"Flavor Physics with Non-invertible Symmetries (NIS): Radiative Neutrino Mass and Beyond"

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{ T. Kobayashi, H. O., H. Otsuka }, JHEP 12 (2025) 111, 2505.14878,

22nd, Aug. 2026 at (Gyeongju) 慶州

Why Radiative Seesaw?

i). Tiny neutrino masses can be explained without introducing

too small dim.less coupling (y, λ $\cdots$) and (or)

↳ leading to relax hierarchies among S.M. fermions!

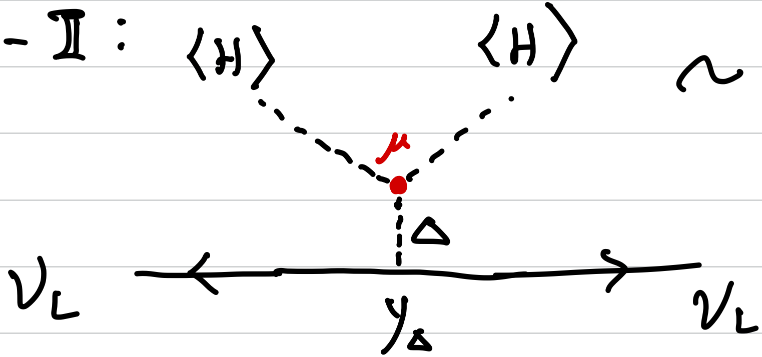
too large mass scale (M_R, $\cdots$)

↳ leading to a good testability at low energy scale!

To make the radiative seesaw's advantage clearer, let's compare with a tree-level neutrino scenario.

$\Rightarrow$ Let's consider type-II seesaw !

Type-II :



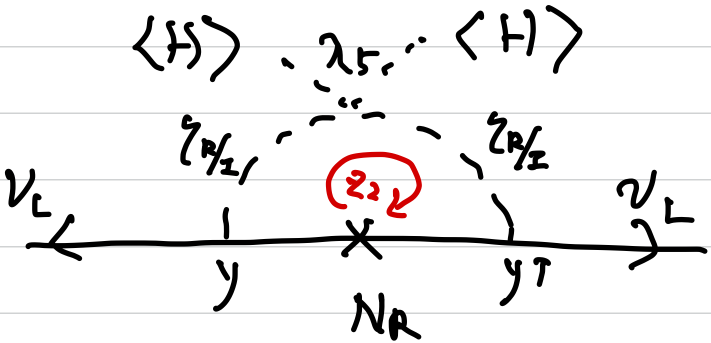
$$\sim y_\Delta \cdot \langle \Delta \rangle \sim y_\Delta \cdot \frac{v_H^2}{m_\Delta^2} \cdot \mu$$

$$= \mathcal{O}(0.1)\,\mathrm{eV}.$$

$v_H \sim 246\,\mathrm{GeV}$.

when $m_\Delta = \mathcal{O}(1\,\mathrm{TeV})$ $\boxed{y_\Delta \cdot \mu \sim \mathcal{O}(10^{-9})\,\mathrm{GeV} ..}$

Ma-model: hep-ph/0601225, prd. 73. (2006) 077301



$$\sim \frac{y\, y^T}{(4\pi)^2} \frac{\delta m^2}{M_R} \cdot \mathcal{O}(1)_{\text{l.f.}}$$

where $\delta m^2 = m^2_{\eta_R} - m^2_{\eta_I} \simeq \lambda_5 v_H^2$.

Order estimations:

$\text{Max}[M_R, m_\eta]$

When loop function $= \mathcal{O}(1)$, $M = \mathcal{O}(1\,\text{TeV})$, $m_\nu = \mathcal{O}(0.1\,\text{eV})$,

$$\boxed{(\therefore)\ y \cdot y^T \cdot \delta m = \mathcal{O}(10^{-3})\ \text{GeV}.}$$

$\Rightarrow$ hierarchy is relaxed by 10^6, compared to type-II.

ii) Linking a Dark Matter candidate (D.M.) to active neutrinos!

$$\begin{cases} \text{New paricles} \cdots \eta\,,\ N_R\,, \\ \text{D.M.} \cdots \eta_{R/I}\,,\ N_{R1}\,. \end{cases}$$

$\Leftarrow Z_2$ odd, it doesn't break at any order!

$\Leftarrow$ They can be D.M. candidate!

@ Specifically

when $\underline{y = \mathcal{O}(1)}$ $\left(\Rightarrow \delta m = \mathcal{O}(10^{-3})\,\mathrm{GeV}.\right)$

$\Rightarrow$ FDM satisfies the relic density and there are high testability!

