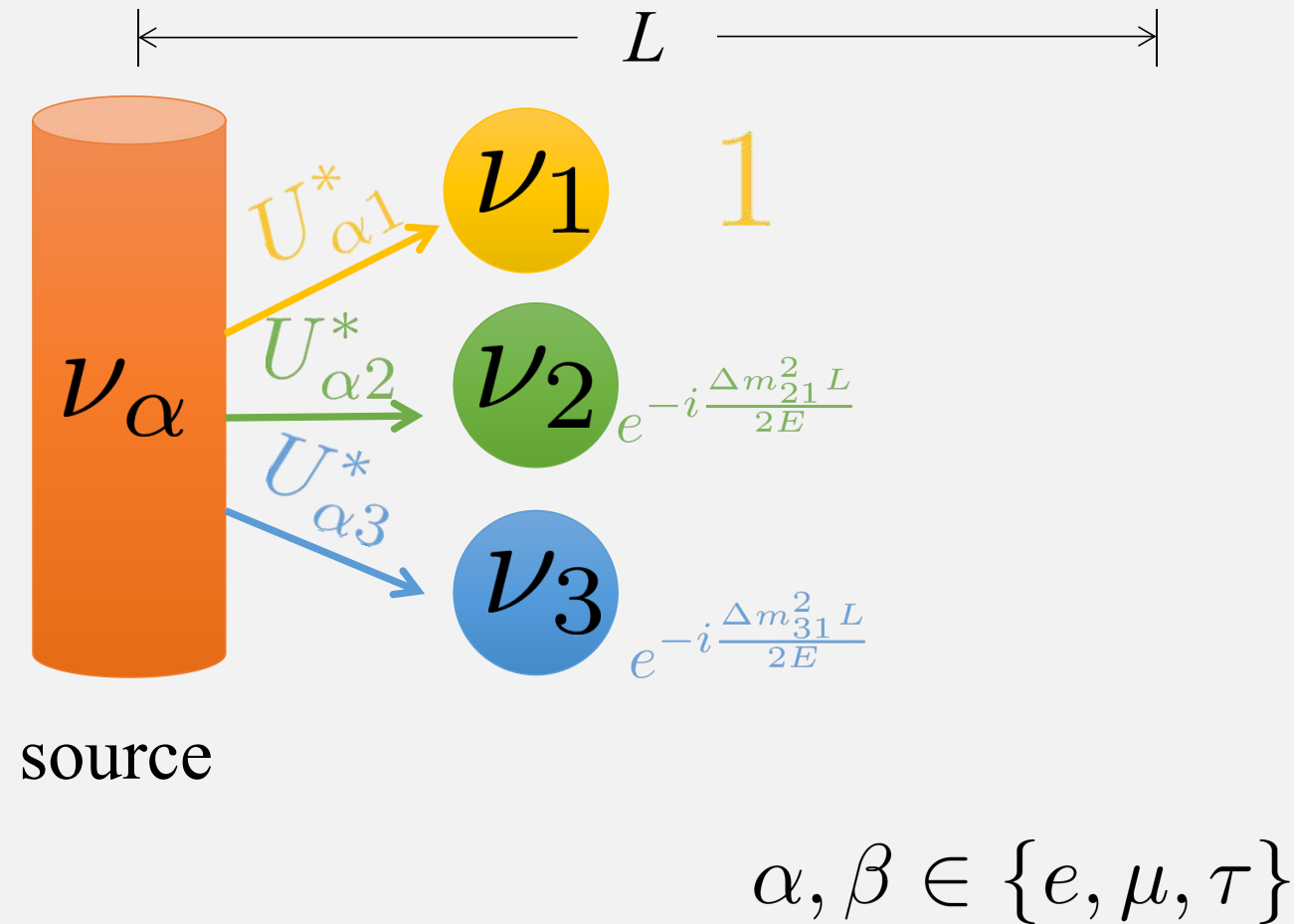
$$\alpha, \beta \in \{e, \mu, \tau\}$$

Neutrino Oscillation in A Nutshell

$$[\Delta m_{21}^2 \equiv m_2^2 - m_1^2, \\ \Delta m_{31}^2 \equiv m_3^2 - m_1^2]$$

Propagation after a distance L :

ν_1	ν_2	ν_3
$e^{-i\frac{m_1^2}{2E}L}$	$e^{-i\frac{m_2^2}{2E}L}$	$e^{-i\frac{m_3^2}{2E}L}$
↓	↓	↓
1	$e^{-i\frac{\Delta m_{21}^2}{2E}L}$	$e^{-i\frac{\Delta m_{31}^2}{2E}L}$

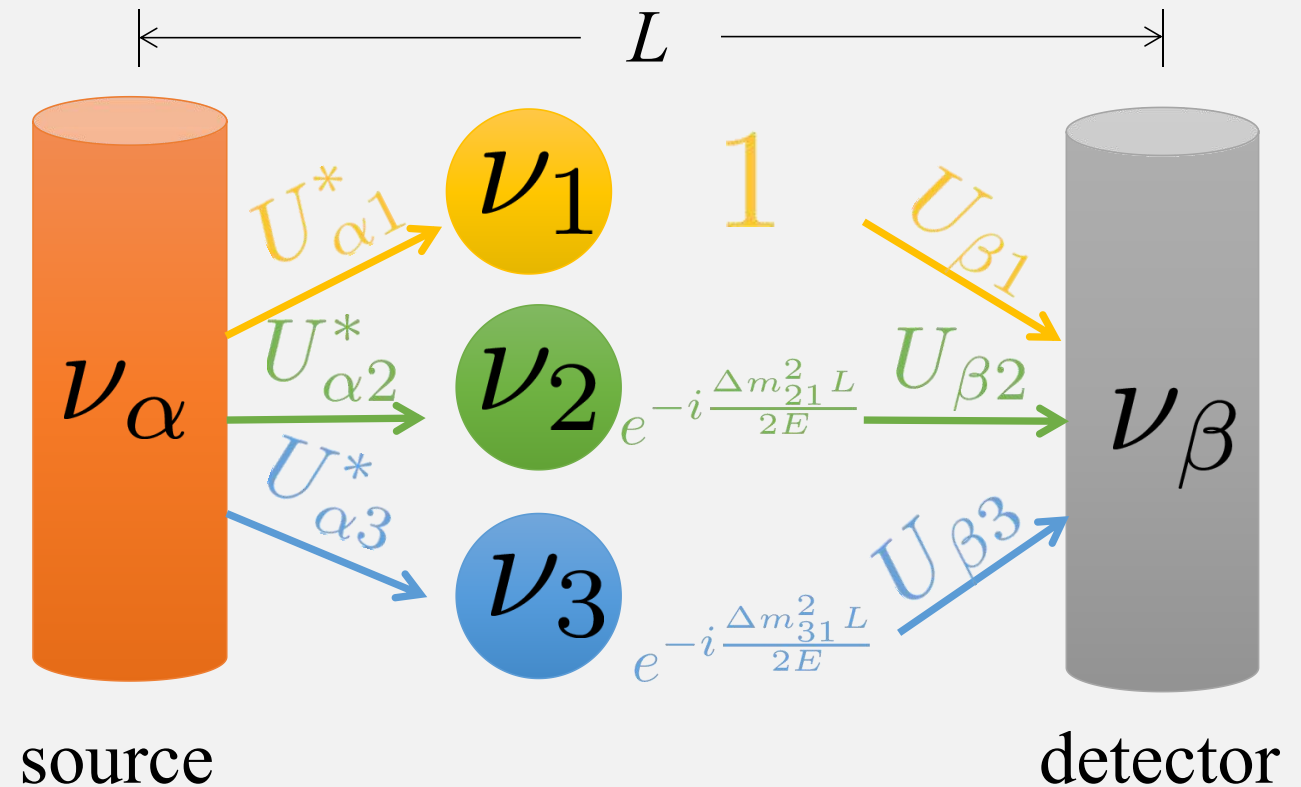


Neutrino Oscillation in A Nutshell

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Detection:

$$\langle \nu_\beta | = U_{\beta 1} \langle \nu_1 | + U_{\beta 2} \langle \nu_2 | + U_{\beta 3} \langle \nu_3 |$$



Neutrino Oscillation in A Nutshell

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Production:

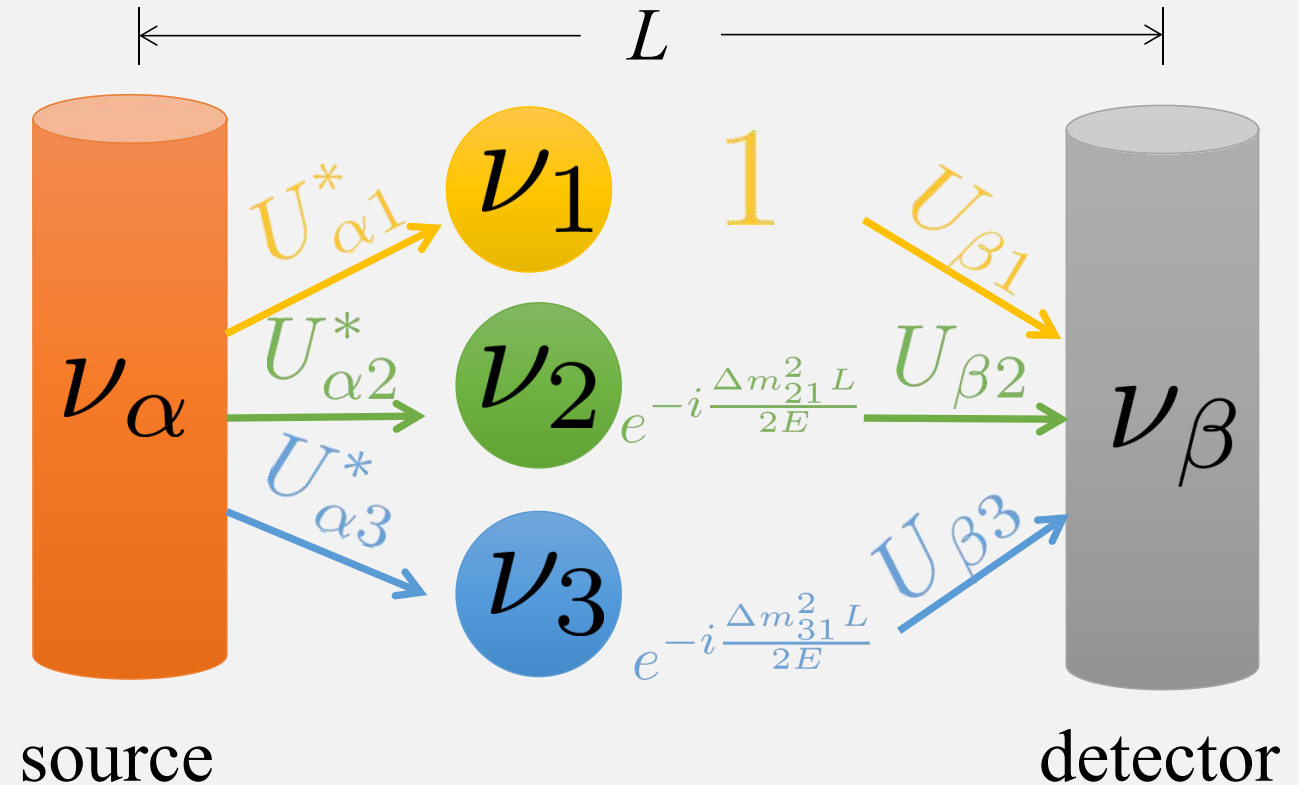
$$|\nu_\alpha\rangle = U_{\alpha 1}^* |\nu_1\rangle + U_{\alpha 2}^* |\nu_2\rangle + U_{\alpha 3}^* |\nu_3\rangle$$

