Inclusive vs exclusive $b \to s\mu\mu$: new physics or hadronic effects?

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Based on:
Huber, Hurth, Jenkins, EL, Qin, Vos, 2007.04191
Huber, Hurth, Jenkins, EL, Qin, Vos, 2306.03134
Huber, Hurth, Jenkins, EL, Qin, Vos, 2404.03517
Capdevila, Alvarez, EL, Matias, 2605.06567

The $b \to s\ell\ell$ anomalies

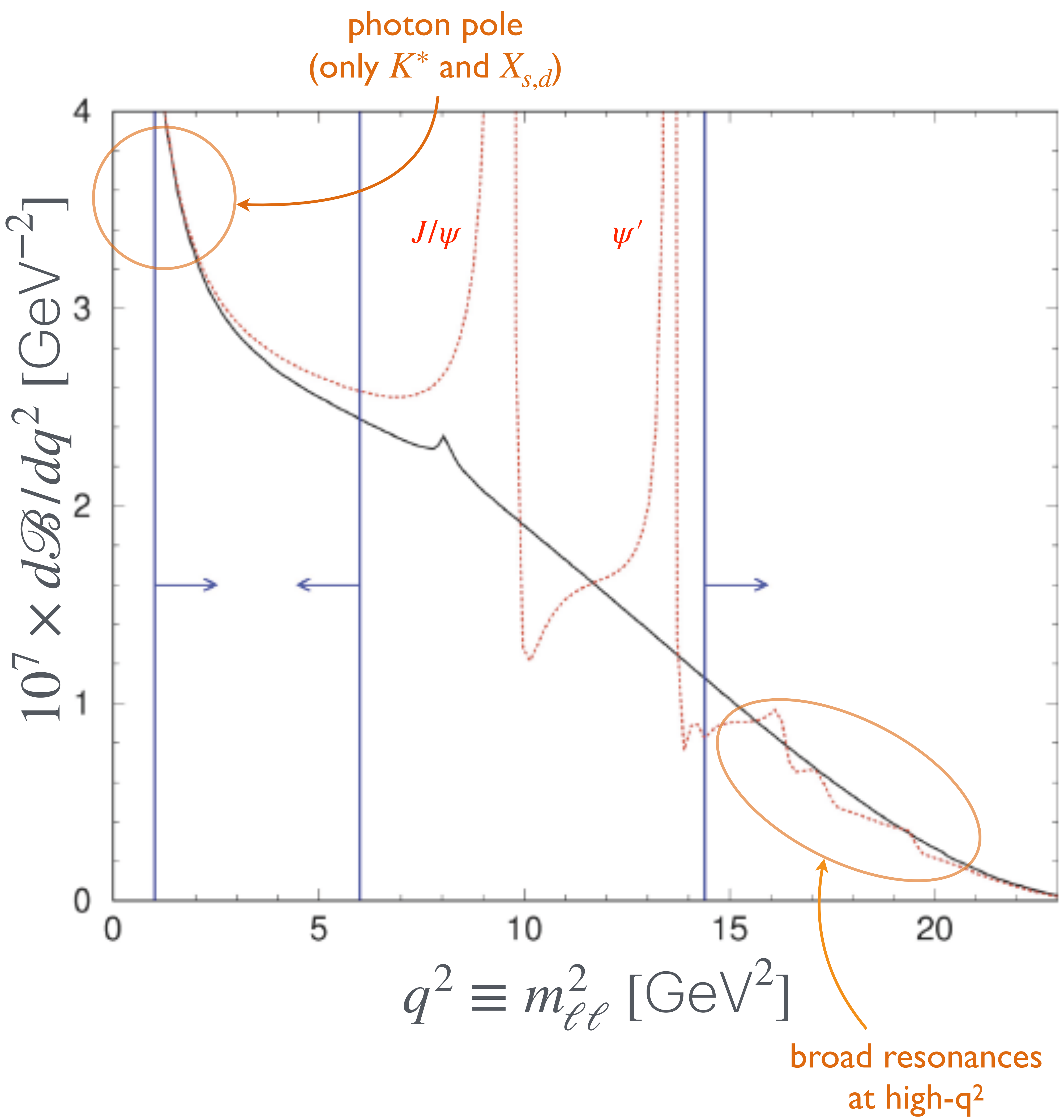
$b \to s\ell\ell$: typical spectrum

- Intermediate charmonium resonances contribute via:
  $B \to (K, K^*, X_s)\psi_{c\bar{c}} \to (K, K^*, X_s)\ell^+\ell^-$

- Contributions of J/ψ and $\psi(2S)$ have to be dropped:
  - Low-q^2 $\Rightarrow$ 1 GeV^2 < q^2 < 6 GeV^2
  - High-q^2 $\Rightarrow$ q^2 > (14.4 or 15) GeV^2

- Dependence on the Wilson Coefficients [$\hat{q}^2 = q^2/m_b^2$]:

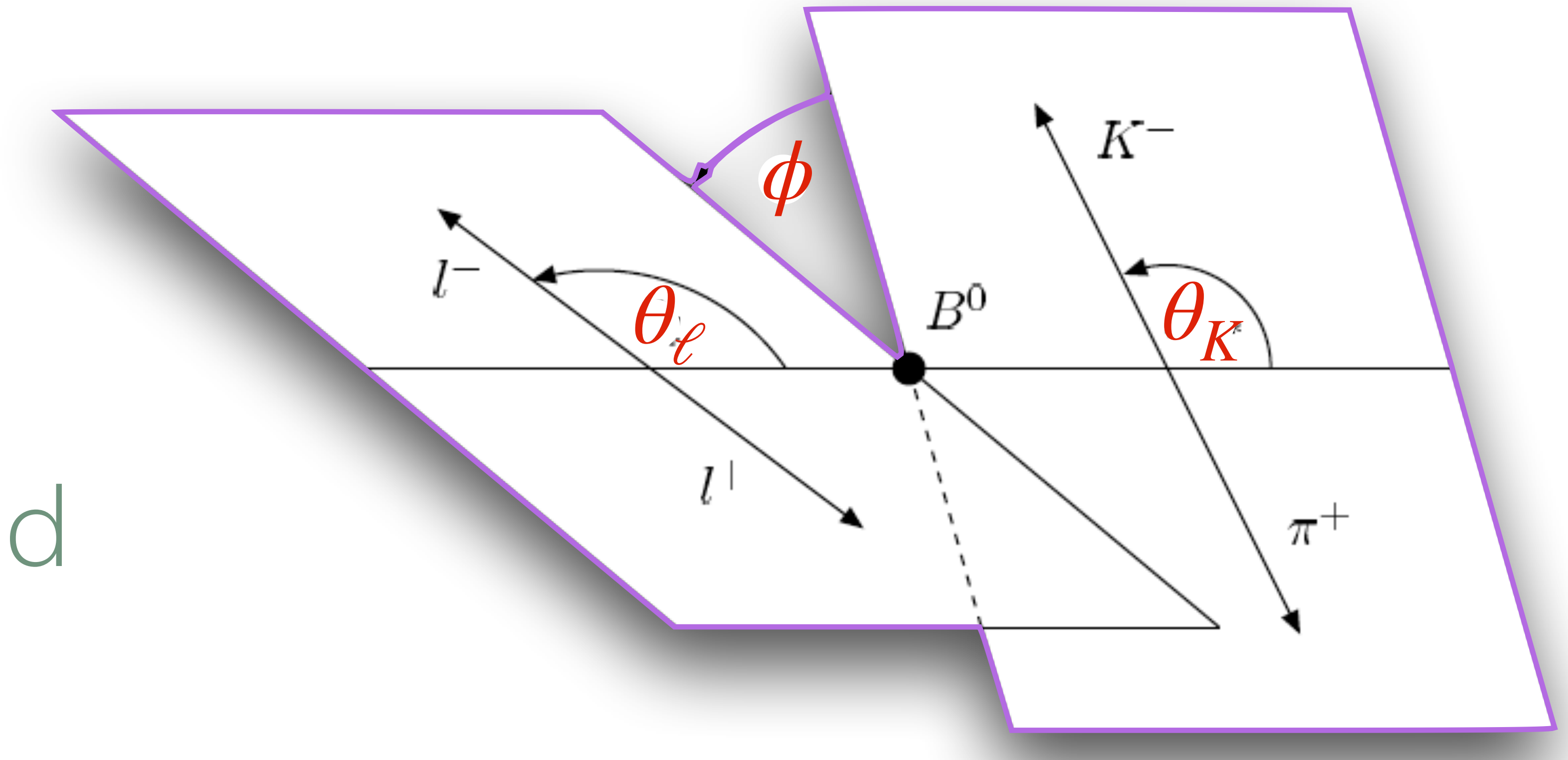
$$\frac{d\Gamma}{d\hat{q}^2} \sim \left(1-\hat{q}^2\right)^2 \left[\left(1+2\hat{q}^2\right)\left(C_9^2 + C_{10}^2\right) + 4\left(1+\frac{2}{\hat{q}^2}\right)C_7^2 + 12 C_7 C_9\right]$$



$B \to K^* \mu\mu$: short distance vs strong phases

- Angular decomposition of $B \to K^* \mu\mu \to K\pi\mu\mu$:

$$\frac{\mathrm{d}\Gamma_{\mathrm{P}}(B^0 \to K^{*0}\mu^+\mu^-)}{\mathrm{d}\cos\theta_\ell \ \mathrm{d}\cos\theta_K \ \mathrm{d}\phi \ \mathrm{d}q^2} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K$$
$$+ J_{2s}\sin^2\theta_K\cos 2\theta_\ell + J_{2c}\cos^2\theta_K\cos 2\theta_\ell$$
$$+ J_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi$$
$$\text{T-even} \quad + J_5\sin 2\theta_K\sin\theta_\ell\cos\phi + J_{6s}\sin^2\theta_K\cos\theta_\ell$$
$$+ J_{6c}\cos^2\theta_K\cos\theta_\ell + J_7\sin 2\theta_K\sin\theta_\ell\sin\phi \quad \text{T-odd}$$
$$+ J_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + J_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$



- Both CP averaged distributions (S-observables) and CP asymmetries (P-observables) have been studied

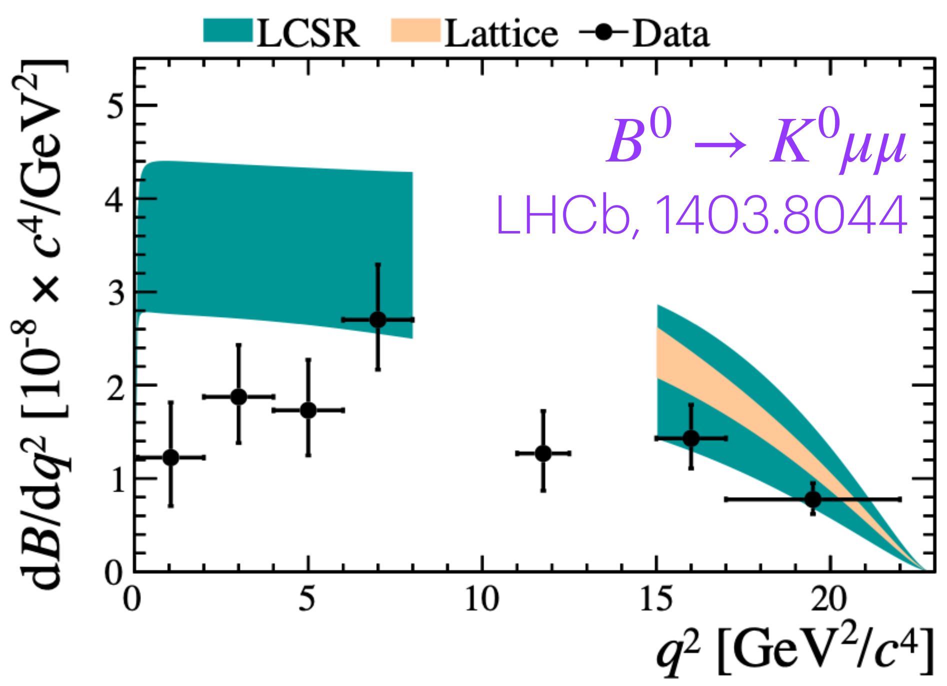
- In terms of transversity amplitudes ($A^{L,R}_{0,\perp,\|}$) the two most important observables are:

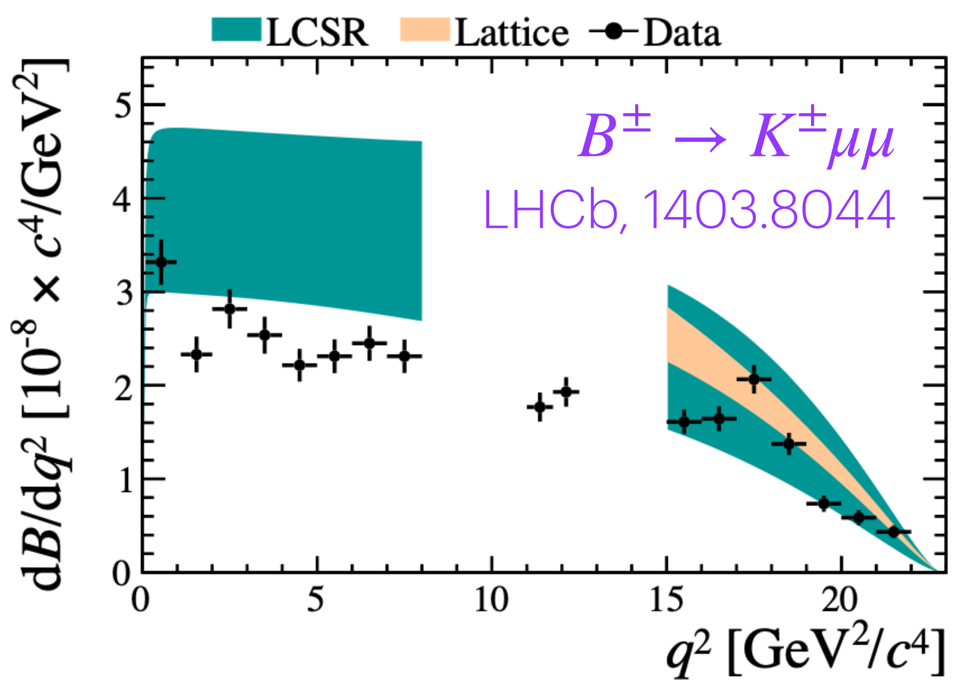
$$\begin{cases} P_5' \propto \mathrm{Re}\left(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}\right) \\ S_7 \propto \mathrm{Im}\left(A_0^L A_\|^{L*} - A_0^R A_\|^{R*}\right) \end{cases}$$

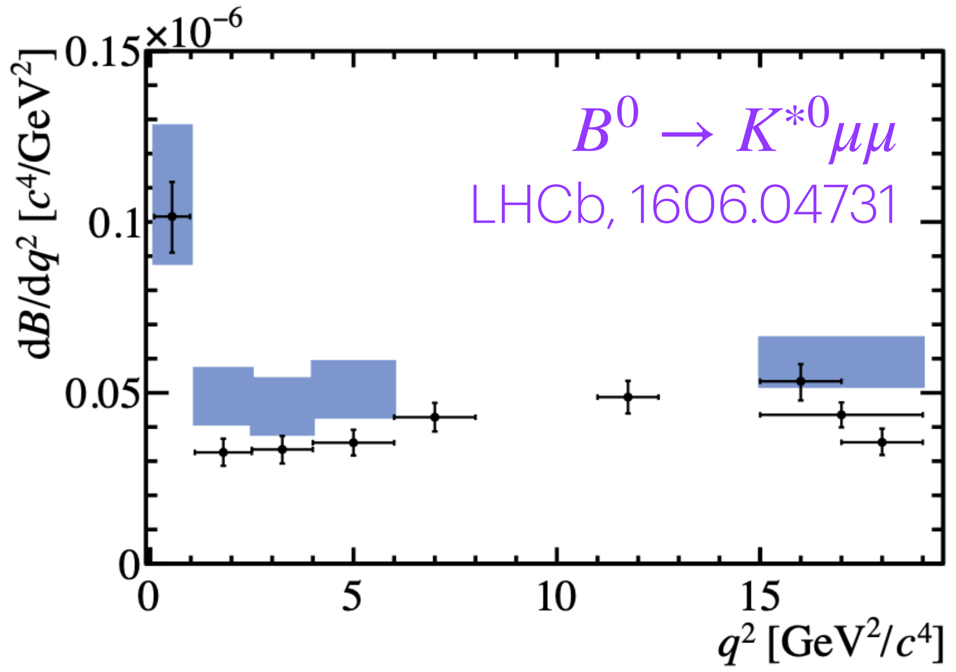
- P_5' is sensitive to short distance physics
  S_7 is sensitive to long distance effects (strong phases)

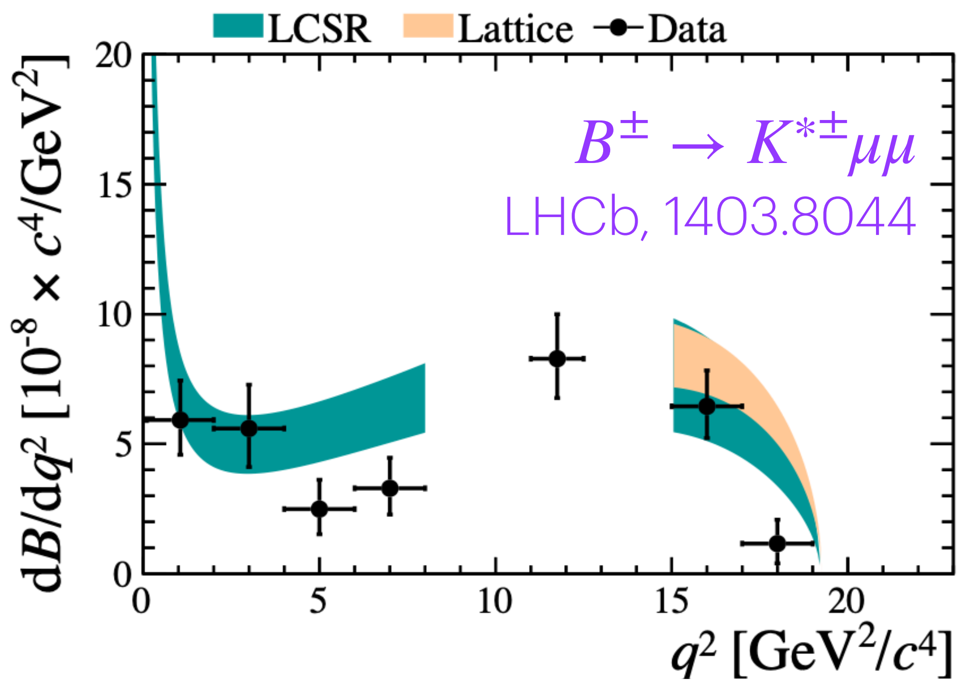
Anomalies in exclusive modes

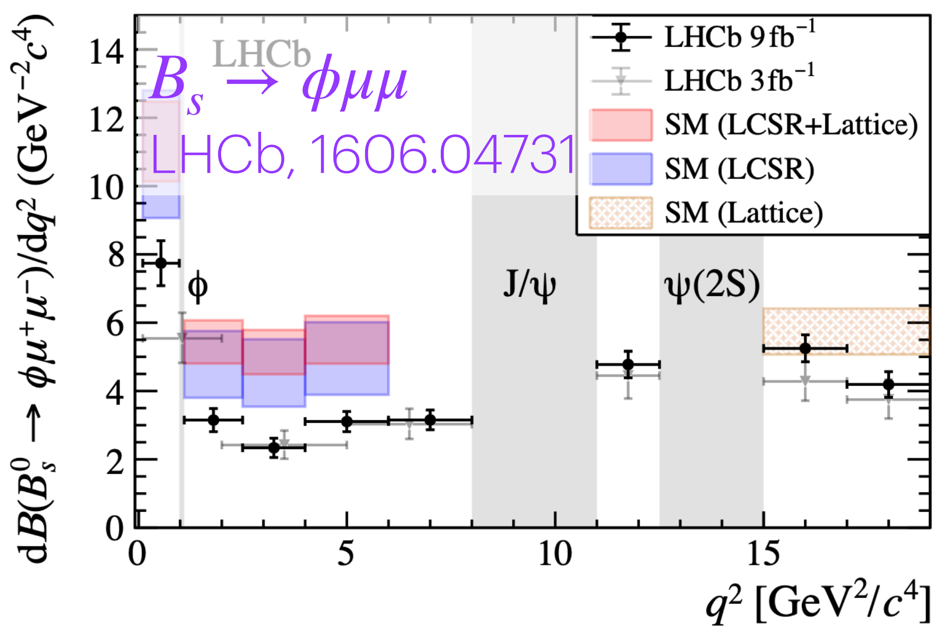
- Branching ratios:

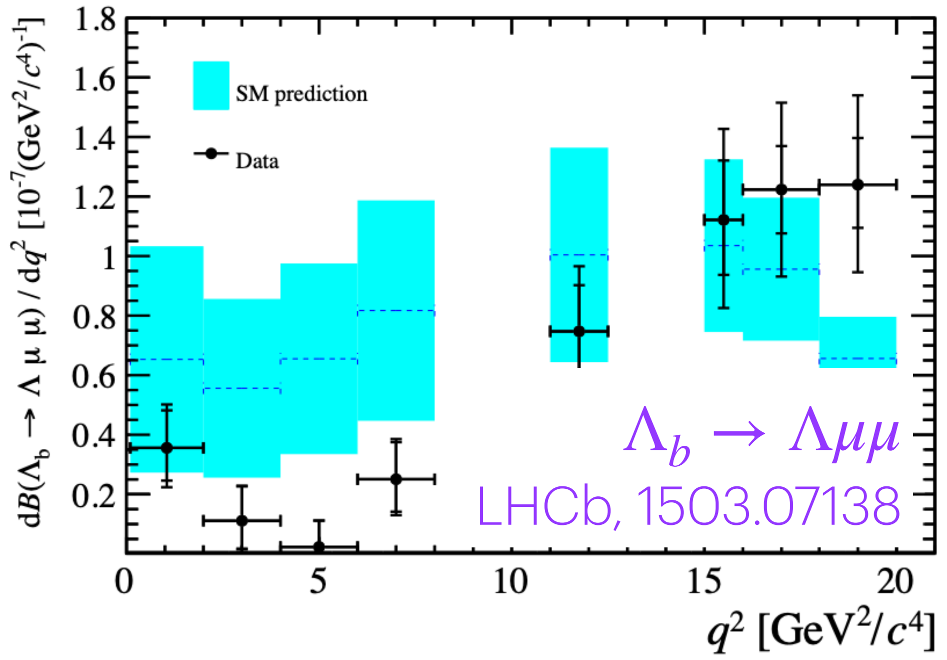




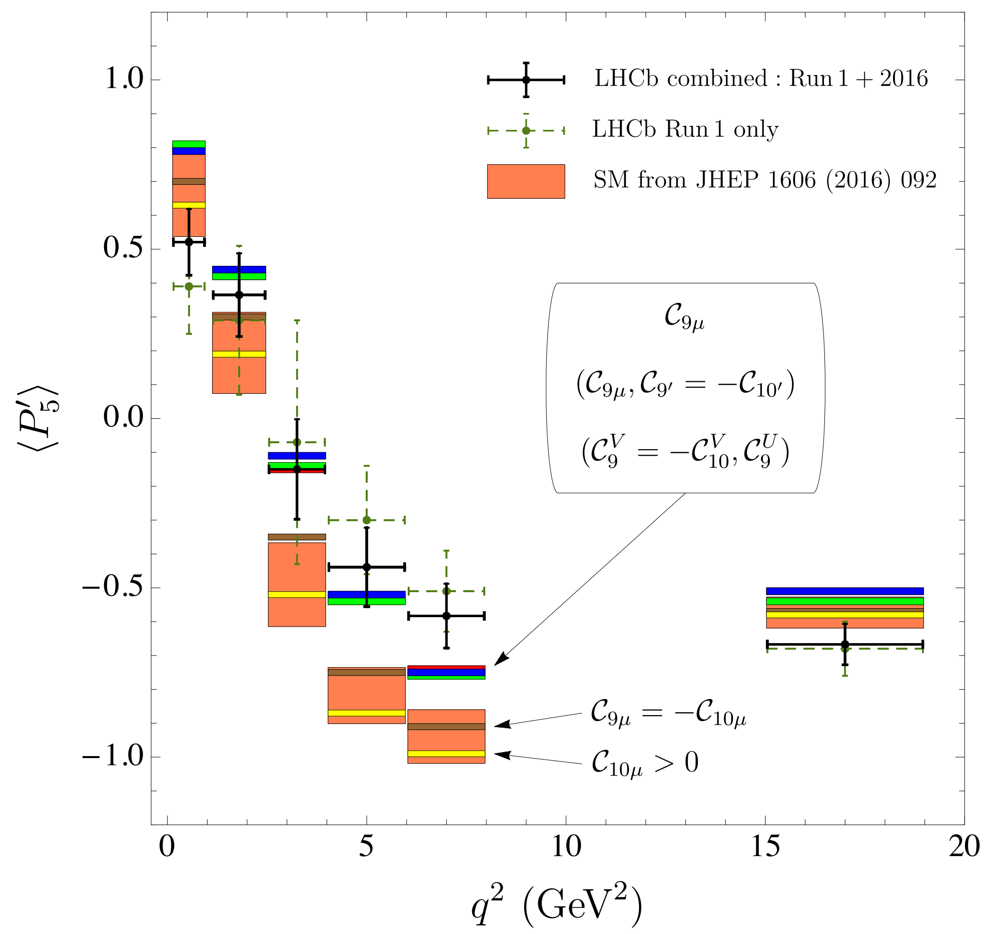


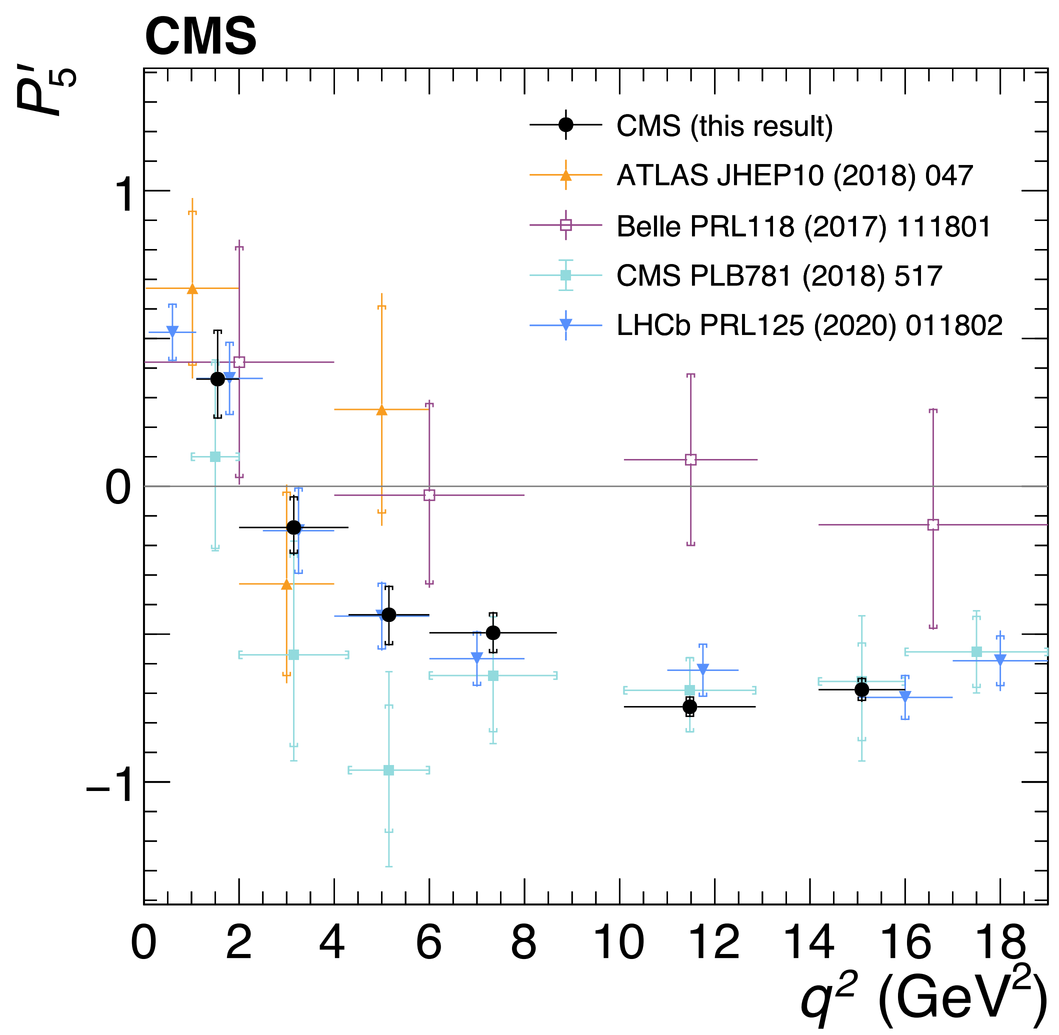




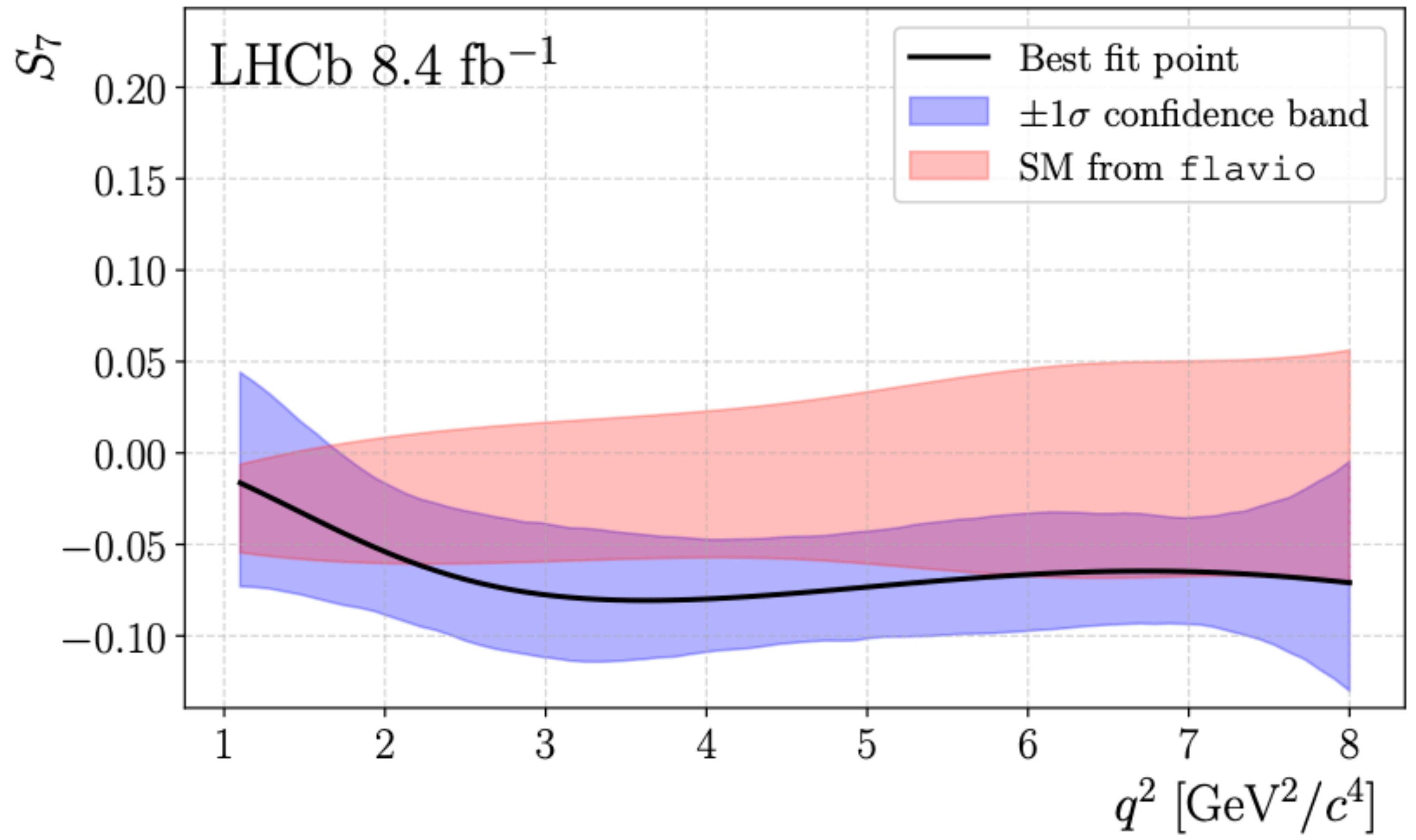


- Angular observables ($B^0 \to K^{*0}\mu\mu$):

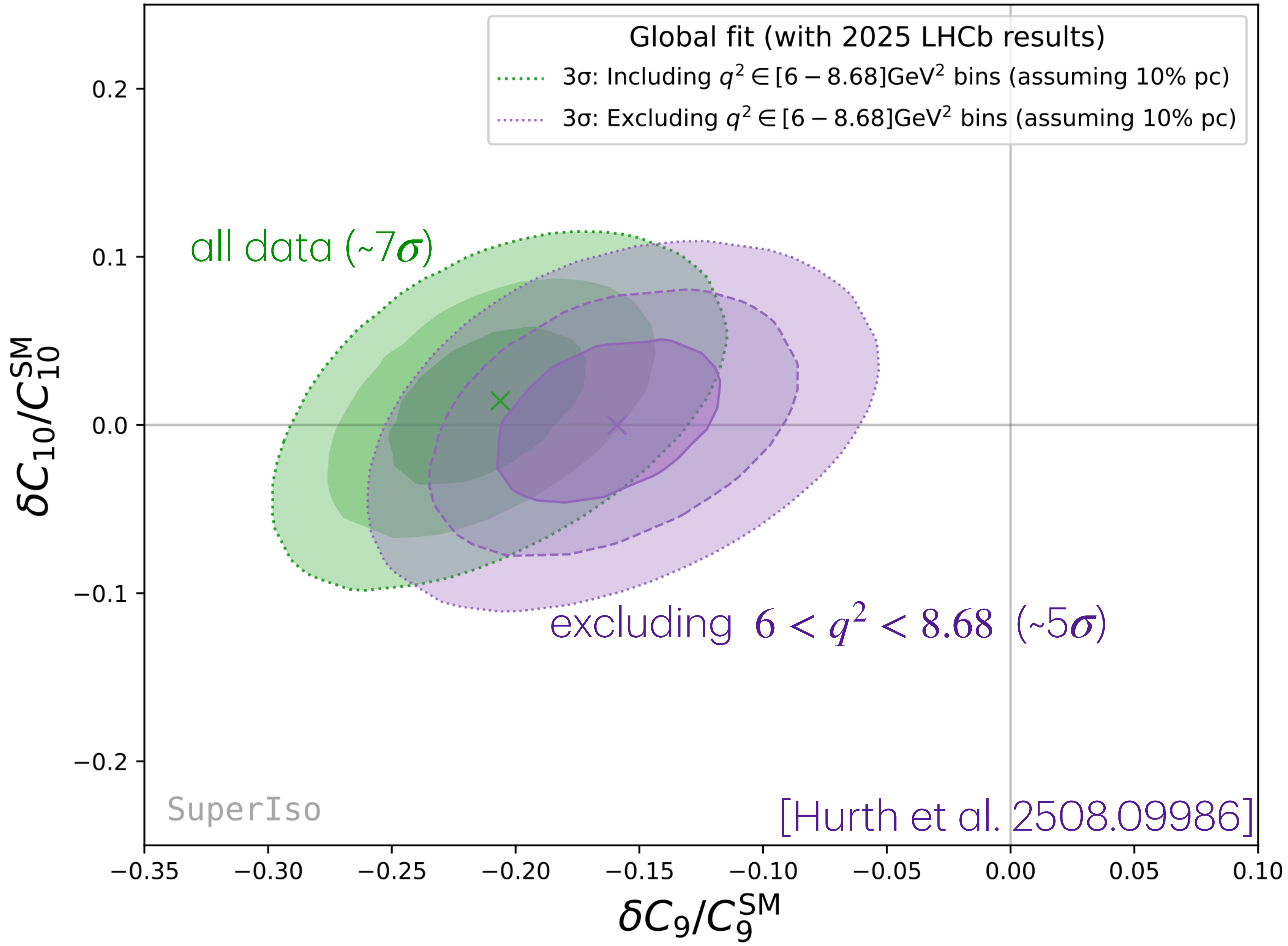




[LHCB 2608.12215.16266]



Global fits



- This fit includes an O(10%) estimate of nonlocal power corrections
- With and without the inclusion of the $q^2 \in [6,8.68]$ GeV^2 bins the fit result is:

$$\delta C_9 = -0.69 \pm 0.12 \quad (4.8\sigma) \qquad \text{[without } q^2 \in [6,8.68]\text{]}$$
$$\delta C_9 = -0.89 \pm 0.11 \quad (7.1\sigma) \qquad \text{[with } q^2 \in [6,8.68]\text{]}$$

- This is an enormous NP effect:

$$\boxed{C_9^{\mathrm{NP}} \sim -\frac{1}{4} C_9^{\mathrm{SM}}}$$

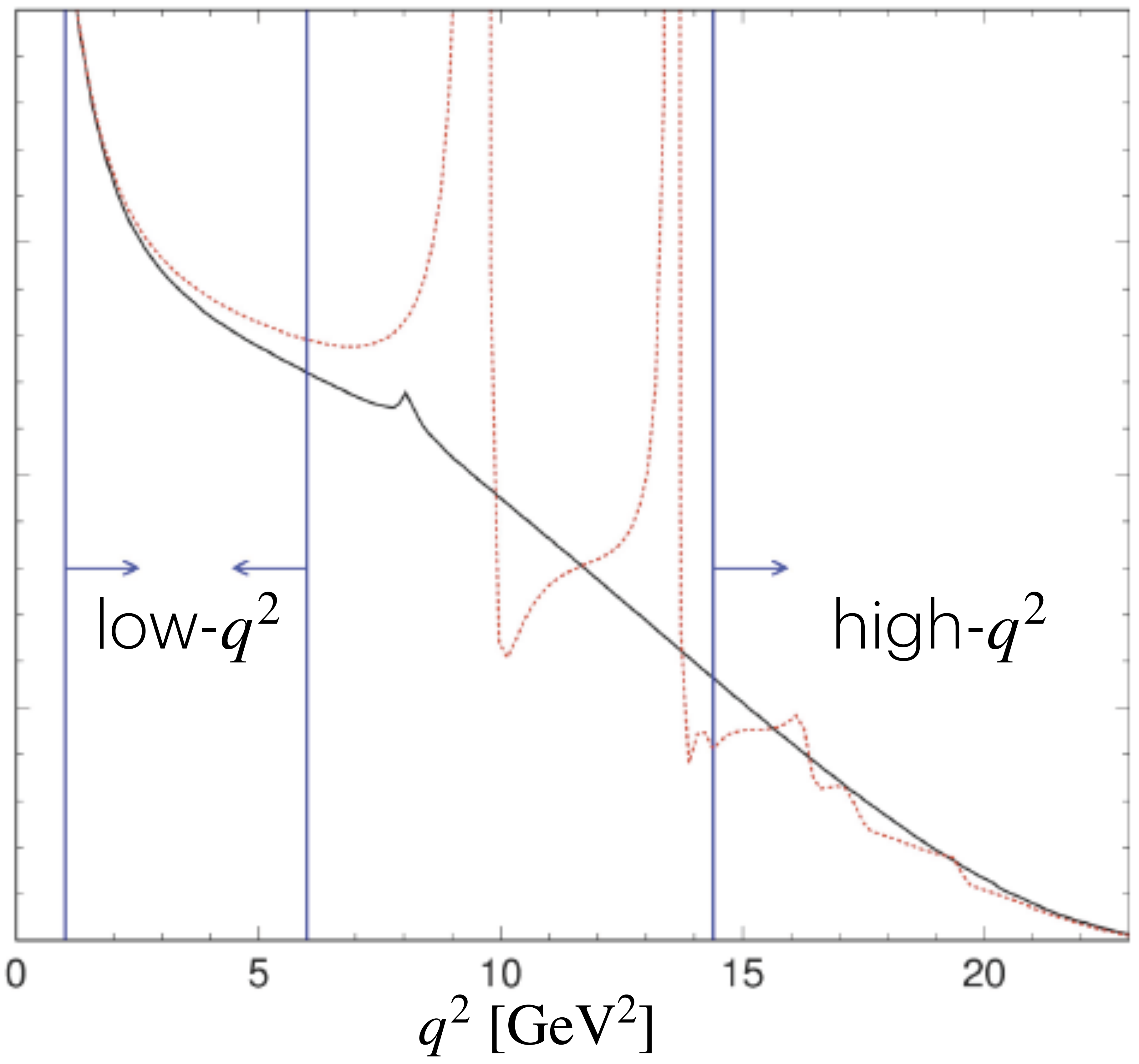
- Good agreement across many fitters:
  - ABCDMN [Algueró et al, 2304.07330]
  - AS/GSSS [Altmannshofer et al, 2212.10497]
  - CFFPSV [Ciuchini et al, 2212.10516]
  - HMMN [Hurth et al, 2104.10058]
  - GRvDV [Gubernari et al, 2206.03797]

Theory overview

- Before calling this new physics, we need to understand how reliably these processes can be calculated.

Exclusive at low-q^2

- Soft-Collinear factorization at leading power (SCET-II)
- Inputs: Form Factors, LCDA
- Leading uncertainties: **non-local power corrections**, form factors (K^*)



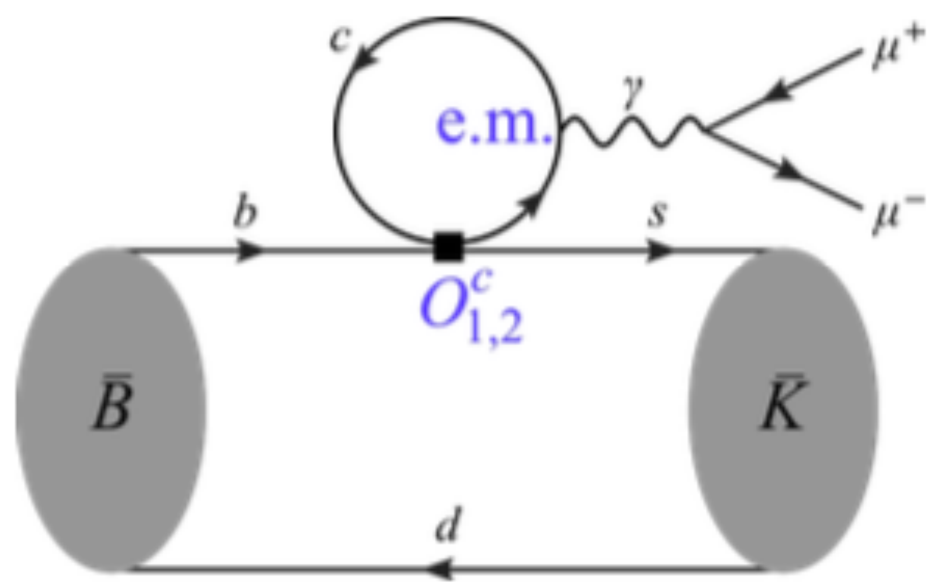
Exclusive at high-q^2

- $1/q^2$ expansion: only integrated spectrum is predicted
- If this expansion can be interpreted as an OPE, the integrated spectrum receives only local power corrections
- Inputs: **Form Factors, HQET matrix elements**

Charming penguins (nonlocal power corrections)

- **Power corrections at low-q^2**

  Within SCET-II, matrix elements, like $\langle K^{(*)} | T J_\mu^{\rm em} O_2 | B \rangle$, are factorized up to subleading power corrections:

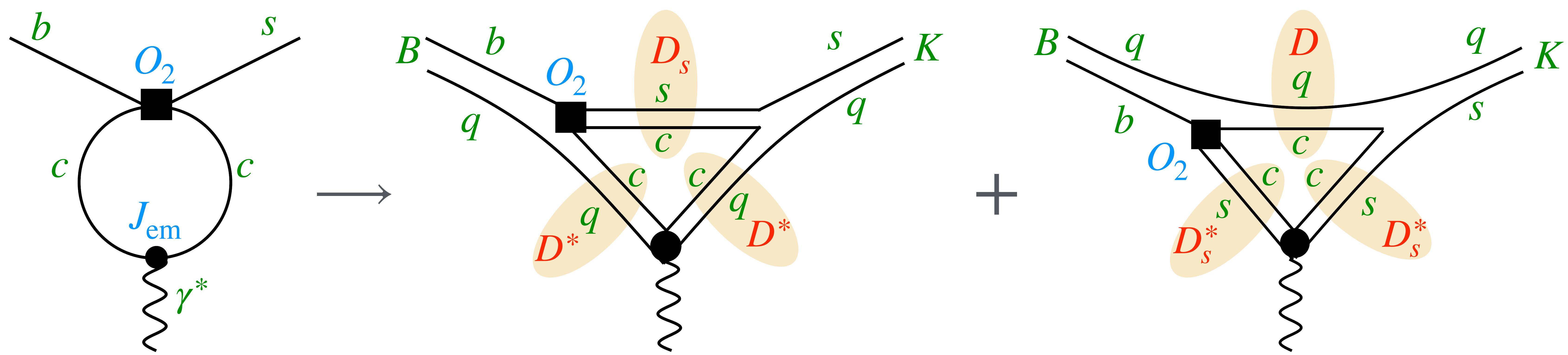


$$\langle K^* | T J^{\rm em} O_2 | B \rangle \sim {\rm FF} + \phi_{K^*} \star J \star \phi_B + \text{power corrections}$$

$10\,\%$

SCET power counting: **10%-20%** $\Longleftrightarrow$ To explain the anomalies: **~500%**
[not a first-principles calculation!]

- Possible enhancement: anomalous thresholds / rescattering



New physics or hadronic effects?

What is $\delta C_9 \sim -1$ telling us?

Short-distance New Physics: $C_9^{\text{New Physics}} \sim -1$

Nonlocal charm: $C_9^{\text{Non Perturbative}} \sim -1$

Lattice QCD

Calculate the nonlocal amplitude directly:

is $C_9^{\text{Non Perturbative}}$ large enough?

Inclusive Decays

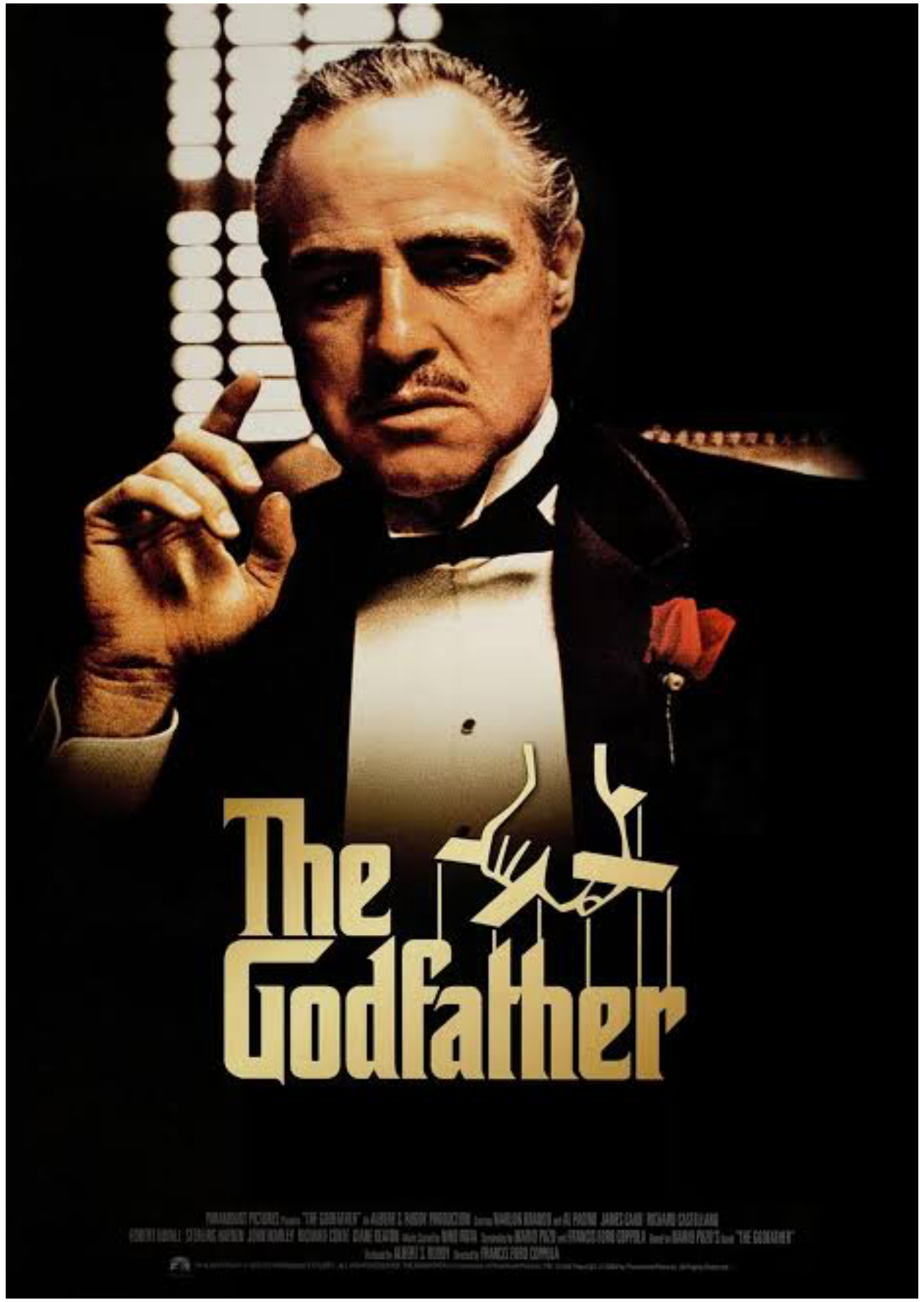
Same short-distance physics, different hadronic uncertainties:

does the same anomaly appear with different hadronic physics?

