Application of a non-holomorphic modular symmetry for neutrino mass models

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1. Introduction

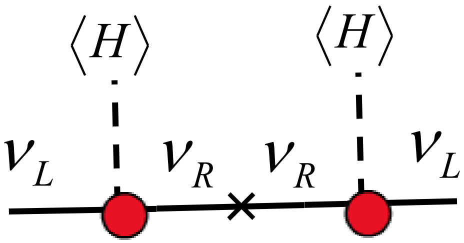
Non-zero neutrino masses require new physics

- ✓ **We need a mechanism to generate neutrino mass**
- ✓ **Why neutrino masses are so small?**

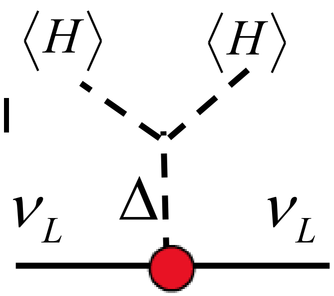
Many mechanisms to explain neutrino masses are proposed

- **Type-I,II,III (Inverse/Linear) Seesaw**

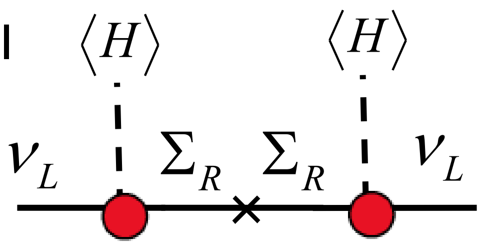
Type-I



Type-II

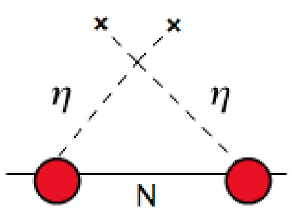


Type-III

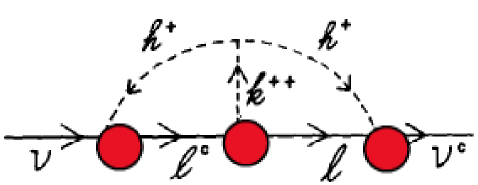


- **Radiative seesaw**

Scotogenic:



Zee-Babu:



Yukawa couplings are source of flavor structure

Many mechanisms to explain neutrino masses

However, they do not explain flavor structure

Structures of Yukawa couplings are not restricted

An introduction of symmetry can constrain flavor structure

Flavor symmetries

- Flavor dependent gauge symmetries
- Non-Abelian discrete symmetries
- **Modular symmetries** This talk
- Non-invertible symmetries
- Etc.

1. Introduction

Neutrino models with modular flavor symmetry

⇩

Explaining neutrino masses + more predictive structure

In particular, we focus on non-holomorphic modular symmetries

Outline of the talk

1. Introduction
2. $\boxed{\text{Modular symmetry}}$
3. A LQ neutrino mass model
4. Summary

Modular flavor symmetry

F. Feruglio, Are neutrino masses modular forms? (2019), “From My Vast Repertoire ...” pp. 227–266, 1706.08749.

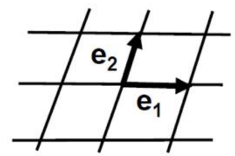
⇨ Attractive framework to describe flavor

Originated from extra dimensional symmetry (e.g. transformation on torus)

Modular transformation & modular group

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d} \quad \Leftrightarrow \quad \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \to \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\gamma} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

From 2307.03384



$$\tau = \frac{e_2}{e_1}$$

This lattice corresponds to Torus

Γ =SL(2,Z) : ad-bc=1, {a,b,c,d} are integer

Generators: $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\quad S^4 = (ST)^3 = 1$

$$\tau \to -\frac{1}{\tau} \qquad \tau \to \tau + 1$$

Finite modular group

$$\Gamma'_N = \Gamma/\Gamma(N)$$
$$\Gamma_N = \Gamma/\pm\Gamma(N)$$

$$\Gamma(N) \equiv \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ (mod\ N) \right\}$$

Non-Abelian discrete symmetry appears : $\Gamma_2 \simeq S_3, \Gamma_3 \simeq A_4,$ etc.

Modular flavor symmetry (A_4 example)

Yukawa couplings can be given by **modular forms** transformed under A_4

$$f_i(\gamma\tau) = (c\tau + d)^k \rho(\gamma)_{ij} f_i(\tau)$$

k: modular weight

ρ: A_4 transformation matrix

$$\left(\tau \longrightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d}\ , \quad \text{where} \quad a, b, c, d \in \mathbb{Z} \quad \text{and} \quad ad - bc = 1, \quad \mathrm{Im}[\tau] > 0\ , \right)$$

Field (with weight k_Φ) is also transformed by

$$\Phi \to (c\tau + d)^{-k_\Phi} \rho^{(\Phi)}(\gamma)\Phi$$

Yukawa term can be invariant under modular transformation

⇨ Sum of modular weight = 0,   invariant under A_4

Ex) $f\Phi_i\Phi_j\Phi_k \quad (k_f - k_{\Phi_i} - k_{\Phi_j} - k_{\Phi_k} = 0)$

Holomorphic & Non-holomorphic cases

Originally modular symmetry is given in supersymmetric framework

⇨ Yukawa interactions are from super-potential (holomorphic)

Non-holomorphic framework is also possible!

B.Y. Qu, G.J. Ding, JHEP08(2024)136

Holomorphic case

- ✓ Yukawa coupling
  - ⇨ Modular form
- ✓ Supersymmetric Lagrangian

Non-holomorphic case

- ✓ Yukawa coupling
  - ⇨ Polyharmonic Maaβ form
  - Satisfying the Laplacian condition

$$\Delta_k Y = 0$$

- ✓ Non-supersymmetric

2. Modular symmetry

Weight k_Y	Polyharmonic Maaß forms $Y_{\mathbf{r}}^{(k_Y)}$
$k_Y = -4$	$Y_{\mathbf{1}}^{(-4)},\ Y_{\mathbf{3}}^{(-4)}$
$k_Y = -2$	$Y_{\mathbf{1}}^{(-2)},\ Y_{\mathbf{3}}^{(-2)}$
$k_Y = 0$	$Y_{\mathbf{1}}^{(0)},\ Y_{\mathbf{3}}^{(0)}$
$k_Y = 2$	$Y_{\mathbf{1}}^{(2)},\ Y_{\mathbf{3}}^{(2)}$
$k_Y = 4$	$Y_{\mathbf{1}}^{(4)},\ Y_{\mathbf{1'}}^{(4)},\ Y_{\mathbf{3}}^{(4)}$
$k_Y = 6$	$Y_{\mathbf{1}}^{(6)},\ Y_{\mathbf{3}I}^{(6)},\ Y_{\mathbf{3}II}^{(6)}$

B.Y. Qu, G.J. Ding, JHEP08(2024)136

There are 0 and negative weight ones

Example of modular form (non-holomorphic version)

B.Y. Qu, G.J. Ding, JHEP08(2024)136

$$Y_3^{(0)} = \begin{pmatrix} Y_{3,1}^{(0)} \\ Y_{3,2}^{(0)} \\ Y_{3,3}^{(0)} \end{pmatrix} \equiv \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

A_4 triplet (modular weight 0)

$$\begin{aligned}
Y^{(0)}_{\mathbf{3},1} &= y - \frac{3\,e^{-4\pi y}}{\pi q} - \frac{9\,e^{-8\pi y}}{2\pi q^2} - \frac{e^{-12\pi y}}{\pi q^3} - \frac{21\,e^{-16\pi y}}{4\pi q^4} - \frac{18\,e^{-20\pi y}}{5\pi q^5} - \frac{3\,e^{-24\pi y}}{2\pi q^6} + \cdots \\
&\quad - \frac{9\log 3}{4\pi} - \frac{3q}{\pi} - \frac{9q^2}{2\pi} - \frac{q^3}{\pi} - \frac{21q^4}{4\pi} - \frac{18q^5}{5\pi} - \frac{3q^6}{2\pi} + \cdots\,, \\
Y^{(0)}_{\mathbf{3},2} &= \frac{27q^{1/3}e^{\pi y/3}}{\pi}\left(\frac{e^{-3\pi y}}{4q} + \frac{e^{-7\pi y}}{5q^2} + \frac{5\,e^{-11\pi y}}{16q^3} + \frac{2\,e^{-15\pi y}}{11q^4} + \frac{2\,e^{-19\pi y}}{7q^5} + \frac{4\,e^{-23\pi y}}{17q^6} + \cdots\right) \\
&\quad + \frac{9q^{1/3}}{2\pi}\left(1 + \frac{7q}{4} + \frac{8q^2}{7} + \frac{9q^3}{5} + \frac{14q^4}{13} + \frac{31q^5}{16} + \frac{20q^6}{19} + \cdots\right)\,, \\
Y^{(0)}_{\mathbf{3},3} &= \frac{9\,q^{2/3}e^{2\pi y/3}}{2\pi}\left(\frac{e^{-2\pi y}}{q} + \frac{7\,e^{-6\pi y}}{4q^2} + \frac{8\,e^{-10\pi y}}{7q^3} + \frac{9\,e^{-14\pi y}}{5q^4} + \frac{14\,e^{-18\pi y}}{13q^5} + \frac{31\,e^{-22\pi y}}{16q^6} + \cdots\right) \\
&\quad + \frac{27q^{2/3}}{\pi}\left(\frac{1}{4} + \frac{q}{5} + \frac{5q^2}{16} + \frac{2q^3}{11} + \frac{2q^4}{7} + \frac{9q^5}{17} + \frac{21q^6}{20} + \cdots\right)\,,
\end{aligned}$$

$$q = e^{2\pi i\tau}$$

$$\tau = x + iy$$

Other modular forms

$$Y_{\mathbf{3}}^{(2)} = \begin{pmatrix} \varepsilon^2(\tau) \\ \sqrt{2}\vartheta(\tau)\varepsilon(\tau) \\ -\vartheta^2(\tau) \end{pmatrix} \equiv \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}$$

Modular weight 2

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + 84q^4 + 72q^5 + \ldots \\ Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + 18q^3 + 14q^4 + \ldots) \\ Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + 4q^3 + 8q^4 + \ldots)\,. \end{aligned}$$

$$q = e^{2\pi i \tau}$$

$$\begin{aligned} Y_{\mathbf{3}}^{(4)} &= (Y_{\mathbf{3}}^{(2)}Y_{\mathbf{3}}^{(2)})_{\mathbf{3}} = (Y_1^2 - Y_2Y_3, Y_3^2 - Y_1Y_2, Y_2^2 - Y_1Y_3)^T\,, \\ Y_{\mathbf{1}}^{(4)} &= (Y_{\mathbf{3}}^{(2)}Y_{\mathbf{3}}^{(2)})_{\mathbf{1}} = Y_1^2 + 2Y_2Y_3\,, \\ Y_{\mathbf{1'}}^{(4)} &= (Y_{\mathbf{3}}^{(2)}Y_{\mathbf{3}}^{(2)})_{\mathbf{1'}} = Y_3^2 + 2Y_1Y_2\,. \end{aligned}$$

Modular weight 4

$$\begin{aligned} Y_{\mathbf{1}}^{(6)} &= (Y_{\mathbf{3}}^{(2)}Y_{\mathbf{3}}^{(4)})_{\mathbf{1}} = Y_1^3 + Y_2^3 + Y_3^3 - 3Y_1Y_2Y_3\,, \\ Y_{\mathbf{3}I}^{(6)} &= Y_{\mathbf{3}}^{(2)}Y_{\mathbf{1}}^{(4)} = (Y_1^3 + 2Y_1Y_2Y_3, Y_1^2Y_2 + 2Y_2^2Y_3, Y_1^2Y_3 + 2Y_3^2Y_2)^T\,, \\ Y_{\mathbf{3}II}^{(6)} &= Y_{\mathbf{3}}^{(2)}Y_{\mathbf{1'}}^{(4)} = (Y_3^3 + 2Y_1Y_2Y_3, Y_3^2Y_1 + 2Y_1^2Y_2, Y_3^2Y_2 + 2Y_2^2Y_1)^T\,. \end{aligned}$$

