

# Predicting the Dark Matter-Baryon Abundance Ratio\*

\*No returns or refunds

**Anson Hook**

University of Maryland

w/ Abhishek Bannerjee  
and Dawid Brzemiński  
2310.07777, 2410.22412



# Dark Matter

**We know VERY little about dark matter**

- 1) Non-relativistic**
- 2) Gravitational Interactions**
- 3) No evidence for any other interactions**
- 4) Adiabatic (small isocurvature) perturbations**
- 5)  $\rho_{DM} \approx 10 \text{ meV}^4$**

# Dark Matter

We know **VERY** little about dark matter

- 1) **Non-relativistic**
- 2) **Gravitational Interactions**
- 3) **No evidence for any other interactions**
- 4) **Adiabatic (small isocurvature) perturbations**
- 5)  $\rho_{DM} \approx 10 \text{ meV}^4$

# Dark Matter

We know **VERY** little about dark matter

- 1) **Non-relativistic**
- 2) **Gravitational Interactions**
- 3) **No evidence for any other interactions**
- 4) **Adiabatic (small isocurvature) perturbations**

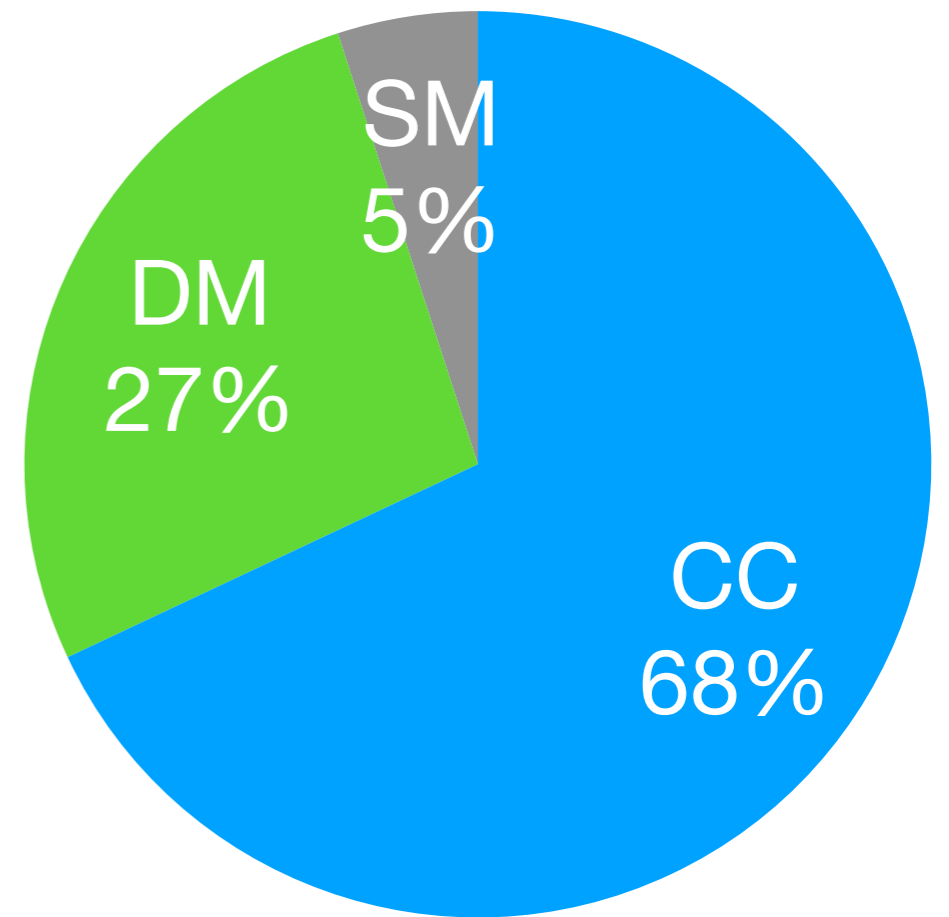
5)  $\rho_{DM} \approx 10 \text{ meV}^4$

**Time dependent and has units**

# Coincidence

$$\frac{\rho_{DM}}{\rho_{baryons}} = 5.36$$

**You Should Care**



# Dimensional Transmutation : Upside

**Some old famous people told us  
that an important problem was\***

$$m_p \ll M_{Pl}$$

**Very elegant solution**

$$m_p = M_{Pl} e^{\mathcal{O}(1)/\alpha_s}$$

**Proton mass exponentially  
sensitive to UV parameters**

\*I was taught as a grad student that the proper response is to nod and agree, and then ignore them

# Dimensional Transmutation : Downside

$$\rho_B = m_p n_B \sim e^{\mathcal{O}(1)/\alpha_s} \dots$$

**Baryon abundance also exponentially sensitive**

$$\rho_D = 5.36 \rho_B$$

**Exponential coincidence - 5.36 demands\* an explanation!**

\*Data speaks to me, I swear I'm not crazy

# Previous Approach

**Only one\* type of solution currently on the market**

$$\rho_D \sim \rho_B$$

$$n_D \sim n_B$$

$$m_D \sim m_B$$

\*We are in an anthropics free zone

# Previous Approach

Only one type of solution currently on the market

$$\rho_D \sim \rho_B$$

$$n_D \sim n_B$$

$$m_D \sim m_B$$

**Mirror world baryogenesis**

**Asymmetric Dark Matter**

~1000 papers

**(Broken) Mirror world**

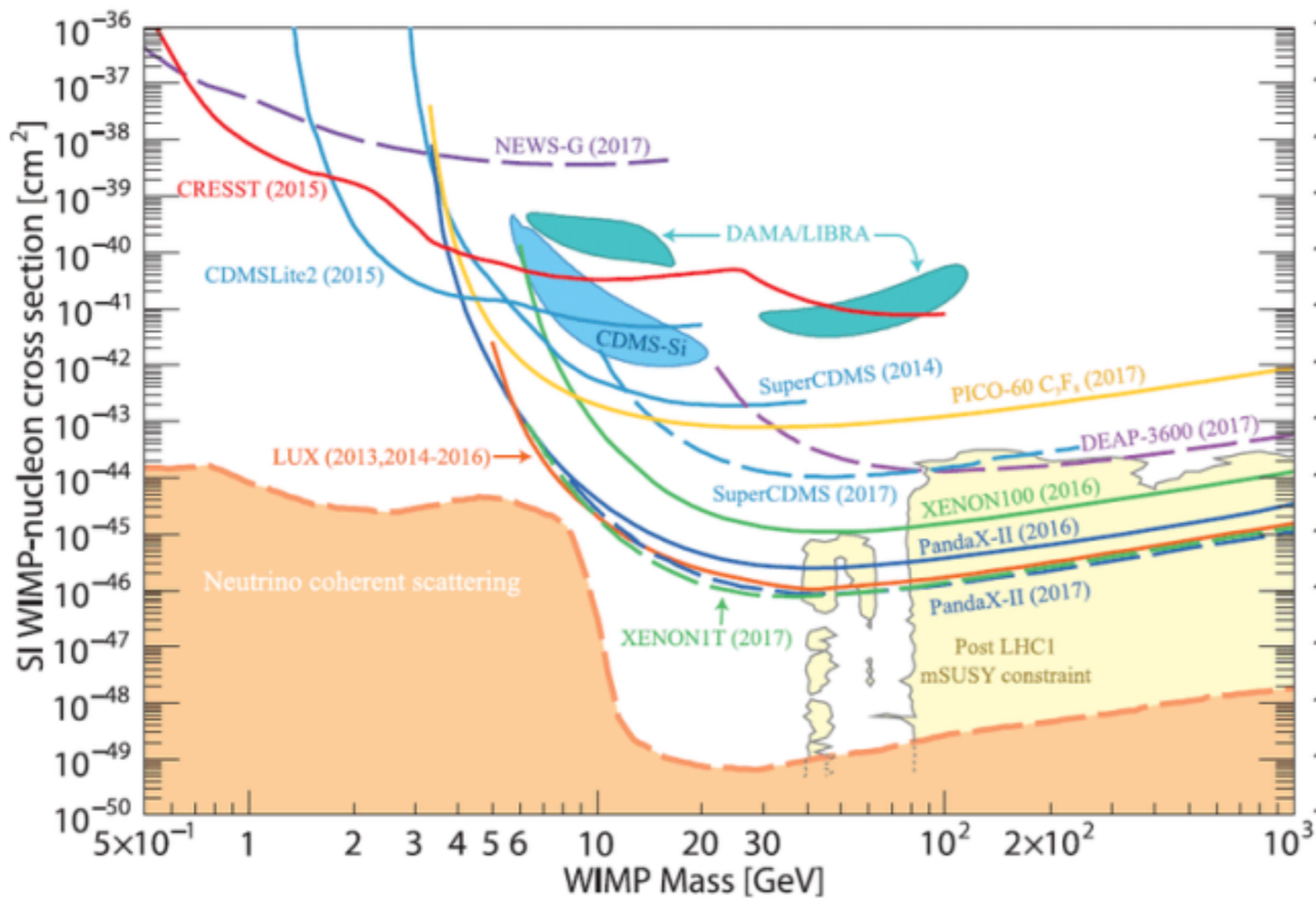
**Coupled CFTs**

**Dark/SM unification**

~10 papers

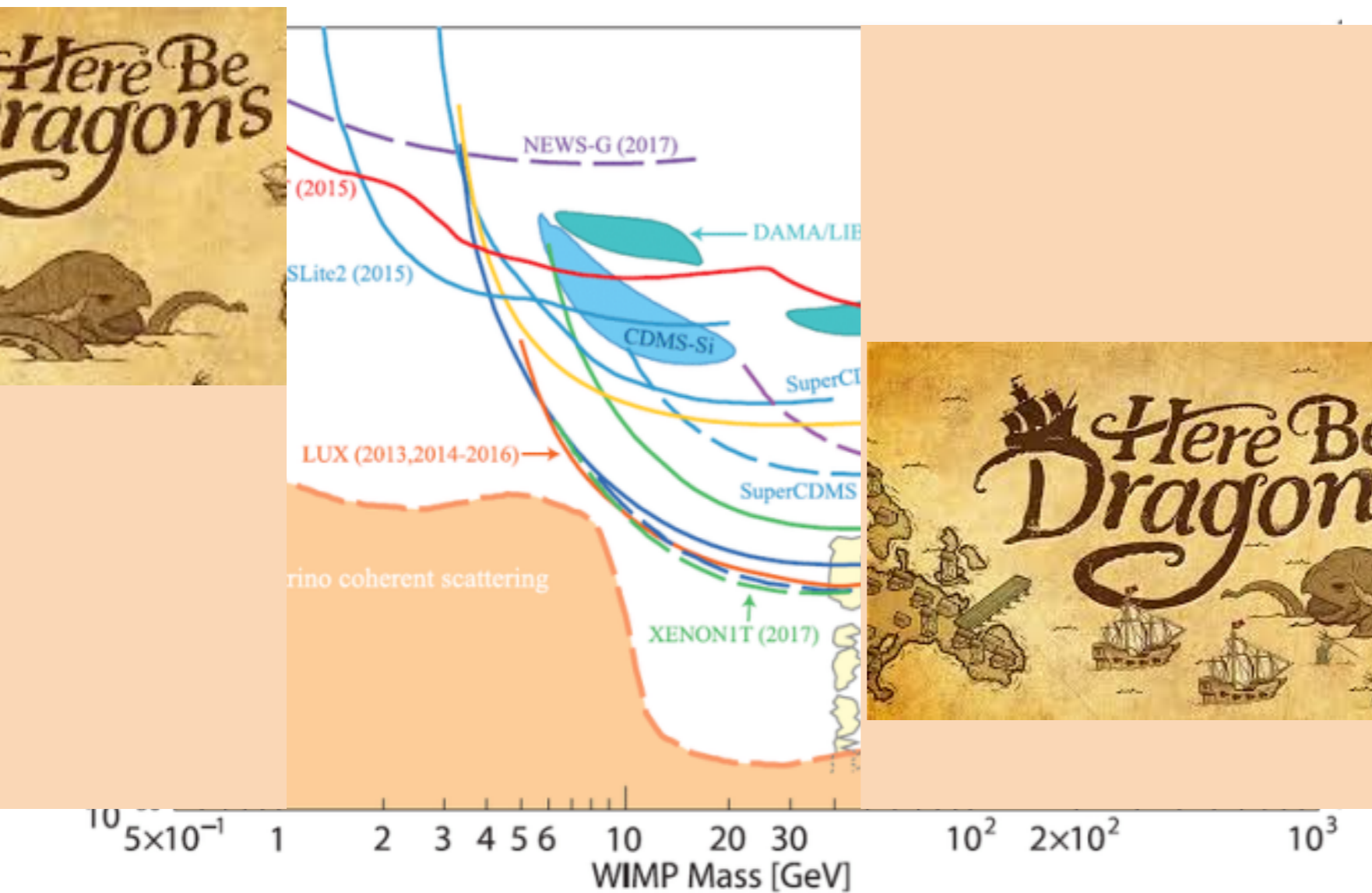
# Goal

Does this imply that dark matter must have its mass around a few GeV?



# Goal

Does this imply that dark matter must have its mass around a few GeV?