Propagation:

$$e^{-i \frac{\Delta m_{21}^2 L}{2E}} \quad e^{-i \frac{\Delta m_{31}^2 L}{2E}}$$

Detection:

$$\langle \nu_\beta | = U_{\beta 1} \langle \nu_1 | + U_{\beta 2} \langle \nu_2 | + U_{\beta 3} \langle \nu_3 |$$



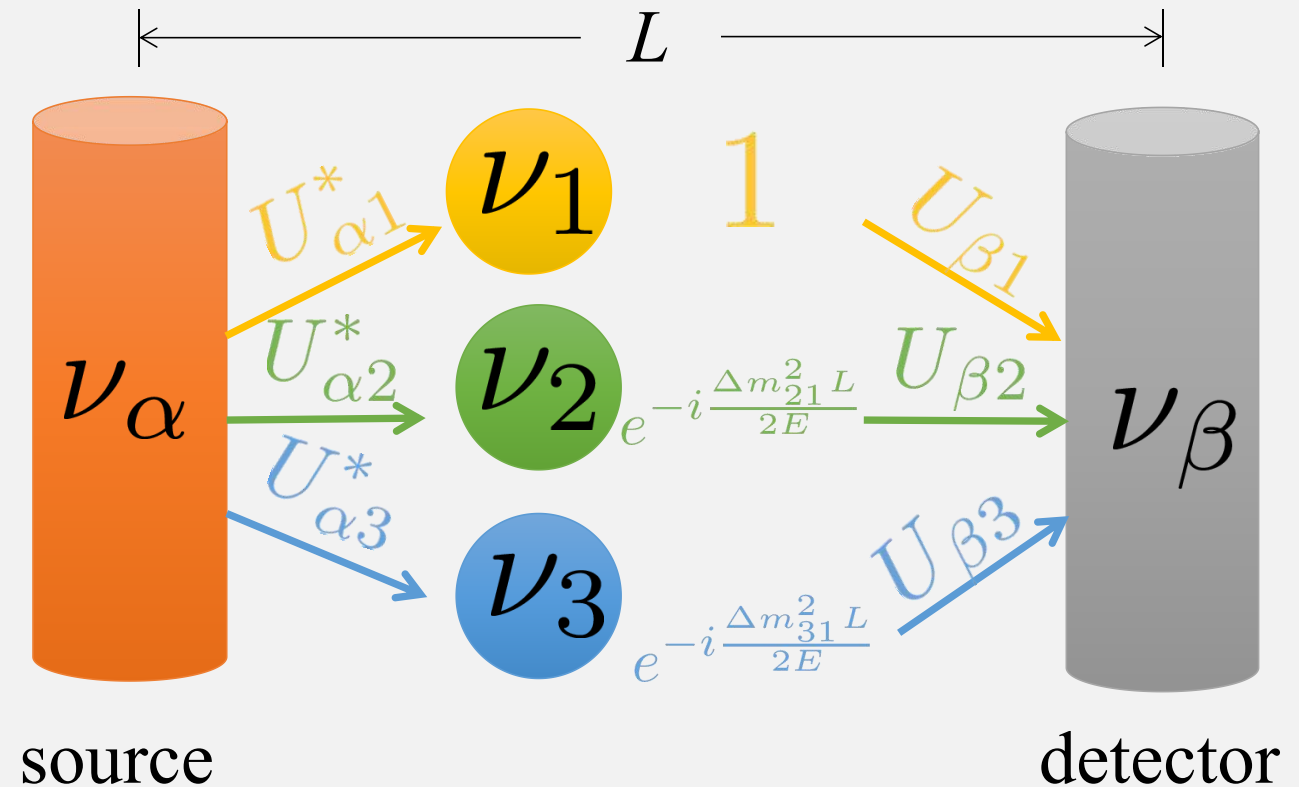
Neutrino Oscillation in A Nutshell

- Oscillation amplitude

$$\begin{aligned}\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = & U_{\alpha 1}^* U_{\beta 1} \\ & + U_{\alpha 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} U_{\beta 2} \\ & + U_{\alpha 3}^* e^{-i \frac{\Delta m_{31}^2 L}{2E}} U_{\beta 3}\end{aligned}$$

- Oscillation probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2$$



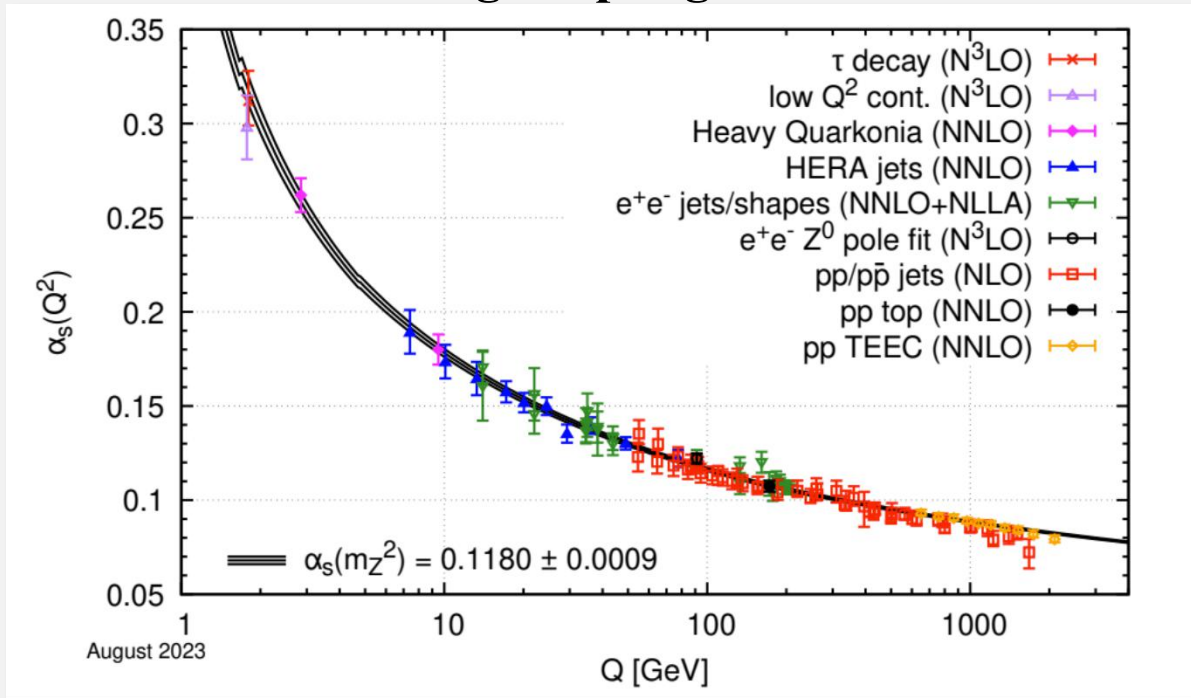
$$\alpha, \beta \in \{e, \mu, \tau\}$$

RG running effect on neutrino oscillation

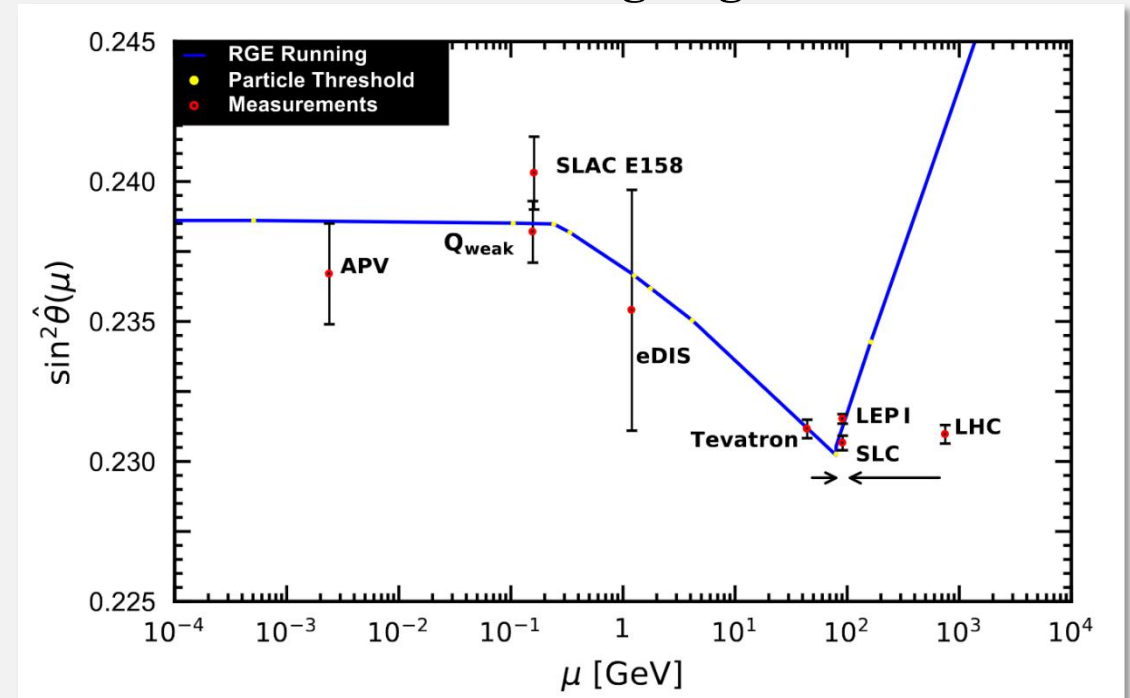
RG Running Effect

The **loop corrections** lead to RG running effect:

strong coupling constant



weak mixing angle

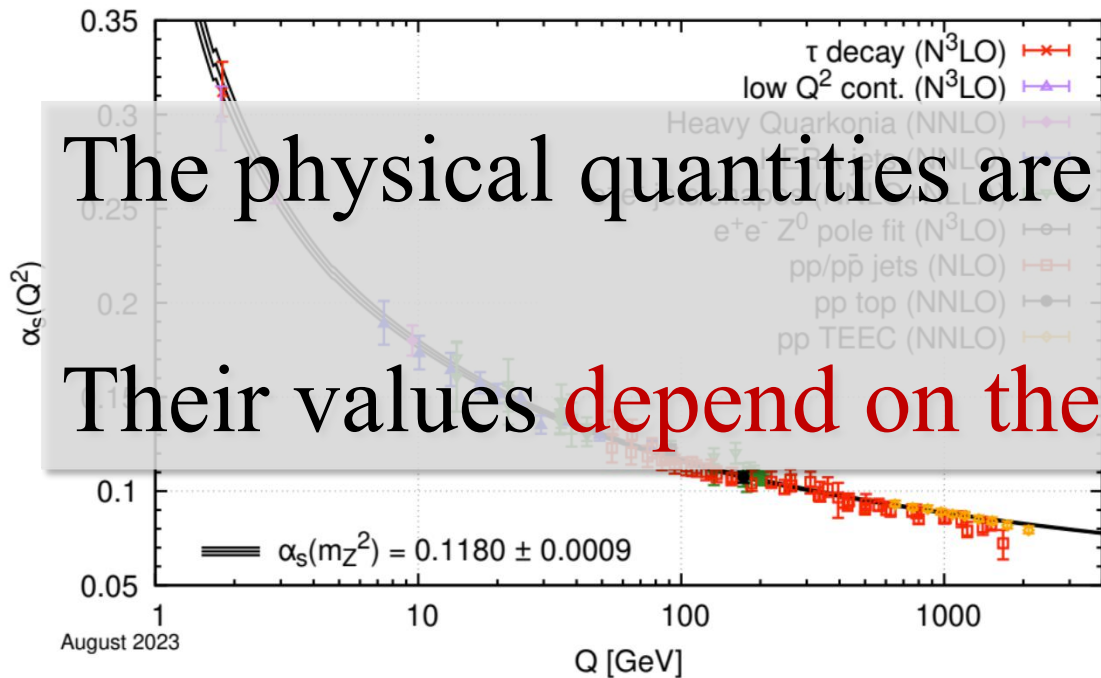


credit: Particle Data Group

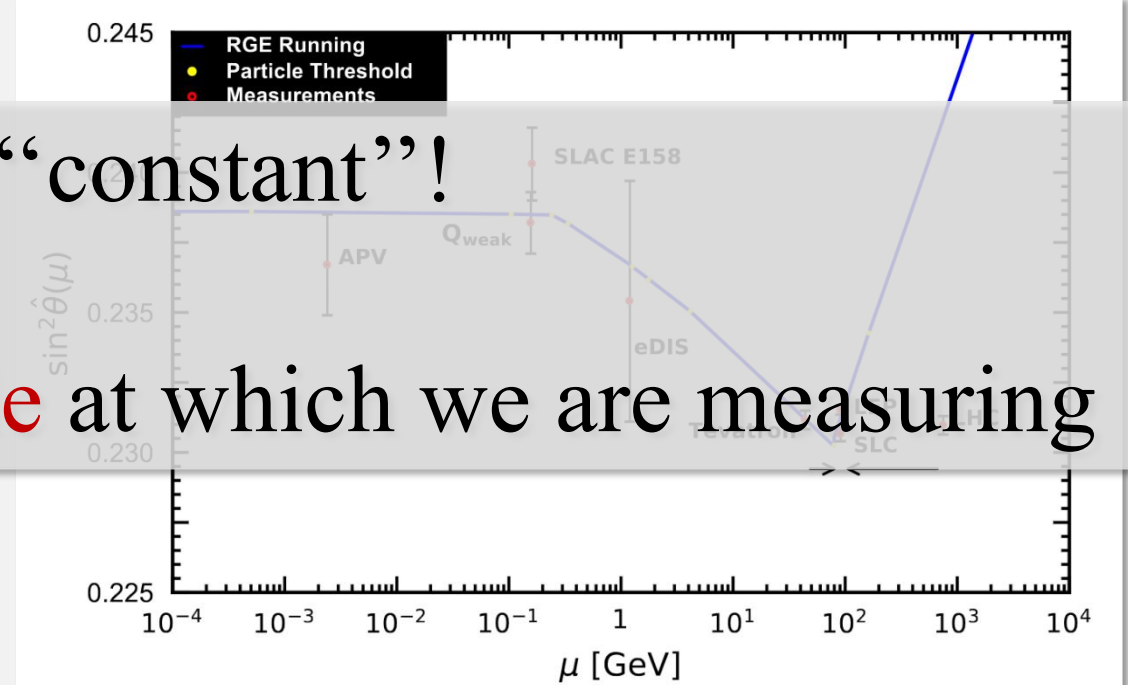
RG Running Effect

The **loop corrections** lead to RG running effect:

strong coupling constant



weak mixing angle

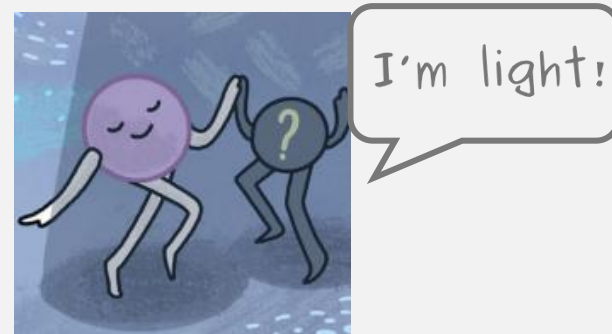


The physical quantities are not “constant”!
Their values **depend on the scale** at which we are measuring

credit: Particle Data Group

RG Running of Neutrino Mixing Parameters

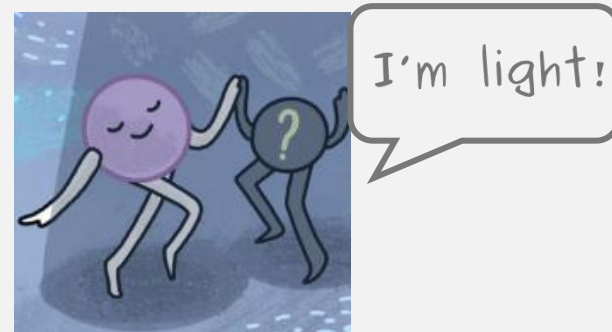
What happens if neutrinos interact with **new light degree of freedoms**?



credit: symmetry

RG Running of Neutrino Mixing Parameters

What happens if neutrinos interact with **new light degree of freedoms**?

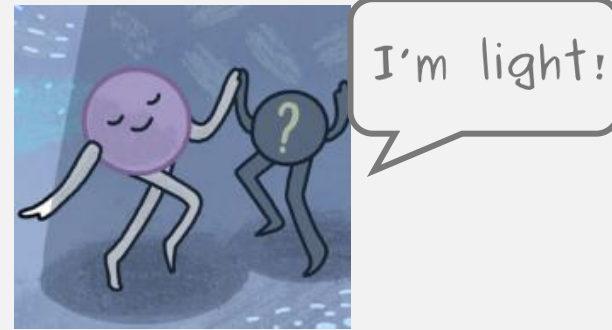


credit: symmetry

Loop corrections can happen at low scales

RG Running of Neutrino Mixing Parameters

What happens if neutrinos interact with **new light degree of freedoms**?



