On the Low-Energy Behavior of Dynamical Gauge Couplings

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CAU-IBS Beyond the Standard Model Workshop 2026

Motivation

- Several constructions utilize the following effective Lagrangian:

$$\mathscr{L} \supset -\frac{1}{2}\left(\frac{1}{g^2} - \frac{\phi}{\Lambda}\right) \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Before BBN/EWPT little is known, dynamics may have differed drastically from standard cosmological evolution

- Gauge couplings drive early-universe dynamics and since $g \to g(\mu)$, different cosmological epochs map to different energy scales

- Varying fine-structure (Lee, et. al., '22, Chluba & Hart, '23, Ghosh, et. al., '25), electroweak confinement (Berger, et. al., '19, Howard, et. al., '21, Bhalla-Ladd, et. al., '25), ultralight scalars (Brzeminski, et. al., '20, Banerjee, et. al., '22, Becker, et. al., '25), early QCD confinement (Ipek & Tait, '18, Lu, et. al., '22)…

- RGE-driven evolution depends only on heavy thresholds, only slow (logarithmic!) variations in energy are possible

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- Varying ... weak confinement (Berger, et. al., '19, Howard, et. al., '21, Bhalla-Ladd, et. al., '25), ultralight ... '25), early QCD confinement (Ipek & Tait, '18, Lu, et. al., '22)…
- RGE-drive... depends only on heavy thresholds, only slow (logarithmic!) variations in energy are possible

Is this interpretation theoretically consistent? Can vacuum shifts induce large variations? How fast can gauge couplings change at low energies?

Varying Gauge Coupling Induced by Singlet Scalar

- Many scenarios introduce singlet scalar ϕ with induced interaction with $SU(N)$ or $U(1)$ gauge boson

$$\mathscr{L} \supset \frac{\phi}{\Lambda} \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Low energy description includes only one extra degree of freedom
- Highly motivated by several UV-completions: Higgs-like, dilation, ultralight scalar (DM), extra dimensions, string theory...
- In this picture, non-trivial scalar dynamics would create a non-trivial profile for the gauge coupling:

$$\mathscr{L} \supset -\frac{1}{2}\left(\frac{1}{g^2} - \frac{\phi}{\Lambda}\right) \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) \equiv \frac{1}{2g_{\mathrm{eff}}^2} \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Drastic changes of couplings implies $\langle\phi\rangle \sim \Lambda$... e.g. confined EW requires $\frac{1}{g^2} \approx \frac{\langle\phi\rangle}{\Lambda} \sim 2.37$

Varying Gauge Coupling Induced by Singlet Scalar...

- The EFT description breaks down in this regime!

$$\mathscr{L} \supset -\frac{1}{2}\left[\frac{1}{g^2} - \sum_n c_n \left(\frac{\phi}{\Lambda}\right)^n\right] \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Require: Low energy containing only singlet ϕ, described by 4D QFT, including the approach to the UV
- Generically can classify possible UV completions into Higgs-like or Extra dimensions

…and the Charged-Matter Portal

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- Require: Low energy containing only singlet ϕ, described by 4D QFT, including the approach to the UV
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- Simplest mechanism: matter charged under the gauge group whose mass tracks a gauge-singlet scalar ϕ

$$\mathscr{L} \supset -(M + y\phi)\bar{\psi}\psi \implies m(\phi) = |M + y\phi|$$

- As ϕ shifts, $m(\phi)$ moves the RG threshold where ψ enters the $\beta(\mu)$, inducing a field-dependent low-energy coupling

The Charged-Matter Portal

- Leading term in the one-loop effective action (Shifman, et. al., '79): $\mathcal{L} \supset -\frac{1}{2g^2(\mu)}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$
- $g(\mu)$ determined by RGE, well known solution: $\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}\ln\left(\frac{\mu}{\Lambda}\right)$
- The slope is fixed by all light charged degrees of freedom present at that scale:

$$\tilde{b} = \frac{1}{4\pi^2}\left[\frac{11}{3}C_2(G) - \frac{2}{3}\sum_f S_2(r_f) - \frac{1}{3}\sum_s S_2(r_s)\right]$$