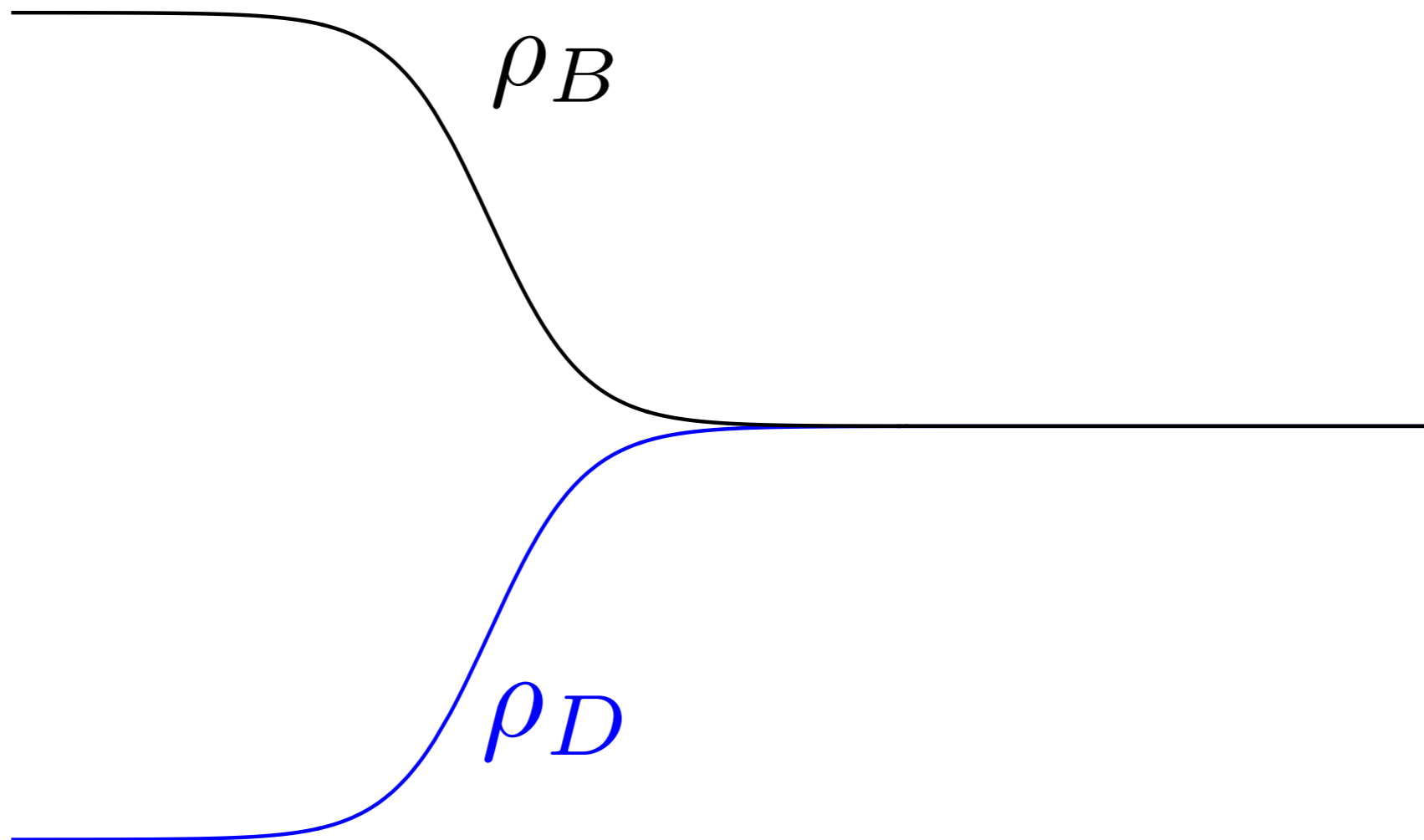
I like other DM candidates\* ...



\*As long as it's unpopular

# Idea Sketch

Couple a scalar to dark matter and baryon mass terms such that at its finite density minimum the energy densities are comparable



# Faux Pas - Tiny bit of Math

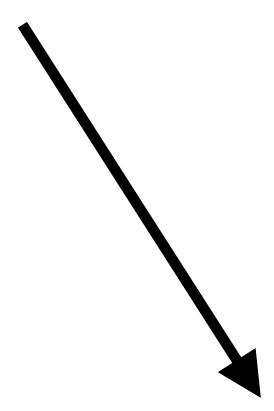
So easy, we'll go over three examples today

SUSY inspired example

$$\mathcal{L} \supset y_1 \phi_1 \psi_1 \psi_1^c + y_2 \phi_2 \psi_2 \psi_2^c - \lambda (\phi_1 \phi_2 - f^2)^2$$

Finite density potential

$$V(\phi) = m_1(\phi_1)n_1 + m_2(\phi_2)n_2$$


$$\phi_1 = f^2 / \phi_2$$

# Faux Pas - Tiny bit of Math

$$V(\phi_1) = y_1 \phi_1 n_1 + y_2 \frac{f^2}{\phi_1} n_2$$

Find Minimum

$$\phi_1 V'(\phi_1) = \rho_1 - \rho_2 = 0$$

$$\rho_1 = \rho_2$$

A little too predictive!

# Faux Pas - Tiny bit of Math

Moduli

$$m_B(\phi) = m_B(0)e^{-c_B\phi/f}$$

$$m_D(\phi) = m_D(0)e^{c_D\phi/f}$$

Imagine somehow we create baryons and dark matter  
Finite density potential

$$V(\phi) = m_D(\phi)n_D + m_B(\phi)n_B$$

# Faux Pas - Tiny bit of Math

$$V(\phi) = m_D(\phi)n_D + m_B(\phi)n_B$$

$$m_B(\phi) = m_B(0)e^{-c_B\phi/f}$$

$$m_D(\phi) = m_D(0)e^{c_D\phi/f}$$

Find Minimum

$$fV'(\phi) = c_D m_D(\phi)n_D - c_B m_B(\phi)n_B = 0$$

$$\rho_D/\rho_B = c_B/c_D$$

5.36 transformed into why are these constants similar

# Greed is Good\*

$$\rho_D/\rho_B = c_B/c_D$$

5.36 transformed into why are these constants similar

$$\rho_D = \rho_B$$

Too predictive!

Best Cases : predicting 5.36

Need to specify dark matter candidate and moduli interactions

\*I'm a man of the times

# Choosing your dark matter

Need a dark matter candidate connected to baryons

So that coupling to baryons and dark matter are related to each other

Simple candidate : QCD axion

Connected to baryons :

$$\mathcal{L} \supset \frac{a}{f_a} G\tilde{G}$$

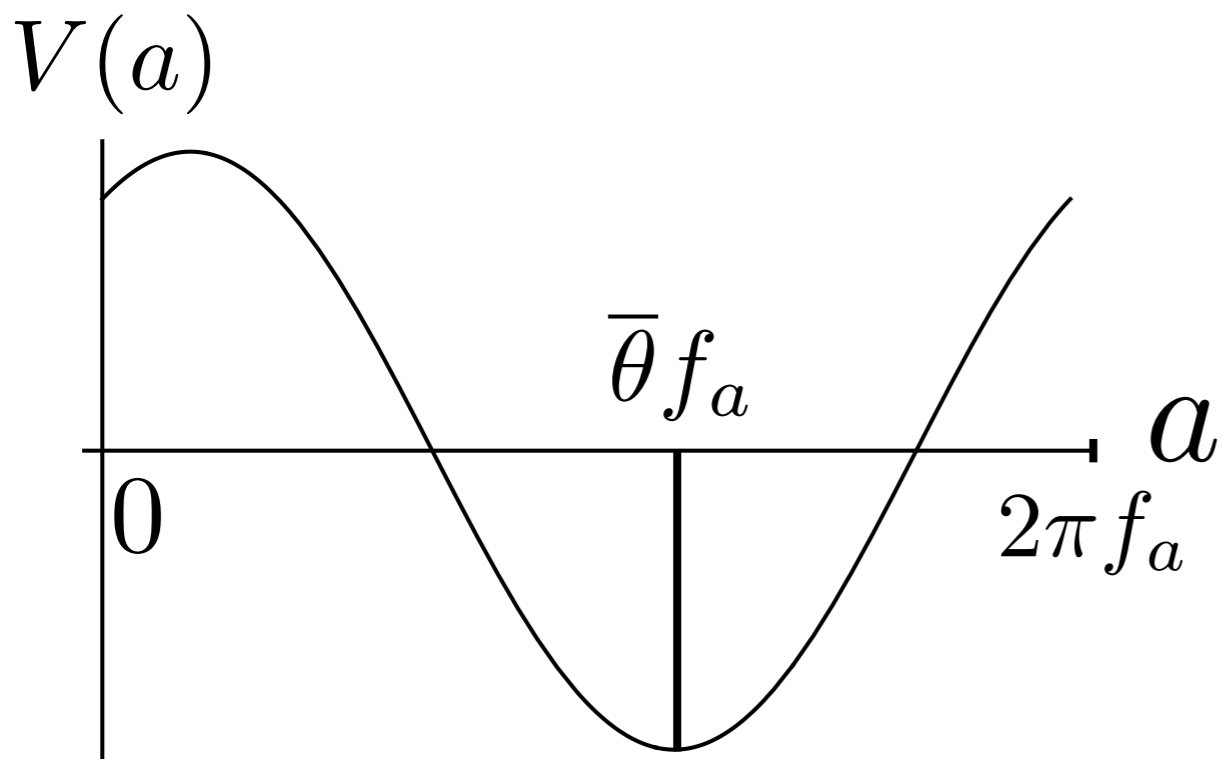
Can be dark matter

# QCD Axion Dark Matter\*

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left( \frac{\bar{\theta} - a/f_a}{2} \right)}$$

Axion dynamically sets  
the neutron EDM to 0

$$m_a \sim m_\pi f_\pi / f_a \propto v^{1/2} \Lambda_{QCD}^{3/2} / f_a$$



$$|d_n| = 3.2 \times 10^{-16} \left( \bar{\theta} - \left\langle \frac{a}{f_a} \right\rangle \right) e \text{ cm}$$

\*I don't read arxiv, axions are unpopular right?

# Simplest UV completion

Every UV completion of the axion involves new colored particles

	$SU(3)_{QCD}$	$SU(N)$
$\psi$	$\square$	$\square$
$\psi^c$	$\overline{\square}$	$\overline{\square}$

$$\mathcal{L} \supset y\Phi\psi\psi^c = yfe^{ia/f_a}\psi\psi^c$$

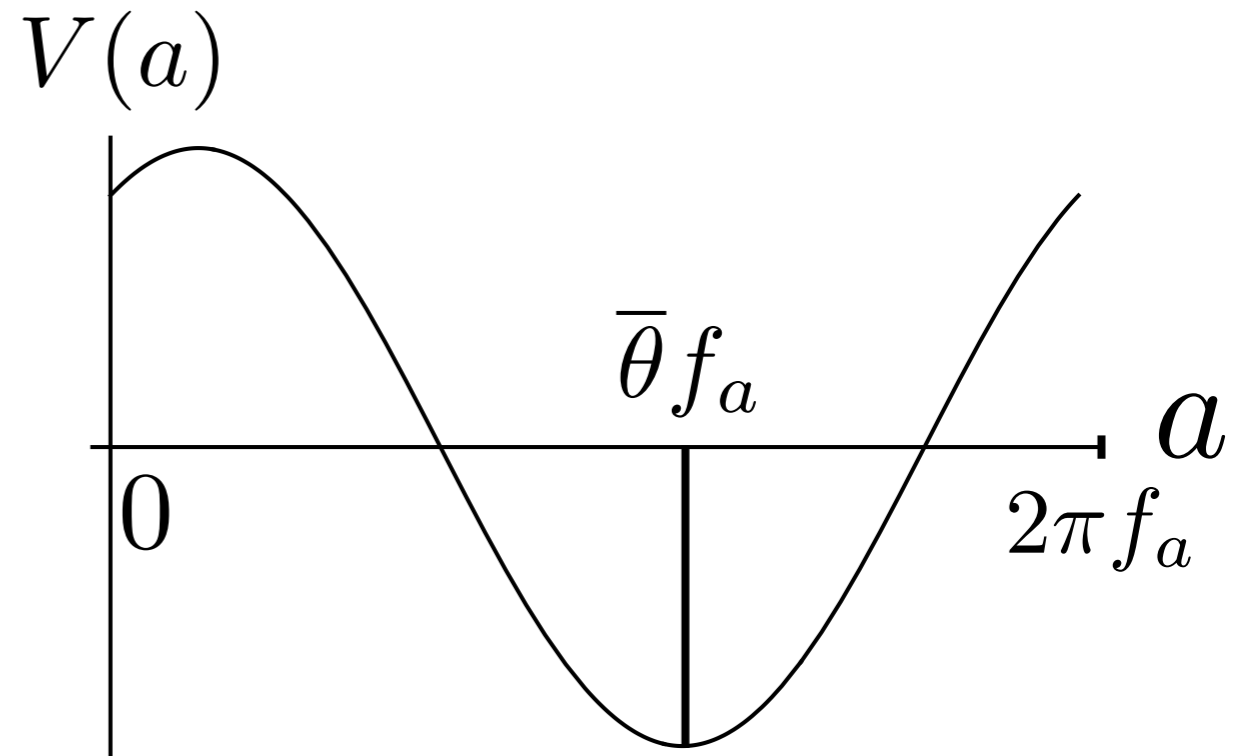
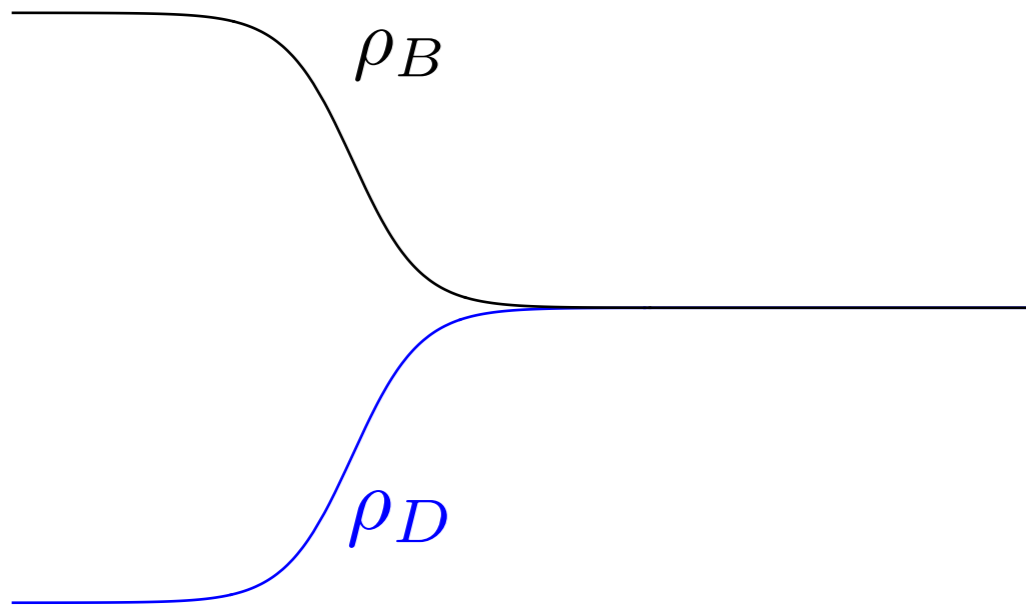
\*Food for thought. Every other example of a composite particle having 0 dipole is solved by a relaxation solution