($LFVs$, $(g-2)_e$, $\cdots$).

— Main topic —

Why non-invertible symmetries (NIS) are applied to neutrino mass models?

A : To provide an origin of zero and/or small couplings in a different manner!

$\Rightarrow$
- Quark and lepton textures (Prof. Tanimoto talk)
- Radiative seesaw models $\Rightarrow$ I will show you later.
- DM stability

Features of non-invertible symmetries (NIS)

Kaidi-Tachikawa-Zhang, SciPost Phys. 17 (2024) 6, 169

(Invertible symmetry + Selection rules)

$\Downarrow$ | $\Downarrow$

Group symmetries such as $\mathbb{Z}_N$

$\Downarrow$

D.M. stability

In principle!

Non-invertible symmetries, i.e., a generator's inverse cannot be defined!

$\Downarrow$

$\boxed{\text{generate small couplings at loop-level!}}$

Examples:

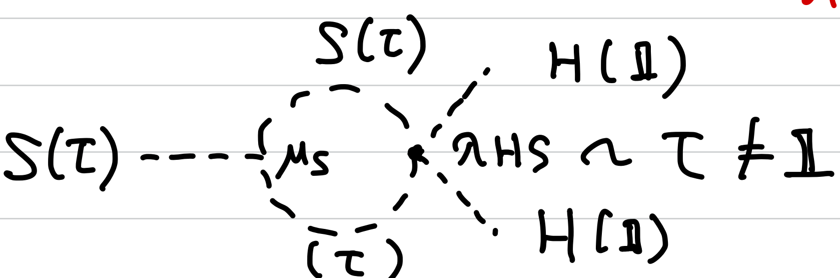
· 1. The simplest one: Fibonacci NIS; $\{\mathbb{1}, \tau\} \in G$

$\tau^2 = \mathbb{1} + \tau$ ⇒ Selection rules only ($\because$) $\tau^{-1} = ?$ "

@ Let's consider a real scalar singlet(s) with τ. (Motoo Suzuki, Ling-Xiao Xu, 2503.19964)

$\boxed{\mu |H|^2 \cdot S}$ is forbidden by Fibo. NIS at tree level.

However, μ is allowed by 1-loop level via $-\mu_S \cdot S^3 + \lambda_{HS} |H|^2 \cdot S^2$.



But the invariance is broken!

⇒ Effective theory!

2. The 2nd simplest one ; Ising NIS $\{\mathbb{1}, \epsilon, \sigma\} \in \mathcal{G}$

- $\epsilon^2 = \mathbb{1}$, $\Rightarrow \{\mathbb{1}, \epsilon\} \simeq \mathbb{Z}_2 \Rightarrow$ Invertible (sub-group) symmetry

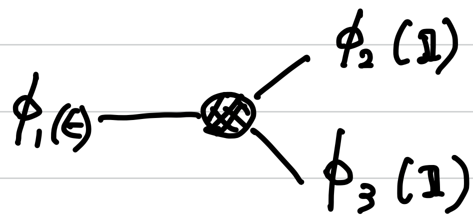
  $\mathbb{Z}_2$ is equivalent to $\mathbb{Z}_2$-odd in neutrino masses!

- $\begin{cases} \epsilon \cdot \sigma = \sigma, \\ \sigma^2 = 1 + \epsilon \end{cases} \Rightarrow \sigma^{-1} = ?$ $\Rightarrow$ Selection rules.

$\Rightarrow$ Small coupling could be induced ...

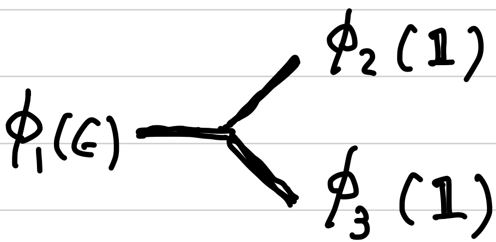
Let's try to construct a small coupling,

focusing on 3-point coupling!