- Distinct coefficients $\tilde{b}_>$ $(m(\phi) < \mu)$ and $\tilde{b}_<$ $(m(\phi) > \mu)$, $g(\mu)$ continuous across thresholds at leading order
- ϕ dependence of the gauge coupling arises when $\mu < m(\phi)$... Running and matching gives

$$\frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}_>\ln\left(\frac{\mu}{\Lambda}\right) + 2\tilde{b}_\Delta\ln\left[\frac{\mu}{m(\phi)}\right]\Theta\left[m(\phi) - \mu\right]$$

Logarithmic, Not Linear

- Re-expressed via the coupling measured today, $g(\mu_0, \phi_0)$:

$$\frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\mu_0,\phi_0)} + 2\tilde{b}_< \ln\left(\frac{\mu}{\mu_0}\right) - 2\tilde{b}_\Delta \ln\left[\frac{\mu}{m(\phi)}\right],\ m(\phi) > \mu,\ m(\phi_0) > \mu_0$$

$$\to \mathscr{L}_{\text{EFT}} = -\frac{1}{2}\left[\frac{1}{g^2(\mu_0,\phi_0)} + \tilde{b}_< \ln\left(\frac{\mu}{\mu_0}\right) - 2\tilde{b}_\Delta \ln\left(\frac{|M + y\phi|}{m(\phi_0)}\right)\right] \text{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

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$$\rightarrow \mathscr{L}_{\rm EFT} = -\frac{1}{2}\left[\frac{1}{g^2(\mu_0,\phi_0)} + \tilde{b}_< \ln\left(\frac{\mu}{\mu_0}\right) - 2\tilde{b}_\Delta \ln\left(\frac{|M + y\phi|}{m(\phi_0)}\right)\right] {\rm Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- The dependence on the scalar is logarithmic, not linear, re-summing the EFT tower keeps it valid for $|\Delta m|/m \gtrsim 1$
- Applies for any charged matter whose mass depends on ϕ; multiple fields summed!
- Small excursions recover the dimension-5 form, but the variation is far slower than linear

$$\mathscr{L} \supset -\frac{1}{2}\left[\frac{1}{g^2(\mu,\phi_0)} - 2\tilde{b}_\Delta \frac{1}{m}\frac{dm}{d\phi}\bigg|_0 (\phi - \phi_0) + \ldots\right] {\rm Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

Couplings in the Early Universe

- Toy abelian model: N_f vector-like fermions + ϕ; a second-order transition profile

$$V(\phi, T) = \frac{\lambda}{2}\left(\phi^2 - \langle\phi\rangle^2\right)^2 + f(\lambda, y, \ldots)\phi^2 T^2$$

$$\Longrightarrow \langle\phi(T)\rangle = \Theta(T_c - T)\langle\phi\rangle\sqrt{1 - \left(\frac{T}{T_c}\right)^2} \qquad m(\phi) = |M \pm y\langle\phi(T)\rangle|$$

- The probed energy scale is the temperature ($\mu \to T$) of the bath; as the universe cools, thresholds shift
- With $N_f = 100$, many distinct UV theories flow to the same IR coupling
- The naïve linear EFT estimate drastically overestimates the achievable variation.

