Can pre-inflationary axions form miniclusters?

2026. 06. 08.

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Work in progress, in collaboration with
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- Temperature-sourced fluctuations
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Axion dark matter substructure

Post-inflationary

PQ symmetry breaks after inflation.

$O(1)$ misalignment variations on horizon scales at PQ breaking.

String + Domain Wall source large $\delta\rho/\rho$

⟶ Miniclusters & Minivoids well-studied

Eggemeier et al. 2212.00560

Pierobon et al. 2311.17367

Pre-inflationary

PQ symmetry breaks before inflation.

$\theta_{\rm init}$ uniform across the observable universe.

No strings or domain wall

⟶ Naïve expectation: smooth axion DM

But: adiabatic $\delta T/T$ from inflation is still there.

Does it source enough fluctuation to seed substructure?

Why this matters

Direct detection

Local DM density is set by substructure

Lensing & pulsar timing

Pre-inflationary is the cleaner window

Single θ_{init} removes string/wall uncertainties

Predictions follow from inflation + QCD

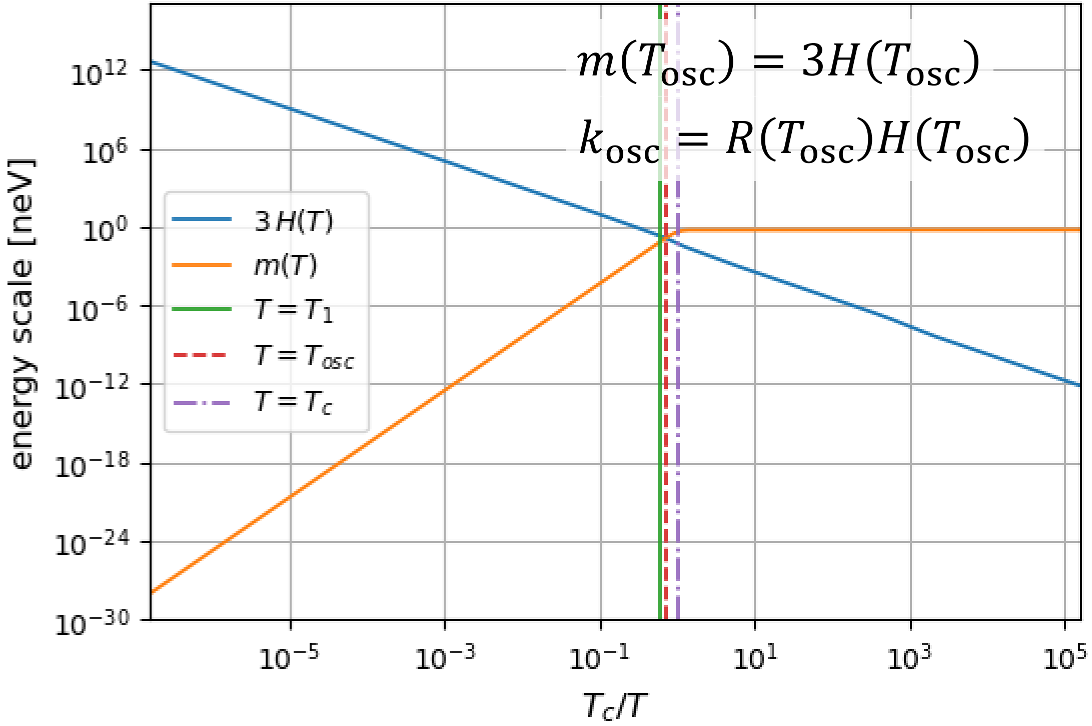
The QCD axion: field and mass

Real scalar $a(x) = f_a\,\theta(x)$, with cosine potential $V = f_a^2 m^2(T)(1 - \cos\theta)$

Temperature-dependent topological susceptibility $\chi(T)$ (Lattice QCD)

$$m^2(T) = \frac{\chi(T)}{f_a^2} = \frac{\chi_0/f_a^2}{1 + \left(\frac{T}{T_c}\right)^b}$$

$\chi_0 = (75.5\ \mathrm{MeV})^4, \qquad T_c = 0.1565\ \mathrm{GeV}, \qquad b = 8.16$



Linearised axion perturbation

Decompose $\theta(t,x) = \bar{\theta}(t) + \delta\theta(t,x)$, $T(t,x) = \bar{T}(t) + \delta T(t,x)$.

Linearise the cosine potential around the homogeneous mode.

Homogenous part

$$\frac{d^2\bar{\theta}}{dt^2} + 3H\frac{d\bar{\theta}}{dt} + m^2(\bar{T})\sin\bar{\theta} = 0$$

Fluctuation mode

$$\frac{\partial^2\widehat{\delta\theta}}{\partial t^2} + 3H\frac{\partial\widehat{\delta\theta}}{\partial t} + \left(\frac{k^2}{R^2} + m^2(\bar{T})\cos\bar{\theta}\right)\widehat{\delta\theta} = \hat{J}$$

Source term

$$\hat{J} = -\bar{T}\frac{dm^2}{dT}(\bar{T})\frac{\widehat{\delta T}}{\bar{T}}\sin\bar{\theta} - 2m^2(\bar{T})\hat{\Psi}\sin\bar{\theta} + \frac{d\bar{\theta}}{dt}\frac{\partial\left(\hat{\Psi} + 3\hat{\Phi}\right)}{\partial t}$$