# Goal

Combine both relaxation mechanisms\*

Relax energy densities such  
that they are equal

QCD Axion DM



\*Nothing bad has ever come from smashing together your two favorite foods, just ask any kid

# Outline



- The Good
  - The basic idea and prediction
- The Bad
  - Cosmology + modifications needed to realize the idea
- The Ugly
  - Model building needed to avoid fine tuning

# First Pass

Introduce one new scalar and one new coupling\*

$$f_a(\phi)$$

New scalar acts as a moduli for the axion decay constant

\*Cool kids ignore hierarchy problems

# First Pass

Also Moduli for QCD

$$\Lambda_{QCD}^{\beta_{QCD}} \propto \prod_f m_f^{2/3} \sim f_a^{2N/3}$$

One coupling forces  $\phi$  to be a moduli for both proton and dark matter (QCD axion)\*

Only 1 free parameter, N, the number of new colored particles

\*Equations will fly fast and furious so just nod and be impressed

# First Pass

Ala dimensional transmutation

$$m_B(\phi) = \Lambda_{QCD}(\phi) \propto f_a(\phi)^{\frac{2N}{27}}$$

$$m_D(\phi) \propto \Lambda_{QCD}^{3/2}(\phi)/f_a(\phi) \propto f_a(\phi)^{\frac{N}{9}-1}$$

$$\frac{\partial V}{\partial \phi} = \frac{\partial f_a}{\partial \phi} \frac{\partial V}{\partial f_a} = 0$$

Don't care what the function  $f_a(\phi)$  is!

# First Pass

$$V(\phi) = m_D(\phi)n_D + m_B(\phi)n_B$$

$$m_B(\phi) \propto f_a(\phi)^{\frac{2N}{27}}$$

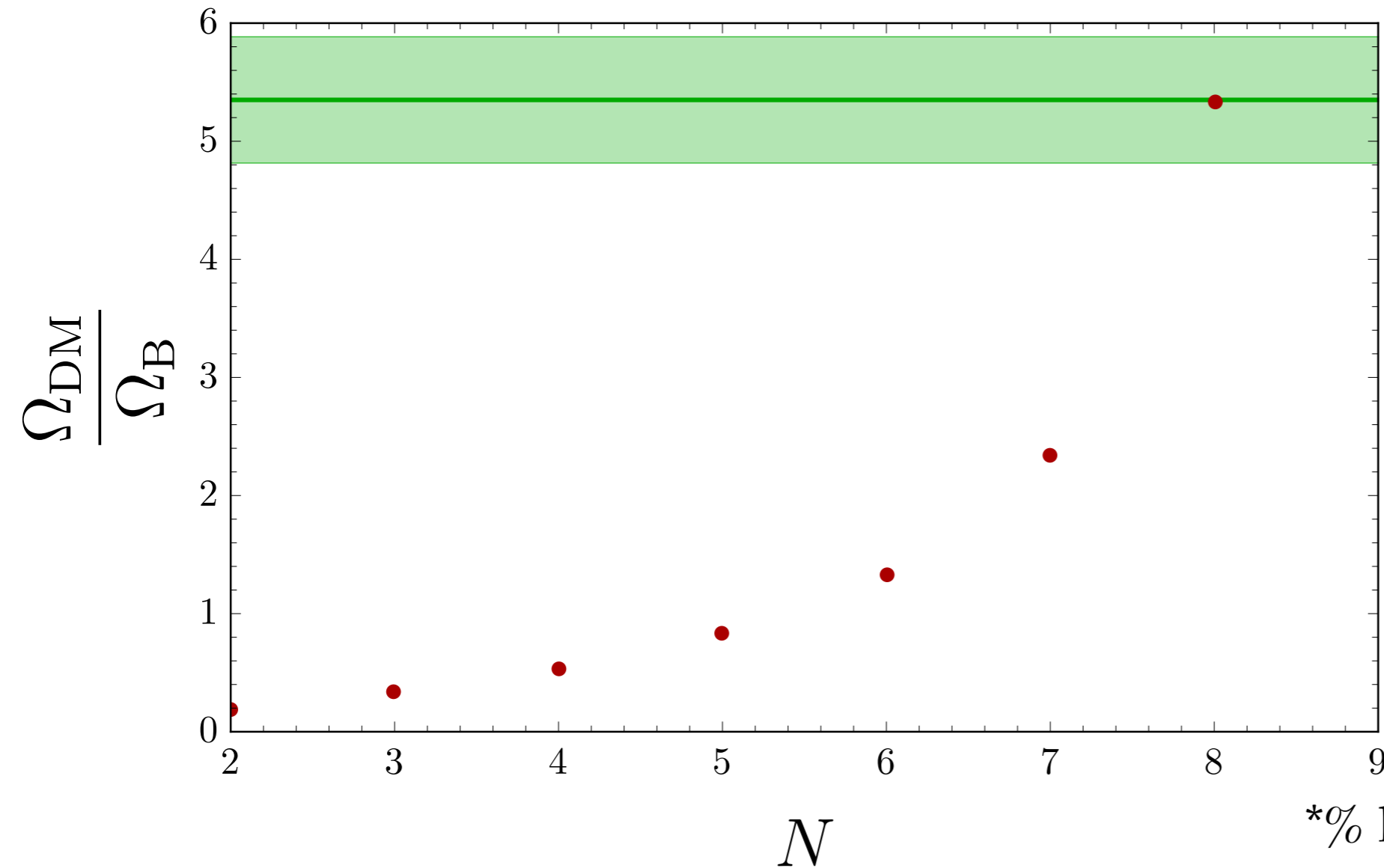
$$m_D(\phi) \propto f_a(\phi)^{\frac{N}{9}-1}$$

$$\rho_D/\rho_B = 2N/(27 - 3N)$$

Cannot accommodate every  
value of the ratio!

# Toy Model

$$\rho_D/\rho_B = c_B/c_D = 2N/(27 - 3N)$$



N = 8 gives 5.33\*

Observed value  
of  $5.36 \pm 0.05^{**}$

\*% level error bars from 2-loop

\*\*Assumes standard cosmology

# Toy Cosmology

Assume DM and Baryons have been produced in a radiation dominated universe and are non-relativistic

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{c_B}{f} m_B^0 e^{c_B \phi/f} n_B + \frac{c_D}{f} m_D^0 e^{-c_D \phi/f} n_D$$

Assume Baryons heavier than a GeV

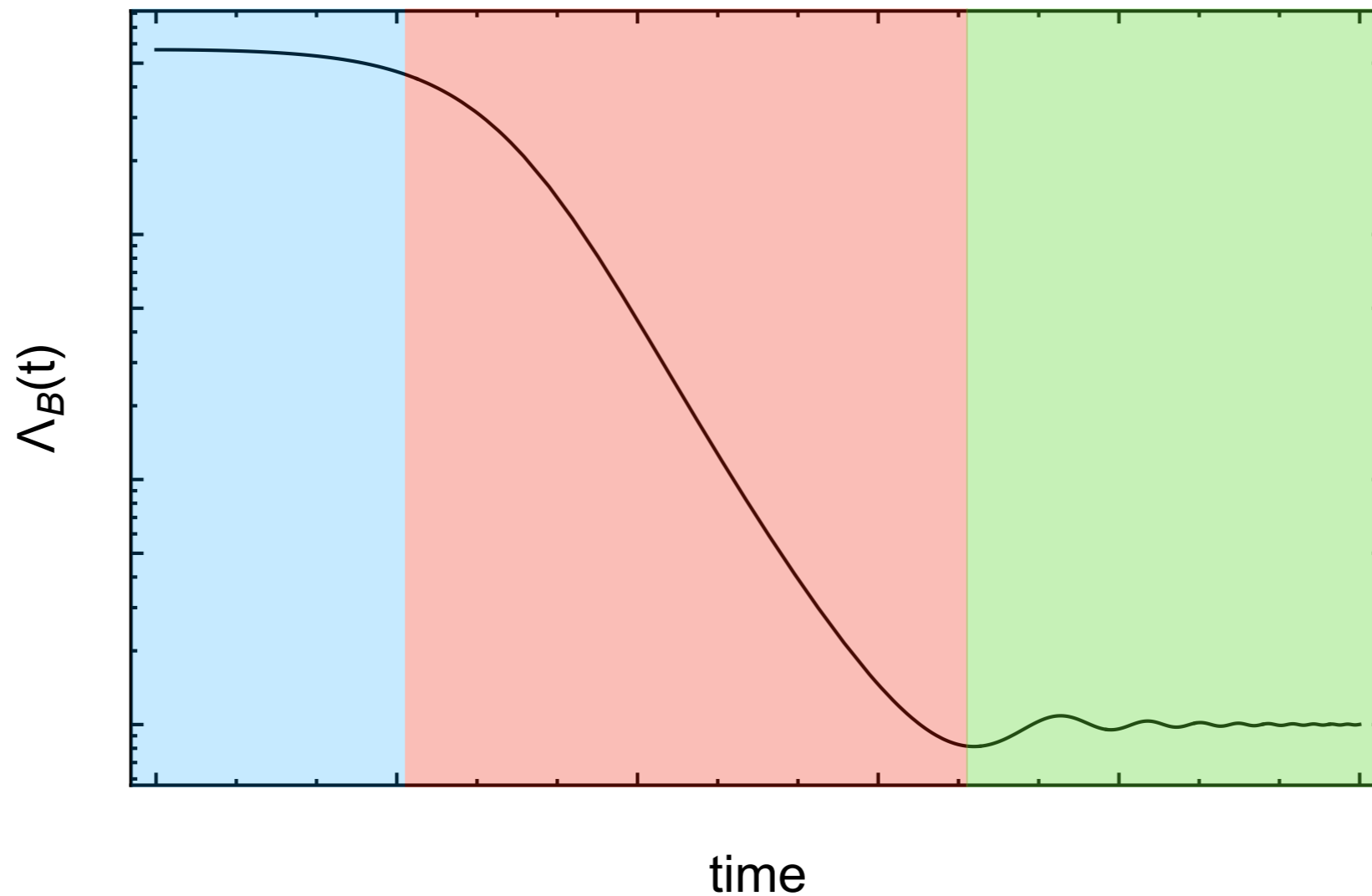
Initially, we have

$$\ddot{\phi} + 3H\dot{\phi} \approx -\frac{c_B}{f} \rho_B(\phi) \qquad m_\phi^2 = V'' \approx \frac{c_B^2}{f^2} \rho_B$$