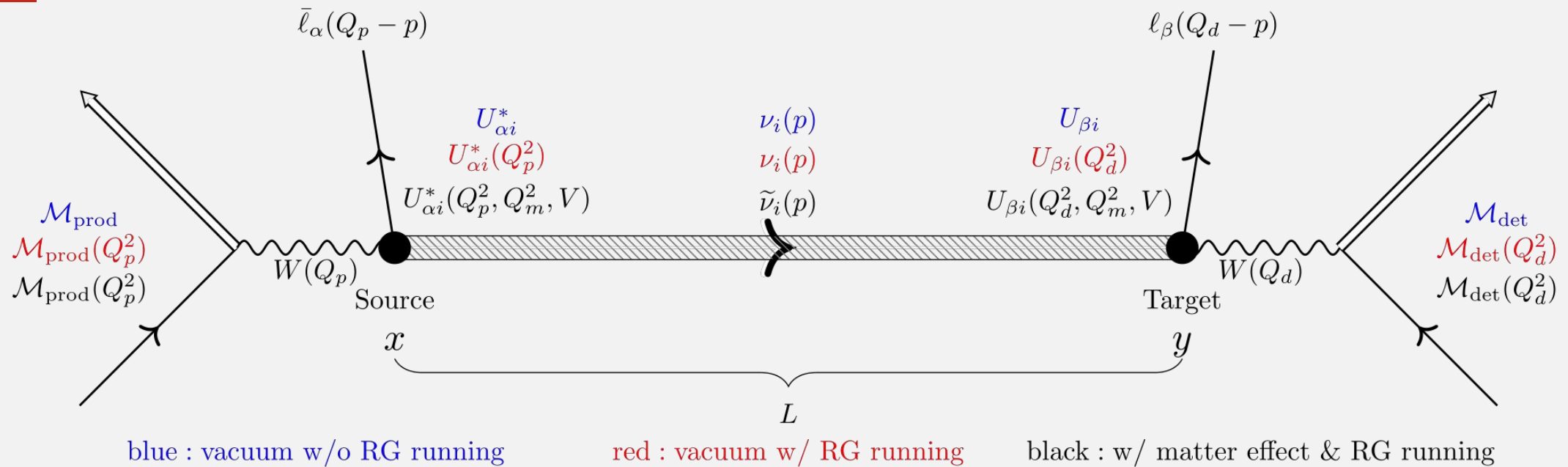
credit: symmetry

Loop corrections can happen at low scales



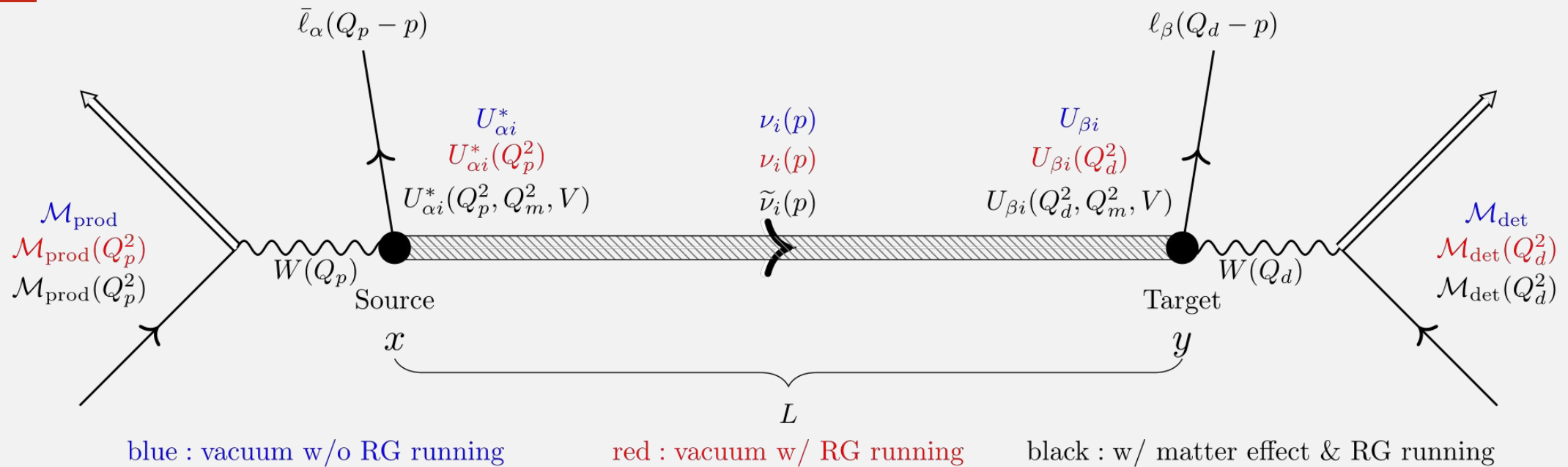
Neutrino mixing parameters become **scale-dependent**: $U \rightarrow U(Q^2)$

Oscillation w/ RG Running Effect



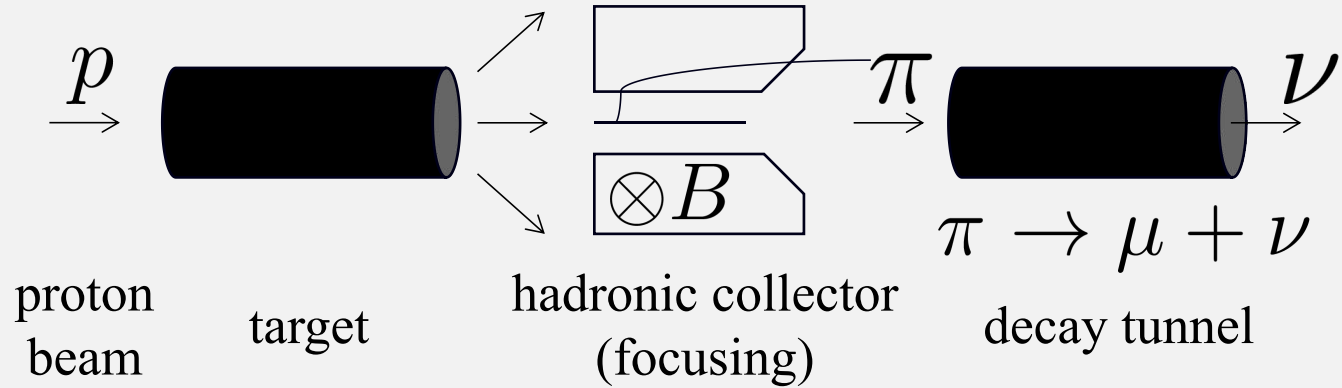
- w/o RG running: $\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E_\nu}} U_{\beta i}$
- w/ RG running: $\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^*(Q_p^2) e^{-im_i^2 \frac{L}{2E_\nu}} U_{\beta i}(Q_d^2)$

Oscillation w/ RG Running Effect

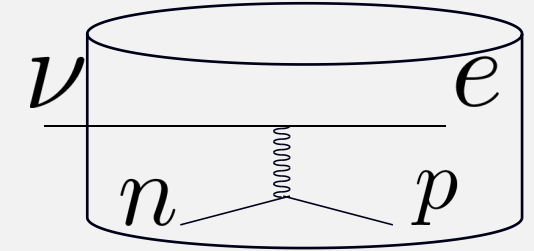


- w/o RG running: $\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E_\nu}} U_{\beta i}$
- w/ RG running: $\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i \boxed{U_{\alpha i}^*(Q_p^2)} e^{-im_i^2 \frac{L}{2E_\nu}} \boxed{U_{\beta i}(Q_d^2)}$

RG Running at Oscillation Experiments



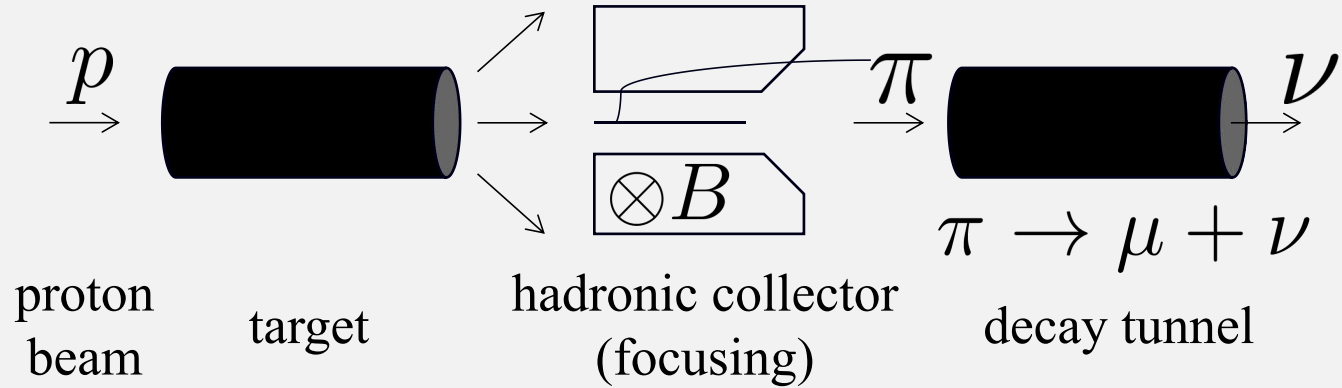
neutrino production
(particle decay)



detector

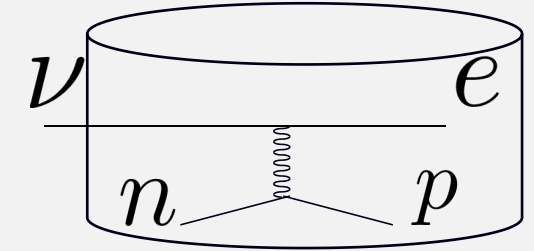
neutrino detection
(scattering)

RG Running at Oscillation Experiments



neutrino production
(particle decay)

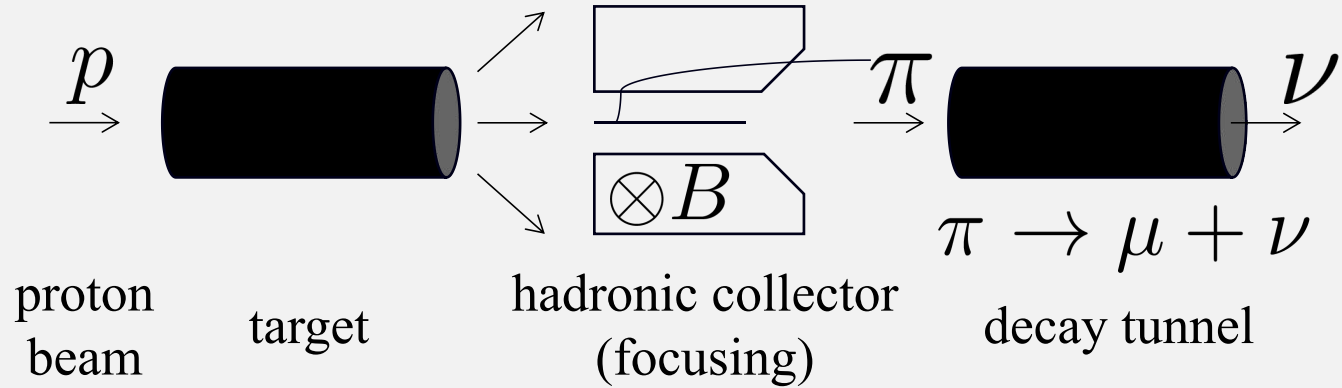
$$U(Q_p^2)$$



neutrino detection
(scattering)

$$U(Q_d^2)$$

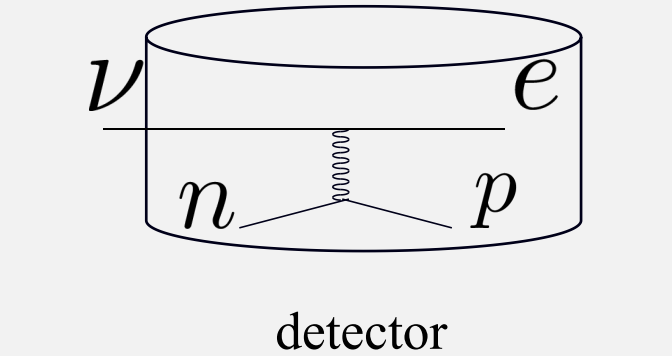
RG Running at Oscillation Experiments



neutrino production
(particle decay)