Modular weight 6

Example of Yukawa interaction under the symmetry

$$Y_3^{(0)}\bar{L}eH \qquad \bar{L}=\begin{pmatrix}\bar{L}_1\\ \bar{L}_2\\ \bar{L}_3\end{pmatrix} \qquad l=\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix} \qquad Y_3^{(0)}=\begin{pmatrix}Y_{3,1}^{(0)}\\ Y_{3,2}^{(0)}\\ Y_{3,3}^{(0)}\end{pmatrix}\equiv\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}$$

A_4 triplet (k_L=0) A_4 triplet (k_e=0) A_4 triplet

$$\begin{pmatrix}\bar{L}_1\\ \bar{L}_2\\ \bar{L}_3\end{pmatrix}\otimes\begin{pmatrix}e_1\\ e_2\\ e_3\end{pmatrix}=\frac{1}{3}\underbrace{\begin{pmatrix}2\bar{L}_1e_1-\bar{L}_2e_3-\bar{L}_3e_2\\ 2\bar{L}_3e_3-\bar{L}_1e_2-\bar{L}_2e_1\\ 2\bar{L}_2e_2-\bar{L}_3e_1-\bar{L}_1e_3\end{pmatrix}_{3_S}}_{(\bar{L}e)_{3S}}\oplus\frac{1}{2}\underbrace{\begin{pmatrix}\bar{L}_2e_3-\bar{L}_3e_2\\ \bar{L}_1e_2-\bar{L}_2e_1\\ \bar{L}_3e_1-\bar{L}_1e_3\end{pmatrix}_{3_A}}_{(\bar{L}e)_{3A}}$$

A_4 invariant term

$$fY_3^{(0)}(\bar{L}e)_{3S}H+gY_3^{(0)}(\bar{L}e)_{3A}H$$
$$=f[y_1(2\bar{L}_1e_1-\bar{L}_2e_3-\bar{L}_3e_2)+y_2(2\bar{L}_2e_2-\bar{L}_3e_1-\bar{L}_1e_2)+y_3(2\bar{L}_3e_3-\bar{L}_1e_2-\bar{L}_2e_1)]$$
$$+g[y_1(\bar{L}_2e_3-\bar{L}_3e_2)+y_2(\bar{L}_3e_1-\bar{L}_1e_2)+y_3(\bar{L}_1e_2-\bar{L}_2e_1)]$$

Example of Yukawa interaction under the symmetry

$$Y_3^{(0)} \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1b_1 + a_2b_3 + a_3b_2)_{\mathbf{1}} \oplus (a_3b_3 + a_1b_2 + a_2b_1)_{\mathbf{1'}}$$
$$\oplus (a_2b_2 + a_1b_3 + a_3b_1)_{\mathbf{1''}}$$
$$\oplus \frac{1}{3}\begin{pmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_3b_1 - a_1b_3 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_3b_1 - a_1b_3 \end{pmatrix}_{\mathbf{3}}$$

et

$$\begin{pmatrix} \bar{L}_1 \\ \bar{L}_2 \\ \bar{L}_3 \end{pmatrix} \otimes \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \underbrace{\frac{1}{3}\begin{pmatrix} 2\bar{L}_1e_1 - \bar{L}_2e_3 - \bar{L}_3e_2 \\ 2\bar{L}_3e_3 - \bar{L}_1e_2 - \bar{L}_2e_1 \\ 2\bar{L}_2e_2 - \bar{L}_3e_1 - \bar{L}_1e_3 \end{pmatrix}_{3_S}}_{(\bar{L}e)_{3S}} \oplus \underbrace{\frac{1}{2}\begin{pmatrix} \bar{L}_2e_3 - \bar{L}_3e_2 \\ \bar{L}_1e_2 - \bar{L}_2e_1 \\ \bar{L}_3e_1 - \bar{L}_1e_3 \end{pmatrix}_{3_A}}_{(\bar{L}e)_{3A}}$$

A_4 invariant term

$$fY_3^{(0)}(\bar{L}e)_{3S}H + gY_3^{(0)}(\bar{L}e)_{3A}H$$
$$= f[y_1(2\bar{L}_1e_1 - \bar{L}_2e_3 - \bar{L}_3e_2) + y_2(2\bar{L}_2e_2 - \bar{L}_3e_1 - \bar{L}_1e_2) + y_3(2\bar{L}_3e_3 - \bar{L}_1e_2 - \bar{L}_2e_1)]$$
$$+g[y_1(\bar{L}_2e_3 - \bar{L}_3e_2) + y_2(\bar{L}_3e_1 - \bar{L}_1e_2) + y_3(\bar{L}_1e_2 - \bar{L}_2e_1)]$$

Holomorphic & Non-holomorphic models

Type-II seesaw example

Holomorphic model (T.Kobayashi, T.N., T.Shimomura PRD 102 (2020) 3, 035019)

	Lepton		Higgs			
	L	(e_R, μ_R, τ_R)	T_1	T_2	H_u	H_d
$SU(2)_L$	$\mathbf{2}$	1	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)_Y$	$-\frac{1}{2}$	1	1	-1	$\frac{1}{2}$	$-\frac{1}{2}$
A_4	3	$1, 1'', 1'$	Model (1), (3): 1 Model (2), (4): $1''$	(1), (3): 1 (2), (4): $1'$	(1), (3): 1 (2), (4): $1'$	1
k_I	Model (1), (2): -1 Model (3), (4): -2	-1 $0[-2]$ for case A[B]	0	0	0	0

Non-Holomorphic model T.N., H.Okada PLB 868 (2025) 139763)

	$\overline{L_L}$	ℓ_R	H	$\Delta^\dagger$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$
$U(1)_Y$	$\frac{1}{2}$	-1	$\frac{1}{2}$	-1
A_4	$\mathbf{3}$	$\{\mathbf{1}\}$	$\mathbf{1}$	$\mathbf{1}$
$-k_I$	$+1$	r	0	$+2$

We can reduce field contents in non-holomophic case

Outline of the talk

1. Introduction
2. Short Review
3. $\boxed{\text{A LQ neutrino mass model}}$
4. Summary

Introduction of Leptoquarks (LQs) is interesting

Examples of scalar LQ couplings:

$$\tilde{R}_2 = \begin{pmatrix} \tilde{R}_{2/3} \\ \tilde{R}_{-1/3} \end{pmatrix}$$

$$\overline{Q_L^c} L_L S_3 \qquad \overline{u_R^c} l_R S_1 \qquad \overline{d_R} L_L \tilde{R}_2 \qquad \overline{Q_L} e_R R_2$$

Etc.

Review of LQ pheno: 1603.04993

- ✓ Meson decays
- ✓ Lepton flavor violations
- ✓ Neutrino mass generation

Connecting quark-lepton sectors

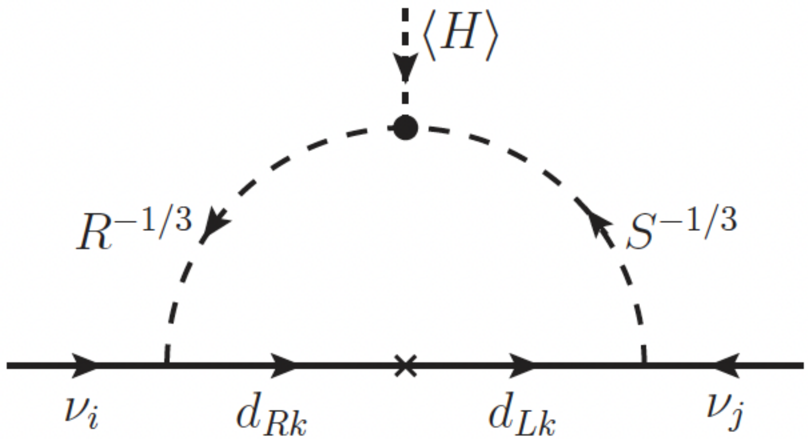
Good to apply modular symmetry

⇨ For both quark and lepton flavor

3. A LQ neutrino mass model

Let us consider a neutrino mass model with LQs

U. Mahanta, PRD 62 (2000), C.K. Chua etc. PLB 479 (2000)



$$L \supset \underline{\tilde{y}\overline{Q^c}P_L L S_1} + w\,\overline{u^c}P_R l S_1$$

$$=\tilde{y}\left(\overline{u^c}P_L l - \overline{d^c}P_L \nu\right)S_1$$

$$+f\left(\bar{d}P_L l\tilde{R}_{2/3} - \bar{d}P_L \nu\tilde{R}_{-1/3}\right)$$

$$+h.c.$$

Taken from 2510.08504

Assuming LQs are sufficiently heavy, two-loop contribution is neglected

There are many free parameters : $\tilde{y}$ and f are 3×3 complex matrices

⇨ Generally 18 complex parameters in total

⇩

Let us apply **A$_4$ modular flavor symmetry** to reduce free parameters

⇨ We can get some prediction in neutrino observables

3. A LQ neutrino mass model

Structure of the model

Charge Assignment :

(a minimal choice
that can fit the data)

	L_L	ℓ_R	Q_L	d_R	u_R	H	η ($\tilde{R}_2$)	S (S_1)
SU(3)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$
$\mathrm{SU}(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$
$\mathrm{U}(1)_Y$	$\frac{1}{2}$	-1	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\{1\}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$-k_I$	0	0	0	0	-6	0	0	0

T.N., H. Okada, X.Y. Wang JHEP09(2025), arXiv: 2504.21404

Yukawa Interactions :

$$\begin{aligned}
-\mathcal{L}_Y = {} & f_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{1}}] + f_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_A}] \\
& + g_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{1}}] + g_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_A}] \\
& + y_1^d[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{1}}] + y_3^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_S}] + y_{3'}^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_A}] \\
& + y_1^\ell[Y_{\mathbf{1}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{1}}] + y_3^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_S}] + y_{3'}^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_A}] \\
& + \sum_{A=1,2} \left[\alpha_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}} u_{R_1} + \beta_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}''} u_{R_{1'}} + \gamma_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}'} u_{R_{1''}}\right] \\
& + \sum_{A=1,2} \left[a_u^A \overline{u^c_{R_1}} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}} + b_u^A \overline{u^c_{R_{1'}}} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}''} + c_u^A \overline{u^c_{R_{1''}}} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}'}\right],
\end{aligned}$$

Scalar Potential

$$\begin{aligned}
V = {} & -\mu_H^2 |H|^2 + \mu_\eta^2 |\eta|^2 + \mu_S^2 |S|^2 + (\mu H^\dagger \eta S + h.c.) \\
& + \lambda_H |H|^4 + \lambda_\eta |\eta|^4 + \lambda_S |S|^4 + \lambda_{H\eta} |H|^2 |\eta|^2 + \lambda_{HS} |H|^2 |S|^2 + \lambda_{\eta S} |\eta|^2 |S|^2
\end{aligned}$$

Leptoquark mass

$$V = -\mu_H^2|H|^2 + \mu_\eta^2|\eta|^2 + \mu_S^2|S|^2 + (\mu H^\dagger \eta S + h.c.)$$
$$+ \lambda_H|H|^4 + \lambda_\eta|\eta|^4 + \lambda_S|S|^4 + \lambda_{H\eta}|H|^2|\eta|^2 + \lambda_{HS}|H|^2|S|^2 + \lambda_{\eta S}|\eta|^2|S|^2$$

$$\Downarrow \qquad \eta = \begin{pmatrix} \eta_{2/3} \\ \eta_{-1/3} \end{pmatrix}$$

S and $\eta_{-1/3}$ mix after EWSB

$$\begin{pmatrix} \eta_{\frac{1}{3}} \\ S \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho_{\frac{1}{3}} \\ \chi_{\frac{1}{3}} \end{pmatrix}$$