# Toy Cosmology

## 3 Stage Cosmology

Frozen Out    Pseudo-Slow Roll    Damped Oscillations



# Toy Cosmology : Frozen Out

$$m_\phi \ll H$$

Not much happens

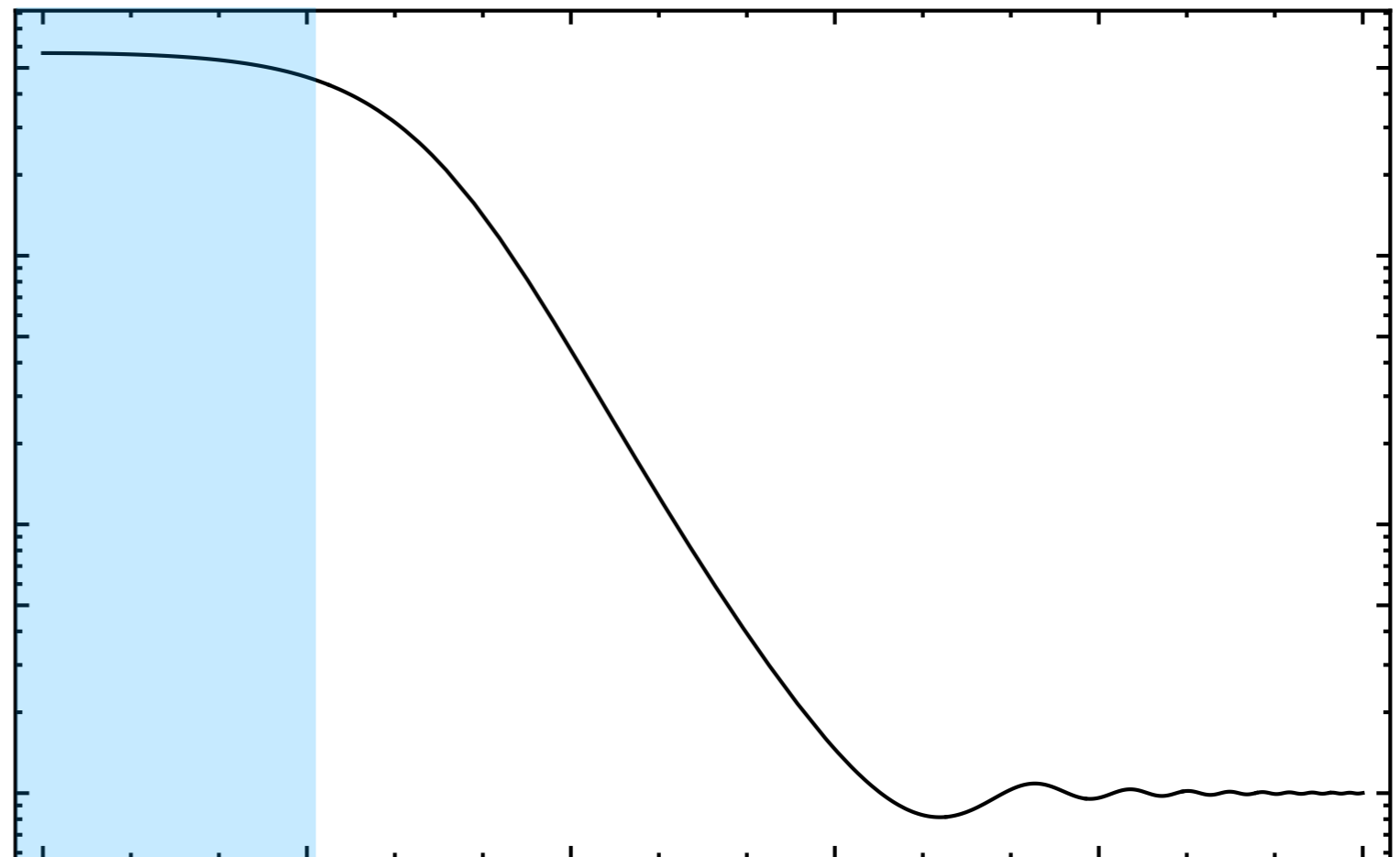
Everything just redshifts as usual

Frozen Out

$$\ddot{\phi} + 3H\dot{\phi} \approx -\frac{c_B}{f}\rho_B(\phi)$$

$$\dot{\phi} \approx 0$$

$\Lambda_B(t)$



time

# Toy Cosmology : Pseudo-Slow roll

$$m_\phi \approx H$$

Schematically

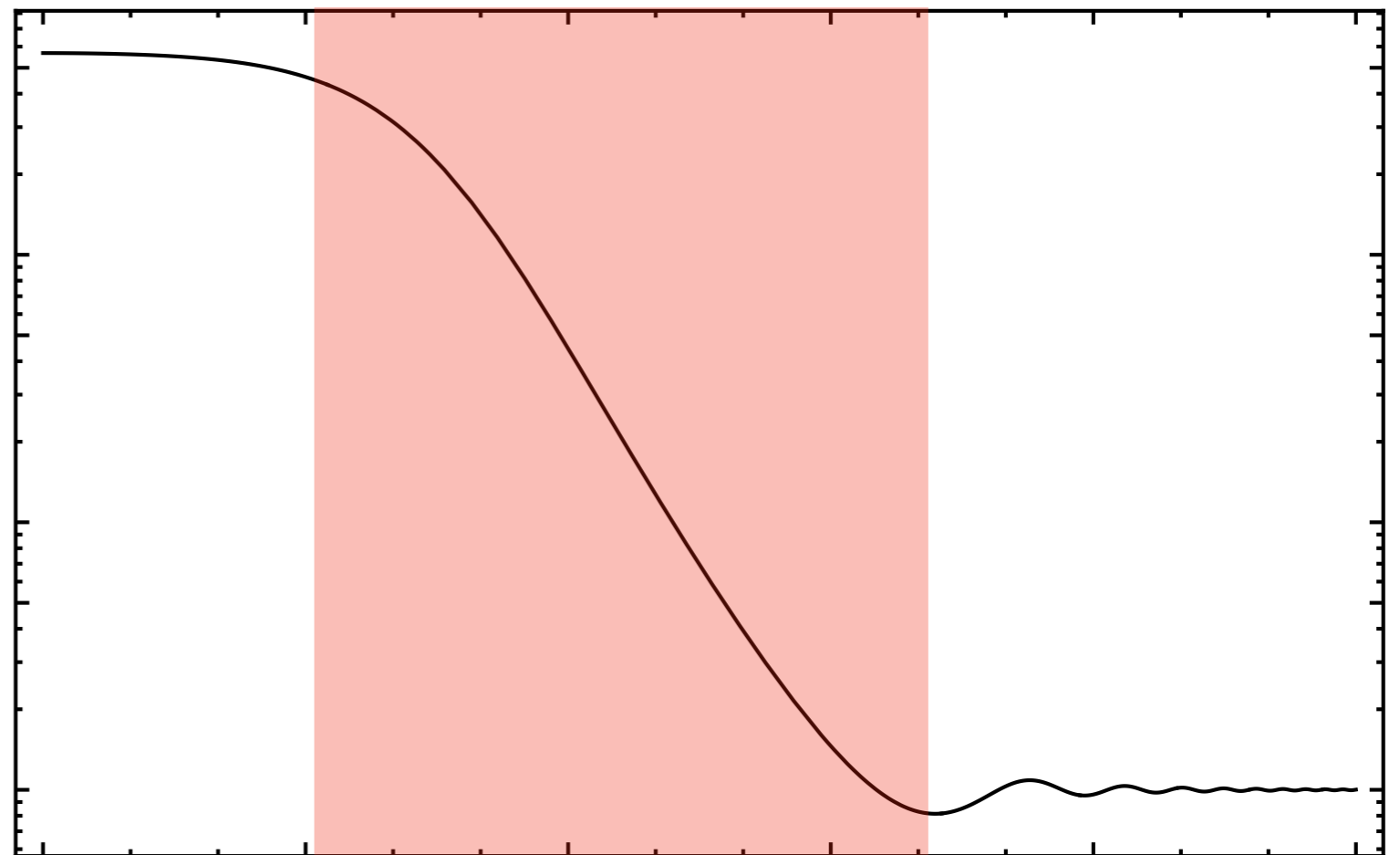
$$\phi \sim \log t$$

So that

$$\ddot{\phi} + 3H\dot{\phi} \approx -\frac{c_B}{f} \rho_B(\phi) \quad \Lambda_B(t)$$
$$\sim 1/t^2 \quad \sim 1/t^2$$

$$\Lambda_{QCD} \sim 1/a(t) \sim 1/\sqrt{t}$$

Pseudo-Slow Roll



time

# Toy Cosmology : Pseudo-Slow roll

$$m_\phi \approx H$$

Check List

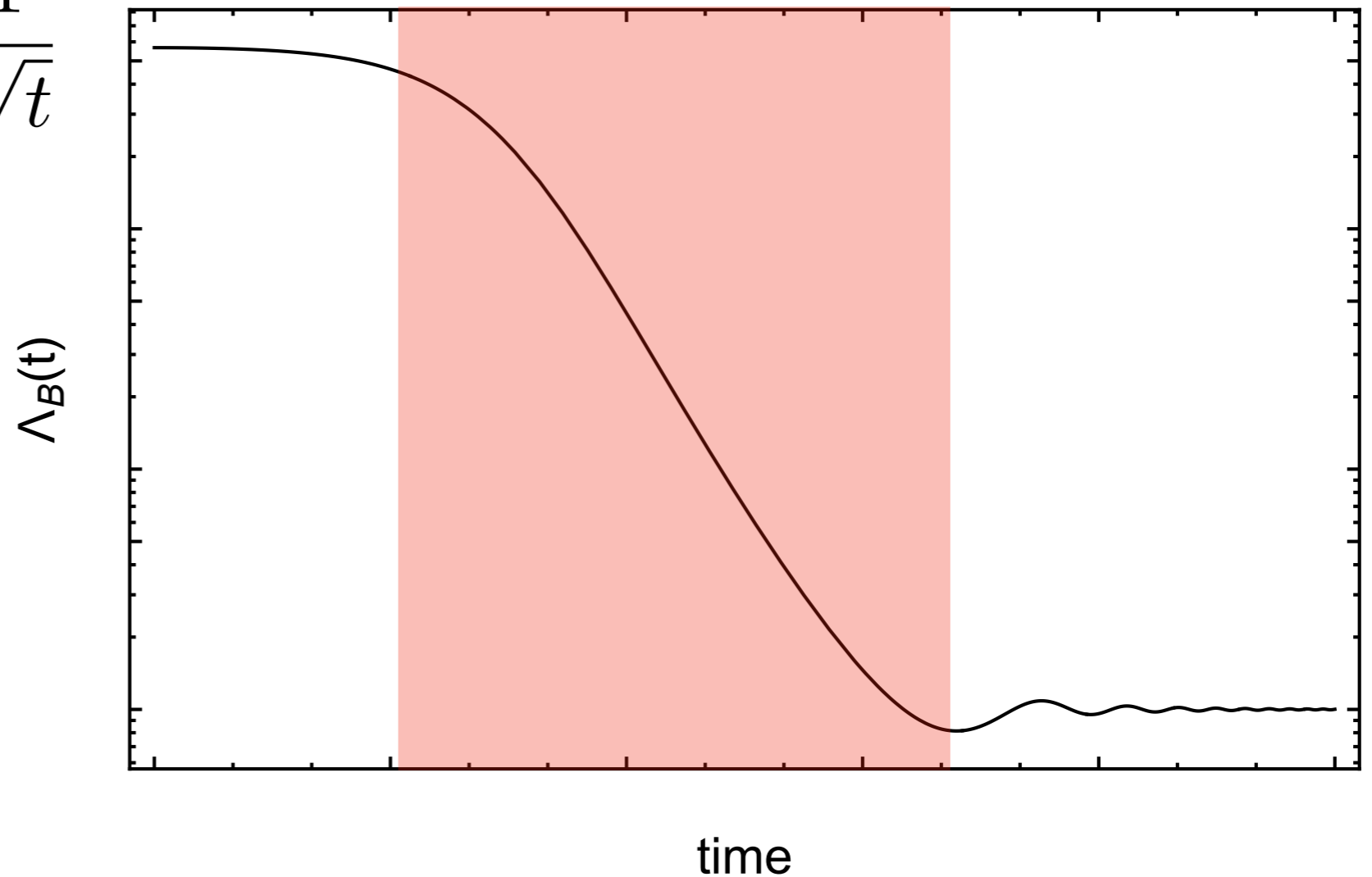
1)  $m_B \sim \Lambda_{QCD} \sim \frac{1}{\sqrt{t}}$