Thermal drive Grav. drive Kinetic

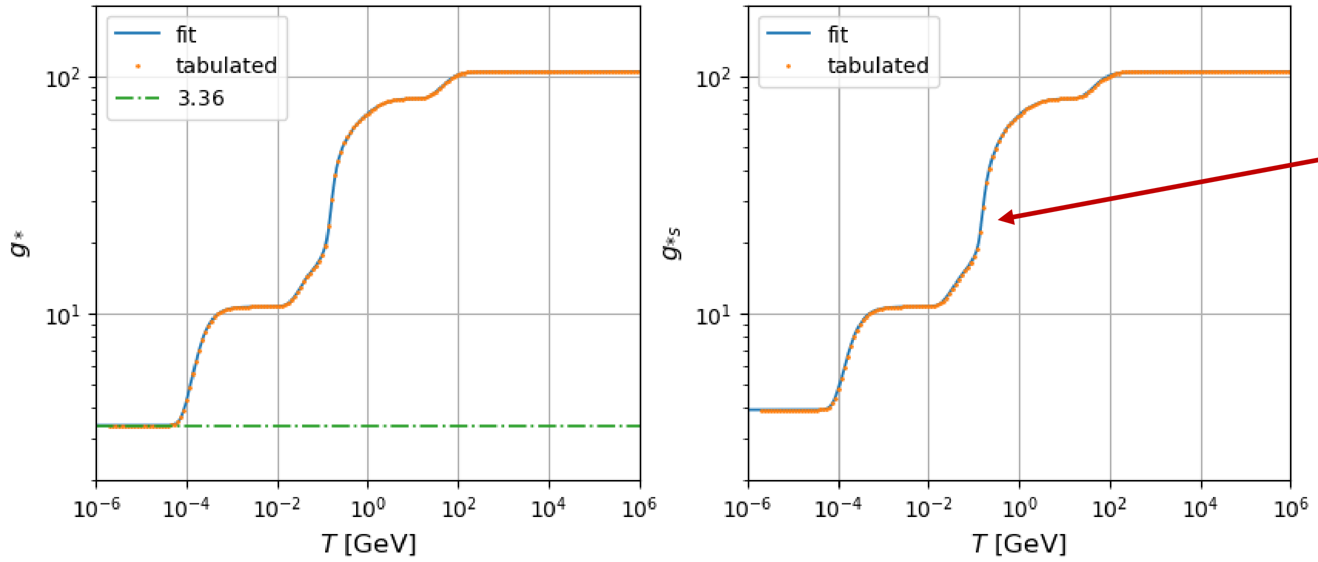
How does $\delta T/T$ excite $\delta\theta$

Source term

$$\hat{J} = -\bar{T}\frac{dm^2}{dT}(\bar{T})\frac{\widehat{\delta T}}{\bar{T}}\sin\bar{\theta} - 2m^2(\bar{T})\hat{\Psi}\sin\bar{\theta} + \frac{d\bar{\theta}}{dt}\frac{\partial\left(\hat{\Psi} + 3\hat{\Phi}\right)}{\partial t}$$

	Thermal drive	Grav. drive	Kinetic
Temperature contrast	$\frac{\widehat{\delta T}}{\bar{T}}$	Sikivie & Wei Xue `22 Dominik & Ahmed `25	
Metric potentials		Φ and Ψ $ds^2 = (1+2\Psi)dt^2 - R^2(1-2\Phi)d\vec{x}^2$ Newtonian gauge	

Inputs: $g_*(T)$, $g_{*s}(T)$, and curvature spectrum



$g_*(T)$ and $g_{*s}(T)$ from Saikawa & Shirai `18

We are looking at the edge

where the source is the largest

$$\bar{T}\frac{dm^2}{dT}(\bar{T})$$

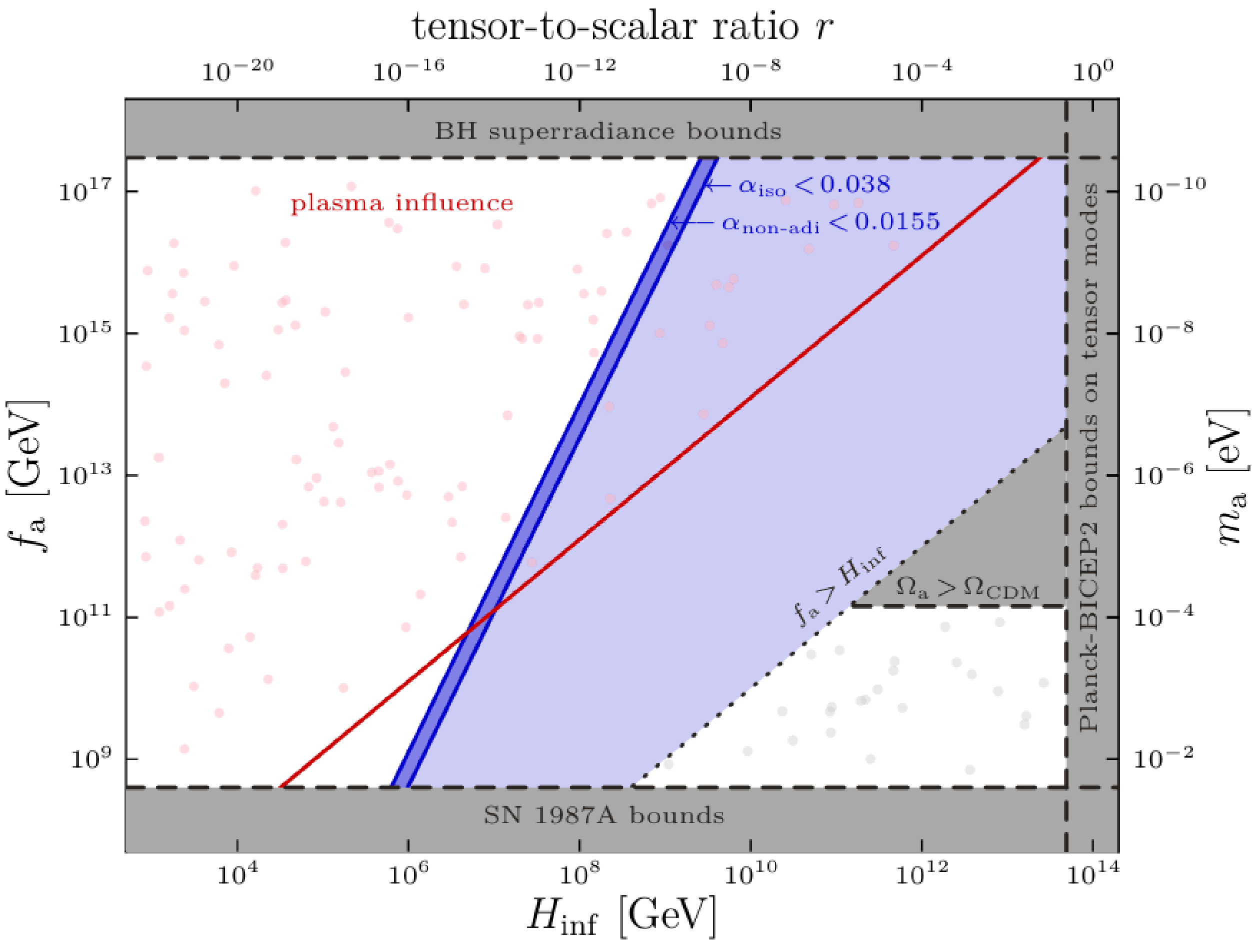
Inflationary input on super-horizon scales:

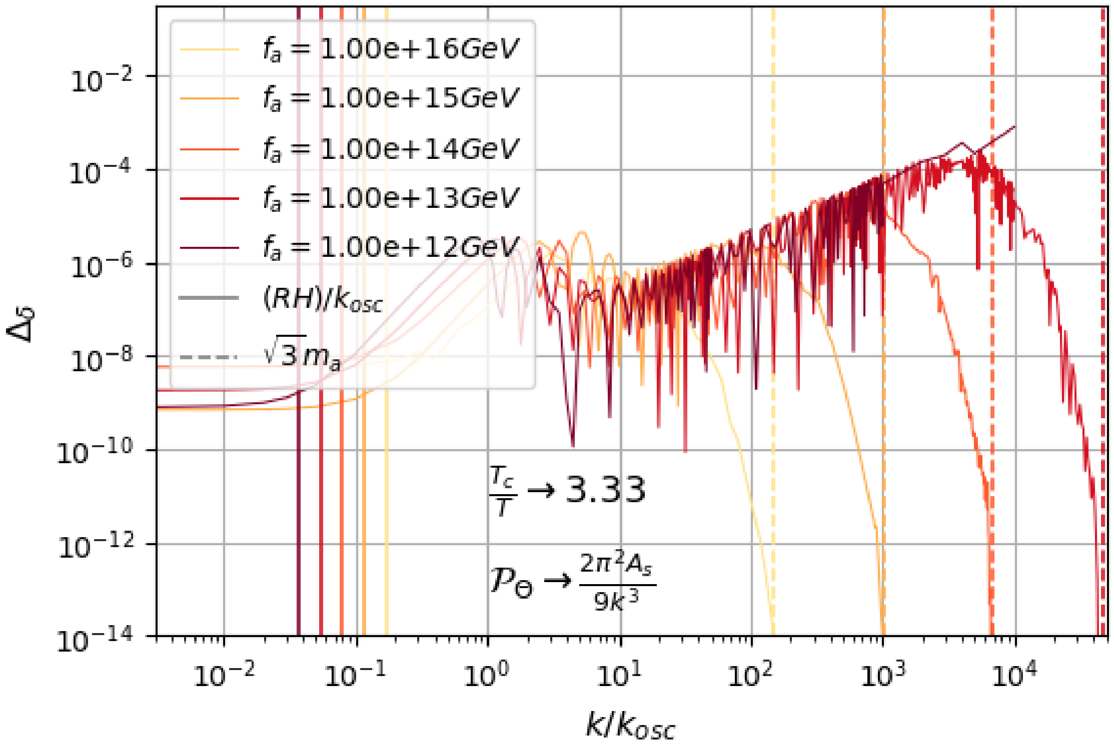
$$\langle\hat{\Theta}(\vec{p})^*\,\hat{\Theta}(\vec{q})\rangle = (2\pi)^3\delta(\vec{p}-\vec{q})\,\mathcal{P}_\Theta(p)$$

$$\mathcal{P}_\Theta(k) \to \frac{1}{9}\frac{2\pi^2}{k^3}A_s \qquad A_s = 2.1\times 10^{-9}$$