Mass eigenstate

Quark mass matrices

$$
\begin{aligned}
-\mathcal{L}_Y = {} & f_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{1}}] + f_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_A}] \\
& + g_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{1}}] + g_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_A}] \\
& + y_1^d[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{1}}] + y_3^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_S}] + y_{3'}^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_A}] \\
& + y_1^\ell[Y_{\mathbf{1}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{1}}] + y_3^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_S}] + y_{3'}^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_A}] \\
& + \sum_{A=1,2} \left[\alpha_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}} u_{R_1} + \beta_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}''} u_{R_{1'}} + \gamma_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}'} u_{R_{1''}}\right] \\
& + \sum_{A=1,2} \left[a_u^A \overline{u_{R_1}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}} + b_u^A \overline{u_{R_{1'}}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}''} + c_u^A \overline{u_{R_{1''}}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}'}\right],
\end{aligned}
$$

$$
M_d = \frac{v}{\sqrt{2}} \begin{pmatrix} y_1^d + 2y_3^d y_1 & (-y_3^d + y_{3'}^d) y_3 & -(y_3^d + y_{3'}^d) y_2 \\ (-y_3^d + y_{3'}^d) y_2 & y_1^d - (y_3^d + y_{3'}^d) y_1 & 2y_3^d y_3 \\ -(y_3^d + y_{3'}^d) y_3 & 2y_3^d y_2 & y_1^d + (-y_3^d + y_{3'}^d) y_1 \end{pmatrix}
$$

$$
Y_3^{(0)} \equiv \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}
$$

$$
Y_{3^1}^{(6)} \equiv \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \end{pmatrix}
$$

$$
M_u = \frac{v}{\sqrt{2}} \left[\begin{pmatrix} \tilde{y}_1 & \tilde{y}_3 & \tilde{y}_2 \\ \tilde{y}_2 & \tilde{y}_1 & \tilde{y}_3 \\ \tilde{y}_3 & \tilde{y}_2 & \tilde{y}_1 \end{pmatrix} + \begin{pmatrix} \tilde{y}'_1 & \tilde{y}'_3 & \tilde{y}'_2 \\ \tilde{y}'_2 & \tilde{y}'_1 & \tilde{y}'_3 \\ \tilde{y}'_3 & \tilde{y}'_2 & \tilde{y}'_1 \end{pmatrix} \begin{pmatrix} \tilde{\alpha} & 0 & 0 \\ 0 & \tilde{\beta} & 0 \\ 0 & 0 & \tilde{\gamma} \end{pmatrix} \right] \begin{pmatrix} \alpha_u^1 & 0 & 0 \\ 0 & \beta_u^1 & 0 \\ 0 & 0 & \gamma_u^1 \end{pmatrix}
$$

$$
Y_{3^2}^{(6)} \equiv \begin{pmatrix} \tilde{y}'_1 \\ \tilde{y}'_2 \\ \tilde{y}'_3 \end{pmatrix}
$$

$\tilde{\alpha} = \alpha_u^2/\alpha_u^1$, $\tilde{\beta} = \beta_u^2/\beta_u^1$ and $\tilde{\gamma} = \gamma_u^2/\gamma_u^1$

Quark mass matrix

$$-\mathcal{L}_Y = f_1[Y_1^{(0)} \otimes (\overline{Q_I^c} S L_I)_1] + f_S[Y_3^{(0)} \otimes (\overline{Q_I^c} S L_I)_{3_S}] + f_A[Y_3^{(0)} \otimes (\overline{Q_I^c} S L_I)_{3_A}]$$

The mass matrices are diagonalized as in the SM

$$\mathrm{diag.}(m_d, m_s, m_b) \equiv V_{dL}^\dagger M_d V_{dR} \text{ and } \mathrm{diag.}(m_u, m_c, m_t) \equiv V_{uL}^\dagger M_u V_{uR}$$

$$d_{L,R} \to V_{dL,dR} d_{L,R}$$
$$u_{L,R} \to V_{uL,uR} u_{L,R}$$

$$V_{CKM} = V_{u_L}^\dagger V_{d_L}$$

$$M_d = \frac{v}{\sqrt{2}} \begin{pmatrix} y_1^d + 2y_3^d y_1 & (-y_3^d + y_{3'}^d) y_3 & -(y_3^d + y_{3'}^d) y_2 \\ (-y_3^d + y_{3'}^d) y_2 & y_1^d - (y_3^d + y_{3'}^d) y_1 & 2y_3^d y_3 \\ -(y_3^d + y_{3'}^d) y_3 & 2y_3^d y_2 & y_1^d + (-y_3^d + y_{3'}^d) y_1 \end{pmatrix}$$

$$M_u = \frac{v}{\sqrt{2}} \left[\begin{pmatrix} \tilde{y}_1 & \tilde{y}_3 & \tilde{y}_2 \\ \tilde{y}_2 & \tilde{y}_1 & \tilde{y}_3 \\ \tilde{y}_3 & \tilde{y}_2 & \tilde{y}_1 \end{pmatrix} + \begin{pmatrix} \tilde{y}'_1 & \tilde{y}'_3 & \tilde{y}'_2 \\ \tilde{y}'_2 & \tilde{y}'_1 & \tilde{y}'_3 \\ \tilde{y}'_3 & \tilde{y}'_2 & \tilde{y}'_1 \end{pmatrix} \begin{pmatrix} \tilde{\alpha} & 0 & 0 \\ 0 & \tilde{\beta} & 0 \\ 0 & 0 & \tilde{\gamma} \end{pmatrix} \right] \begin{pmatrix} \alpha_u^1 & 0 & 0 \\ 0 & \beta_u^1 & 0 \\ 0 & 0 & \gamma_u^1 \end{pmatrix}$$

$$\tilde{\alpha} = \alpha_u^2/\alpha_u^1,\ \tilde{\beta} = \beta_u^2/\beta_u^1 \text{ and } \tilde{\gamma} = \gamma_u^2/\gamma_u^1$$

$$Y_3^{(0)} \equiv \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$Y_{3^1}^{(6)} \equiv \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \end{pmatrix}$$

$$Y_{3^2}^{(6)} \equiv \begin{pmatrix} \tilde{y}'_1 \\ \tilde{y}'_2 \\ \tilde{y}'_3 \end{pmatrix}$$

3. A LQ neutrino mass model

Charged lepton mass matrix

$$
\begin{aligned}
-\mathcal{L}_Y = & f_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{1}}] + f_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_A}] \\
& + g_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{1}}] + g_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_A}] \\
& + y_1^d[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{1}}] + y_3^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_S}] + y_{3'}^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_A}] \\
& \boxed{+ y_1^\ell[Y_{\mathbf{1}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{1}}] + y_3^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_S}] + y_{3'}^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_A}]} \\
& + \sum_{A=1,2} \left[\alpha_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}} u_{R_1} + \beta_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}''} u_{R_{1'}} + \gamma_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}'} u_{R_{1''}}\right] \\
& + \sum_{A=1,2} \left[a_u^A \overline{u_{R_1}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}} + b_u^A \overline{u_{R_{1'}}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}''} + c_u^A \overline{u_{R_{1''}}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}'}\right],
\end{aligned}
$$

$$\Downarrow$$

$$
M_\ell = \frac{v}{\sqrt{2}} \begin{pmatrix} y_1^\ell + 2y_3^\ell y_1 & (-y_3^\ell + y_{3'}^\ell) y_3 & -(y_3^\ell + y_{3'}^\ell) y_2 \\ (-y_3^\ell + y_{3'}^\ell) y_2 & y_1^\ell - (y_3^\ell + y_{3'}^\ell) y_1 & 2y_3^\ell y_3 \\ -(y_3^\ell + y_{3'}^\ell) y_3 & 2y_3^\ell y_2 & y_1^\ell + (-y_3^\ell + y_{3'}^\ell) y_1 \end{pmatrix}
$$

The mass matrix is diagonalized by

$$\mathrm{diag.}(m_e, m_\mu, m_\tau) \equiv V_L^\dagger M_\ell V_R \qquad\qquad l_{L,R} \to V_{L,R} l_{L,R}$$

Interactions in generating neutrino mass

$$
\begin{aligned}
-\mathcal{L}_Y =\; & f_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{1}}] + f_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L^c} S L_L)_{\mathbf{3}_A}] \\
& + g_1[Y_{\mathbf{1}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{1}}] + g_S[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_S}] + f_A[Y_{\mathbf{3}}^{(0)} \otimes (\overline{d_R} \eta L_L)_{\mathbf{3}_A}] \\
& + y_1^d[Y_{\mathbf{1}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{1}}] + y_3^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_S}] + y_{3'}^d[Y_{\mathbf{3}}^{(0)} \otimes (\overline{Q_L} H d_R)_{\mathbf{3}_A}] \\
& + y_1^\ell[Y_{\mathbf{1}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{1}}] + y_3^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_S}] + y_{3'}^\ell[Y_{\mathbf{3}}^{(0)} \otimes (\overline{L_L} H \ell_R)_{\mathbf{3}_A}] \\
& + \sum_{A=1,2} \left[\alpha_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}} u_{R_1} + \beta_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}''} u_{R_{1'}} + \gamma_u^A [Y_{3A}^{(6)} \overline{Q_L} \tilde{H}]_{\mathbf{1}'} u_{R_{1''}}\right] \\
& + \sum_{A=1,2} \left[a_u^A \overline{u_{R_1}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}} + b_u^A \overline{u_{R_{1'}}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}''} + c_u^A \overline{u_{R_{1''}}^c} [Y_{3A}^{(6)} S \ell_R]_{\mathbf{1}'}\right],
\end{aligned}
$$

$$
-\mathcal{L}_Y \supset \overline{Q_{L_i}^c} f_1 F_{ij} L_j S + \overline{d_{R_i}} g_1 G_{ij} L_j \eta + h.c.,
$$

$$
F_{ij} = \begin{pmatrix}
1 + 2f_S/f_1 y_1 & (-f_S + f_A)/f_1 y_3 & -(f_S + f_A)/f_1 y_2 \\
-(f_S + f_A)/f_1 y_3 & 2f_S/f_1 y_2 & 1 - (f_S - f_A)/f_1 y_1 \\
(-f_S + f_A)/f_1 y_2 & 1 - (f_S + f_A)/f_1 y_1 & 2f_S/f_1 y_3
\end{pmatrix},
$$

$$
G_{ij} = \begin{pmatrix}
1 + 2g_S/g_1 y_1 & (-g_S + g_A)/g_1 y_3 & -(g_S + g_A)/g_1 y_2 \\
(-g_S + g_A)/g_1 y_2 & 1 - (g_S + g_A)/g_1 y_1 & 2g_S/g_1 y_3 \\
-(g_S + g_A)/g_1 y_3 & 2g_S/g_1 y_2 & 1 + (-g_S + g_A)/g_1 y_1
\end{pmatrix}.
$$

3. A LQ neutrino mass model

Interactions in generating neutrino mass

$$-\mathcal{L}_Y \supset \overline{Q^c_{L_i}} f_1 F_{ij} L_j S + \overline{d_{R_i}} g_1 G_{ij} L_j \eta + h.c.,$$

$$F_{ij} = \begin{pmatrix} 1+2f_S/f_1y_1 & (-f_S+f_A)/f_1y_3 & -(f_S+f_A)/f_1y_2 \\ -(f_S+f_A)/f_1y_3 & 2f_S/f_1y_2 & 1-(f_S-f_A)/f_1y_1 \\ (-f_S+f_A)/f_1y_2 & 1-(f_S+f_A)/f_1y_1 & 2f_S/f_1y_3 \end{pmatrix},$$

$$G_{ij} = \begin{pmatrix} 1+2g_S/g_1y_1 & (-g_S+g_A)/g_1y_3 & -(g_S+g_A)/g_1y_2 \\ (-g_S+g_A)/g_1y_2 & 1-(g_S+g_A)/g_1y_1 & 2g_S/g_1y_3 \\ -(g_S+g_A)/g_1y_3 & 2g_S/g_1y_2 & 1+(-g_S+g_A)/g_1y_1 \end{pmatrix}.$$

