



Electromagnetic signatures of axion dark matter halos around primordial black holes

Ongoing work (2609.xxxx)

Junghoon Joh (SungKyunKwan Univ.)

With Ki-Young Choi (SKKU) and Kenji Kadota (HIAS, UCAS)

The 5th workshop on Symmetry and Structure of the Universe, 2026-08-27

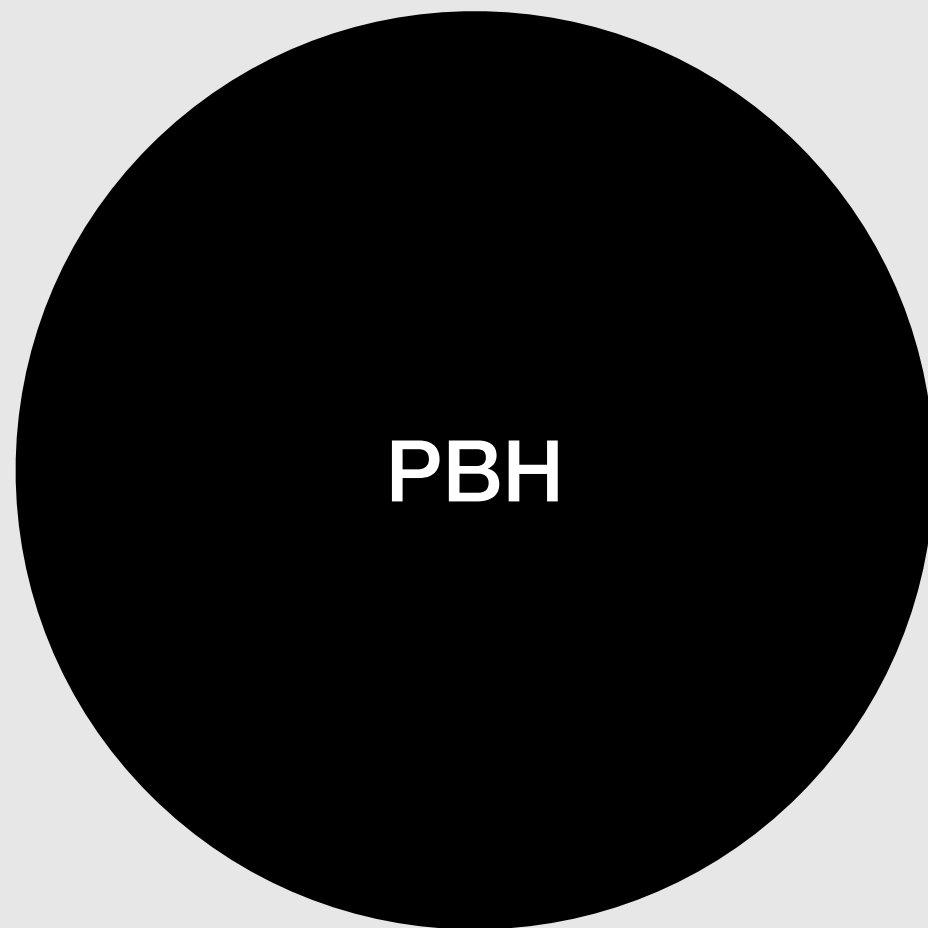
Motivation

The search for PBH-ALP DM co-existence

- **Coexistence:** The dark matter (DM) can be composed of both *Primordial Black Holes (PBHs)* and *Axion-Like Particles (ALPs)*.
- **ALP Halo:** Around PBHs, ALPs can form halos due to gravitational attraction. It is denser than the NFW profile, which is formed in MD era.
- **Photon Enhancement:** The high ALP density can *stimulate photon production* from ALP decay.
- **Observable Flux:** This enhancement produces a *detectable photon flux* that reaches Earth, providing a signature for searching for or constraining both PBHs and ALPs.

ALP Halo around PBH

Dark Matter: PBH + ALP

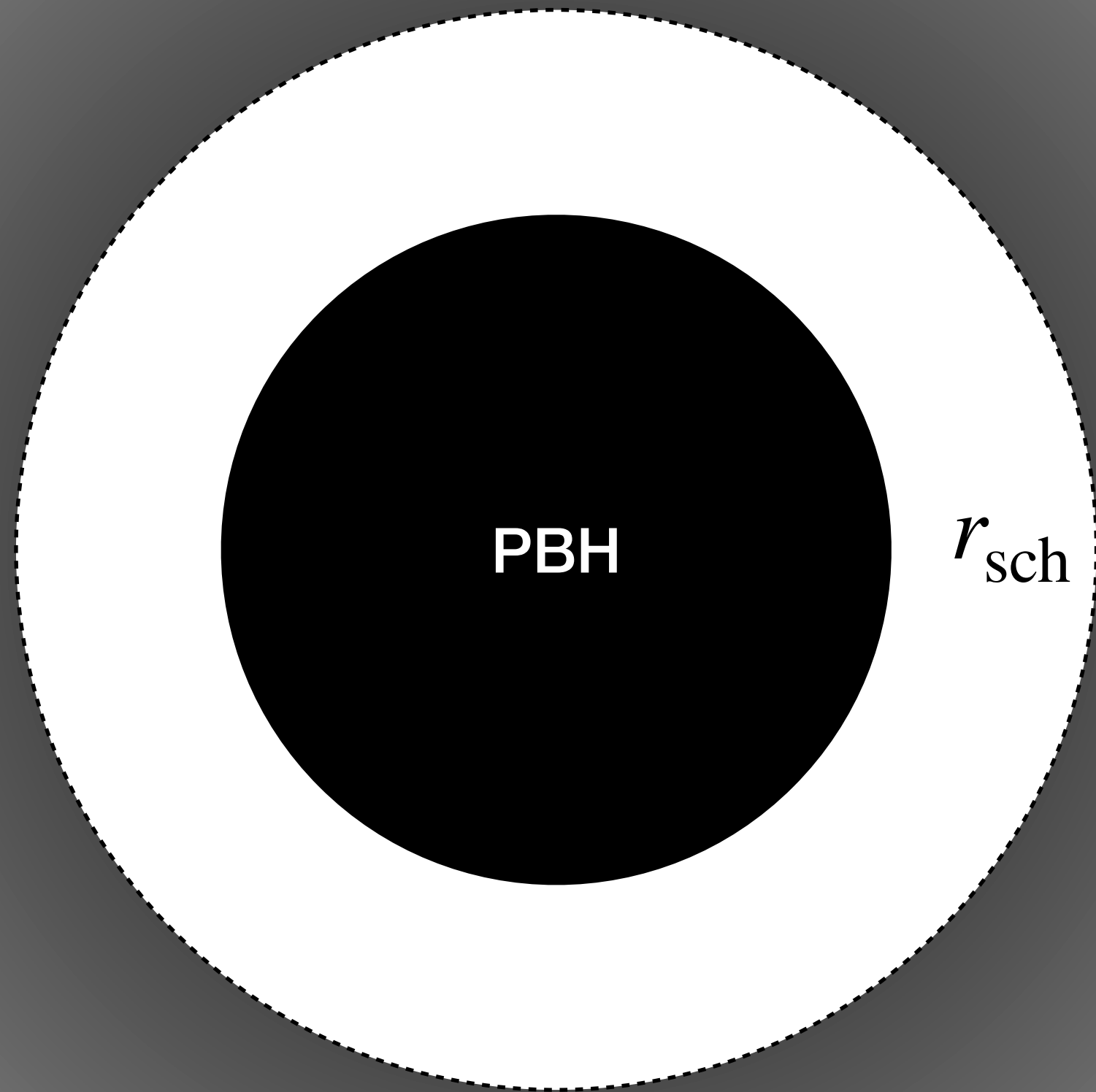


ALP Halo around PBH

Dark Matter: PBH + ALP



ALP Halo around PBH



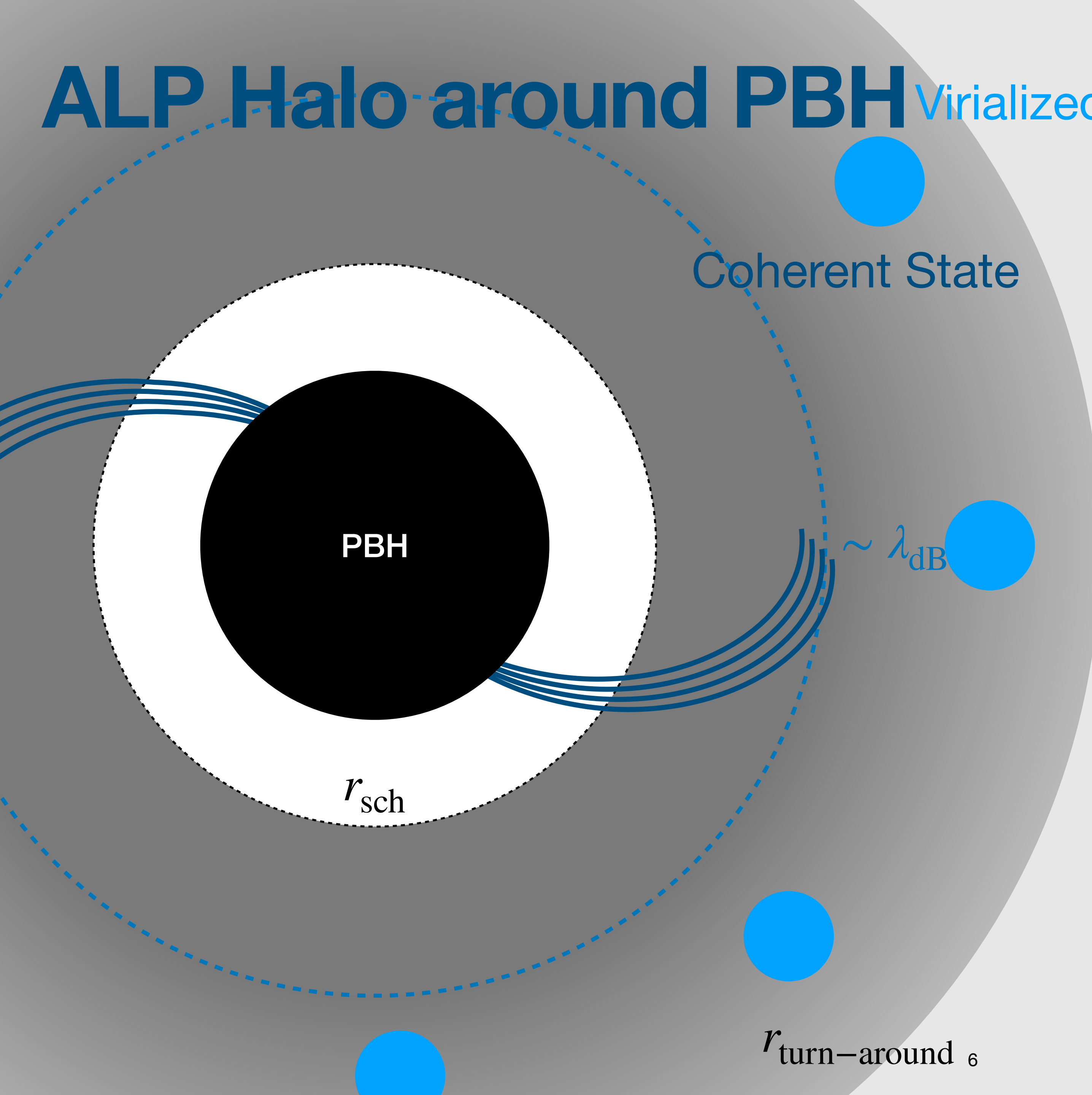
$$r_{\text{turn-around}} \simeq (r_{\text{sch}} t^2)^{1/3}$$

: Gravity = Expansion

[Adamek et al., Phys.Rev.D 100 \(2019\) 2, 023506](#)

ALP Halo around PBH

Virialized Halo

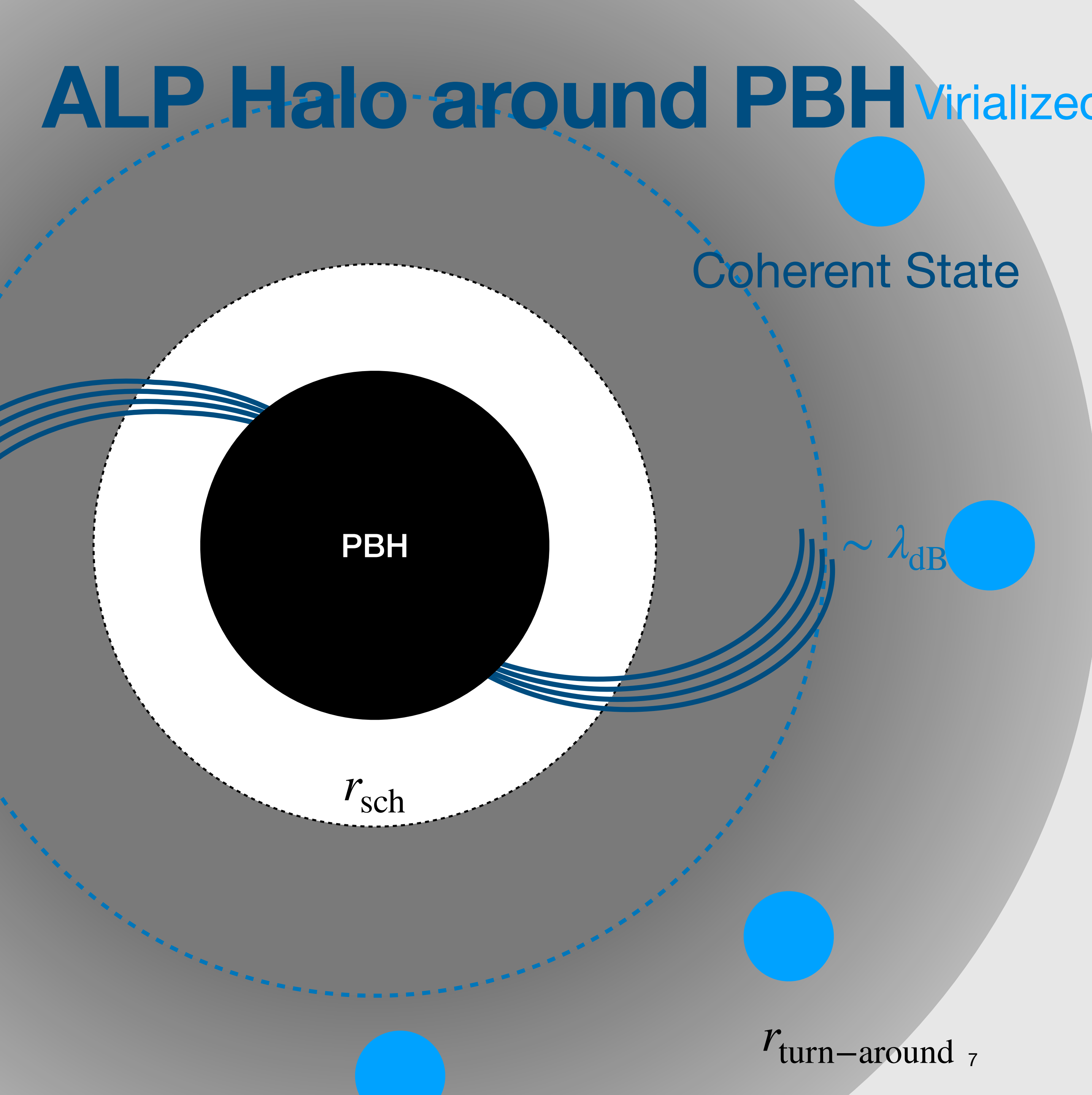


ALP Halo around PBH

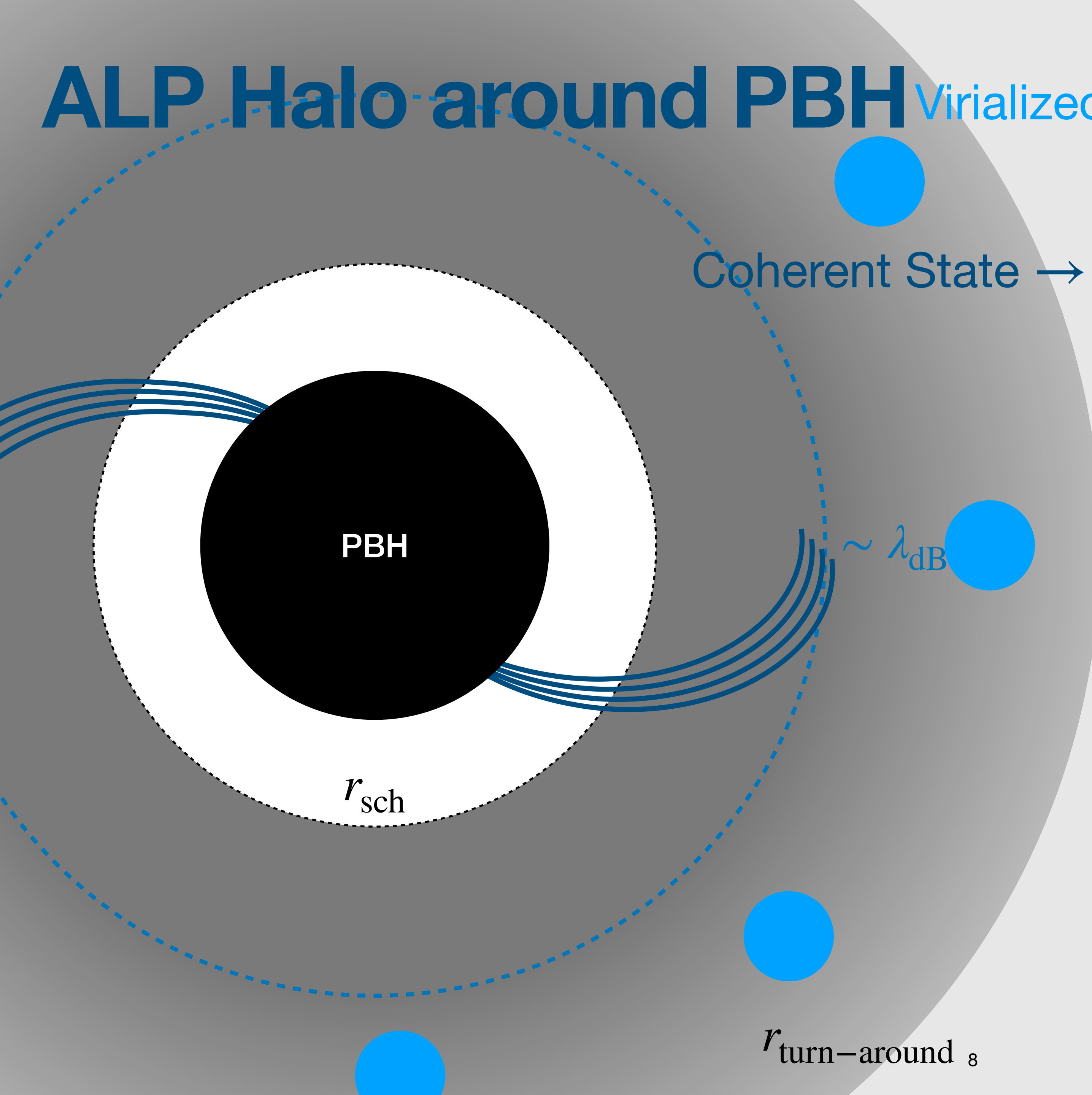
Virialized Halo $\rightarrow$ Spike density profile $\propto r^{-9/4}$