2)  $\rho_B \sim \frac{1}{t^2}$

3)  $m_\phi = H$

4)  $\frac{1}{2}\dot{\phi}^2 = \frac{1}{2}\rho_B$

Pseudo-Slow Roll



# Toy Cosmology : Damped Oscillations

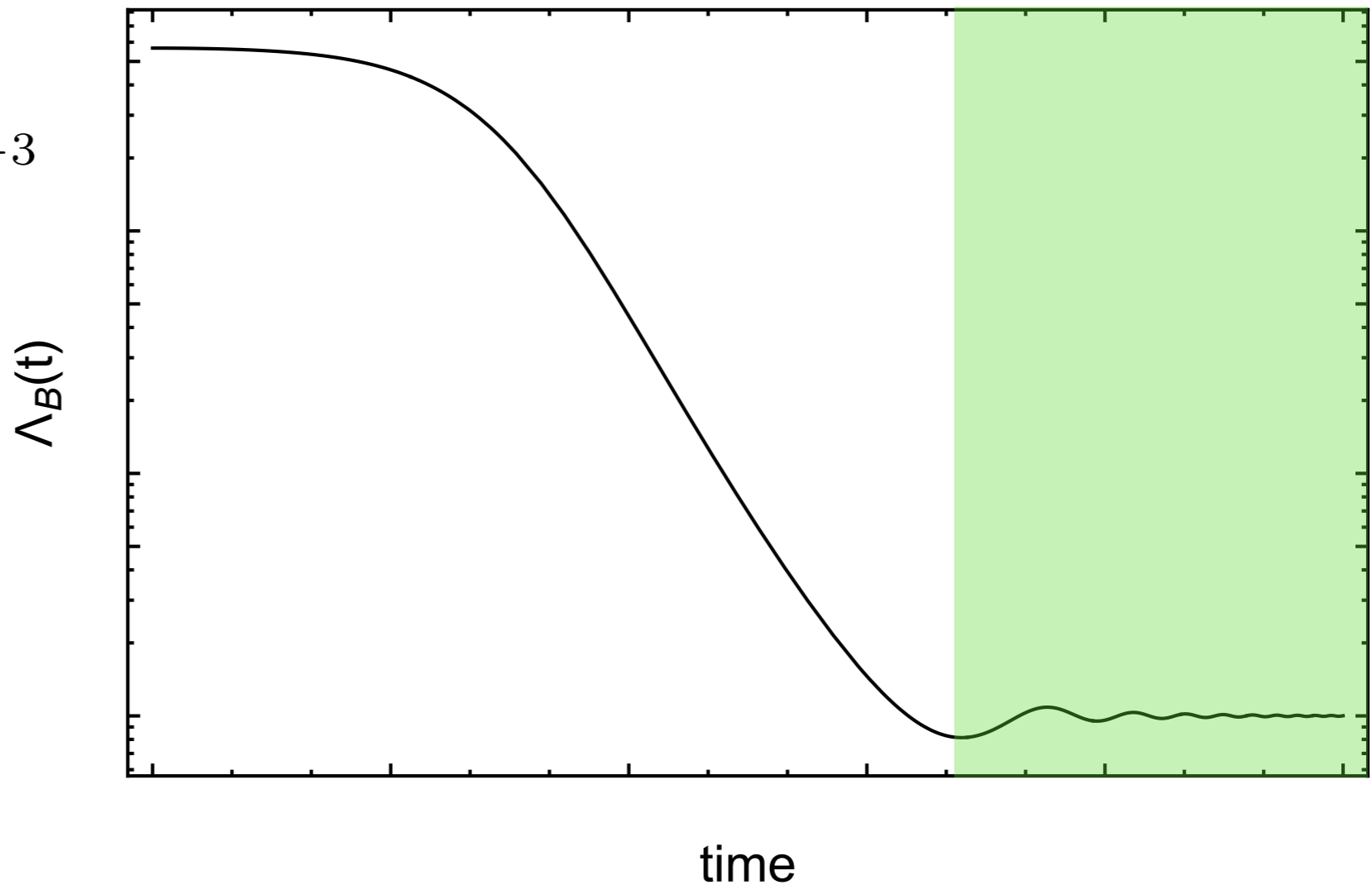
$$m_\phi \gg H$$

Dark Matter becomes  
important

Damped Oscillations

$$m_\phi^2 = \frac{c_B^2}{f^2} \rho_B + \frac{c_D^2}{f^2} \rho_D \sim a^{-3}$$

$$\rho_\phi = m_\phi n_\phi \sim \frac{1}{a^{4.5}}$$



# Outline



- ~~The Good~~
  - ~~The basic idea and prediction~~
- The Bad
  - Cosmology + modifications needed to realize the idea
- The Ugly
  - Model building needed to avoid fine tuning

# Adding the Real World

Beautiful idea - obviously realized in nature\*

In fact, this model works *perfectly* with all known data  
except one small problem

\*An approach that has never failed before

# Adding the Real World

Beautiful idea - obviously realized in nature

**It's very excluded**

Massless moduli have many issues



# What to do?

## Honorable Attempt

Move on with our lives



## Rescue It

See if it was worth it afterwards

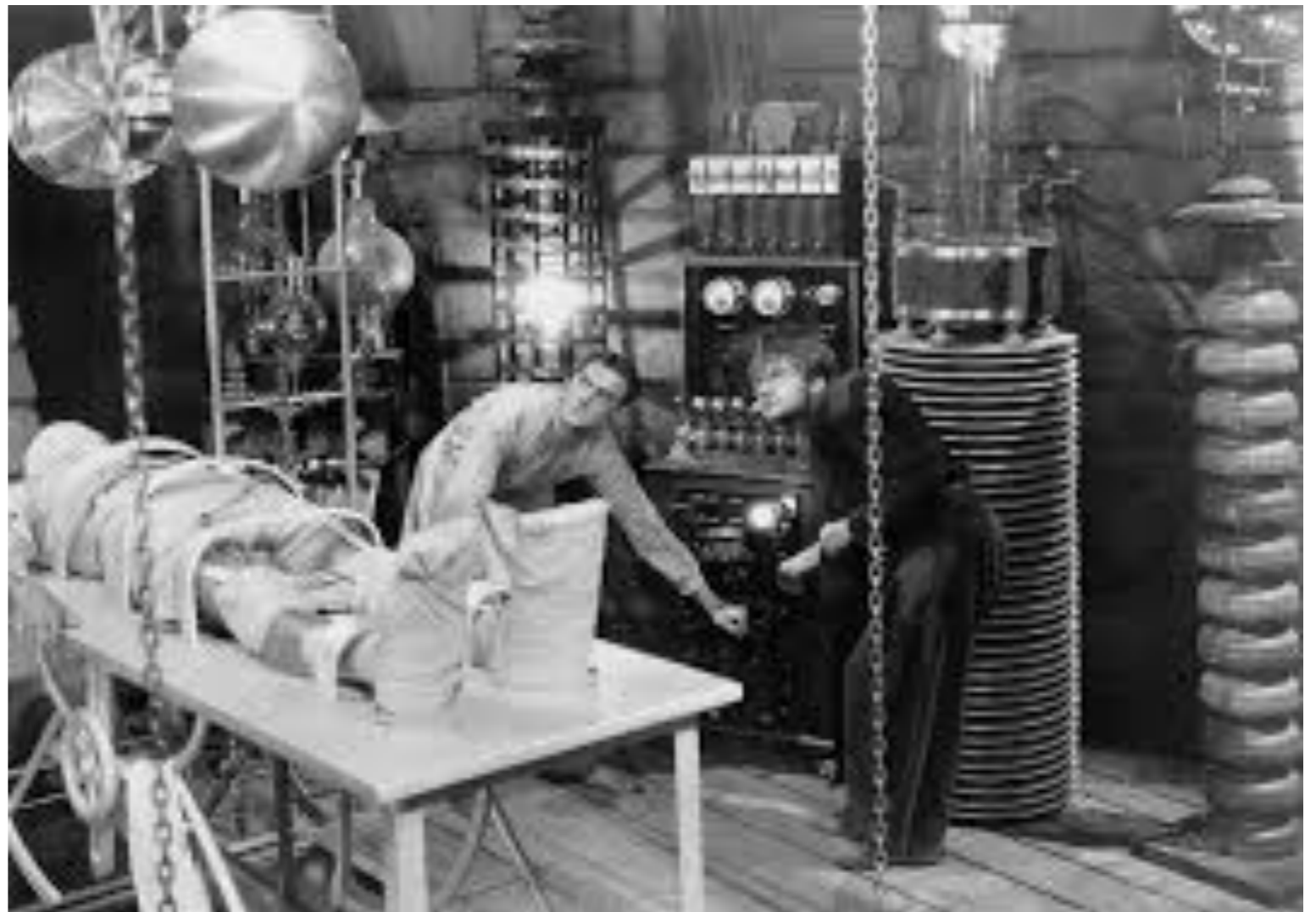


# Adding the Real World

**It's very excluded\***

Early time ( $T > \text{MeV}$ )  
solution

Late time ( $T < \text{MeV}$ )  
problems



\*Never stopped a model builder before, won't stop me now

# SM example : Challenges

Two basic issues with implementing this in the SM

1. Its very excluded : New massless forces are very excluded by fifth force searches, proton mass isn't dark matter density dependent, etc.
2. Temperature effects

# SM example : Challenges

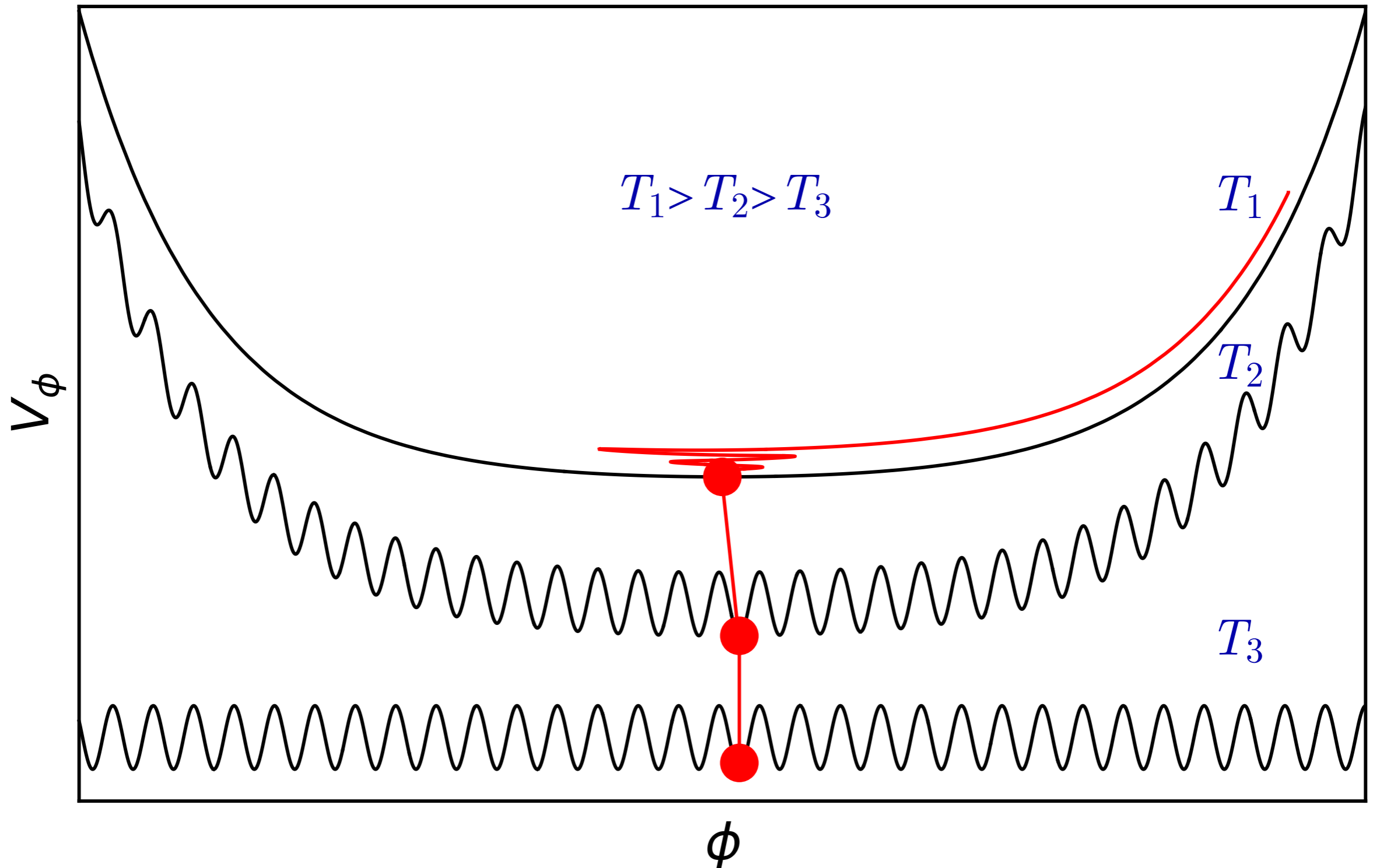
## Solution

Add a mass/bare  
potential

$$V = e^{c_B \phi / f} \rho_B + \Lambda^4(T_D) \cos \left( \frac{\phi}{F} + \theta \right)$$