Exclusive/Inclusive Ratios

Compare exclusive and inclusive directly:

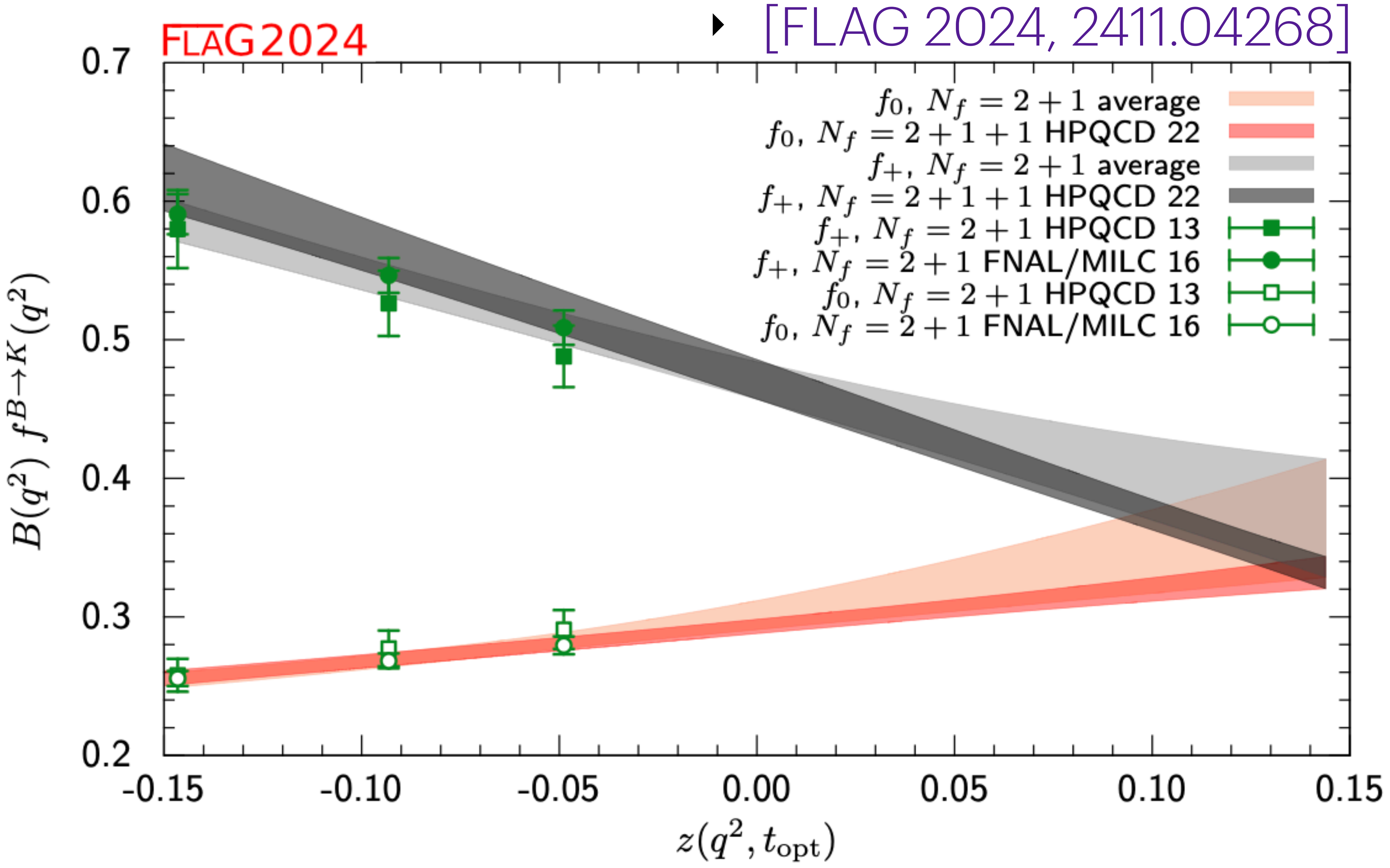
is the effect universal or exclusive specific?



Part 1: The role of lattice QCD

Form factors

- If an operator contains a lepton pair: $\langle K^{(*)}\ell\ell\,|\,(\bar{s}_L\gamma^\mu b_L)(\ell\gamma_\mu\ell)\,|\,B\rangle \propto \langle K^{(*)}\,|\,\bar{s}_L\gamma^\mu b_L\,|\,B\rangle$ ~ Form Factors
- **Light-Cone Sum Rules** (low-q²): some uncertainties have to be ball-parked but allow access to all form factors
  [Bharucha, Straub, Zwicky, 1503.05534]
- **Lattice QCD** (high-q²)
  - $B \to K$: multiple calculations
    - [FLAG 2024, 2411.04268]

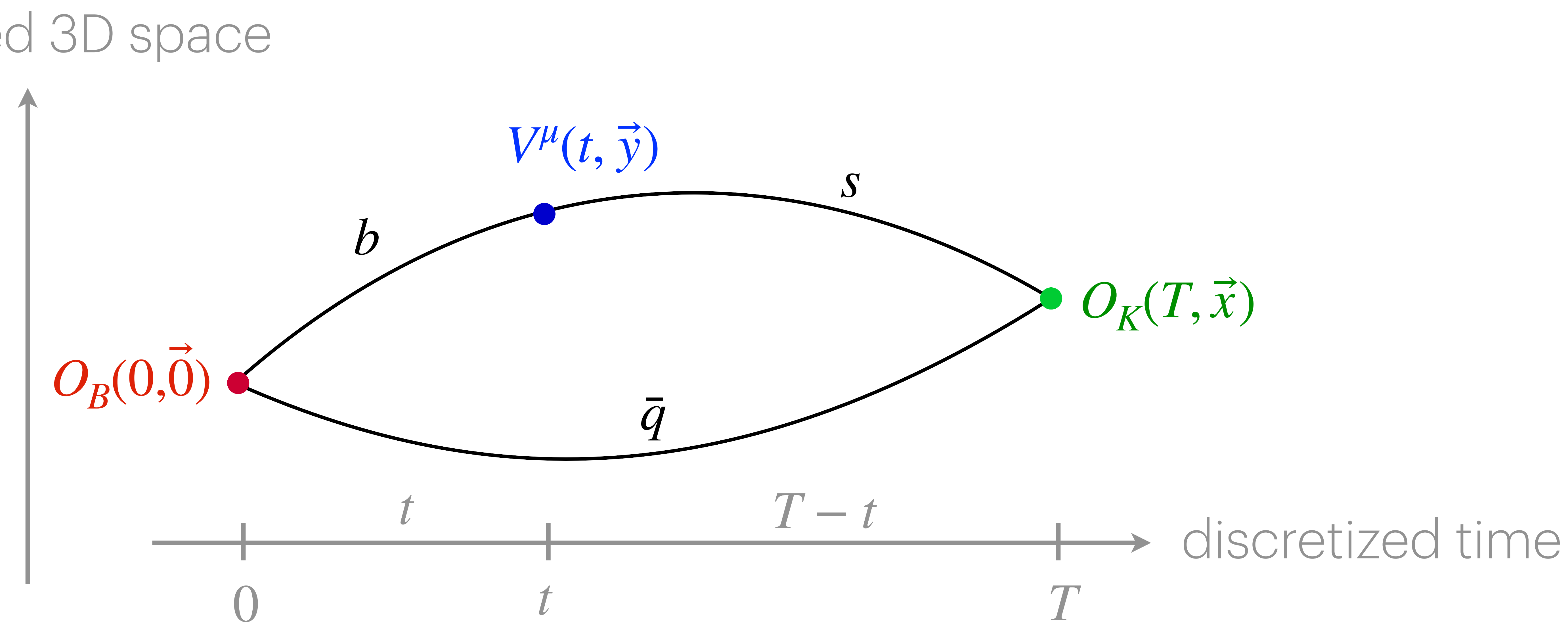


  - $B \to K^*$: complicated by the resonant nature of the K^*
    - Calculations with stable resonances
      [Horgan, Liu, Meinel, Wingate, 1310.3722 and 1501.00367]
      [RBC/UKQCD 2016 - preliminary results]
    - Calculations based on the Lüscher
      [Rendon et al, 2006.14035 - $K\pi$ scattering amplitudes]
      [Leskovec et al, 2403.19543, 2501.00903 - $B \to \rho$ form factors
      [Boyle et al, 2406.19194 - mass and width of K^* and ρ]
      [Li et al, 2603.16266]
      [Di Carlo, Erben, Tsang et al, ongoing]
    - **Currently a combination of Lattice and LCSR is used**
      [Gubernari, Reboud, Dyk, Virto, 2305.06301]

$B \to K$ form factors

- For stable mesons the correlator is dominated at large **Euclidean time** by the meson ground state:

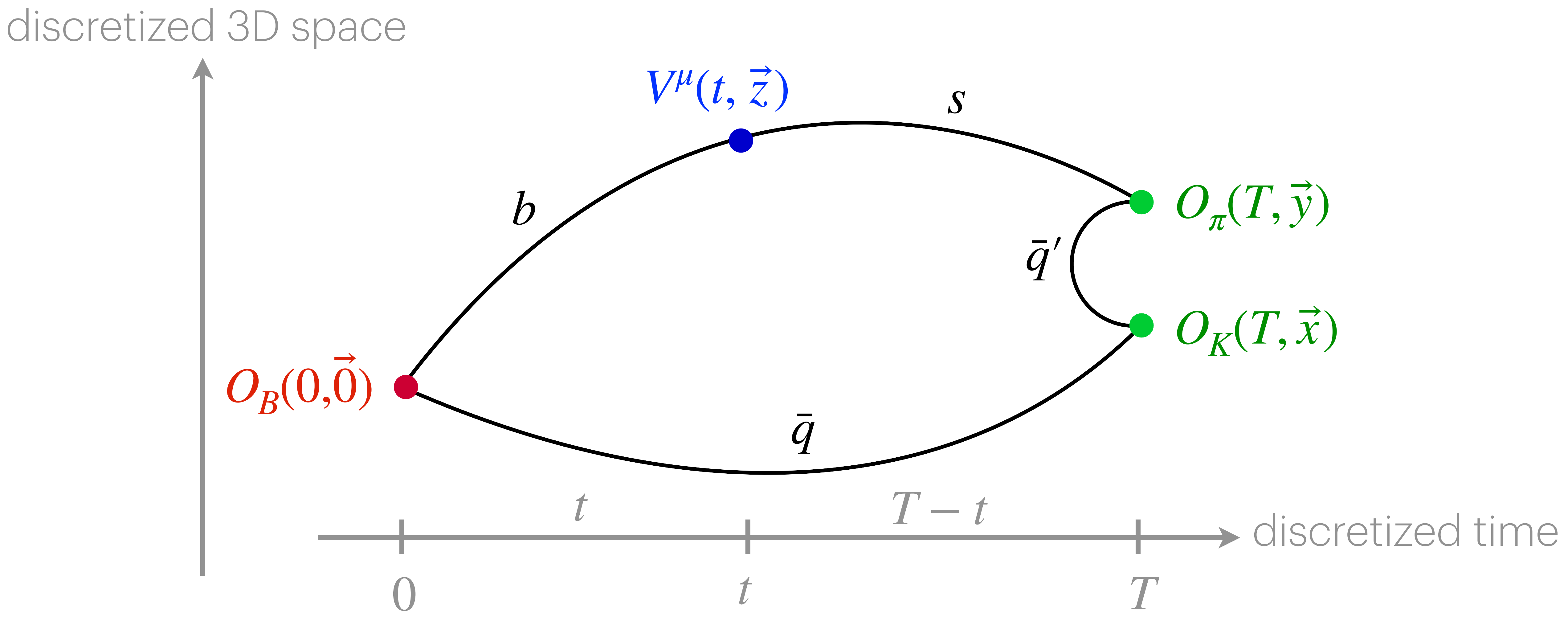
$$C_3^{B\to K}(t,T;\vec{p}_K) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_K\cdot(\vec{x}-\vec{y})}\,\langle 0\,|\,\mathcal{O}_K(T,\vec{x})\;V^\mu(t,\vec{y})\;\mathcal{O}_B^\dagger(0,\vec{0})\,|\,0\rangle \sim Z_K\,Z_B^*\,\langle K(\vec{p}_K)\,|\,V^\mu\,|\,B(\vec{0})\rangle\; e^{-E_K(T-t)}\;e^{-M_B t}$$



- ▷ We need the point-to-all propagators for the mesons (to obtain $Z_{K,B}$) and the three point function itself
- ▷ Each propagator requires solving $3_{\rm color}\times 4_{\rm spins} = 12$ lattice Dirac equations, whose sparse matrix has dimension $n = 3_{\rm color}\times 4_{\rm spins}\times N^3_{\rm space}\times N_{\rm time} = 3\times 4\times 64^3\times 128 \simeq 4\times 10^8$

$B \to K^*$ form factors

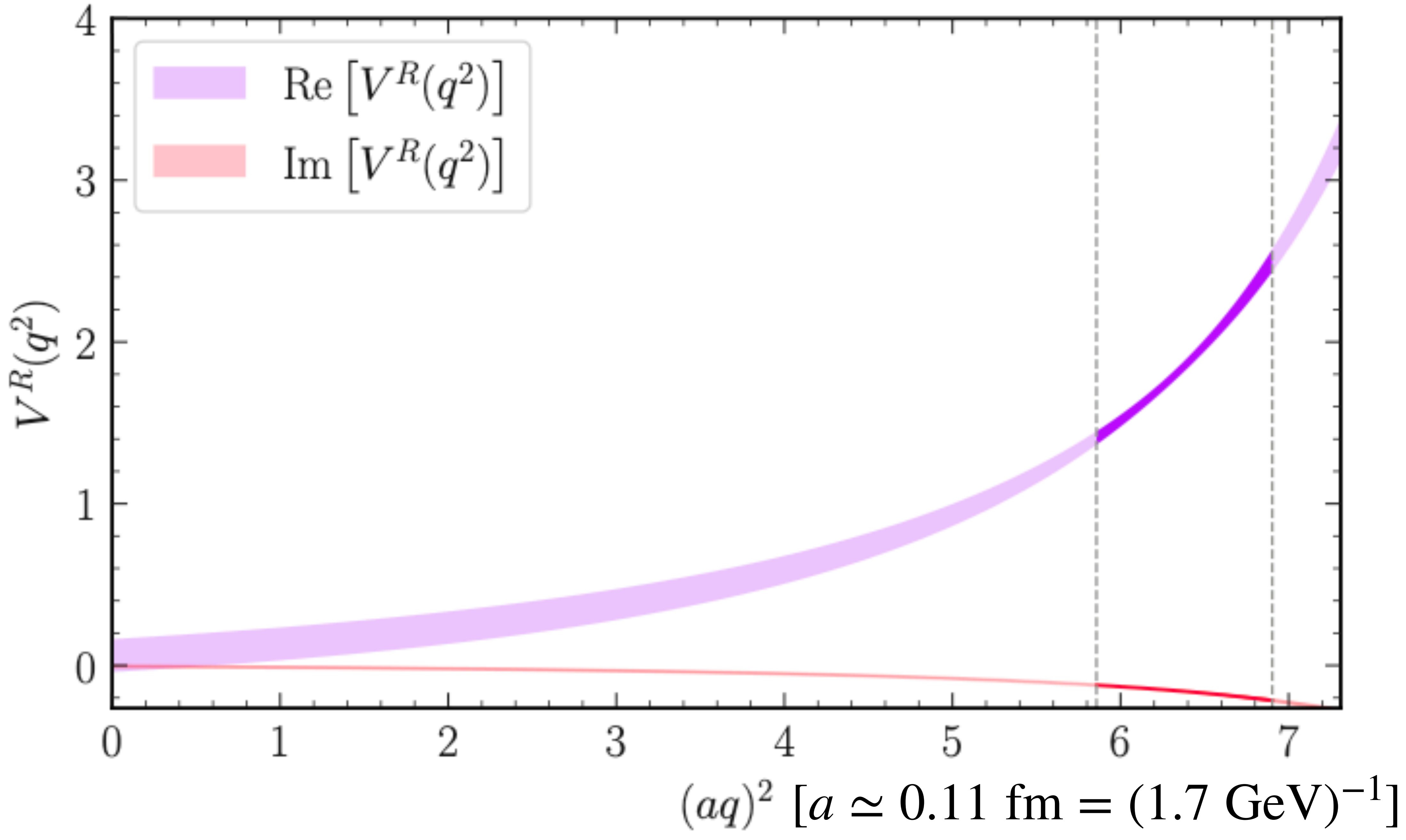
- For resonances ($\rho \to \pi\pi$, $\phi \to KK$, $K^* \to K\pi$) one relies on the Lüscher formalism:
  - **Interactions modify the finite volume energy levels**
  - Calculate the **2-point correlators** between interpolators (which include the K^* **and all possible $K\pi$ states**)
  - The **Lüscher quantization condition** allows the extraction of the phase shifts
  - **Analytic continuation** of the phase shifts to the second Riemann sheet allows the extraction of the resonance parameters
  - The form factors are extracted from the **3-point correlators** involving the external current



- The all-to-all (on a fixed time slice) propagator ($\bar{q}'$) requires, in principle, solving $12 \times N^3 \simeq 3 \times 10^6$ lattice Dirac equations
- Distillation reduces this to $O(10^3)$, which is feasible with current computing technology

$B \to K^*$ form factors

- $B \to \rho$ **form factors for** $m_\pi = 320$ MeV
  [Leskovec et al, 2501.00903]



- ρ **and** K^* **resonance parameters at the physical point**

$$\sqrt{s_\rho} = \begin{cases} (761 - 765) - i\,(71 - 74) & \text{[PDG]} \\ 796(5)(50) - i\,96(5)(15) & \text{[Erben et al., 2406.19193(4)]} \end{cases}$$

$$\sqrt{s_{K^*}} = \begin{cases} 890(14) - i\,26(6) & \text{[PDG]} \\ 893(2)(54) - i\,26(1)(6) & \text{[Erben et al., 2406.19193(4)]} \\ 883(22) - i\,20(13) & \text{[Li et al. 2603.16266]} \end{cases}$$

[The PDG determinations are T-matrix pole parameters]

- K^* **form factors pursued by two groups:**
  [Jevsenak, Meinel, Leskovec]
  [Black, Boyle, Di Carlo, Erben, Gülpers, Hansen, Lachini, Mukherjee, Portelli]

Can lattice QCD calculate the charming penguins?

- **Lattice-QCD** breakthrough via **Spectral Density reconstruction**
- Sketch of the mechanism:

$$H^\mu(q_0,\vec q) = i\int d^4x\, e^{iqx}\,\langle K(\vec p_K)\,|\,TJ^\mu_{\rm em}(x)O^c_{1,2}(0)\,|\,B(\vec 0)\rangle$$

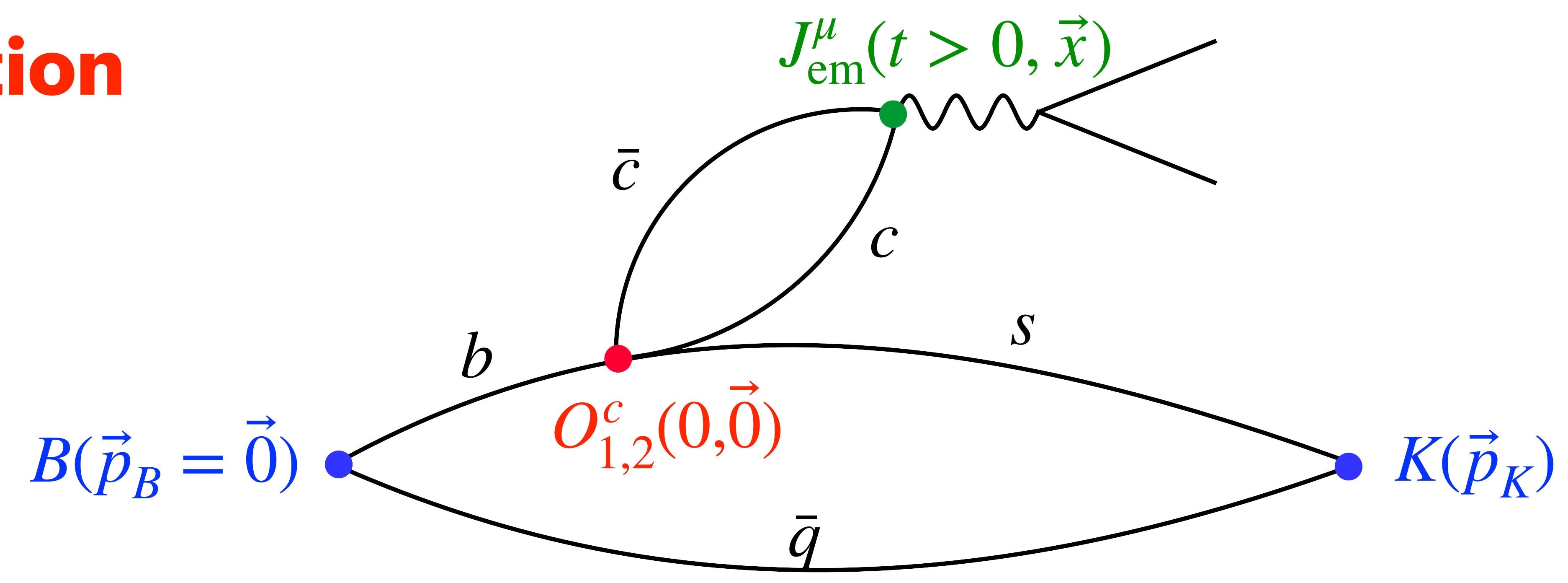
$$\overset{t>0}{=} i\int_0^\infty dt\, e^{iq_0 t}\underbrace{\int \frac{dE}{2\pi} e^{-i(E-E_K)t}\,\rho^\mu(E,\vec q)}_{\equiv C^\mu(t,\vec q)}$$

$$\overset{[m_B=q^0+E_K]}{=} \int_{E^*}^\infty \frac{dE}{2\pi}\lim_{\epsilon->0}\underbrace{\frac{1}{E-m_B-i\epsilon}}_{\simeq \sum_{n=1}^N g_n(\epsilon)e^{-anE}}\,\rho^\mu(E,\vec q)$$

$$= \lim_{\epsilon->0}\sum_{n=1}^N g_n(\epsilon)\,e^{-anE_K}\underbrace{\int_{E^*}^\infty\frac{dE}{2\pi}e^{-an(E-E_K)}\,\rho^\mu(E,\vec q)}_{C^\mu_{\rm euclid}(na,\vec q)}$$