Quinlan, et al., *Astrophys.J.* 440 (1995), 554-564

Gondolo, et al., *Nucl.Phys.B Proc.Suppl.* 87 (2000), 87-89



ALP Halo around PBH



Virialized Halo $\rightarrow$ Spike density profile $\propto r^{-9/4}$

Quinlan, et al., *Astrophys.J.* 440 (1995), 554-564

Gondolo, et al., *Nucl.Phys.B Proc.Suppl.* 87 (2000), 87-89

Coherent State $\rightarrow$ Soliton-like Halo $\propto e^{-r/r_0}$

Palomares-Chavez, et al., *Phys.Rev.D* 112 (2025) 2, 023545

ALP Halo around PBH

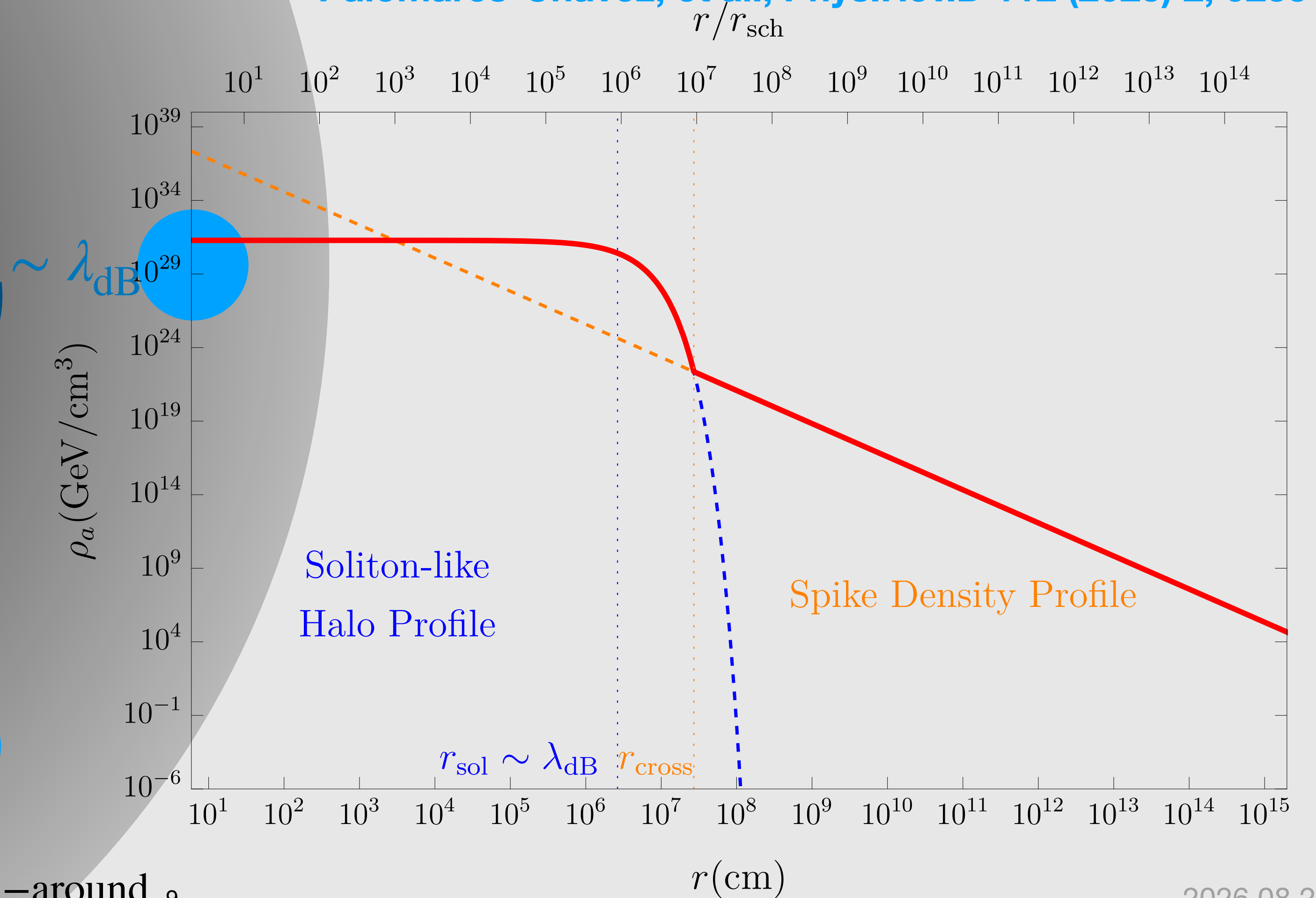
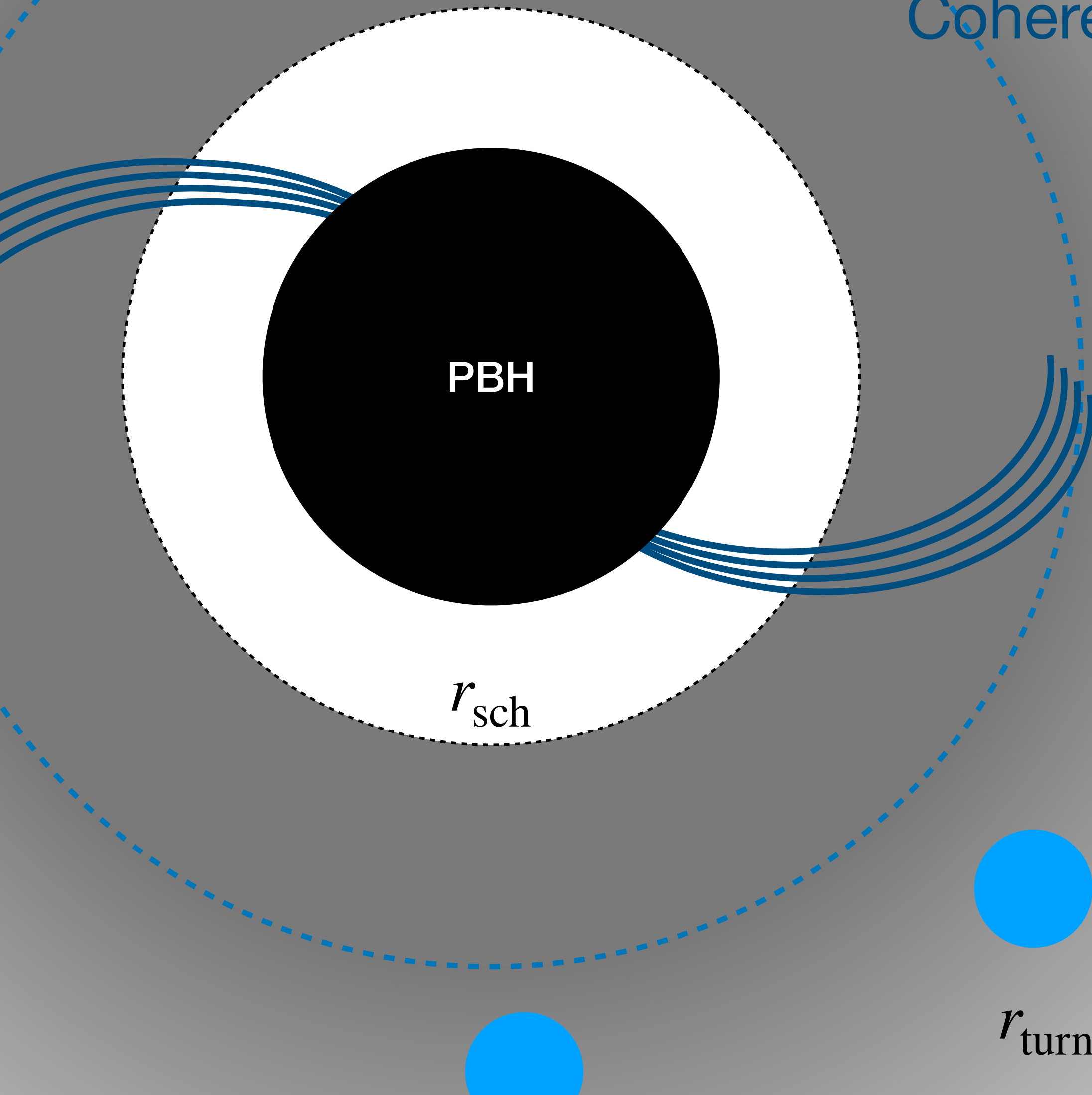
Virialized Halo $\rightarrow$ Spike density profile $\propto r^{-9/4}$

Quinlan, et al., *Astrophys.J.* 440 (1995), 554-564

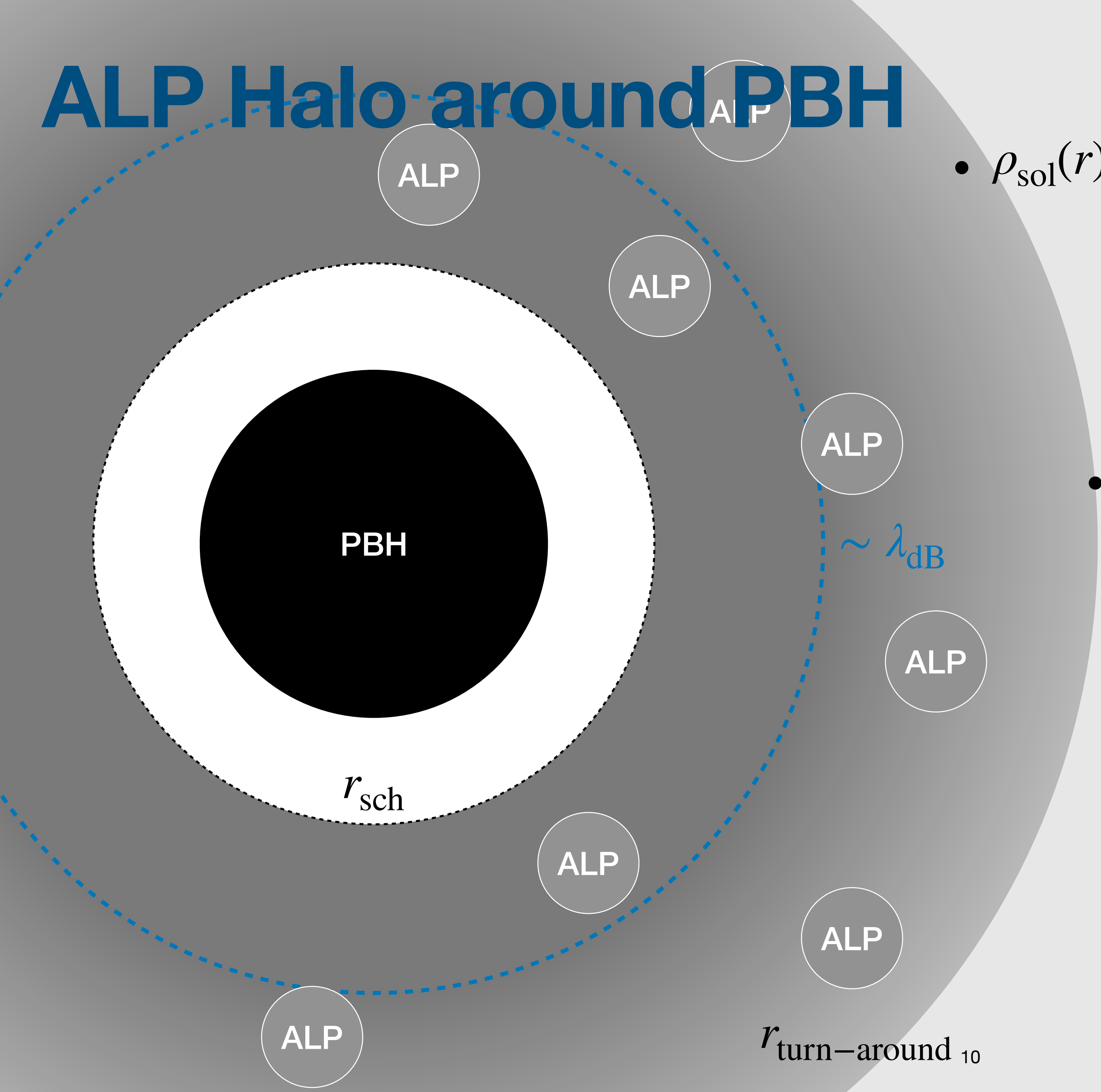
Gondolo, et al., *Nucl.Phys.B Proc.Suppl.* 87 (2000), 87-89

Coherent State $\rightarrow$ Soliton-like Halo $\propto e^{-r/r_0}$

Palomares-Chavez, et al., *Phys.Rev.D* 112 (2025) 2, 023545



ALP Halo around PBH



Aghaie, et al., Phys.Rev.D 109 (2024) 10, 103030

- $$\rho_{\text{sol}}(r) = \frac{\epsilon\, m_a^2}{\pi G} \left(GM_{\text{PBH}}m_a\right)^4 \exp\left[-2GM_{\text{PBH}}m_a^2r\right]$$

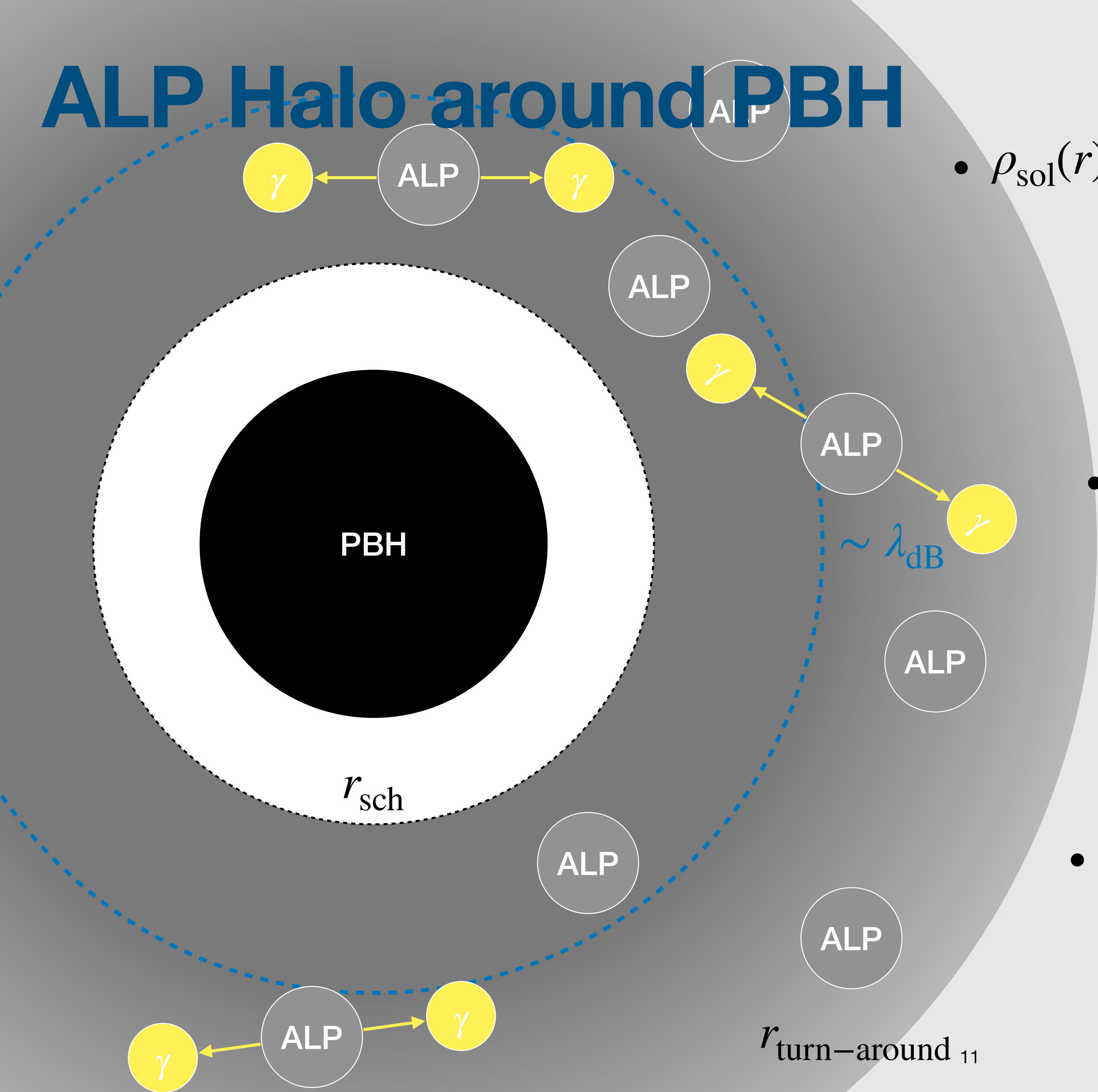
where $r_{\text{sch}} \leq r \lesssim \lambda_{\text{dB}}$

Quinlan, et al.,Astrophys.J. 440 (1995), 554-564
 Gondolo, et al., Nucl.Phys.B Proc.Suppl. 87 (2000), 87-89

- $$\rho_{\text{spike}}(r) = f_{a/\text{DM}}\frac{\rho_{\text{eq}}}{2} \left(\frac{M_{\text{PBH}}}{M_{\odot}}\right)^{3/4} \left(\frac{\tilde{r}}{r}\right)^{9/4}$$

where $\lambda_{\text{dB}} \lesssim r \leq r_{\text{turn-around}}$

ALP Halo around PBH



Aghaie, et al., Phys.Rev.D 109 (2024) 10, 103030

- $$\rho_{\text{sol}}(r) = \frac{\epsilon m_a^2}{\pi G} \left(GM_{\text{PBH}} m_a \right)^4 \exp \left[-2 GM_{\text{PBH}} m_a^2 r \right]$$

where $r_{\text{sch}} \leq r \lesssim \lambda_{\text{dB}}$

Quinlan, et al.,Astrophys.J. 440 (1995), 554-564
 Gondolo, et al., Nucl.Phys.B Proc.Suppl. 87 (2000), 87-89

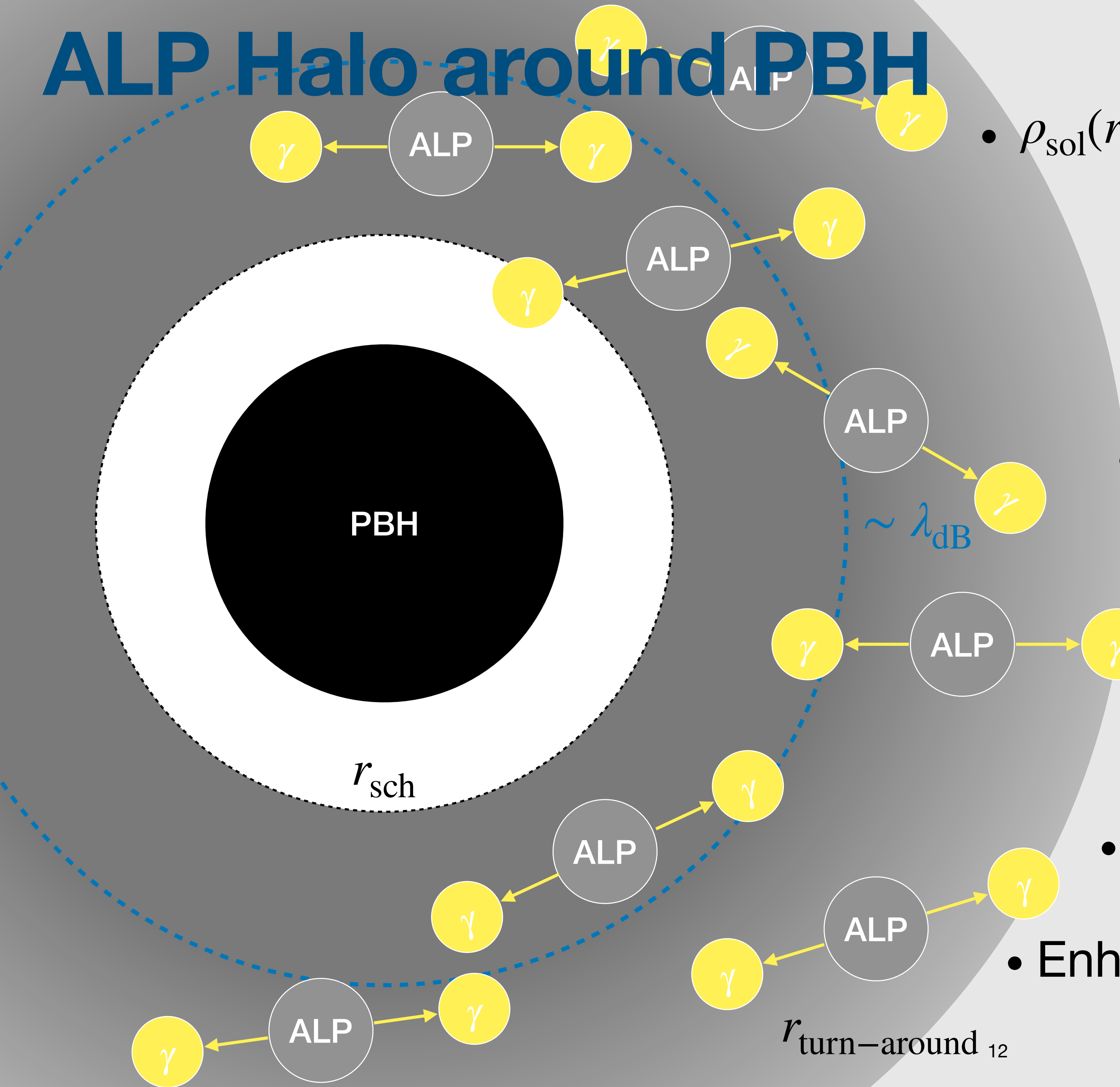
- $$\rho_{\text{spike}}(r) = f_{a/\text{DM}} \frac{\rho_{\text{eq}}}{2} \left(\frac{M_{\text{PBH}}}{M_{\odot}} \right)^{3/4} \left(\frac{\tilde{r}}{r} \right)^{9/4}$$

where $\lambda_{\text{dB}} \lesssim r \leq r_{\text{turn-around}}$

- $$\mathcal{L}_{a\gamma\gamma} \supset \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- $$\omega_{\gamma} \simeq m_a/2$$