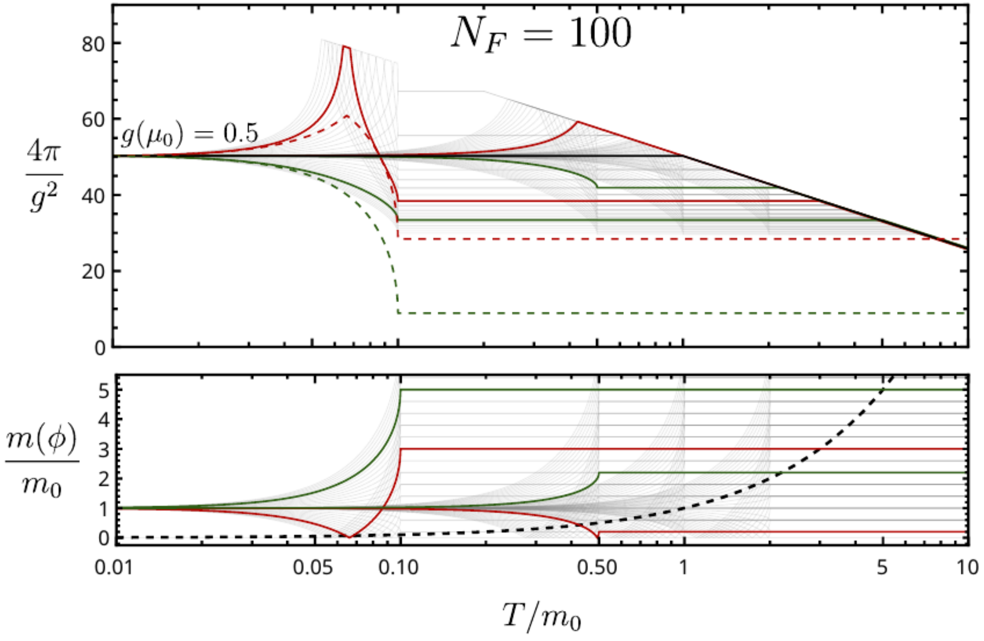


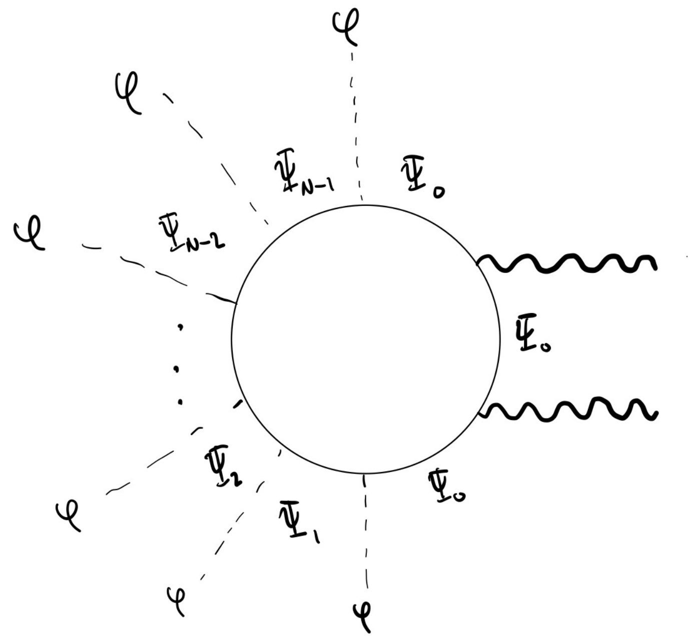
Fig. 1 Coupling fixed in the IR; dashed = naïve linear

Symmetries on the Scalar

- When ϕ transforms non-trivially under a symmetry, dependence of the effective coupling can be modified
- Consider theory with ϕ and N_f fermions ψ_n, $n = 0,1,2,\ldots, N_f - 1$ and invariant under $\mathbb{Z}_{N_f}$ symmetry
  $\phi \to e^{2i\pi/N_f}\phi,\ \psi_n \to e^{-2i\pi n/N_f}\psi_n$

$$\mathscr{L} = -\sum_{k=0}^{N-1}\left[M_k\bar{\psi}_k\psi_k + y_k\phi\bar{\psi}_k\psi_{k+1} + Y\bar{\psi}_0\phi\psi_{N_f} + \mathrm{h.c.}\right]$$

$$\Longrightarrow \frac{1}{g^2(\mu,\phi)} = \frac{1}{g^2(\Lambda)} + 2\tilde{b}_> \ln\left(\frac{\mu}{\Lambda}\right) + 2\tilde{b}_\Delta \ln\left[\frac{\mu^{N_f}}{|\det\, m(\phi)|}\right]$$



- The symmetry is encoded in $|\det\, m(\phi)|$ ... e.g. $N_f = 3$:

$$\det\, m(\phi) = M_0M_1M_2 - M_0|y_1\phi|^2 - M_1|y_2\phi|^2 - M_2|y_0\phi|^2 + (y_0y_1Y\phi^3 + \mathrm{h.c.})$$

- ϕ^3 term $\propto Y$… $Y \to 0$ restores a larger $U(1)$ symmetry.