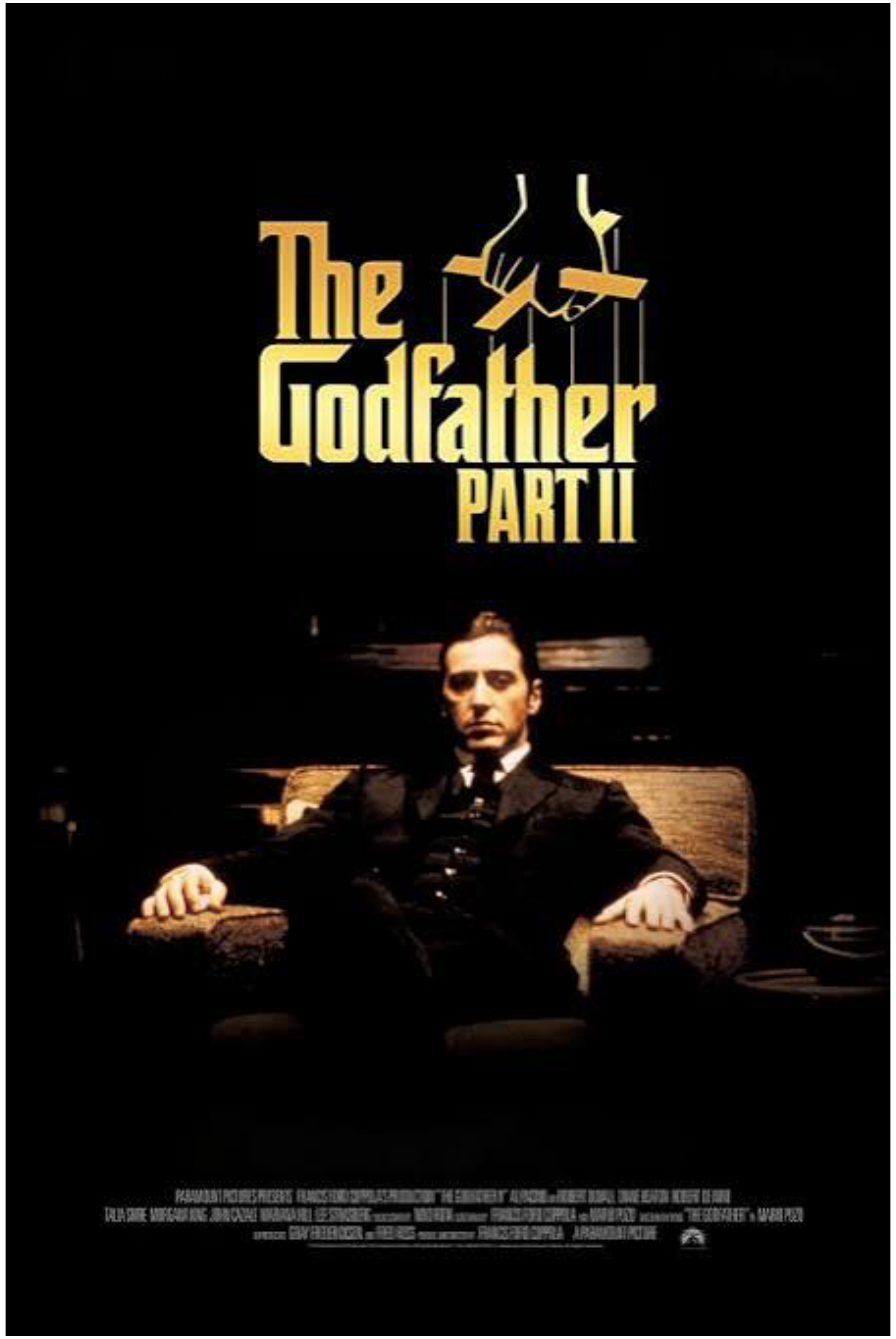
$$= \lim_{\epsilon->0}\sum_{n=1}^N g_n(\epsilon)\,e^{-anE_K}\,\boxed{C^\mu_{\rm euclid}(na,\vec q)}$$

from Lattice QCD



ϵ: smaller than the relevant spectral structures, but $\gtrsim 100$ MeV to control reconstruction/statistical errors

[Hansen, Meyer, Robaina 1704.08993]
[Hansen, Lupo, Tantalo 1903.06476]
[Bailas, Hashimoto, Ishikawa 2001.11779]
[Frezzotti et al. 2306.07228]
[Frezzotti et al. 2402.03262]
[Frezzotti et al. 2508.03655]

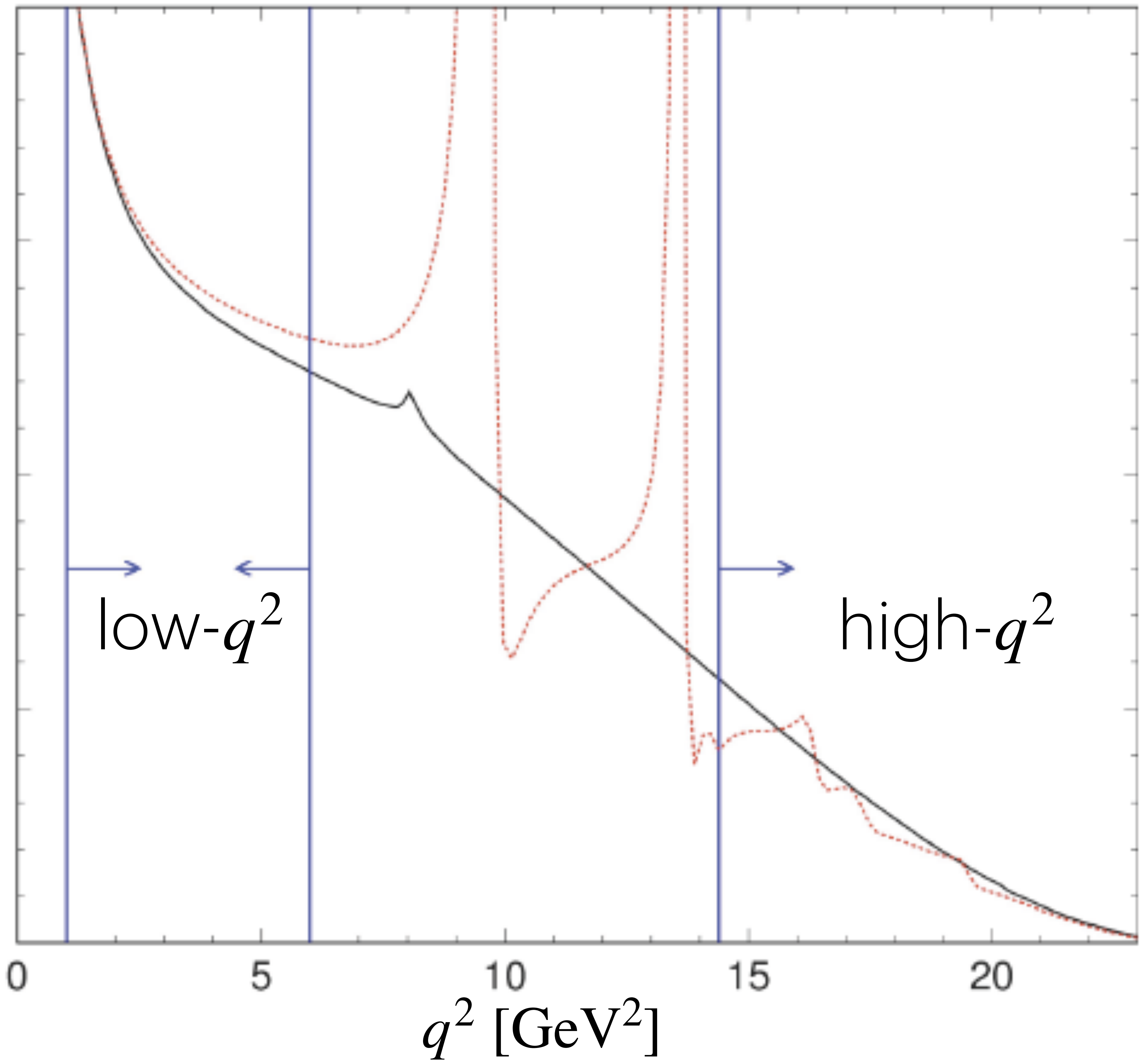


Part 2: The role of inclusive decays

Theory overview

Inclusive at low-q²

- OPE
- Parametric inputs under control
- Leading uncertainties: **resolved photon contributions** (SCET-I)



Inclusive at high-q²

- OPE breaks down but the ratio $b \to s\ell\ell / b \to u\ell\nu$ **with the same q² cut** is well behaved
- Leading uncertainties: $1/m_b^3$ HQET matrix elements (Weak annihilation)

Inclusive decays: theoretically clean observables

- At leading power the inclusive rate is free of hadronic uncertainties:

$$\Gamma[B \to X_s \ell\ell] = \Gamma[b \to X_s \ell\ell] + O\left(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}, \frac{\Lambda^2_{QCD}}{m_c^2} \cdots\right)$$

Low q^2

- The OPE works
- Nonfactorizable resolved contributions remain: they depend on poorly known subleading B-meson shape functions and lead to a residual uncertainty $\sim 5\,\%$ (resolved contributions)

$\mathscr{B}[1,6]_{\ell\ell} = (17.4 \pm 1.3) \times 10^{-7}$ **[7.4%]**

scale & resolved contributions

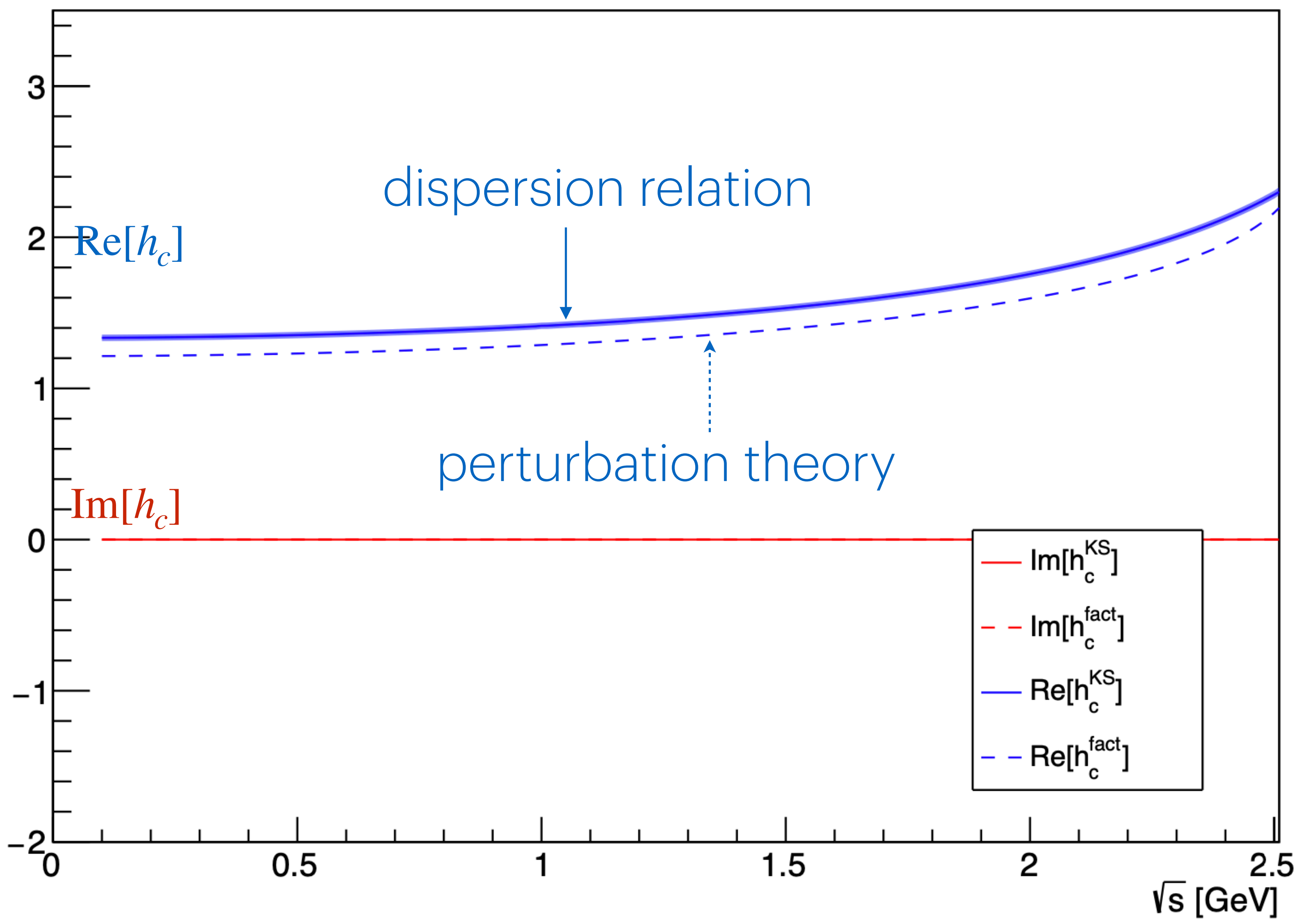
High q^2

- Ordinary OPE deteriorates near the endpoint of the spectrum

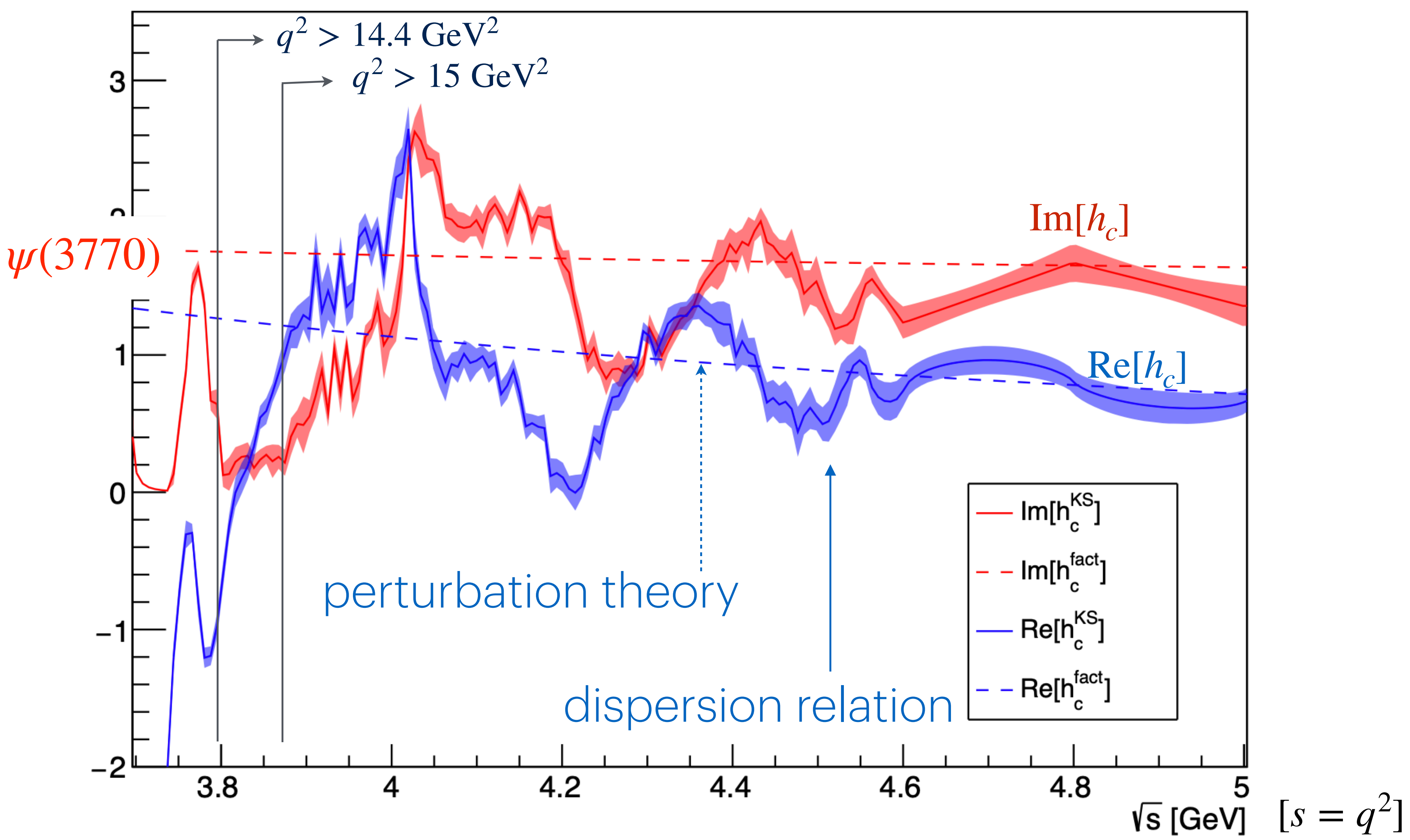
Charm effects in inclusive decays

- We use $e^+e^- \to$ hadrons data to describe **exactly** the factorizable charming penguin effects:

Low-q^2



High-q^2



- Factorizable effects are small ($\simeq 2\,\%$)
- Nonfactorizable effects are described in terms of subleading B meson shape functions and are $\sim 5\,\%$ (resolved contributions)

- Factorizable effects are larger ($\simeq -10\,\%$)
- Nonfactorizable effects are local $\sim \Lambda^2_{\rm QCD}/m_b^2$

Charm effects in inclusive decays

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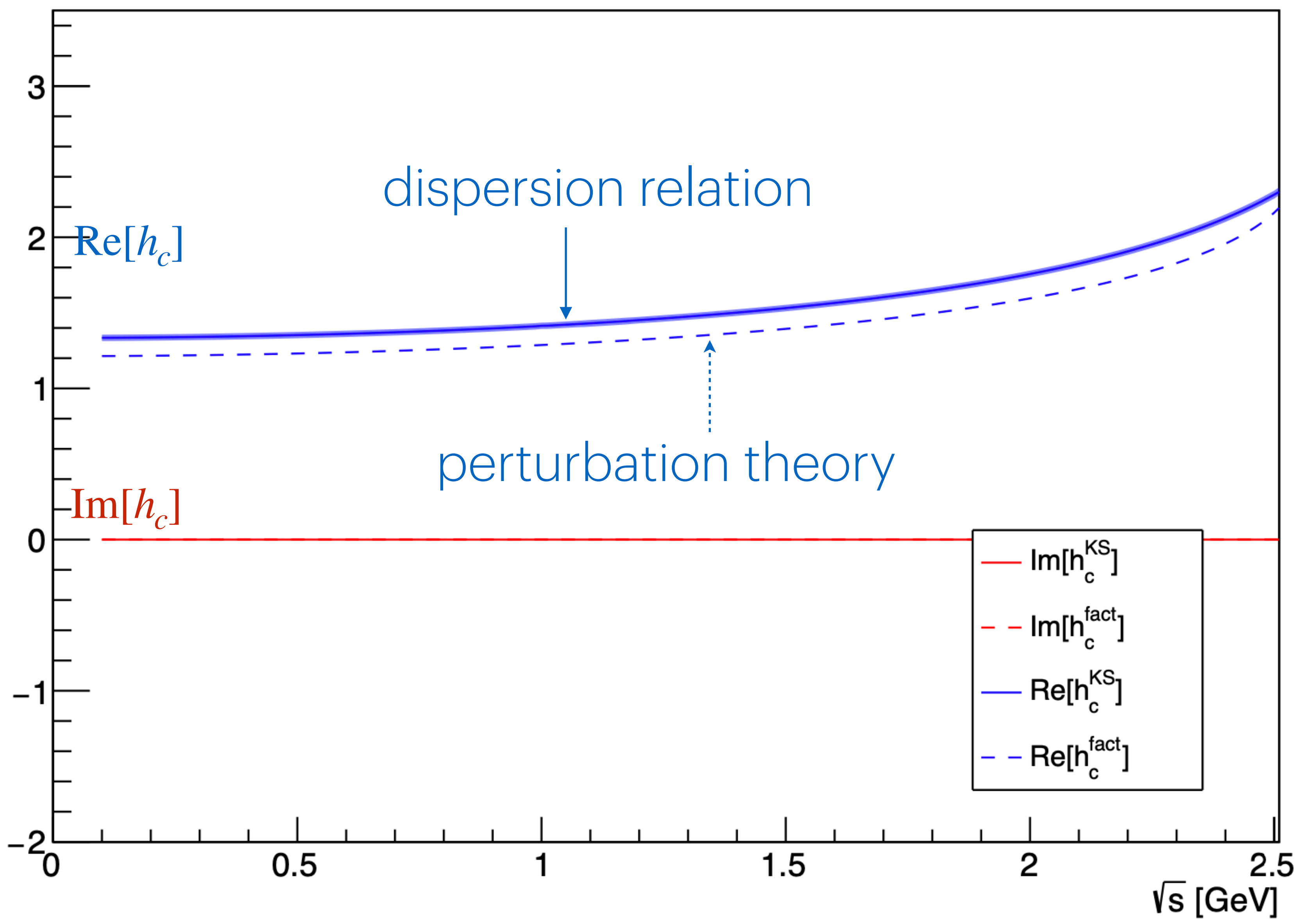
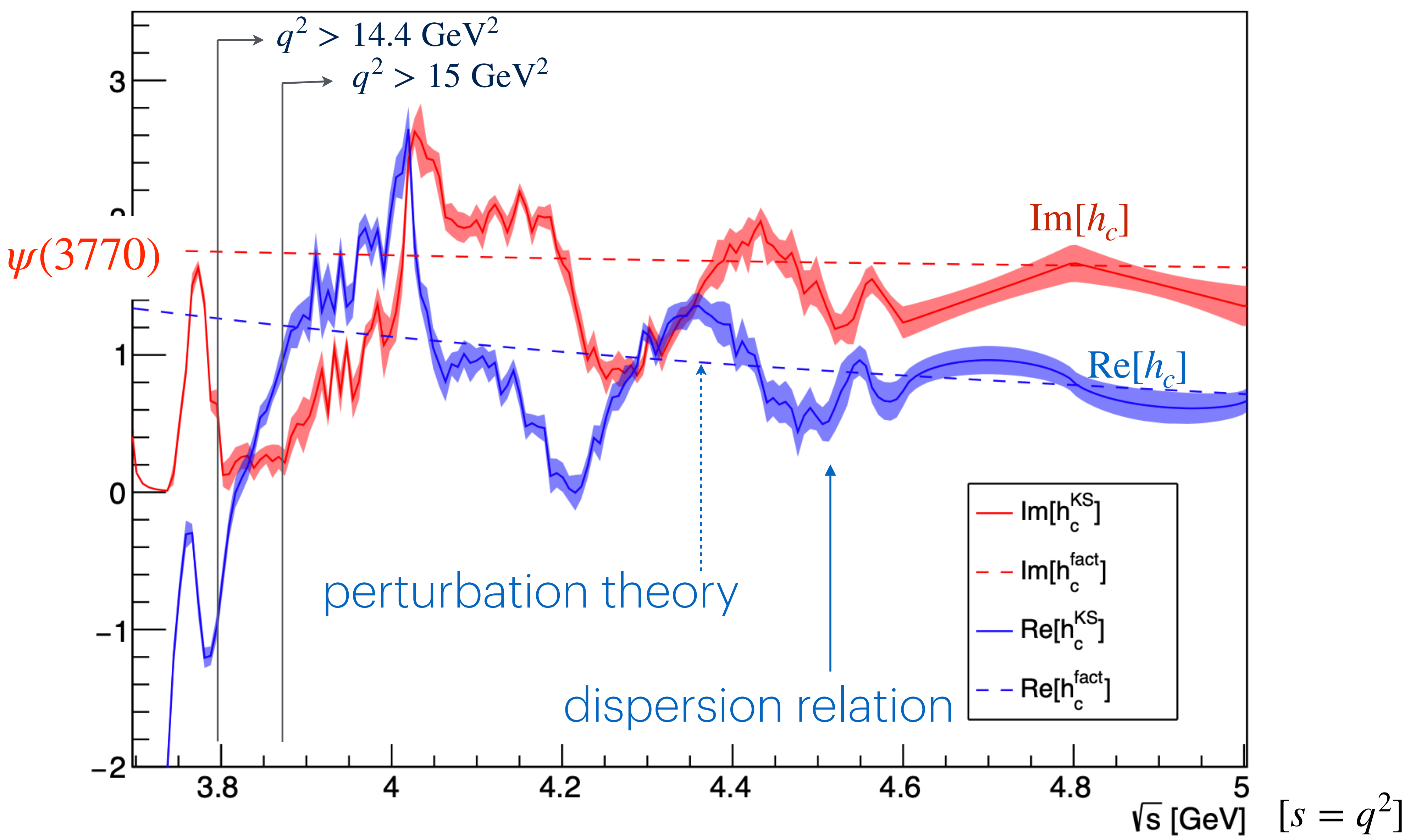
Low-q^2



High-q^2



- Factorizable effects are small ($\simeq 2\,\%$)
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- Factorizable effects are larger ($\simeq -10\,\%$)
- Nonfactorizable effects are local $\sim \Lambda_{\rm QCD}^2/m_b^2$

These effects correspond to the problematic nonlocal power corrections in exclusive decays