ALP Halo around PBH



Aghaie, et al., Phys.Rev.D 109 (2024) 10, 103030

- $\rho_{\text{sol}}(r) = \frac{\epsilon m_a^2}{\pi G} (GM_{\text{PBH}} m_a)^4 \exp[-2GM_{\text{PBH}} m_a^2 r]$
where $r_{\text{sch}} \leq r \lesssim \lambda_{\text{dB}}$

Quinlan, et al., Astrophys.J. 440 (1995), 554-564
Gondolo, et al., Nucl.Phys.B Proc.Suppl. 87 (2000), 87-89

- $\rho_{\text{spike}}(r) = f_{a/\text{DM}} \frac{\rho_{\text{eq}}}{2} \left(\frac{M_{\text{PBH}}}{M_{\odot}} \right)^{3/4} \left(\frac{\tilde{r}}{r} \right)^{9/4}$
where $\lambda_{\text{dB}} \lesssim r \leq r_{\text{turn-around}}$

- $\mathcal{L}_{a\gamma\gamma} \supset \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

- $\omega_{\gamma} \simeq m_a/2$

- Enhancement condition: $f_{\gamma} > 1$

Boltzmann Equation in ALP Halo

- Boltzmann equations for photons:

$$\frac{dn_\gamma}{dt} = 2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) - \frac{8\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3} \right) - \frac{n_\gamma}{t_{\text{esc}}}$$

- Boltzmann equations for axions:

$$\frac{dn_a}{dt} = -\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) + \frac{8\pi^2 v_{a,\max} \Gamma_a}{3m_a^3} n_\gamma^2 + \frac{n_{a,0} - n_a}{t_{\text{dyn}}}$$

- Boltzmann equations for sterile axions ($E_a > E_{a,\max}$):

$$\frac{dn_s}{dt} = \frac{4\pi^2 \Gamma_a}{m_a^3} n_\gamma^2$$

- * Momentum distribution is flat
- * Axion energy range: $m_a \leq E_a \leq E_{a,\max}$
- * Photon energy range: $\omega_{\gamma,\min} \leq \omega_\gamma \leq \omega_{\gamma,\max}$

Boltzmann Equation in ALP Halo

- Boltzmann equations for photons:

$$\frac{dn_\gamma}{dt} = 2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) - \frac{8\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3} \right) - \frac{n_\gamma}{t_{\text{esc}}}$$

- Boltzmann equations for axions:

$$\frac{dn_a}{dt} = -\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) + \frac{8\pi^2 v_{a,\max} \Gamma_a}{3m_a^3} n_\gamma^2 + \frac{n_{a,0} - n_a}{t_{\text{dyn}}}$$

- Boltzmann equations for sterile axions ($E_a > E_{a,\max}$):

$$\frac{dn_s}{dt} = \frac{4\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \quad * \text{ Production of photons} \rightarrow \text{Spontaneous decay / Stimulated decay}$$

Boltzmann Equation in ALP Halo

- Boltzmann equations for photons:

$$\frac{dn_\gamma}{dt} = \underbrace{2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right)}_{\text{Production of photons}} - \underbrace{\frac{8\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3} \right)}_{\text{Annihilation of photons}} - \frac{n_\gamma}{t_{\text{esc}}}$$

- Boltzmann equations for axions:

$$\frac{dn_a}{dt} = - \underbrace{\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right)}_{\text{Spontaneous decay / Stimulated decay}} + \underbrace{\frac{8\pi^2 v_{a,\max} \Gamma_a}{3m_a^3} n_\gamma^2}_{\text{Annihilation of photons}} + \frac{n_{a,0} - n_a}{t_{\text{dyn}}}$$

- Boltzmann equations for sterile axions ($E_a > E_{a,\max}$):

$$\frac{dn_s}{dt} = \underbrace{\frac{4\pi^2 \Gamma_a}{m_a^3} n_\gamma^2}_{\text{Annihilation of photons}}$$

* Production of photons → Spontaneous decay / Stimulated decay

* Annihilation of photons → Axion outside halo / Axion inside halo

Boltzmann Equation in ALP Halo

- Boltzmann equations for photons:

$$\frac{dn_\gamma}{dt} = \underbrace{2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right)}_{\text{Production of photons}} - \underbrace{\frac{8\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3} \right)}_{\text{Annihilation of photons}} - \underbrace{\frac{n_\gamma}{t_{\text{esc}}}}_{\text{Escape of photons}}$$

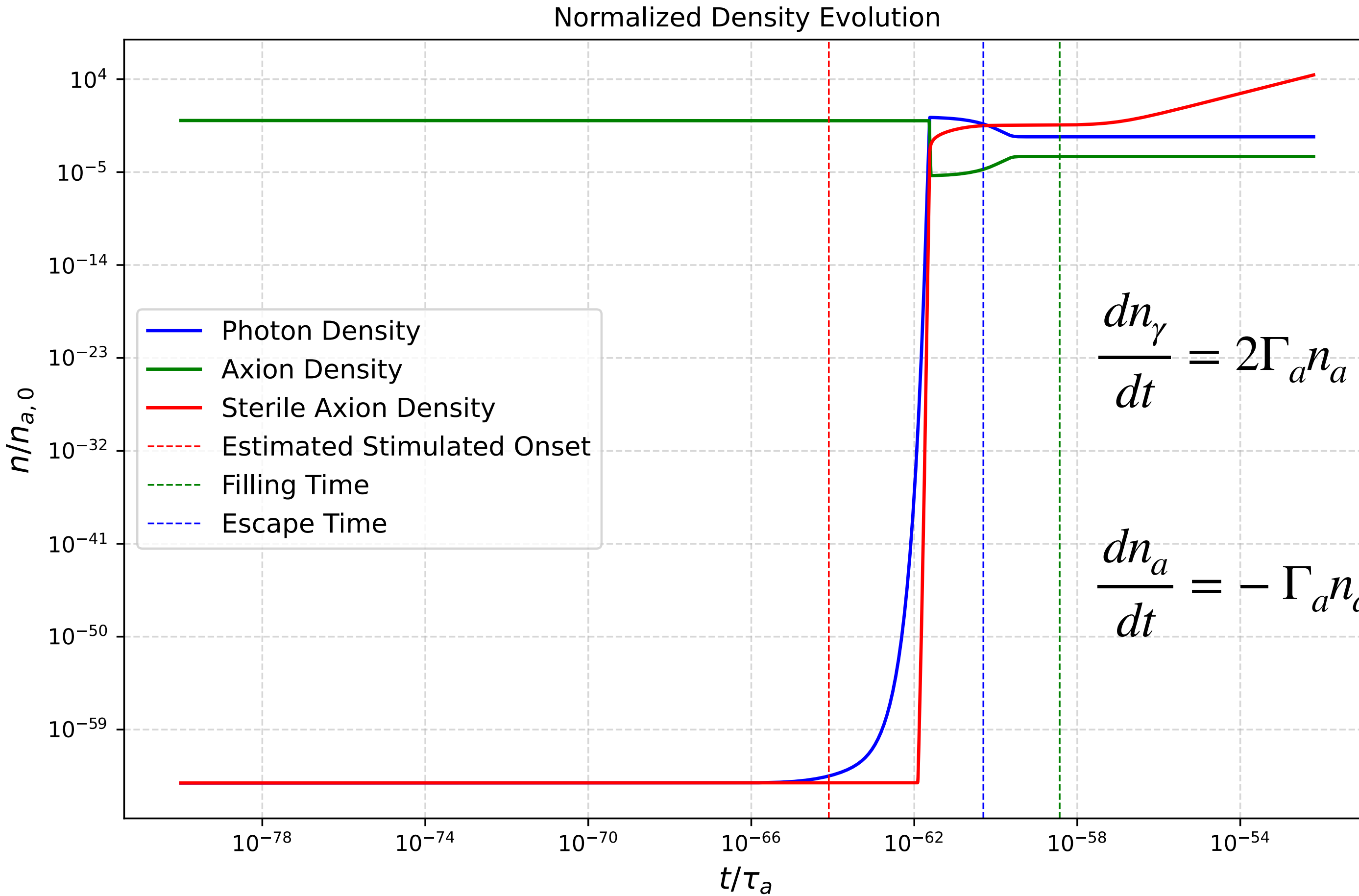
- Boltzmann equations for axions:

$$\frac{dn_a}{dt} = - \underbrace{\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right)}_{\text{Spontaneous decay / Stimulated decay}} + \underbrace{\frac{8\pi^2 v_{a,\max} \Gamma_a}{3m_a^3} n_\gamma^2}_{\text{Axion outside halo / Axion inside halo}} + \underbrace{\frac{n_{a,0} - n_a}{t_{\text{dyn}}}}_{\text{Filling of axions from background}}$$

- Boltzmann equations for sterile axions ($E_a > E_{a,\max}$):

$$\frac{dn_s}{dt} = \underbrace{\frac{4\pi^2 \Gamma_a}{m_a^3} n_\gamma^2}_{\text{Production of photons}} \quad \begin{array}{l} * \text{ Production of photons} \rightarrow \text{Spontaneous decay / Stimulated decay} \\ * \text{ Annihilation of photons} \rightarrow \text{Axion outside halo / Axion inside halo} \\ * \text{ Escape of photons} \end{array} \quad \begin{array}{l} * \text{ Filling of axions from background} \end{array}$$

Example Result of Enhancement Case

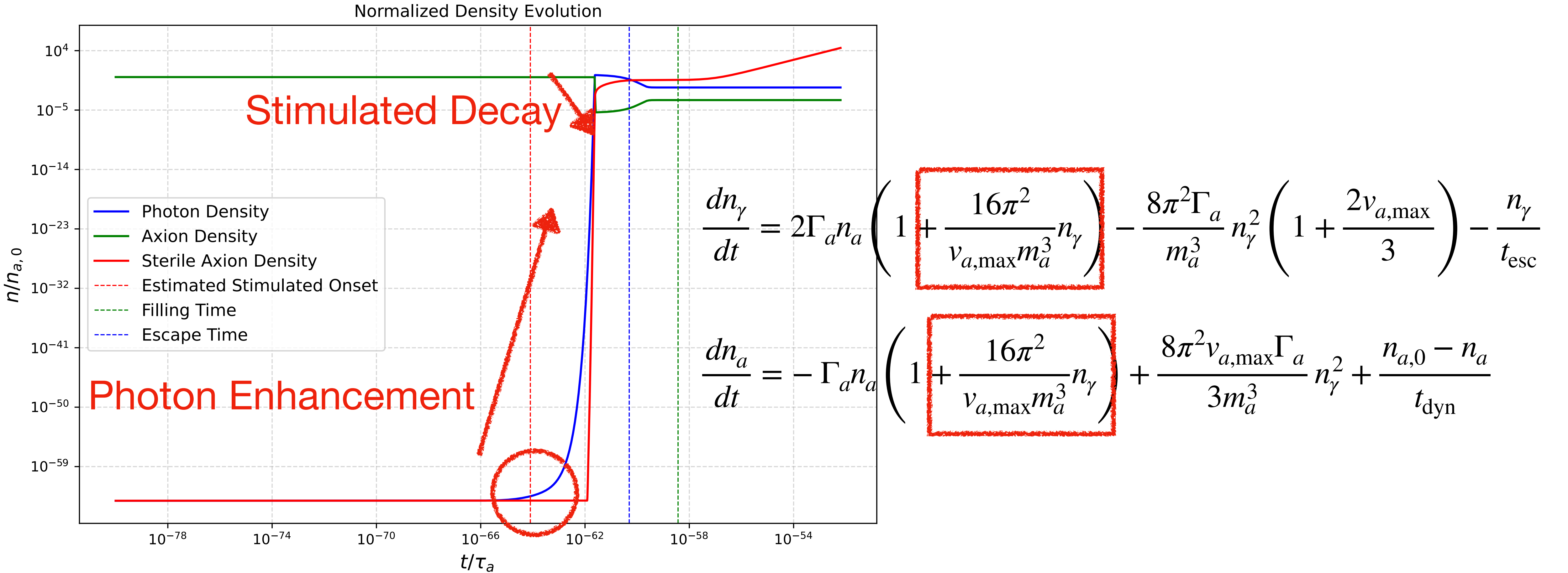


$$\frac{dn_\gamma}{dt} = 2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) - \frac{8\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3} \right) - \frac{n_\gamma}{t_{\text{esc}}}$$

$$\frac{dn_a}{dt} = -\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) + \frac{8\pi^2 v_{a,\max} \Gamma_a}{3m_a^3} n_\gamma^2 + \frac{n_{a,0} - n_a}{t_{\text{dyn}}}$$

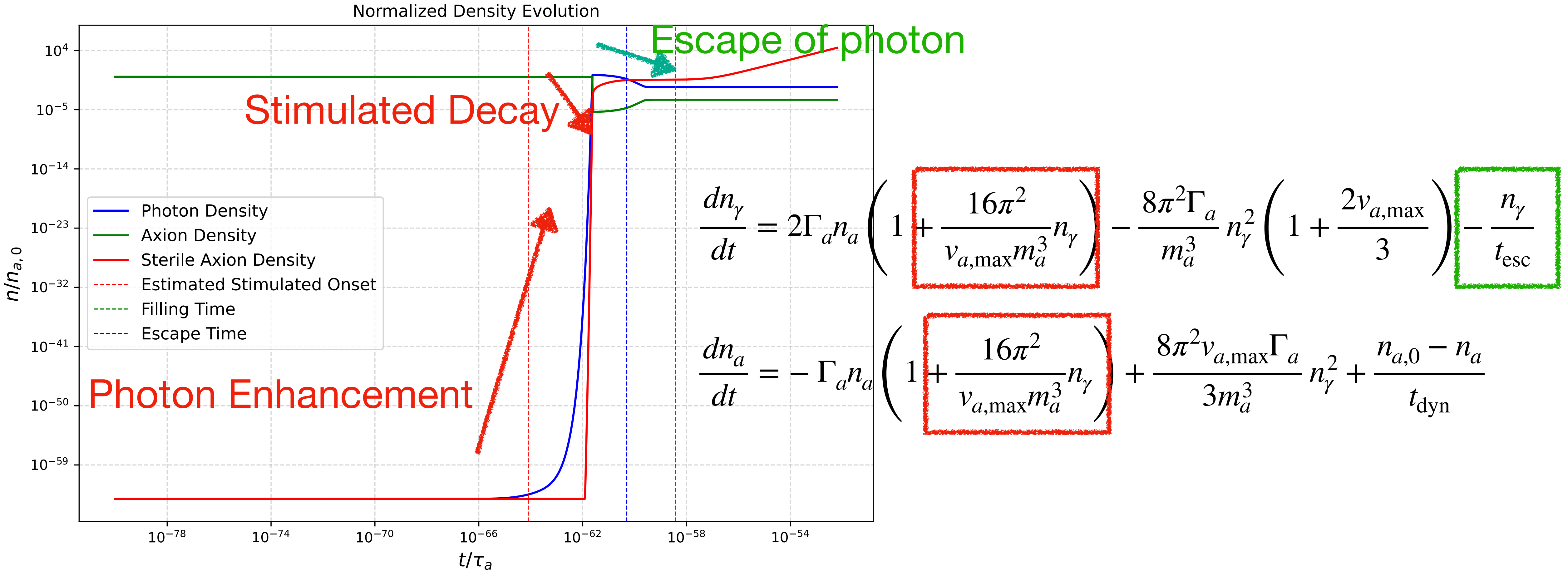
$$m_{\text{PBH}} = 10^{-5} M_\odot, m_a = 10^{-8} \text{eV}, g_{a\gamma\gamma} = 10^{-12} \text{GeV}^{-1}, r = 10^3 r_{\text{sch}}$$

Example Result of Enhancement Case



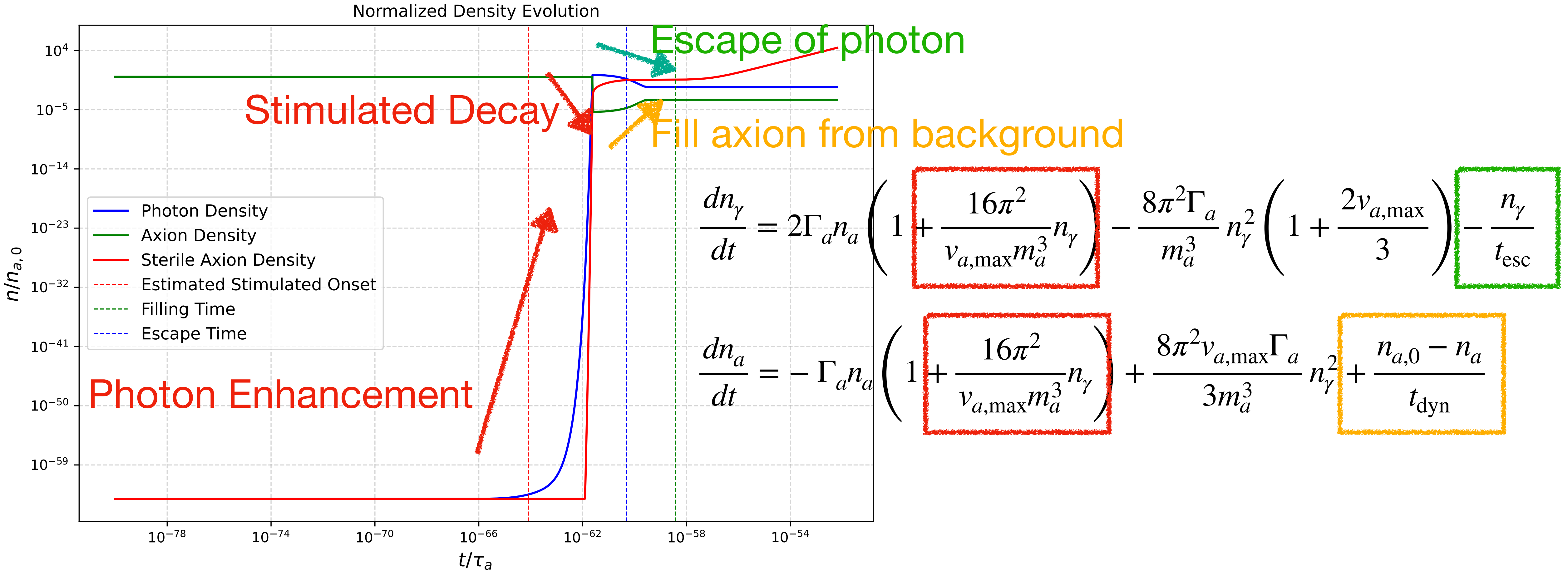
$$m_{\text{PBH}} = 10^{-5} M_\odot, m_a = 10^{-8} \text{eV}, g_{a\gamma\gamma} = 10^{-12} \text{GeV}^{-1}, r = 10^3 r_{\text{sch}}$$