$$\hat{\Theta} = -\frac{1}{2}\hat{\Phi} = -\frac{1}{2}\hat{\Psi}$$

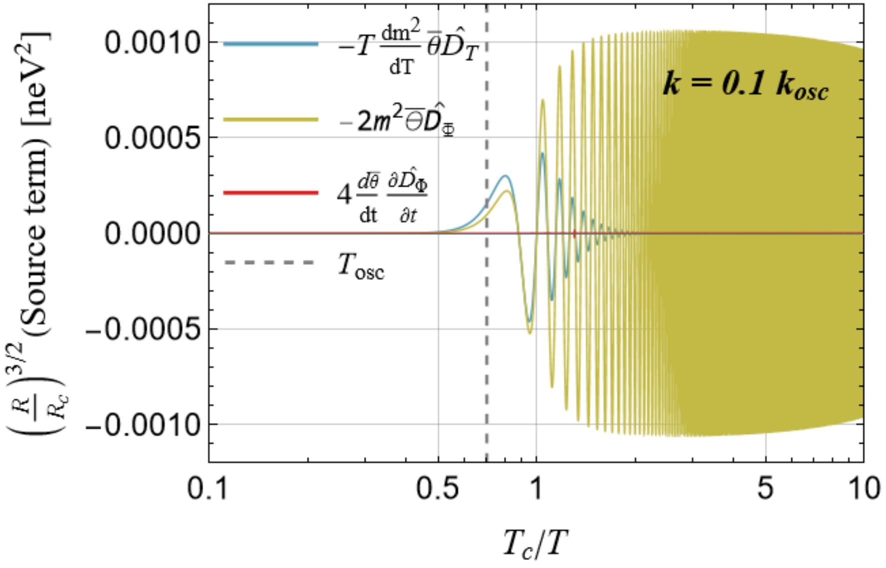
Density contrast power spectrum

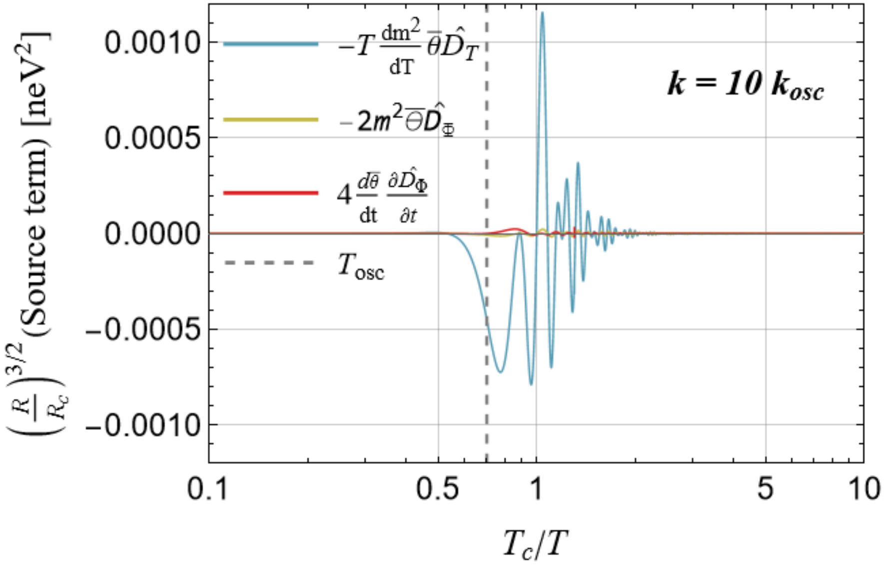




Dominik & Ahmed `25

Density contrast power spectrum





Source of $\widehat{\delta\theta}$

$$\hat{J} = -\bar{T}\frac{dm^2}{dT}(\bar{T})\frac{\widehat{\delta T}}{\bar{T}}\sin\bar{\theta} - 2m^2(\bar{T})\hat{\Psi}\sin\bar{\theta} + \frac{d\bar{\theta}}{dt}\frac{\partial(\hat{\Psi}+3\hat{\Phi})}{\partial t}$$

Dominant in the resonance regime

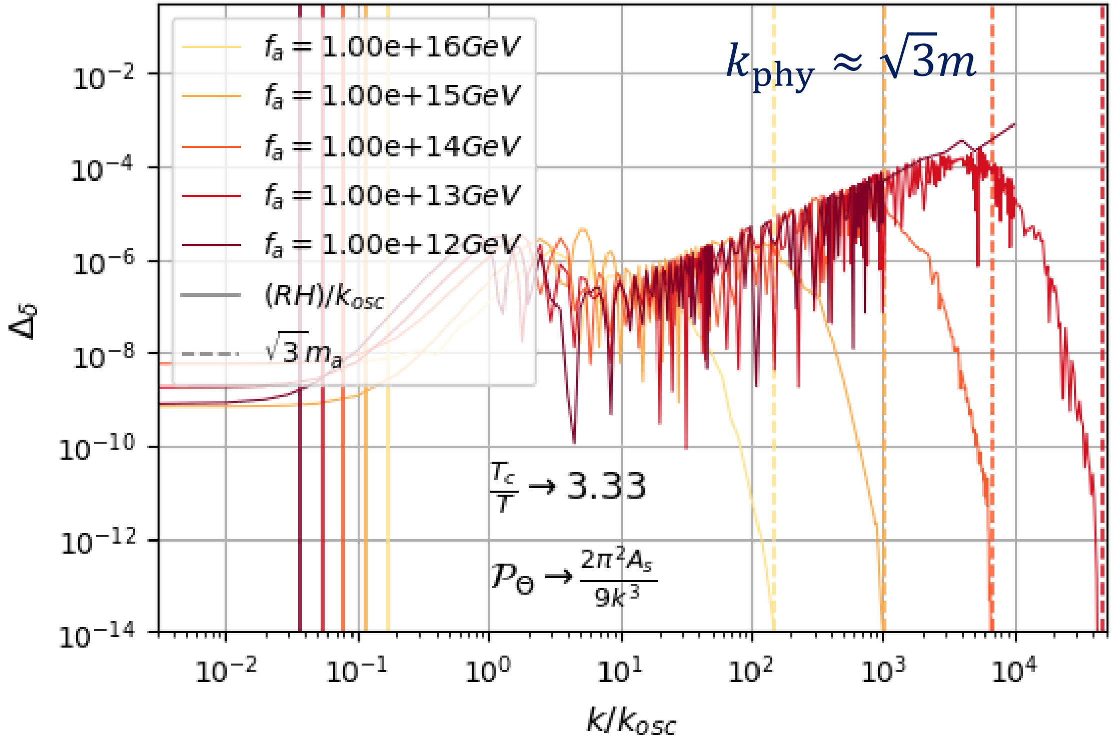
After horizon entry around QCD

$$\hat{\Theta} \sim \cos\left(\frac{k}{\sqrt{3}}\int^t \frac{dt'}{R(t')}\right) \sim \cos\left(\frac{k_{\mathrm{phy}}}{\sqrt{3}}t + \delta'\right)$$

Latest resonance condition ⟶ $k_{\mathrm{phy}} \approx \sqrt{3}m$

$$\sqrt{m^2 + k_{\mathrm{phy}}^2} \approx m + \frac{k_{\mathrm{phy}}}{\sqrt{3}}$$

Density contrast power spectrum



⟶ Would this fluctuation collapse?

Condition for growth

$\mathcal{O}(1)$ density contrast

$k < k_{QJ}$ (Quantum Jeans' scale)