# SM example : Challenges

Potential



# SM example : Challenges

Two basic issues with implementing this in the SM

1. Fifth forces



2. Temperature effects\*

\*anything not forbidden is allowed

# SM example : Challenges

## Finite Temperature

1. Scanning the QCD scale inevitably leads to scanning the fine structure constant

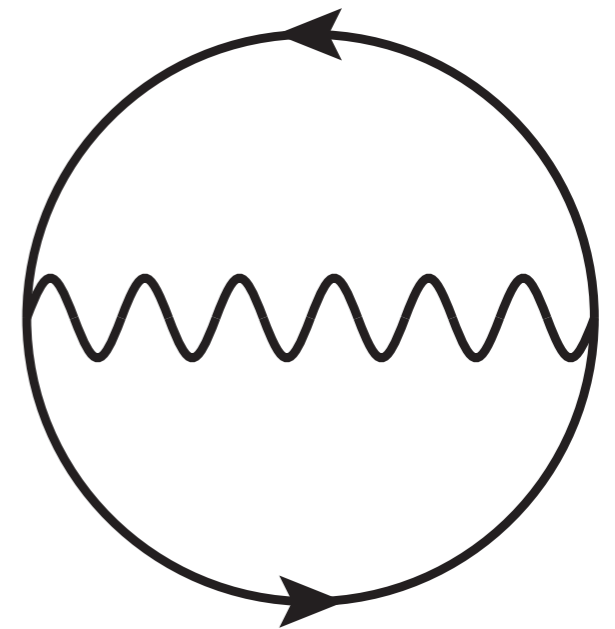
$$\frac{\phi}{f} \frac{c_B \beta_{QCD}}{32\pi^2} G^2 \quad \longrightarrow \quad \mathcal{O}(1) \frac{\phi}{f} \frac{c_B}{32\pi^2} F^2$$

# SM example : Challenges

## Finite Temperature

2. Free energy at low energies depends on fine structure constant\*

$$f = \frac{5}{288} e^2(\phi) T^4 \sim 10^{-6} T^4 \frac{c_B \phi}{f}$$

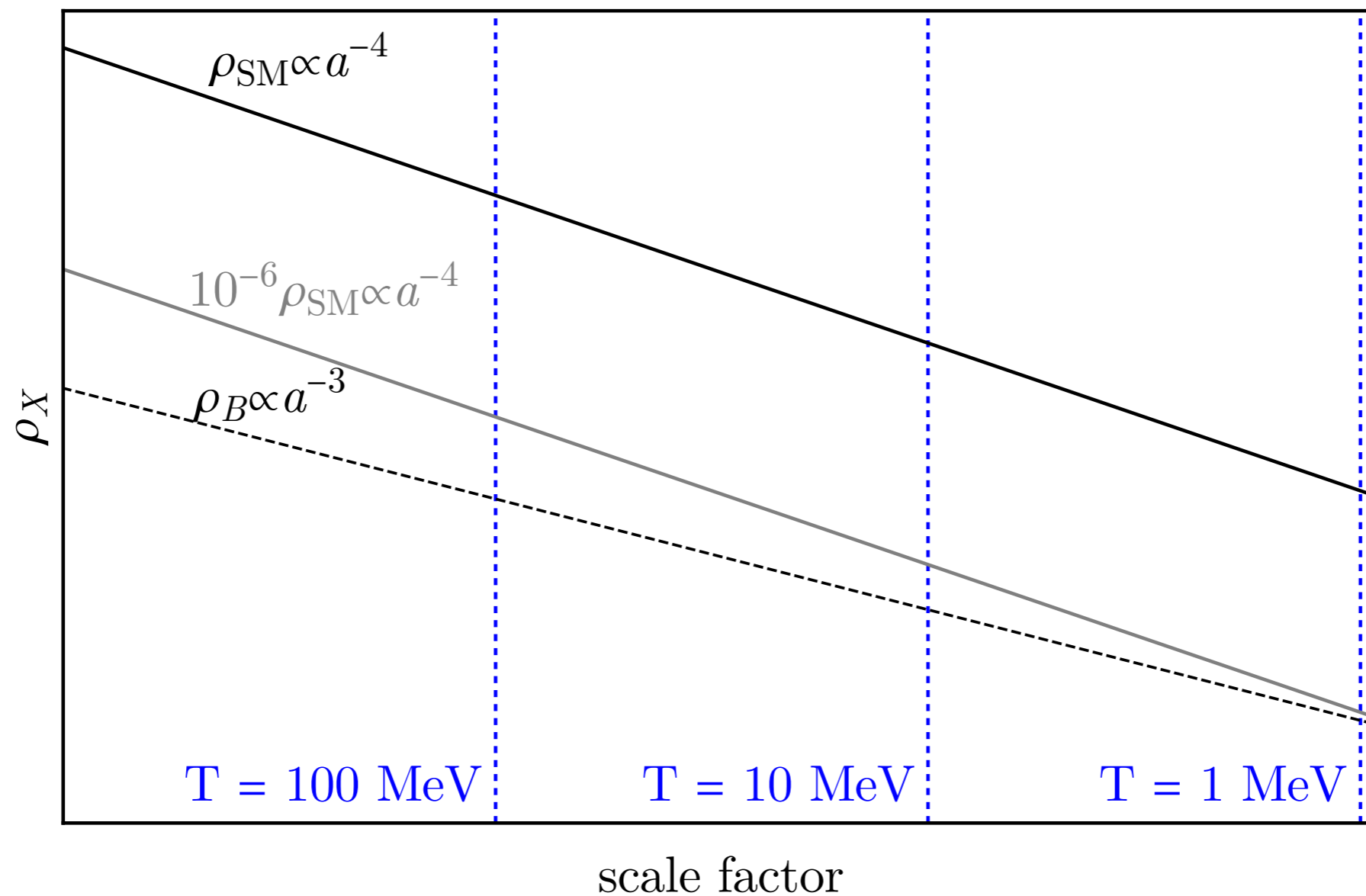


\*If you want to brag about doing a 2-loop integral, do this one. It factorizes into two separate 1-loop integrals