Inclusive at high-q^2: restoring theoretical control

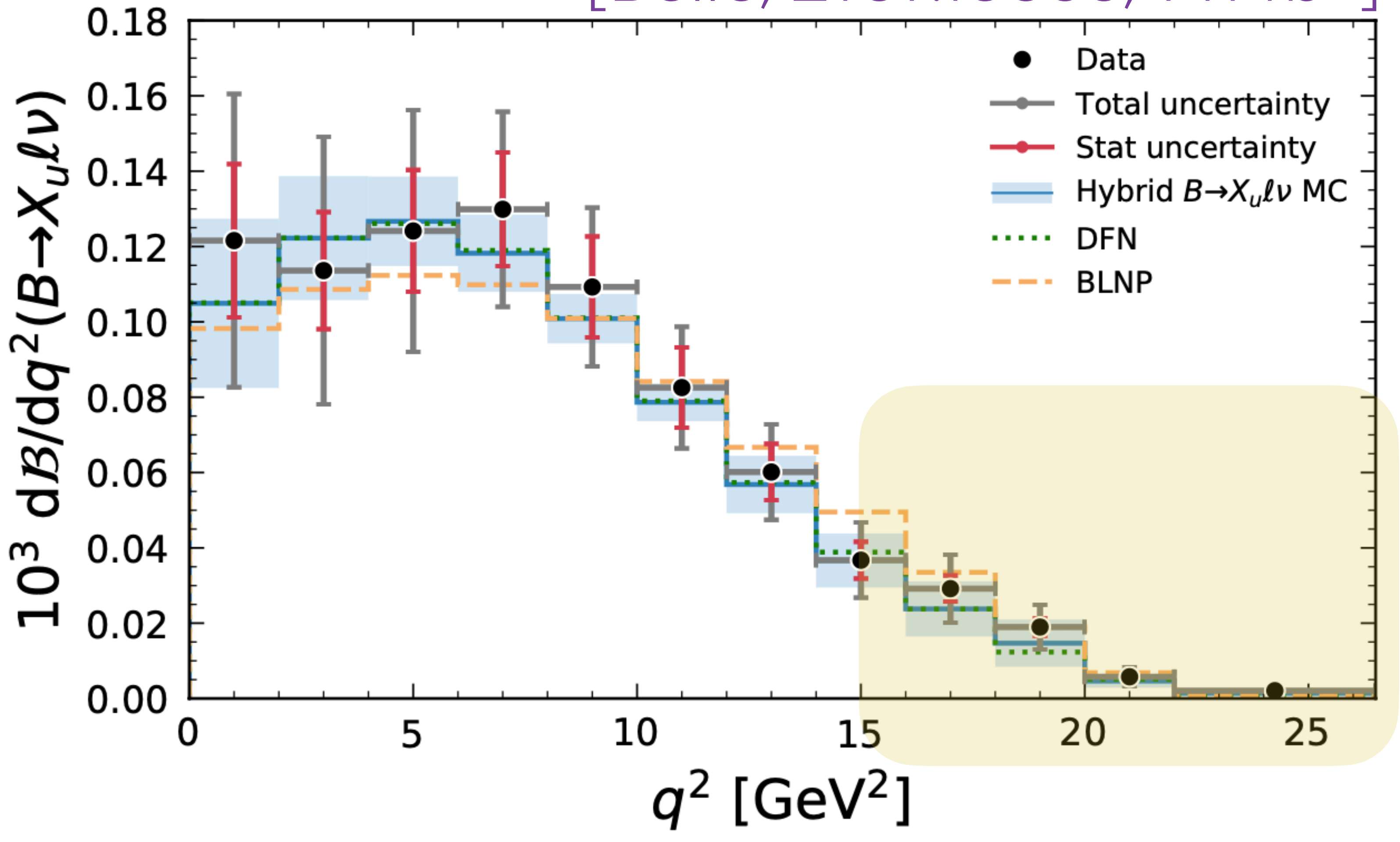
- At high-q^2, the OPE for the branching ratio breaks down, but the same occurs in $B \to X_u \ell \nu$
- The following ratio is found to have a well behaved OPE:

$$\mathcal{R}(q_0^2) = \frac{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{dq^2}}{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B} \to X_u \ell \nu)}{dq^2}} \xrightarrow{\text{SM prediction}} \mathcal{R}(15) = (17.00 \pm 1.9) \times 10^{-4} \quad \textbf{[7.2\%]}$$

V_{ub} & local power corrections

- Using the Belle measurement: $\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})[>15]_{\rm exp} = (1.52 \pm 0.28) \times 10^{-4}$ **[18.4%]**

[Belle, 2107.13855, 711 fb^{-1}]



$$\mathcal{B}[>15]_{{\rm SM},\mathcal{R}} = \mathcal{R}(15)_{\rm SM} \times \mathcal{B}(B \to X_u \ell \bar{\nu})[>15]_{\rm exp}$$
$$= (4.10 \pm 0.81) \times 10^{-7} \quad [19.8\%]$$

dominated by the $B \to X_u \ell \nu$ partial rate

Theory is no longer the bottleneck: $B \to X_u \ell \nu$ is

Inclusive at high-q² from LHCb

- For $q^2 > 15\ \mathrm{GeV}^2$ the inclusive rate is saturated by $X_s = K,\ K^*,\ (K\pi)_{\text{S-wave}},\ K\pi\pi$ modes
- This allows for a measurement of the inclusive at high-q^2 using LHCb measurements
  [Isidori, Polonsky, Tinari, 2305.03076; Huber, Hurth, Jenkins, EL, Qin, Vos, 2404.03517]
- All these modes have been measured:

$K + K^*$: $(2.628 \pm 0.094) \times 10^{-7}$

$(K\pi)_{\text{S-wave}}$: $(0.11 \pm 0.04) \times 10^{-7}$

$K\pi\pi$: $(0.06 \pm 0.04) \times 10^{-7}$ (using isospin)

$K(n\pi)_{n>2}$: $(0.00 \pm 0.04) \times 10^{-7}$ (estimate)

[LHCb 1403.8044 (3 fb^{-1})]
[LHCb 1408.1137 (3 fb^{-1})]
[LHCb 2512.18053 (8.4 fb^{-1})]
[CMS 2401.07090]

$$\mathscr{B}[>15]_{\text{exp}} = (2.80 \pm 0.12) \times 10^{-7}$$

[Capdevila, Alvarez, EL, Matias, 2605.06567]

Inclusive at high-q²: SM prediction as sum of exclusives

- The saturation of the high-q^2 rate by $X_s = K,\ K^*,\ (K\pi)_{\text{s-wave}},\ K\pi\pi$ modes suggests the possibility of obtaining a SM prediction as sum of exclusive modes

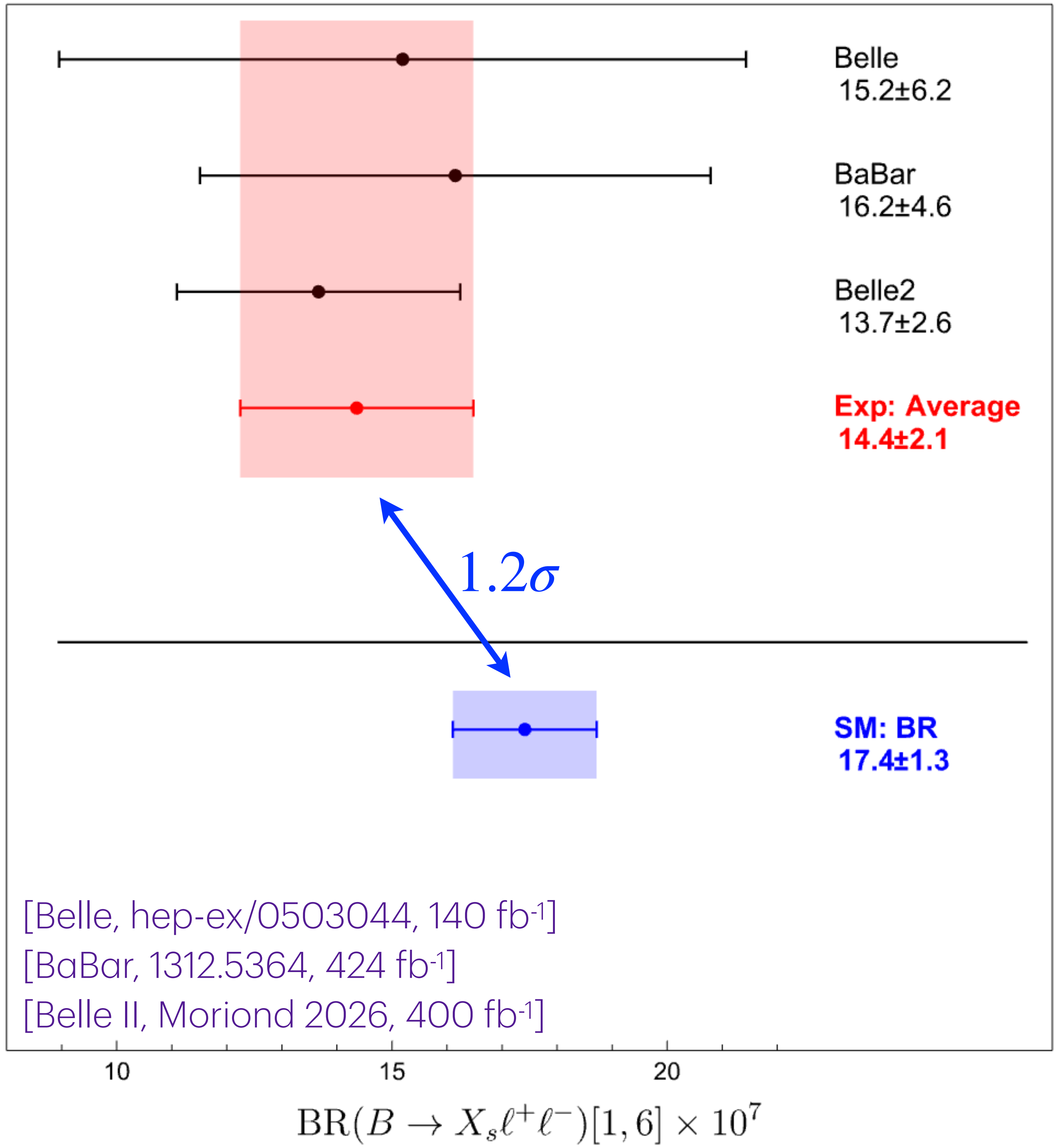
- The dominant contribution comes from $K + K^*$: $(3.75 \pm 0.28) \times 10^{-7}$ [Alguero et al. 2304.07330]

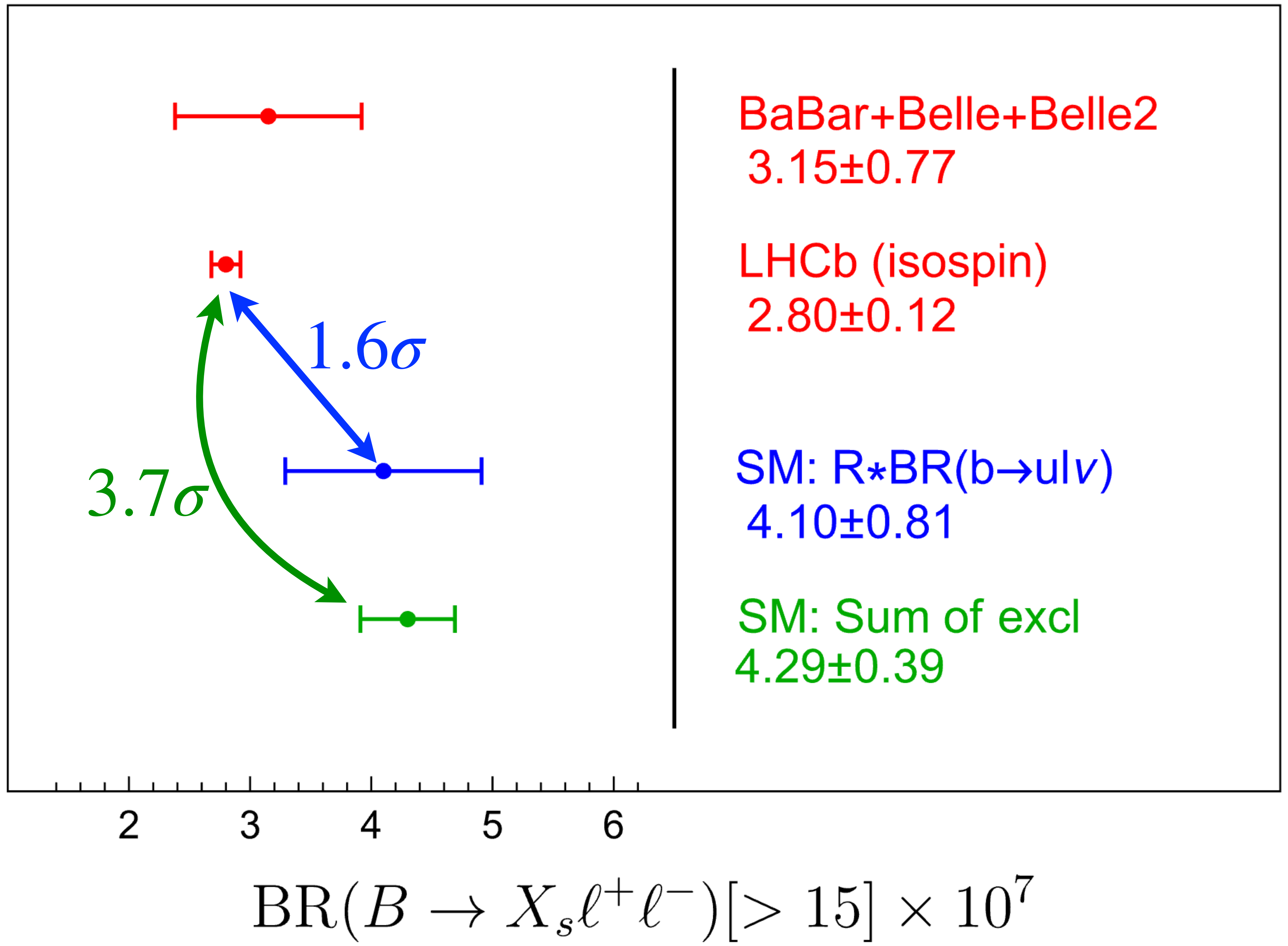
- The $K\pi$ contribution can be estimated using Heavy Hadron ChiPT: $(0.55 \pm 0.23) \times 10^{-7}$ [Isidori et al, 2305.03076]
  For comparison, the total nonresonant determination is $(0.176 \pm 0.070) \times 10^{-7}$

- The total SM prediction as sum of exclusive reads:

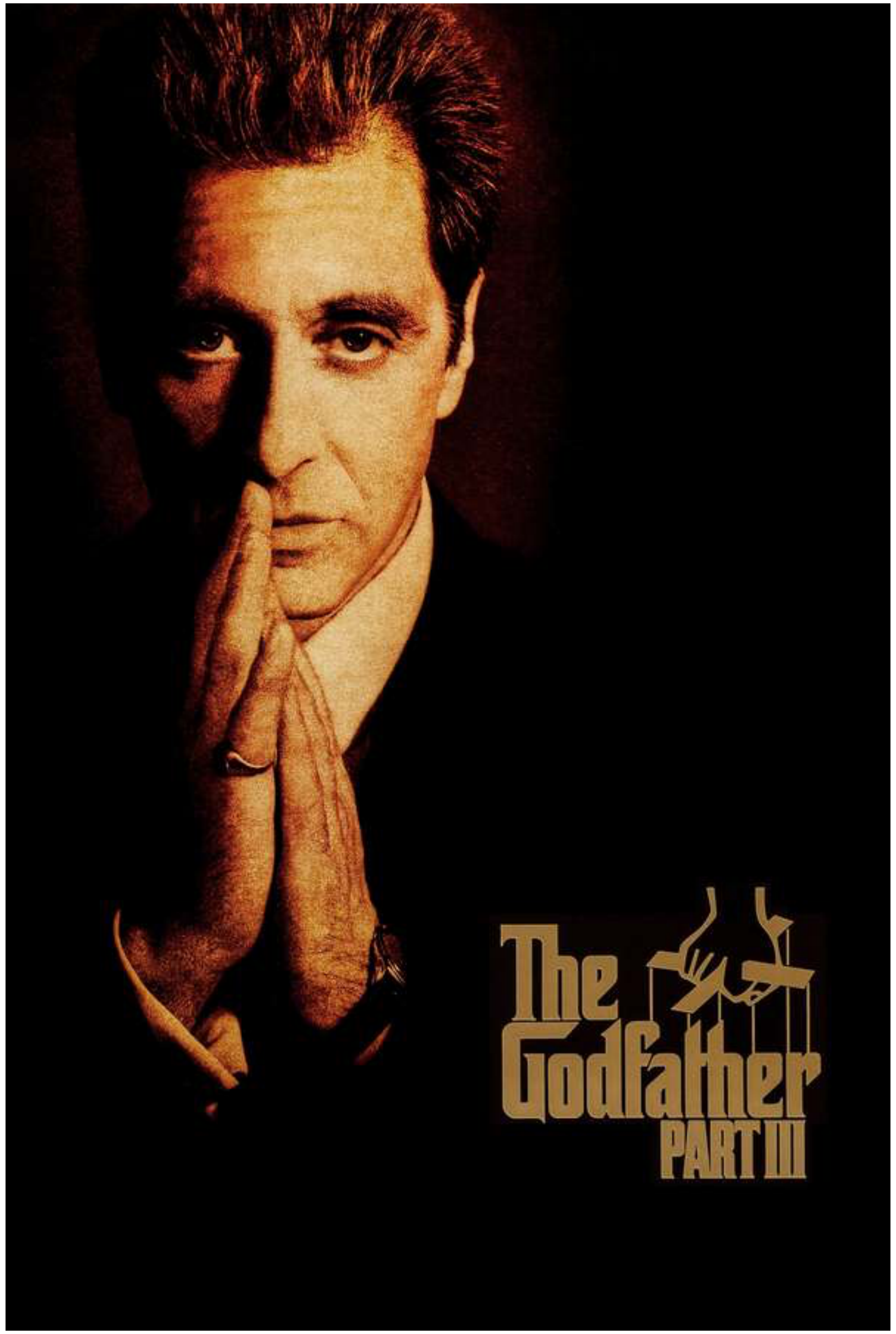
$$\mathscr{B}[>15]_{\mathrm{SM},\sum \mathrm{excl}} = (4.30 \pm 0.39) \times 10^{-7}$$

Current status: experiment vs SM





- The inclusive measurements at both low and high q^2 show a slight deficit relative to the fully inclusive SM predictions, which are independent of the exclusive calculations



Part 3: A new strategy

Exclusive/inclusive ratios as a charm diagnostic

- While exclusive modes could be affected by large $c\bar{c}$-loop effects, this is not the case for inclusive modes:

	$K^{(*)}\mu\mu$	$X_s\mu\mu$
C_9^{NP}	✓	✓
charming penguins	✓	no corresponding enhancement

- This suggests the introduction of the following "exclusive fraction" observables:

$$\mathscr{F}_{K^{(*)}}^{[1.1,6]} = \frac{\mathscr{B}(B \to K^{(*)}\mu\mu)_{[1.1,6]}}{\mathscr{B}(B \to X_s\ell\ell)_{[1,6]}} \qquad \mathscr{F}_{K^{(*)}}^{>15} = \frac{\mathscr{B}(B \to K^{(*)}\mu\mu)_{>15}}{\mathscr{B}(B \to X_s\ell\ell)_{>15}}$$

- **Universal short-distance NP largely cancels; exclusive-specific rescattering does not**
- **Can be measured by LHCb without need of external normalizations**

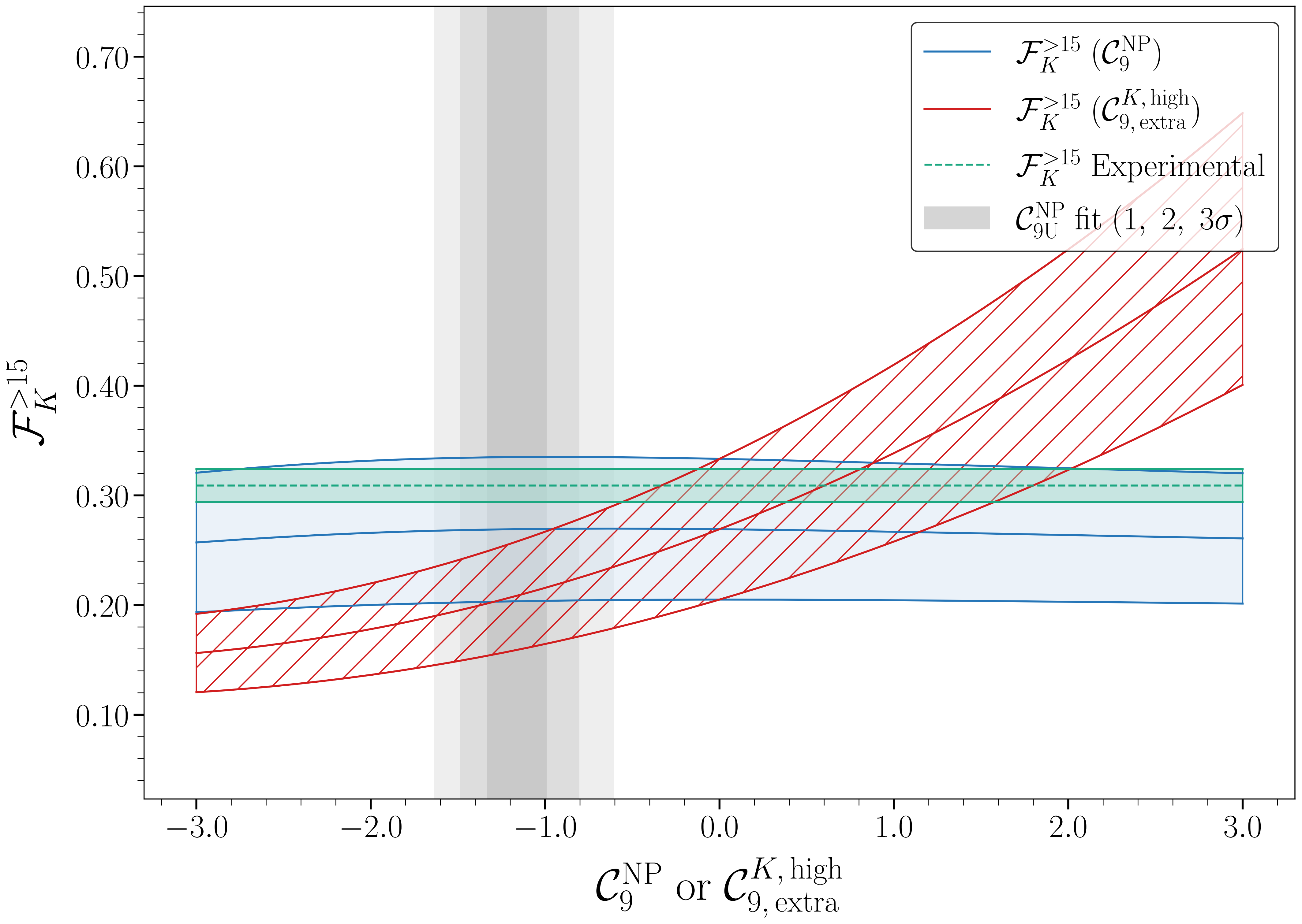
- Dependence on the NP and $c\bar{c}$-resonant contributions to C_9 :