Example Result of Enhancement Case



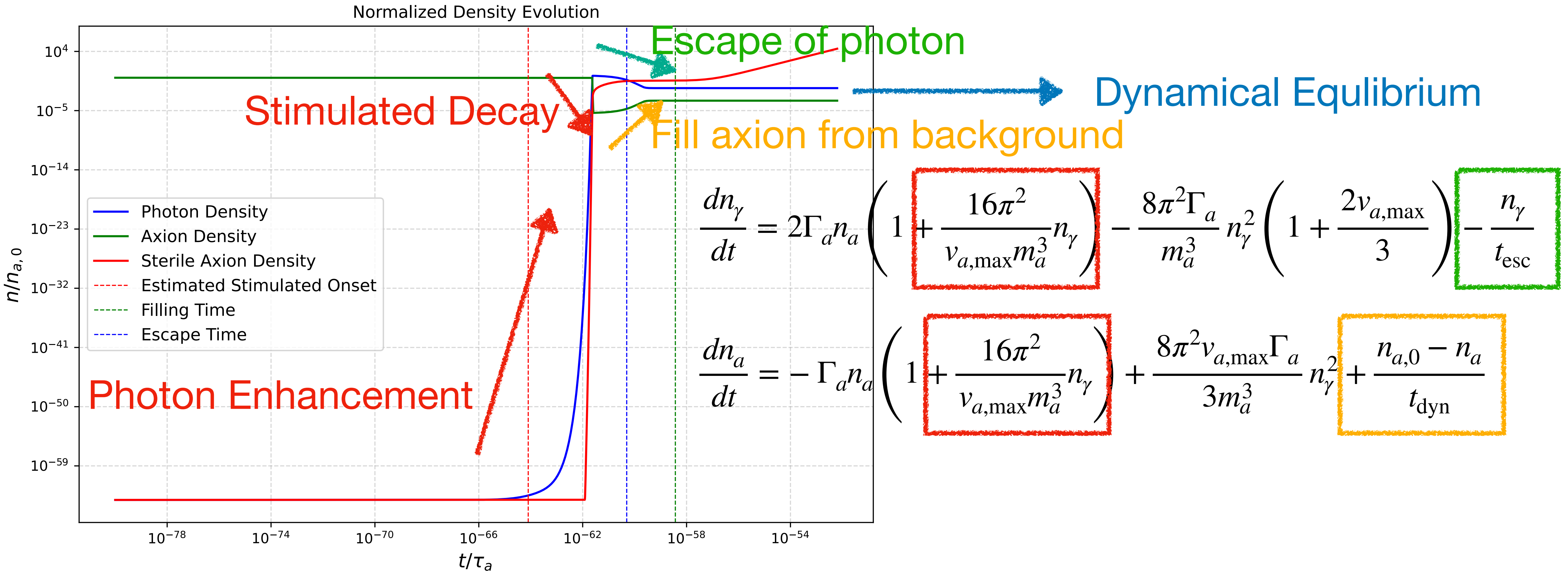
$$m_{\text{PBH}} = 10^{-5} M_\odot, m_a = 10^{-8} \text{eV}, g_{a\gamma\gamma} = 10^{-12} \text{GeV}^{-1}, r = 10^3 r_{\text{sch}}$$

Example Result of Enhancement Case



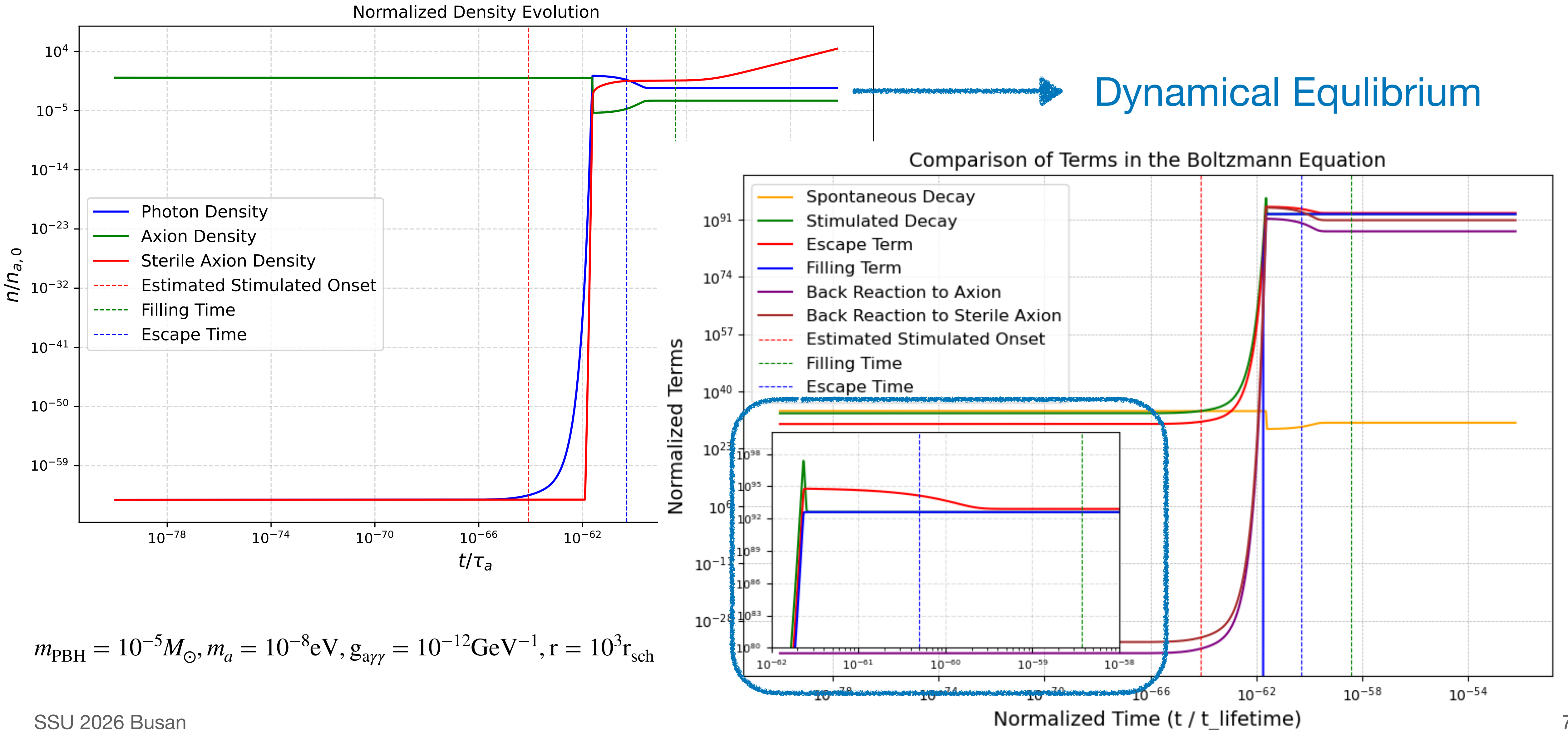
$$m_{\text{PBH}} = 10^{-5} M_\odot, m_a = 10^{-8} \text{eV}, g_{a\gamma\gamma} = 10^{-12} \text{GeV}^{-1}, r = 10^3 r_{\text{sch}}$$

Example Result of Enhancement Case

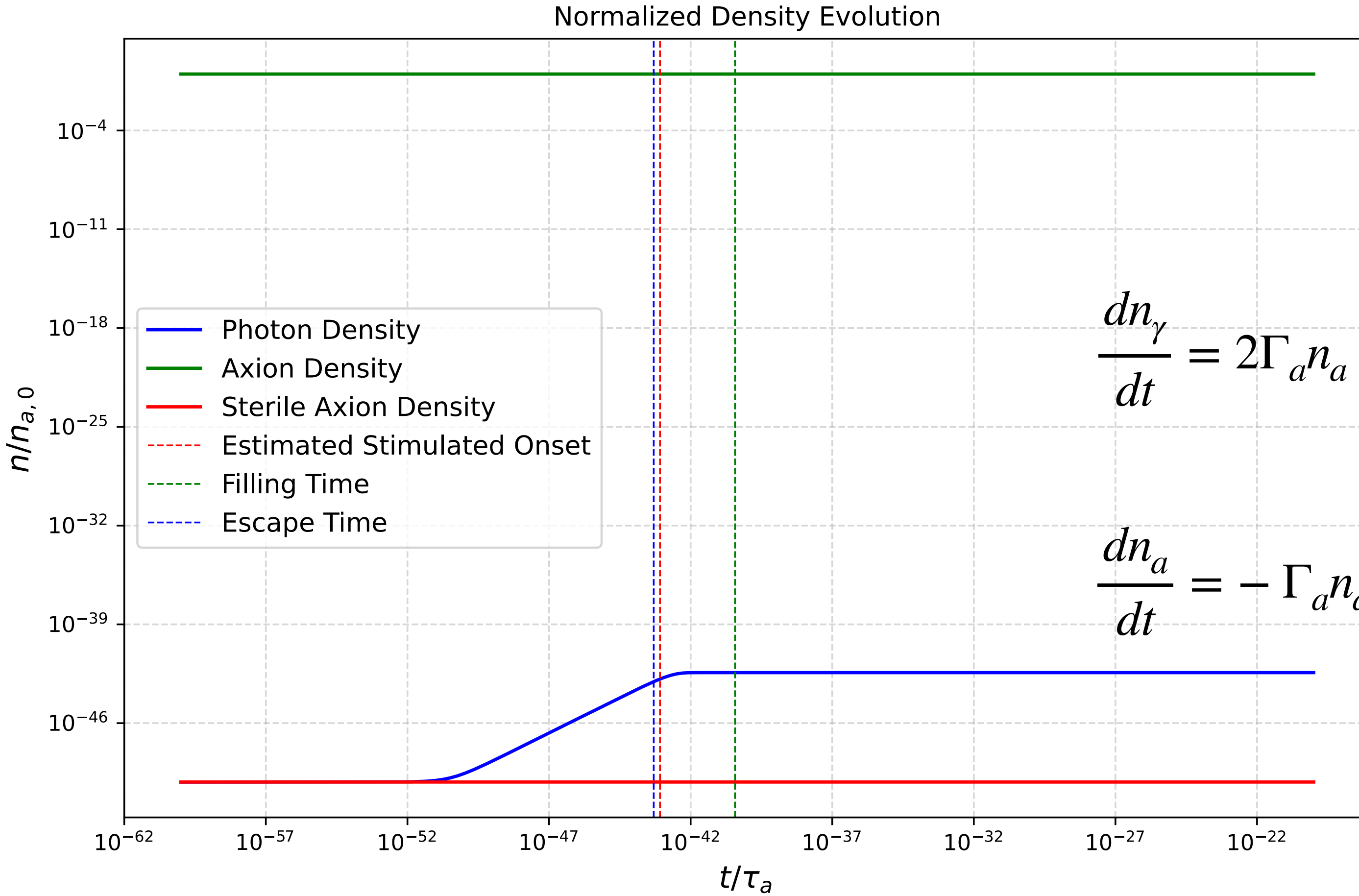


$$m_{\text{PBH}} = 10^{-5} M_\odot, m_a = 10^{-8} \text{eV}, g_{a\gamma\gamma} = 10^{-12} \text{GeV}^{-1}, r = 10^3 r_{\text{sch}}$$

Example Result of Enhancement Case



Example Result of Non-Enhancement Case

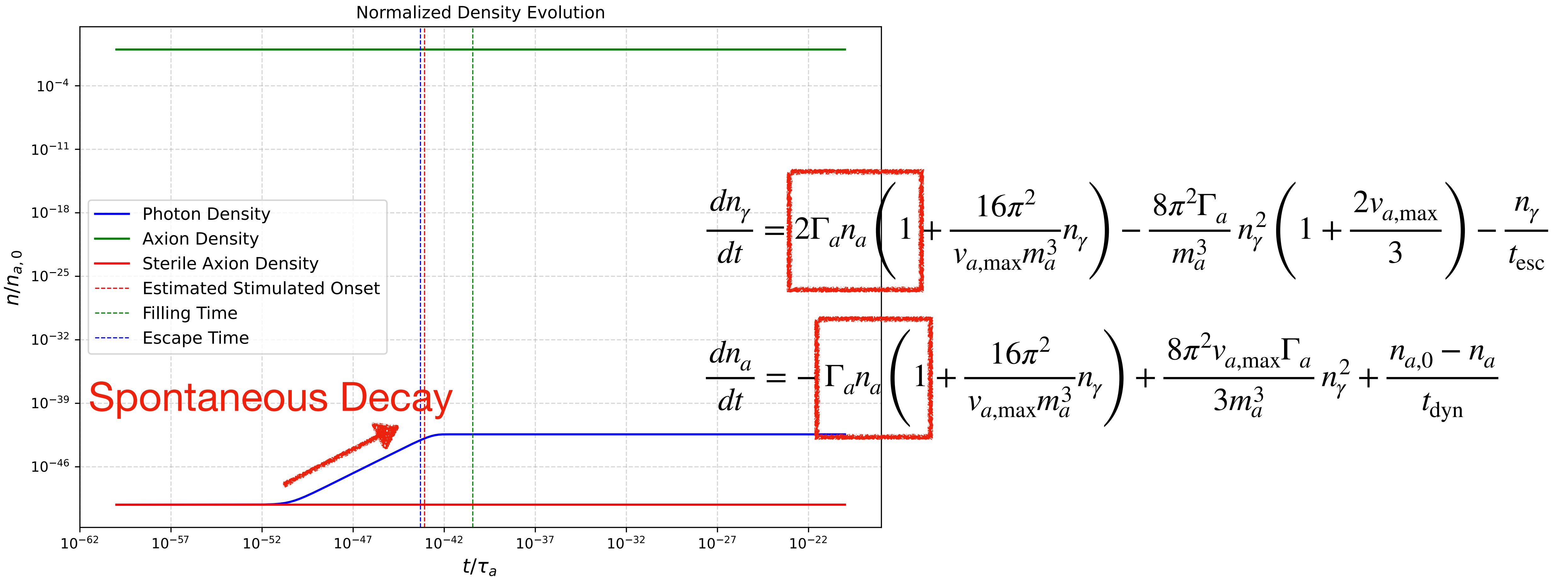


$$\frac{dn_\gamma}{dt} = 2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) - \frac{8\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\max}}{3} \right) - \frac{n_\gamma}{t_{\text{esc}}}$$

$$\frac{dn_a}{dt} = -\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\max} m_a^3} n_\gamma \right) + \frac{8\pi^2 v_{a,\max} \Gamma_a}{3m_a^3} n_\gamma^2 + \frac{n_{a,0} - n_a}{t_{\text{dyn}}}$$

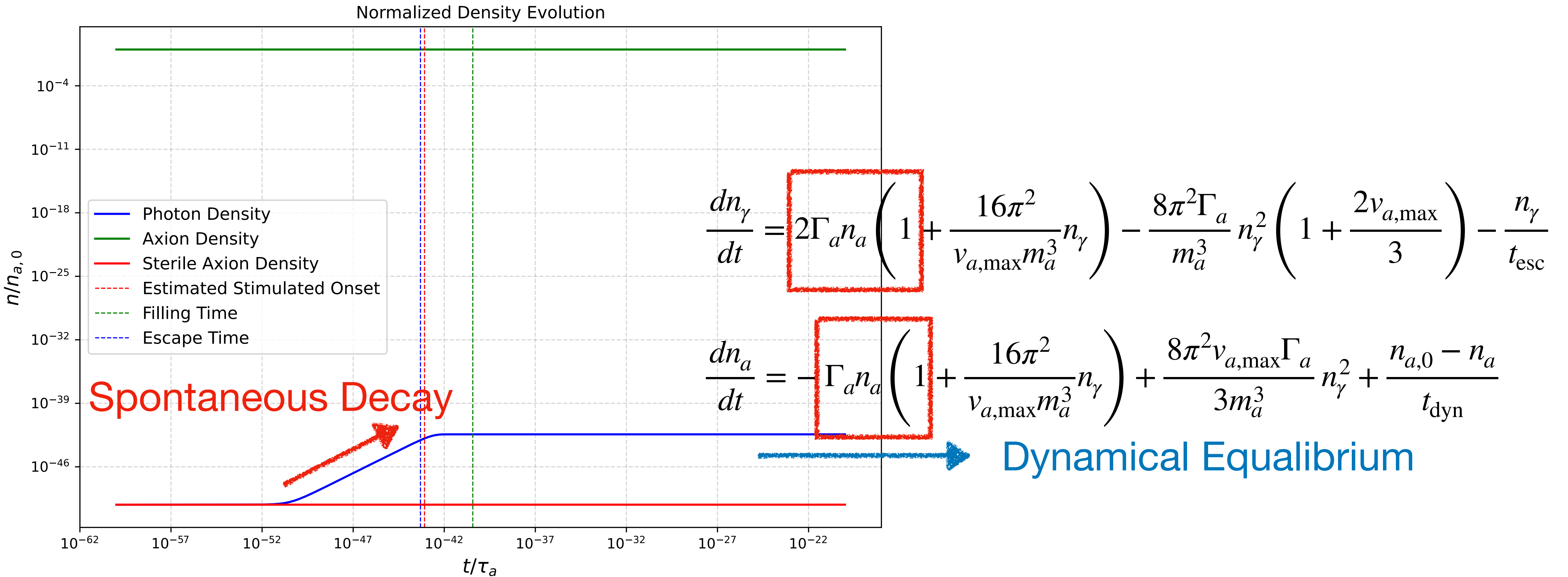
$$m_{\text{PBH}} = 1M_\odot, m_a = 10^{-6}\text{eV}, g_{a\gamma\gamma} = 10^{-10}\text{GeV}^{-1}, r = 10^5 r_{\text{sch}}$$

Example Result of Non-Enhancement Case



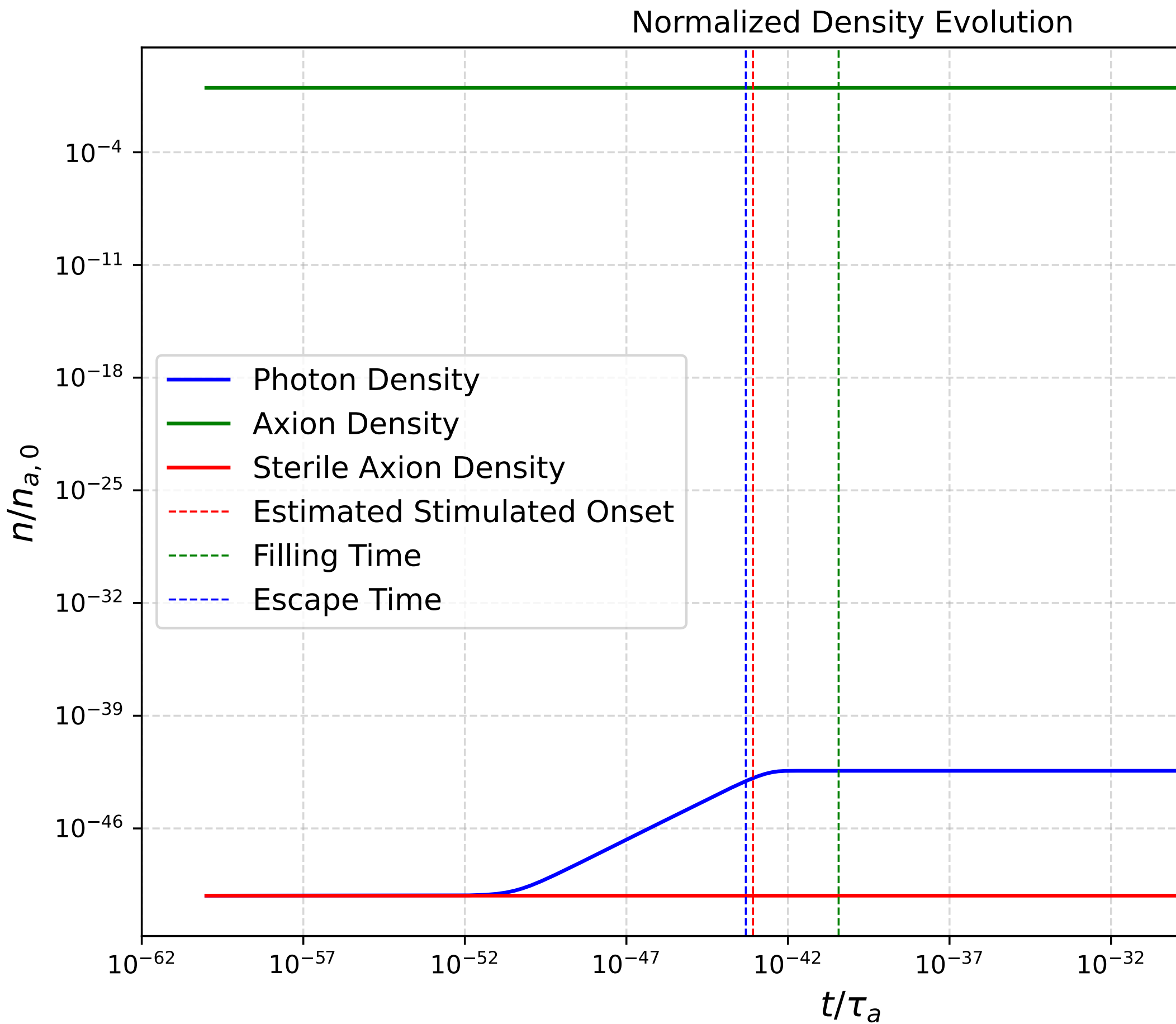
$$m_{\text{PBH}} = 1M_\odot, m_a = 10^{-6}\text{eV}, g_{a\gamma\gamma} = 10^{-10}\text{GeV}^{-1}, r = 10^5 r_{\text{sch}}$$