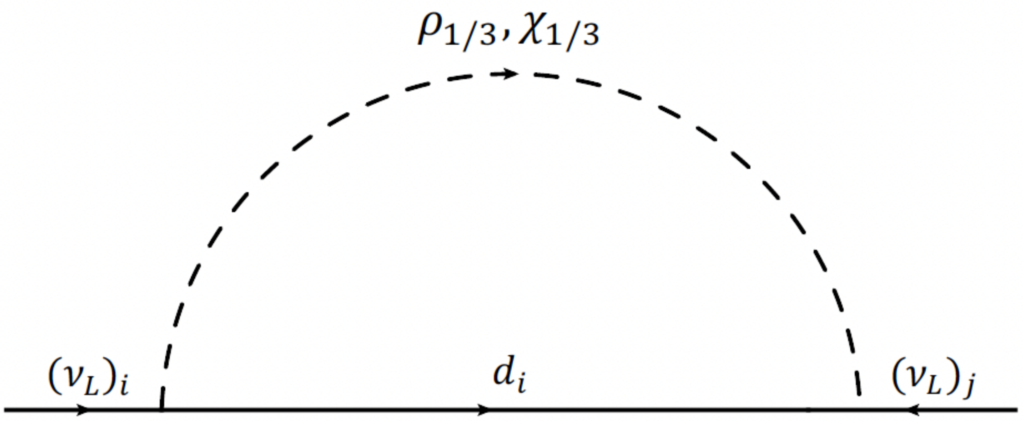
$$d_{L,R} \to V_{dL,dR} d_{L,R} \qquad \begin{pmatrix} \eta_{\frac{1}{3}} \\ S \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho_{\frac{1}{3}} \\ \chi_{\frac{1}{3}} \end{pmatrix}$$

$$\mathcal{L}_Y \supset f_1 \overline{d^{m\,c}_{L_a}} \tilde{F}_{ai} \nu_{L_i} (-s_\alpha \rho_{\frac{1}{3}} + c_\alpha \chi_{\frac{1}{3}}) + g_1 \overline{d^m_{R_a}} \tilde{G}_{ai} \nu_{L_i} (c_\alpha \rho_{-\frac{1}{3}} + s_\alpha \chi_{-\frac{1}{3}})$$

$$\tilde{F}_{ai} \equiv (V^T_{dL})_{aj} F_{ji}, \quad \tilde{G}_{ai} \equiv (V^\dagger_{dR})_{aj} G_{ji},$$

Written in mass basis

One-loop diagram generating neutrino mass



Analytical formula of neutrino mass matrix

$$(\mathcal{M}_\nu)_{ij} = f_1 g_1 \frac{N_c s_\alpha c_\alpha}{2(4\pi)^2}\left(1-\frac{m_\rho^2}{m_\chi^2}\right)\sum_{a=1}^{3}\left[(\tilde{F}^T)_{ja} m_{d_a}\tilde{G}_{ai}+(\tilde{G}^T)_{ja} m_{d_a}\tilde{F}_{aj}\right]F_I(r_\rho, r_{d_a}),$$

$$F_I(r_1,r_2) = \frac{r_1(r_2-1)\ln(r_1)-r_2(r_1-1)\ln(r_2)}{(r_1-1)(r_2-1)(r_1-r_2)},$$

$$r_f = m_f^2/m_\chi^2$$

Free parameters

- Original model (without modular symmetry)

$$\{\underbrace{M_e, M_d, M_u}_{\text{SM fermion mass matrices}}, f, \tilde{y}\} \qquad 5 \times (3 \times 3) = 45 \text{ complex elements}$$

$$\Downarrow$$

- This model (with modular symmetry)

$$\{f_1, f_S, f_A, g_1, g_S, g_A, y_1^d, y_3^d, y_{3'}^d, y_1^l, y_3^l, y_{3'}^l, \alpha_u^1, \alpha_u^2, \beta_u^1, \beta_u^2, \gamma_u^1, \gamma_u^2\}$$

18 complex parameters

\# of free parameters is well reduced

Numerical Analysis

We search for parameters to fit the experimental data

Original free coupling parameters

$$\{f_1, f_S, f_A, g_1, g_S, g_A, y_1^d, y_3^d, y_{3'}^d, y_1^l, y_3^l, y_{3'}^l, \alpha_u^1, \alpha_u^2, \beta_u^1, \beta_u^2, \gamma_u^1, \gamma_u^2\}$$

$$\tilde{\alpha} = \alpha_u^2/\alpha_u^1,\ \tilde{\beta} = \beta_u^2/\beta_u^1 \text{ and } \tilde{\gamma} = \gamma_u^2/\gamma_u^1$$

$$\{f_1, f_S, f_A, g_1, g_S, g_A, \underbrace{y_1^d, y_3^d, y_{3'}^d}_{\text{d-type}}, \underbrace{y_1^l, y_3^l, y_{3'}^l}_{\text{lepton}}, \underbrace{\alpha_u^1, \beta_u^1, \gamma_u^1}_{\text{u-type}}, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}\}$$

Used to fit d-type quark mass

Used to fit u-type quark mass

Used to fit charged lepton mass

Then we scan other parameters $\{|\tilde{\alpha}|, |\tilde{\beta}|, |\tilde{\gamma}|\} \in [10^{-5}, 10^2]$

$\{|f_S|, |f_A|, |g_S|, |g_A|\} \in [10^{-5}, 1]$, $\{m_\rho, m_\chi\} \in [10^3, 10^5]$ [GeV], $s_\alpha \in [10^{-3}, 1]$

Modulus τ in fundamental domain

Estimate chi-square value $\Delta\chi^2 = \sum_i \left(\frac{O_i^{\rm obs} - O_i^{\rm th}}{\delta O_i^{\rm exp}}\right)^2$

CKM mixing angles
+
Neutrino observables

□ Current fit from various neutrino oscillation experiment data

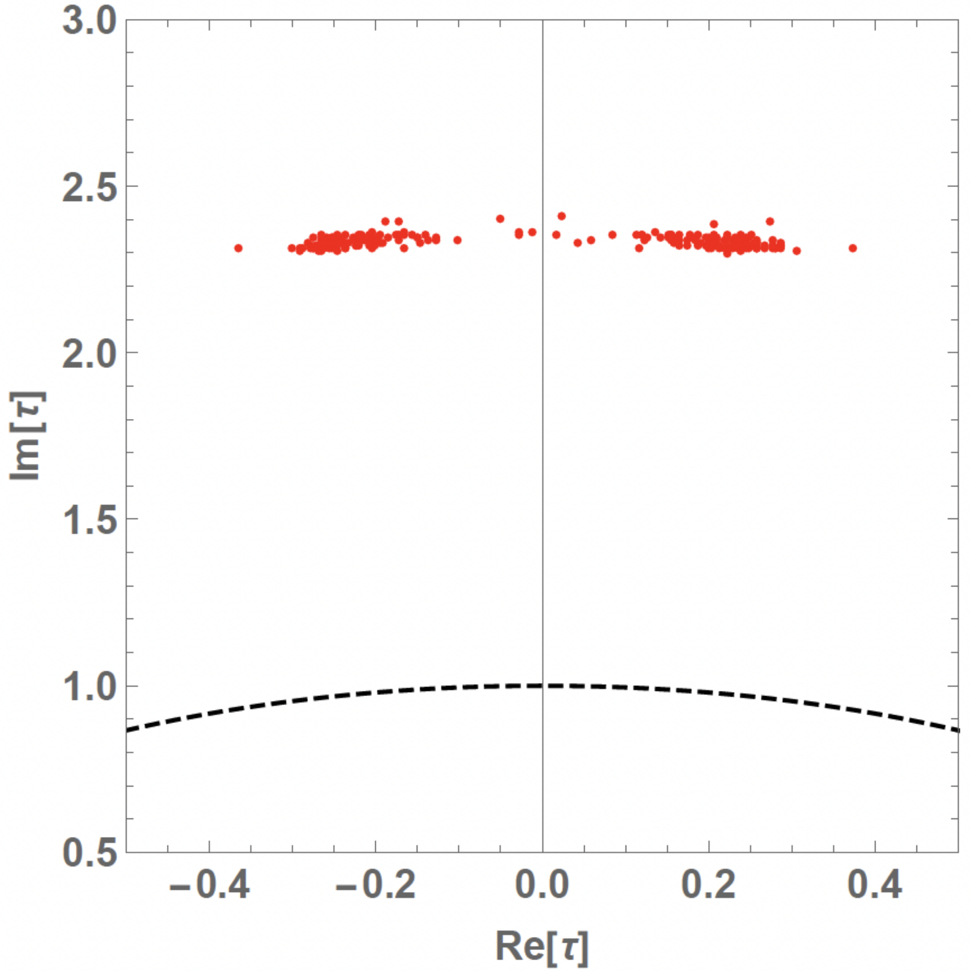
NuFit 6.0: http://www.nu-fit.org/?q=node/294 *I. Estavan et. al. JHEP* 12 (2025) 216, arXiv:2410.05380

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
IC24 with SK atmospheric data	$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
	$\delta_{\mathrm{CP}}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

$m_1<m_2<m_3$ $m_3<m_1<m_2$

Results

Modulus

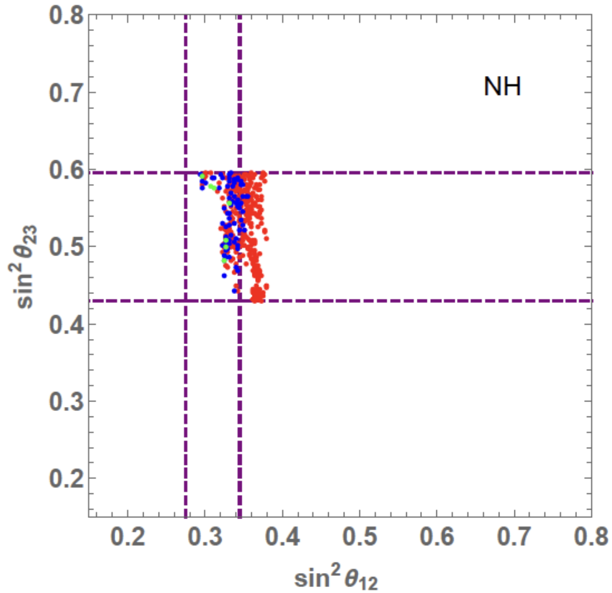


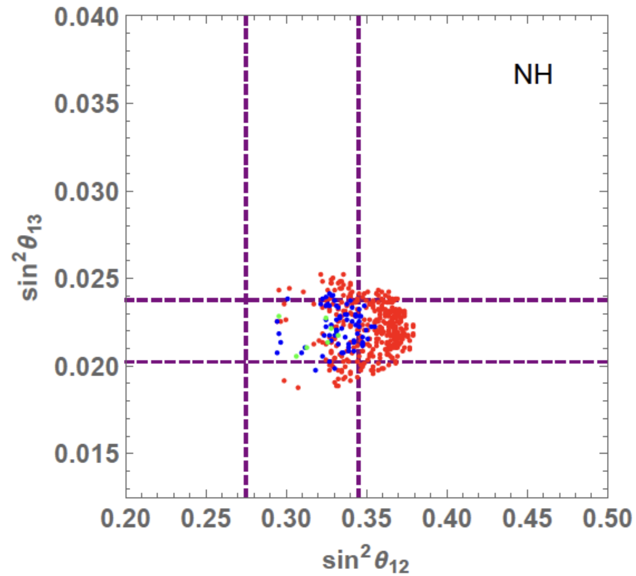
- ✓ Only limited region can accommodate observed data