$$U(Q_p^2)$$

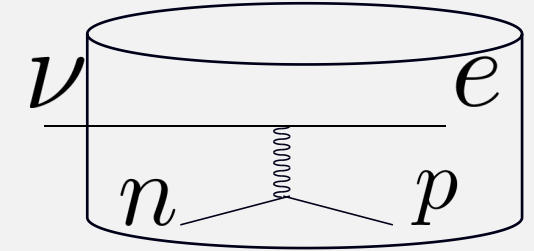
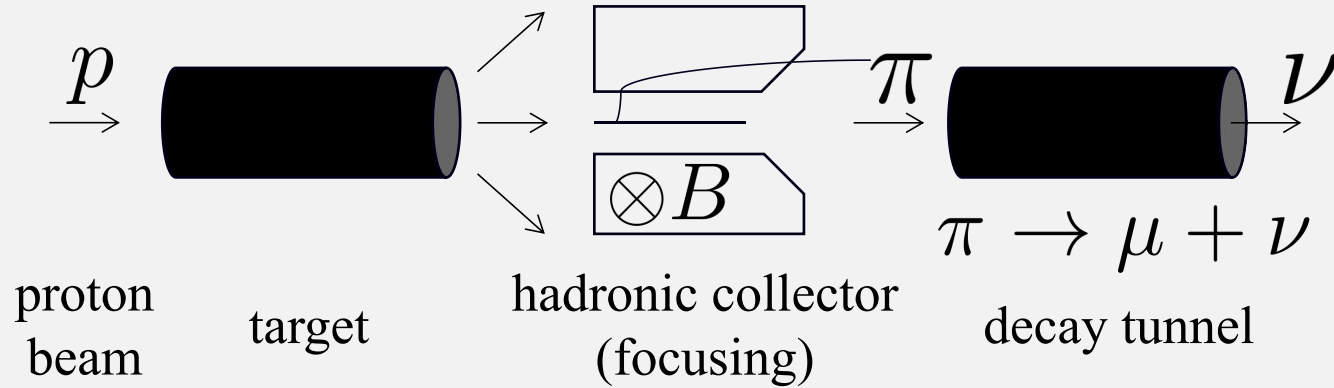
$\neq$



neutrino detection
(scattering)

$$U(Q_d^2)$$

RG Running at Oscillation Experiments



neutrino production
(particle decay)

$$U(Q_p^2)$$

$\neq$

neutrino detection
(scattering)

$$U(Q_d^2)$$

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i \boxed{U_{\alpha i}^*(Q_p^2)} e^{-im_i^2 \frac{L}{2E_\nu}} \boxed{U_{\beta i}(Q_d^2)}$$

Experimental sensitivities

We take a **model-independent** parametrization

$$\frac{dX}{d \ln |Q^2|} = \beta_X$$

Solution:

$$X(Q^2) \approx X(Q_0^2) + \beta_X \ln \left(\left| \frac{Q^2}{Q_0^2} \right| \right)$$

We take a **model-independent** parametrization

$$\theta_{ij}(Q^2) \equiv \theta_{ij}(Q_0^2) + \boxed{\beta_{ij}} \ln \left(\left| \frac{Q^2}{Q_0^2} \right| \right)$$

$$\delta_D(Q^2) \equiv \delta_D(Q_0^2) + \boxed{\beta_\delta} \ln \left(\left| \frac{Q^2}{Q_0^2} \right| \right)$$

β parameters **quantify** the RG running effect

For model-dependent case, refer to

K. S. Babu, V. Brdar, A. de Gouvêa, P. A.N. Machado, arXiv: 2108.11961

Pheno Results: Zero-distance Effect

- w/o RG running effect:

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^* e^{-i m_i^2 \frac{L}{2E_\nu}} U_{\beta i} \xrightarrow{L \rightarrow 0} \sum_i U_{\alpha i}^* U_{\beta i} = \delta_{\alpha\beta}$$

no flavor transition in the zero-distance limit!

Pheno Results: Zero-distance Effect

- w/o RG running effect:

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E_\nu}} U_{\beta i} \xrightarrow{L \rightarrow 0} \sum_i U_{\alpha i}^* U_{\beta i} = \delta_{\alpha\beta}$$

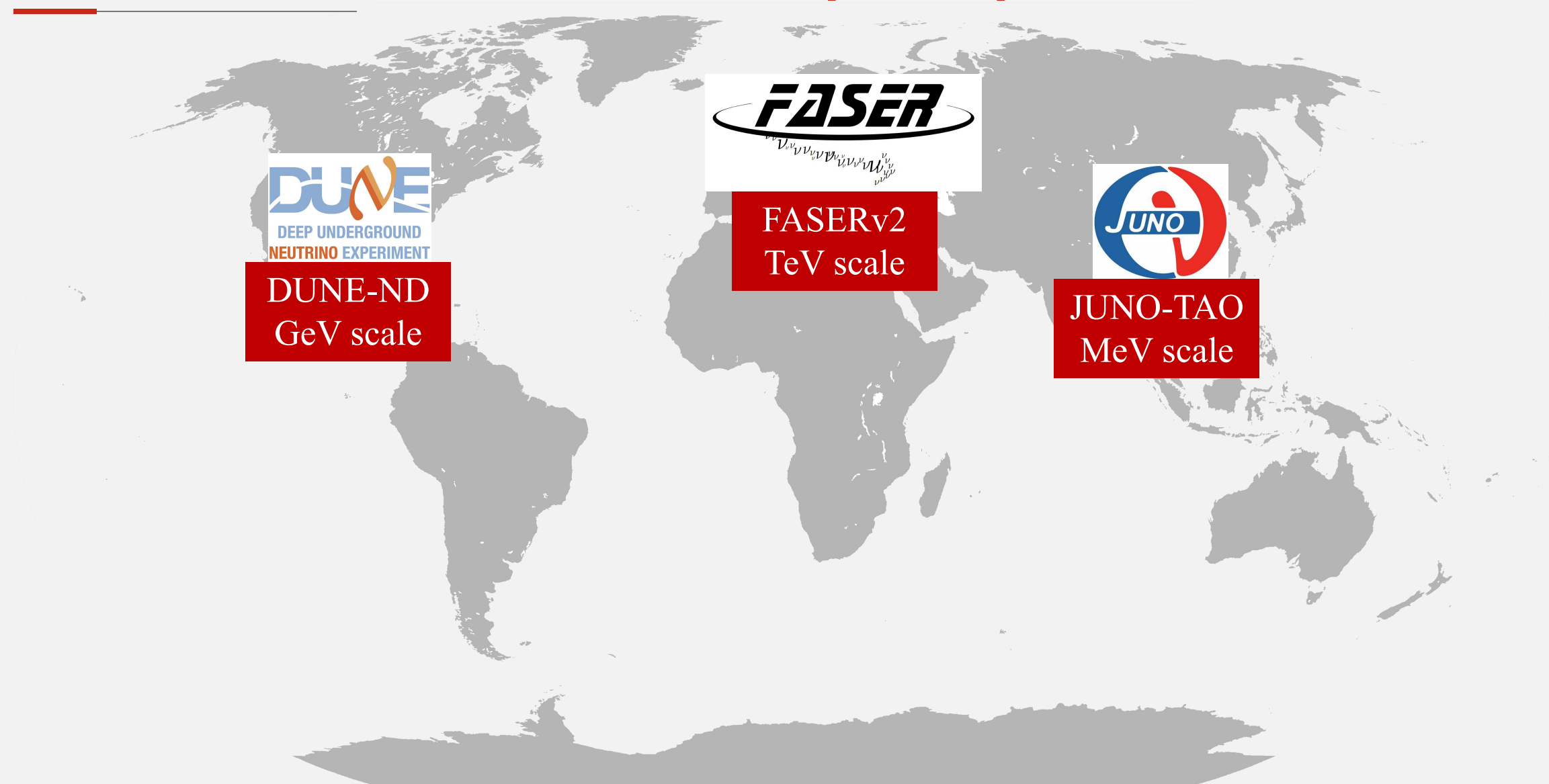
no flavor transition in the zero-distance limit!

- w/ RG running effect:

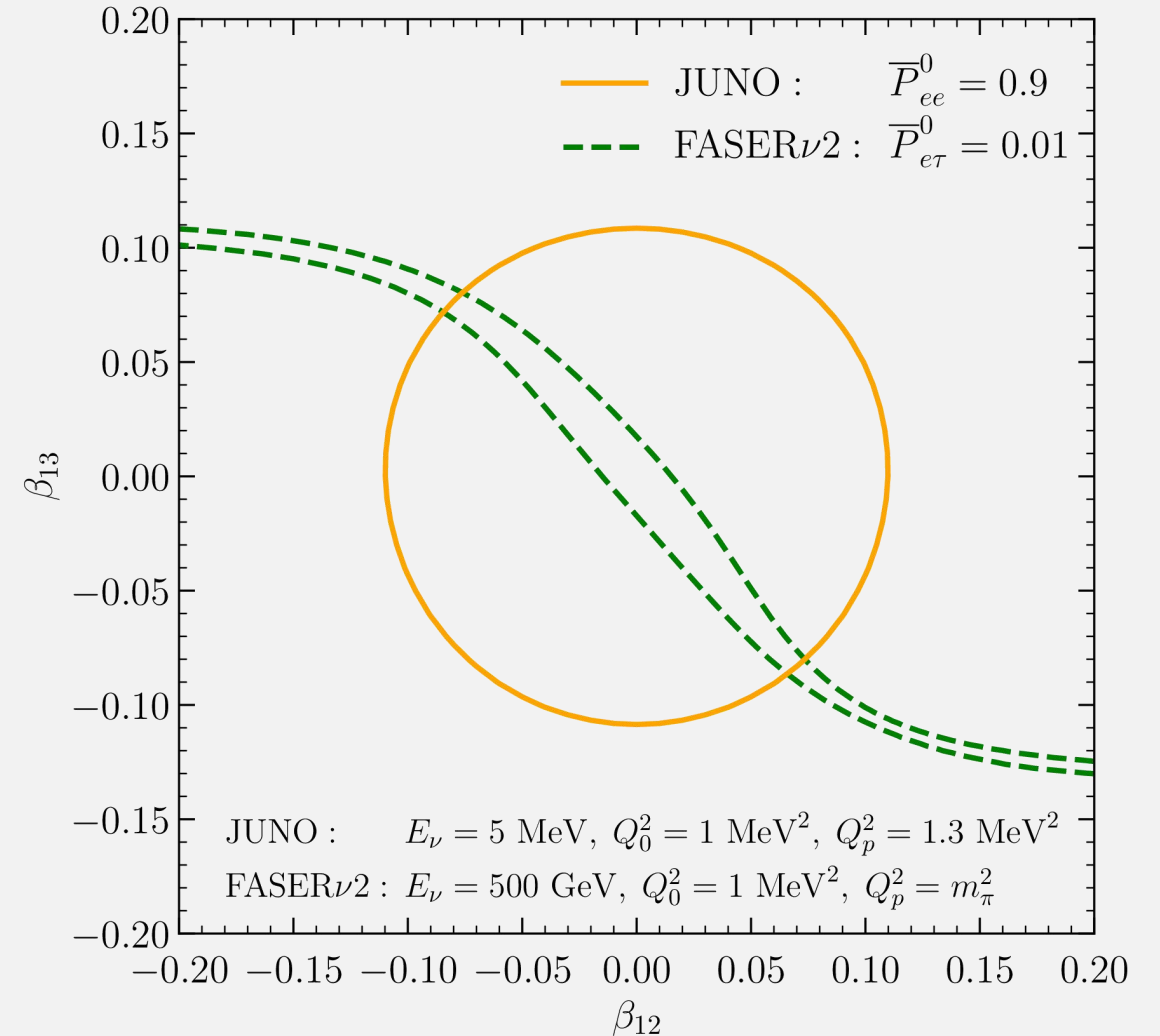
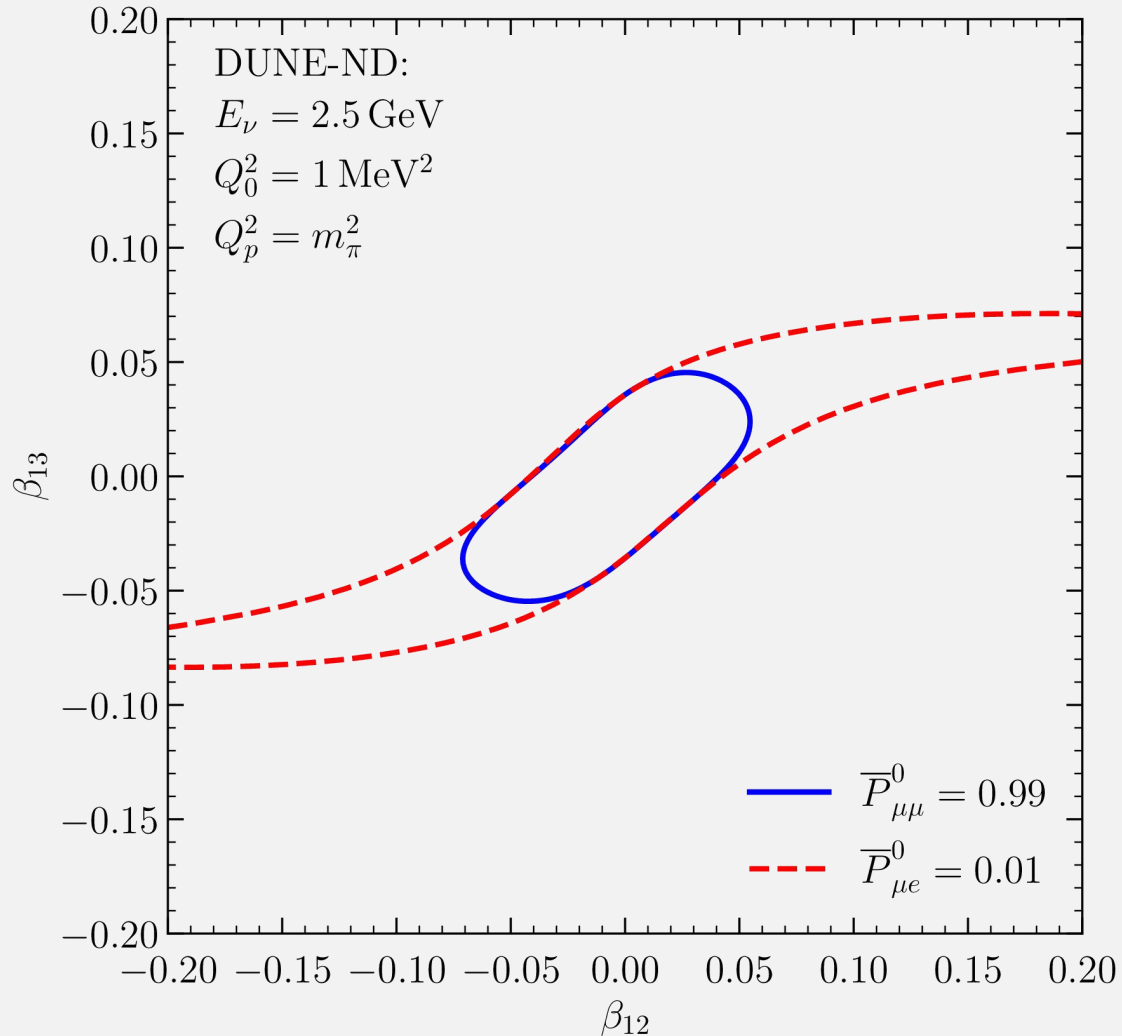
$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}^*(Q_p^2) e^{-im_i^2 \frac{L}{2E_\nu}} U_{\beta i}(Q_d^2) \xrightarrow{L \rightarrow 0} \sum_i U_{\alpha i}^*(Q_p^2) U_{\beta i}(Q_d^2) \neq \delta_{\alpha\beta}$$

flavor transition in the zero-distance limit!