Example Result of Non-Enhancement Case

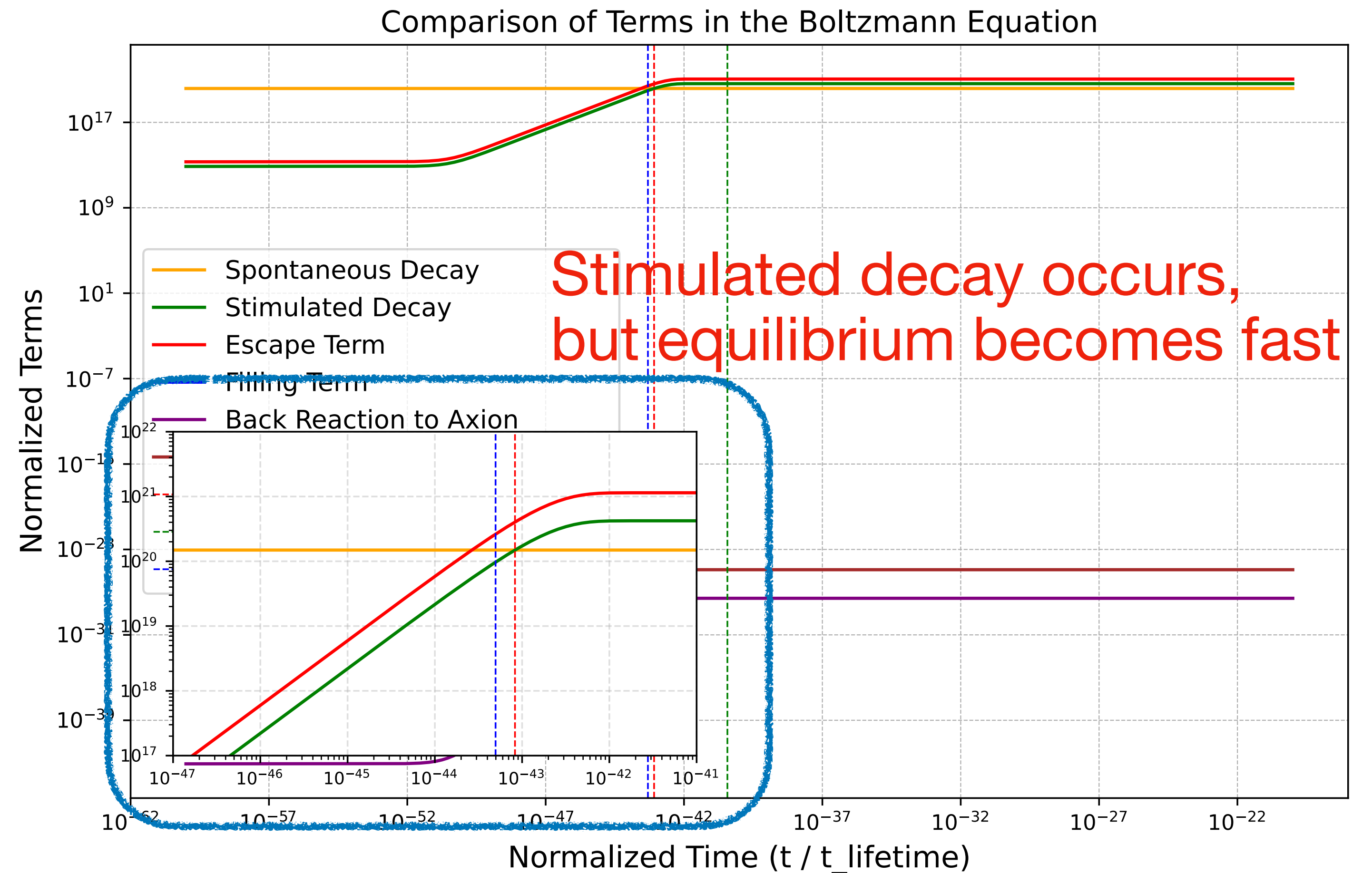


$$m_{\text{PBH}} = 1M_\odot, m_a = 10^{-6}\text{eV}, g_{a\gamma\gamma} = 10^{-10}\text{GeV}^{-1}, r = 10^5 r_{\text{sch}}$$

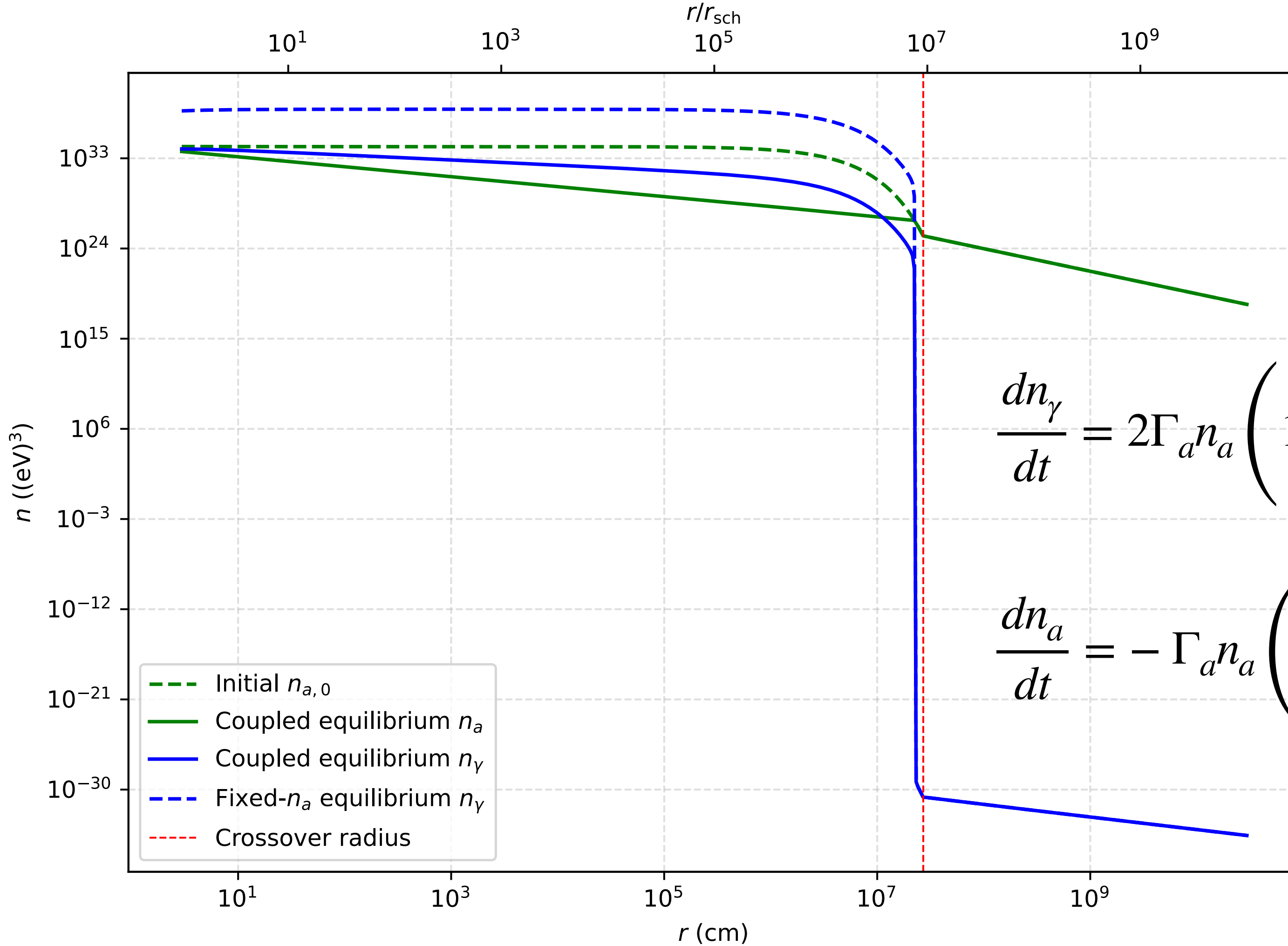
Example Result of Non-Enhancement Case



$$m_{\text{PBH}} = 1M_{\odot}, m_a = 10^{-6}\text{eV}, g_{a\gamma\gamma} = 10^{-10}\text{GeV}^{-1}, r = 10^5 r_{\text{sch}}$$



Example Result of Dynamical Equilibrium

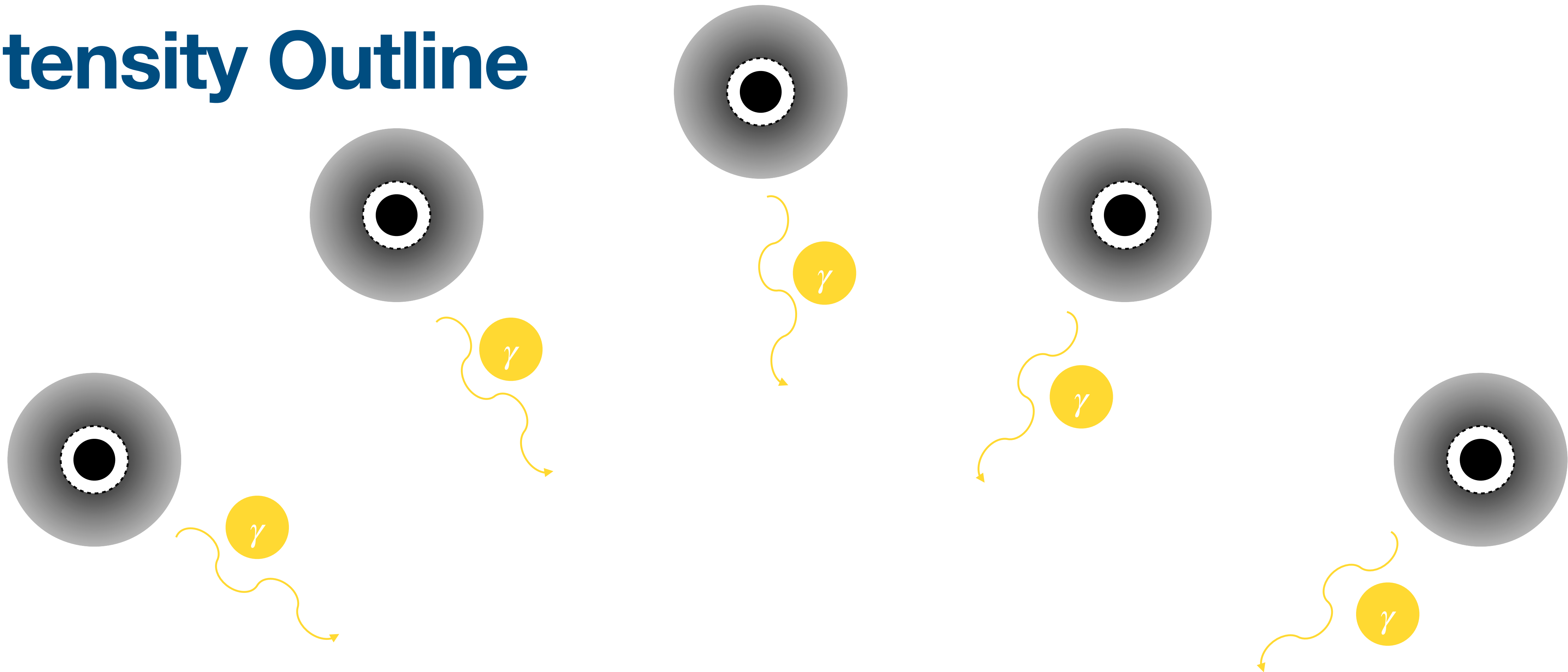


$$\frac{dn_\gamma}{dt} = 2\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\text{max}} m_a^3} n_\gamma \right) - \frac{8\pi^2 \Gamma_a}{m_a^3} n_\gamma^2 \left(1 + \frac{2v_{a,\text{max}}}{3} \right) - \frac{n_\gamma}{t_{\text{esc}}} = 0$$

$$\frac{dn_a}{dt} = -\Gamma_a n_a \left(1 + \frac{16\pi^2}{v_{a,\text{max}} m_a^3} n_\gamma \right) + \frac{8\pi^2 v_{a,\text{max}} \Gamma_a}{3m_a^3} n_\gamma^2 + \frac{n_{a,0} - n_a}{t_{\text{dyn}}} = 0$$

$$m_{\text{PBH}} = 10^{-5} M_\odot, m_a = 10^{-8} \text{eV}, g_{a\gamma\gamma} = 10^{-12} \text{GeV}^{-1}, r = 10^3 r_{\text{sch}}$$

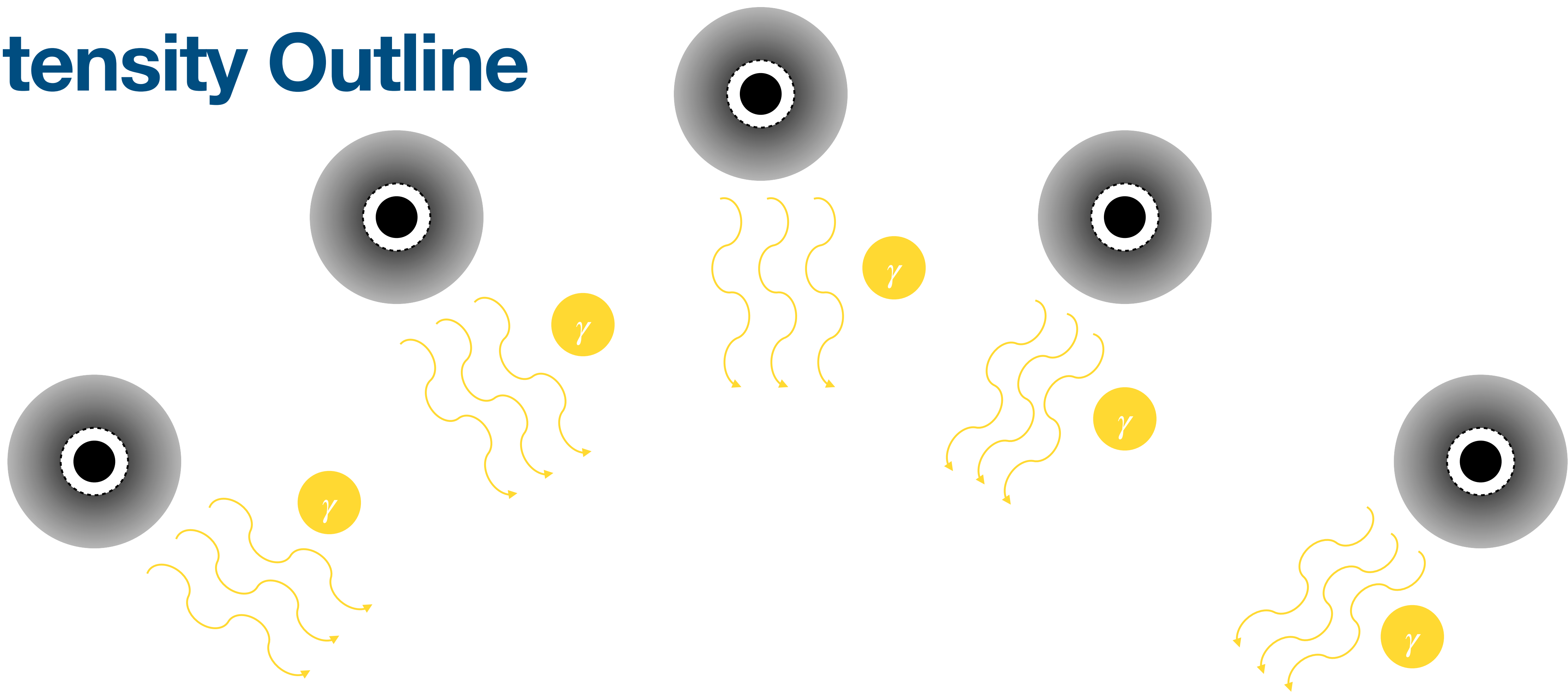
Intensity Outline



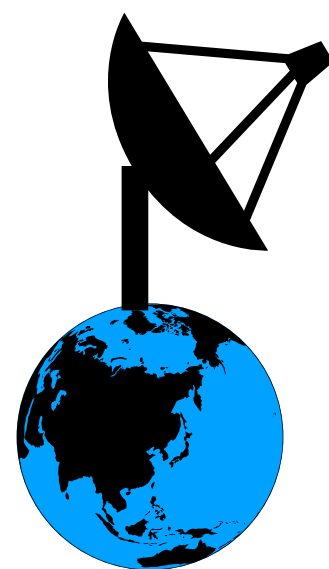
No enhancement,

Observable?

Intensity Outline



Enhancement,



Observable!

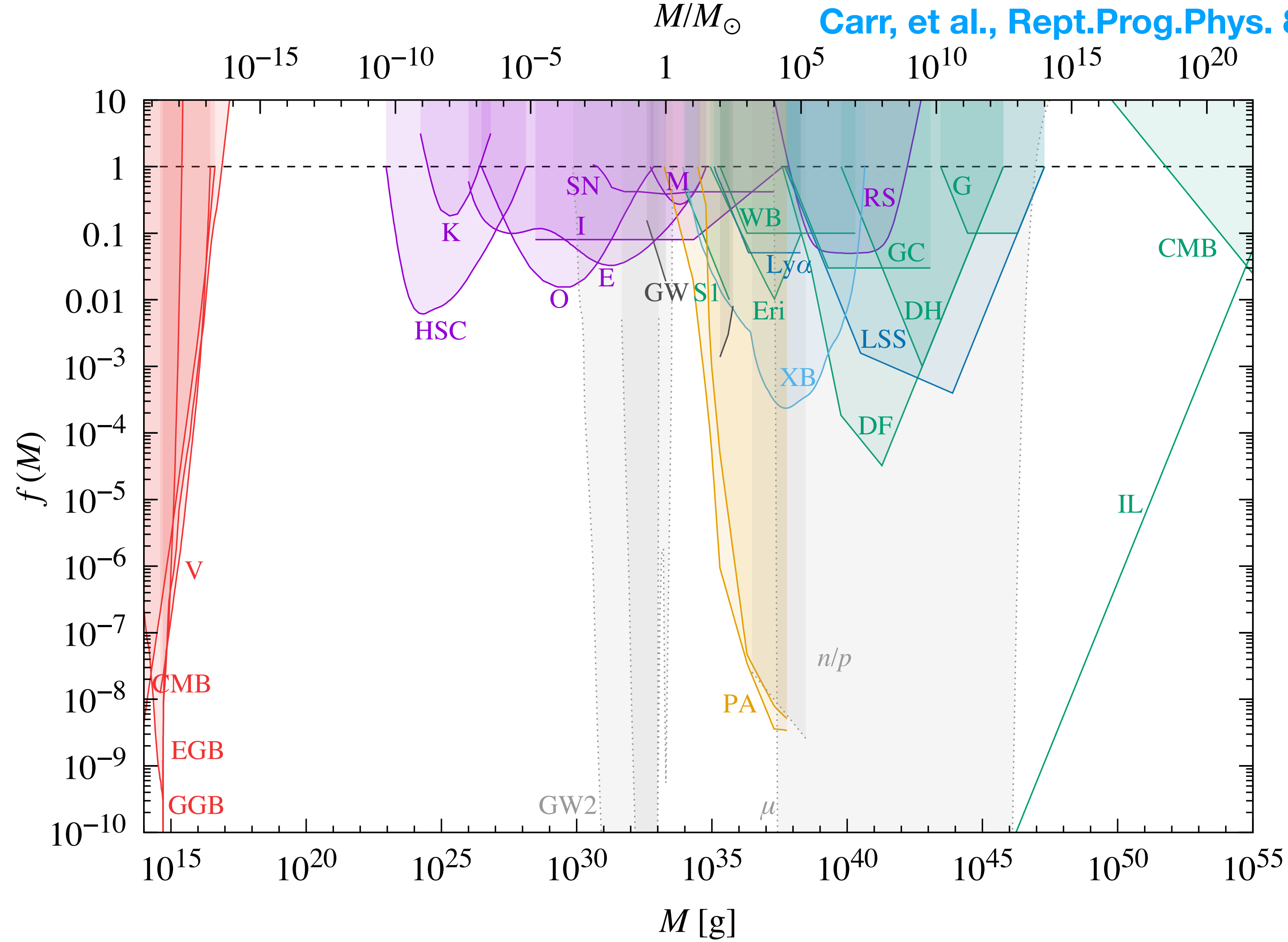
Summary

- **Verified Mechanism:** Using the *Boltzmann Equation*, we confirmed that ALP halos surrounding PBHs lead to significant *photon enhancement*.
- **Future Work:**
 - **PBH and ALP Phase Space:** Identifying the PBH–ALP parameter space where photon production is enhanced.
 - **Observational Signature:** Predicting detectable radio signals from PBH halos and deriving constraints on PBHs and ALPs.

Back-Up

PBH constraints

Carr, et al., Rept.Prog.Phys. 84 (2021) 11, 116902



Boltzmann Equation momentum distribution. ver.