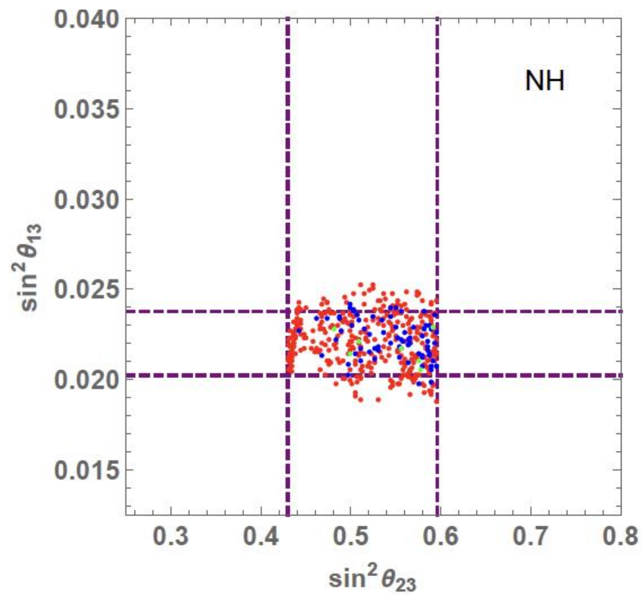
3. A LQ neutrino mass model

Results

Mixing Angles (NH)







Red points: χ^2 value corresponds to 5σ-3σ

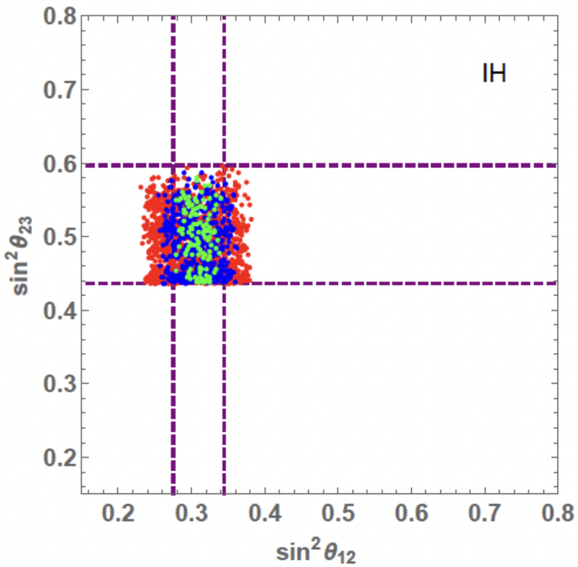
Blue points: χ^2 value corresponds to 3σ-1σ

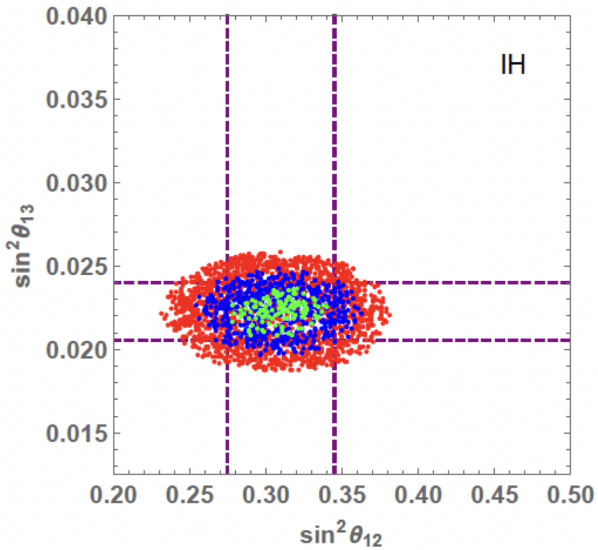
Green points: χ^2 value corresponds to <1σ

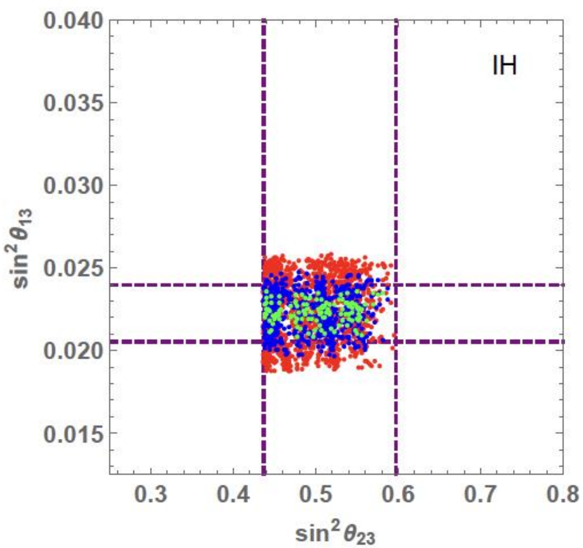
✓ Some tendency for s_{12}

Results

Mixing Angles (IH)





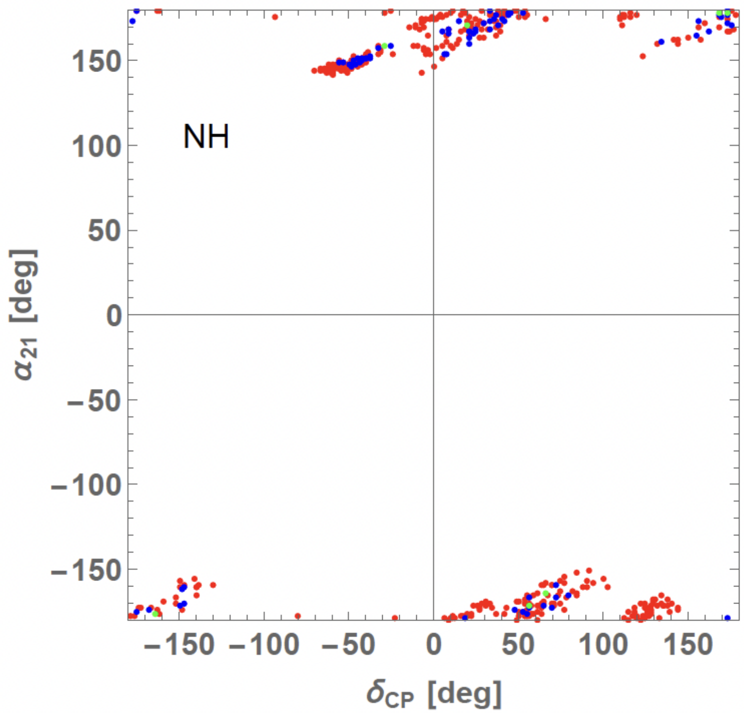


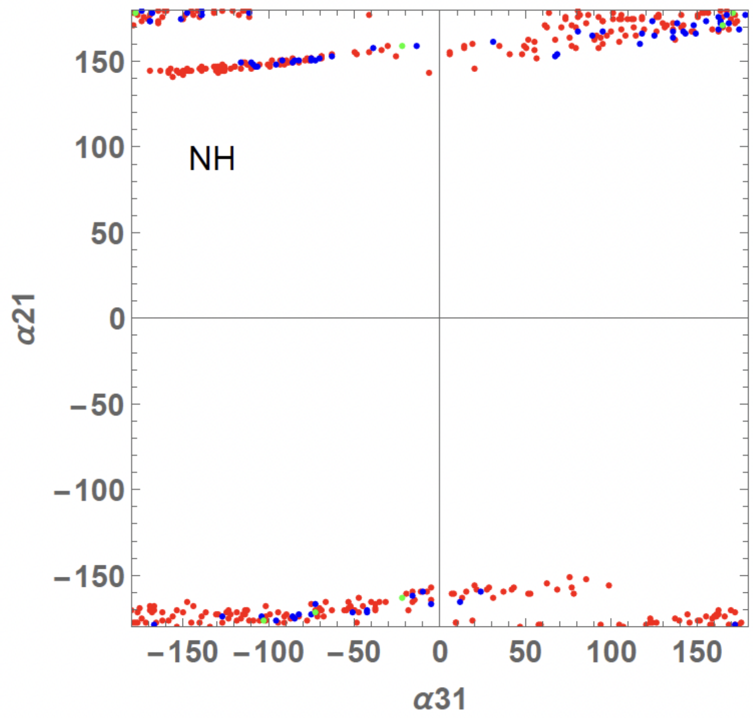
- Color of points are the same as previous figures
  - ✓ Little tendency for s_{23}

3. A LQ neutrino mass model

Results

Phases (NH)