# SM example : Challenges

$$V(\phi) \sim \frac{c_B}{f} \rho_B + \frac{c_B}{f} 10^{-6} \rho_{SM}$$

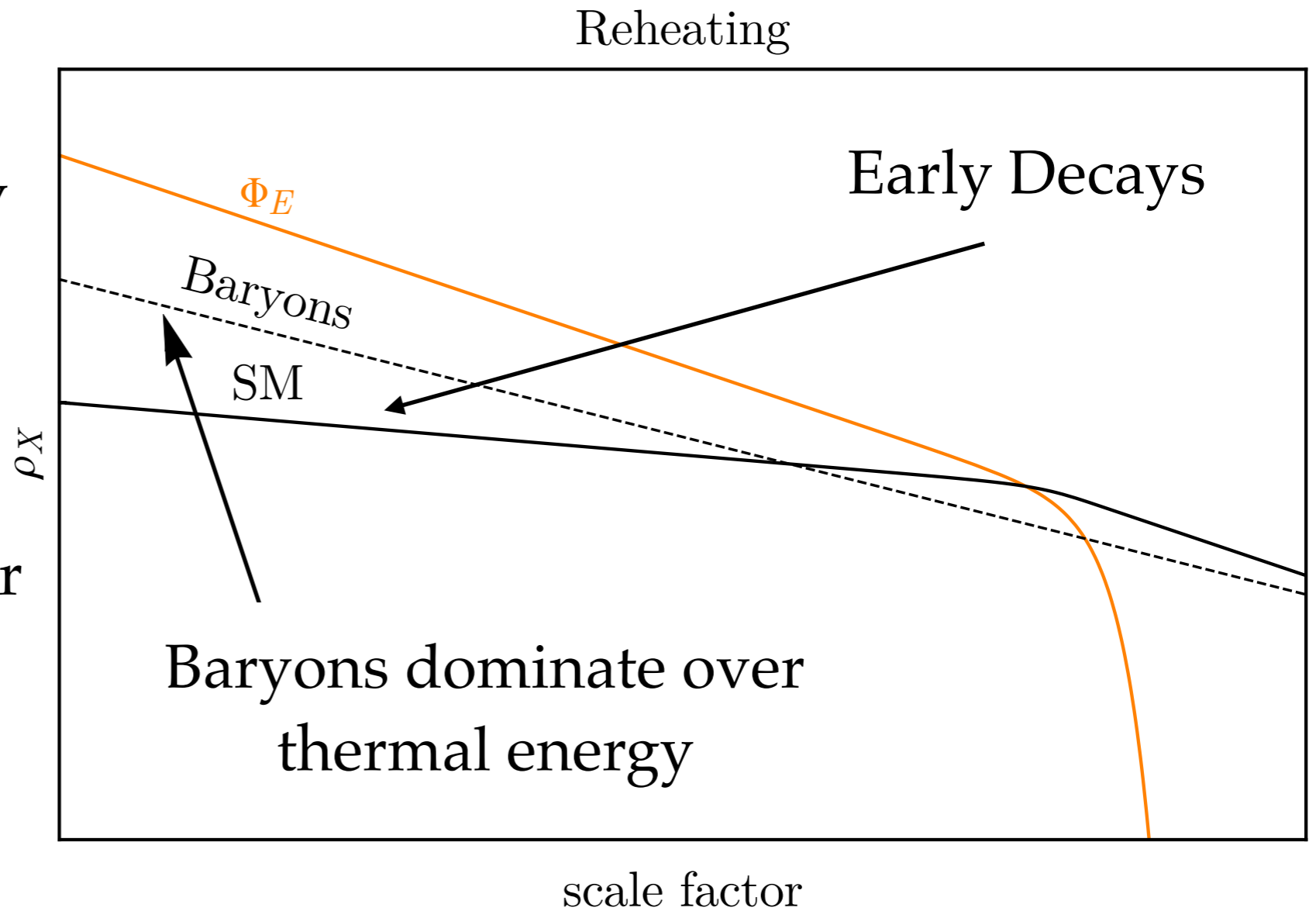
Standard Cosmology



# SM example : Challenges

Non Standard Cosmology  
Entropy Dump

Need large Baryon number  
asymmetry

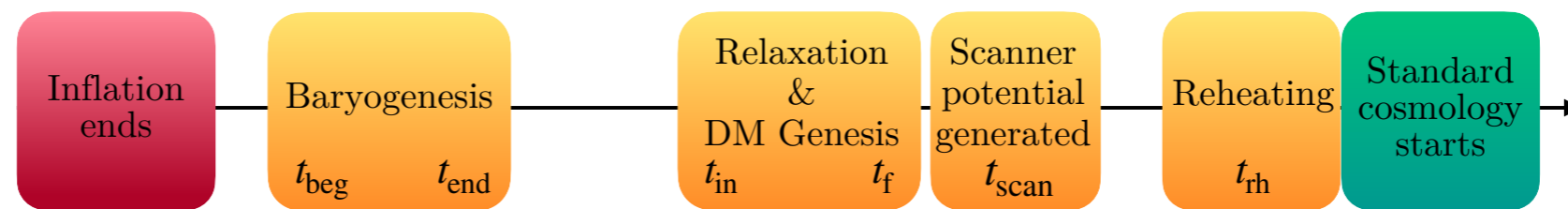


# History

Proton Mass falls as  $1/a$

Inflaton decays into  
reheaton/entropy dump

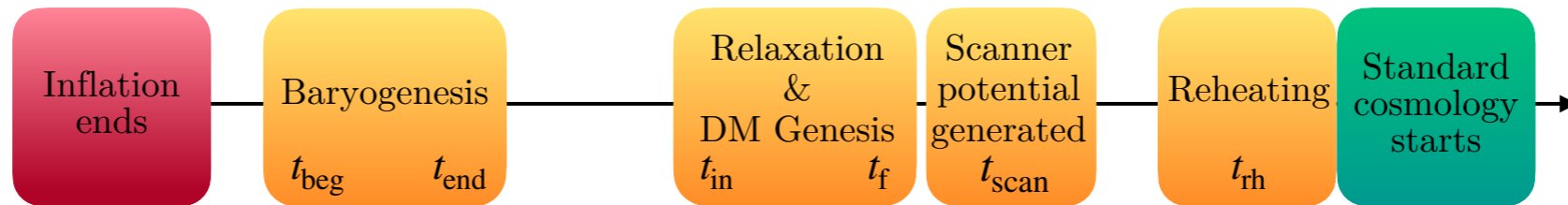
Back to normal



Lock everything  
into place

Need to very over-produce and be non-thermal  
Affleck-Dine

# History - Inflation Ends

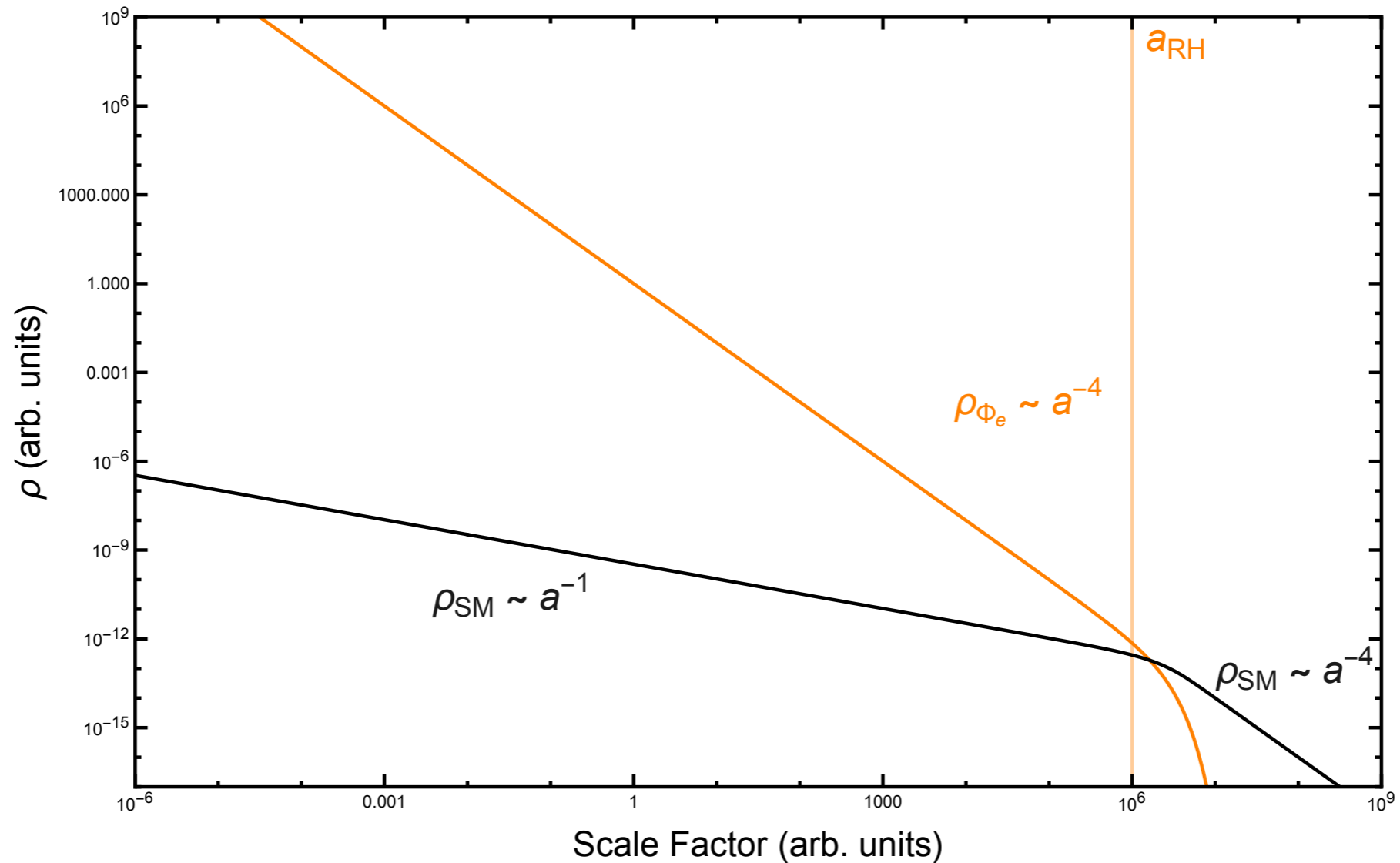
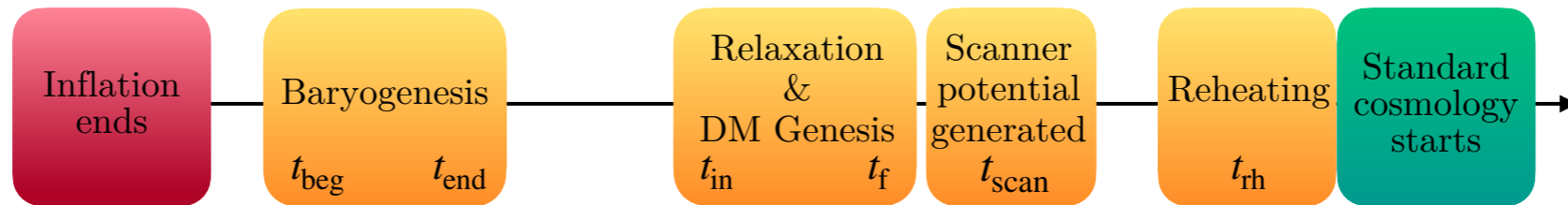


Inflaton decays into  $\Phi_E$

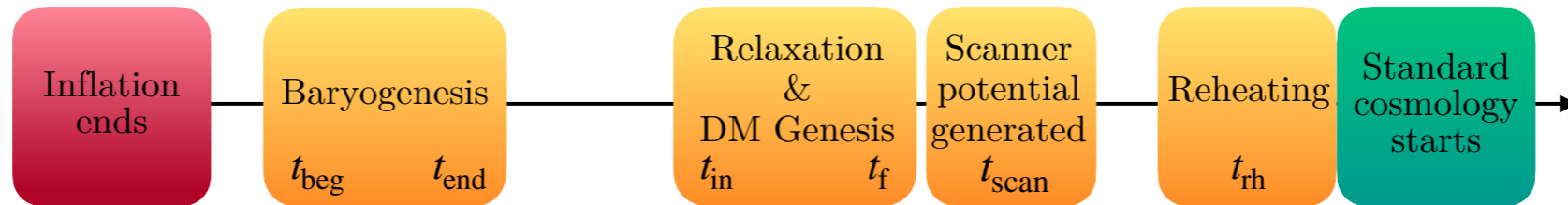
$\Phi_E$  dominates the energy of the universe until it decays into the SM

$$\mathcal{L} = \kappa_E |H|^2 \Phi_E \quad \Gamma_{\Phi_E} \simeq \frac{\kappa_E^2}{8\pi M_{\Phi_E} \gamma_E}$$

# History - Inflation Ends



# History - Baryogenesis



Simplest Affleck - Dine Baryogenesis Model\*

$$V = m_{\Phi}^2 |\Phi|^2 + \epsilon m_{\Phi}^2 (\Phi^2 + \Phi^{\dagger,2}) + y\Phi\psi\psi$$

Gives a non-zero decay width

$$\gamma = \Gamma_{\Phi}/m_{\Phi}$$

Misalignment, initially  $\Phi_E$  has a vev and when  $H \sim m_{\Phi_E}$  it starts to oscillate like a particle

\*The defining feature of AD models is how complicated they are. As such, I hesitate to call this an AD model

# History - Baryogenesis



## Simplest Affleck - Dine Baryogenesis Model

$$V = m_{\Phi}^2 |\Phi|^2 + \epsilon m_{\Phi}^2 (\Phi^2 + \Phi^{\dagger,2}) + y\Phi\psi\psi$$

$$n_B = Q_B n_{AD} \mathcal{F}$$

Scalar field converted into baryons

$$\mathcal{F} = \frac{4\epsilon\gamma(2 + \gamma^2)}{4\gamma^2 + \gamma^4 + 16\epsilon^2} \rightarrow \frac{1}{2}$$

If  $\epsilon = \gamma/2$ , condensate oscillates into all particles before decaying to baryons

# History - DM Genesis



Misalignment mechanism for the QCD axion

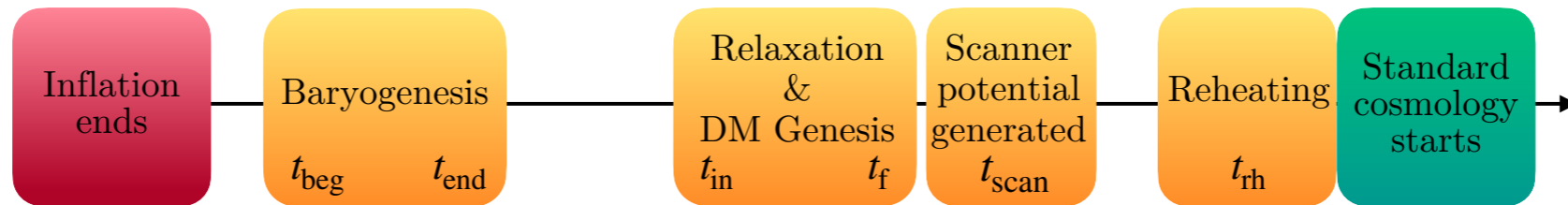
Relaxation changes mass and decay constant of axion

$$\theta(t) \equiv \frac{a(t)}{f_a(t)} \quad \ddot{\theta} + \left[ 3 \frac{\dot{R}}{R} + 2 \frac{\dot{f}_a}{f_a} \right] \dot{\theta} + m_a^2(t) \sin \theta = 0$$

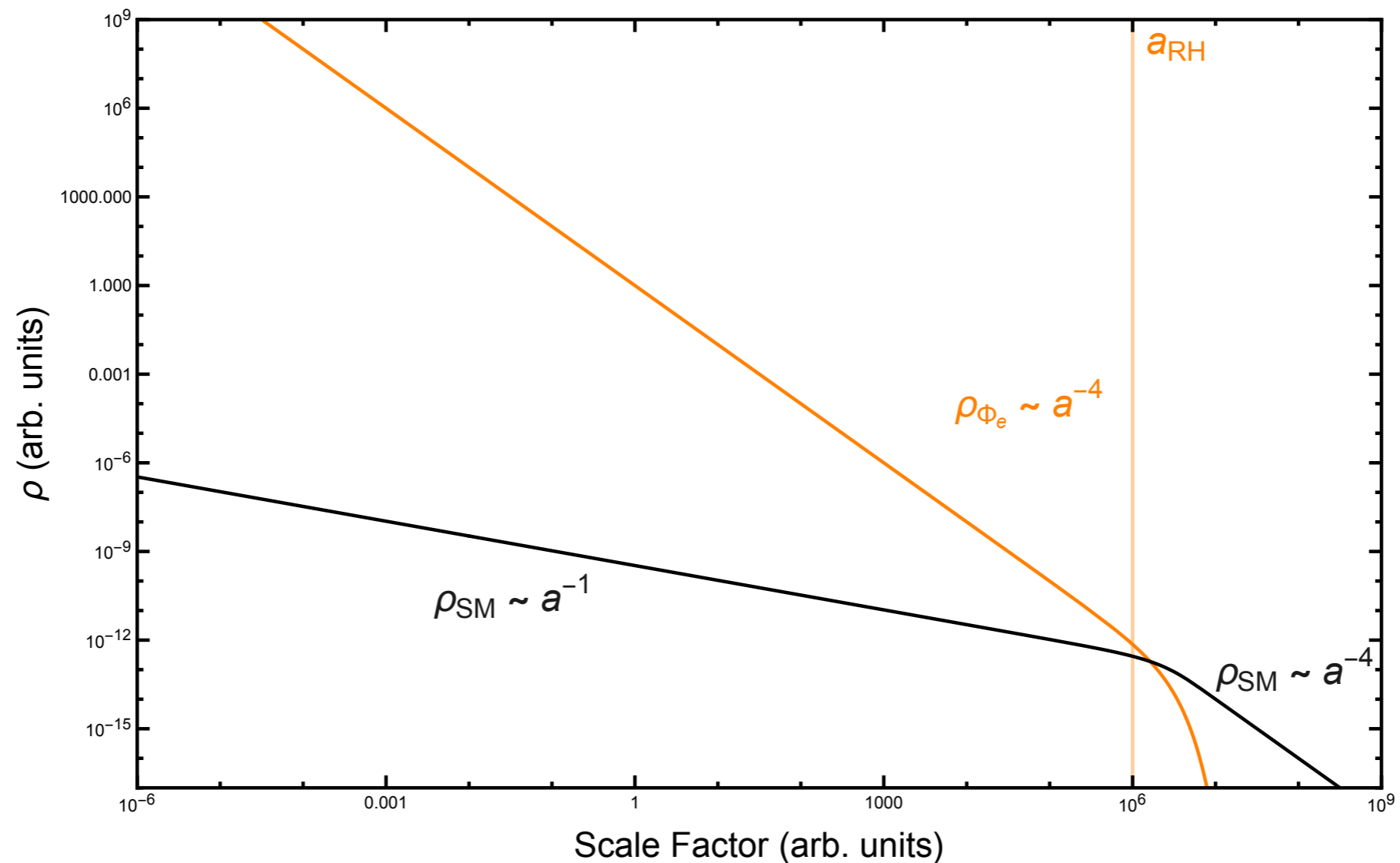
$$\rho_a(t) \simeq \frac{1}{2} m_a^2(t) f_a^2(t) \theta_0^2(t) \simeq m_a(t) n_a(t)$$

Despite everything, co-moving number density is conserved and mass changes as expected