(F)	(A)
$\sigma\epsilon\epsilon$	$\boxed{\sigma\sigma\epsilon}$
$\sigma\sigma\sigma$	$\sigma\sigma\epsilon\epsilon$
$\epsilon\mathbb{1}\mathbb{1}$	$\sigma\sigma\sigma\sigma$
$\sigma\mathbb{1}\mathbb{1}$	$\epsilon\epsilon\epsilon\epsilon$
$\epsilon\epsilon\epsilon$	
$\sigma\sigma\sigma\epsilon$	
$\epsilon\epsilon\epsilon\sigma$	
$\vdots$	

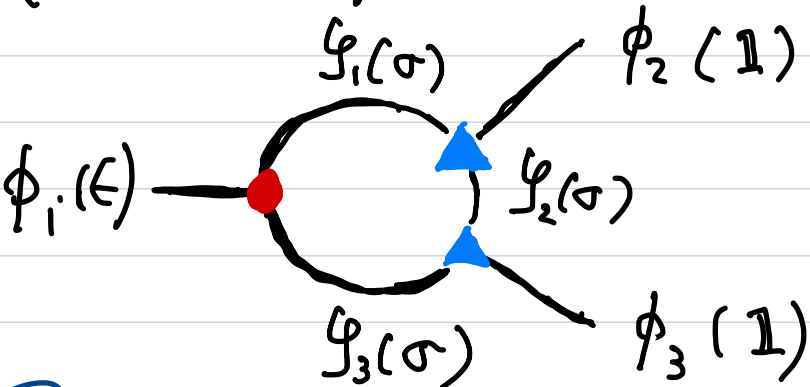
(Tree-level)



Forbidden!

$\epsilon \not\ni \mathbb{1}$.

(One-loop-level)



● … $\sigma\sigma\epsilon \ni \mathbb{1}$.

▲ … $\sigma\sigma \ni \mathbb{1}$.

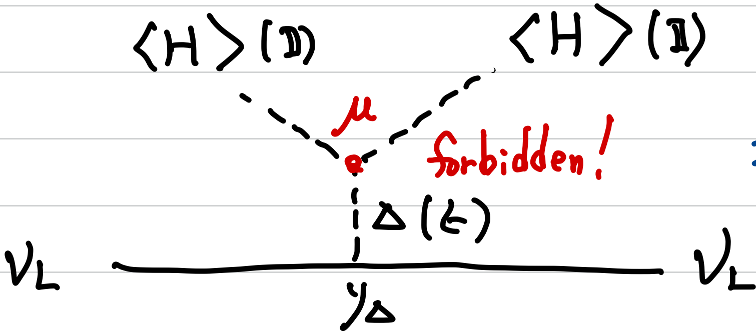
Allowed!

Three-point coupling can be obtained by a one-loop effect!!

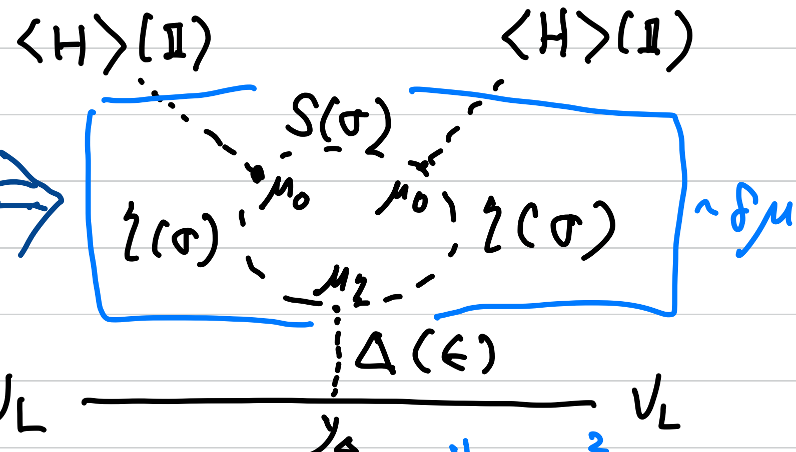
Note: This kind of model should be treated as an effective theory!

Let's apply Ising NIS to type-II Seesaw!

Outside $\begin{cases} \phi_1 \longrightarrow \Delta(\epsilon) \\ \phi_2, \phi_3 \longrightarrow H(\mathbb{1}) \end{cases}$, Inside $\begin{cases} \varphi_2 \longrightarrow S(\sigma) \\ \varphi_1, \varphi_3 \longrightarrow \eta(\sigma) \end{cases}$



$\Rightarrow$



$y_\Delta \cdot \mu \sim \mathcal{O}(10^{-9})\,\mathrm{GeV}.$

$y_\Delta \cdot \delta\mu \sim \frac{y_\Delta}{(4\pi)^2} \frac{\mu_0^2 \mu_2}{m_\Delta^2} \cdot \mathcal{O}(1)_{\text{1-lp.}}$

(※) Only η can be D.M., since S decays into S.M.