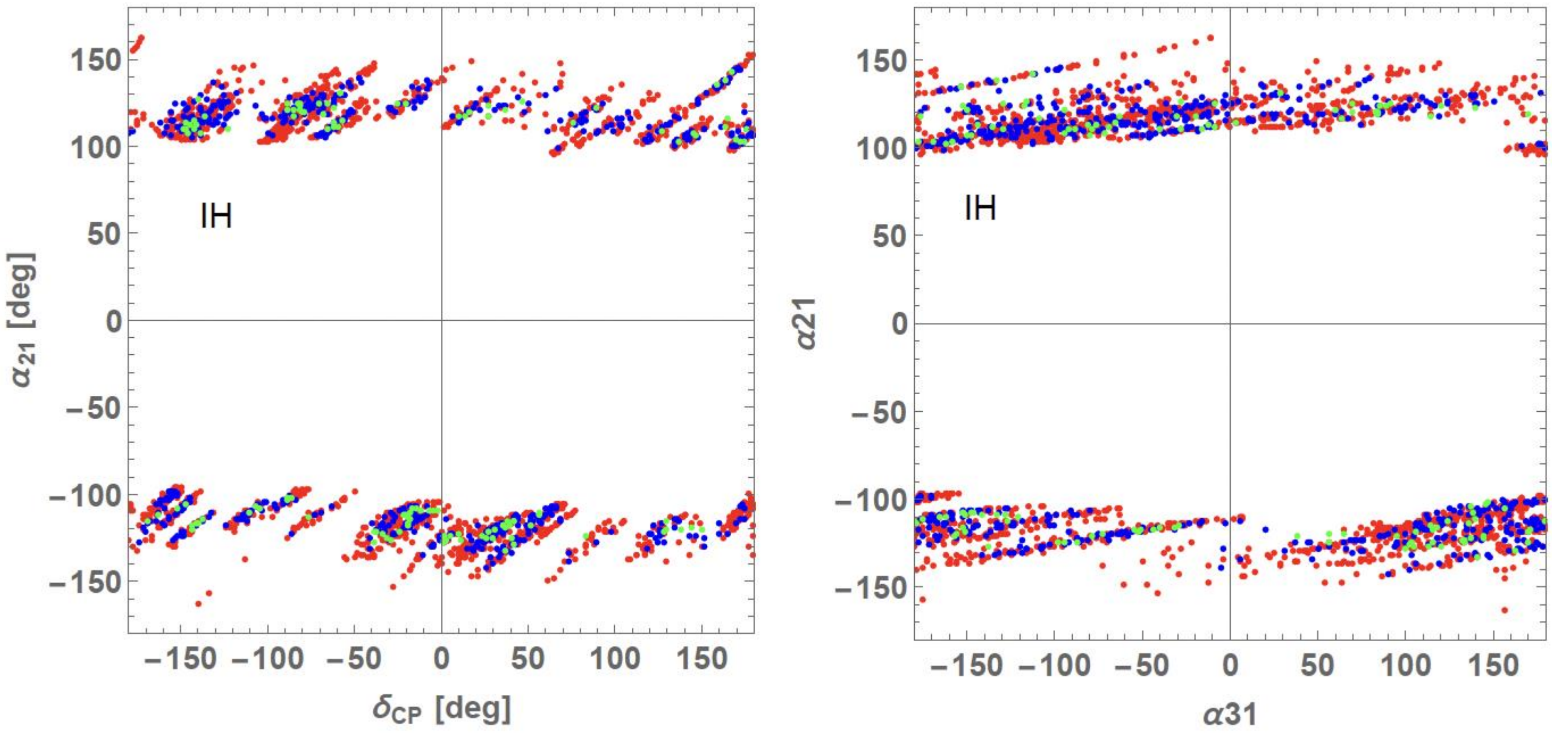




✓ Possible region of Majorana phase is limited

Results

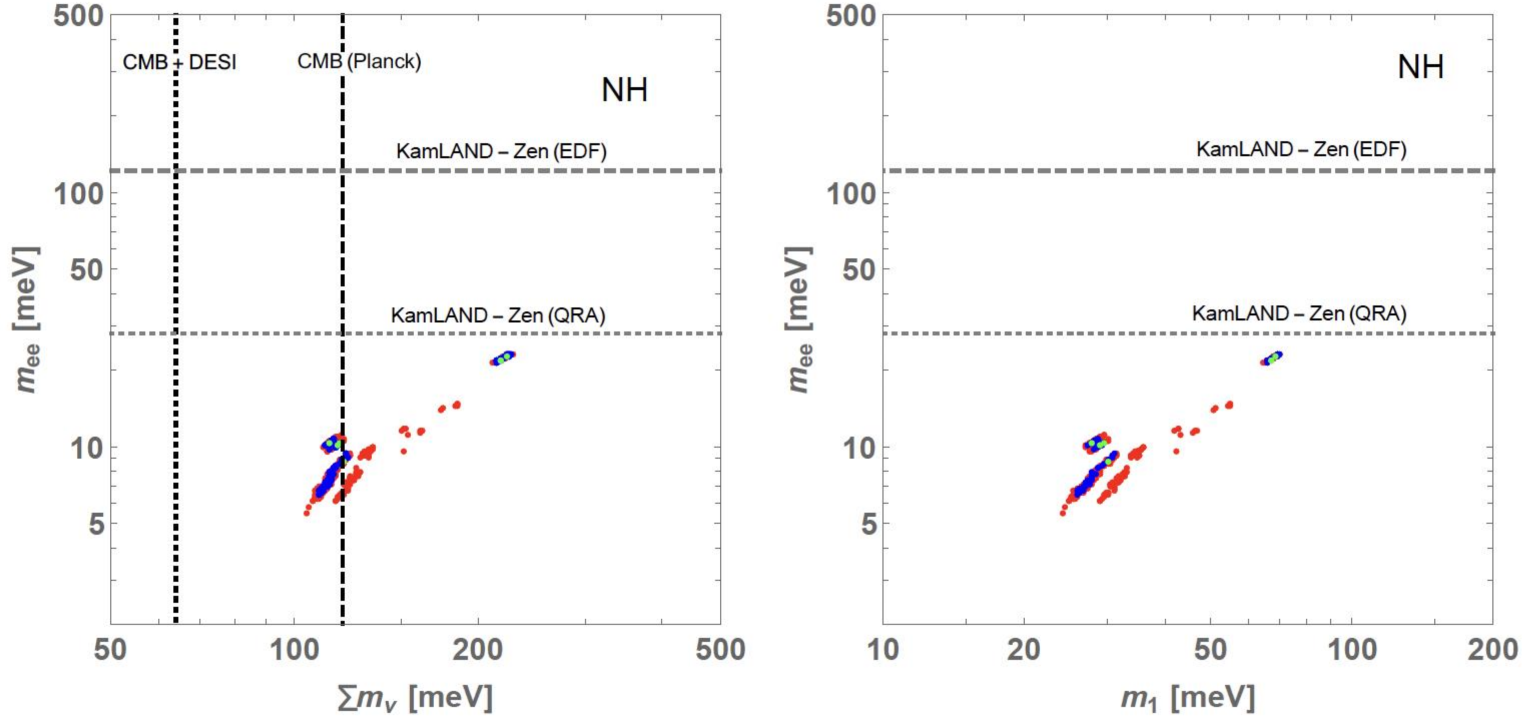
Phases (IH)



✓ Possible region of Majorana phase is limited

Results

Sum of mass and 0νββ (NH)

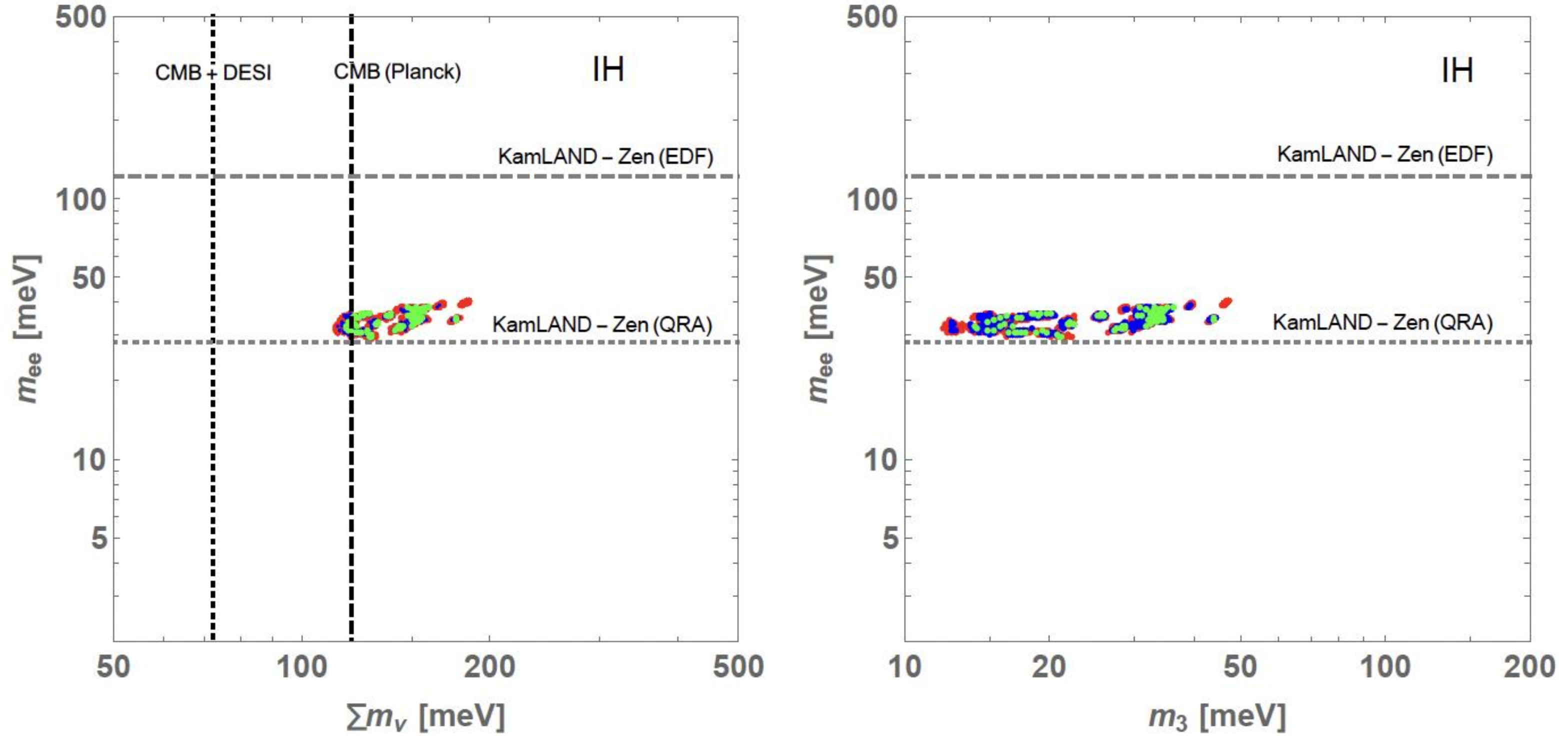


✓ Realized value of observables are restricted

3. A LQ neutrino mass model

Results

Sum of mass and 0vββ (IH)



✓ Realized value of observables are restricted

✓ It could be tested in future 0vββ observations

Summary and Discussions

❑ Neutrino mass and flavor symmetry

- ✓ Generation of neutrino masses requires new physics
- ✓ Mass generation mechanisms do not explain flavor structure
- ✓ Introduction of flavor symmetry would help to understand flavor

◻ Neutrino mass generation in a flavor symmetric model

- ✓ We consider one-loop neutrino mass generation with leptoquarks
- ✓ Application of modular flavor symmetry (non-holomorphic A_4)
- ✓ We got some predictions due to the symmetry

◻ Future direction

- ✓ Explore other phenomenology under the symmetry

Appendix

Types of leptoquarks

$(SU(3), SU(2), U(1))$	Spin	Symbol	Type	F
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	0	S_3	$LL\,(S_1^L)$	-2
$(\mathbf{3}, \mathbf{2}, 7/6)$	0	R_2	$RL\,(S_{1/2}^L)$, $LR\,(S_{1/2}^R)$	0
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	$\tilde{R}_2$	$RL\,(\tilde{S}_{1/2}^L)$, $\overline{LR}\,(\tilde{S}_{1/2}^{\overline{L}})$	0
$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	0	$\tilde{S}_1$	$RR\,(\tilde{S}_0^R)$	-2
$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	0	S_1	$LL\,(S_0^L)$, $RR\,(S_0^R)$, $\overline{RR}\,(S_0^{\overline{R}})$	-2
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	0	$\bar{S}_1$	$\overline{RR}\,(\bar{S}_0^R)$	-2
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3	$LL\,(V_1^L)$	0
$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	1	V_2	$RL\,(V_{1/2}^L)$, $LR\,(V_{1/2}^R)$	-2
$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	1	$\tilde{V}_2$	$RL\,(\tilde{V}_{1/2}^L)$, $\overline{LR}\,(\tilde{V}_{1/2}^{\overline{R}})$	-2
$(\mathbf{3}, \mathbf{1}, 5/3)$	1	$\tilde{U}_1$	$RR\,(\tilde{V}_0^R)$	0
$(\mathbf{3}, \mathbf{1}, 2/3)$	1	U_1	$LL\,(V_0^L)$, $RR\,(V_0^R)$, $\overline{RR}\,(V_0^{\overline{R}})$	0
$(\mathbf{3}, \mathbf{1}, -1/3)$	1	$\bar{U}_1$	$\overline{RR}\,(\bar{V}_0^R)$	0

Table from 1603.04993

There are various types of LQs with different gauge charges

Different interaction structures

Leptoquarks from GUT

LEPTOQUARK	$SU(5)$
$S_3 \equiv (\overline{\mathbf{3}}, \mathbf{3}, 1/3)$	$\overline{\mathbf{45}}, \overline{\mathbf{70}}$
$R_2 \equiv (\mathbf{3}, \mathbf{2}, 7/6)$	$\overline{\mathbf{45}}, \overline{\mathbf{50}}$
$\tilde{R}_2 \equiv (\mathbf{3}, \mathbf{2}, 1/6)$	$\mathbf{10}, \mathbf{15}, \mathbf{40}$
$\tilde{S}_1 \equiv (\overline{\mathbf{3}}, \mathbf{1}, 4/3)$	$\mathbf{45}$
$S_1 \equiv (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$	$\overline{\mathbf{5}}, \overline{\mathbf{45}}, \overline{\mathbf{50}}, \overline{\mathbf{70}}$
$\bar{S}_1 \equiv (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$	$\mathbf{10}, \mathbf{40}$
$U_3 \equiv (\mathbf{3}, \mathbf{3}, 2/3)$	$\overline{\mathbf{35}}, \overline{\mathbf{40}}$
$V_2 \equiv (\overline{\mathbf{3}}, \mathbf{2}, 5/6)$	$\mathbf{24}, \mathbf{75}$
$\tilde{V}_2 \equiv (\overline{\mathbf{3}}, \mathbf{2}, -1/6)$	$\overline{\mathbf{10}}, \overline{\mathbf{40}}$
$\tilde{U}_1 \equiv (\mathbf{3}, \mathbf{1}, 5/3)$	$\mathbf{75}$
$U_1 \equiv (\mathbf{3}, \mathbf{1}, 2/3)$	$\overline{\mathbf{10}}, \overline{\mathbf{40}}$
$\bar{U}_1 \equiv (\mathbf{3}, \mathbf{1}, -1/3)$	$\mathbf{5}, \mathbf{45}, \mathbf{50}, \mathbf{70}$