Non-relativistic wave description

$$i\partial_t\psi + \frac{\nabla^2}{2mR^2}\psi - m\Phi\psi = 0$$

Two scales compete in the dispersion relation

Gradient pressure

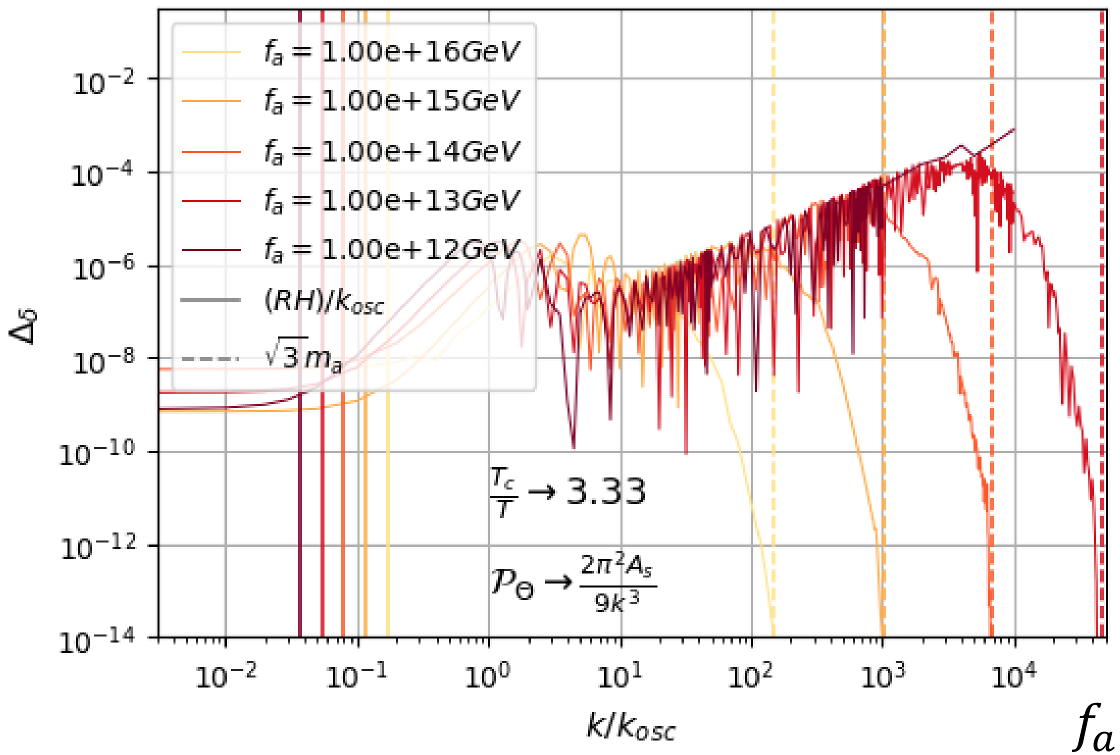
$$\omega_{\rm grad} \sim \frac{k^2}{2mR^2}$$

self-gravity

$$\omega_{\rm grav} \sim \sqrt{4\pi G\bar{\rho}}$$

$$\omega_{\rm grad} = \omega_{\rm grav} \longrightarrow k_{\rm QJ} = R(16\pi G\bar{\rho}m^2)^{1/4}$$

Density contrast power spectrum



In the scanned region, $\frac{k_{\rm peak}}{R(T_c)} \approx m$.

Condition for growth

$\mathcal{O}(1)$ density contrast

$k < k_{QJ}$ (Quantum Jeans' scale)

$$k_{QJ} = R(16\pi G\bar{\rho}m^2)^{1/4}$$

$$\frac{k_{QJ}}{k_{\rm peak}} \approx 0.15 \times \left(\frac{R_0/R}{3400}\right)^{-\frac{1}{4}} \left(\frac{m}{0.57\ \rm neV}\right)^{-\frac{1}{2}} \left(\frac{\Omega_{DM}h^2}{0.12}\right)^{\frac{1}{4}}$$

$f_a \to 10^{16}$ GeV ⟶ $m \approx 0.57$ neV, smaller f_a only worsen.

⟶ Requires the redshift $z < 0.7$ for $k < k_{QJ}$.

For such large f_a, density should grow by $\times 10^3$.

⟶ Requires the redshift $z < 2.4$

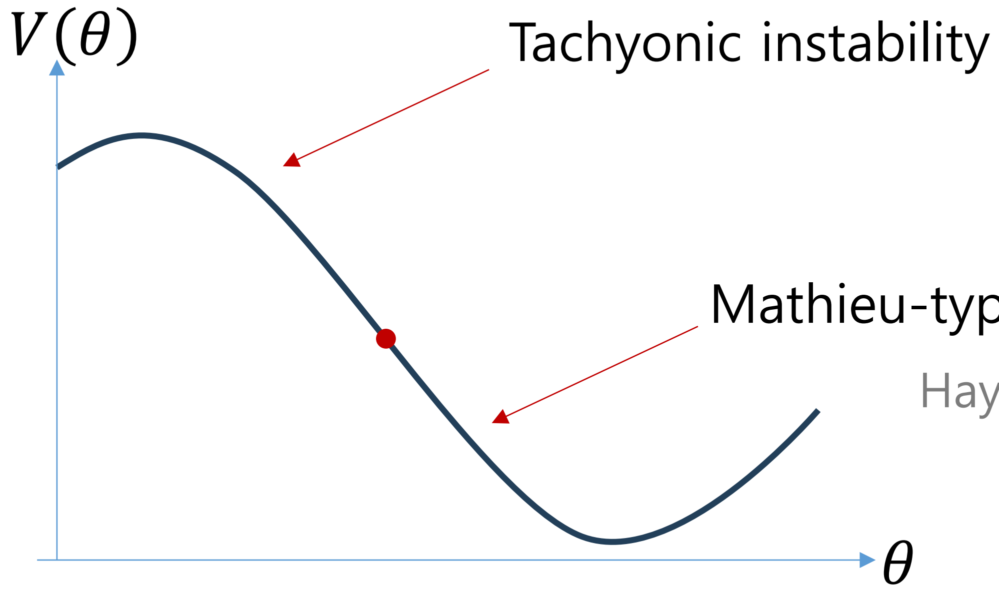
Smaller f_a scenario

For $f_a \leq 10^{11}$ GeV, $\bar{\theta}_{\rm init}$ must be tuned to π. $\bar{\theta}_{\rm init} \approx 0.65$ for $f_a \approx 10^{12}$ GeV

Oscillation is delayed, and Hubble drag becomes smaller at oscillation.

The anharmonicity of the potential becomes important even after the oscillation.