Summary and Conclusion:

- Non-invertible symmetries are new types of effective symmetries that potentially possess

invertible symmetry ($\mathbb{Z}_N$) and selection rules ($\tau^2 = \mathbb{1} + \tau$).

(~ non-invertible part)

Specifically, the selection rules ($\tau^2 = 1 + \tau$ under Fibo. NIS) play a crucial role in generating a small coupling via loop-effects!

(◦ Furthermore, a stable D.M. can also be achieved from a NIS, depending on model building.)

- We show two simplest examples (Fibo, Ising NIS) and demonstrate how to generate a tiny coupling μ in case of type-II Seesaw extended by Ising NIS.

- But, their applications to particle phenomenology have only started over the past year, and only relatively simple NIS have been explored.

Your contributions are highly expected!

— Many thanks!

- Sub-dominant slide:

  $\mathbb{Z}_3$ Tambara-Yamagami symmetry (TY).

- Generators: $\{\mathbb{1}, C, C^*, N\}$ Commutable!

- Rules: $C^2 = C^*$, $(C^*)^2 = C$, $C \cdot C^* = \mathbb{1}$, $\Rightarrow \{\mathbb{1}, C, C^*\} = \mathbb{Z}_3$.

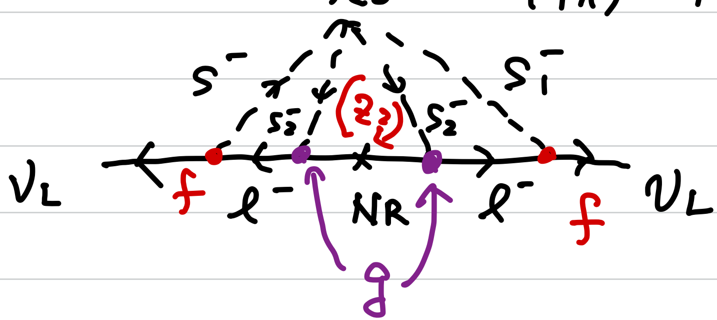
  Groupification.

  $C^{(*)} \cdot N = N$,

  $N^2 = 1 + C + C^* \Rightarrow$

Another radiative seesaw (including D. M.).

KNT $\quad \lambda_0 \sim \lambda_0 \dfrac{fggf}{(4\pi)^4} \dfrac{m_\ell^2}{M_R} \cdot \mathcal{O}(1)_{3\text{-}lp.}$



$\left\{ \begin{array}{l} \text{New particles: } N_R,\ S_1^\pm,\ S_2^\pm, \\ \quad \text{D.M. } \cdots\ N_{R1}, \\ \text{Neutrinos} \cdots\ m_{\nu\,\text{lightest}} = 0,\ \dfrac{1}{(4\pi)^4} \text{ suppression!} \end{array} \right.$

· Under the same condition as Ma-model $(f = g)$,

① $\lambda_0 = \mathcal{O}(1), \Rightarrow f = g = \mathcal{O}(10^{-2})$

② $f = g = \mathcal{O}(1) \Rightarrow \lambda_0 = \mathcal{O}(10^{-6})$.

Groupification:

Def: $Gr(A) = A / Com(A)^{\infty}$:

where $Com(A) = \{ x \mid x \prec y\bar{y} \text{ for some } y \}$

ex): Ising fusion rule: $A = \{ \mathbb{1}, \epsilon, \sigma \}$,

$\epsilon\bar{\epsilon} = \epsilon^2 = \mathbb{1}$, $\sigma\bar{\sigma} = \sigma^2 = 1 + \epsilon$,

1, because of so simple NIS.

$(\because)$ $Com(A) = \{ \mathbb{1}, \epsilon \}$, $\Rightarrow Com(A)^{\infty} = \{ \mathbb{1}, \epsilon \}$

$\Rightarrow Gr(A) = A / Com(A) = \{ \mathbb{1}, \sigma \} \cong \mathbb{Z}_2$.

Note : This $\widetilde{\mathbb{Z}}_2$ is not same as the $\mathbb{Z}_2$

that cosists of $\{ \mathbb{1}, \epsilon \}$ as a invertible

sub-group symmetry !!

$$\widetilde{\mathbb{Z}}_2 \neq \mathbb{Z}_2 .$$

But, these $\mathbb{Z}_2$ are dual relation !

$\Rightarrow A/\{\mathbb{1}, \sigma\} = \{\mathbb{1}, \epsilon\} \leftrightarrows A/\{\mathbb{1}, \epsilon\} = \{\mathbb{1}, \sigma\}.$