Table from 1603.04993

LQs could be embedded in GUT

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

Yukawa interactions of the LQs

- $SU(2)_L$ singlet scalar S_1: ($\bar{3}$, 1, 1/3)

$$L \supset \tilde{y}\overline{Q^c}P_L L S_1 + w\,\overline{u^c}P_R l S_1 + z\,\overline{Q^c}P_L Q S_1^* + z\,\overline{u^c}P_L d S_1^* + h.c.$$

- $SU(2)_L$ doublet scalar $\tilde{R}_2$: (3, 2, 1/6)

$$L \supset f\,\bar{d}P_L L\tilde{R}_2 + h.c.$$

$$\tilde{R}_2 = \begin{pmatrix} \tilde{R}_{2/3} \\ \tilde{R}_{-1/3} \end{pmatrix}$$

- $SU(2)_L$ triplet scalar S_3: ($\bar{3}$, 3, 1/3)

$$L \supset w\overline{Q^c}P_L L S_3 + h.c.$$

$$S_3 = \begin{pmatrix} \delta_{1/3}/\sqrt{2} & \delta_{4/3} \\ \delta_{-2/3} & -\delta_{1/3}/\sqrt{2} \end{pmatrix}$$

Yukawa interactions of the LQs

- $SU(2)_L$ singlet scalar S_1: $(\bar{3}, 1, 1/3)$

$$L \supset \underline{\tilde{y}\overline{Q^c}P_L L S_1} + w\,\overline{u^c}P_R l S_1 + z\,\overline{Q^c}P_L Q S_1^* + z\,\overline{u^c}P_L d S_1^* + h.c.$$

$$=\tilde{y}\left(\overline{u^c}P_L l - \overline{d^c}P_L \nu\right)S_1$$

- $SU(2)_L$ doublet scalar $\tilde{R}_2$: $(3, 2, 1/6)$

$$L \supset f\bar{d}P_L L\tilde{R}_2 + h.c.$$

$$= f\left(\bar{d}P_L l\tilde{R}_{2/3} - \bar{d}P_L \nu\tilde{R}_{-1/3}\right)$$

$$\tilde{R}_2 = \begin{pmatrix} \tilde{R}_{2/3} \\ \tilde{R}_{-1/3} \end{pmatrix}$$

- $SU(2)_L$ triplet scalar S_3: $(\bar{3}, 3, 1/3)$

$$S_3 = \begin{pmatrix} \delta_{1/3}/\sqrt{2} & \delta_{4/3} \\ \delta_{-2/3} & -\delta_{1/3}/\sqrt{2} \end{pmatrix}$$

$$L \supset w\overline{Q^c}P_L L S_3 + h.c.$$

$$= w\left(\overline{u^c}P_L \nu\delta_{-2/3} - \frac{1}{\sqrt{2}}\overline{u^c}P_L l\delta_{\frac{1}{3}} - \frac{1}{\sqrt{2}}\overline{d^c}P_L \nu\delta_{\frac{1}{3}} + \overline{d^c}P_L l\delta_{4/3}\right)$$

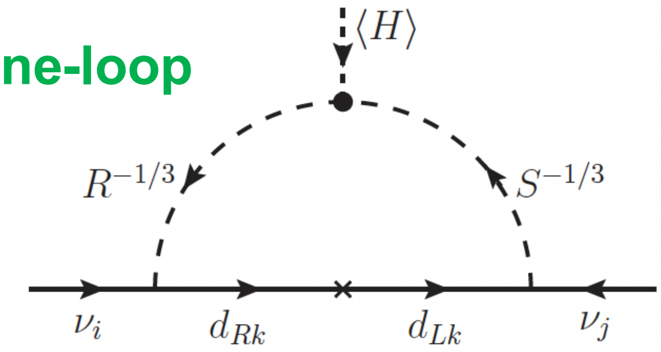
+h.c.

These interactions induce various phenomena

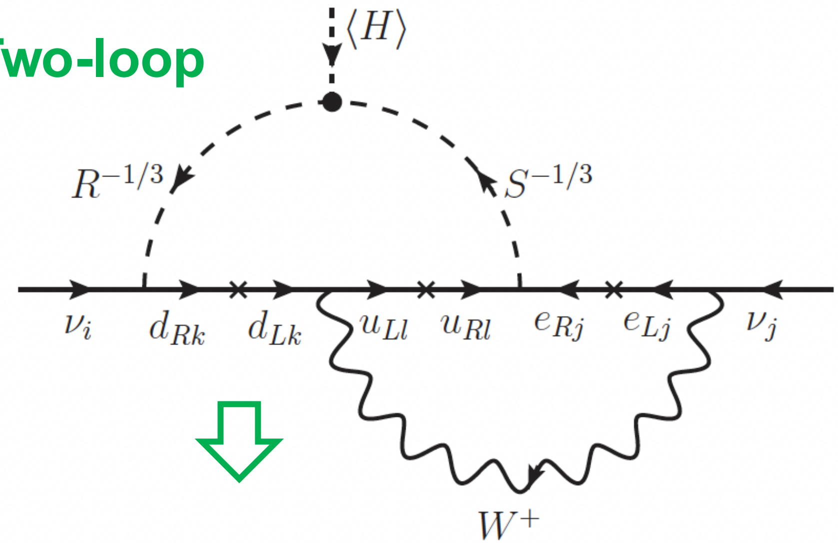
Neutrino mass generation

Taken from 2510.08504

One-loop



Two-loop



Generated Operators

$LLHH$

$LLHHHH^*$

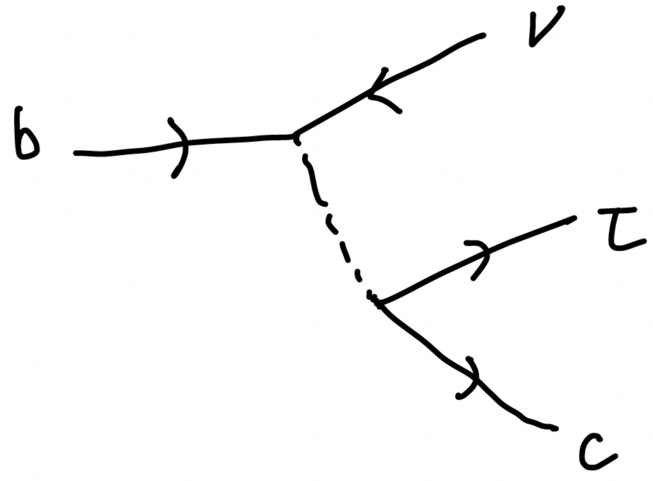
Majorana neutrino mass can be generated

$$(M_\nu)_{ji} = \frac{3 s_{2\theta}}{16\pi^2} m_b \Big\{ \Big[(\lambda^{d^T})_{jk}(D_d)_k \lambda_{ki} + (\lambda^T)_{jk}(D_d)_k \lambda^d_{ki}\Big] I^{(1)}_k$$
$$+ \frac{g^2 m_t m_\tau}{16\pi^2 M_1^2}\Big[(D_\ell)_j(\lambda^{R^T})_{jl}(D_u)_l V_{lk}(D_d)_k\lambda_{ki} I^{(2)}_{jkl} + (\lambda^T)_{jk}(D_d)_k(V^T)_{kl}(D_u)_l\lambda^R_{li}(D_\ell)_i I^{(2)}_{ikl}\Big]\Big\}.$$

Two-loop diagram can be sizable if LQ mass is not much higher than EW scale

Lepton non-universality in meson decay

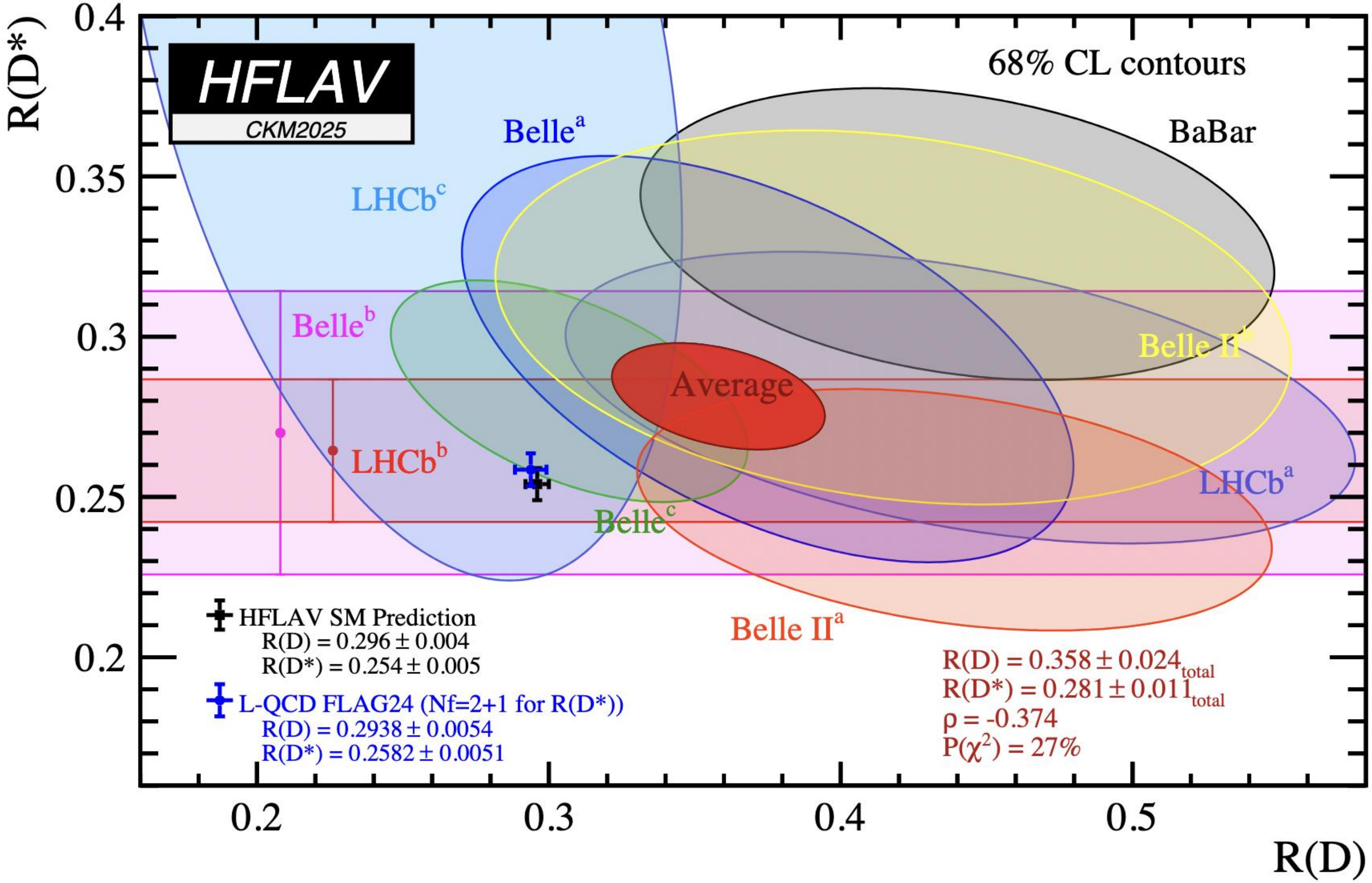
- $b \to c\tau\nu$



$$L \supset \underline{\tilde{y}\overline{Q^c}P_L L S_1} + w\,\overline{u^c}P_R l S_1 + h.c.$$
$$= \tilde{y}\left(\overline{u^c}P_L l - \overline{d^c}P_L \nu\right)S_1$$