- ALP energy distribution:

$$f_a(E_a, r) = \begin{cases} (2\pi)^3 \mathcal{N}(v_{\text{esc}}) \frac{\rho_a(r)}{m_a} \frac{1}{2\pi m_a^2 \sigma_v^2} e^{-\frac{E_a^2 - m_a^2}{2m_a^2 \sigma_v^2}} & \text{where } E_a \lesssim E_{\text{max}}, \\ 0 & \text{where } E_a > E_{\text{max}} \end{cases}$$

:Truncated Gaussian distribution

King, Astron.J. 71 (1966), 64

Eroshenko, Astron.Lett. 42 (2016) 6, 347-356

- ALP energy range:

$$|\vec{v}_a| \leq v_{\text{esc}} = \sqrt{\frac{2GM_{\text{PBH}}}{r}} \Rightarrow E_a \leq E_{a,\text{max}} = \frac{m_a}{\sqrt{1 - v_{\text{esc}}^2}}$$

- Photon energy range:

$$\omega_{\gamma,\text{min}} \leq \omega_{\gamma} \leq \omega_{\gamma,\text{max}} \quad \text{where } \omega_{\gamma,\text{max}/\gamma,\text{min}} = \frac{E_{a,\text{max}} \pm \sqrt{E_{a,\text{max}}^2 - m_a^2}}{2} = \frac{m_a(1 \pm v_{\text{esc}})}{2\sqrt{1 - v_{\text{esc}}^2}}$$

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω :

$$\frac{df_{\lambda}(\omega)}{dt} = \underbrace{\frac{m_a \Gamma_a}{\omega^2} \int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a) (1 + f_{\lambda}(\omega) + f_{\lambda}(E_a - \omega))}_{\text{Production of photons}} - \underbrace{\frac{m_a \Gamma_a}{\omega^2} \int_{m_a - \omega}^{\infty} d\omega'' f_{\lambda}(\omega) f_{\lambda}(\omega'') - \frac{3}{2r} f_{\lambda}(\omega)}_{\text{Loss of photons}}$$

Production of photons

Loss of photons

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω :

$$\frac{df_{\lambda}(\omega)}{dt} = \frac{m_a \Gamma_a}{\omega^2} \int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a) \underbrace{\left(1 + \underbrace{f_{\lambda}(\omega)}_{\text{Stimulated decay}} + \underbrace{f_{\lambda}(E_a - \omega)}_{\text{Spontaneous decay}} \right)}_{\text{Production of photons}} - \underbrace{\frac{m_a \Gamma_a}{\omega^2} \int_{m_a - \omega}^{\infty} d\omega'' f_{\lambda}(\omega) f_{\lambda}(\omega'')}_{\text{Loss of photons}} - \underbrace{\frac{3}{2r} f_{\lambda}(\omega)}_{\text{Photon escape}}$$

Production of photons
Loss of photons

Stimulated decay
Back-reaction
Photon escape

Spontaneous decay

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω :

$$\frac{df_\lambda(\omega)}{dt} = \frac{m_a \Gamma_a}{\omega^2} \int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a) \left(1 + f_\lambda(\omega) + f_\lambda(E_a - \omega) \right) - \frac{m_a \Gamma_a}{\omega^2} \int_{m_a - \omega}^{\infty} d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} f_\lambda(\omega)$$

Production of photons

Stimulated decay

Spontaneous decay

Loss of photons

Back-reaction

Photon escape

Range of ALP energy to produce photon with momentum ω

Range of photon momentum to produce ALP by annihilating with a photon ω

Boltzmann Equation momentum distribution. ver.

- Boltzmann equation for a photon with momentum ω :

$$\frac{df_\lambda(\omega)}{dt} = \frac{m_a \Gamma_a}{\omega^2} \int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a) \underbrace{\left(1 + f_\lambda(\omega) + f_\lambda(E_a - \omega) \right)}_{\text{Production of photons}} - \frac{m_a \Gamma_a}{\omega^2} \int_{m_a - \omega}^{\infty} d\omega'' \underbrace{f_\lambda(\omega) f_\lambda(\omega'')}_{\text{Loss of photons}} - \frac{3}{2r} f_\lambda(\omega)$$

Stimulated decay
Spontaneous decay
Back-reaction
Photon escape

Range of ALP energy to produce photon with momentum ω

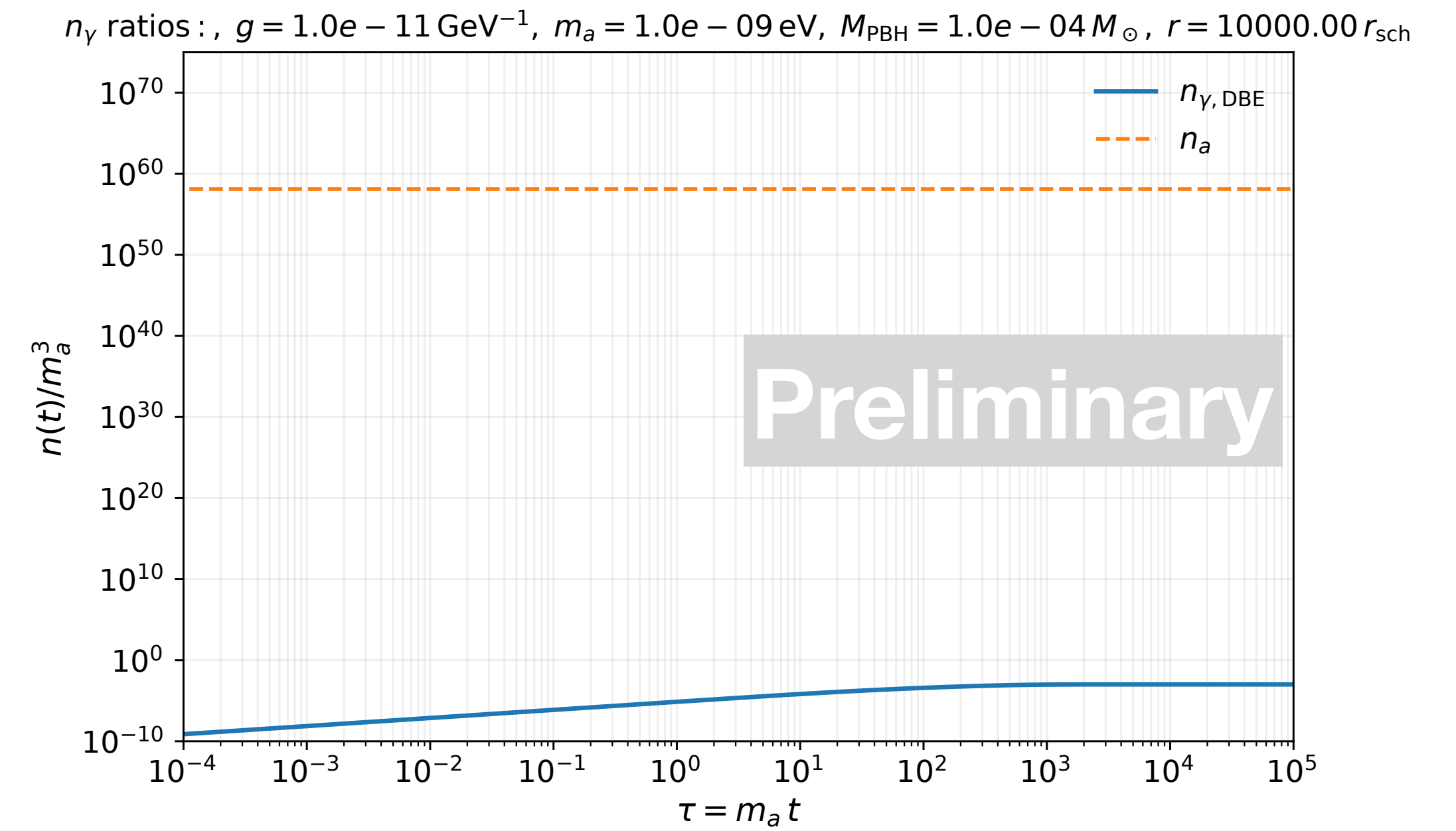
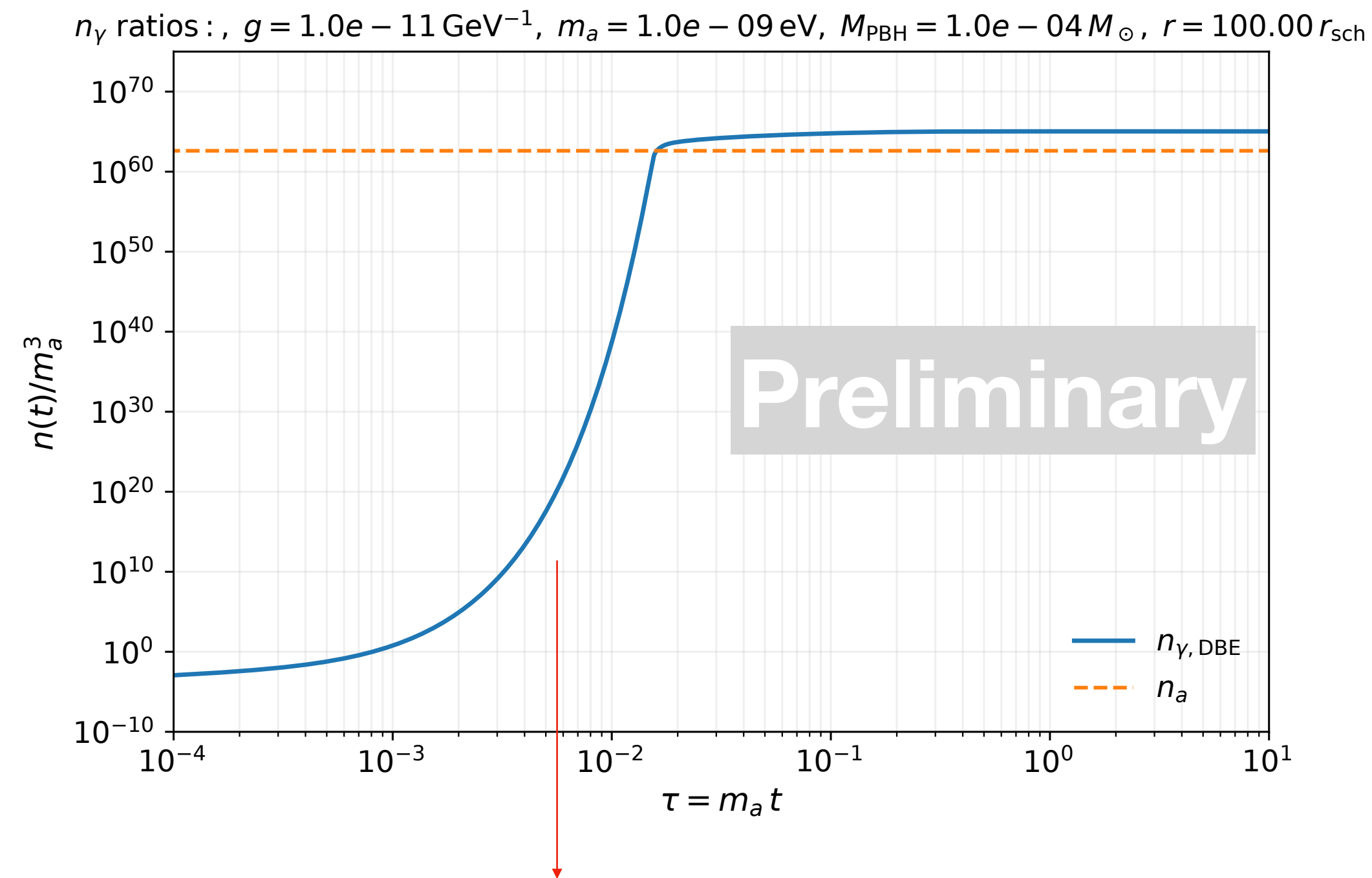
Range of photon momentum to produce ALP by annihilating with a photon ω

- Number density: $n_\gamma(r) = \sum_{\lambda=\pm} \frac{1}{(2\pi)^3} \int d^3\vec{\omega} f_\lambda(r, \vec{\omega})$, $n_a(r) = \frac{\rho_a(r)}{m_a}$

Assumption: Constant on time

Results of Enhancement/Non-Enhancement (md ver.)

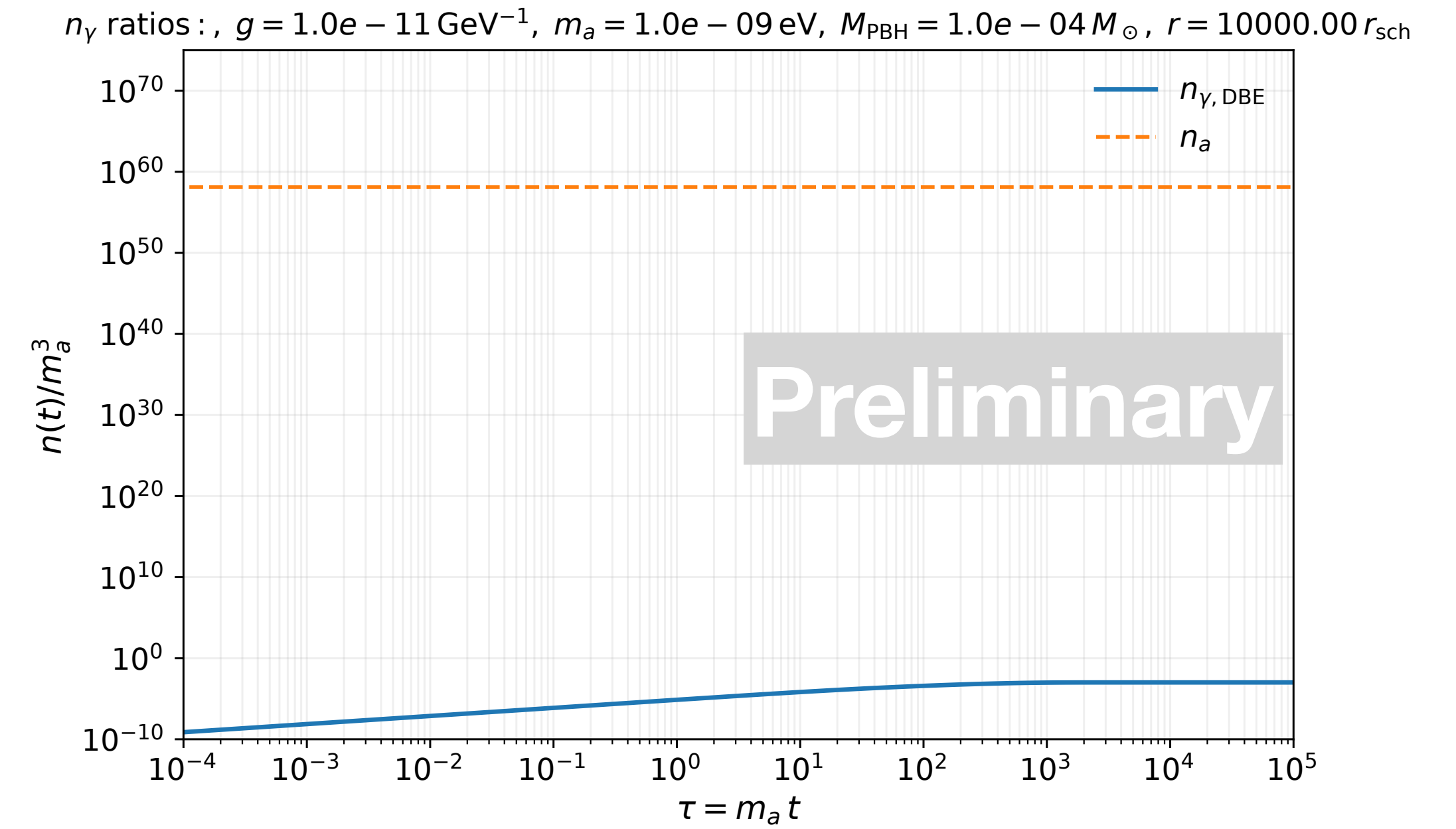
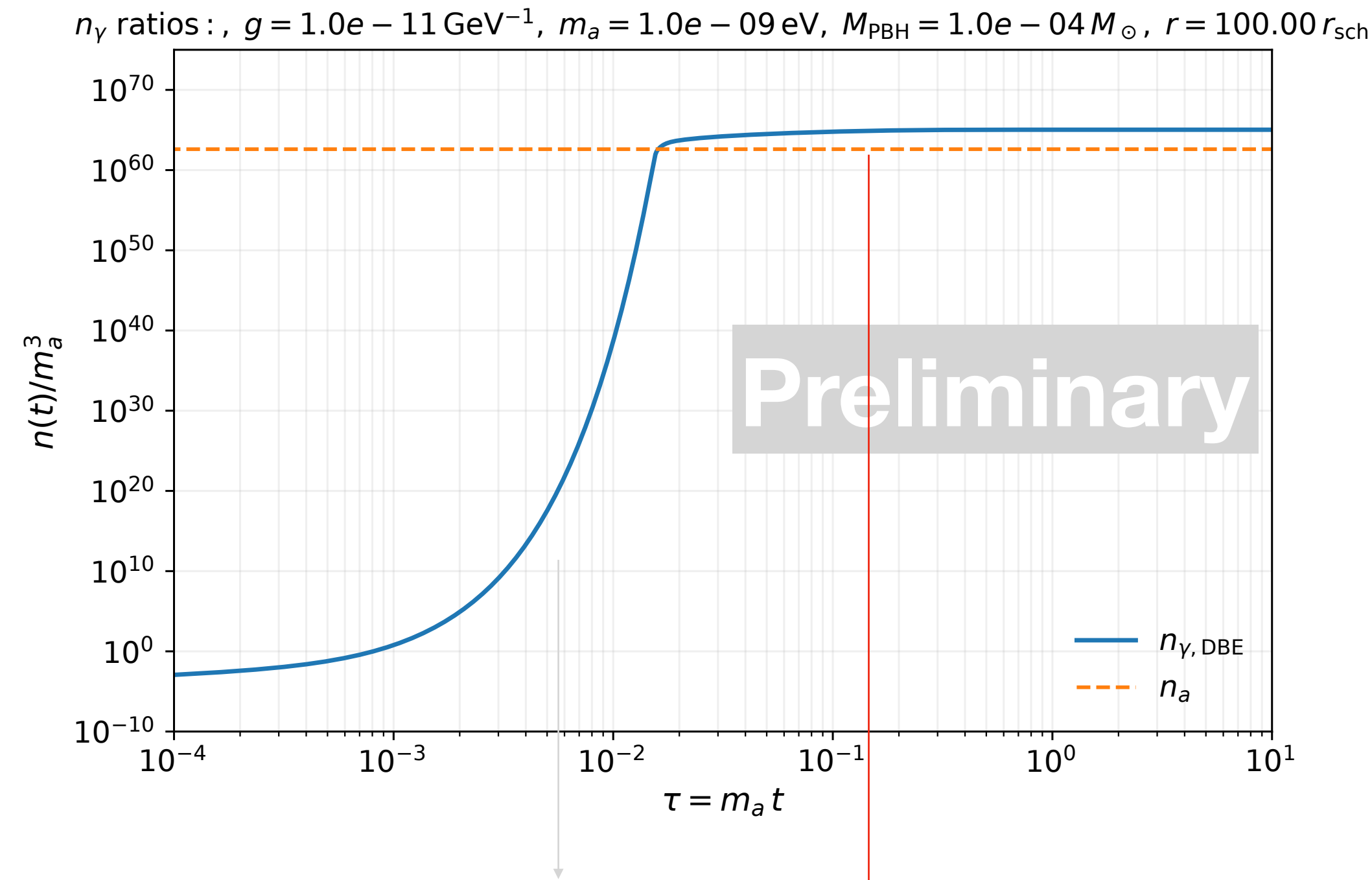
$$\frac{dn_\gamma}{dt} = \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{E_{\min}}^\infty dE_a f_a(E_a) \left(1 + f_\lambda(\omega) + f_\lambda(E_a - \omega) \right) - \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{m_a - \omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} n_\gamma,$$



Stimulated decay is dominant → Enhancement occurs

Results of Enhancement/Non-Enhancement (md ver.)

$$\frac{dn_\gamma}{dt} = \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{E_{\min}}^\infty dE_a f_a(E_a) \left(1 + f_\lambda(\omega) + f_\lambda(E_a - \omega) \right) - \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{m_a - \omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} n_\gamma,$$