Enhanced Symmetry → Higher Operators

- Generalizing... $\det\, m(\phi) \supset \prod_i M_i |y_{i+1}\phi|^2 + Y\phi^{N_f}$
- Expand $\phi = \phi_0 + \Delta\phi$...

$$\ln|\det\, m(\phi)| = \ln|\det\, m(\phi_0)| + \mathrm{Re}\left[\mathrm{Tr}\left(m^{-1}\frac{dm}{d\phi}\right)_{\phi_0}\right]\Delta\phi + \frac{1}{2}\mathrm{Re}\left[\mathrm{Tr}\left(m^{-1}\frac{d^2m}{d\phi^2}\right)_{\phi_0} - \mathrm{Tr}\left(m^{-1}\frac{dm}{d\phi}\right)^2_{\phi_0}\right](\Delta\phi)^2$$

- Generic point: the trace is non-zero → leading dependence is linear (a dimension-5 operator).
- Enhanced-symmetry point (e.g. $\phi_0 = 0$): leading dependence is quadratic or higher (dimension-6+).
- Qualitatively different low-energy EFT... significant phenomenological implications (Brzeminski, et. al., '20, Banerjee, et. al., '22, Becker, et. al., '25, Ghosh, et. al., '25)

Why Logarithms? Gauge Invariance

- Gauge invariance forbids mass/kinetic mixing of gauge bosons with neutral fields (exception: Abelian kinetic mixing)
- Up to dimension four, gauge bosons couple only to charged matter, which feeds back at one loop
- To change the coupling we must change charged matter… only masses and non-gauge couplings may vary
- At one loop, only masses reach the gauge running
- How does a moving mass threshold enter the coupling?

The Logarithm Is Generic

- $G_> \supseteq G_<$ matched at massive threshold m... arises from scale-invariant interactions
- Treat the threshold as spontaneous breaking of scale invariance by ϕ
- Above m have renormalizable theory, no ϕ dependence; scale anomaly coefficient at one-loop: $\partial_\mu D^\mu = \tilde{b}_> \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$
- Below $m(\phi)$ expand in powers of g:

$$\mathscr{L}_{\rm eff} \supset -\frac{1}{2}\left[\frac{1}{g^2(\mu)} + f(\phi) + \mathcal{O}(g^2)\right] \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Scale anomaly coefficient below threshold:

$$\partial_\mu D^\mu = \left(\tilde{b}_< - \frac{1}{2}\frac{df}{d\ln\phi}\right) \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

- Matching the light-field anomaly across the threshold fixes $f(\phi)$:

$$\frac{df}{d(\ln\phi)} = -2(\tilde{b}_< - \tilde{b}_>) = -2\tilde{b}_\Delta \implies f(\phi) = \text{constant} - 2\tilde{b}_\Delta \ln\phi$$

Yukawas Vary at Tree Level

- Yukawas are not protected by gauge invariance
- Froggatt–Nielsen-style completion (Froggatt & Nielsen, '79, Leurer, et. al., '92, Leurer, et. al., '93):

$$-\mathscr{L} \supset \xi^c \Phi(y\eta + Y\psi) + (m + f\phi)\psi^c \eta + (M + F\phi)\psi^c \psi$$

	η	ξ^c	ψ	ψ^c	Φ	ϕ
$SU(N)$	$\mathbb{1}$	$\bar{\boldsymbol{N}}$	$\mathbb{1}$	$\mathbb{1}$	$\boldsymbol{N}$	$\mathbb{1}$
$U(1)$	1	-1	1	-1	0	0

- Integrate out ψ and re-sum ϕ insertions:

$$-\mathscr{L}_{\text{eff}} \supset \xi^c \Phi \eta \left(y + \frac{Yf\phi}{M + F\phi} \right) \equiv y(\phi)\xi^c \Phi \eta \qquad y(\phi) = y + \frac{fY\phi}{M + F\phi}$$