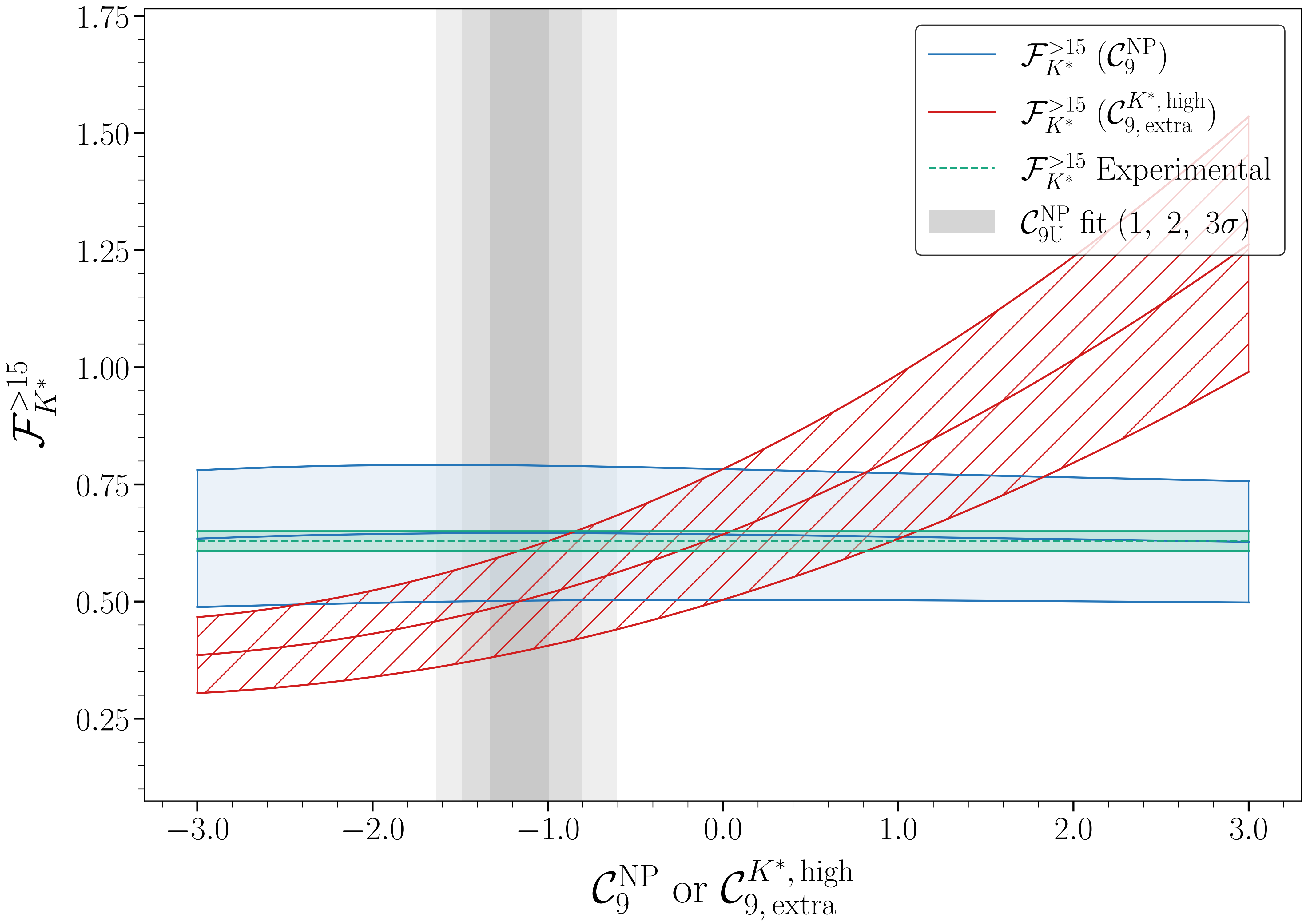
$$\mathscr{F}_K^{>15} = (0.270 \pm 0.064) \cdot \frac{1 + 0.228\,(C_9^{\mathrm{NP}} + C_{9,\mathrm{res}}^K) + 0.0325\,(C_9^{\mathrm{NP}} + C_{9,\mathrm{res}}^K)^2}{1 + 0.234\,C_9^{\mathrm{NP}} + 0.0293\,C_9^{\mathrm{NP}2}}$$

Exclusive fractions: high-q^2

- At high-q^2, the exclusive fractions already discriminate between the two scenarios

[Capdevila, Alvarez, EL, Matias, 2605.06567]





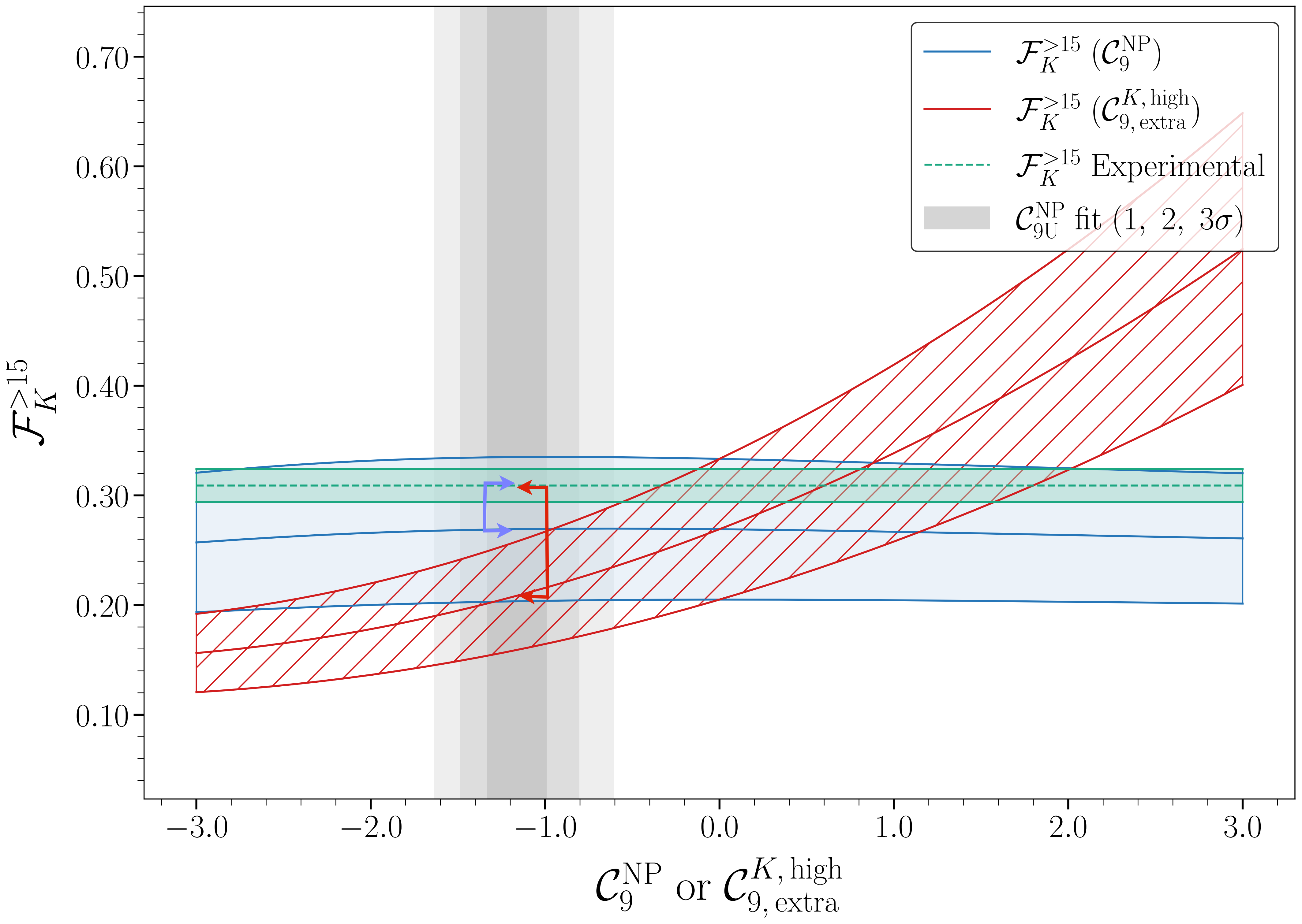
$$\mathcal{F}_K^{>15} = \begin{cases} 0.309(15) & \text{LHCb} \\ 0.271(64) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (-1.16, 0) \\ 0.210(50) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (0, -1.16) \end{cases}$$

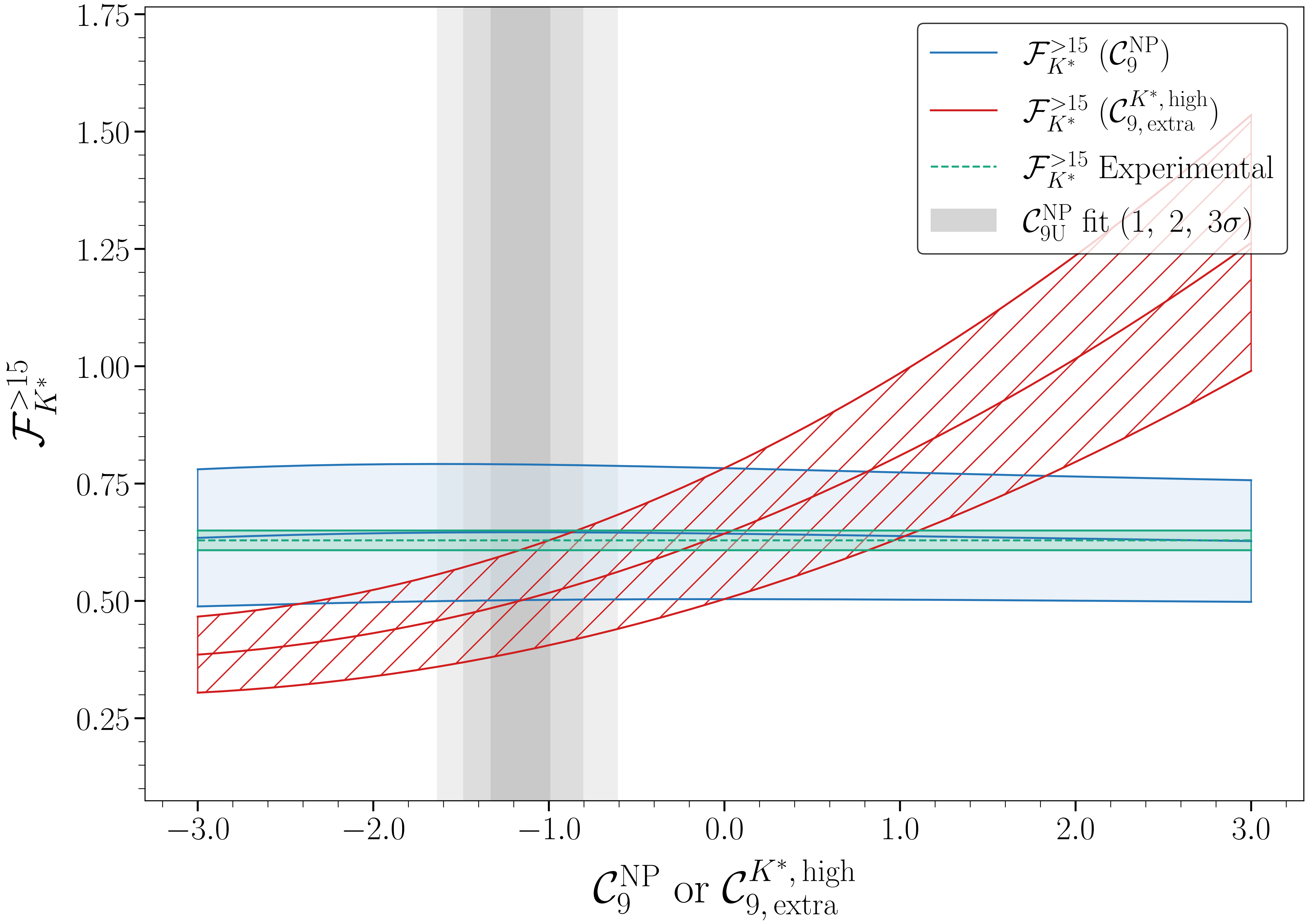
$$\mathcal{F}_{K^*}^{>15} = \begin{cases} 0.629(21) & \text{LHCb} \\ 0.65(14) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (-1.16, 0) \\ 0.50(11) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (0, -1.16) \end{cases}$$

Exclusive fractions: high-q^2

- At high-q^2, the exclusive fractions already discriminate between the two scenarios

[Capdevila, Alvarez, EL, Matias, 2605.06567]





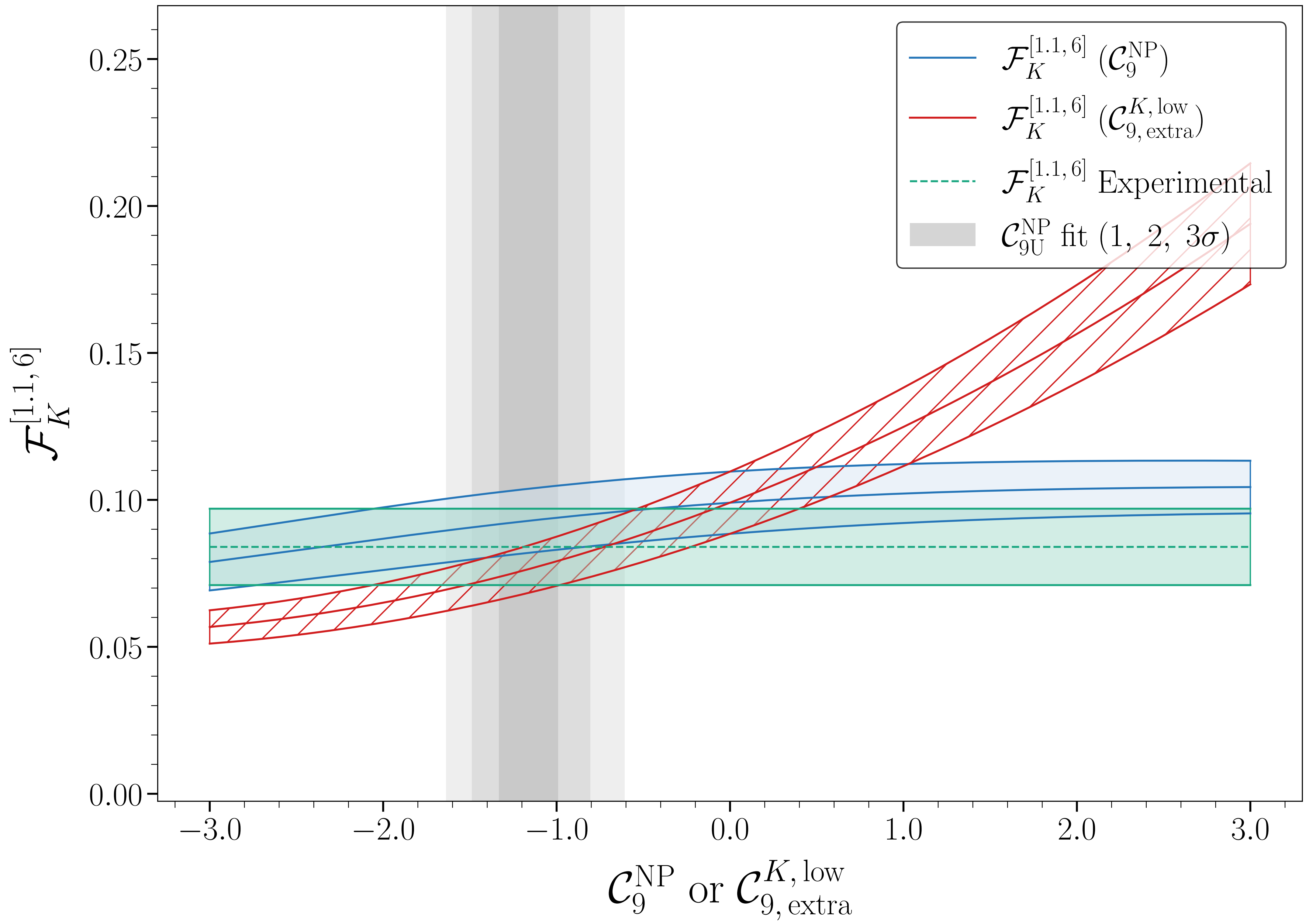
$$\mathcal{F}_K^{>15} = \begin{cases} 0.309(15) & \text{LHCb} \\ 0.271(64) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (-1.16, 0) \\ 0.210(50) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (0, -1.16) \end{cases} \quad 0.6\sigma \quad 1.9\sigma$$

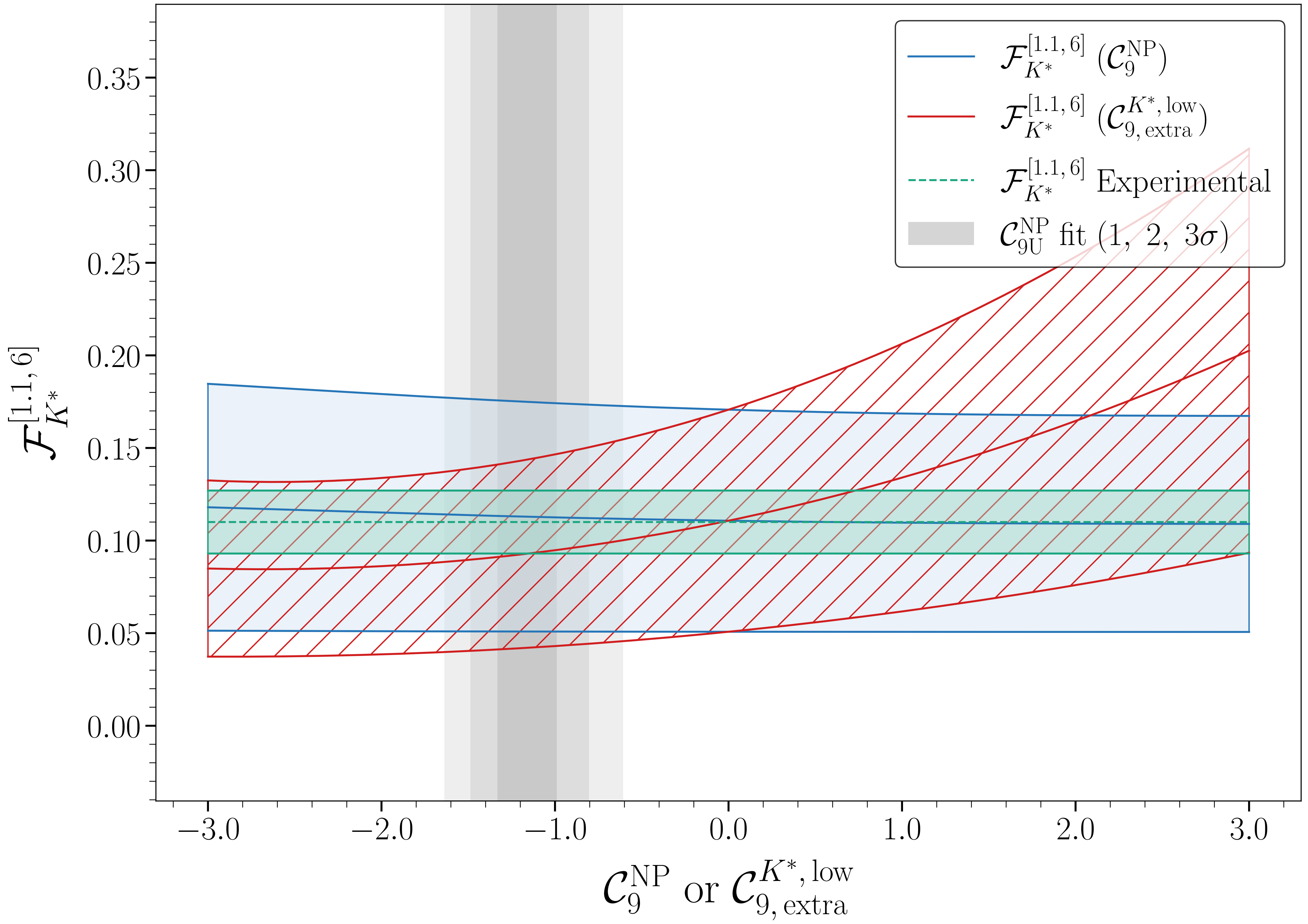
$$\mathcal{F}_{K^*}^{>15} = \begin{cases} 0.629(21) & \text{LHCb} \\ 0.65(14) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (-1.16, 0) \\ 0.50(11) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (0, -1.16) \end{cases}$$

Exclusive fractions: low-q^2

- At low-q^2, current uncertainties are still large; the discrimination is limited

[Capdevila, Alvarez, EL, Matias, 2605.06567]





$$\mathcal{F}_K^{[1.1,6]} = \begin{cases} 0.084(13) & \text{B-factories, LHCb} \\ 0.0928(98) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (-1.16, 0) \\ 0.0764(81) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (0, -1.16) \end{cases}$$

$$\mathcal{F}_{K^*}^{[1.1,6]} = \begin{cases} 0.110(17) & \text{B-factories, LHCb} \\ 0.112(62) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (-1.16, 0) \\ 0.092(51) & (\mathcal{C}_9^{\rm NP}, \mathcal{C}_9^{\rm cc}) = (0, -1.16) \end{cases}$$

Future expectations

- Without updated $B \to J/\psi K^{(*)}$ measurements from Belle II, all exclusive branching ratios will be eventually dominated by the normalizations. In this scenario we find:

$$\delta\mathcal{F}_K^{>15} = 4.9\,\% \longrightarrow 3.1\,\%$$

$$\delta\mathcal{F}_{K^*}^{>15} = 3.3\,\% \longrightarrow 2.1\,\%$$

- LHCb can determine these ratios **without making use of normalizations!**

- Future precision is driven by Belle II $B \to X_u \ell \nu$, not by the $J/\psi K^{(*)}$ normalization uncertainty that will eventually limit absolute exclusive branching fractions.

Summary

- Exclusive $b \to s\mu\mu$ data show a large, coherent deviation from the SM — unless nonlocal hadronic effects are much larger than expected:

$$\delta C_9 \sim -1 \quad \Longrightarrow \quad \text{New Physics or Nonlocal charm loops?}$$

Lattice QCD

Directly test whether nonlocal charm can be large enough

$B \to K^*$ form factors are becoming accessible.

Spectral reconstruction may allow a first-principles calculation of nonlocal $c\bar{c}$ loops.

Inclusive Decays

The deficit does not disappear when the hadronic environment changes

Same short-distance physics ($C_{7,9,10}$), different nonperturbative uncertainties.

Low and high-q^2 data currently also tend to lie below the SM.

Exclusive/Inclusive Ratios

Universal $C_9^{\rm NP}$ largely cancels; exclusive rescattering does not

Already measurable precisely at LHCb.