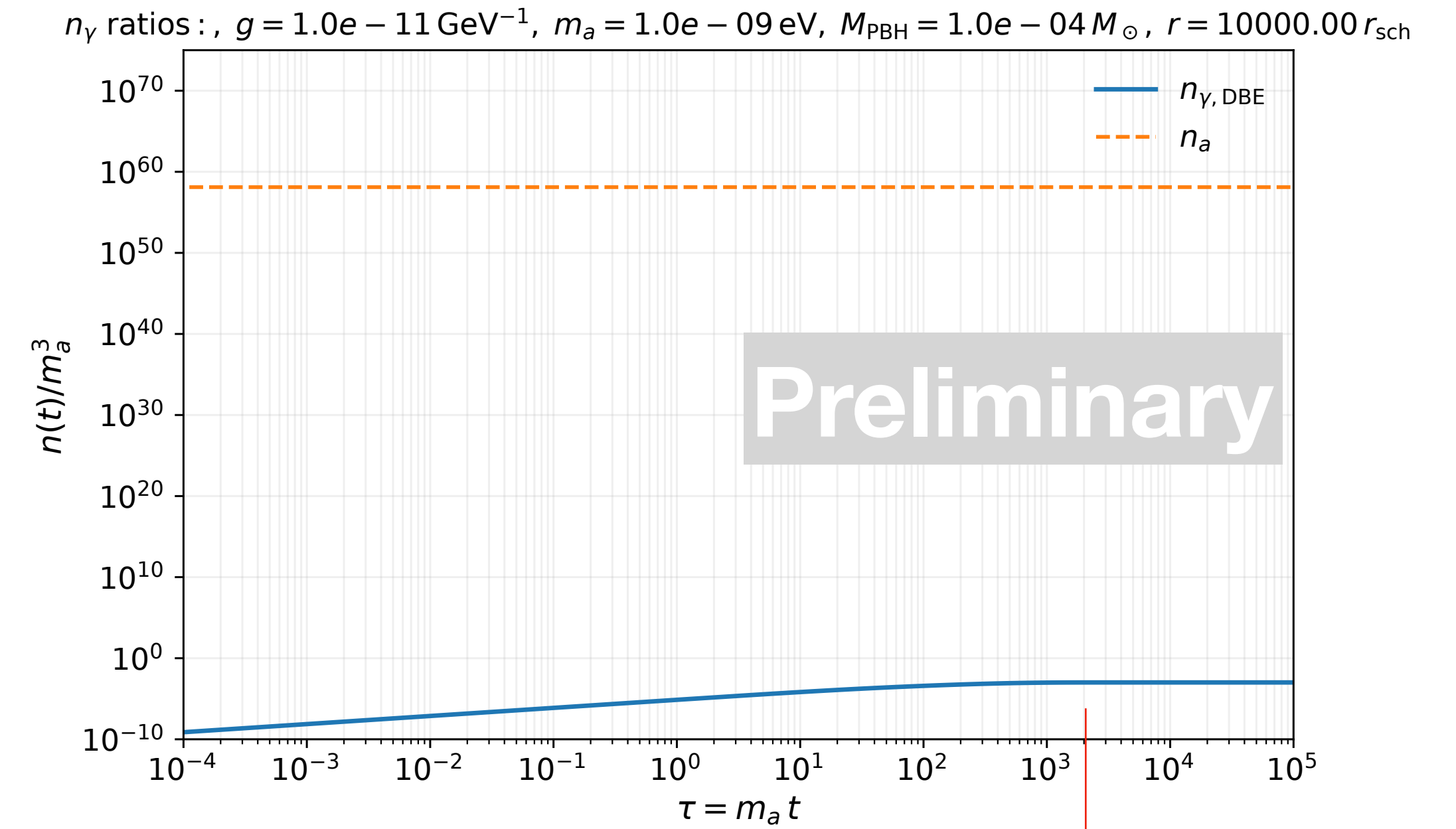
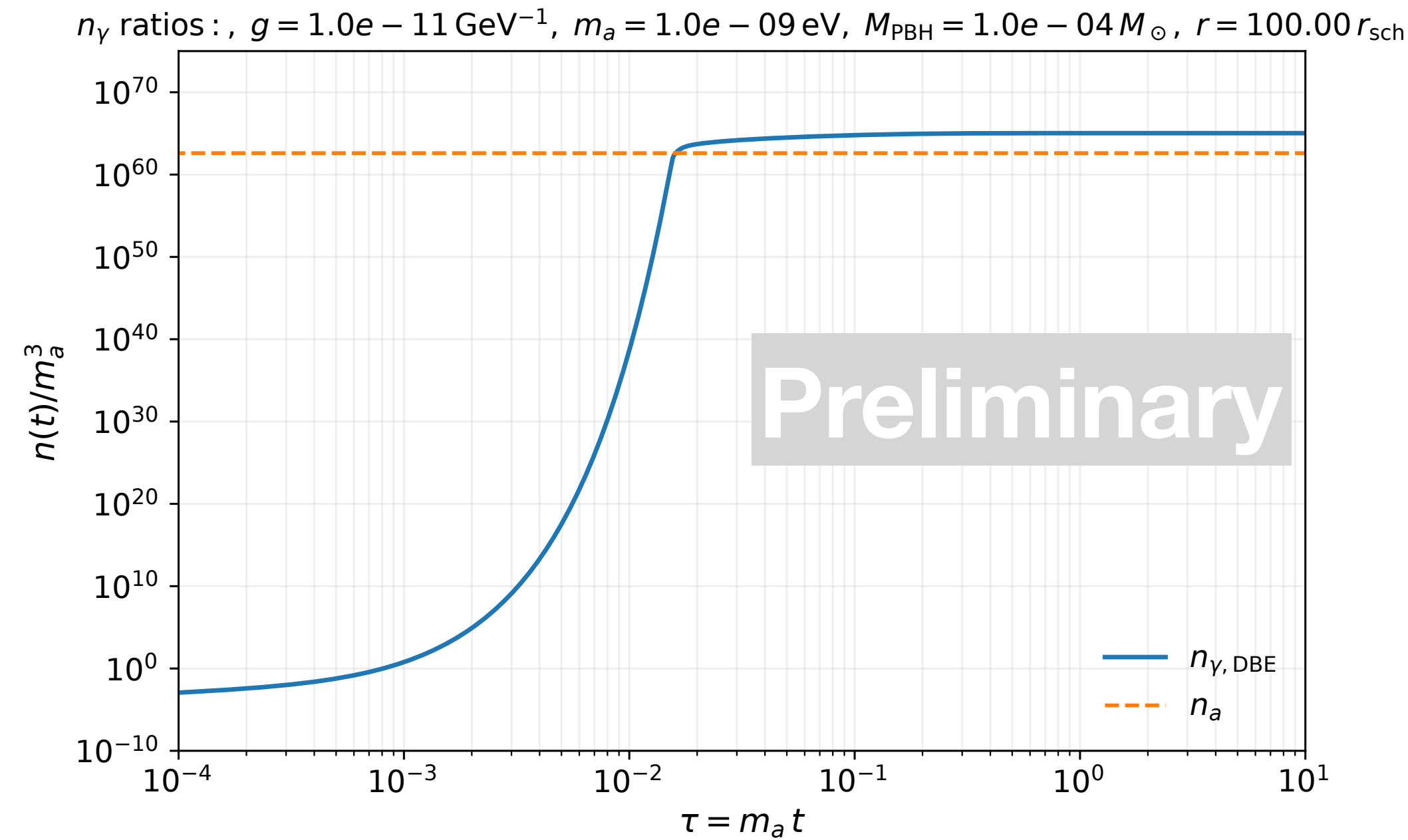


Stimulated decay is dominant → Enhancement occurs

Stimulated decay = **Back-reaction** → Number density reaches a constant

Results of Enhancement/Non-Enhancement (md ver.)

$$\frac{dn_\gamma}{dt} = \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{E_{\min}}^\infty dE_a f_a(E_a) (1 + f_\lambda(\omega) + f_\lambda(E_a - \omega)) - \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{m_a - \omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} n_\gamma,$$

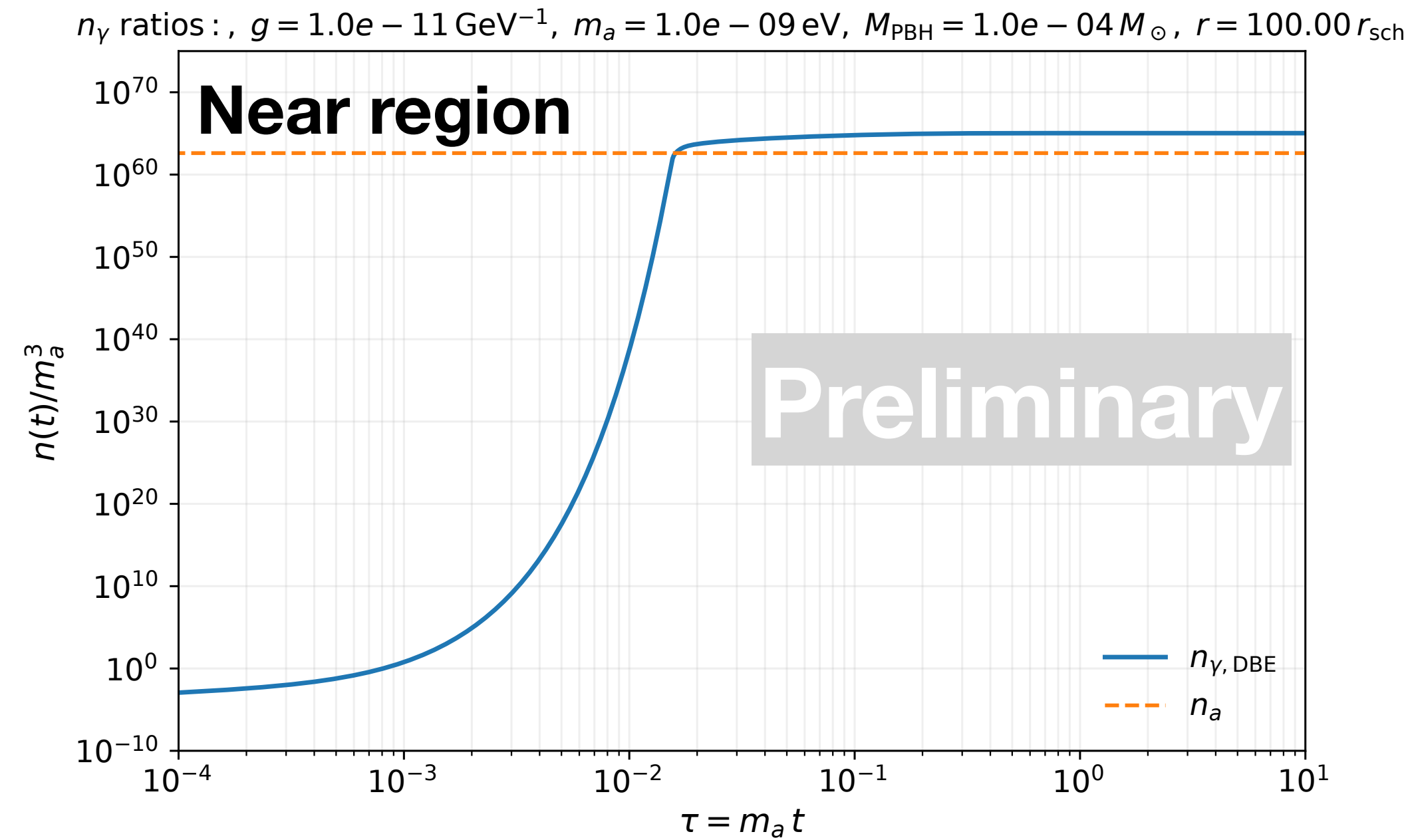


Escape is dominant → No enhancement occurs

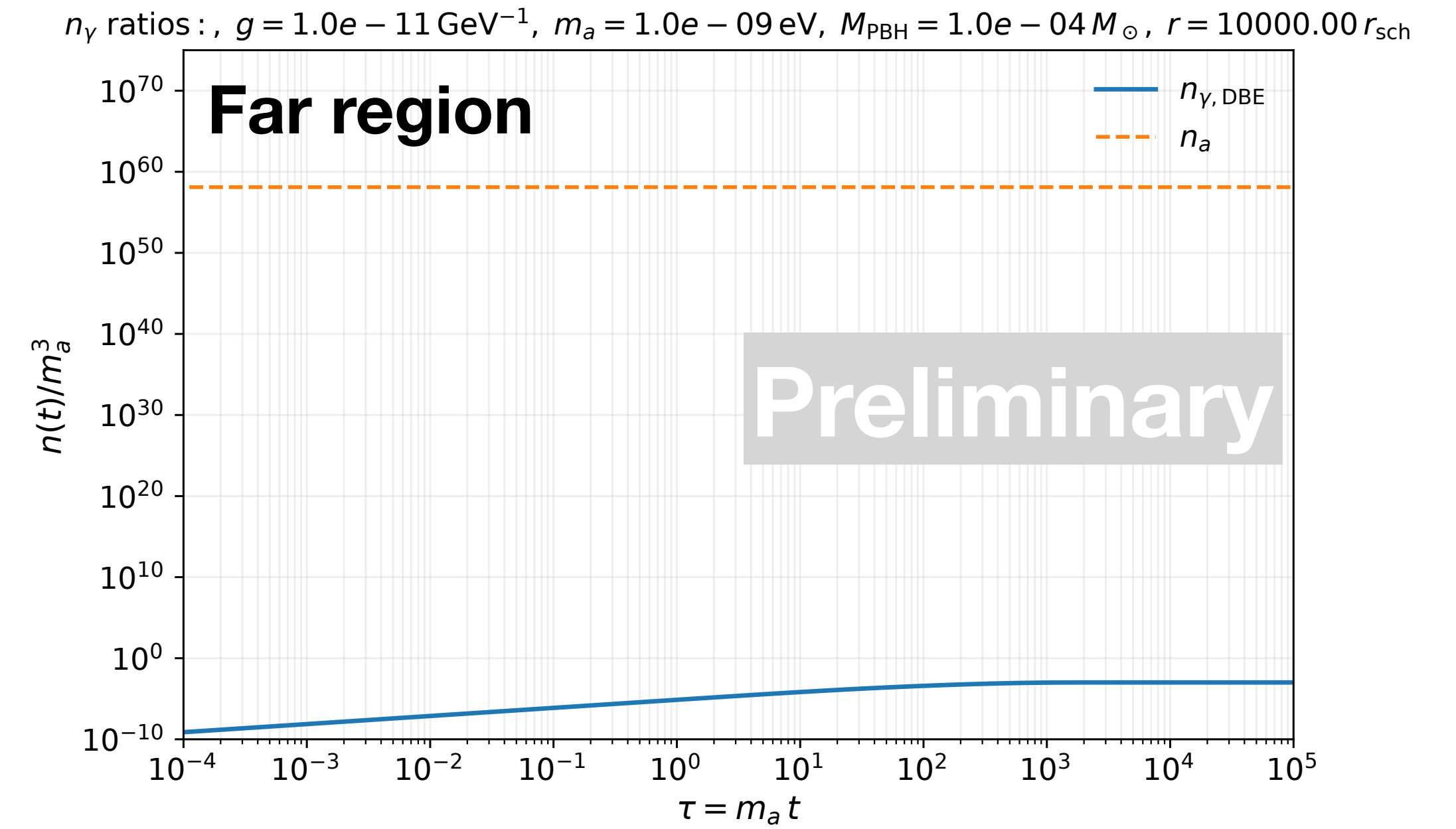
Decay = Escape → Number density reaches a constant

Results of Enhancement/Non-Enhancement (md ver.)

$$\frac{dn_\gamma}{dt} = \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{E_{\min}}^\infty dE_a f_a(E_a) \left(1 + f_\lambda(\omega) + f_\lambda(E_a - \omega) \right) - \frac{m_a \Gamma_a}{\pi^2} \int_0^\infty d\omega \int_{m_a - \omega}^\infty d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} n_\gamma,$$



Stimulated decay > Photon escape

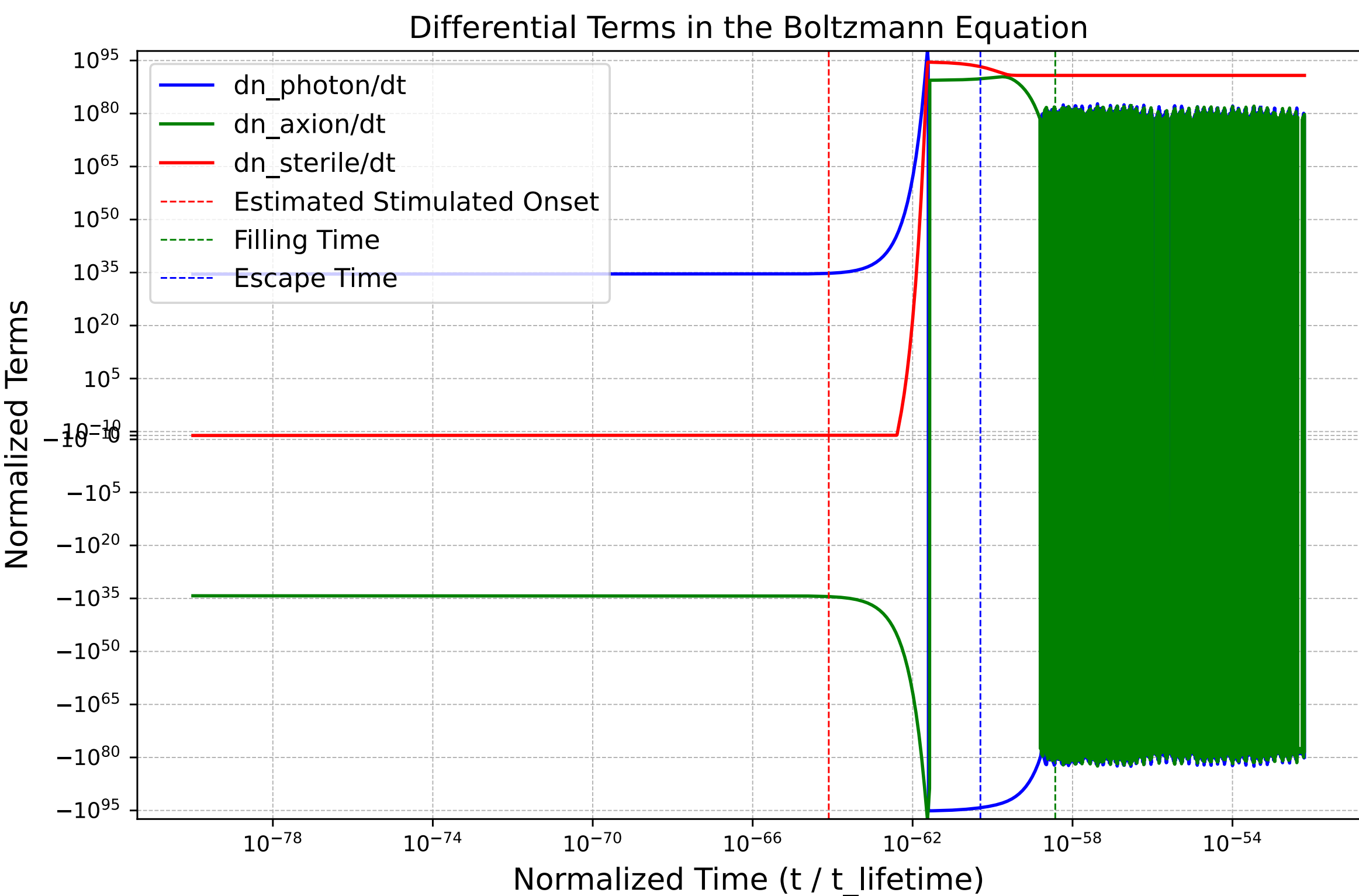
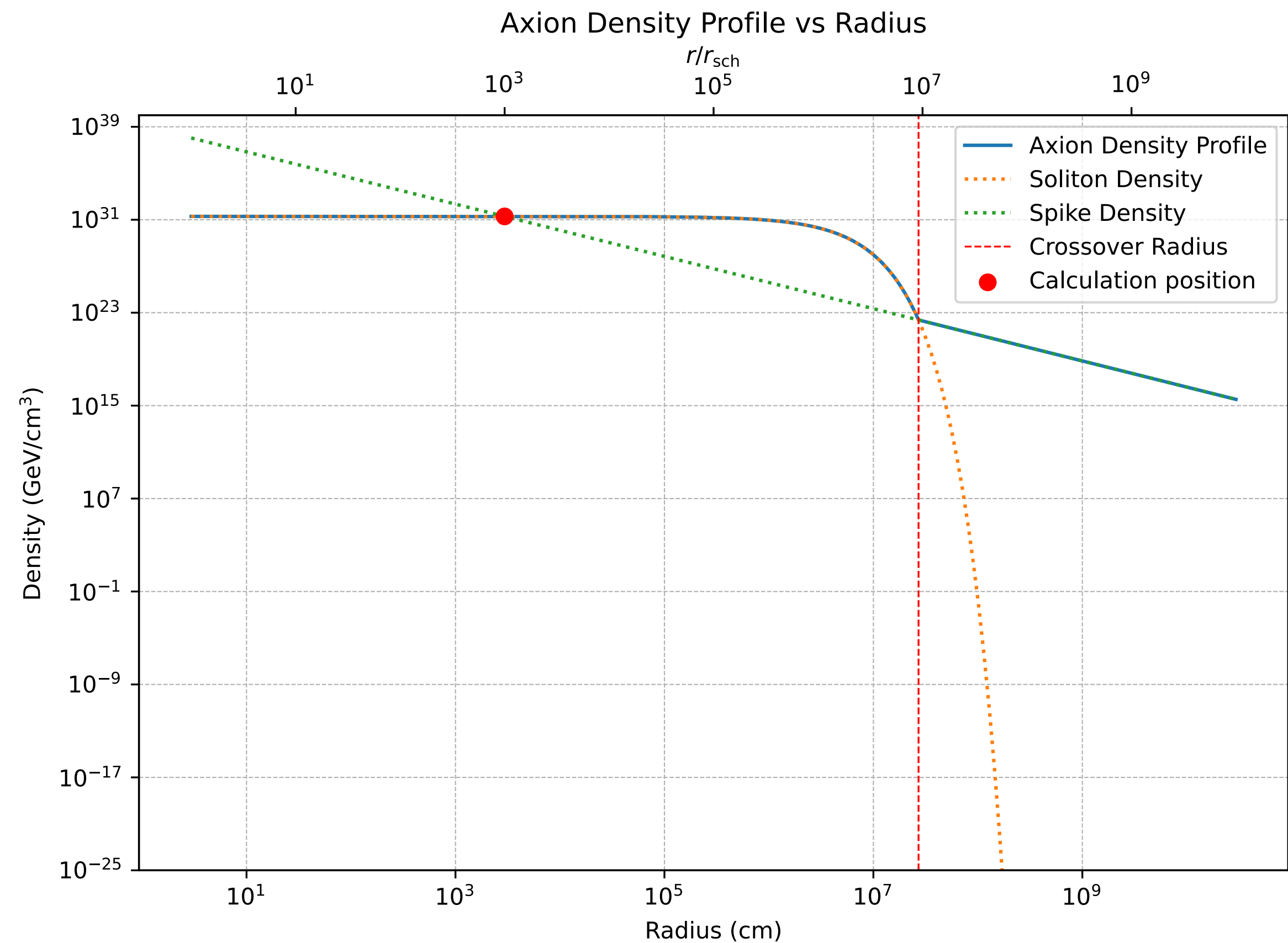