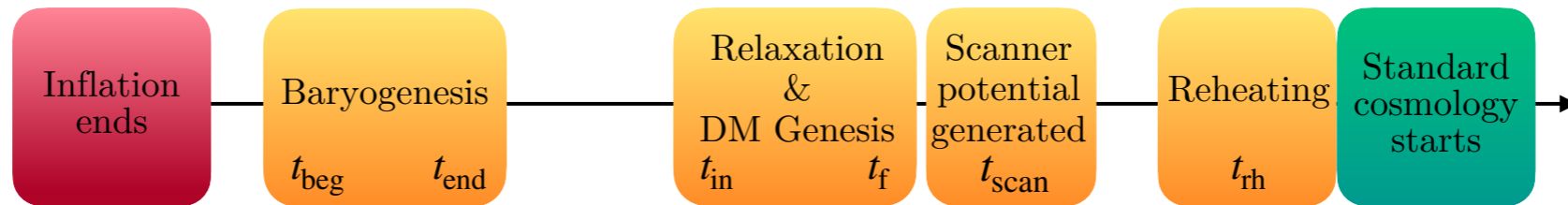
# History - Relaxation



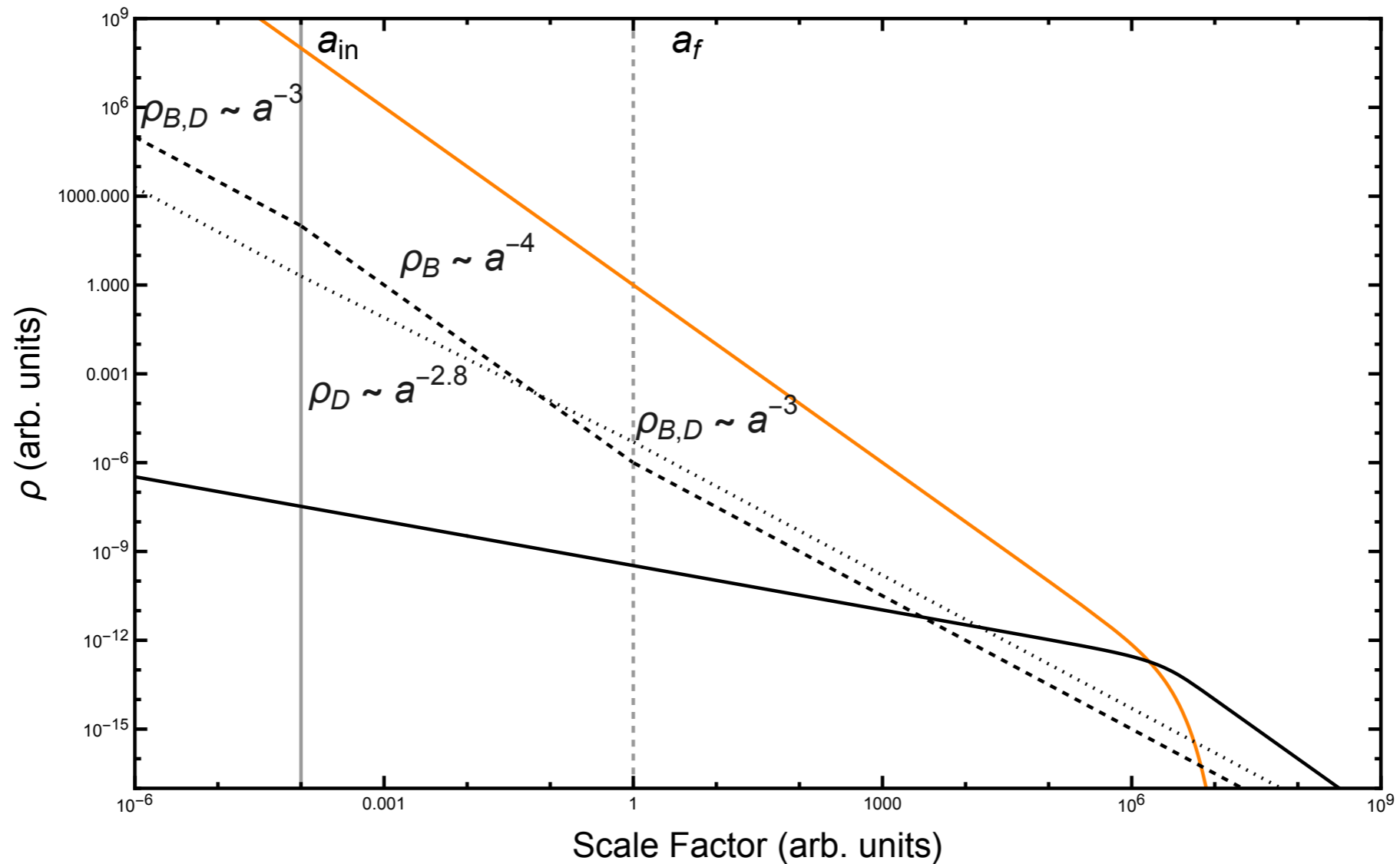
Occurs as mentioned before



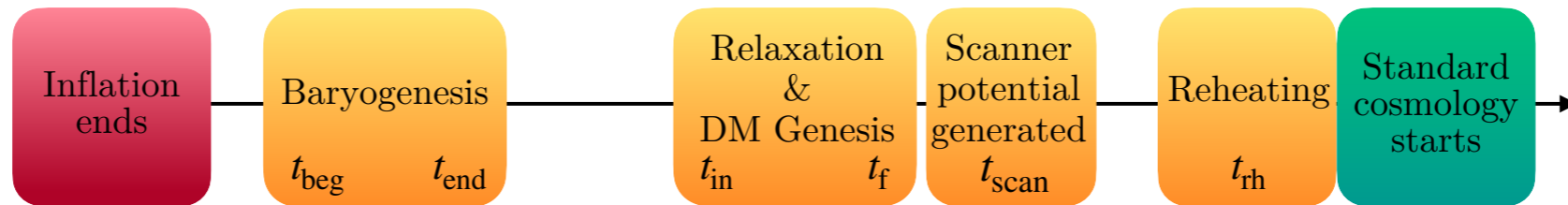
# History - Relaxation



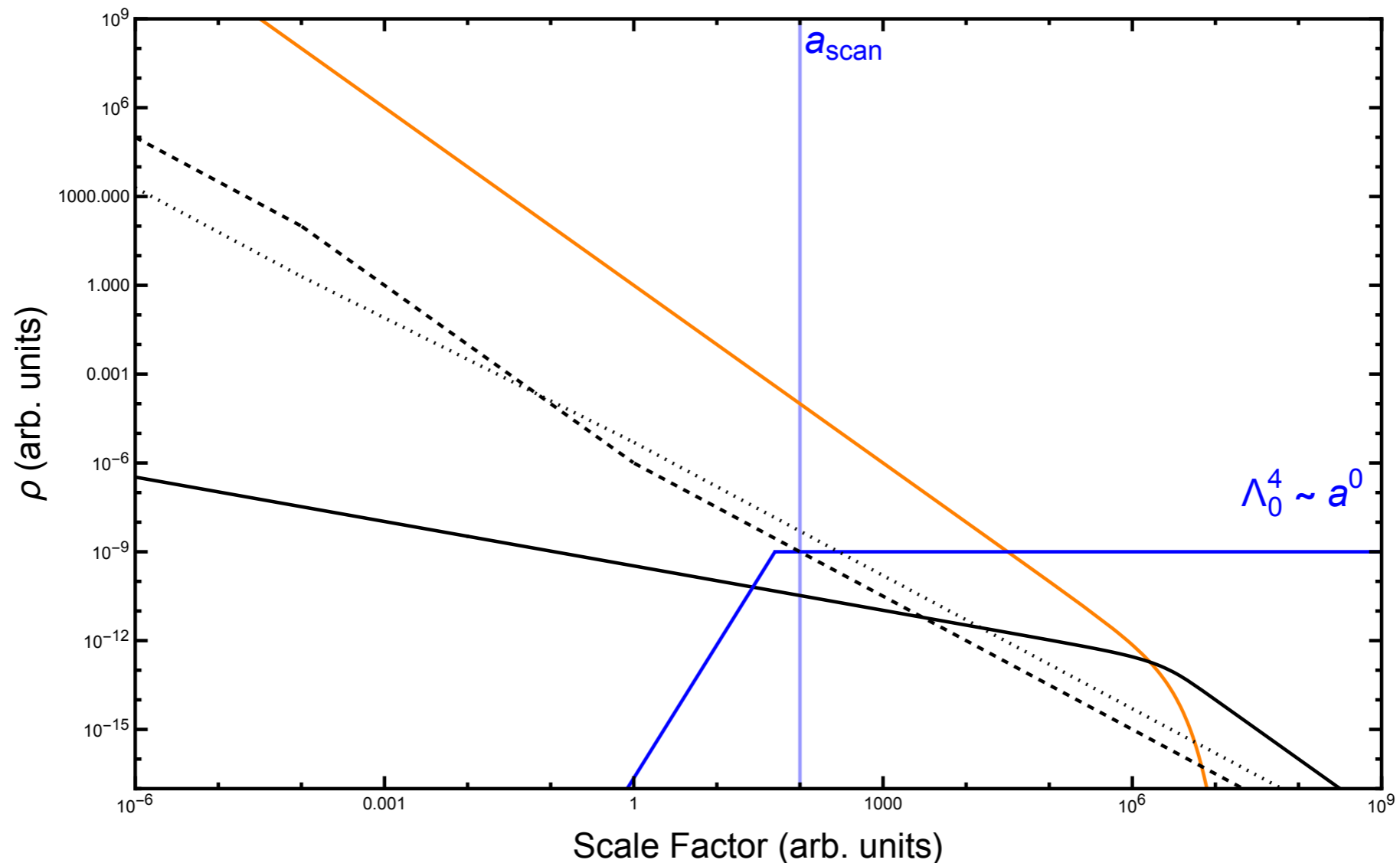
Occurs as mentioned before



# History - Scanner Potential



Scanner potential locks ratio in place



# Constraints

## Plethora of restrictions

- EFT :  $f_a < f_\phi$
- Baryons non-relativistic at start of relaxation
- Thermal (pion) contribution to  $\phi$  potential is subdominant to baryons
- Initial axion misalignment not larger than  $2\pi$
- Parametric resonance causing friction
- $\Delta N_{\text{eff}}$  and overclose universe
- 5th force and supernova cooling

# Some example numbers

Nailed in place @ late times

$$m_\phi \sim \text{MeV} \quad \Lambda_\phi \sim 5 \text{ GeV}$$

Adjuster dynamics

$$F \sim 10^4 \text{ GeV}$$

Clockwork

Coupling to baryons

$$f_\phi / c_B \sim 10^{12} \text{ GeV}$$

Reheat Temperature

$$T_{RH} \sim 10 \text{ MeV}$$

Dark Matter / Baryon  
Properties

$$m_{B,\text{in}} \lesssim 10^3 \text{ GeV}$$

$$10^8 \text{ GeV} < f_a < 10^{11} \text{ GeV}$$

Only works if you under-  
produce axions

# Outline



- ~~The Good~~
  - ~~The basic idea and prediction~~
- ~~The Bad~~
  - ~~Cosmology + modifications needed to realize the idea~~
- The Ugly
  - Model building needed to avoid fine tuning

# Vacuum Potential

$$\mathcal{L}_{\text{toy}} = \frac{\phi}{f} G_N^2$$

Non-shift symmetric potentials tend to lead to large vacuum potentials

$$\mathcal{L} = f^4 F\left(\frac{\phi}{f}\right) + \Lambda_{QCD,N}^4 e^{\phi/f}$$

These are all larger than the finite density contribution that dictates relaxation

# PNGBs of $Z_N$

Start with a NGB of a U(1)

$$\frac{\phi}{f} = 2\pi + \frac{\phi}{f}$$

Make it non-linearly realize a  $Z_N$  symmetry

$$\frac{\phi}{f} \rightarrow \frac{\phi}{f} + \frac{2\pi}{N} \qquad V\left(\frac{\phi}{f}\right) = V\left(\frac{\phi}{f} + \frac{2\pi}{N}\right)$$

# PNGBs of $Z_N$

Any potential for this PNGB can be written as

$$V(\phi) \propto \sum_{k=0}^{N-1} F\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

Written in this form, if  $F$  is independent of  $N$ ,  
then this is a Riemann Sum!

# PNGBs of $Z_N$

$$V(\phi) \propto \sum_{k=0}^{N-1} F\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right) = \frac{N}{2\pi} \int_0^{2\pi} F(\theta) d\theta + \mathcal{O}(N^0)$$

Large N limit is independent of the PNGB

Expected since as N goes to infinity, you have a continuous shift symmetry

# Convergence Theorems

Riemann sums have a lot of theorems associated with them

$$E_N(F) = \int_0^{2\pi} F(\theta) d\theta - \frac{2\pi}{N} \sum_{k=0}^{N-1} F\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

The potential for the PNGB comes entirely from the error in the Riemann sum in approximating an integral

# Euler-Maclaurin Theorem

If the function  $F$  is analytic

Then there is a strip around the real axis from  $-i a$  to  $i a$   
where  $F$  is a holomorphic function with a bound  $M$

$$|E_N(F)| \leq \frac{4\pi M}{e^{Na} - 1}$$

Well behaved potentials where no particle becomes massless result in exponentially suppressed PNCB masses

# Light $G^2$ scalar

Consider  $N$  decoupled QCDs all with one new vector-like fermion

QCDs / Fermions exchange under  $Z_N$

$$G_k \rightarrow G_{k+1} \quad \psi_k \rightarrow \psi_{k+1}$$

Scalar (non)-linearly realizes the  $Z_N$

$$\Phi \rightarrow e^{2\pi i/N} \Phi \quad \phi/f \rightarrow \phi/f + 2\pi/N$$

$$\Phi = f e^{i\phi/f}$$

# Light $G^2$ scalar

$$\mathcal{L} \supset \sum_j \left( M + ye^{\frac{2\pi j}{N}\Phi} \right) \bar{\psi}_j \psi_j$$

Yukawa coupled scalar

$$V_{CW} = \sum_j \Lambda^2 M_j^2 + M_j^4 \log M_j^2 / \Lambda^2$$
$$\propto M^4 \left( \frac{yf}{M} \right)^N \cos\left( \frac{N\phi}{f} \right)$$

Exponentially suppressed potential

# Conclusion

**The baryon/DM coincidence problem is a very troubling problem**

So little known about DM, need to have some opinion on this

**Scalar adjusts DM and baryon masses until energy densities are about equal**

New solution that applies to almost all models of DM!

Interesting experimental signatures!

In the context of QCD axion dark matter - Postdicts the value of the dark matter - baryon abundance ratio!