Pheno Results: Effect at Multiple Exps

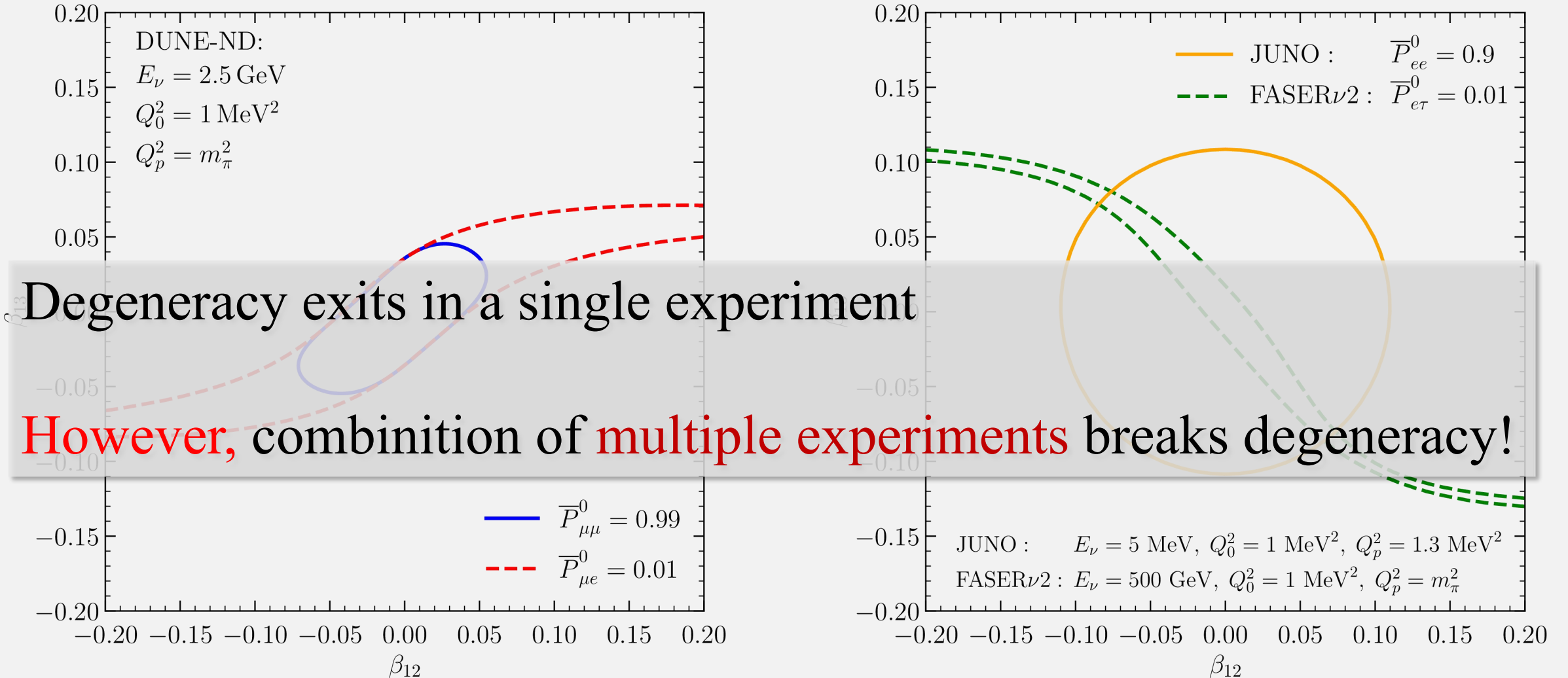


Pheno Results: Effect at Multiple Exps



Denton, Ge, CFK, Pasquini, arXiv: 2606.26720

Pheno Results: Effect at Multiple Exps



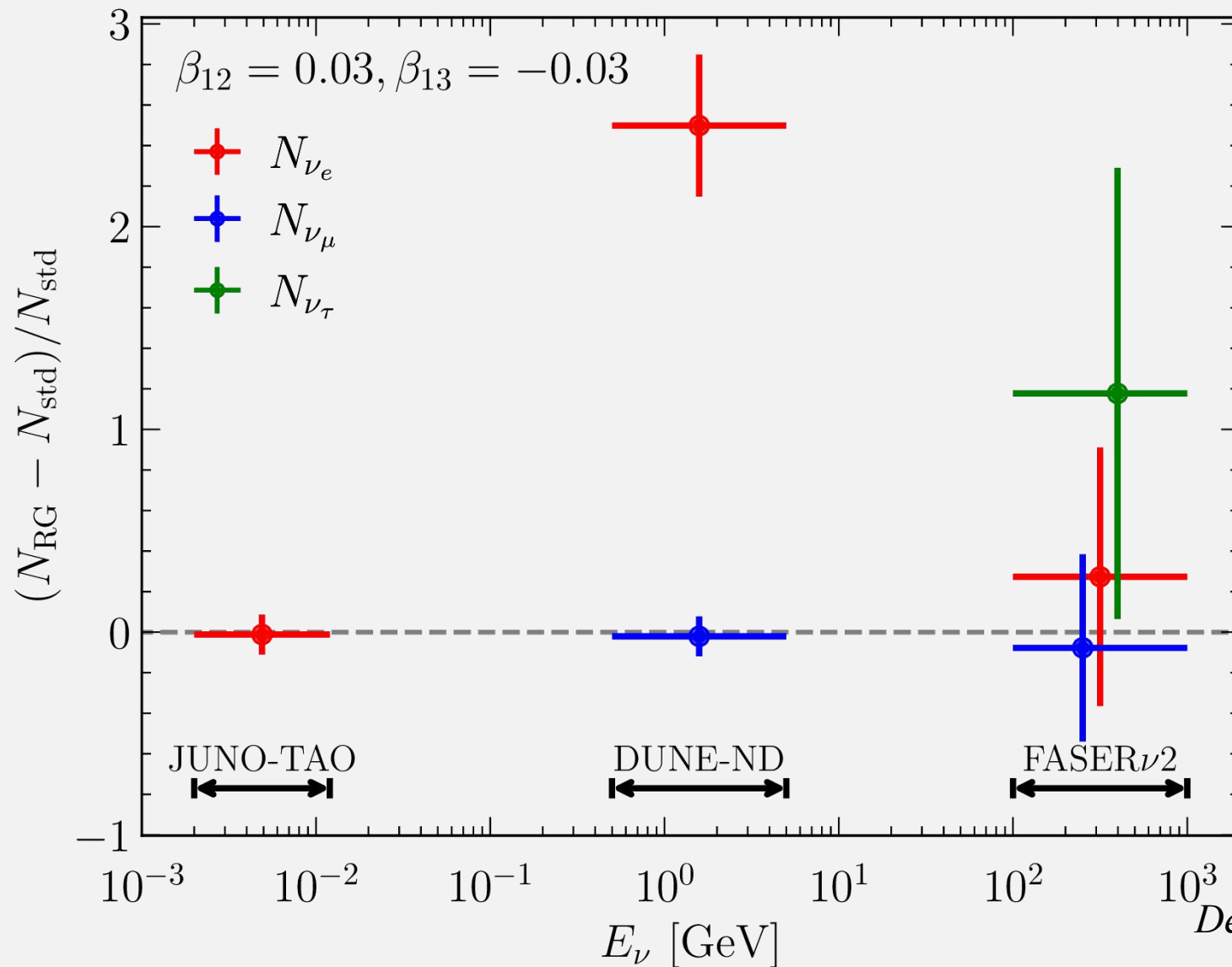
Denton, Ge, CFK, Pasquini, arXiv: 2606.26720

Pheno Results: Effect at Multiple Exps

Exp.	Oscillation Channels	$\overline{\ln^2 Q_d^2 / Q_p^2 }$
JUNO-TAO	$\bar{\nu}_e \rightarrow \bar{\nu}_e$	5 ($E_\nu = 3.6 \text{ MeV}$)
DUNE-ND	$\nu_\mu \rightarrow \nu_{e,\mu}$ & $\bar{\nu}_\mu \rightarrow \bar{\nu}_{e,\mu}$ $\nu_e \rightarrow \nu_{e,\mu}$ & $\bar{\nu}_e \rightarrow \bar{\nu}_{e,\mu}$	12 ($E_\nu = 2.5 \text{ GeV}$)
FASER ν 2	$\nu_\mu \rightarrow \nu_{e,\mu,\tau}$ & $\bar{\nu}_\mu \rightarrow \bar{\nu}_{e,\mu,\tau}$ $\nu_e \rightarrow \nu_{e,\mu,\tau}$ & $\bar{\nu}_e \rightarrow \bar{\nu}_{e,\mu,\tau}$ $\nu_\tau \rightarrow \nu_{e,\mu,\tau}$ & $\bar{\nu}_\tau \rightarrow \bar{\nu}_{e,\mu,\tau}$	82 ($E_\nu = 0.5 \text{ TeV}$)