- What can settle this in the next $\lesssim 5$ years?
  - **LHCb**: high-q^2 inclusive reconstruction + precise $\mathscr{F}$ ratios
  - **Belle II**: $B \to X_s \ell\ell$, $B \to X_u \ell\nu$ and normalization modes
  - **Lattice QCD**: $B \to K^*$ form factors + nonlocal charm effects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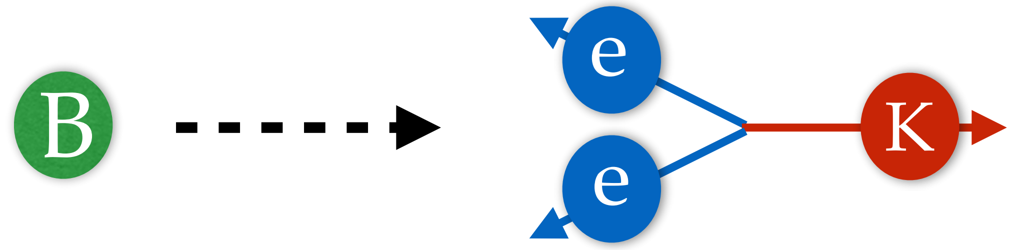
General framework for exclusive decays

- The central problem is the calculation of matrix elements of the type: $\langle K^{(*)}\ell\ell \,|\, O(y) \,|\, B\rangle \approx \langle K^{(*)} \,|\, T\, J^{\rm em}_\mu(x)\; O(y) \,|\, B\rangle$
- If $O(y)$ contains a lepton pair this is trivial: $\langle K^{(*)}\ell\ell \,|\, (\bar s_L \gamma^\mu b_L)(\ell\gamma_\mu \ell) \,|\, B\rangle \propto \langle K^{(*)} \,|\, \bar s_L \gamma^\mu b_L \,|\, B\rangle \sim$ Form Factors
- At low-q² the large energy of the K(*) introduces three scales: m_b^2, $\Lambda_{\rm QCD} m_b$ and $\Lambda_{\rm QCD}^2$:

$$\langle K^{(*)} \,|\, T J^{\rm em}_\mu(x) O(y) \,|\, B\rangle \sim C \times [\,\text{Form Factor} + \phi_B \star J \star \phi_{K^{(*)}}] + O\left(\Lambda_{\rm QCD}/m_b\right)$$

lattice QCD +
Light Cone Sum Rules

nonlocal power corrections

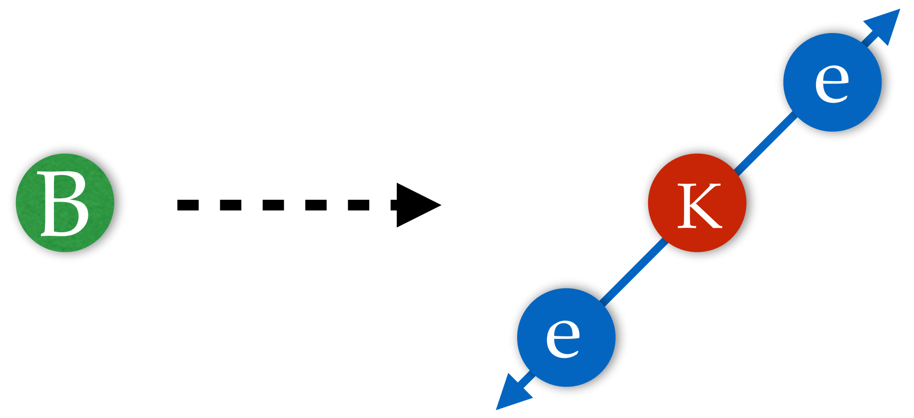


Soft-Collinear Effective Theory

- At high-q² the K(*) does not recoil, $(x-y)^2 \sim 1/q^2 \sim 1/m_b^2$:

$$\langle K^{(*)} \,|\, T J^{\rm em}_\mu(x) O(y) \,|\, B\rangle \sim C \times [\,\text{Form Factor}] + O\left(\Lambda/m_b\right)$$

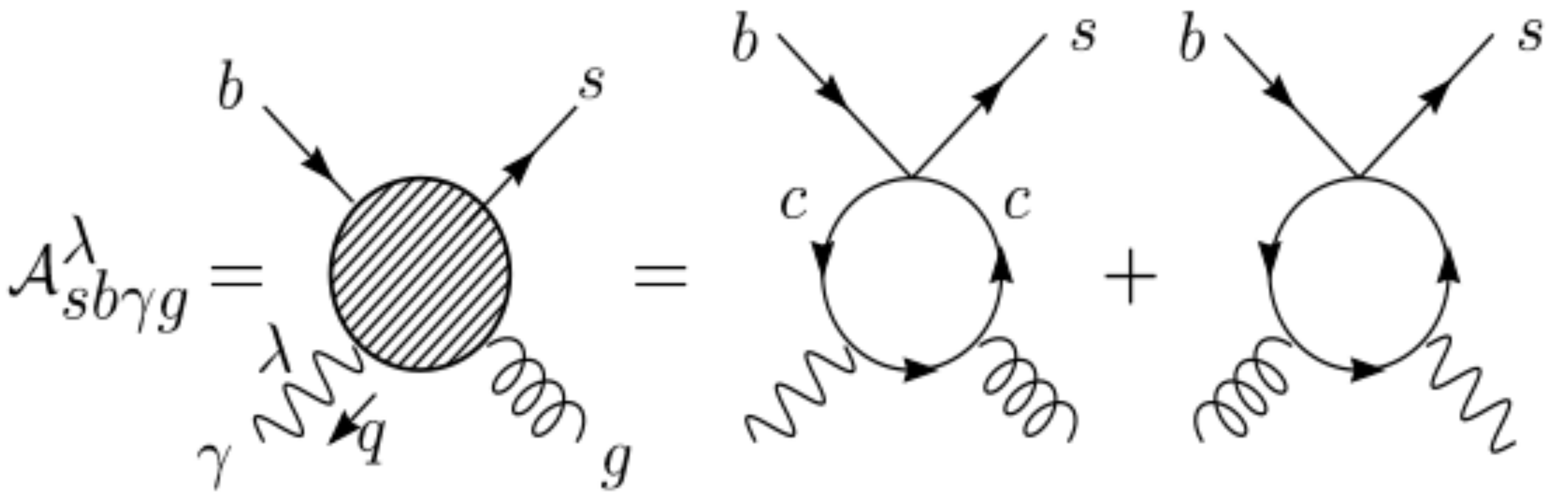
local power corrections



$1/q^2$ expansion

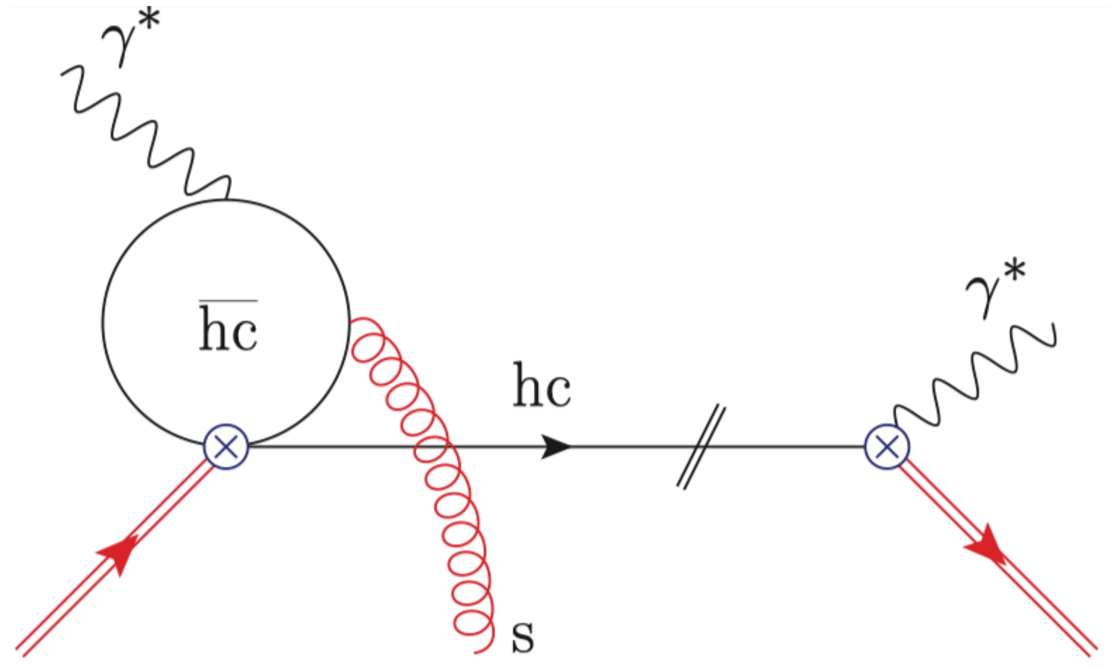
$B \to X_s \ell\ell$: resonances

- Non-factorizable effects at **high-q²** can be calculated in perturbation theory for the integrated rate, they are **local** and scale as $\Lambda_{\rm QCD}^2/q^2$ [Buchalla, Isidori, Rey]:



$$A^\lambda_{sb\gamma g} = \ldots \Longrightarrow \frac{\langle B|\bar b \sigma_{\mu\nu} G^{\mu\nu} b|B\rangle}{m_c^2}\left(-6\frac{m_c^2}{q^2}\right) \sim \frac{\lambda_2}{q^2}$$

- Non-factorizable effects at **low-q²** and with a cut on m_X, are **nonlocal** and can treated using SCET-I [Hurth, Benzke, Fickinger, Turczyk]:



  - Power corrections remain non-local after m_X cut is released
    $\Rightarrow$ so-called resolved contributions
  - Depend on mostly unknown subleading B shape functions
  - A rough estimate yields an irreducible uncertainty of about 5%

Inclusive: theory summary

issues

- **NNLO$_{QCD}$ + NLO$_{QED}$**
- $c\bar{c}$ **resonances**: included using e^+e^- data via a dispersion relation (Krüger-Sehgal mechansim)
- **QED radiation**: soft/collinear photons treatment at B-factories and LHCb needs to be taken into account
- m_X **cuts**: at low-q² removal of double semileptonic background, $b \to (c \to s\ell\nu)\ell\nu$, requires extrapolation in m_X

errors

- Dominant uncertainties:
  - **Low-q²** ⇒ **resolved photon contributions** (irreducible 5%) **and scale** (N³LO)
  - **High-q² (BR)** ⇒ **power corrections** (OPE breakdown)
  - **High-q² ($b \to s\ell\ell / b \to u\ell\nu$ ratio)** ⇒ **parametric** (CKM) and **power corrections** (5x smaller than in BR!)

results

$$\mathcal{B}[1,6]_{\mu\mu} = 17.29\,(1 \pm 4.4\%_{\rm scale} \pm 1.1\%_{m_t} \pm 2.3\%_{C,m_c} \pm 1.2\%_{m_b} \pm 0.5\%_{\alpha_s} \pm 0.1\%_{\rm CKM} \pm 1.5\%_{{\rm BR}_{\rm sl}} \underbrace{\pm 0.7\%_{\lambda_2} \pm 5\%_{\rm resolved}}_{5.1\,\%}) \times 10^{-7}$$
$$= (17.29 \pm 1.28) \times 10^{-7} \quad [7.4\%]$$

$$\mathcal{B}[>15]_{\rm no\ QED} = 2.59\,(1 \pm 8.1\%_{\rm scale} \pm 1.2\%_{m_t} \pm 1.9\%_{C,m_c} \pm 7.3\%_{m_b} \pm 0.2\%_{\alpha_s} \pm 0.08\%_{\rm CKM} \pm 1.5\%_{{\rm BR}_{\rm sl}} \underbrace{\pm 3.9\%_{\lambda_2} \pm 10\%_{\rho_1} \pm 21\%_{f_{u,s}}}_{23.5\,\%}) \times 10^{-7}$$
$$= (2.59 \pm 0.68) \times 10^{-7} \quad [26\%]$$

$$\mathcal{R}(15)_{\rm no\ QED} = 27.00\,(1 \pm 0.93\%_{\rm scale} \pm 1.1\%_{m_t} \pm 0.41\%_{C,m_c} \pm 0.63\%_{m_b} \pm 0.56\%_{\alpha_s} \pm 4.3\%_{\rm CKM} \underbrace{\pm 0.26\%_{\lambda_2} \pm 1.4\%_{\rho_1} \pm 5.3\%_{f_{u,s}}}_{5.5\,\%}) \times 10^{-4}$$
$$= (27.0 \pm 1.9) \times 10^{-4} \quad [7.2\%]$$

4x reduction

$B \to X_s \ell\ell$ measurement at high-q^2 as sum of exclusive

- $B \to \left[K + K^* + (K\pi)_{\text{S-wave}} + K\pi\pi\right] \mu^+\mu^-$ saturate $B \to X_s\mu^+\mu^-$ for $q^2 > 15\ \text{GeV}^2$
  This allows for a measurement of the inclusive at high-q^2 using LHCb measurements
  [Isidori, Polonsky, Tinari, 2305.03076; Huber, Hurth, Jenkins, EL, Qin, Vos, 2404.03517]

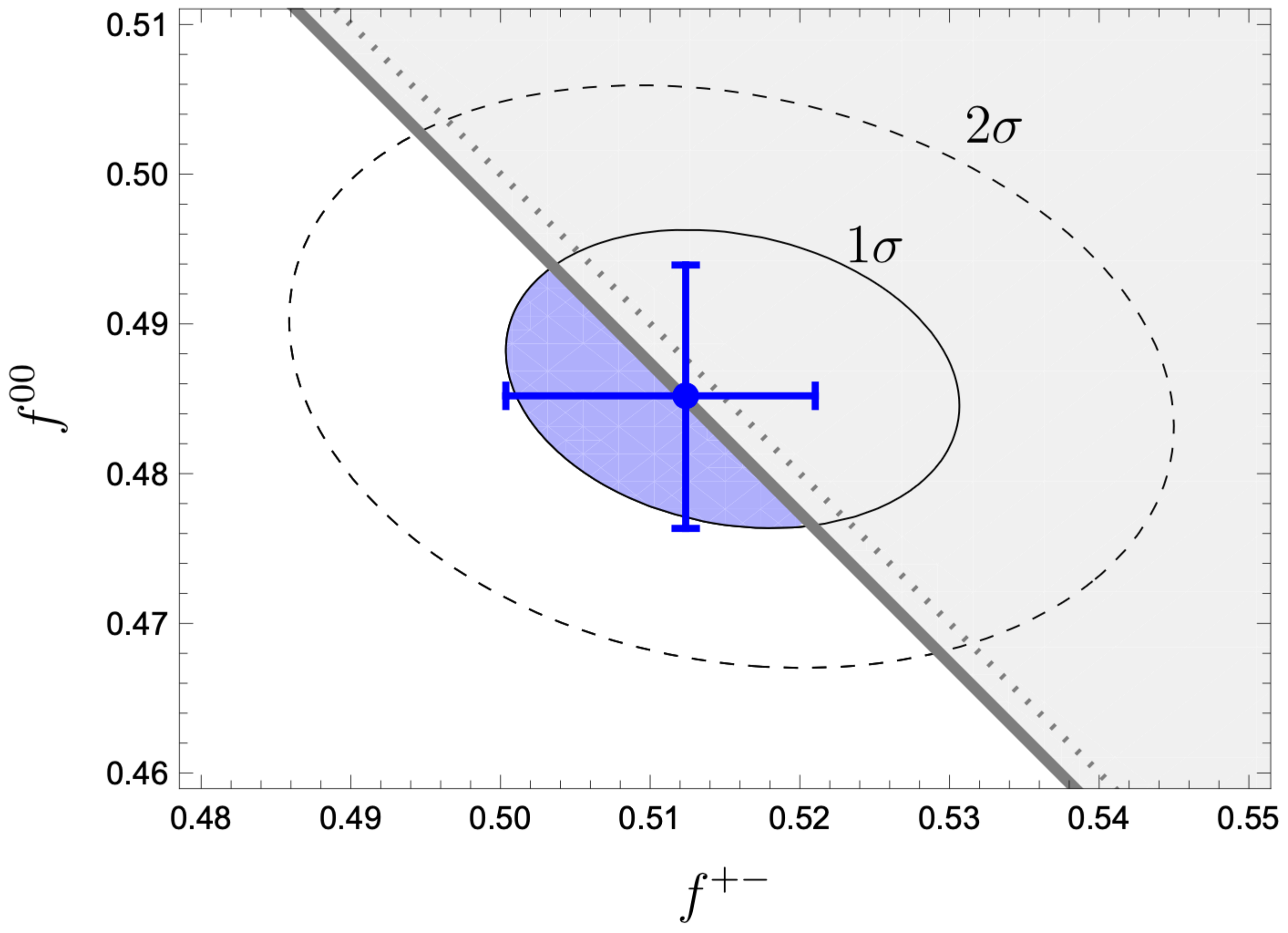
- Limitations to phenomenological uses of this measurement are
  - branching ratios of charged an neutral $B \to J/\psi K^{(*)}$ (used as normalizations by LHCb)
  - $B^0 \to X_u \ell\nu$ branching ratio for $q^2 > 15\ \text{GeV}^2$ (required for a clean theoretical prediction for $B \to X_s\mu^+\mu^-$)

  $\Rightarrow$ both normalizations will be measured with improved accuracy at Belle II

- In the following we present
  - The combination of all existing LHCb and CMS measurements to produce the inclusive at high-q^2
  - The comparison between this experimental determination and two SM predictions
  - New observables that can help disentangle Non Perturbative charm contributions from New Physics ones

Impact of $\mathscr{B}(\Upsilon(4S) \to B^+B^-) \neq \mathscr{B}(\Upsilon(4S) \to B^0\bar{B}^0) \neq 1/2$

- Almost all but few of the most recent Belle measurements assume $f^{+-} = f^{00} = 1/2$
- This issues have been recently addressed in:
  [Bernlochner, Jung, Khan, Landsberg, Ligeti 2306.04686]
  [Jung, Schacht 2604.08391]
  [Belle II, Michele Mantovano @ Challenges in Semileptonic B decays 2026, Bad Honnef]
- Currently PDG and HFLAV ignore this problem and simply average all measurements together (irrespectively to whether the $f^{+-} = f^{00} = 1/2$ assumption was or was not made (!)



$$\begin{cases} f^{+-} = 0.511 \pm 0.010 \\ f^{00} = 0.4851 \pm 0.0088 \end{cases}$$

[Symmetrized averages]

$$\mathcal{B}(B^0 \to K^{*0} J/\psi)_{\rm avg} = (13.23 \pm 0.54) \times 10^{-4}.$$

$$\mathcal{B}(B^0 \to K^{*0} J/\psi)_{\rm PDG} = (12.65 \pm 0.46) \times 10^{-4}.$$

$$\mathcal{B}(B^0 \to K^{*0} J/\psi)_{\rm HFLAV} = (12.78 \pm 0.44) \times 10^{-4}.$$

- Similarly for all other normalization modes [Capdevila, Alvarez, EL, Matias, to appear]

$\mathscr{B}(B \to K^{(*)}\mu\mu)$ averages

- For each measurement, reconstruct what is truly measured: $\mathscr{R}(K^{(*)}) \equiv \mathscr{B}(B \to K^{(*)}\mu\mu)/\mathscr{B}(B \to K^{(*)}J/\psi)$
- Average (if more than one measurement exist, i.e. $B^+ \to K^+\mu\mu$ we have LHCb and CMS)
- Reintroduce the normalization calculated as detailed above

For instance for $q^2 \in [1,6]\ \mathrm{GeV}^2$ we find:

K^+

$$\mathscr{R}(K^+)^{\mathrm{low}}_{\mathrm{LHCb}} = (1.188 \pm 0.046) \times 10^{-4}$$
$$\mathscr{R}(K^+)^{\mathrm{low}}_{\mathrm{CMS}} = (1.228 \pm 0.051) \times 10^{-4}$$
$$\mathscr{R}(K^+)^{\mathrm{low}}_{\mathrm{avg}} = (1.206 \pm 0.034) \times 10^{-4}$$
$$\mathscr{B}(B^+ \to K^+\mu\mu)^{\mathrm{low}}_{\mathrm{avg}} = (1.228 \pm 0.048) \times 10^{-7}$$

K^0

$$\mathscr{R}(K^0)^{\mathrm{low}}_{\mathrm{LHCb}} = (0.995 \pm 0.18) \times 10^{-4}$$
$$\mathscr{B}(B^0 \to K^0\mu\mu)^{\mathrm{low}}_{\mathrm{avg}} = (0.90 \pm 0.16) \times 10^{-7}$$

$$\mathscr{B}(B \to K\mu\mu)^{\mathrm{low}}_{\mathrm{avg}} = (1.202 \pm 0.046) \times 10^{-7}$$

K^{*+}

$$\mathscr{R}(K^{*+})^{\mathrm{low}}_{\mathrm{LHCb}} = (1.265 \pm 0.27) \times 10^{-4}$$
$$\mathscr{B}(B^+ \to K^{*+}\mu\mu)^{\mathrm{low}}_{\mathrm{avg}} = (1.77 \pm 0.40) \times 10^{-7}$$

K^{*0}

$$\mathscr{B}(B^{*0} \to K^{*0}\mu\mu)^{\mathrm{low}}_{\mathrm{avg}} = (1.578 \pm 0.081) \times 10^{-7}$$

For $B \to K^{*0}\mu\mu$, LHCb adopts a normalization which is adjusted to $m_{K^+\pi^-} \in [0.7459, 1.0959]\ \mathrm{GeV}$.
This quantity can only be reconstructed from the most recent Belle measurement which adopts the HFLAV values for f^{+-} and f^{00}

$$\mathscr{B}(B \to K^*\mu\mu)^{\mathrm{low}}_{\mathrm{avg}} = (1.585 \pm 0.079) \times 10^{-7}$$

$\mathcal{B}(B \to K^{(*)}\mu\mu)$ averages

- $q^2 > 15\ \mathrm{GeV}^2$

K^+

$$\mathcal{R}(K^+)^{\rm high}_{\rm LHCb} = (0.849 \pm 0.035) \times 10^{-4}$$
$$\mathcal{R}(K^+)^{\rm high}_{\rm CMS} = (0.909 \pm 0.042) \times 10^{-4}$$
$$\mathcal{R}(K^+)^{\rm high}_{\rm avg} = (0.874 \pm 0.027) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to K^+\mu\mu)^{\rm high}_{\rm avg} = (0.890 \pm 0.036) \times 10^{-7}$$

K^0

$$\mathcal{R}(K^0)^{\rm high}_{\rm LHCb} = (0.72 \pm 0.12) \times 10^{-4}$$
$$\mathcal{B}(B^0 \to K^0\mu\mu)^{\rm high}_{\rm avg} = (0.65 \pm 0.11) \times 10^{-7}$$

$$\mathcal{B}(B \to K\mu\mu)^{\rm high}_{\rm avg} = (0.866 \pm 0.035) \times 10^{-7}$$

K^{*+}

$$\mathcal{R}(K^{*+})^{\rm high}_{\rm LHCb} = (1.11 \pm 0.22) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to K^{*+}\mu\mu)^{\rm high}_{\rm avg} = (1.56 \pm 0.32) \times 10^{-7}$$

K^{*0}

$$\mathcal{B}(B^{*0} \to K^{*0}\mu\mu)^{\rm high}_{\rm avg} = (1.780 \pm 0.091) \times 10^{-7}$$

$$\mathcal{B}(B \to K^{*}\mu\mu)^{\rm high}_{\rm avg} = (1.763 \pm 0.088) \times 10^{-7}$$

$K^+\pi^+\pi^-$

$$\mathcal{R}(K^+\pi^+\pi^-)^{\rm high}_{\rm LHCb} = (0.70 \pm 0.45) \times 10^{-5}$$
$$\mathcal{B}(B^+ \to K^+\psi(2S)) = (6.27 \pm 0.24) \times 10^{-4}$$
$$\mathcal{B}(B^+ \to K^+\pi^+\pi^-\mu\mu)^{\rm high}_{\rm avg} = (0.043 \pm 0.028) \times 10^{-7}$$

$$\mathcal{B}(B^+ \to (K\pi\pi)^+\mu\mu)^{\rm high}_{\rm avg} = \frac{3}{2}\mathcal{B}(B^+ \to K^+\pi^+\pi^-\mu\mu)^{\rm high}_{\rm avg} = (0.065 \pm 0.042) \times 10^{-7}$$

$$\mathcal{B}(B \to K\pi\pi\mu\mu)^{\rm high}_{\rm avg} = \frac{\tau^+ + \tau^0}{2\tau^+}\mathcal{B}^+(B \to (K\pi\pi)^+\mu\mu)^{\rm high}_{\rm avg} = (0.062 \pm 0.040) \times 10^{-7}$$

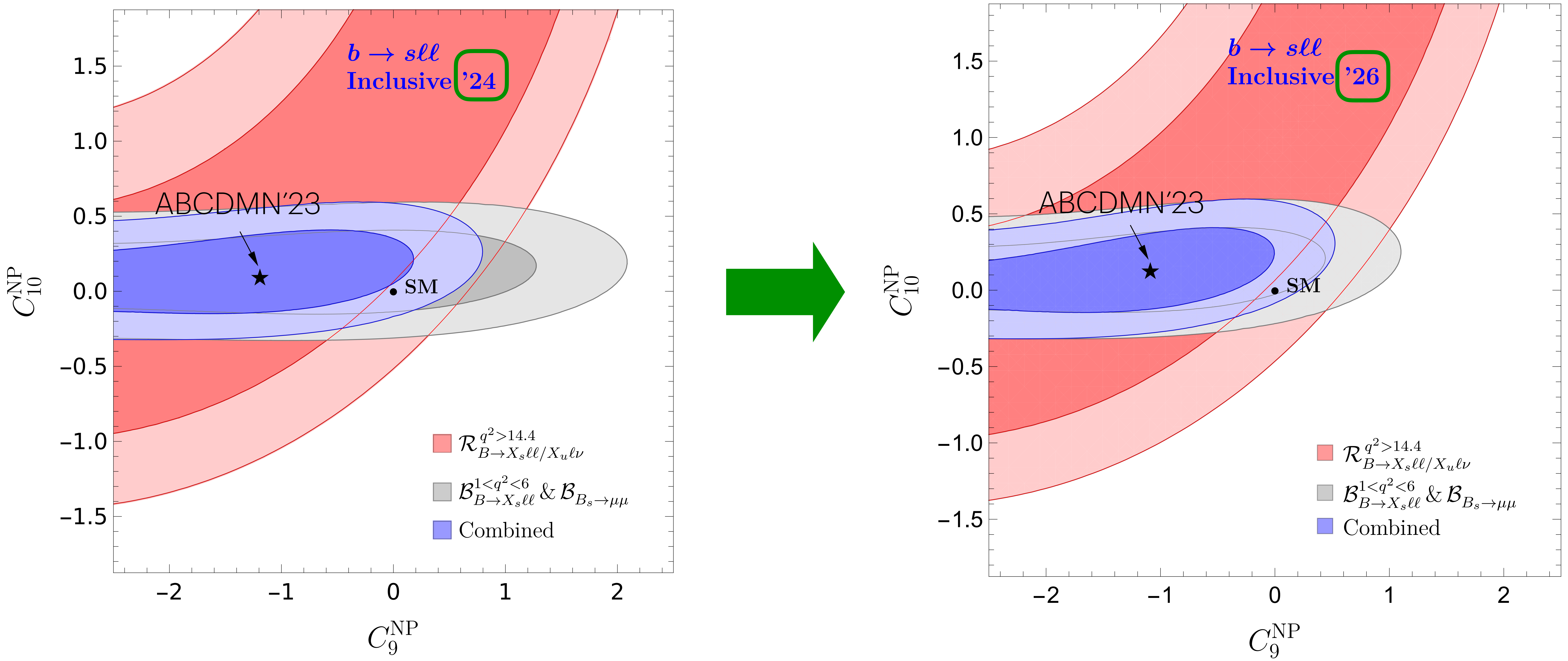
$(K^+\pi^-)_S$

$$\mathcal{B}(B^0 \to (K^+\pi^+)_S\mu\mu)^{\rm high}_{\rm avg} = (0.07 \pm 0.03) \times 10^{-7}$$

$$\mathcal{B}(B^0 \to (K\pi)^0_S\mu\mu)^{\rm high}_{\rm avg} = \frac{3}{2}\mathcal{B}(B^0 \to (K^+\pi^-)_S\mu\mu)^{\rm high}_{\rm avg} = (0.11 \pm 0.04) \times 10^{-7}$$