- A genuine linear dependence (EFT restructuring once $F\phi \gtrsim M$)

Faster-than-Linear Yukawas

- Add a second heavy pair protected by a $\mathbb{Z}_2$; the leading insertion is now ϕ^2:

$$-\mathscr{L}_{\text{eff}} \supset \xi^c \Phi \eta \left(y + \frac{Y f F_{12} \phi^2}{M_1 M_2 - F_{12} F_{21} \phi^2} \right) \equiv y(\phi) \xi^c \Phi \eta \qquad y(\phi) = y + \frac{Y f F_{12} \phi^2}{M_1 M_2 - F_{12} F_{21} \phi^2}$$

- Larger discrete symmetries push the leading power higher, giving

$$y(\phi) \simeq Y \left[\frac{\phi}{\Lambda(\phi)} \right]^p$$

- Exponent controlled by symmetry... power-law sensitivity, not from any logarithm!

Caveat at Two-loops: Enter the Yukawa Coupling

- At two loops the Yukawa enters the gauge β-function:

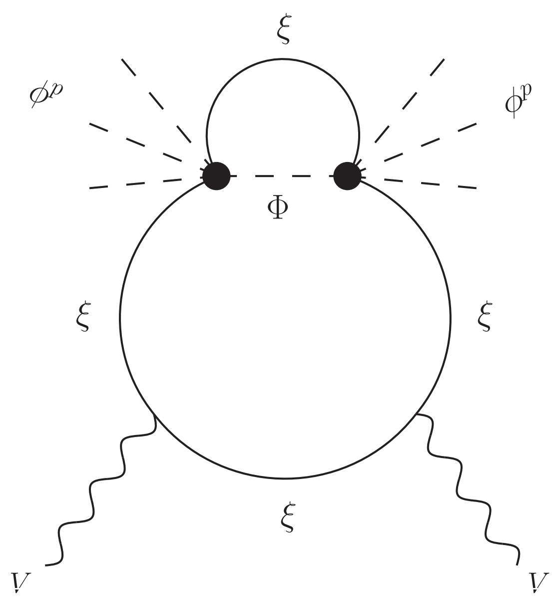
$$\frac{dg}{dt} \simeq -\frac{b_1}{(4\pi)^2}\left[1+\frac{b_2^{(y)}}{b_1(4\pi)^2}y^2\right]g^3$$

- Integrating (with Yukawa as effective β coefficient):

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + \frac{2b_1}{(4\pi)^2}\left[1+\frac{b_2^{(y)}}{b_1(4\pi)^2}y^2\right]\ln\left(\frac{\mu}{\Lambda}\right)$$

- With $y(\phi) \simeq Y\left[\frac{\phi}{\Lambda(\phi)}\right]^p$, the gauge coupling inherits a power-law piece:

$$\left(\frac{1}{g^2}\right)_{2-\text{loop}} \propto y^2(\phi)\ln\left(\frac{\mu}{\Lambda}\right) \sim Y^2\left[\frac{\phi}{\Lambda(\phi)}\right]^{2p}\ln\left(\frac{\mu}{\Lambda}\right)$$



Gauge vs. Yukawa: The Contrast

GAUGE COUPLINGS

$$\mathscr{L} \supset -\frac{1}{2}\left[\frac{1}{g^2(\mu)} - c\ln\left(1+\frac{\Delta m(\phi)}{M}\right)\right]$$

- Protected by gauge invariance
- Enter only at one loop, only through mass thresholds
- Sensitivity is logarithmic in ϕ
- Large change needs many states/big excursions

YUKAWA COUPLINGS

$$y(\phi) \simeq Y\left[\frac{\phi}{\Lambda(\phi)}\right]^p$$

- Unprotected can be modified at tree-level
- Symmetry and mass hierarchy set power p
- Sensitivity is power-law in ϕ
- Feed back into gauge coupling at two loops

Bottom line: *in 4D, gauge couplings are logarithmically stiff; reaching power-law variation costs a two-loop Yukawa channel (or Extra Dimensions)*