Denton, Ge, **CFK**, Pasquini
arXiv: 2606.26720

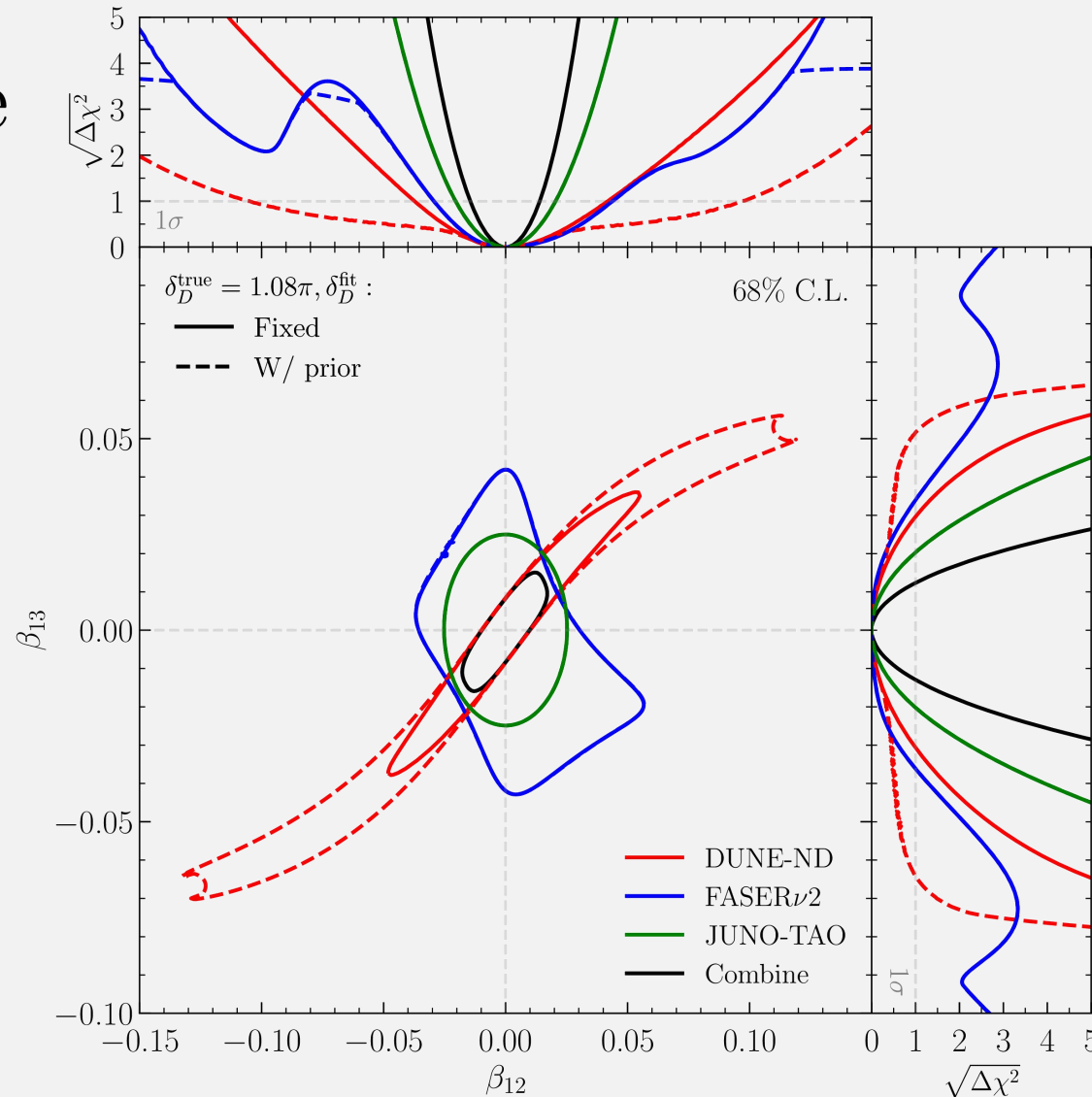
Pheno Results: Effect at Multiple Exps



Denton, Ge, **CFK**, Pasquini
arXiv: 2606.26720

Pheno Results: Effect at Multiple Exps

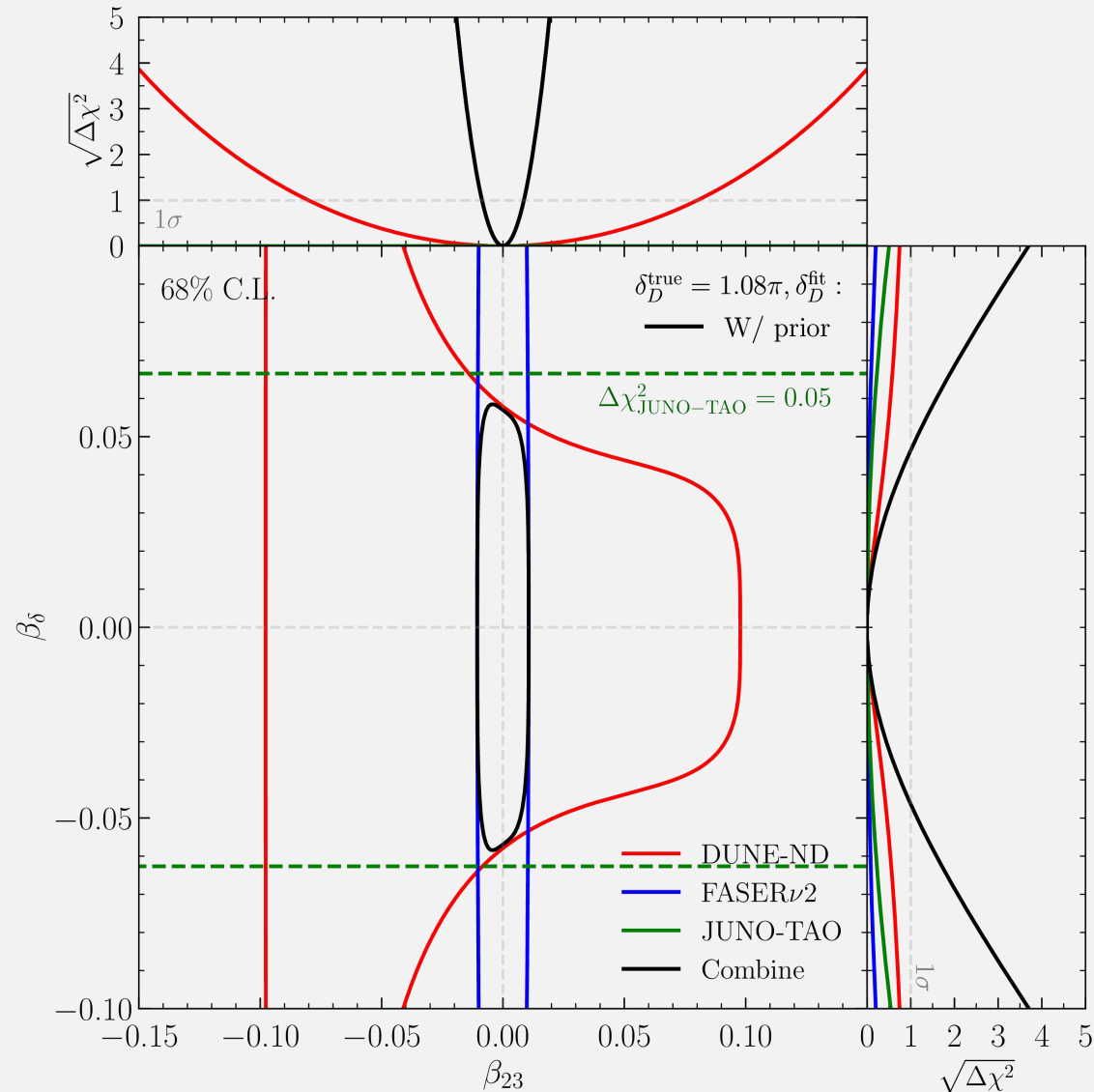
$\beta_{12} - \beta_{13}$ space



Denton, Ge, **CFK**, Pasquini
arXiv: 2606.26720

Pheno Results: Effect at Multiple Exps

$\beta_{23} - \beta_{\delta}$ space

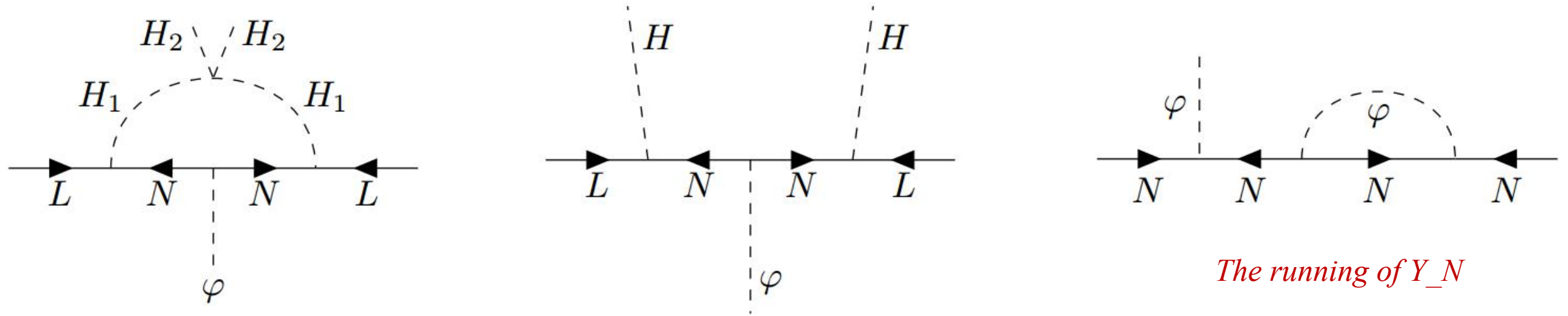


Denton, Ge, **CFK**, Pasquini
arXiv: 2606.26720

Thank you for your attention!

- BSM induced RG running effect can modify neutrino oscillation
- It can affect experimental sensitivities to neutrino mixing parameters
- It's desirable to combine experimental data at different energy scales

Back up



The running of Y_N

FIG. 2: *Left*: Feynman diagram associated to the generation of neutrino masses in a scotogenic-like neutrino mass model in which $\mathcal{M} \propto Y_N$ (Model 1). *Middle*: Feynman diagram associated to the generation of neutrino masses in an inverse seesaw model in which $\mathcal{M} \propto Y_N^{-1}$ (Model 2). *Right*: Representative Feynman diagram responsible for the running of Y_N in both Model 1 and Model 2. See text for details.

$$-\mathcal{L}_\nu^{(1)} = \bar{L}Y_\nu \tilde{H}_1 N_R + \varphi \bar{N}_R^c Y_N N_R + \text{h.c.},$$

$$-\mathcal{L}_\nu^{(2)} = \bar{L}Y_\nu \tilde{H} N_R + \varphi \bar{N}_R^c Y_N N_R + \text{h.c.},$$

$$M_\nu \simeq \frac{v_\varphi}{16\sqrt{2}\pi^2} Y_\nu Y_N Y_\nu^T \ln \frac{M_H^2}{M_A^2}.$$

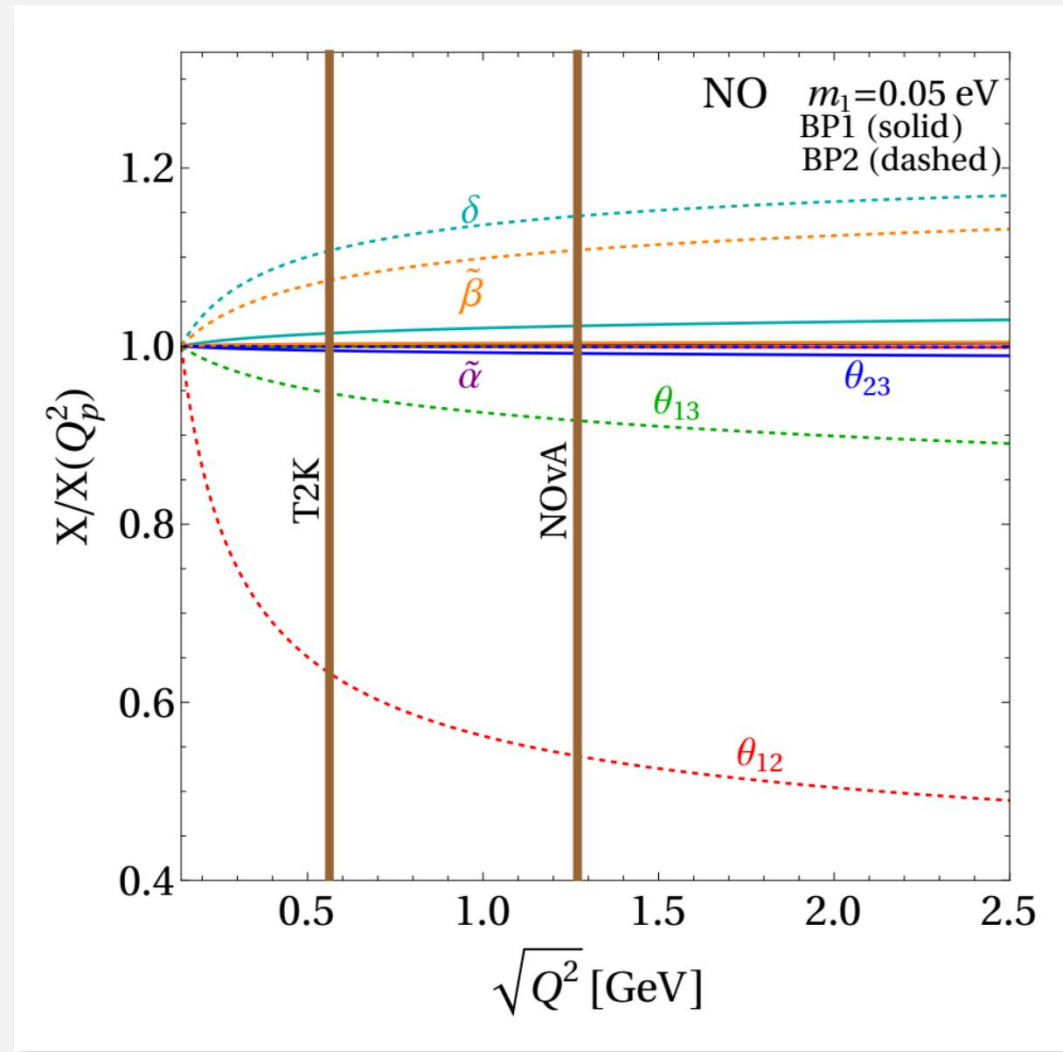
$$M_\nu = \frac{\sqrt{2}v^2}{4v_\varphi} Y_\nu (Y_N)^{-1} Y_\nu^T.$$

*K. S. Babu et al,
arXiv: 2108.11961*

$$16\pi^2\beta(Y_N) \equiv 16\pi^2 \frac{dY_N}{d\ln|Q|} = 4Y_N \left[Y_N^2 + \frac{1}{2}\text{Tr}(Y_N^2) \right].$$

Casas-Ibarra parametrization

$$Y_\nu = \frac{1}{\sqrt{C}} Y_N^{-x/2} R \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U(Q_p^2),$$



K. S. Babu et al,
arXiv: 2108.11961

zero-distance limit

$$1/E_\nu \ll L \ll 2E_\nu/\Delta m_{ij}^2$$

$$i \frac{\partial}{\partial \tau_i} |\nu_i(\tau_i)\rangle = m_i |\nu_i(\tau_i)\rangle \quad .$$

$$|\nu_i(\tau_i)\rangle = e^{-im_i\tau_i} |\nu_i(0)\rangle \quad .$$

$$m_i\tau_i = E_it - p_iL \quad .$$

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad .$$

$$m_i\tau_i \cong E(t - L) + \frac{m_i^2}{2E}L \quad .$$

$$\nu_i \equiv \nu_i(Q^2 = m_{\nu_i}^2)$$

*K. S. Babu et al,
arXiv: 2108.11961*

energy reconstruction:

$$E_{\nu}^{\text{rec}} \equiv E_{\ell}^{\text{rec}} + E_{\text{had}}^{\text{rec}}$$

momentum transfer reconstruction:

$$Q_{\text{rec}}^2 \equiv |(p_{\nu}^{\text{rec}} - p_{\ell}^{\text{rec}})^2| = 2E_{\nu}^{\text{rec}}(E_{\ell}^{\text{rec}} - |\mathbf{p}_{\ell}^{\text{rec}}| \cos \theta_{\ell}^{\text{rec}}) - m_{\ell}^2,$$