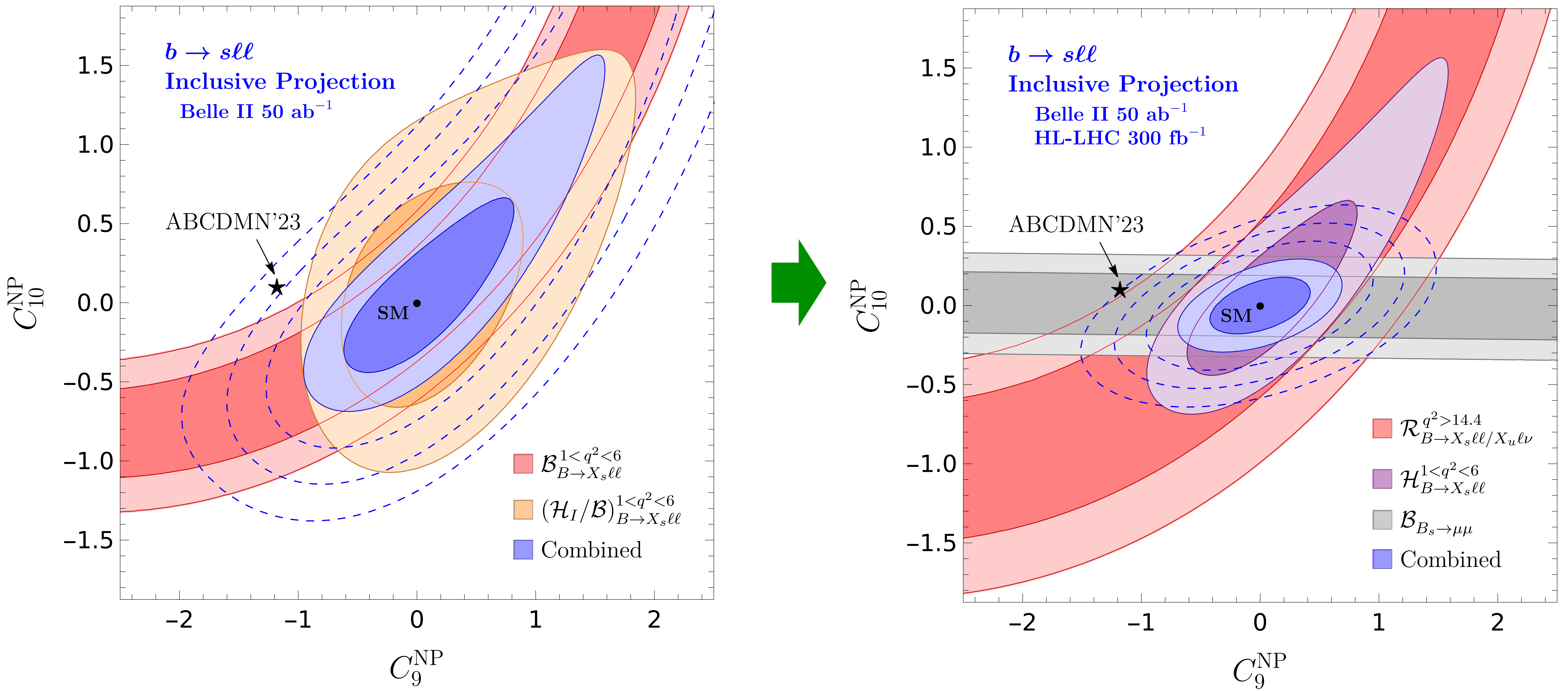
$$\mathcal{B}(B \to (K\pi)_S\mu\mu)^{\rm high}_{\rm avg} = \frac{\tau^+ + \tau^0}{2\tau^+}\mathcal{B}(B^0 \to (K\pi)^0_S\mu\mu)^{\rm high}_{\rm avg} = (0.114 \pm 0.042) \times 10^{-7}$$

Experiment vs SM: current status



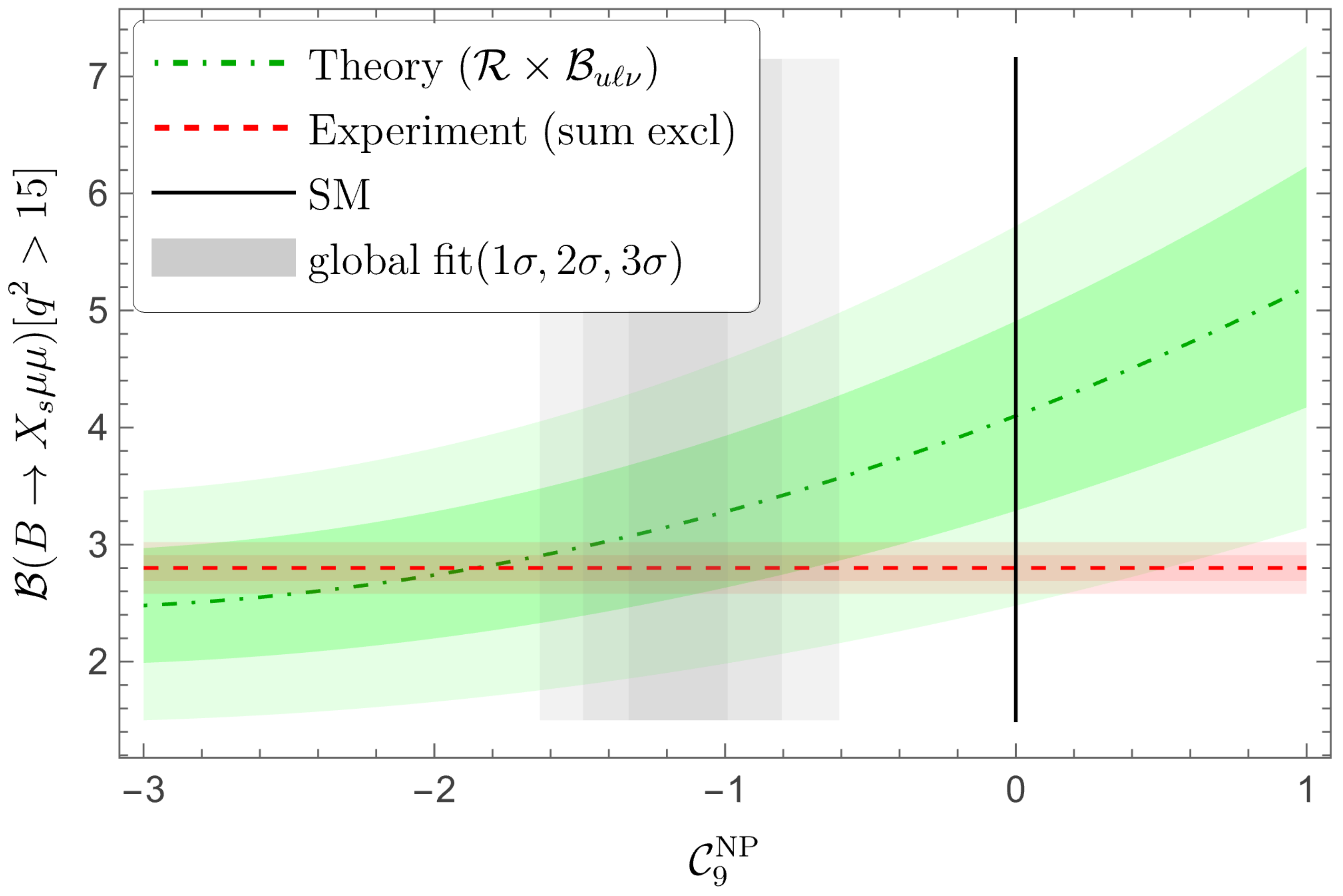
- Update using new inclusive BR at low-q^2 from Belle II and exclusive $B^0 \to K^{*0}\mu\mu$ at high-q^2 from LHCb

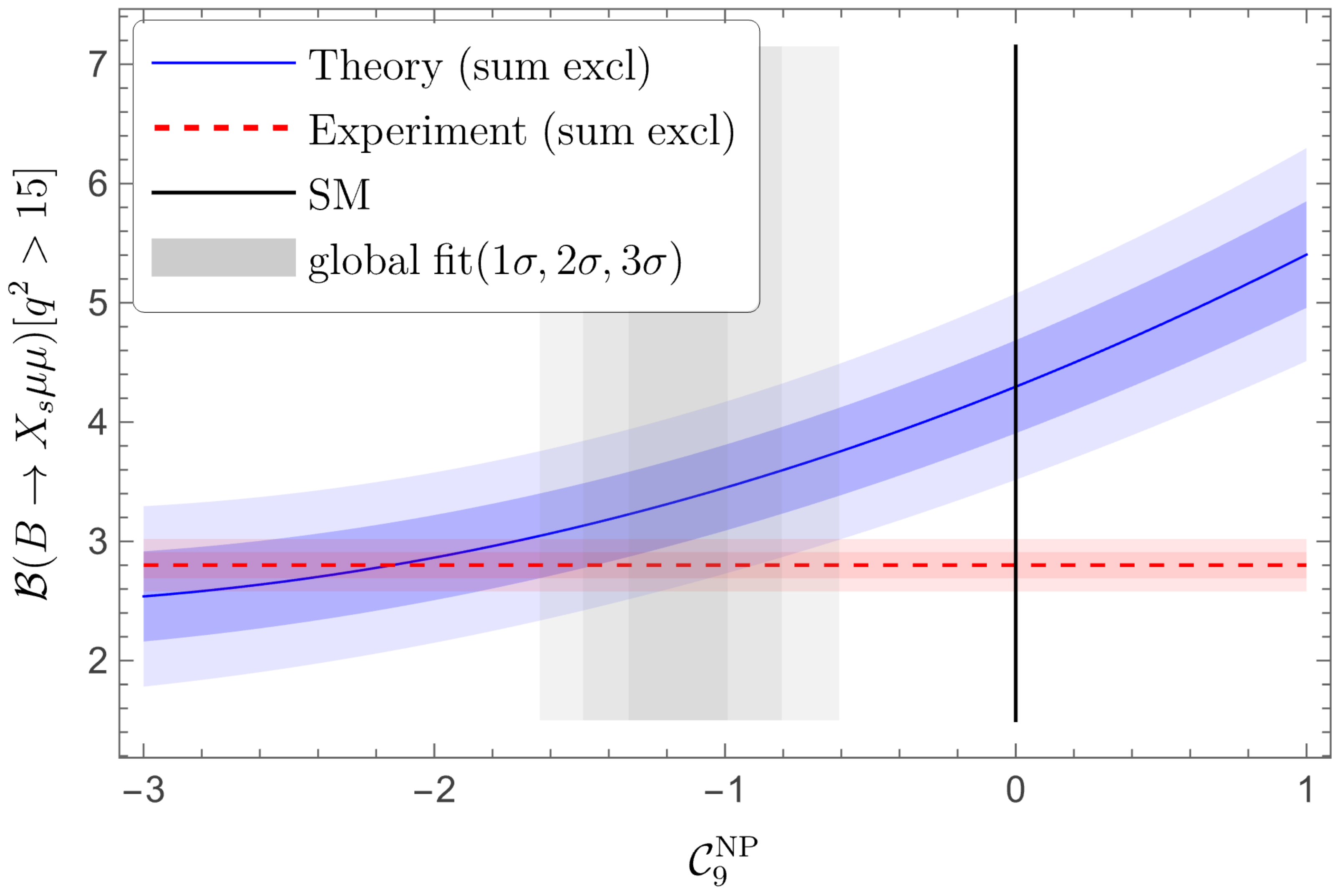
Projected constraints: Belle II and LHCb

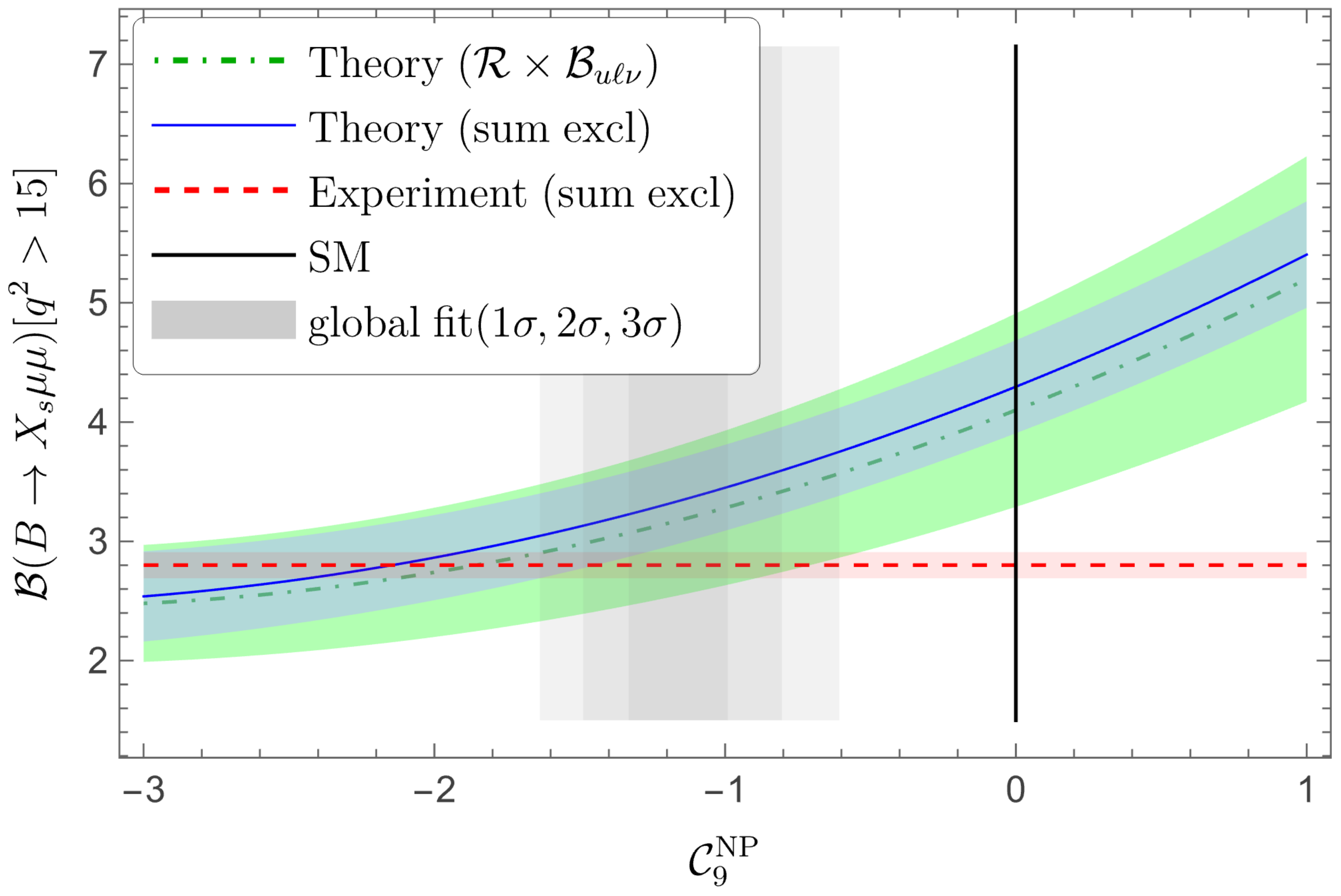


Current status: experiment vs SM

- Comparison of the inclusive $q^2 > 15\ \mathrm{GeV}^2$ branching ratio from
  - Sum of exclusive (experiment): $\mathscr{B}[>15]_{\rm exp} = (2.80 \pm 0.12) \times 10^{-7}$
  - Fully inclusive ($\mathscr{R} \times \mathscr{B}_{u\ell\nu}$): $\mathscr{B}[>15]_{\rm SM,\mathscr{R}} = (4.10 \pm 0.81) \times 10^{-7}$
  - Sum of exclusive (theory): $\mathscr{B}[>15]_{\rm SM, \sum excl} = (4.30 \pm 0.39) \times 10^{-7}$







Future expectations

- LHCb can determine these ratios **without making use of normalizations!**

  For instance, for the $\mathscr{B}(K^*\mu\mu)/\mathscr{B}(K\mu\mu)$ ratio, schematically one gets:

$$\frac{K^*}{K} = \begin{cases} \dfrac{\langle K^{*0}, K^{*+}\rangle}{\langle K^0, K^+\rangle} \sim \dfrac{K^{*0}}{K^+} = \dfrac{R_{*0}}{R_+}\overbrace{\dfrac{\mathscr{B}_{*0}}{\mathscr{B}^+}}^{\text{normalizations}} \to \sqrt{2}\delta_{\text{small}} \oplus^2 \delta\mathscr{B} \\ \langle \dfrac{K^{*0}}{K^0}, \dfrac{K^{*+}}{K^+}\rangle \sim \dfrac{\delta_{\text{large}}}{\sqrt{2}} \sim \dfrac{4\delta_{\text{small}}}{\sqrt{2}} = 2\sqrt{2}\delta_{\text{small}} \end{cases} \Longrightarrow \begin{cases} \sqrt{2}\delta_+ \to 2\sqrt{2}\delta_+ \\ \delta\mathscr{B} \quad \to 0 \end{cases}$$

$B \to X_s \nu\bar{\nu}$: SM prediction

- Only one operator contributes:

$$\mathscr{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{\alpha_{\rm em}(M_Z)}{2\sin^2\theta_W(M_Z)} X_t \, \bar{b}_L \gamma_\mu s_L \sum \bar{\nu}_L \gamma^\mu \nu_L$$

$$X_t = 1.471 \pm 0.010_{\rm scale} \pm 0.006_{\rm param} = 1.471 \pm 0.012$$ [NLO QCD + NLO EW]

- Most important parameters are the top and bottom quark masses ($\mathrm{BR} \propto m_t^4 m_b^5$)
  - $m_t^{\rm MC} = 172.56(31)\ \mathrm{GeV} \Longrightarrow m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}}) = 162.87(17)_{\rm MSR}(30)_{M_t}(50)_{\rm MC}\ \mathrm{GeV} = 162.87(60)\ \mathrm{GeV}$

    MSR scale dependence

    relation between MC and MSR masses

    parametric uncertainty from $m_t^{\rm MC}$
  - $m_b^{\rm kin} = 4.573(12)\ \mathrm{GeV}$ (from $B \to X_c \ell\nu$ fits [Finauri, Gambino]

- N3LO hadronic matrix element identical to $B \to X_u \ell \nu$

$B \to X_s \nu\bar{\nu}$: SM prediction

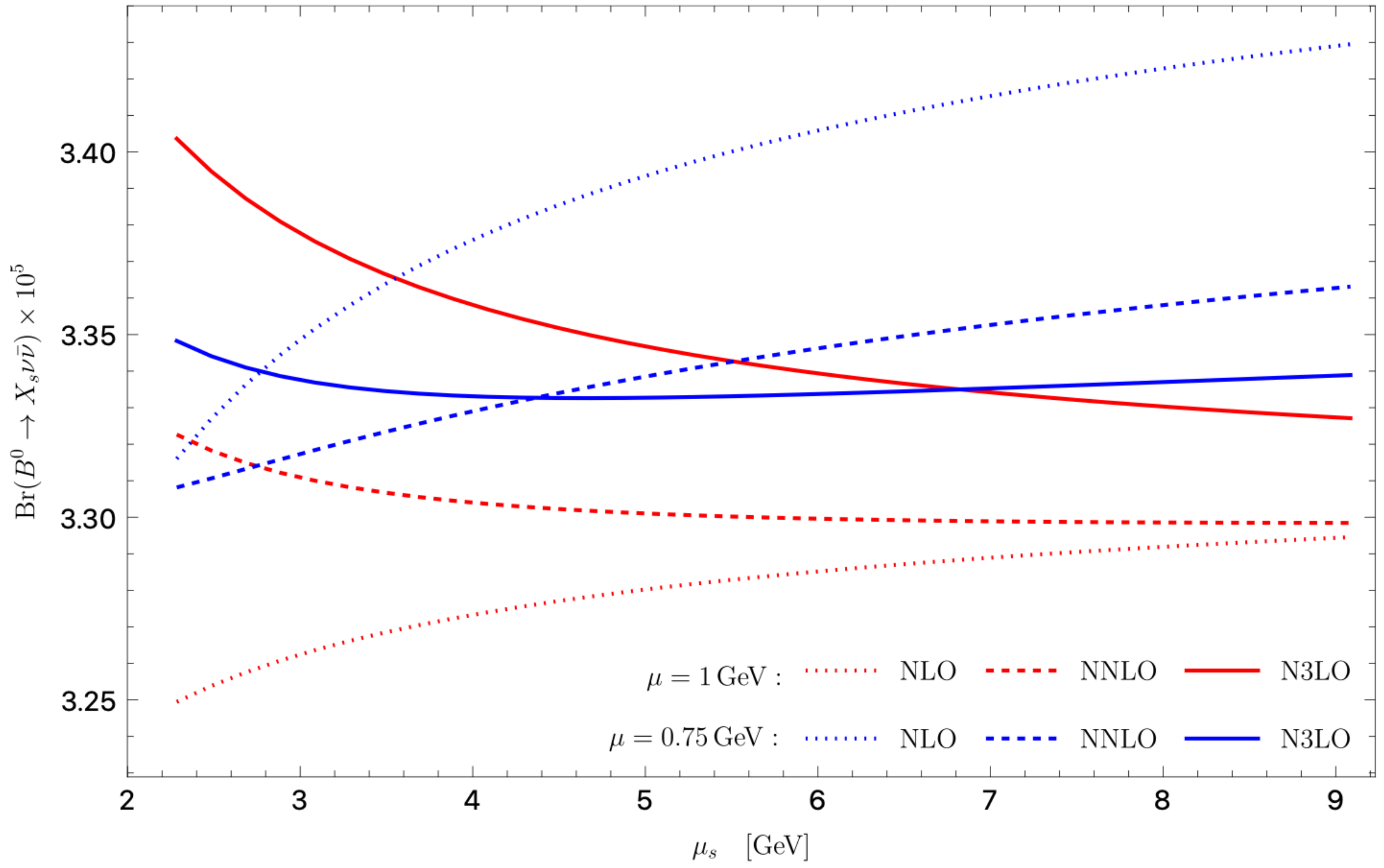
- Using $|V_{cb}| = 0.04197(48)$ from inclusive $B \to X_c \ell \nu$ [Finauri, Gambino] we obtain: [Fael, Jenkins, EL, Polonsky, 2512.19138]

$$\mathcal{B}(B^0 \to X_s \nu\bar{\nu}) = \left[3.351 \pm 0.058_{V_{cb}} \pm 0.029_{\text{param}} \pm 0.043_{\text{HQE}} \pm 0.065_{\text{scale}}\right] \times 10^{-5}$$

- First search at Belle II with $365\ \text{fb}^{-1}$ [2511.10980]:

$$\mathcal{B}(B \to X_s \nu\bar{\nu}) = \left[8.8^{+8.5}_{-8.2}(\text{stat})^{+12.6}_{-10.8}(\text{syst})\right] \times 10^{-5} \implies \mathcal{B}(B \to X_s \nu\bar{\nu}) < 3.2 \times 10^{-4} \ @ 90\,\%\,\text{C.L.}$$

- Low scale dependence of the total branching ratio for various kinetic scales ($\mu_{\text{kin}} = [0.75, 1]$ GeV):



$B \to X_s \nu\bar{\nu}$ VS $B \to X_s \ell\ell$

- SMEFT contributions to the $C_{9,10,L}$ Wilson coefficients:

$$\Delta C_9 \propto \tilde{C}^{+}_{lq} + (4s^2_W - 1)\tilde{C}^{+}_{\varphi q} + \tilde{C}_{qe} \simeq \tilde{C}^{+}_{lq} + \tilde{C}_{qe},$$

$$\Delta C_{10} \propto -\tilde{C}^{+}_{lq} + \tilde{C}^{+}_{\varphi q} + \tilde{C}_{qe} \ ,$$

$$\Delta C_L \propto \tilde{C}^{-}_{lq} + \tilde{C}^{+}_{\varphi q} \ .$$