Naoya et al. `22



Hayato et al. `19

Smaller f_a scenario

Large scale hierarchy

Completion of QCD cross-over takes $\sim H(T_c)^{-1} \approx (0.018\,\mathrm{neV})^{-1}$

$\times 10^6$

Axion oscillation time scale $\sim m^{-1} \approx (5.7 \times 10^4\,\mathrm{neV})^{-1}$

Yet, estimation of axion fluctuation amplitude, not only the density,

is important for Schrodinger-Poisson initialisation.

$$i\partial_t\psi + \frac{\nabla^2}{2mR^2}\psi - m\Phi\psi = 0$$

$$\nabla^2\Phi = \frac{4\pi G m}{R}\left(|\psi|^2 - \langle|\psi|^2\rangle\right)$$

$\longleftarrow$

$$a(x) = \frac{1}{\sqrt{2m\,R^3}}\left(\psi(x)\,e^{-imt} + \psi^*(x)\,e^{imt}\right)$$

Smaller f_a scenario

Slow-envelope formulation

Absorbing the fast phase

$$\varphi(t) = \int^t \omega(t')\, dt' \qquad \omega(t) \to m(t)$$

$$\widehat{\delta\theta} = \frac{1}{\sqrt{2\omega R^3}}\left(A(t,\vec{k})\, e^{-i\varphi} + A^*(t,-\vec{k})\, e^{i\varphi}\right)$$

$$\bar{\theta} = \frac{1}{\sqrt{2\omega R^3}}\left(B(t)\, e^{-i\varphi} + B^*(t)\, e^{i\varphi}\right)$$

Laurent expand the equation of motion in $e^{-i\varphi}$

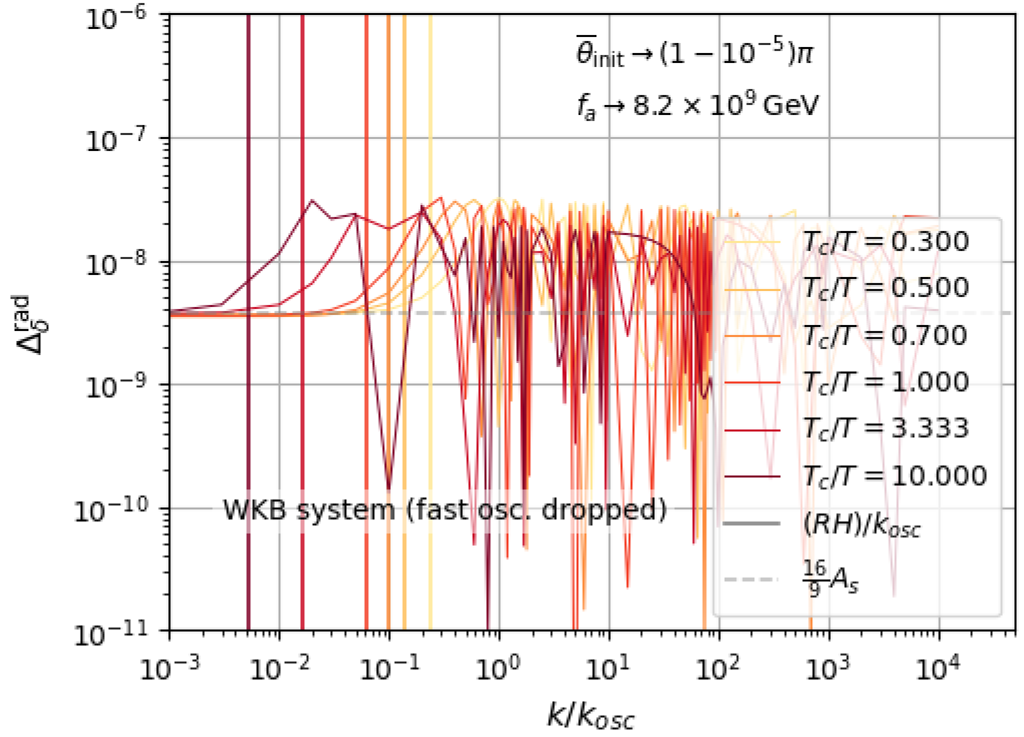
$$\frac{d^2\bar{\theta}}{dt^2} + 3H\frac{d\bar{\theta}}{dt} + m^2(\bar{T})\sin\bar{\theta} = 0$$

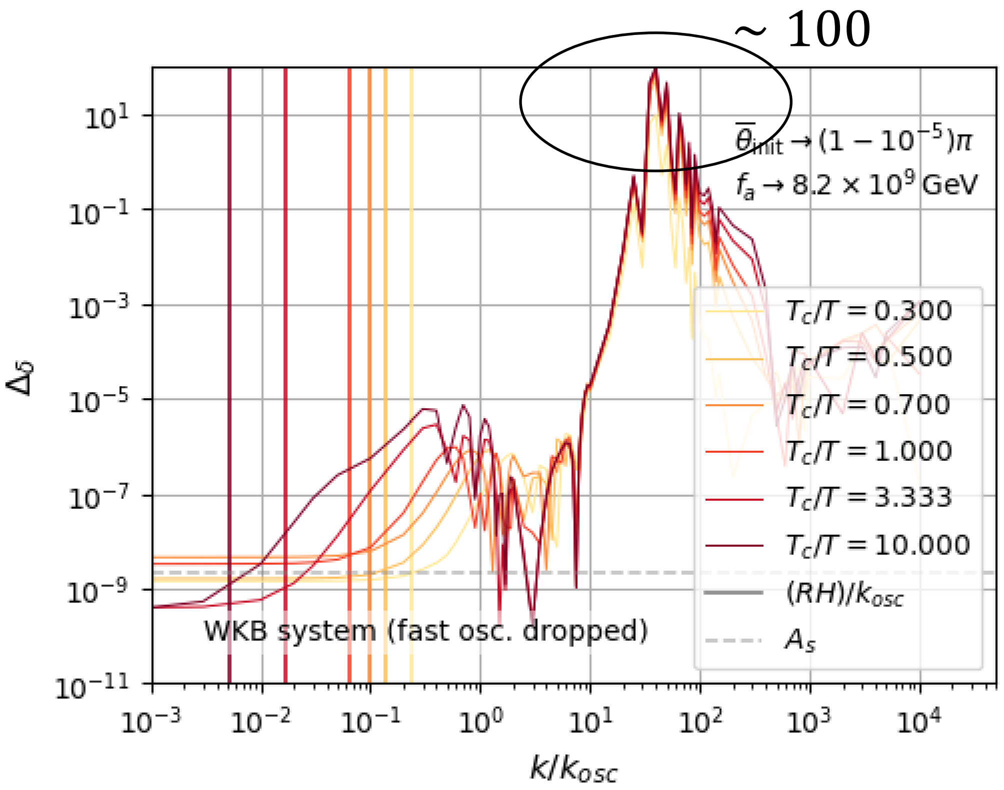
$$\frac{\partial^2\widehat{\delta\theta}}{\partial t^2} + 3H\frac{\partial\widehat{\delta\theta}}{\partial t} + \left(\frac{k^2}{R^2} + m^2(\bar{T})\cos\bar{\theta}\right)\widehat{\delta\theta} = \hat{J}$$

⟶ $e^{-in\varphi}$ terms on both sides.