Contribution of S_1 LQ induce lepton non-universality

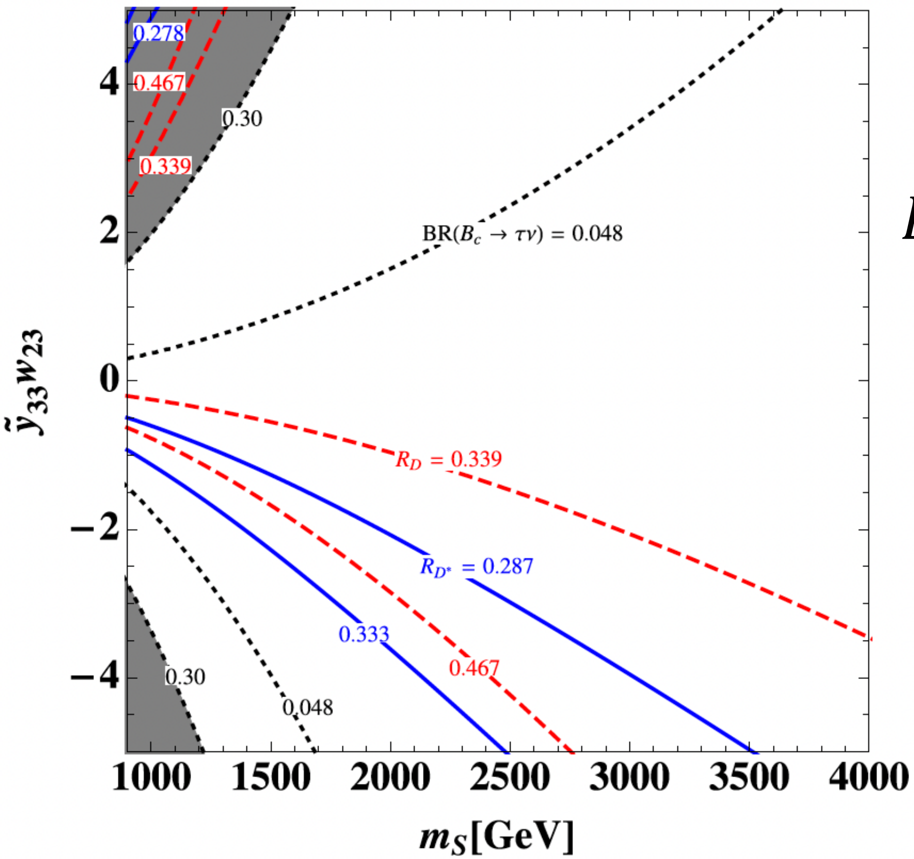
$$\Rightarrow \quad R(D) = \frac{BR(\bar{B} \to D\tau\bar{\nu})}{BR(\bar{B} \to Dl\bar{\nu})} \qquad R(D^*) = \frac{BR(\bar{B} \to D^*\tau\bar{\nu})}{BR(\bar{B} \to D^*l\bar{\nu})}$$



3.8 σ tension with SM prediction

Exp Average: $R(D) = 0.358 \pm 0.024$
$R(D^*) = 0.281 \pm 0.011$

https://hflav-eos.web.cern.ch/hflav-eos/semi/ckm25/html/RDsDsstar/RDRDs.html



LQ explanation of R(D) & R(D*)

$$L \supset \tilde{y}\overline{Q^c}P_L L S_1 + w\,\overline{u^c}P_R l S_1 + h.c.$$
$$=\tilde{y}\left(\overline{u^c}P_L l - \overline{d^c}P_L \nu\right)S_1$$

C.H. Chen, T.N., H.Okada
PLB 774 (2017) 456-464

R(D) = 0.358 ± 0.024
R(D*) = 0.281 ± 0.011

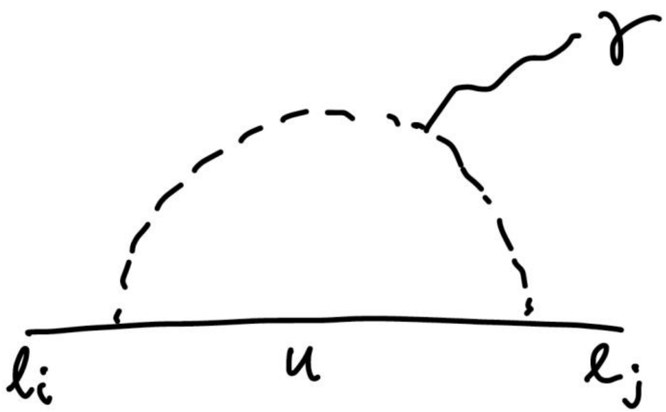
✓ TeV scale LQ can explain the tension

Lepton Flavor Violation (LFV)

Ex) Singlet Leptoquark S_1

$$L \supset \underline{\tilde{y}\overline{Q^c}P_L L S_1} + w\,\overline{u^c}P_R l S_1 + h.c.$$

$$=\tilde{y}\left(\overline{u^c}P_L l - \overline{d^c}P_L \nu\right)S_1$$



$$BR(\ell_b \to \ell_a \gamma) = \frac{48\pi^3 \alpha_{\mathrm{em}} C_{ba}}{G_F^2 m_{\ell_b}^2}\left(|(a_R)_{ab}|^2 + |(a_L)_{ab}|^2\right)$$

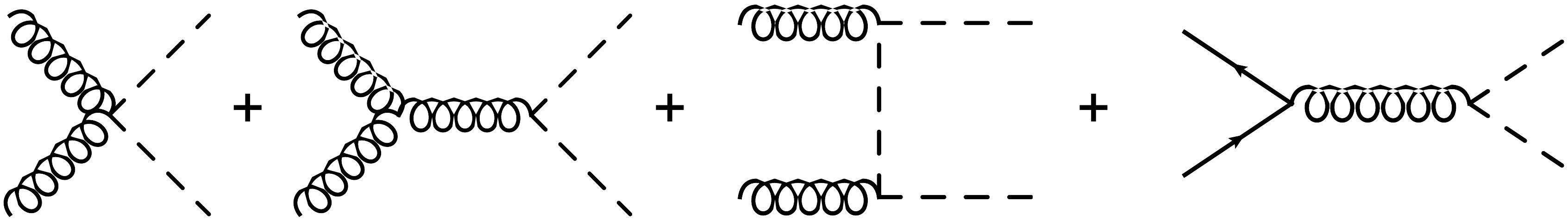
$$a_R \sim \frac{3}{(4\pi)^2}\int dx\,dy\,dz\,\delta(1-x-y-z)\,m_q w \tilde{y}\left(\frac{1}{3}\frac{x}{\Delta(m_t, m_S)_{ab}} + \frac{2}{3}\frac{1-x}{\Delta(m_S, m_t)_{ab}}\right)$$

$$\Delta(m_1, m_2)_{ab} \approx x m_1^2 + (y+z) m_2^2$$

Collider physics

✓ LQ production at the LHC

LQs can be produced via QCD processes at the LHC



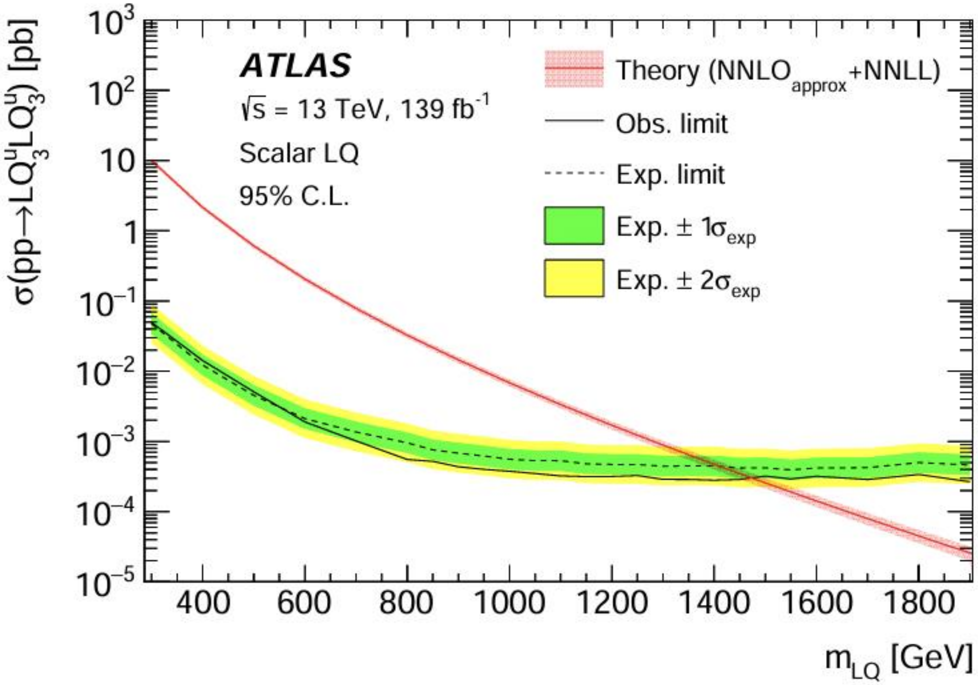
The production cross section is sizable

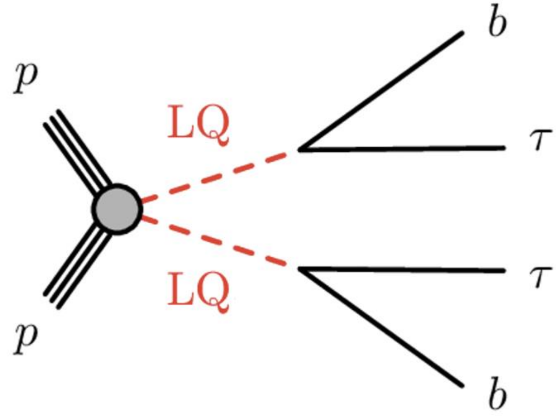
- ➢ $\sigma_{pp\to LQLQ} \sim$ few $\times$ 10 fb for m_{LQ} = 1 TeV

❖ LQs decays into quarks + leptons

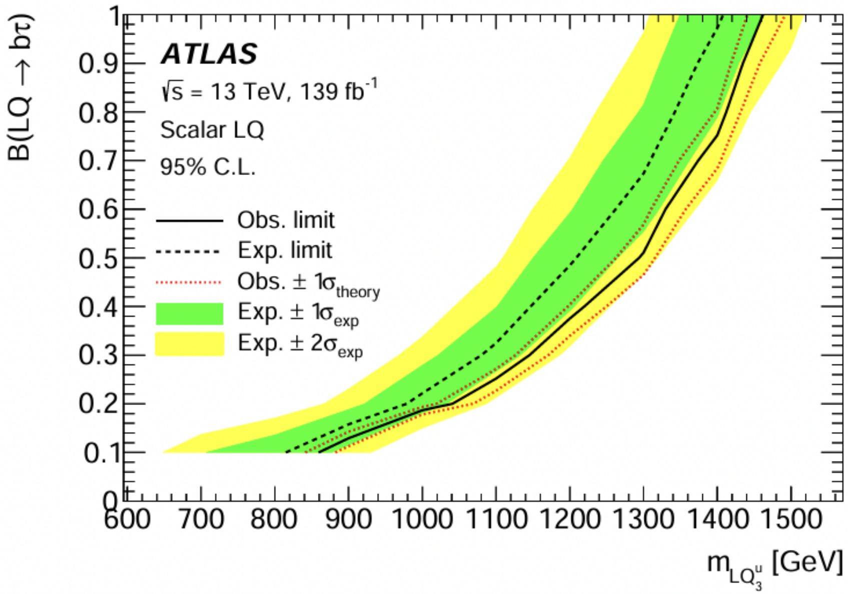
❖ BRs of LQ depend on Yukawa coupling

❖ Model structure would be tested by measuring BRs





ATLAS, EPJC 83 (2023) 11, 1075



LQ mass is constrained up to ~ 1.5 TeV