Stimulated decay < Photon escape

r_{enh}

Example Result of Enhancement Case

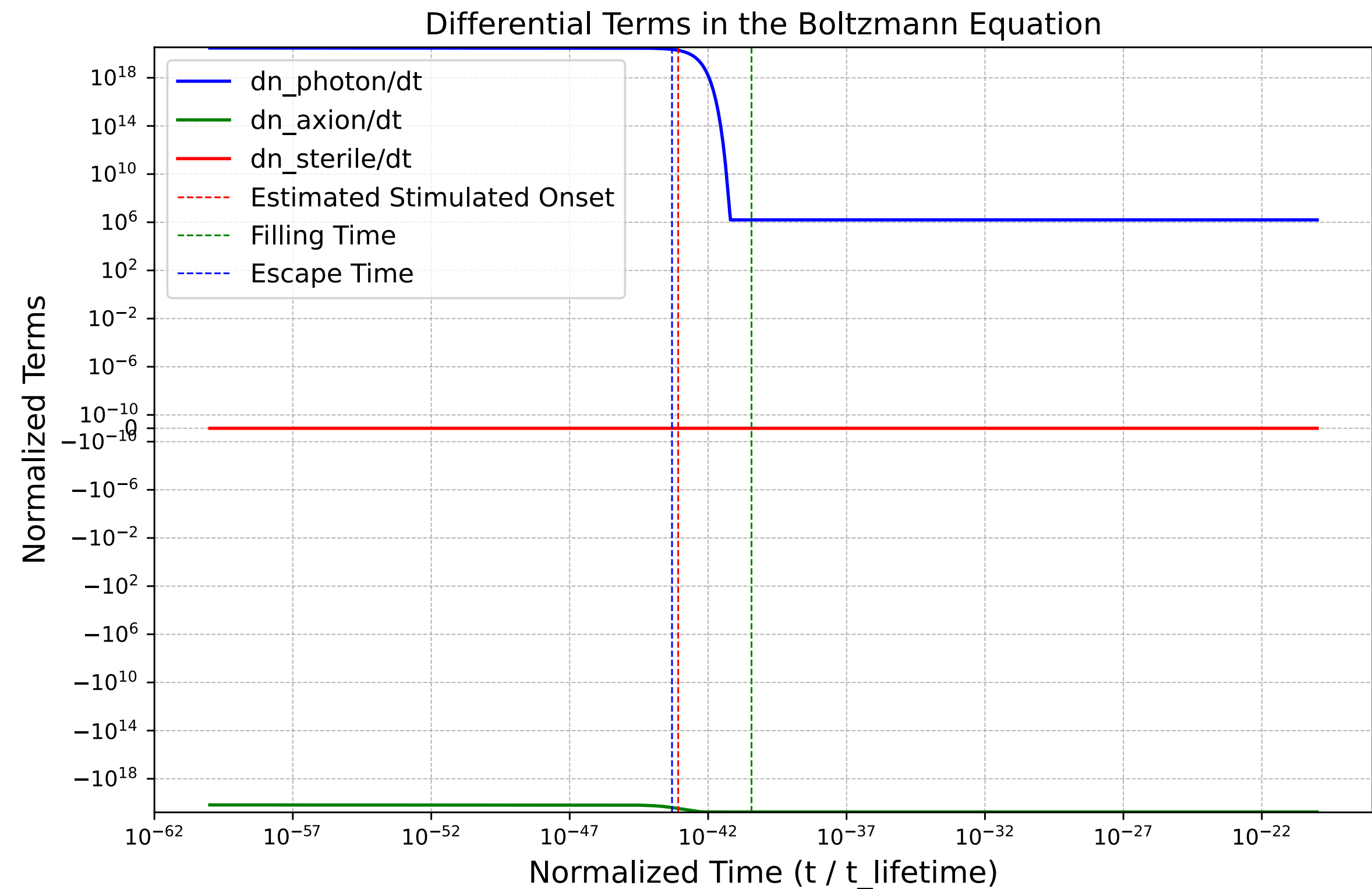
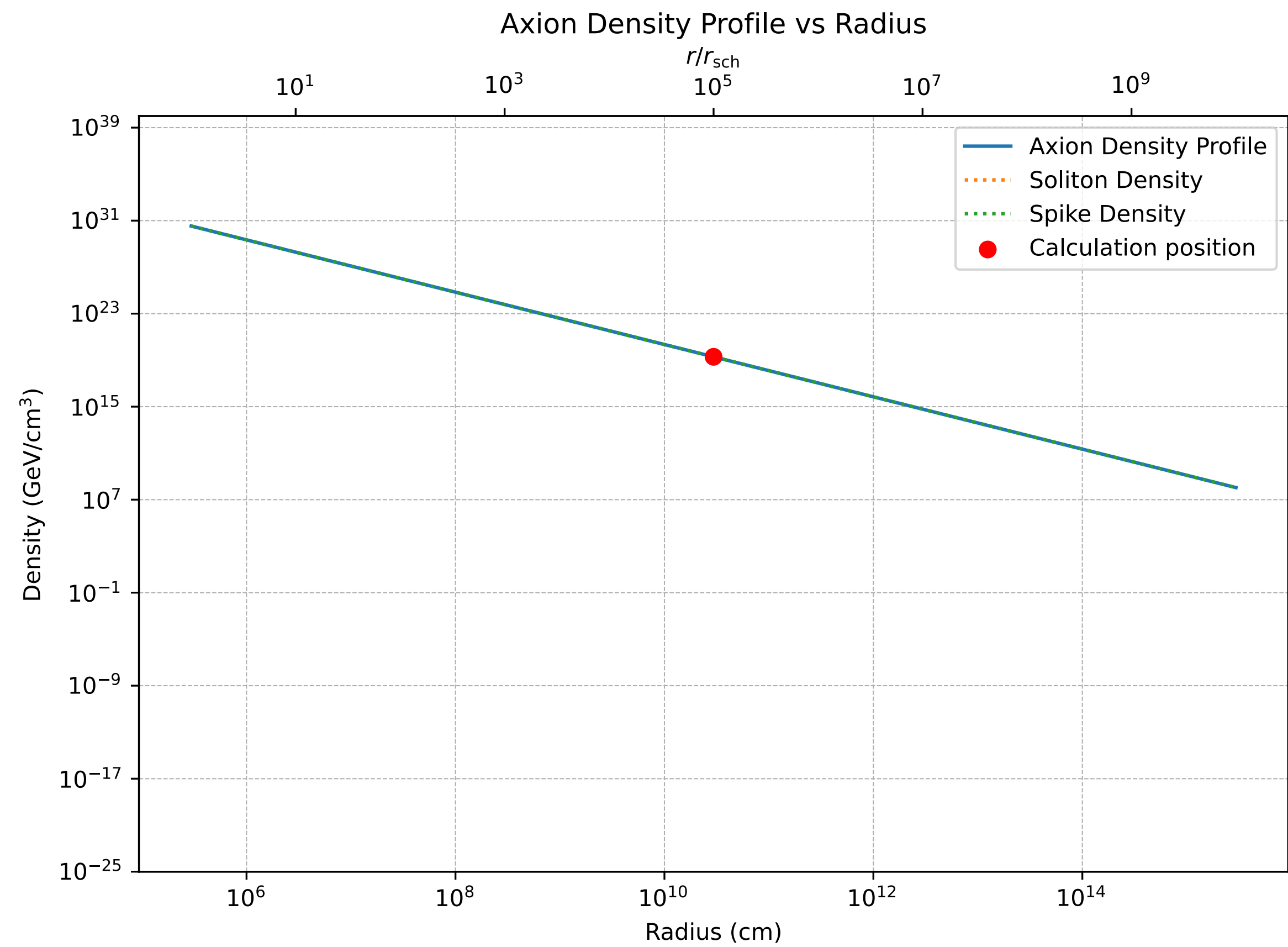
$$m_{\text{PBH}} = 10^{-5} M_{\odot}, m_a = 10^{-8} \text{eV} \quad g_{a\gamma\gamma} = 10^{-12} \text{GeV}^{-1}$$



Axion energy density and calculation point

Differential of number densities

Example Result of Non-Enhancement Case



Axion energy density and calculation point

Differential of number densities

Intensity

Carr, et al., Mon.Not.Roy.Astron.Soc. 506 (2021) 3, 3648-3661
 Carr, et al., Phys.Rev.D 81 (2010), 104019
 Ferraz, et al., JCAP 07 (2022) 07, 026

- Photon number density:

$$\frac{dn_\gamma}{dt}(E_\gamma, t) = n_{\text{PBH}}(t) \frac{dN_\gamma}{dt}(E, t) \simeq n_{\text{PBH}}(t) E_\gamma \frac{d^2 N_\gamma}{dt dE_\gamma}(E, t)$$

Logarithmic binning approximation $\Delta E \simeq E$

- Intensity:

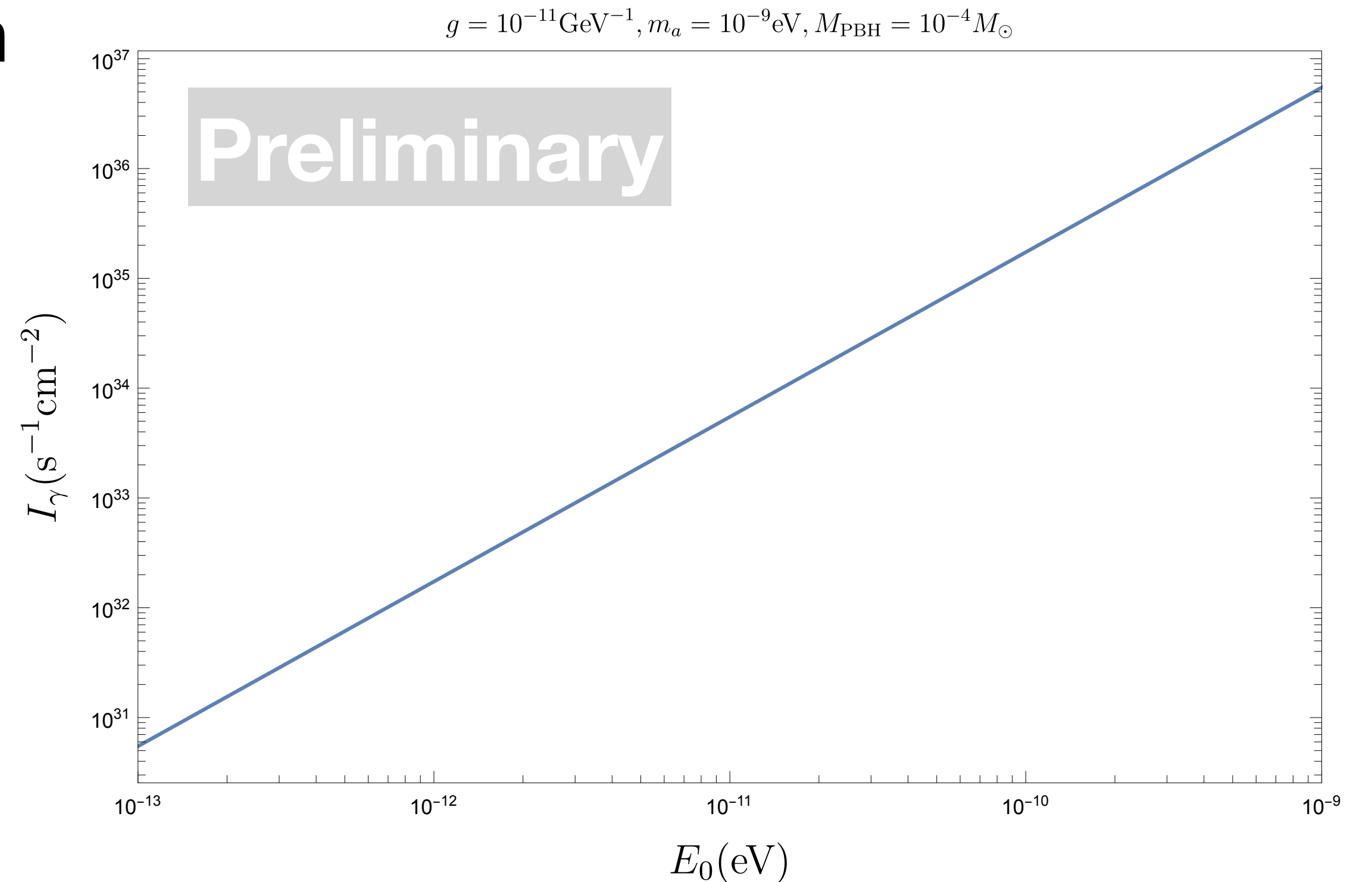
$$I \simeq \frac{dI}{dE} E = \frac{1}{4\pi} \rho_{c,0} \Omega_{\text{PBH},0} E_0 \int_{t_{\text{rec}}}^{t_{\text{cloud}}} dt \frac{(1+z)}{M(t)} \frac{d^2 N_\gamma}{dE dt}(E_0(1+z))$$

$$\simeq \frac{3}{4\pi^2} \rho_{c,0} \Omega_{\text{PBH},0} \frac{E_0^3}{M_{\text{PBH}}} \int_{t_i}^{t_f} dt (1+z)^3 \int_{100 r_{\text{sch}}}^{r_0(t)} drr f_\lambda(E_0(1+z), r) dr$$

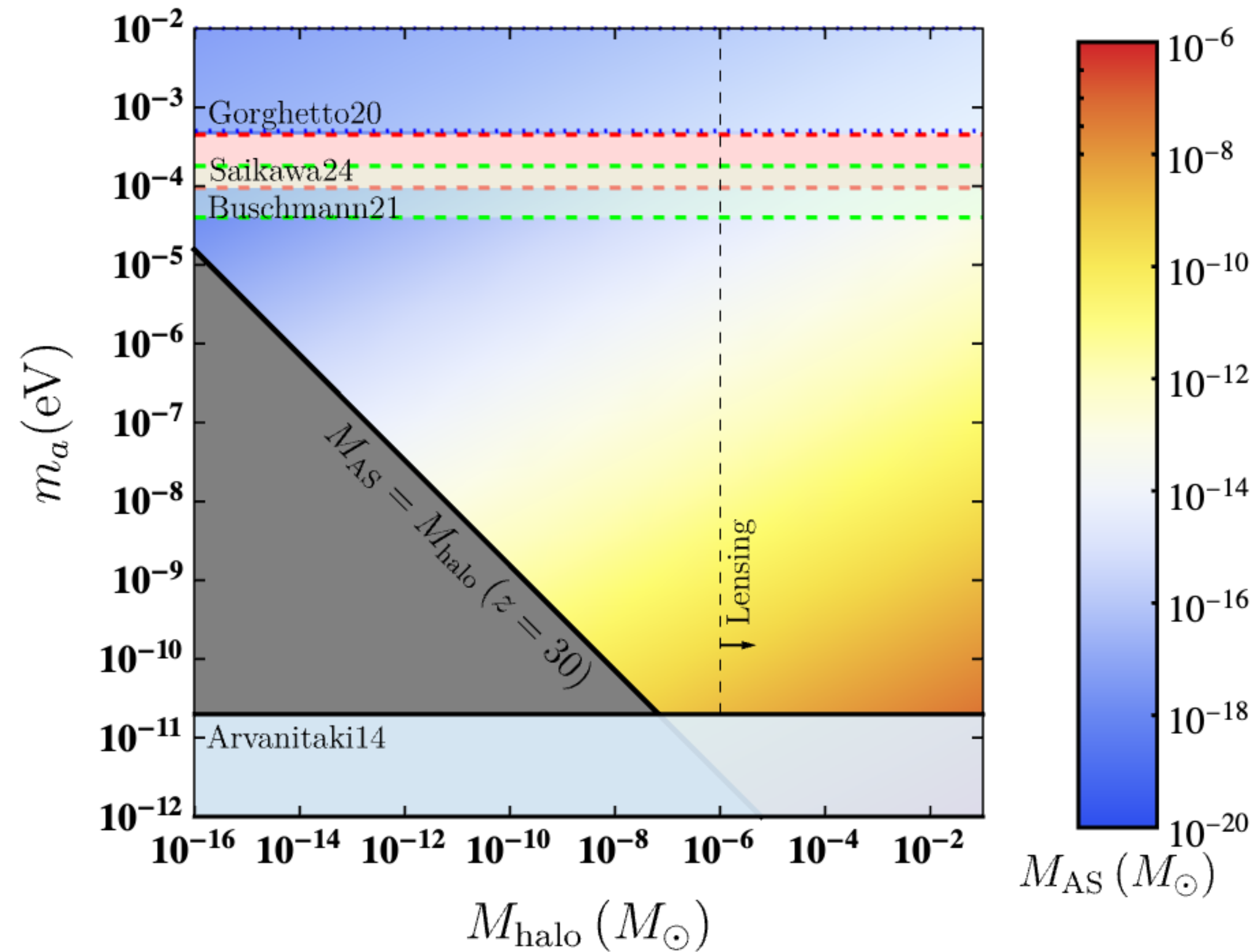
Intensity

$$\frac{df_\lambda(\omega)}{dt} = \frac{m_a \Gamma_a}{\omega^2} \int_{E_{\min}(\omega)}^{\infty} dE_a f_a(E_a) (1 + f_\lambda(\omega) + f_\lambda(E_a - \omega)) - \frac{m_a \Gamma_a}{\omega^2} \int_{m_a - \omega}^{\infty} d\omega'' f_\lambda(\omega) f_\lambda(\omega'') - \frac{3}{2r} f_\lambda(\omega)$$

$$I \simeq \frac{3}{4\pi^2} \rho_{c,0} \Omega_{\text{PBH},0} \frac{E_0^3}{M_{\text{PBH}}} \int_{t_i}^{t_f} dt (1+z)^3 \int_{100r_{\text{sch}}}^{r_0(t)} dr r f_\lambda(E_0(1+z), r) dr$$



Axion Star in ALP Halo around PBH



Yin, et al., JCAP 10 (2024), 013