Take only the $e^{-0\cdot i\cdot\varphi}$ term.

Density contrast power spectrum: $f_a = 8.2 \times 10^9$ GeV $\quad \bar{\theta}_{\rm init} = (1 - 10^{-5})\pi$



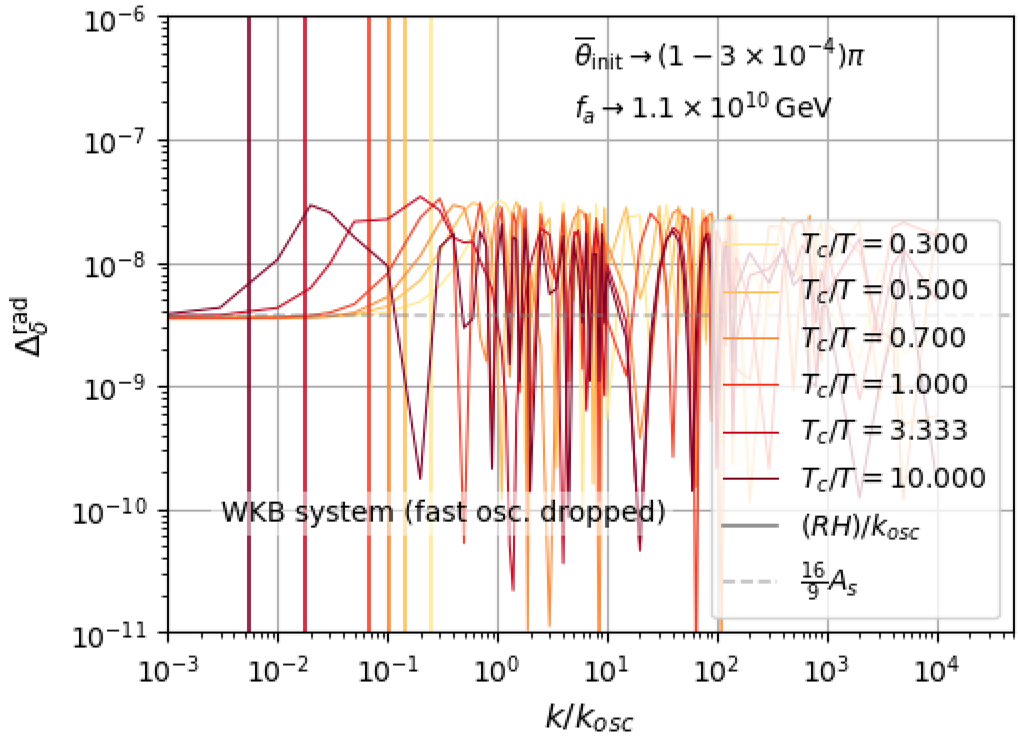


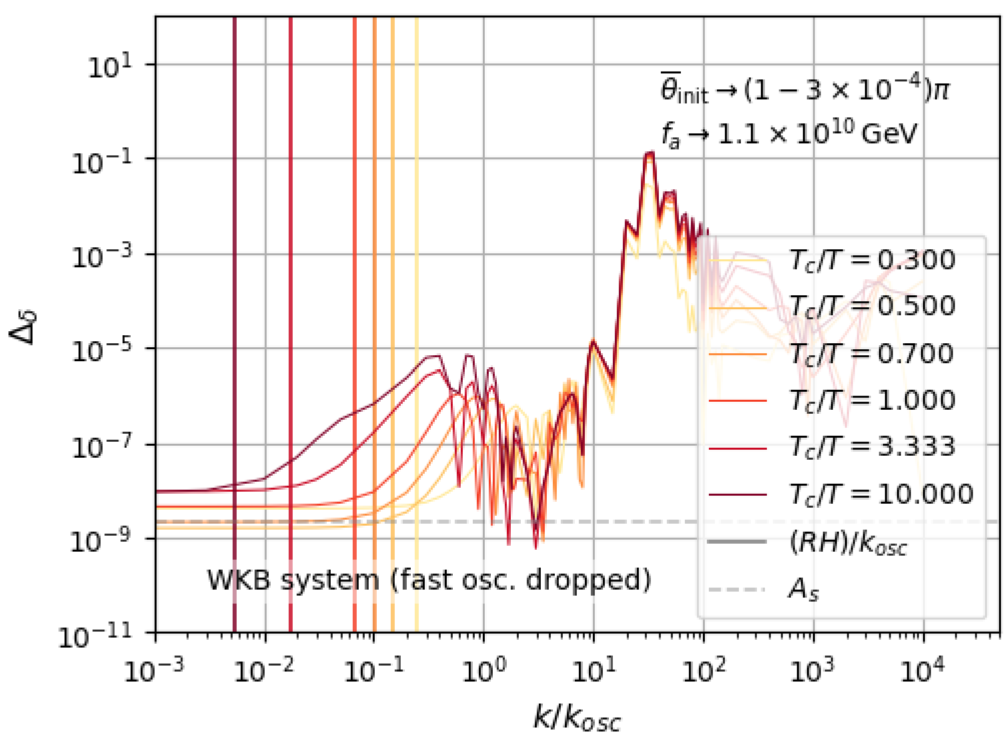
$k_{QJ}(T_{\rm MRE}) = 23000\ {\rm pc}^{-1}$ ← $k_{\rm peak} = 3500\ {\rm pc}^{-1}$

$\times 10$

Density contrast power spectrum: $f_a = 1.1 \times 10^{10}$ GeV $\bar{\theta}_{\rm init} = (1 - 3 \times 10^{-4})\pi$

The peak height is not very sensitive to f_a, but to $\bar{\theta}_{\rm init}$.





$k_{QJ}(T_{\rm MRE}) = 20000\ {\rm pc}^{-1}$ ← $k_{\rm peak} = 2300\ {\rm pc}^{-1}$

$\times 10$

Why this benchmark is interesting

$$f_a = 1.1 \times 10^{10}\ \text{GeV}$$

$k_{\text{peak}} = 2300\ \text{pc}^{-1}$

$\times$ 10 at MRE

$k_{QJ}(T_{\text{MRE}}) = 20000\ \text{pc}^{-1}$

$\delta_{\text{peak}} \approx 0.1$

Peak will readily start to collapse at $\frac{R}{R_{\text{MRE}}} \sim 10 \quad z = 340$ → Perfect target for S-P simulation.

The linear perturbation theory result can be trusted down to MRE.

Clean environment (first star at $z < 65$, first galaxy at $z < 20$) Naoz et at. `06

The story in one picture

10^9 10^{11} 10^{13} 10^{15} 10^{16} GeV

Temperature fluctuation sources $\delta T/T \rightarrow \delta\theta$

anharmonicity ($\bar{\theta}_{\mathrm{init}} \rightarrow \delta\theta$)

Low f_a

Potential collapse
at early time

Intermediate f_a

High f_a

Potential collapse
only at very late time

$z < 2.4$

Telling pre- and post-inflationary scenarios apart

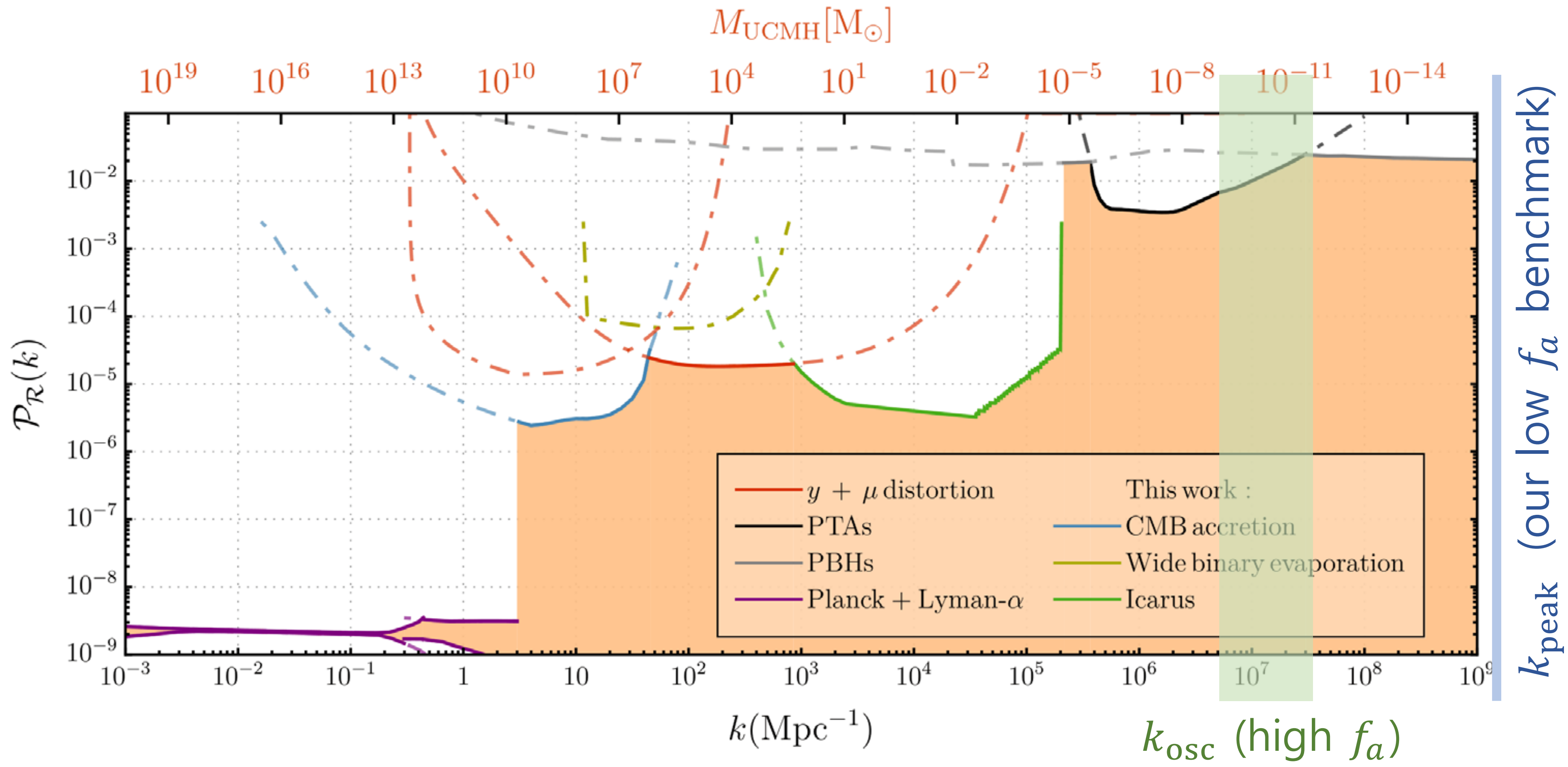
Peak scale is similar

k_{peak} is pushed to $\sim k_{\text{QJ}}$ at MRE in post-inflationary scenario as well. Gorghetto et al. \`24

Yet,

Discriminant	Post-inflationary	Pre-inflationary (high f_a)	Pre-inflationary (low f_a)
IR slope of $\Delta_\delta(k)$	$\propto k^3$ (causality)	plateau (inherited from adiabatic input)	plateau + resonance peak
Collapse redshift	$z \approx z_{\text{MRE}} \approx 3400$	$z < 2.4$ (only just collapses today)	$z \sim 340$
Minicluster density	$\sim 140\, \delta_{\text{MRE}}^3 (1 + \delta_{\text{MRE}})\, \bar{\rho}_{MRE}$ Kolb & Tkachev \`94	$8 - 9$ orders of magnitude less dense	$0 - 3$ orders of magnitude less dense

How wildly can pre-inflationary scenario vary



Torsten Bringmann et al. `25

Summary

- **Possibility of having pre-inflationary axion Minicluster formation is explored at linear level.**
- **Effect of gravitational potential and realistic QCD cross-over era are considered.**
- **The cluster formation window for high $10^{12}\ \mathrm{GeV} \le f_a \le 10^{16}\ \mathrm{GeV}$ is very narrow.**
- **Low axion decay constant (e.g. $1.1 \times 10^{10}\ \mathrm{GeV}$) scenario makes a perfect target for S-P.**